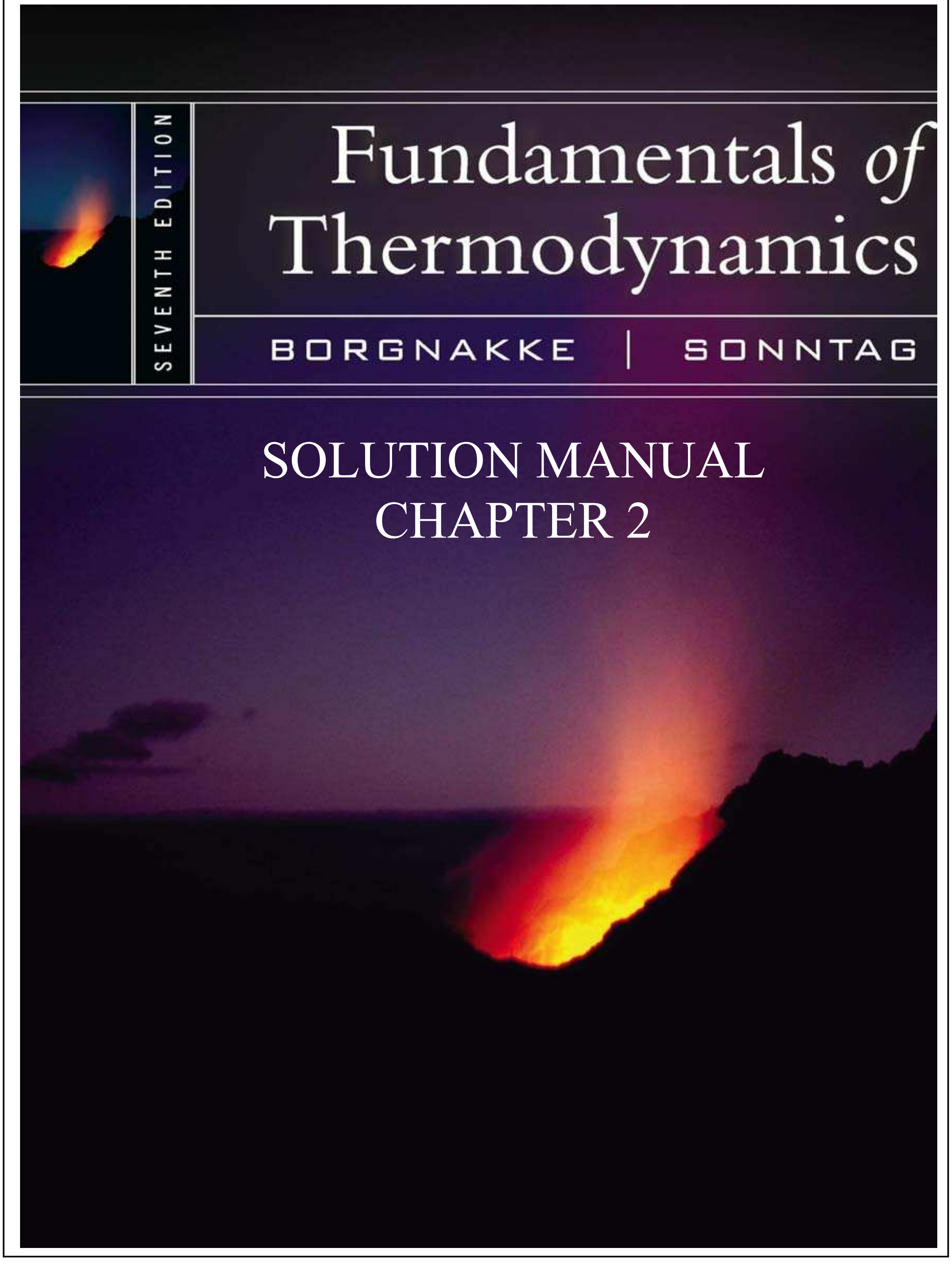


Fundamentals *of* Thermodynamics

SEVENTH EDITION

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 2

CONTENT

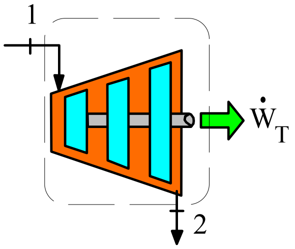
In-Text Concept Questions

2.a

Make a control volume around the turbine in the steam power plant in Fig. 1.1 and list the flows of mass and energy that are there.

Solution:

We see hot high pressure steam flowing in at state 1 from the steam drum through a flow control (not shown). The steam leaves at a lower pressure to the condenser (heat exchanger) at state 2. A rotating shaft gives a rate of energy (power) to the electric generator set.

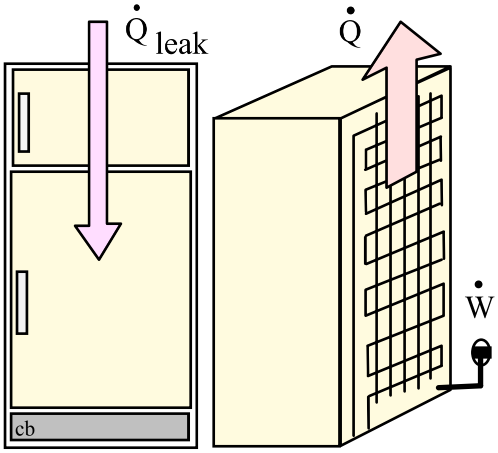


2.b

Take a control volume around your kitchen refrigerator and indicate where the components shown in Figure 1.6 are located and show all flows of energy transfer.

Solution:

The valve and the cold line, the evaporator, is inside close to the inside wall and usually a small blower distributes cold air from the freezer box to the refrigerator room.



The black grille in the back or at the bottom is the condenser that gives heat to the room air.

The compressor sits at the bottom.

2.c

Why do people float high in the water when swimming in the Dead Sea as compared with swimming in a fresh water lake?

As the dead sea is very salty its density is higher than fresh water density. The buoyancy effect gives a force up that equals the weight of the displaced water. Since density is higher the displaced volume is smaller for the same force.

2.d

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases, what happens to the density and specific volume?

Solution:

The density is seen to decrease as the temperature increases.

$$\Delta\rho = -\Delta T/2$$

Since the specific volume is the inverse of the density $v = 1/\rho$ it will increase.

2.e

A car tire gauge indicates 195 kPa; what is the air pressure inside?

The pressure you read on the gauge is a gauge pressure, ΔP, so the absolute pressure is found as

`

$$P = P_o + \Delta P = 101 + 195 = 296 \text{ kPa}$$

2.f

Can I always neglect ΔP in the fluid above location A in figure 2.12? What does that depend on?

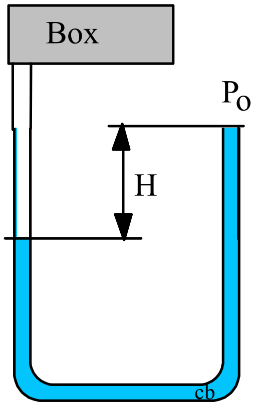
If the fluid density above A is low relative to the manometer fluid then you neglect the pressure variation above position A, say the fluid is a gas like air and the manometer fluid is like liquid water. However, if the fluid above A has a density of the same order of magnitude as the manometer fluid then the pressure variation with elevation is as large as in the manometer fluid and it must be accounted for.

2.g

A U tube manometer has the left branch connected to a box with a pressure of 110 kPa and the right branch open. Which side has a higher column of fluid?

Solution:

Since the left branch fluid surface feels 110 kPa and the right branch surface is at 100 kPa you must go further down to match the 110 kPa. The right branch has a higher column of fluid.

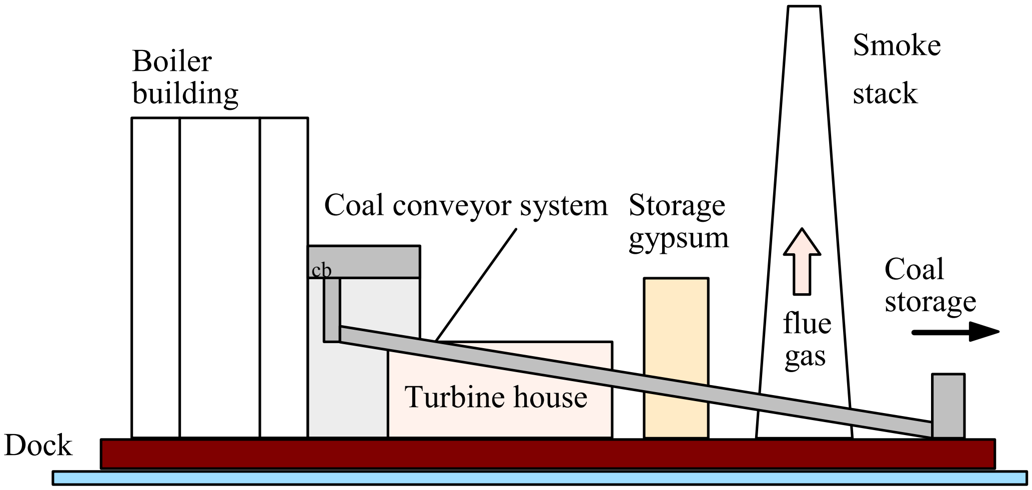


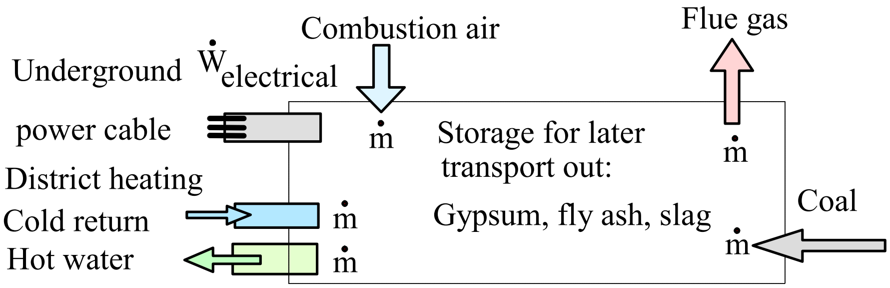
Concept Problems

2.1

Make a control volume around the whole power plant in Fig. 1.2 and with the help of Fig. 1.1 list what flows of mass and energy are in or out and any storage of energy. Make sure you know what is inside and what is outside your chosen C.V.

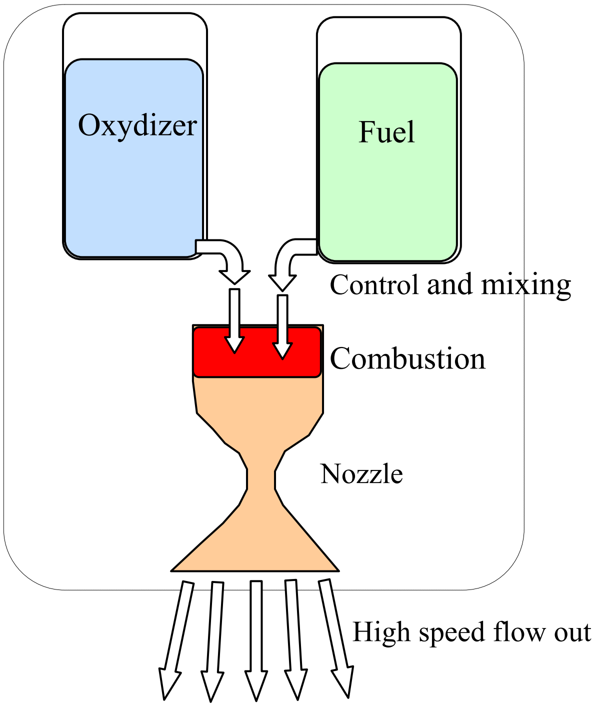
Solution:





2.2

Take a control volume around the rocket engine in Fig. 1.12. Identify the mass flows and where you have significant kinetic energy and where storage changes.



We have storage in both tanks are reduced, mass flows out with modest velocities.

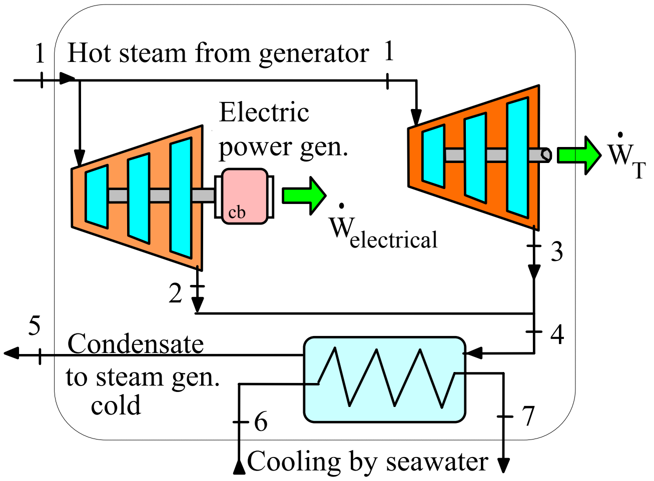
Energy conversion in the combustion process.

gas at high pressure expands towards lower pressure outside and thus accelerates to high velocity with significant kinetic energy flowing out.

2.3

Make a control volume that includes the steam flow around in the main turbine loop in the nuclear propulsion system in Fig.1.3. Identify mass flows (hot or cold) and energy transfers that enter or leave the C.V.

Solution:



The electrical power also leaves the C.V. to be used for lights, instruments and to charge the batteries.

2.4

Separate the list P, F, V, v, ρ, T, a, m, L, t, and **V** into intensive, extensive, and non-properties.

Solution:

Intensive properties are independent upon mass: P, v, ρ, T
Extensive properties scales with mass: V, m
Non-properties: F, a, L, t, **V**

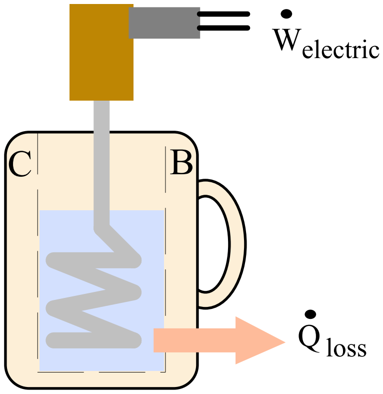
Comment: You could claim that acceleration a and velocity **V** are physical properties for the dynamic motion of the mass, but not thermal properties.

2.5

An electric dip heater is put into a cup of water and heats it from 20°C to 80°C. Show the energy flow(s) and storage and explain what changes.

Solution:

Electric power is converted in the heater element (an electric resistor) so it becomes hot and gives energy by heat transfer to the water. The water heats up and thus stores energy and as it is warmer than the cup material it heats the cup which also stores some energy. The cup being warmer than the air gives a smaller amount of energy (a rate) to the air as a heat loss.



2.6

Water in nature exists in different phases such as solid, liquid and vapor (gas). Indicate the relative magnitude of density and specific volume for the three phases.

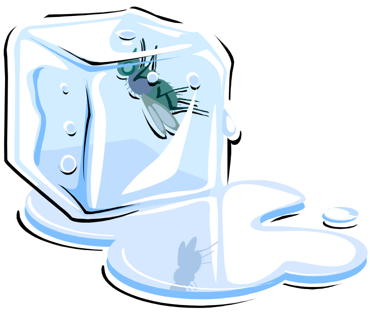
Solution:

Values are indicated in Figure 2.7 as density for common substances. More accurate values are found in Tables A.3, A.4 and A.5

Water as solid (ice) has density of around 900 kg/m^3

Water as liquid has density of around 1000 kg/m^3

Water as vapor has density of around 1 kg/m^3 (sensitive to P and T)



Ice cube

Liquid drop

Cloud*

* Steam (water vapor) can not be seen what you see are tiny drops suspended in air from which we infer that there was some water vapor before it condensed.

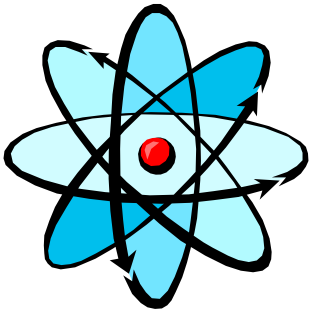
2.7

Is density a unique measure of mass distribution in a volume? Does it vary? If so, on what kind of scale (distance)?

Solution:

Density is an average of mass per unit volume and we sense if it is not evenly distributed by holding a mass that is more heavy in one side than the other. Through the volume of the same substance (say air in a room) density varies only little from one location to another on scales of meter, cm or mm. If the volume you look at has different substances (air and the furniture in the room) then it can change abruptly as you look at a small volume of air next to a volume of hardwood.

Finally if we look at very small scales on the order of the size of atoms the density can vary infinitely, since the mass (electrons, neutrons and positrons) occupy very little volume relative to all the empty space between them.



2.8

Density of fibers, rock wool insulation, foams and cotton is fairly low. Why is that?

Solution:

All these materials consist of some solid substance and mainly air or other gas. The volume of fibers (clothes) and rockwool that is solid substance is low relative to the total volume that includes air. The overall density is

$$\rho = \frac{m}{V} = \frac{m_{solid} + m_{air}}{V_{solid} + V_{air}}$$

where most of the mass is the solid and most of the volume is air. If you talk about the density of the solid only, it is high.

2.9

How much mass is there approximately in 1 L of engine oil? Atmospheric air?

Solution:

A volume of 1 L equals 0.001 m^3, see Table A.1. From Table A.4 the density is 885 kg/m^3 so we get

$$m = \rho V = 885 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = \mathbf{0.885 \ kg}$$

For the air we see in Figure 2.7 that density is about 1 kg/m^3 so we get

$$m = \rho V = 1 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = \mathbf{0.001 \ kg}$$

A more accurate value from Table A.5 is $\rho = 1.17$ kg/m^3 at 100 kPa, 25^{o}C.

2.10

Can you carry 1 m^3 of liquid water?

Solution:

The density of liquid water is about 1000 kg/m^3 from Figure 2.7, see also Table A.3. Therefore the mass in one cubic meter is

$$m = \rho V = 1000 \text{ kg/m}^3 \times 1 \text{ m}^3 = 1000 \text{ kg}$$

and we can not carry that in the standard gravitational field.

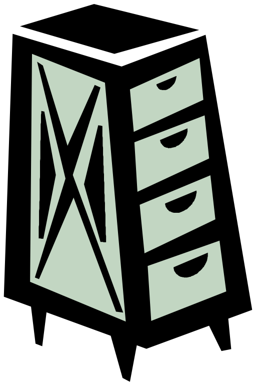
2.11

A heavy cabinet has four adjustable feet on it. What feature of the feet will ensure that they do not make dents in the floor?

Answer:
The area that is in contact with the floor supports the total mass in the gravitational field.

$$F = PA = mg$$

so for a given mass the smaller the area is the larger the pressure becomes.



2.12

The pressure at the bottom of a swimming pool is evenly distributed. Suppose we look at a cast iron plate of 7272 kg lying on the ground with an area of 100 m^2. What is the average pressure below that? Is it just as evenly distributed as the pressure at the bottom of the pool?

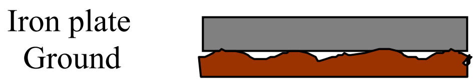
Solution:

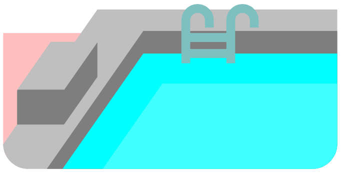
The pressure is force per unit area from page 25:

$$P = F/A = mg/A = 7272 \text{ kg} \times (9.81 \text{ m/s}^2) / 100 \text{ m}^2 = \mathbf{713.4 \text{ Pa}}$$

The iron plate being cast can be reasonable plane and flat, but it is stiff and rigid. However, the ground is usually uneven so the contact between the plate and the ground is made over an area much smaller than the 100 m^2. Thus the local pressure at the contact locations is much larger than the quoted value above.

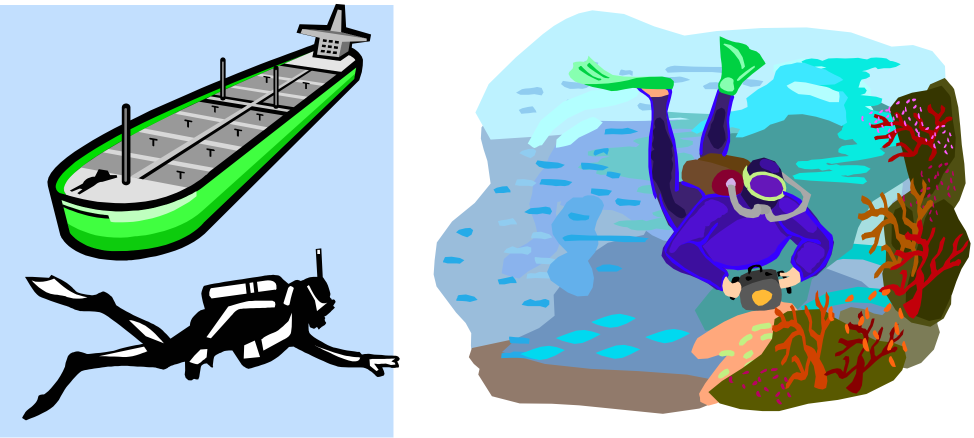
The pressure at the bottom of the swimming pool is very even due to the ability of the fluid (water) to have full contact with the bottom by deforming itself. This is the main difference between a fluid behavior and a solid behavior.





2.13

Two divers swim at 20 m depth. One of them swims right in under a supertanker; the other stays away from the tanker. Who feels a greater pressure?



Solution:

Each one feels the local pressure which is the static pressure only a function of depth.

$$P_{ocean} = P_0 + \Delta P = P_0 + \rho g H$$

So they feel exactly the same pressure.

2.14

A manometer with water shows a ΔP of $P_o/10$; what is the column height difference?

Solution:

$$\Delta P = P_o/10 = \rho H g$$

$$H = P_o/(10\ \rho\ g) = \frac{101.3 \times 1000\ \text{Pa}}{10 \times 997\ \text{kg/m}^3 \times 9.80665\ \text{m/s}^2}$$

$= \mathbf{1.036\ m}$

2.15

A water skier does not sink too far down in the water if the speed is high enough. What makes that situation different from our static pressure calculations?

The water pressure right under the ski is not a static pressure but a static plus dynamic pressure that pushes the water away from the ski. The faster you go, the smaller amount of water is displaced but at a higher velocity.

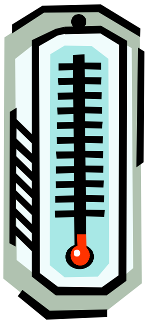
2.16

What is the smallest temperature in degrees Celsuis you can have? Kelvin?

Solution:

The lowest temperature is absolute zero which is at zero degrees Kelvin at which point the temperature in Celsius is negative

$$T_K = 0 \text{ K} = -273.15\ ^{o}\text{C}$$



2.17

Convert the formula for water density in In-text Concept Question "e" to be for T in degrees Kelvin.

Solution:

$$\rho = 1008 - T_C/2 \quad [\text{kg/m}^3]$$

We need to express degrees Celsius in degrees Kelvin

$$T_C = T_K - 273.15$$

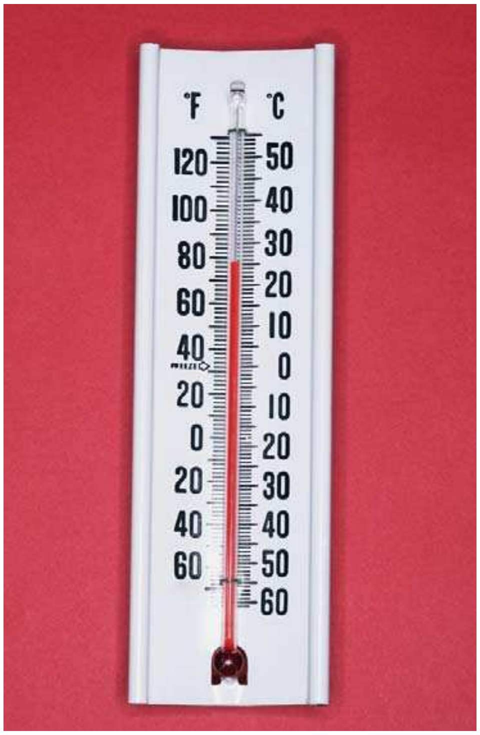
and substitute into formula

$$\rho = 1008 - T_C/2 = 1008 - (T_K - 273.15)/2 = 1144.6 - T_K/2$$

2.18

A thermometer that indicates the temperature with a liquid column has a bulb with a larger volume of liquid, why is that?

The expansion of the liquid volume with temperature is rather small so by having a larger volume expand with all the volume increase showing in the very small diameter column of fluid greatly increases the signal that can be read.



Properties and units

2.19

An apple “weighs” 60 g and has a volume of 75 cm^3 in a refrigerator at 8^{o}C. What is the apple density? List three intensive and two extensive properties of the apple.

Solution:

$$\rho = \frac{m}{V} = \frac{0.06}{0.000\ 075}\ \frac{\text{kg}}{\text{m}^3} = 800\ \frac{\text{kg}}{\text{m}^3}$$

Intensive

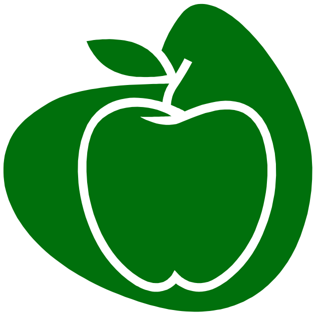
$$\rho = 800\ \frac{\text{kg}}{\text{m}^3}; \qquad v = \frac{1}{\rho} = 0.001\ 25\ \frac{\text{m}^3}{\text{kg}}$$

$$T = 8°\text{C}; \qquad P = 101\ \text{kPa}$$

Extensive

$$m = 60\ \text{g} = 0.06\ \text{kg}$$

$$V = 75\ \text{cm}^3 = 0.075\ \text{L} = 0.000\ 075\ \text{m}^3$$



2.20

A steel cylinder of mass 2 kg contains 4 L of liquid water at 25°C at 200 kPa. Find the total mass and volume of the system. List two extensive and three intensive properties of the water

Solution:

Density of steel in Table A.3: $\rho = 7820 \text{ kg/m}^3$

Volume of steel: $V = m/\rho = \dfrac{2 \text{ kg}}{7820 \text{ kg/m}^3} = 0.000\ 256 \text{ m}^3$

Density of water in Table A.4: $\rho = 997 \text{ kg/m}^3$

Mass of water: $m = \rho V = 997 \text{ kg/m}^3 \times 0.004 \text{ m}^3 = 3.988 \text{ kg}$

Total mass: $m = m_{steel} + m_{water} = 2 + 3.988 = \mathbf{5.988 \text{ kg}}$

Total volume: $V = V_{steel} + V_{water} = 0.000\ 256 + 0.004$

$= \mathbf{0.004\ 256 \ \ m^3 = 4.26 \ L}$

Extensive properties: m, V
Intensive properties: ρ (or $v = 1/\rho$), T, P

2.21

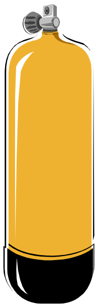
A storage tank of stainless steel contains 7 kg of oxygen gas and 5 kg of nitrogen gas. How many kmoles are in the tank?

Table A.2: $M_{O2} = 31.999$; $M_{N2} = 28.013$

$$n_{O2} = m_{O2} / M_{O2} = \frac{7}{31.999} = 0.21876 \text{ kmol}$$

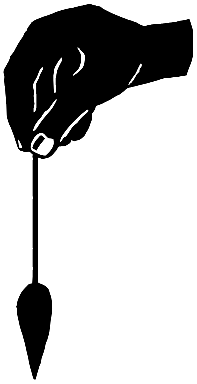
$$n_{O2} = m_{N2} / M_{N2} = \frac{5}{28.013} = 0.17848 \text{ kmol}$$

$$n_{tot} = n_{O2} + n_{N2} = 0.21876 + 0.17848 = \mathbf{0.3972\ kmol}$$



2.22

One kilopond (1 kp) is the weight of 1 kg in the standard gravitational field. How many Newtons (N) is that?



F = ma = mg

1 kp = 1 kg × 9.807 m/s^2 = **9.807 N**

Force and Energy

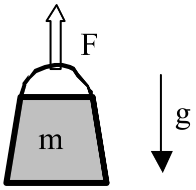
2.23

The “standard” acceleration (at sea level and 45° latitude) due to gravity is $9.80665\ \mathrm{m/s^2}$. What is the force needed to hold a mass of 2 kg at rest in this gravitational field? How much mass can a force of 1 N support?

Solution:

$$ma = 0 = \sum F = F - mg$$

$$F = mg = 2\ \mathrm{kg} \times 9.80665\ \mathrm{m/s^2} = \mathbf{19.613\ N}$$



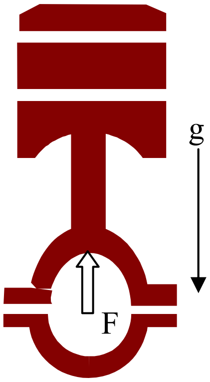
$$F = mg \qquad \Rightarrow$$

$$m = \frac{F}{g} = \frac{1\ \mathrm{N}}{9.80665\ \mathrm{m/s^2}} = \mathbf{0.102\ kg}$$

2.24

A steel piston of 2.5 kg is in the standard gravitational field where a force of 25 N is applied vertically up. Find the acceleration of the piston.

Solution:



$$F_{up} = ma = F - mg$$

$$a = \frac{F - mg}{m} = \frac{F}{m} - g$$

$$= \frac{25\ \text{N}}{2.5\ \text{kg}} - 9.807\ \text{m/s}^2$$

$$= \mathbf{0.193\ ms^{-2}}$$

2.25

When you move up from the surface of the earth the gravitation is reduced as $g = 9.807 - 3.32 \times 10^{-6} z$, with z as the elevation in meters. How many percent is the weight of an airplane reduced when it cruises at 11 000 m?

Solution:

$$g_o = 9.807 \text{ ms}^{-2}$$

$$g_H = 9.807 - 3.32 \times 10^{-6} \times 11\,000 = 9.7705 \text{ ms}^{-2}$$

$$W_o = m\, g_o \;\; ; \;\; W_H = m\, g_H$$

$$W_H/W_o = g_H/g_o = \frac{9.7705}{9.807} = 0.9963$$

Reduction = 1 – 0.9963 = 0.0037 or **0.37%**

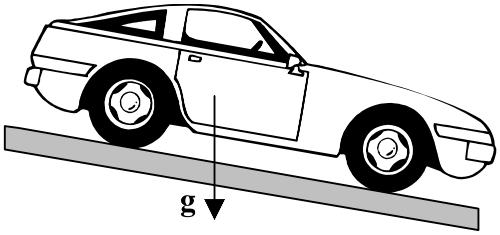
i.e. we can neglect that for most applications.

2.26

A model car rolls down an incline with a slope so the gravitational "pull" in the direction of motion is one third of the standard gravitational force (see Problem 2.23). If the car has a mass of 0.06 kg find the acceleration.

Solution:

$$ma = \sum F = mg / 3$$
$$a = mg / 3m = g/3$$
$$= 9.80665 \text{ (m/s}^2\text{) } / 3$$
$$= \mathbf{3.27\ m/s^2}$$



This acceleration does not depend on the mass of the model car.

2.27

A van drives at 60 km/h and is brought to a full stop with constant deceleration in 5 seconds. If the total car and driver mass is 2075 kg find the necessary force.

Solution:

Acceleration is the time rate of change of velocity.

$$a = \frac{d\mathbf{V}}{dt} = \frac{60 \times 1000}{3600 \times 5} = 3.333 \text{ m/s}^2$$

$ma = \sum F$;

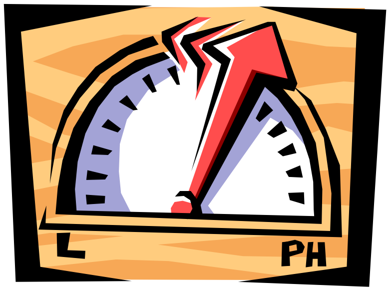
$F_{net} = ma = 2075 \text{ kg} \times 3.333 \text{ m/s}^2 = \mathbf{6916\ N}$

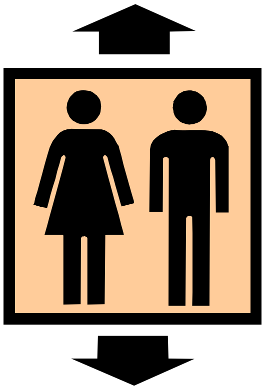
2.28

An escalator brings four people of total 300 kg, 25 m up in a building. Explain what happens with respect to energy transfer and stored energy.

Solution:

The four people (300 kg) have their potential energy raised, which is how the energy is stored. The energy is supplied as electrical power to the motor that pulls the escalator with a cable.





2.29

A car of mass 1775 kg travels with a velocity of 100 km/h. Find the kinetic energy. How high should it be lifted in the standard gravitational field to have a potential energy that equals the kinetic energy?

Solution:

Standard kinetic energy of the mass is

$$\text{KIN} = \tfrac{1}{2}\,\text{m}\,\mathbf{V}^2 = \tfrac{1}{2} \times\ 1775\ \text{kg} \times \left(\frac{100 \times 1000}{3600}\right)^2\ \text{m}^2/\text{s}^2$$

$$= \tfrac{1}{2} \times 1775 \times 27.778\ \text{Nm} = 684\ 800\ \text{J}$$

$$= \mathbf{684.8\ kJ}$$

Standard potential energy is

POT = mgh

$$\text{h} = \tfrac{1}{2}\,\text{m}\,\mathbf{V}^2\ /\ \text{mg}\ = \frac{684\ 800\ \text{Nm}}{1775\ \text{kg} \times 9.807\ \text{m/s}^2} = \mathbf{39.3\ m}$$

2.30

A 1500-kg car moving at 20 km/h is accelerated at a constant rate of 4 m/s^2 up to a speed of 75 km/h. What are the force and total time required?

Solution:

$$a = \frac{d\mathbf{V}}{dt} = \frac{\Delta\mathbf{V}}{\Delta t} \quad => \qquad \Delta t = \frac{\Delta\mathbf{V}}{a} = \frac{(75 - 20)\ 1000}{3600 \times 5} = \mathbf{3.82\ sec}$$

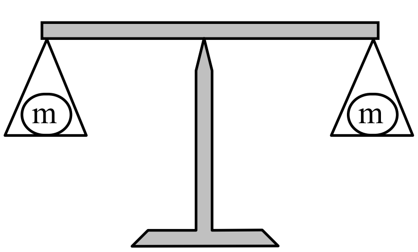
$$F = ma = 1500\text{ kg} \times 4\text{ m/s}^2 = \mathbf{6000\ N}$$

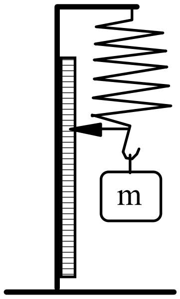
2.31

On the moon the gravitational acceleration is approximately one-sixth that on the surface of the earth. A 5-kg mass is “weighed” with a beam balance on the surface on the moon. What is the expected reading? If this mass is weighed with a spring scale that reads correctly for standard gravity on earth (see Problem 2.23), what is the reading?

Solution:

Moon gravitation is: $g = g_{earth}/6$





Beam Balance Reading is **5 kg**
This is mass comparison

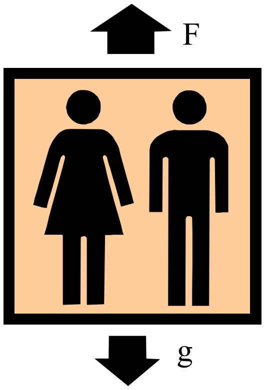
Spring Balance Reading is in kg units
Force comparison length $\propto F \propto g$

Reading will be $\mathbf{\frac{5}{6}}$ **kg**

2.32

The escalator cage in Problem 2.28 has a mass of 500 kg in addition to the people. How much force should the cable pull up with to have an acceleration of 1 m/s^2 in the upwards direction?

Solution:



The total mass moves upwards with an acceleration plus the gravitations acts with a force pointing down.

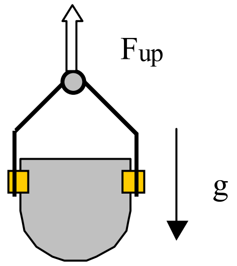
$$ma = \sum F = F - mg$$

$$F = ma + mg = m(a + g)$$
$$= (500 + 300)\text{ kg} \times (1 + 9.81)\text{ m/s}^2$$
$$= \mathbf{8648\ N}$$

2.33

A bucket of concrete of total mass 200 kg is raised by a crane with an acceleration of 2 m/s^2 relative to the ground at a location where the local gravitational acceleration is 9.5 m/s^2. Find the required force.

Solution:



$$F = ma = F_{up} - mg$$

$$F_{up} = ma + mg = 200\ (\ 2 + 9.5\ ) = \mathbf{2300\ N}$$

2.34

A bottle of 12 kg steel has 1.75 kmole of liquid propane. It accelerates horizontal with 3 m/s^2, what is the needed force?

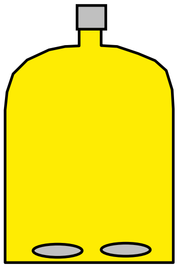
Solution:

The molecular weight for propane is $M = 44.094$ from Table A.2. The force must accelerate both the container mass and the propane mass.

$$m = m_{steel} + m_{propane} = 12 + (1.75 \times 44.094) = 90.645 \text{ kg}$$

$$ma = \sum F \quad \Rightarrow$$

$$F = ma = 90.645 \text{ kg} \times 3 \text{ m/s}^2 = \mathbf{271.9\ N}$$

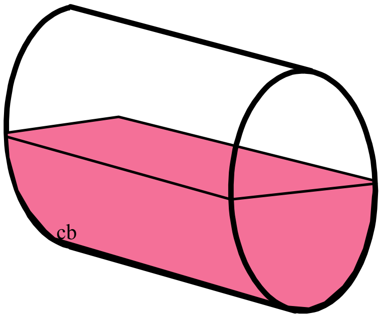


Specific Volume

2.35

A 15-kg steel gas tank holds 300 L of liquid gasoline, having a density of 800 kg/m^3. If the system is decelerated with 2g what is the needed force?

Solution:



$m = m_{tank} + m_{gasoline}$

$= 15 \text{ kg} + 0.3 \text{ m}^3 \times 800 \text{ kg/m}^3$

$= 255 \text{ kg}$

$F = ma = 255 \text{ kg} \times 2 \times 9.81 \text{ m/s}^2$

$=$ **5003 N**

2.36

A power plant that separates carbon-dioxide from the exhaust gases compresses it to a density of 110 kg/m^3 and stores it in an un-minable coal seam with a porous volume of 100 000 m^3. Find the mass they can store.

Solution:

$$m = \rho V = 110 \text{ kg/m}^3 \times 100\,000 \text{ m}^3 = 11 \times 10^6 \text{ kg}$$

Just to put this in perspective a power plant that generates 2000 MW by burning coal would make about 20 million tons of carbon-dioxide a year. That is 2000 times the above mass so it is nearly impossible to store all the carbon-dioxide being produced.

2.37

A 1 m^3 container is filled with 400 kg of granite stone, 200 kg dry sand and 0.2 m^3 of liquid 25°C water. Use properties from tables A.3 and A.4. Find the average specific volume and density of the masses when you exclude air mass and volume.

Solution:

Specific volume and density are ratios of total mass and total volume.

$$m_{liq} = V_{liq}/v_{liq} = V_{liq}\,\rho_{liq} = 0.2\text{ m}^3 \times 997\text{ kg/m}^3 = 199.4\text{ kg}$$

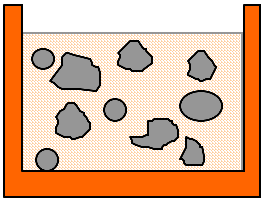
$$m_{TOT} = m_{stone} + m_{sand} + m_{liq} = 400 + 200 + 199.4 = 799.4\text{ kg}$$

$$V_{stone} = mv = m/\rho = 400\text{ kg}/\ 2750\text{ kg/m}^3 = 0.1455\text{ m}^3$$

$$V_{sand} = mv = m/\rho = 200/\ 1500 = 0.1333\text{ m}^3$$

$$V_{TOT} = V_{stone} + V_{sand} + V_{liq}$$

$$= 0.1455 + 0.1333 + 0.2 = 0.4788\text{ m}^3$$



$$v = V_{TOT}/m_{TOT} = 0.4788/799.4 = \mathbf{0.000599\ m^3/kg}$$

$$\rho = 1/v = m_{TOT}/V_{TOT} = 799.4/0.4788 = \mathbf{1669.6\ kg/m^3}$$

2.38

One kilogram of diatomic oxygen (O_2 molecular weight 32) is contained in a 500-L tank. Find the specific volume on both a mass and mole basis (v and $\bar{v}$).

Solution:

From the definition of the specific volume

$$v = \frac{V}{m} = \frac{0.5}{1} = \mathbf{0.5\ m^3/kg}$$

$$\bar{v} = \frac{V}{n} = \frac{V}{m/M} = M\,v = 32 \times 0.5 = \mathbf{16\ m^3/kmol}$$

2.39

A tank has two rooms separated by a membrane. Room A has 1 kg air and volume 0.5 m^3, room B has 0.75 m^3 air with density 0.8 kg/m^3. The membrane is broken and the air comes to a uniform state. Find the final density of the air.

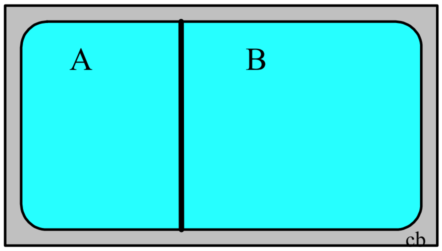
Solution:

Density is mass per unit volume

$$m = m_A + m_B = m_A + \rho_B V_B = 1 + 0.8 \times 0.75 = 1.6 \text{ kg}$$

$$V = V_A + V_B = 0.5 + 0.75 = 1.25 \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{1.6}{1.25} = \mathbf{1.28\ kg/m^3}$$



2.40

A 5 m^3 container is filled with 900 kg of granite (density 2400 kg/m^3) and the rest of the volume is air with density 1.15 kg/m^3. Find the mass of air and the overall (average) specific volume.

Solution:

$$m_{air} = \rho V = \rho_{air} (V_{tot} - \frac{m_{granite}}{\rho})$$

$$= 1.15 [5 - \frac{900}{2400}] = 1.15 \times 4.625 = \mathbf{5.32\ kg}$$

$$v = \frac{V}{m} = \frac{5}{900 + 5.32} = \mathbf{0.005\ 52\ m^3/kg}$$

Comment: Because the air and the granite are not mixed or evenly distributed in the container the overall specific volume or density does not have much meaning.

Pressure

2.41

The hydraulic lift in an auto-repair shop has a cylinder diameter of 0.2 m. To what pressure should the hydraulic fluid be pumped to lift 40 kg of piston/arms and 700 kg of a car?

Solution:

Force acting on the mass by the gravitational field

$$F\downarrow = ma = mg = 740 \times 9.80665 = 7256.9\ \text{N} = 7.257\ \text{kN}$$

Force balance: $F\uparrow = (P - P_0)\ A = F\downarrow \quad => \quad P = P_0 + F\downarrow / A$

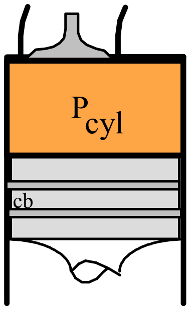
$$A = \pi\ D^2\ (1 / 4) = 0.031416\ \text{m}^2$$

$$P = 101\ \text{kPa} + \frac{7.257\ \text{kN}}{0.031416\ \text{m}^2} = \mathbf{332\ kPa}$$

2.42

A valve in a cylinder has a cross sectional area of 11 cm^2 with a pressure of 735 kPa inside the cylinder and 99 kPa outside. How large a force is needed to open the valve?

$$
\begin{aligned}
F_{net} &= P_{in}A - P_{out}A \\
&= (735 - 99)\ \text{kPa} \times 11\ \text{cm}^2 \\
&= 6996\ \text{kPa cm}^2 \\
&= 6996 \times \frac{\text{kN}}{\text{m}^2} \times 10^{-4}\ \text{m}^2 \\
&= \mathbf{700\ N}
\end{aligned}
$$



2.43

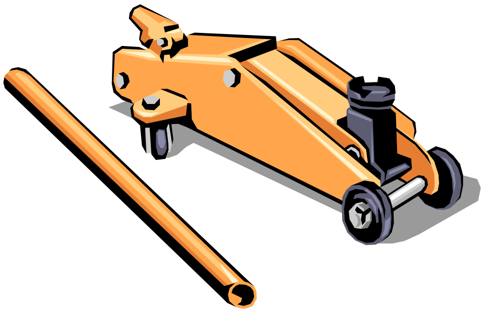
A hydraulic lift has a maximum fluid pressure of 500 kPa. What should the piston-cylinder diameter be so it can lift a mass of 850 kg?

Solution:

With the piston at rest the static force balance is

$$F\uparrow = P\,A = F\downarrow = mg$$
$$A = \pi\, r^2 = \pi\, D^2/4$$
$$PA = P\,\pi\, D^2/4 = mg \quad \Rightarrow \quad D^2 = \frac{4mg}{P\,\pi}$$

$$D = 2\sqrt{\frac{mg}{P\pi}} = 2\sqrt{\frac{850\text{ kg} \times 9.807\text{ m/s}^2}{500\text{ kPa} \times \pi \times 1000\text{ (Pa/kPa)}}} = \mathbf{0.146\ m}$$



2.44

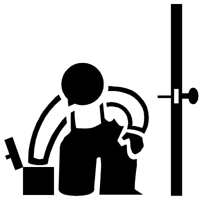
A laboratory room keeps a vacuum of 0.1 kPa. What net force does that put on the door of size 2 m by 1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside}\ A - P_{inside}\ A = \Delta P\ A = 0.1\ \text{kPa} \times 2\ \text{m} \times 1\ \text{m} = \mathbf{200\ N}$$

Remember that kPa is kN/m^2.



$P_{abs} = P_o - \Delta P$

$\Delta P = 0.1$ kPa

2.45

A vertical hydraulic cylinder has a 125-mm diameter piston with hydraulic fluid inside the cylinder and an ambient pressure of 1 bar. Assuming standard gravity, find the piston mass that will create a pressure inside of 1500 kPa.

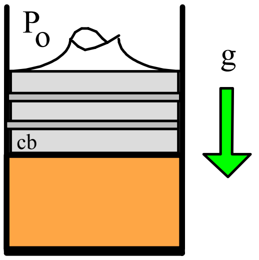
Solution:

Force balance:

$$F\uparrow = PA = F\downarrow = P_0A + m_pg;$$

$$P_0 = 1 \text{ bar} = 100 \text{ kPa}$$

$$A = (\pi/4)\ D^2 = (\pi/4)\ \times 0.125^2 = 0.01227 \text{ m}^2$$



$$m_p = (P - P_0)\frac{A}{g} = (\ 1500 - 100\ ) \times 1000 \times \frac{0.01227}{9.80665} = \mathbf{1752\ kg}$$

2.46

A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 100 kg resting on the stops, as shown in Fig. P2.46. With an outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston?

Solution:

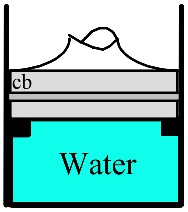
The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface.

Force balance: $F\uparrow = F\downarrow = PA = m_p g + P_0 A$

Now solve for P (divide by 1000 to convert to kPa for 2nd term)

$$P = P_0 + \frac{m_p g}{A} = 100 \text{ kPa} + \frac{100 \times 9.80665}{0.01 \times 1000} \text{ kPa}$$

$$= 100 \text{ kPa} + 98.07 \text{ kPa} = \mathbf{198\ kPa}$$



2.47

A cannon-ball of 5 kg acts as a piston in a cylinder of 0.15 m diameter. As the gun-powder is burned a pressure of 7 MPa is created in the gas behind the ball. What is the acceleration of the ball if the cylinder (cannon) is pointing horizontally?

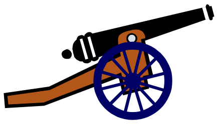
Solution:

The cannon ball has 101 kPa on the side facing the atmosphere.

$$ma = F = P_1 \times A - P_0 \times A = (P_1 - P_0) \times A$$

$$= (7000 - 101)\ \text{kPa} \times \pi\ (0.15^2 / 4)\ \text{m}^2 = 121.9\ \text{kN}$$

$$a = \frac{F}{m} = \frac{121.9\ \text{kN}}{5\ \text{kg}} = \mathbf{24\ 380\ m/s^2}$$



2.48

Repeat the previous problem for a cylinder (cannon) pointing 40 degrees up relative to the horizontal direction.

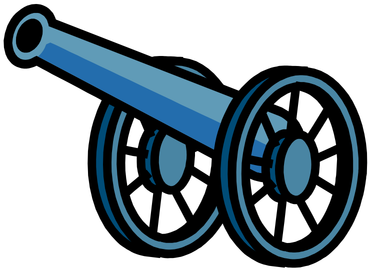
Solution:

$$ma = F = (P_1 - P_0)\, A - mg \sin 40^0$$

$$ma = (7000 - 101\ )\ \text{kPa} \times \pi \times (\ 0.15^2 / 4\ )\ \text{m}^2 - 5 \times 9.807 \times 0.6428\ \text{N}$$

$$= 121.9\ \text{kN} - 31.52\ \text{N} = 121.87\ \text{kN}$$

$$a = \frac{F}{m} = \frac{121.87\ \text{kN}}{5\ \text{kg}} = \mathbf{24\ 374\ m/s^2}$$



2.49

A large exhaust fan in a laboratory room keeps the pressure inside at 10 cm water relative vacuum to the hallway. What is the net force on the door measuring 1.9 m by 1.1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$\begin{aligned} F &= P_{\text{outside}}\,A - P_{\text{inside}}\,A = \Delta P \times A \\ &= 10 \text{ cm } H_2O \times 1.9 \text{ m} \times 1.1 \text{ m} \\ &= 0.10 \times 9.80638 \text{ kPa} \times 2.09 \text{ m}^2 \\ &= \mathbf{2049\ N} \end{aligned}$$

Table A.1: 1 m H_2O is 9.80638 kPa and kPa is kN/m^2.

2.50

A tornado rips off a 100 m^2 roof with a mass of 1000 kg. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

Solution:

The net force on the roof is the difference between the forces on the two sides as the pressure times the area

$$F = P_{inside}\ A - P_{outside} A = \Delta P\ A$$

That force must overcome the gravitation mg, so the balance is

$$\Delta P\ A = mg$$

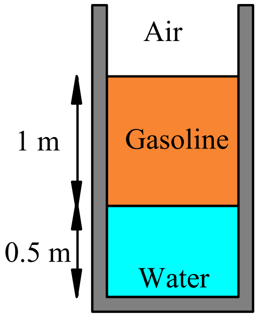
$$\Delta P = mg/A = (1000\text{ kg} \times 9.807\text{ m/s}^2\ )/100\text{ m}^2 = \mathbf{98\ Pa = 0.098\ kPa}$$

Remember that kPa is kN/m^2.

2.51

A 2.5 m tall steel cylinder has a cross sectional area of 1.5 m^2. At the bottom with a height of 0.5 m is liquid water on top of which is a 1 m high layer of gasoline. This is shown in Fig. P2.51. The gasoline surface is exposed to atmospheric air at 101 kPa. What is the highest pressure in the water?

Solution:



The pressure in the fluid goes up with the depth as

$$P = P_{top} + \Delta P = P_{top} + \rho g h$$

and since we have two fluid layers we get

$$P = P_{top} + [(\rho h)_{gasoline} + (\rho h)_{water}]\ g$$

The densities from Table A.4 are:

$$\rho_{gasoline} = 750\ kg/m^3; \quad \rho_{water} = 997\ kg/m^3$$

$$P = 101 + [750 \times 1 + 997 \times 0.5]\ \frac{9.807}{1000} = \mathbf{113.2\ kPa}$$

2.52

What is the pressure at the bottom of a 5 m tall column of fluid with atmospheric pressure 101 kPa on the top surface if the fluid is

a) water at 20°C b) glycerine 25°C or c) gasoline 25°C

Solution:

Table A.4: $\rho_{H2O} = 997\ \text{kg/m}^3$; $\rho_{Glyc} = 1260\ \text{kg/m}^3$; $\rho_{gasoline} = 750\ \text{kg/m}^3$

$\Delta P = \rho g h$ $P = P_{top} + \Delta P$

a) $\Delta P = \rho g h = 997\times 9.807\times 5 = 48\ 888\ \text{Pa}$

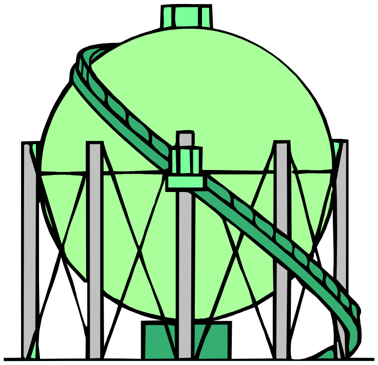
$P = 101 + 48.99 =$ **149.9 kPa**

b) $\Delta P = \rho g h = 1260\times 9.807\times 5 = 61\ 784\ \text{Pa}$

$P = 101 + 61.8 =$ **162.8 kPa**

c) $\Delta P = \rho g h = 750\times 9.807\times 5 = 36\ 776\ \text{Pa}$

$P = 101 + 36.8 =$ **137.8 kPa**



2.53

At the beach, atmospheric pressure is 1025 mbar. You dive 15 m down in the ocean and you later climb a hill up to 250 m elevation. Assume the density of water is about 1000 kg/m^3 and the density of air is 1.18 kg/m^3. What pressure do you feel at each place?

Solution:

$\Delta P = \rho g h,$

Units from A.1: 1 mbar = 100 Pa (1 bar = 100 kPa).

$$P_{ocean} = P_0 + \Delta P = 1025 \times 100 + 1000 \times 9.81 \times 15$$

$$= 2.4965 \times 10^5 \text{ Pa} = \mathbf{250\ kPa}$$

$$P_{hill} = P_0 - \Delta P = 1025 \times 100 - 1.18 \times 9.81 \times 250$$

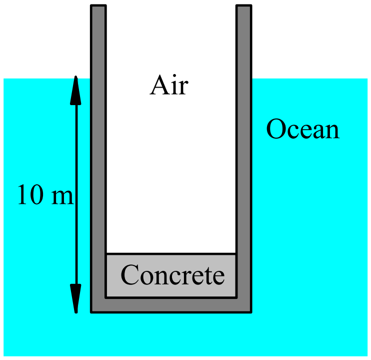
$$= 0.99606 \times 10^5 \text{ Pa} = \mathbf{99.61\ kPa}$$

2.54

A steel tank of cross sectional area 3 m^2 and 16 m tall weighs 10 000 kg and it is open at the top. We want to float it in the ocean so it sticks 10 m straight down by pouring concrete into the bottom of it. How much concrete should I put in?

Solution:

The force up on the tank is from the water pressure at the bottom times its area. The force down is the gravitation times mass and the atmospheric pressure.



$$F\uparrow = PA = (\rho_{ocean}gh + P_0)A$$

$$F\downarrow = (m_{tank} + m_{concrete})g + P_0A$$

The force balance becomes

$$F\uparrow = F\downarrow = (\rho_{ocean}gh + P_0)A = (m_{tank} + m_{concrete})g + P_0A$$

Solve for the mass of concrete

$$m_{concrete} = (\rho_{ocean}hA - m_{tank}) = 997 \times 10 \times 3 - 10\ 000 = \mathbf{19\ 910\ kg}$$

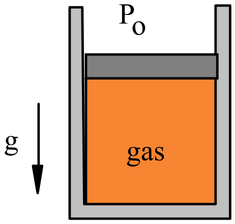
Notice: The first term is the mass of the displaced ocean water. The net force up is the weight (mg) of this mass called bouyancy, P_0 cancel.

2.55

A piston, m_p= 5 kg, is fitted in a cylinder, A = 15 cm^2, that contains a gas. The setup is in a centrifuge that creates an acceleration of 25 m/s^2 in the direction of piston motion towards the gas. Assuming standard atmospheric pressure outside the cylinder, find the gas pressure.

Solution:

Force balance: $F\uparrow = F\downarrow = P_0A + m_pg = PA$



$$P = P_0 + \frac{m_pg}{A}$$

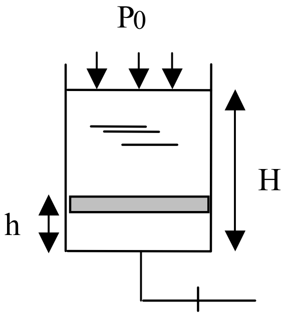
$$= 101.325 + \frac{5 \times 25}{1000 \times 0.0015} \frac{\text{kPa kg m/s}^2}{\text{Pa m}^2}$$

= **184.7 kPa**

2.56

Liquid water with density ρ is filled on top of a thin piston in a cylinder with cross-sectional area *A* and total height *H,* as shown in Fig. P2.56. Air is let in under the piston so it pushes up, spilling the water over the edge. Derive the formula for the air pressure as a function of piston elevation from the bottom, *h.*

Solution:



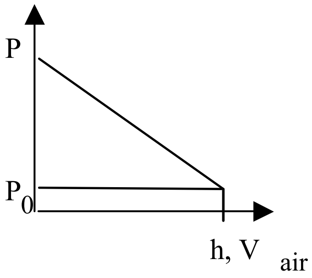
Force balance

Piston: F↑ = F↓

$$PA = P_0A + m_{H_2O}g$$

$$P = P_0 + m_{H_2O}g/A$$

$$\mathbf{P = P_0 + (H - h)\rho g}$$



Manometers and Barometers

2.57

You dive 5 m down in the ocean. What is the absolute pressure there?

Solution:

The pressure difference for a column is from Eq.2.2 and the density of water is from Table A.4.

$$\Delta P = \rho g H$$
$$= 997 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 5 \text{ m}$$
$$= 48\,903 \text{ Pa} = 48.903 \text{ kPa}$$

$$P_{ocean} = P_0 + \Delta P$$
$$= 101.325 + 48.903$$
$$= \mathbf{150\ kPa}$$

2.58

A barometer to measure absolute pressure shows a mercury column height of 725 mm. The temperature is such that the density of the mercury is 13 550 kg/m^3. Find the ambient pressure.

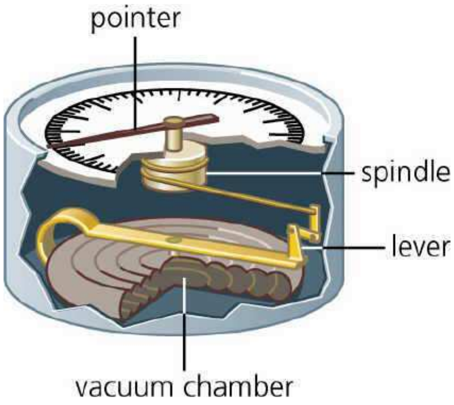
Solution:

Hg : $L = 725 \text{ mm} = 0.725 \text{ m}$; $\rho = 13\,550 \text{ kg/m}^3$

The external pressure P balances the column of height L so from Fig. 2.10

$$P = \rho L g = 13\,550 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2 \times 0.725 \text{ m} \times 10^{-3} \text{ kPa/Pa}$$

= **96.34 kPa**



This is a more common type that does not involve mercury as the wall mounted unit to the left.

2.59

The density of atmospheric air is about 1.15 kg/m^3, which we assume is constant. How large an absolute pressure will a pilot see when flying 2000 m above ground level where the pressure is 101 kPa.

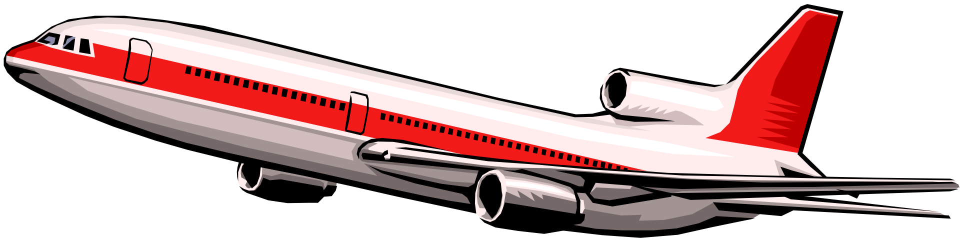
Solution:

Assume g and ρ are constant then the pressure difference to carry a column of height 1500 m is from Fig.2.10

$$\Delta P = \rho g h = 1.15 \text{ kg/m}^3 \times 9.807 \text{ ms}^{-2} \times 2000 \text{ m}$$
$$= 22\,556 \text{ Pa} = 22.6 \text{ kPa}$$

The pressure on top of the column of air is then

$$P = P_0 - \Delta P = 101 - 22.6 = \mathbf{78.4\ kPa}$$



2.60

A differential pressure gauge mounted on a vessel shows 1.25 MPa and a local barometer gives atmospheric pressure as 0.96 bar. Find the absolute pressure inside the vessel.

Solution:

Convert all pressures to units of kPa.

$P_{gauge} = 1.25 \text{ MPa} = 1250 \text{ kPa};$

$P_0 = 0.96 \text{ bar} = 96 \text{ kPa}$

$P = P_{gauge} + P_0 = 1250 + 96 = \mathbf{1346\ kPa}$

2.61

A manometer shows a pressure difference of 1 m of liquid mercury. Find ΔP in kPa.

Solution:

Hg : L = 1 m; $\rho = 13\ 580\ \mathrm{kg/m^3}$ from Table A.4 (or read Fig 2.8)

The pressure difference ΔP balances the column of height L so from Eq.2.2

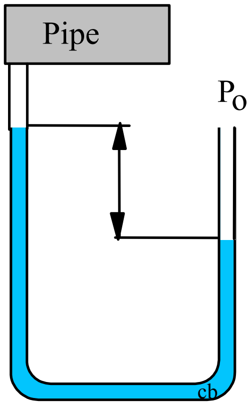
$$\Delta P = \rho\, g\, L = 13\ 580\ \mathrm{kg/m^3} \times 9.80665\ \mathrm{m/s^2} \times 1.0\ \mathrm{m} \times 10^{-3}\ \mathrm{kPa/Pa}$$

$$= \mathbf{133.2\ kPa}$$

2.62

Blue manometer fluid of density 925 kg/m^3 shows a column height difference of 3 cm vacuum with one end attached to a pipe and the other open to P_0 = 101 kPa. What is the absolute pressure in the pipe?

Solution:



Since the manometer shows a vacuum we have

$$P_{PIPE} = P_0 - \Delta P$$

$$\Delta P = \rho g h = 925 \times 9.807 \times 0.03$$

$$= 272.1 \text{ Pa} = 0.272 \text{ kPa}$$

$$P_{PIPE} = 101 - 0.272 = \mathbf{100.73 \text{ kPa}}$$

2.63

What pressure difference does a 10 m column of atmospheric air show?

Solution:

The pressure difference for a column is from Eq.2.2

$$\Delta P = \rho g H$$

So we need density of air from Fig.2.7, $\rho = 1.2\ \text{kg/m}^3$

$$\Delta P = 1.2\ \text{kg/m}^3 \times 9.81\ \text{ms}^{-2} \times 10\ \text{m} = 117.7\ \text{Pa} = \mathbf{0.12\ kPa}$$

2.64

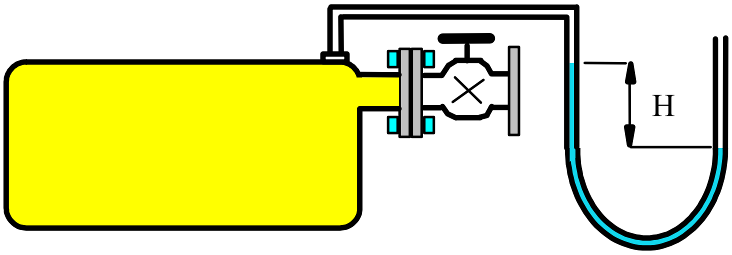
The absolute pressure in a tank is 85 kPa and the local ambient absolute pressure is 97 kPa. If a U-tube with mercury, density 13550 kg/m^3, is attached to the tank to measure the vacuum, what column height difference would it show?

Solution:

$$\Delta P = P_0 - P_{tank} = \rho g\ H$$

$$H = (\ P_0 - P_{tank}\ ) / \rho g = [(97 - 85\ ) \times 1000\ ] / (13550 \times 9.80665)$$

$$= \mathbf{0.090\ m = 90\ mm}$$



2.65

The pressure gauge on an air tank shows 75 kPa when the diver is 10 m down in the ocean. At what depth will the gauge pressure be zero? What does that mean?

Ocean H_20 pressure at 10 m depth is

$$P_{water} = P_0 + \rho Lg = 101.3 + \frac{997 \times 10 \times 9.80665}{1000} = 199 \text{ kPa}$$

Air Pressure (absolute) in tank

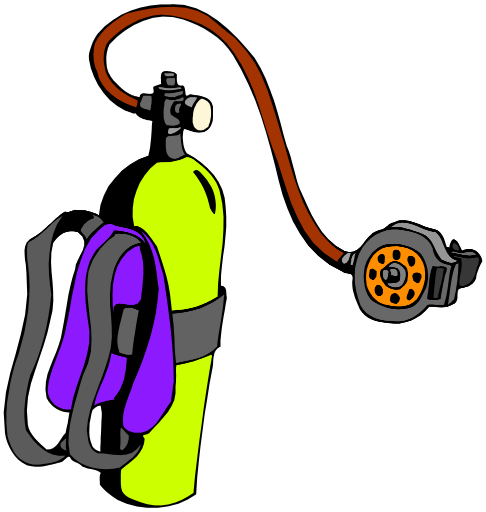
$$P_{tank} = 199 + 75 = 274 \text{ kPa}$$

Tank Pressure (gauge) reads zero at H_20 local pressure

$$274 = 101.3 + \frac{997 \times 9.80665}{1000} L$$

$$L = \mathbf{17.66 \ m}$$

At this depth you will have to suck the air in, it can no longer push itself through a valve.



2.66

An exploration submarine should be able to go 4000 m down in the ocean. If the ocean density is 1020 kg/m^3 what is the maximum pressure on the submarine hull?

Solution:

Assume we have atmospheric pressure inside the submarine then the pressure difference to the outside water is

$$\Delta P = \rho L g = (1020\ \text{kg/m}^3 \times 4000\ \text{m} \times 9.807\ \text{m/s}^2) / 1000$$
$$= 40\ 012\ \text{kPa} \approx \mathbf{40\ MPa}$$

2.67

A submarine maintains 101 kPa inside it and it dives 240 m down in the ocean having an average density of 1030 kg/m^3. What is the pressure difference between the inside and the outside of the submarine hull?

Solution:

Assume the atmosphere over the ocean is at 101 kPa, then ΔP is from the 240 m column water.

$$\Delta P = \rho L g$$

$$= (1030\ kg/m^3 \times\ 240\ m \times\ 9.807\ m/s^2)\ /\ 1000 = \mathbf{2424\ kPa}$$

2.68

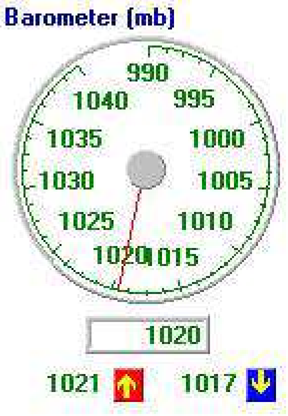
Assume we use a pressure gauge to measure the air pressure at street level and at the roof of a tall building. If the pressure difference can be determined with an accuracy of 1 mbar (0.001 bar) what uncertainty in the height estimate does that corresponds to?

Solution:

$\rho_{air} = 1.169 \text{ kg/m}^3$ from Table A.5

$\Delta P = 0.001 \text{ bar} = 100 \text{ Pa}$

$$L = \frac{\Delta P}{\rho g} = \frac{100}{1.169 \times 9.807} = \mathbf{8.72\ m}$$



2.69

A barometer measures 760 mmHg at street level and 735 mmHg on top of a building. How tall is the building if we assume air density of 1.15 kg/m^3?

Solution:

$$\Delta P = \rho g H$$

$$H = \Delta P/\rho g = \frac{760 - 735}{1.15 \times 9.807} \frac{\text{mmHg}}{\text{kg/m}^2\text{s}^2} \frac{133.32 \text{ Pa}}{\text{mmHg}} = \mathbf{295\ m}$$

2.70

An absolute pressure gauge attached to a steel cylinder shows 135 kPa. We want to attach a manometer using liquid water a day that P_{atm} = 101 kPa. How high a fluid level difference must we plan for?

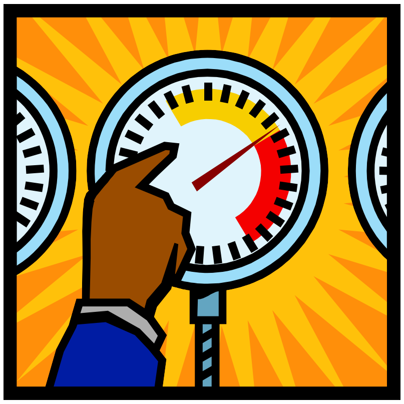
Solution:

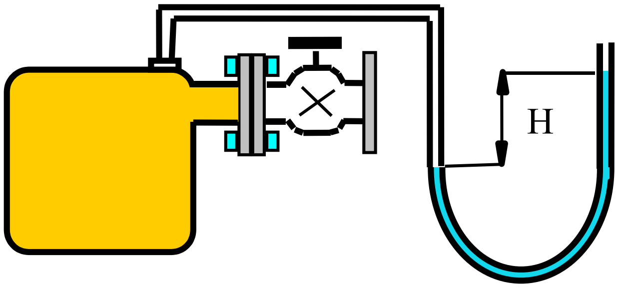
Since the manometer shows a pressure difference we have

$$\Delta P = P_{CYL} - P_{atm} = \rho L g$$

$$L = \Delta P / \rho g = \frac{(135 - 101) \text{ kPa}}{997 \text{ kg m}^{-3} \times 10 \times 9.807 \text{ m/s}^2} \frac{1000 \text{ Pa}}{\text{kPa}}$$

$$= \mathbf{3.467 \text{ m}}$$



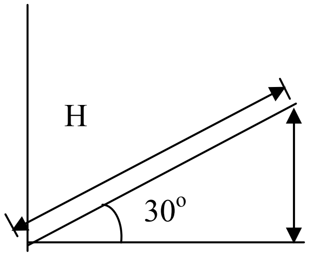


2.71

A U-tube manometer filled with water, density 1000 kg/m^3, shows a height difference of 25 cm. What is the gauge pressure? If the right branch is tilted to make an angle of 30° with the horizontal, as shown in Fig. P2.71, what should the length of the column in the tilted tube be relative to the U-tube?

Solution:

Same height in the two sides in the direction of g.



$$\Delta P = F/A = mg/A = V\rho g/A = h\rho g$$
$$= 0.25 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.807 \text{m/s}^2$$
$$= 2452.5 \text{ Pa}$$
$$= \mathbf{2.45\ kPa}$$

$$h = H \times \sin 30°$$
$$\Rightarrow H = h/\sin 30° = 2h = \mathbf{50\ cm}$$

2.72

A pipe flowing light oil has a manometer attached as shown in Fig. P2.72. What is the absolute pressure in the pipe flow?

Solution:

Table A.3: $\rho_{oil} = 910\ \text{kg/m}^3$; $\rho_{water} = 997\ \text{kg/m}^3$

$$P_{BOT} = P_0 + \rho_{water}\, g\, H_{tot} = P_0 + 997\ \text{kg/m}^3 \times 9.807\ \text{m/s}^2 \times 0.8\ \text{m}$$
$$= P_o + 7822\ \text{Pa}$$

$$P_{PIPE} = P_{BOT} - \rho_{water}\, g\, H_1 - \rho_{oil}\, g\, H_2$$
$$= P_{BOT} - 997 \times 9.807 \times 0.1 - 910 \times 9.807 \times 0.2$$
$$= P_{BOT} - 977.7\ \text{Pa} - 1784.9\ \text{Pa}$$

$$P_{PIPE} = P_o + (7822 - 977.7 - 1784.9)\ \text{Pa}$$
$$= P_o + 5059.4\ \text{Pa} = 101.325 + 5.06 = \mathbf{106.4\ kPa}$$

2.73

The difference in height between the columns of a manometer is 200 mm with a fluid of density 900 kg/m^3. What is the pressure difference? What is the height difference if the same pressure difference is measured using mercury, density 13600 kg/ m^3, as manometer fluid?

Solution:

$$\Delta P = \rho_1 g h_1 = 900 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.2 \text{ m} = 1765.26 \text{ Pa} = \mathbf{1.77\ kPa}$$

$$h_{Hg} = \Delta P/\ (\rho_{hg}\ g) = (\rho_1\ gh_1)\ /\ (\rho_{hg}\ g) = \frac{900}{13600} \times 0.2$$

$$= \mathbf{0.0132\ m = 13.2\ mm}$$

2.74

Two cylinders are filled with liquid water, $\rho = 1000\ \mathrm{kg/m^3}$, and connected by a line with a closed valve. A has 100 kg and B has 500 kg of water, their cross-sectional areas are $A_A = 0.1\ \mathrm{m^2}$ and $A_B = 0.25\ \mathrm{m^2}$ and the height h is 1 m. Find the pressure on each side of the valve. The valve is opened and water flows to an equilibrium. Find the final pressure at the valve location.

Solution:

$$V_A = v_{H_2O} m_A = m_A/\rho = 0.1 = A_A h_A \quad => \quad h_A = 1\ \mathrm{m}$$

$$V_B = v_{H_2O} m_B = m_B/\rho = 0.5 = A_B h_B \quad => \quad h_B = 2\ \mathrm{m}$$

$$P_{VB} = P_0 + \rho g(h_B + H) = 101325 + 1000 \times 9.81 \times 3 = 130\ 755\ \mathrm{Pa}$$

$$P_{VA} = P_0 + \rho g h_A = 101325 + 1000 \times 9.81 \times 1 = 111\ 135\ \mathrm{Pa}$$

Equilibrium: same height over valve in both

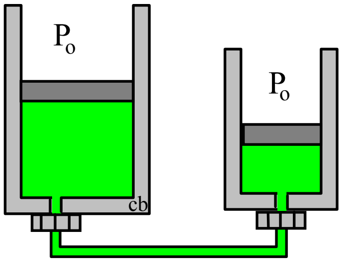
$$V_{tot} = V_A + V_B = h_2 A_A + (h_2 - H)A_B \Rightarrow h_2 = \frac{h_A A_A + (h_B + H)A_B}{A_A + A_B} = 2.43\ \mathrm{m}$$

$$P_{V2} = P_0 + \rho g h_2 = 101.325 + (1000 \times 9.81 \times 2.43)/1000 = \mathbf{125.2\ kPa}$$

2.75

Two piston/cylinder arrangements, A and B, have their gas chambers connected by a pipe. Cross-sectional areas are $A_A = 75$ cm^2 and $A_B = 25$ cm^2 with the piston mass in A being $m_A = 25$ kg. Outside pressure is 100 kPa and standard gravitation. Find the mass m_B so that none of the pistons have to rest on the bottom.

Solution:



Force balance for both pistons: $F\uparrow = F\downarrow$

A: $m_{PA}g + P_0A_A = PA_A$

B: $m_{PB}g + P_0A_B = PA_B$

Same P in A and B gives no flow between them.

$$\frac{m_{PA}g}{A_A} + P_0 = \frac{m_{PB}g}{A_B} + P_0$$

$\Rightarrow m_{PB} = m_{PA}\ A_A/\ A_B = 25 \times 25/75 =$ **8.33 kg**

2.76

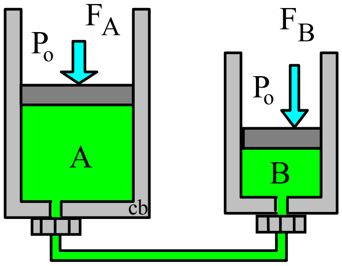
Two hydraulic piston/cylinders are of same size and setup as in Problem 2.75, but with negligible piston masses. A single point force of 250 N presses down on piston A. Find the needed extra force on piston B so that none of the pistons have to move.

Solution:

$A_A = 75\ \text{cm}^2$;

$A_B = 25\ \text{cm}^2$

No motion in connecting pipe: $P_A = P_B$



Forces on pistons balance

$$P_A = P_0 + F_A / A_A = P_B = P_0 + F_B / A_B$$

$$F_B = F_A \times \frac{A_B}{A_A} = 250 \times \frac{25}{75} = \mathbf{83.33\ N}$$

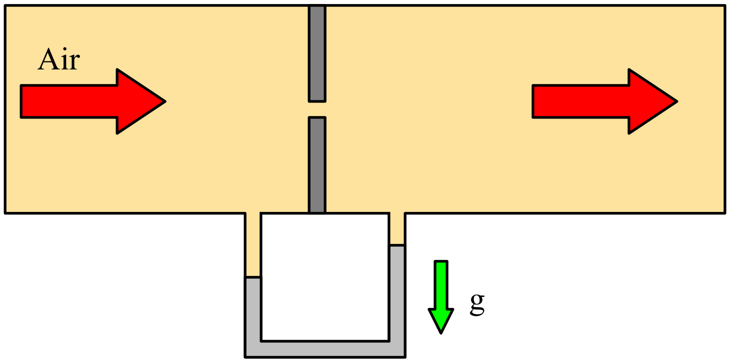
2.77

A piece of experimental apparatus is located where $g = 9.5\ \text{m/s}^2$ and the temperature is 5°C. An air flow inside the apparatus is determined by measuring the pressure drop across an orifice with a mercury manometer (see Problem 2.79 for density) showing a height difference of 200 mm. What is the pressure drop in kPa?

Solution:

$$\Delta P = \rho g h\ ; \qquad \rho_{Hg} = 13600\ \ \text{kg/m}^3$$

$$\Delta P = 13\,600\ \text{kg/m}^3 \times 9.5\ \text{m/s}^2 \times 0.2\ \text{m} = 25840\ \text{Pa} = \mathbf{25.84\ kPa}$$

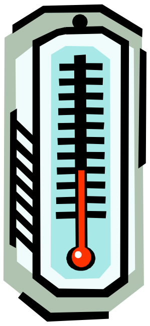


Temperature

2.78

What is a temperature of –5$^{\mathrm{o}}$C in degrees Kelvin?

Solution:



The offset from Celsius to Kelvin is 273.15 K, so we get

$$T_K = T_C + 273.15 = -5 + 273.15$$
$$= \mathbf{268.15\ K}$$

2.79

The density of mercury changes approximately linearly with temperature as

$$\rho_{Hg} = 13595 - 2.5\,T \ \text{kg/ m}^3 \qquad T \ \text{ in Celsius}$$

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho\, L\, g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The manometer fluid density from the given formula gives

$$\rho_{su} = 13595 - 2.5 \times 35 = 13507.5 \text{ kg/m}^3$$

$$\rho_{w} = 13595 - 2.5 \times (-15) = 13632.5 \ \text{ kg/m}^3$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\ \text{m/s}^2} = 0.7549 \text{ m}$$

$$L_{w} = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\ \text{m/s}^2} = 0.7480 \text{ m}$$

$\Delta L = L_{su} - L_{w} =$ **0.0069 m = 6.9 mm**

2.80

A mercury thermometer measures temperature by measuring the volume expansion of a fixed mass of liquid Hg due to a change in the density, see problem 2.35. Find the relative change (%) in volume for a change in temperature from 10°C to 20°C.

Solution:

From 10°C to 20°C

At 10°C : $\rho_{Hg} = 13595 - 2.5 \times 10 = 13570 \ \text{kg/m}^3$

At 20°C : $\rho_{Hg} = 13595 - 2.5 \times 20 = 13545 \ \text{kg/m}^3$

The volume from the mass and density is: $V = m/\rho$

$$\text{Relative Change} = \frac{V_{20} - V_{10}}{V_{10}} = \frac{(m/\rho_{20}) - (m/\rho_{10})}{m/\rho_{10}}$$

$$= \frac{\rho_{10}}{\rho_{20}} - 1 = \frac{13570}{13545} - 1 = \mathbf{0.0018\ (0.18\%)}$$

2.81

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases 10^{o}C how much deeper does a 1 m layer of water become?

Solution:

The density change for a change in temperature of 10^{o}C becomes

$$\Delta\rho = -\Delta T/2 = -5 \text{ kg/m}^3$$

from an ambient density of

$$\rho = 1008 - T/2 = 1008 - 25/2 = 995.5 \text{ kg/m}^3$$

Assume the area is the same and the mass is the same $m = \rho V = \rho AH$, then we have

$$\Delta m = 0 = V\Delta\rho + \rho\Delta V \quad \Rightarrow \quad \Delta V = - V\Delta\rho/\rho$$

and the change in the height is

$$\Delta H = \frac{\Delta V}{A} = \frac{H\Delta V}{V} = \frac{-H\Delta\rho}{\rho} = \frac{-1 \times (-5)}{995.5} = \mathbf{0.005\ m}$$

barely measurable.

2.82

Using the freezing and boiling point temperatures for water in both Celsius and Fahrenheit scales, develop a conversion formula between the scales. Find the conversion formula between Kelvin and Rankine temperature scales.

Solution:

$$T_{Freezing} = 0\ ^{o}C = 32\ F; \qquad T_{Boiling} = 100\ ^{o}C = 212\ F$$

$$\Delta T = 100\ ^{o}C = 180\ F \;\Rightarrow\; T_{oC} = (T_F - 32)/1.8 \quad \text{or} \quad T_F = 1.8\ T_{oC} + 32$$

For the absolute K & R scales both are zero at absolute zero.

$$T_R = 1.8 \times T_K$$

2.83

The atmosphere becomes colder at higher elevation. As an average the standard atmospheric absolute temperature can be expressed as $T_{atm} = 288 - 6.5 \times 10^{-3}\, z$, where z is the elevation in meters. How cold is it outside an airplane cruising at 12 000 m expressed in Kelvin and in Celsius?

Solution:

For an elevation of $z = 12\,000$ m we get

$$T_{atm} = 288 - 6.5 \times 10^{-3}\, z = \mathbf{210\ K}$$

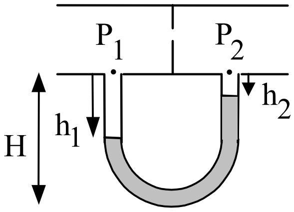
To express that in degrees Celsius we get

$$T_C = T - 273.15 = \mathbf{-63.15^oC}$$

2.84

Repeat problem 2.77 if the flow inside the apparatus is liquid water, $\rho \cong 1000$ kg/m^3, instead of air. Find the pressure difference between the two holes flush with the bottom of the channel. You cannot neglect the two unequal water columns.

Solution:



Balance forces in the manometer:

$$(H - h_2) - (H - h_1) = \Delta h_{Hg} = h_1 - h_2$$

$$P_1A + \rho_{H2O}h_1gA + \rho_{Hg}(H - h_1)gA$$
$$= P_2A + \rho_{H2O}h_2gA + \rho_{Hg}(H - h_2)gA$$

$$\Rightarrow P_1 - P_2 = \rho_{H2O}(h_2 - h_1)g + \rho_{Hg}(h_1 - h_2)g$$

$$P_1 - P_2 = \rho_{Hg}\Delta h_{Hg}g - \rho_{H2O}\Delta h_{Hg}g = 13\,600 \times 0.2 \times 9.5 - 1000 \times 0.2 \times 9.5$$
$$= 25\,840 - 1900 = 23940 \text{ Pa} = \mathbf{23.94\ kPa}$$

2.85

A dam retains a lake 6 m deep. To construct a gate in the dam we need to know the net horizontal force on a 5 m wide and 6 m tall port section that then replaces a 5 m section of the dam. Find the net horizontal force from the water on one side and air on the other side of the port.

Solution:

$P_{bot} = P_0 + \Delta P$

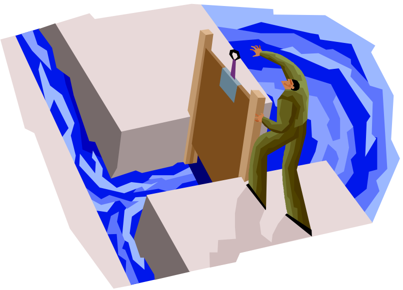
$\Delta P = \rho g h = 997 \times 9.807 \times 6 = 58\,665 \text{ Pa} = 58.66 \text{ kPa}$

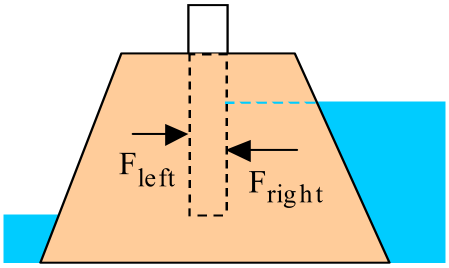
Neglect ΔP in air

$F_{net} = F_{right} - F_{left} = P_{avg} A - P_0 A$

$P_{avg} = P_0 + 0.5\ \Delta P$ Since a linear pressure variation with depth.

$F_{net} = (P_0 + 0.5\ \Delta P)A - P_0 A = 0.5\ \Delta P\ A = 0.5 \times 58.66 \times 5 \times 6 =$ **880 kN**

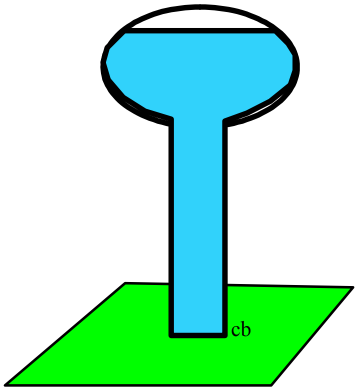




2.86

In the city water tower, water is pumped up to a level 25 m above ground in a pressurized tank with air at 125 kPa over the water surface. This is illustrated in Fig. P2.86. Assuming the water density is 1000 kg/m^3 and standard gravity, find the pressure required to pump more water in at ground level.

Solution:



$$\Delta P = \rho L g$$

$$= 1000 \text{ kg/m}^3 \times 25 \text{ m} \times 9.807 \text{ m/s}^2$$

$$= 245\,175 \text{ Pa} = 245.2 \text{ kPa}$$

$$P_{\text{bottom}} = P_{\text{top}} + \Delta P$$

$$= 125 + 245.2$$

$$= \mathbf{370\ kPa}$$

2.87

The main waterline into a tall building has a pressure of 600 kPa at 5 m elevation below ground level. How much extra pressure does a pump need to add to ensure a water line pressure of 200 kPa at the top floor 150 m above ground?

Solution:

The pump exit pressure must balance the top pressure plus the column ΔP. The pump inlet pressure provides part of the absolute pressure.

$P_{\text{after pump}} = P_{\text{top}} + \Delta P$

$\Delta P = \rho g h = 997 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times (150 + 5) \text{ m}$

$= 1\,515\,525 \text{ Pa} = 1516 \text{ kPa}$

$P_{\text{after pump}} = 200 + 1516 = 1716 \text{ kPa}$

$\Delta P_{\text{pump}} = 1716 - 600 = \mathbf{1116\ kPa}$

2.88

Two cylinders are connected by a piston as shown in Fig. P2.88. Cylinder A is used as a hydraulic lift and pumped up to 500 kPa. The piston mass is 25 kg and there is standard gravity. What is the gas pressure in cylinder B?

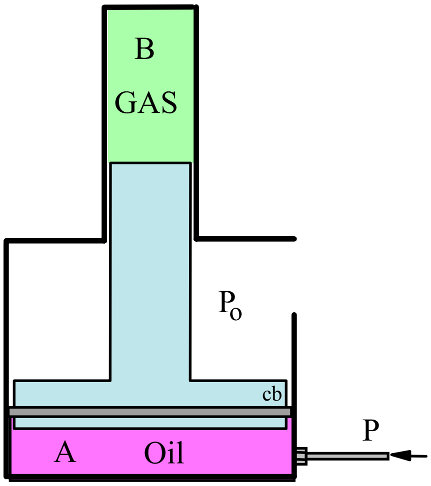
Solution:

Force balance for the piston: $P_BA_B + m_pg + P_0(A_A - A_B) = P_AA_A$

$A_A = (\pi/4)0.1^2 = 0.00785 \text{ m}^2$; $A_B = (\pi/4)0.025^2 = 0.000\,491 \text{ m}^2$

$$P_BA_B = P_AA_A - m_pg - P_0(A_A - A_B) = 500\times 0.00785 - (25 \times 9.807/1000)$$
$$- 100\,(0.00785 - 0.000\,491) = 2.944 \text{ kN}$$

$P_B = 2.944/0.000\,491 = 5996 \text{ kPa} =$ **6.0 MPa**



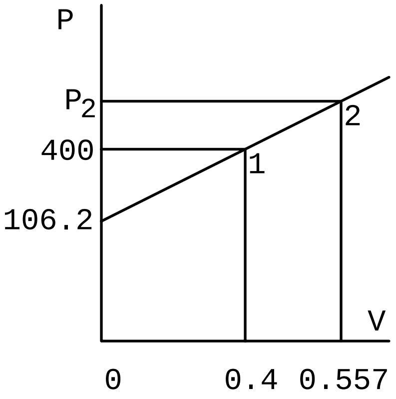
2.89

A 5-kg piston in a cylinder with diameter of 100 mm is loaded with a linear spring and the outside atmospheric pressure of 100 kPa as shown in Fig. P2.89. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 400 kPa with volume 0.4 L. The valve is opened to let some air in, causing the piston to rise 2 cm. Find the new pressure.

Solution:

A linear spring has a force linear proportional to displacement. F = k x, so the equilibrium pressure then varies linearly with volume: P = a + bV, with an intersect a and a slope b = dP/dV. Look at the balancing pressure at zero volume (V -> 0) when there is no spring force $F = PA = P_oA + m_pg$ and the initial state. These two points determine the straight line shown in the P-V diagram.

Piston area = $A_p = (\pi/4) \times 0.1^2 = 0.00785 \text{ m}^2$



$$a = P_0 + \frac{m_pg}{A_p} = 100 \text{ kPa} + \frac{5 \times 9.80665}{0.00785} \text{ Pa}$$

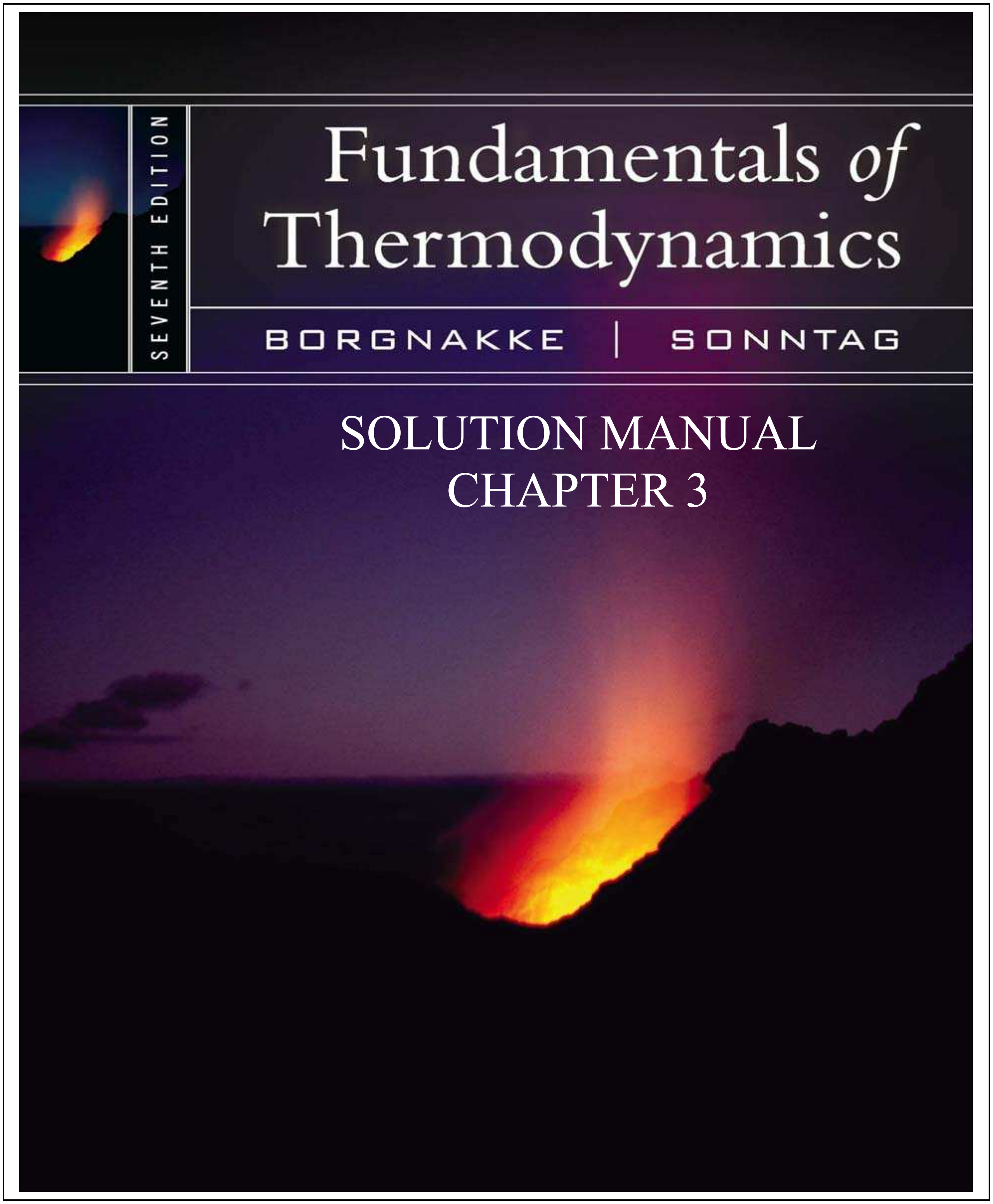
= 106.2 kPa intersect for zero volume.

$$V_2 = 0.4 + 0.00785 \times 20 = 0.557 \text{ L}$$

$$P_2 = P_1 + \frac{dP}{dV} \Delta V$$

$$= 400 + \frac{(400\text{-}106.2)}{0.4 - 0} (0.557 - 0.4)$$

= **515.3 kPa**



SEVENTH EDITION

Fundamentals of Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 3

CONTENT CHAPTER 3

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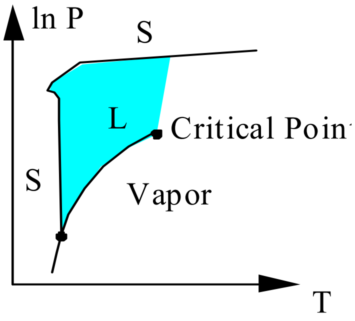
In-Text Concept Questions

3.a

If the pressure is smaller than P_{sat} at a given T, what is the phase?

Refer to the phase diagrams in Figures 3.6 and 3.7. For a lower P you are below the vaporization curve and that is the superheated vapor region. You have the gas phase.



3.b

An external water tap has the valve activated by a long spindle so the closing mechanism is located well inside the wall. Why is that?

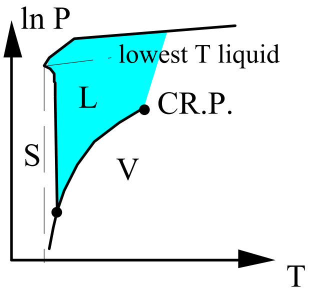
Solution:

By having the spindle inside the wall the coldest location with water when the valve is closed is kept at a temperature above the freezing point. If the valve spindle was outside there would be some amount of water that could freeze while it is trapped inside the pipe section potentially rupturing the pipe.

3.c

What is the lowest temperature (approximately) at which water can be liquid?

Look at the phase diagram in Fig. 3.7. At the border between ice I, ice III and the liquid region is a triple point which is the lowest T where you can have liquid. From the figure it is estimated to be about 255 K i.e. at -18^{o}C.



T ≈ 255 K ≈ - 18°C

3.d

Some tools should be cleaned in water at a least 150°C. How high a P is needed?

Solution:

If I need liquid water at 150°C I must have a pressure that is at least the saturation pressure for this temperature.

Table B.1.1: 150°C, $P_{sat} = 475.9$ kPa.

3.e

Water at 200 kPa has a quality of 50%. Is the volume fraction $V_g/V_{tot} < 50\%$ or $> 50\%$?

This is a two-phase state at a given pressure without looking in the table we know that v_f is much smaller than v_g.

From the definition of quality we get the masses from total mass, m, as

$$m_f = (1 - x)\, m, \qquad m_g = x\, m$$

The volumes are

$$V_f = m_f\, v_f = (1 - x)\, m\, v_f, \qquad V_g = m_g\, v_g = x\, m\, v_g$$

So when half the mass is liquid and the other half is vapor the liquid volume is much smaller that the vapor volume.

3.f

Why are most of the compressed liquid or solid regions not included in the printed tables?

For the compressed liquid and the solid phases the specific volume and thus density is nearly constant. These surfaces are very steep nearly constant v and there is then no reason to fill up a table with the same value of v for different P and T.

3.g

Why is it not typical to find tables for Ar, He, Ne or air like an Appendix B table?

The temperature at which these substances are close to the two-phase region is very low. For technical applications with temperatures around atmospheric or higher they are ideal gases. Look in Table A.2 and we can see the critical temperatures as

Ar : 150.8 K	He: 5.19 K	Ne: 44.4 K

It requires a special refrigerator in a laboratory to bring a substance down to these cryogenic temperatures.

3.h

What is the percent change in volume as liquid water freezes? Mention some effects in nature and for our households the volume change can have.

The density of water in the different phases can be found in Tables A.3 and A.4 and in Table B.1.

From Table B.1.1 $v_f = 0.00100 \text{ m}^3/\text{kg}$

From Table B.1.5 $v_i = 0.0010908 \text{ m}^3/\text{kg}$

Percent change: $$100 \frac{v_i - v_f}{v_f} = 100 \times \frac{0.0010908 - 0.001}{0.001} = 9.1 \text{ \% increase}$$

Liquid water that seeps into cracks or other confined spaces and then freezes will expand and widen the cracks. This is what destroys any porous material exposed to the weather on buildings, roads and mountains. It can burst water pipes and crack engine blocks (that is why you put anti-freeze in it).

3.i

How accurate is it to assume that methane is an ideal gas at room conditions?

From Table A.2: $T_c = 190.4$ K, $P_c = 4.60$ MPa

So at room conditions we have much higher $T > T_c$ and $P << P_c$ so this is the ideal gas region. To confirm look in Table B.7.2

$$100 \text{ kPa}, 300 \text{ K}, \quad v = 1.55215 \text{ m}^3\text{/kg}$$

Find the compressibility factor (R from Table A.5) as

$$Z = Pv/RT = \frac{100 \text{ kPa} \times 1.55215 \text{ m}^3\text{/kg}}{0.5183 \text{ kJ/kg-K} \times 300 \text{ K}} = 0.99823$$

so Z is 1 with an accuracy of 0.2% better than most measurements can be done.

3.j

I want to determine a state of some substance, and I know that P = 200 kPa; is it helpful to write PV = mRT to find the second property?

NO. You need a second property.

Notice that two properties are needed to determine a state. The EOS can give you a third property if you know two, like (P,T) gives v just as you would get by entering a table with a set (P,T). This EOS substitutes for a table when it is applicable.

3.k

A bottle at 298 K should have liquid propane; how high a pressure is needed? (use Fig. D.1)

To have a liquid the pressure must be higher than or equal to the saturation pressure. There is no printed propane table so we use the compressibility chart and Table A.2

Propane Table A.2: $T_c = 369.8$ K, $P_c = 4.25$ MPa

The reduced temperature is:

$$T_r = \frac{T}{T_c} = \frac{298}{369.8} = 0.806,$$

for which we find in Fig. D.1: $P_{r\ sat} = 0.25$

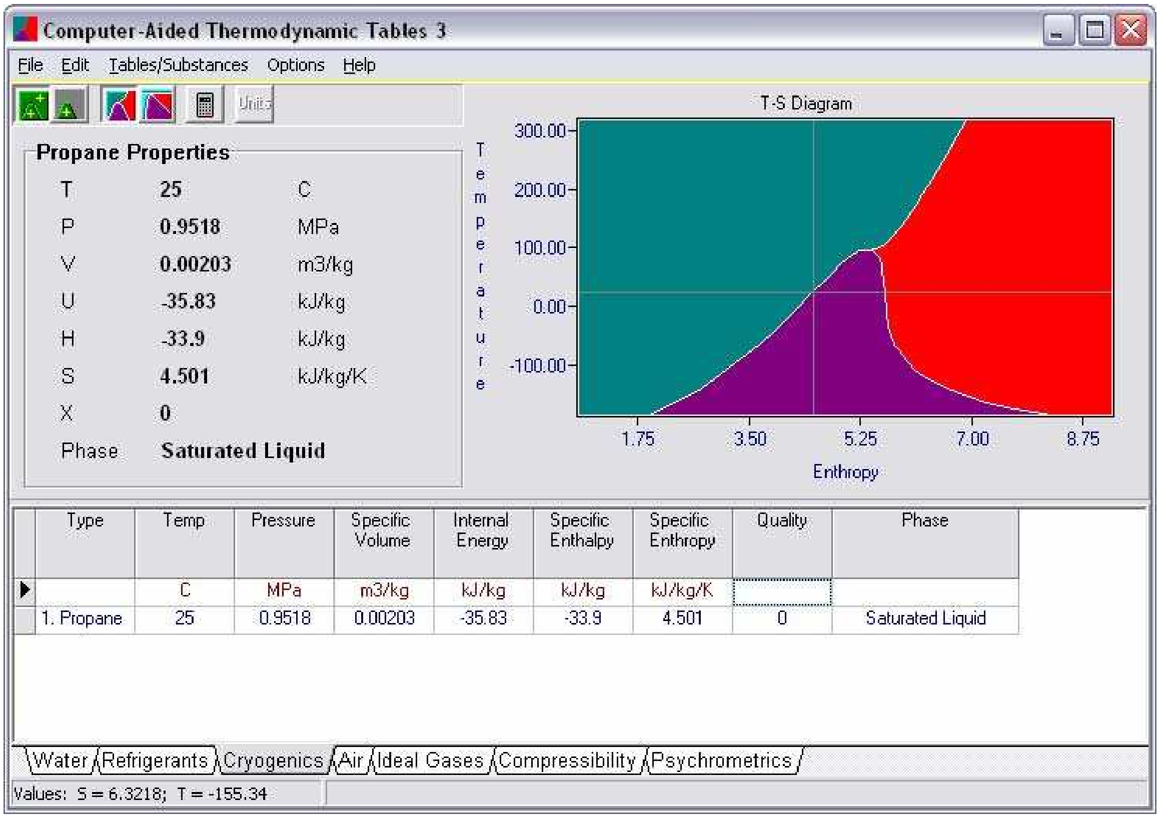
$$P = P_{r\ sat}\, P_c = 0.25 \times 4.25 \text{ MPa} = \mathbf{1.06\ MPa}$$

3.1

A bottle at 298 K should have liquid propane; how high a pressure is needed? (use the software)

To have a liquid the pressure must be higher than or equal to the saturation pressure. There is no printed propane table but the software has propane included

Start CATT3, select cryogenic substances, propane
select calculator, select T, x = (25C, 0) ⇨ P = **0.9518 MPa**



Propane Properties

T	25	C
P	0.9518	MPa
V	0.00203	m3/kg
U	-35.83	kJ/kg
H	-33.9	kJ/kg
S	4.501	kJ/kg/K
X	0	
Phase	Saturated Liquid	

Type	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Enthropy	Quality	Phase
	C	MPa	m3/kg	kJ/kg	kJ/kg	kJ/kg/K		
1. Propane	25	0.9518	0.00203	-35.83	-33.9	4.501	0	Saturated Liquid

Values: S = 6.3218; T = -155.34

Concept Problems

3.1

Are the pressures in the tables absolute or gauge pressures?

Solution:

The behavior of a pure substance depends on the absolute pressure, so P in the tables is absolute.

3.2

What is the minimum pressure for which I can have liquid carbon dioxide?

Look at the phase diagram in Fig. 3.6. The minimum P in the liquid phase is at the triple point. From Table 3.2 this is at **520 kPa** (a similar around 4-500 kPa is seen in Fig. 3.6).

3.3

When you skate on ice, a thin liquid film forms under the skate. How can that be?

The ice is at some temperature below the freezing temperature for the atmospheric pressure of 100 kPa = 0.1 MPa and thus to the left of the fusion line in the solid ice I region of Fig. 3.7. As the skate comes over the ice the pressure is increased dramatically right under the blade so it brings the state straight up in the diagram crossing the fusion line and brings it into a liquid state at same temperature.

The very thin liquid film under the skate changes the friction to be viscous rather than a solid to solid contact friction. Friction is thus significantly reduced.

Comment: The latest research has shown that the pressure may not be enough to generate the liquid, but that such a liquid layer always exist on an ice surface, maybe only a few molecules thick (dependent upon temperature).

3.4

At a higher elevation like in mountains the pressure is lower, what effect does that have for cooking food?

A lower pressure means that water will boil at a lower temperature, see the vaporization line in Fig. 3.7, or in Table B.1.2 showing the saturated temperature as a function of the pressure. You therefore must increase the cooking time a little.

3.5

Water at room temperature and room pressure has $v \approx 1 \times 10^n$ m^3/kg what is n?

See Table B.1.1 or B.1.2 to determine it is in the liquid phase
(you should know this already).
Table A.4 or from B1.1 at 20^{o}C: $n = -3$ ($v = 0.00100$ m^3/kg)

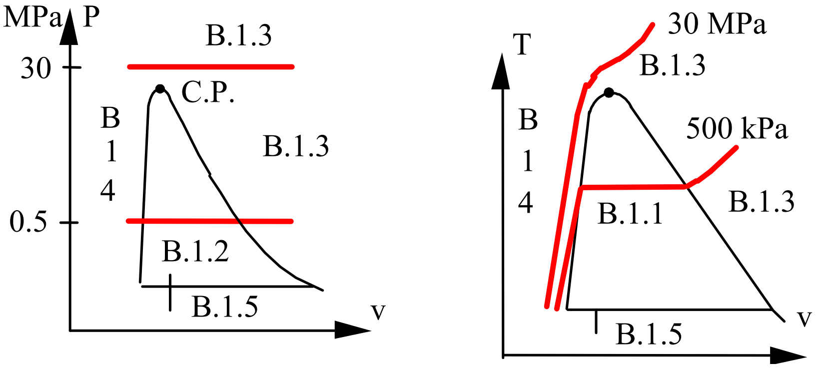
3.6

In Example 3.1 b is there any mass at the indicated specific volume? Explain.

This state is a two-phase mixture of liquid and vapor. There is no mass at the indicated state, the v value is an average for all the mass, so there is some mass at the saturated vapor state (fraction is the quality x) and the remainder of the mass is saturated liquid (fraction 1-x).

3.7

Sketch two constant-pressure curves (500 kPa and 30 000 kPa) in a T-v diagram and indicate on the curves where in the water tables you see the properties.



The 30 MPa line in Table B.1.4 starts at 0°C and table ends at 380°C, the line is continued in Table B.1.3 starting at 375°C and table ends at 1300°C.

The 500 kPa line in Table B.1.4 starts at 0.01°C and table ends at the saturated liquid state (151.86°C). The line is continued in Table B.1.3 starting at the saturated vapor state (151.86°C) continuing up to 1300°C.

3.8

If I have 1 L ammonia at room pressure and temperature (100 kPa, $20^{\circ}C$) how much mass is that?

Ammonia Tables B.2:

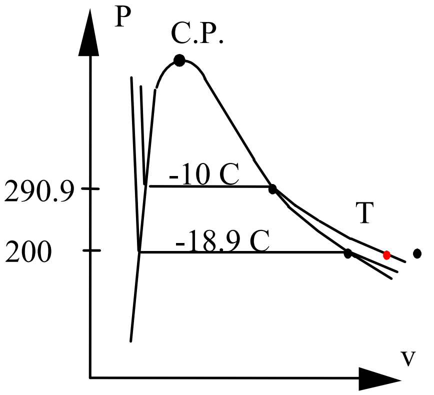
B.2.1 $P_{sat} = 857.5$ kPa at $20^{\circ}C$ so superheated vapor.

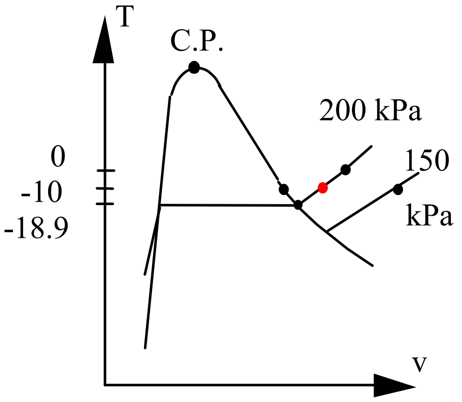
B.2.2 $v = 1.4153\ m^3/kg$ under subheading 100 kPa

$$m = \frac{V}{v} = \frac{0.001\ m^3}{1.4153\ m^3/kg} = \mathbf{0.000\ 706\ kg = 0.706\ g}$$

3.9

Locate the state of ammonia at 200 kPa, -10°C. Indicate in both the P-v and the T-v diagrams the location of the nearest states listed in the printed Table B.2





3.10

Why are most of the compressed liquid or solid regions not included in the printed tables?

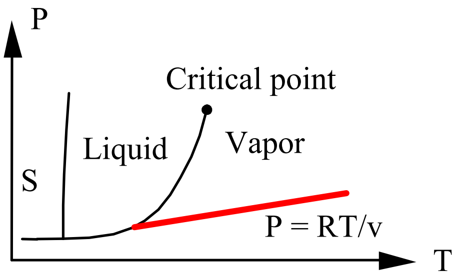
For the compressed liquid and the solid phases the specific volume and thus density is nearly constant. These surfaces are very steep nearly constant v and there is then no reason to fill up a table with the same value of v for different P and T.

3.11

How does a constant v-process look like for an ideal gas in a P-T diagram?

For an ideal gas: Pv = RT so then P = (R/v) T

Constant v is a straight line with slope (R/v) in the P-T diagram



3.12

If $v = RT/P$ for an ideal gas what is the similar equation for a liquid?

The equation for a liquid is: $v = \text{Constant} = v_o$

If you include that v increases a little with T then: $v = v_o + C\,(T - T_o)$
where C is a small constant.

3.13

How accurate (find Z) is it to assume propane is an ideal gas a room conditions?

The propane table is not printed in appendix B so to get this information we either must use the computer software or the generalized charts.

From Table A.2 $P_c = 4250$ kPa, $T_c = 370$ K

The reduced properties: $P_r = \frac{101}{4250} = 0.024$, $T_r = \frac{293}{370} = 0.792$

From Fig. D.1: $Z = 0.98$

So it is an ideal gas within 2% accuracy.

A modest pressure natural gas container.

3.14

With $T_r = 0.80$ what is the ratio of v_g/v_f using Fig. D.1 or Table D.4

Since the two specific volumes are at the same P and T we have

$$Pv_f = Z_f RT \quad \text{and} \quad Pv_g = Z_g RT$$

so the ratio is

$$\frac{v_g}{v_f} = \frac{Z_g}{Z_f} = \frac{0.807}{0.042} = 19.2$$

3.15

To solve for v given (P, T) in Eq 3.9 what mathematical problem do you have?

From Eq. 3.9 you notice it is non-linear in specific volume. Multiplying through with the common denominator we get an equation cubic in v. We thus have to find the roots for a cubic equation (zero points in a polynomial of order three). If you have ever seen the exact solution to that you realize it will be easier to solve by trial and error and select the proper root(s) (it cannot be negative).

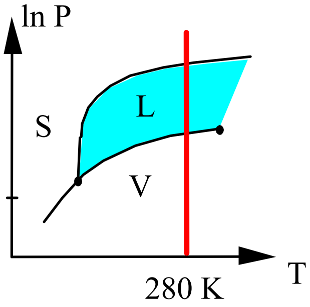
Phase Diagrams, Triple and Critical Points

3.16

Carbon dioxide at 280 K can be in different phases. Indicate the pressure range you have for each of the three phases (vapor, liquid and solid).

Look at the P-T phase diagram in Fig. 3.6 at 280 K:

$P < 4000$ kPa	vapor
$4000 \text{ kPa} < P < 400$ MPa	liquid
$400 \text{ MPa} < P$	solid



3.17

Modern extraction techniques can be based on dissolving material in supercritical fluids such as carbon dioxide. How high are pressure and density of carbon dioxide when the pressure and temperature are around the critical point? Repeat for ethyl alcohol.

Solution:

CO_2 :

Table A.2: $P_c =$ **7.38 MPa**, $T_c = 304$ K, $v_c = 0.00212$ m^3/kg

$\rho_c = 1/v_c = 1/0.00212 =$ **472** $\mathbf{kg/m^3}$

C_2H_5OH:

Table A.2: $P_c =$ **6.14 MPa**, $T_c = 514$ K, $v_c = 0.00363$ m^3/kg

$\rho_c = 1/v_c = 1/0.00363 =$ **275** $\mathbf{kg/m^3}$

3.18

The ice cap on the North Pole could be 1000 m thick with a density of 920 kg/m^3. Find the pressure at the bottom and the corresponding melting temperature.

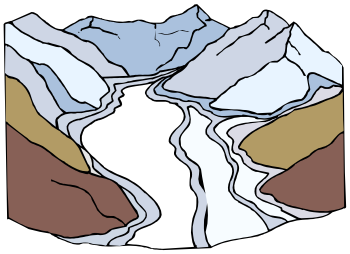
Solution:

$\rho_{ICE} = 920$ kg/m^3

$\Delta P = \rho g H = 920 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2 \times 1000 = 9\,022\,118$ Pa

$P = P_o + \Delta P = 101.325 + 9022 =$ **9123 kPa**

See figure 3.7 liquid solid interphase => **$T_{LS} = -1$°C**



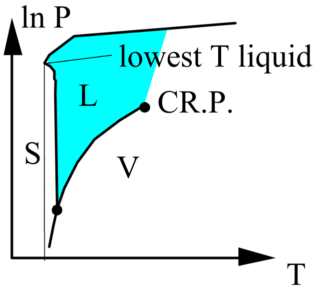
3.19

Find the lowest temperature at which it is possible to have water in the liquid phase. At what pressure must the liquid exist?

Solution:

There is no liquid at lower temperatures than on the fusion line, see Fig. 3.7, saturated ice III to liquid phase boundary is at

T ≈ 263K ≈ - 10°C and
P ≈ 210 MPa



3.20

Water at 27°C can exist in different phases dependent upon the pressure. Give the approximate pressure range in kPa for water being in each one of the three phases vapor, liquid or solid.

Solution:

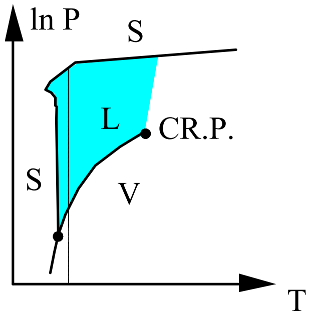
The phases can be seen in Fig. 3.7, a sketch of which is shown to the right.

$T = 27\ ^\circ C = 300\ K$

From Fig. 3.6:

$P_{VL} \approx 4 \times 10^{-3}\ \text{MPa} = 4\ \text{kPa}$,

$P_{LS} = 10^3\ \text{MPa}$



$0 < P < 4$ kPa VAPOR

0.004 MPa $< P < 1000$ MPa LIQUID

$P > 1000$ MPa SOLID (ICE)

3.21

Dry ice is the name of solid carbon dioxide. How cold must it be at atmospheric (100 kPa) pressure? If it is heated at 100 kPa what eventually happens?

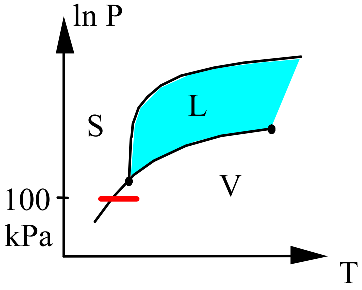
Solution:

The phase boundaries are shown in Figure 3.6

At 100 kPa the carbon dioxide is solid if **T < 190 K**

It goes directly to a vapor state without becoming a liquid hence its name.

The 100 kPa is below the triple point.



3.22

What is the lowest temperature in Kelvin for which you can see metal as a liquid if the metal is *a.* silver *b.* copper

Solution:

Assume the two substances have a phase diagram similar to Fig. 3.6, then we can see the triple point data in Table 3.2

$T_a = 961^oC =$ **1234 K**

$T_b = 1083^oC =$ **1356 K**

3.23

A substance is at 2 MPa, 17°C in a rigid tank. Using only the critical properties can the phase of the mass be determined if the substance is nitrogen, water or propane?

Solution:

Find state relative to critical point properties which are from Table A.2:

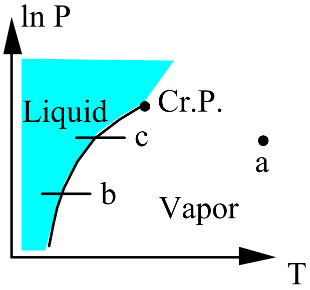
a)	Nitrogen	N_2	: 3.39 MPa	126.2 K
b)	Water	H_2O	: 22.12 MPa	647.3 K
c)	Propane	C_3H_8	: 4.25 MPa	369.8 K

State is at 17 °C = 290 K and 2 MPa < P_c for all cases:

N_2 : $T \gg T_c$ Superheated vapor P < Pc

H_2O : $T \ll T_c$; $P \ll P_c$ you cannot say.

C_3H_8 : $T < T_c$; $P < P_c$ you cannot say



3.24

Give the phase for the following states.

Solution:

a. CO_2 $T = 40^\circ C$ $P = 0.5$ MPa Table A.2

$T > T_c$ => also $P << P_c$

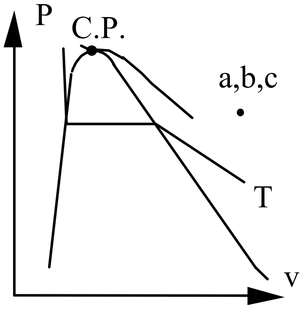
superheated vapor assume ideal gas Table A.5

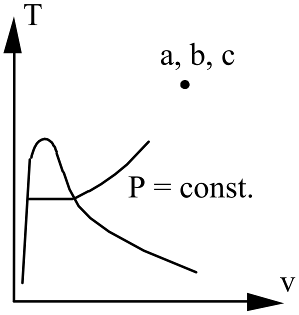
b. Air $T = 20^\circ C$ $P = 200$ kPa Table A.2

superheated vapor assume ideal gas Table A.5

c. NH_3 $T = 170^\circ C$ $P = 600$ kPa Table B.2.2 or A.2

$T > T_c$ => **superheated vapor**





General Tables

3.25

Give the phase for the following states.

Solution:

a. H_2O $T = 260^\circ C$ $P = 5$ MPa Table B.1.1 **or** B.1.2

B.1.1 For given T read: $P_{sat} = 4.689$ MPa

$P > P_{sat}$ => **compressed liquid**

B.1.2 For given P read: $T_{sat} = 264^\circ C$

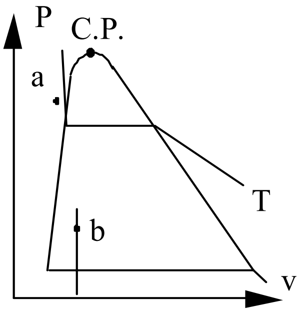
$T < T_{sat}$ => **compressed liquid**

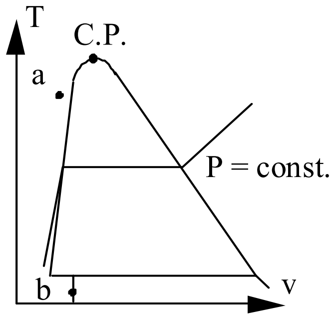
b. H_2O $T = -2^\circ C$ $P = 100$ kPa Table B.1.1 $T < T_{triple\ point}$

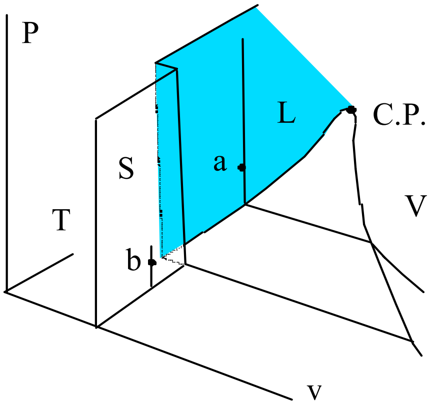
Table B.1.5 at $-2^\circ C$ read: $P_{sat} = 0.518$ kPa

since $P > P_{sat}$ => **compressed solid**

Note state b in P-v, see the 3-D figure, is up on the solid face.







3.26

Determine the phase of the substance at the given state using Appendix B tables

a)	Water	100°C, 500 kPa
b)	Ammonia	-10°C, 150 kPa
c)	R-410a	0°C, 350 kPa

Solution:

a) From Table B.1.1 $P_{sat}(100^\circ C) = 101.3$ kPa

500 kPa > P_{sat} then it is compressed liquid

OR from Table B.1.2 $T_{sat}(500\text{ kPa}) = 152^\circ C$

$100^\circ C < T_{sat}$ then it is subcooled liquid = compressed liquid

b) Ammonia NH_3 :

Table B.2.1: $P < P_{sat}(-10\ ^\circ C) = 291$ kPa

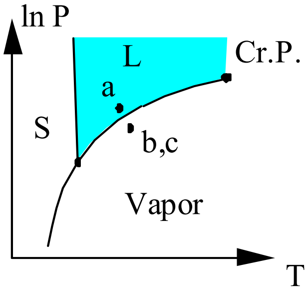
Superheated vapor

c) R-410a

Table B.4.1: $P < P_{sat}(0\ ^\circ C) = 799$ kPa

Superheated vapor.

The S-L fusion line goes slightly to the left for water. It tilts slightly to the right for most other substances.



3.27

Determine whether water at each of the following states is a compressed liquid, a superheated vapor, or a mixture of saturated liquid and vapor.

a. $P = 10$ MPa, $v = 0.003$ m^3/kg b. 1 MPa, 190°C

c. 200°C, 0.1 m^3/kg d. 10 kPa, 10°C

Solution:

For all states start search in table B.1.1 (if T given) or B.1.2 (if P given)

a. $P = 10$ MPa, $v = 0.003$ m^3/kg so look in B.1.2 at 10 MPa

$v_f = 0.001452$; $v_g = 0.01803$ m^3/kg,

$\Rightarrow$ $v_f < v < v_g$ $\Rightarrow$ so mixture of liquid and vapor.

b. 1 MPa, 190°C : Only one of the two look-ups is needed

B.1.1: $P < P_{sat} = 1254.4$ kPa so it is superheated vapor

B.1.2: $T > T_{sat} = 179.91$°C so it is superheated vapor

c. 200°C, 0.1 m^3/kg: look in B.1.1

$v_f = 0.001156$ m^3/kg ; $v_g = 0.12736$ m^3/kg,

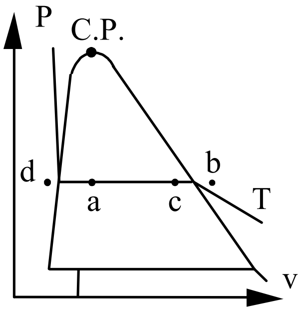
$\Rightarrow$ $v_f < v < v_g$ $\Rightarrow$ so mixture of liquid and vapor.

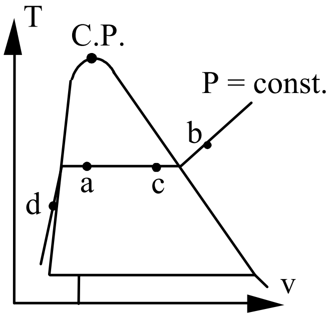
d. 10 kPa, 10°C : Only one of the two look-ups is needed

From B.1.1: $P > P_g = 1.2276$ kPa so compressed liquid

From B.1.2: $T < T_{sat} = 45.8$°C so compressed liquid

States shown are placed relative to the two-phase region, not to each other.





3.28

For water at 100 kPa with a quality of 10%, find the volume fraction of vapor.

This is a two-phase state at a given pressure:

Table B.1.2: $v_f = 0.001\,043\ \mathrm{m^3/kg}$, $v_g = 1.6940\ \mathrm{m^3/kg}$

From the definition of quality we get the masses from total mass, m, as

$$m_f = (1 - x)\,m, \qquad m_g = x\,m$$

The volumes are

$$V_f = m_f\,v_f = (1 - x)\,m\,v_f, \qquad V_g = m_g\,v_g = x\,m\,v_g$$

So the volume fraction of vapor is

$$\text{Fraction} = \frac{V_g}{V} = \frac{V_g}{V_g + V_f} = \frac{x\,m\,v_g}{x\,m\,v_g + (1 - x)m\,v_f}$$

$$= \frac{0.1 \times 1.694}{0.1 \times 1.694 + 0.9 \times 0.001043} = \frac{0.1694}{0.17034} = \mathbf{0.9945}$$

Notice that the liquid volume is only about 0.5% of the total. We could also have found the overall $v = v_f + x v_{fg}$ and then $V = m\,v$.

3.29

Determine whether refrigerant R-410a in each of the following states is a compressed liquid, a superheated vapor, or a mixture of saturated liquid and vapor.

Solution:

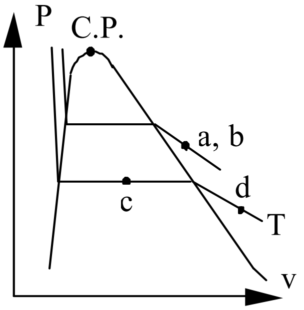
All cases are seen in Table B.4.1

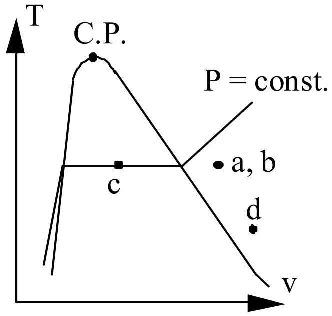
a.	50°C, 0.05 m^3/kg	From table B.4.1 at 50°C $v_g = 0.00707\ m^3/kg$ since $v > v_g$ we have **superheated vapor**
b.	1.0 MPa, 20°C	From table B.4.1 at 20°C $P_g = 909.9$ kPa since $P < P_g$ we have **superheated vapor**
c.	0.1 MPa, 0.1 m^3/kg	From table B.4.1 at 0.1 MPa (use 101 kPa) $v_f = 0.00074$ and $v_g = 0.2395\ m^3/kg$ as $v_f < v < v_g$ we have a **mixture of liquid & vapor**
d	−20°C, 200 kPa	**superheated vapor**, $P < P_g = 400$ kPa at -20°C

3.30

Place the states in Problem 3.29 in a sketch of the P-v diagram.

States shown are placed relative to the two-phase region, not to each other.





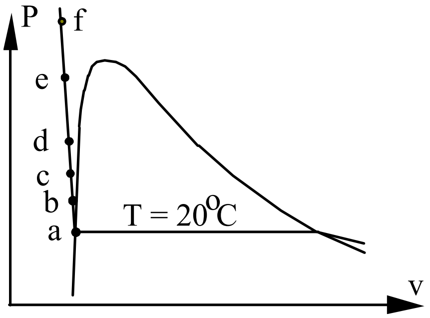
3.31

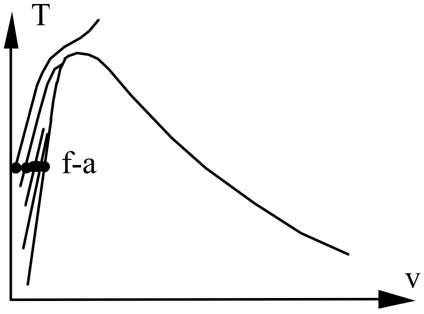
Refer to Fig. 3.18. How much is the change in liquid specific volume for water at 20°C as you move up from state i towards state j reaching 15 000 kPa?

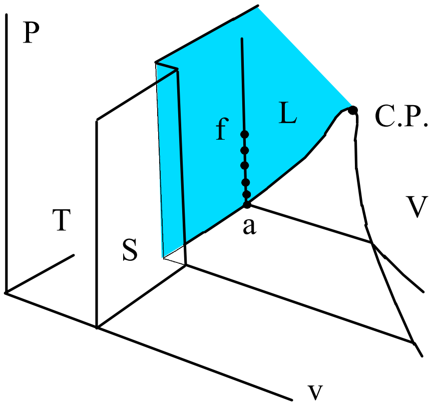
State “i”, here “a”, is saturated liquid and up is then compressed liquid states

a	Table B.1.1:	$v_f = 0.001\ 002\ m^3/kg$	at	2.34 kPa
b	Table B.1.4:	$v_f = 0.001\ 002\ m^3/kg$	at	500 kPa
c	Table B.1.4:	$v_f = 0.001\ 001\ m^3/kg$	at	2000 kPa
d	Table B.1.4:	$v_f = 0.001\ 000\ m^3/kg$	at	5000 kPa
e	Table B.1.4:	$v_f = 0.000\ 995\ m^3/kg$	at	15 000 kPa
f	Table B.1.4:	$v_f = 0.000\ 980\ m^3/kg$	at	50 000 kPa

Notice how small the changes in v are for very large changes in P.







3.32

Fill out the following table for substance ammonia:
Solution:

	P [kPa]	T [^oC]	v [m^3/kg]	x
a)	**1200**	50	0.1185	**Undefined**
b)	**2033**	50	**0.0326**	0.5

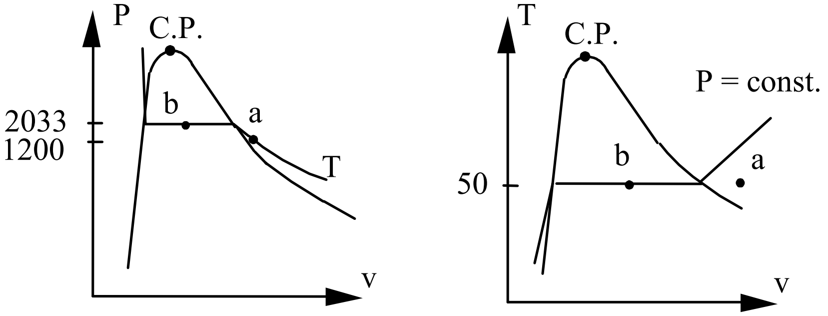
a) B.2.1 $v > v_g$ => superheated vapor Look in B.2.2

b) B.2.1 $P = P_{sat} = 2033$ kPa

$$v = v_f + x\ v_{fg} = 0.001777 + 0.5 \times 0.06159 = 0.0326\ m^3/kg$$

3.33

Place the two states a-b listed in Problem 3.32 as labeled dots in a sketch of the P-v and T-v diagrams.
Solution:



3.34

Give the phase and the missing property of P, T, v and x.

a. R-134a $T = -20^{o}C$, $P = 150$ kPa

b. R-134a $P = 300$ kPa, $v = 0.072\ m^3/kg$

Solution:

a) B.5.1 $P > P_{sat} = 133.7$ kPa $\Rightarrow$ **compressed liquid**

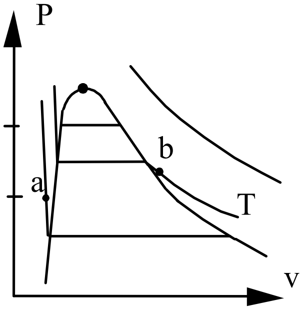
$v \sim v_f = 0.000738\ m^3/kg$

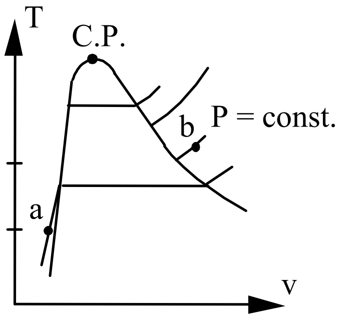
x = undefined

b) B.5.2 $v > v_g$ at 300 kPa $\Rightarrow$ **superheated vapor**

$$T = 10 + (20 - 10)\left(\frac{0.072 - 0.07111}{0.07441 - 0.07111}\right) = 12.7°C$$

x = undefined





3.35

Fill out the following table for substance water:
Solution:

	P [kPa]	T [^oC]	v [m^3/kg]	x
a)	500	20	**0.001002**	**Undefined**
b)	500	**151.86**	0.20	**0.532**
c)	1400	200	**0.14302**	**Undefined**
d)	**8581**	300	**0.01762**	0.8

a) Table B.1.1 $P > P_{sat}$ so it is compressed liquid => Table B.1.4

b) Table B.1.2 $v_f < v < v_g$ so two phase L + V

$$x = \frac{v - v_f}{v_{fg}} = (0.2 - 0.001093) / 0.3738 = 0.532$$

$$T = T_{sat} = 151.86^oC$$

c) Only one of the two look-up is needed

Table B.1.1 200°C $P < P_{sat} =$ => superheated vapor

Table B.1.2 1400 kPa $T > T_{sat} = 195^oC$

Table B.1.3 sub-table for 1400 kPa gives the state properties

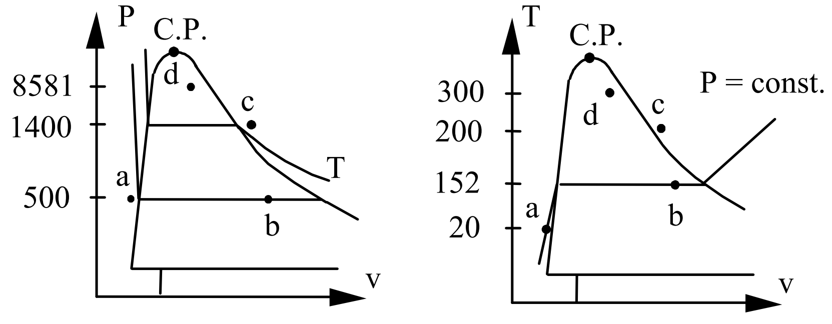
d) Table B.1.1 since quality is given it is two-phase

$$v = v_f + x \times v_{fg} = 0.001404 + 0.8 \times 0.02027 = 0.01762 \text{ m}^3\text{/kg}$$

3.36

Place the four states a-d listed in Problem 3.35 as labeled dots in a sketch of the P-v and T-v diagrams.

Solution:



3.37

Determine the specific volume for R-410a at these states:

a. –15°C, 500 kPa
b. 20°C, 1000 kPa
c. 20°C, quality 25%

a) Table B.4.1: $P > P_{sat} = 480.4$ kPa, so compressed liquid.

$v \approx v_f = 0.000815\ m^3/kg$

b) Table B.4.1: $P < P_{sat} = 1444$ kPa, so superheated vapor

Table B.4.2: $v = 0.02838\ m^3/kg$

c) Table B.4.1: $v_f = 0.000815\ m^3/kg$, $v_{fg} = 0.01666\ m^3/kg$ so

$$v = v_f + x\ v_{fg} = 0.000815 + 0.25 \times 0.01666 = 0.00498\ m^3/kg$$

3.38

Give the missing property of P, T, v and x for CH_4 at:

a. $T = 155$ K, $v = 0.04\ m^3/kg$

b. $T = 350$ K, $v = 0.25\ m^3/kg$

a) B.7.1 $v < v_g = 0.04892\ m^3/kg \quad \Rightarrow \quad$ 2-phase

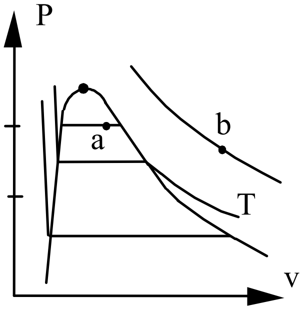
$$x = \frac{v - v_f}{v_{fg}} = \frac{0.04 - 0.002877}{0.04605} = \mathbf{0.806}$$

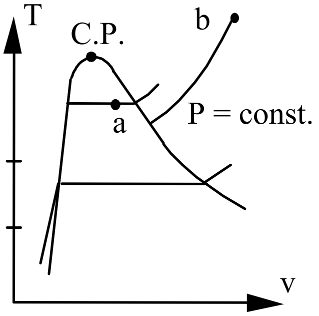
$P = P_{sat} = \mathbf{1296\ kPa}$

b) B.7.1 $T > T_c$ and $v >> v_c \quad \Rightarrow$ superheated vapor B.7.2

located between 600 & 800 kPa

$$P = 600 + 200\,\frac{0.25 - 0.30067}{0.2251 - 0.30067} = \mathbf{734\ kPa}$$





3.39

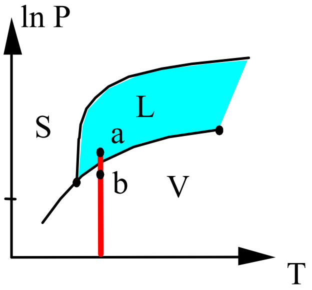
Give the specific volume of carbon-dioxide at -20^{o}C for 2000 kPa and repeat for 1400 kPa.

Table B.3.1: -20^{o}C P_{sat} = 1969 kPa,

at 2000 kPa state is compressed liquid: $v = v_f = 0.000969$ m^3/kg

at 1400 kPa state is superheated vapor: $v = 0.0296$ m^3/kg

The 2000 kPa is above and
the 1400 kPa is below
the vaporization line.



3.40

Calculate the following specific volumes

a.	CO_2	10°C, 80% quality
b.	Water	4 MPa, 90% quality
c.	Nitrogen	120 K, 60% quality

Solution:

All states are two-phase with quality given. The overall specific volume is given by Eq.3.1 or 3.2

$$v = v_f + x\ v_{fg} = (1\text{-}x)v_f + x\ v_g$$

a. CO_2 10°C, 80% quality in Table B.3.1

$$v = 0.001161 + x \times 0.00624 = 0.006153\ \mathrm{m^3/kg}$$

b. Water 4 MPa, 90% quality in Table B.1.2

$$v = 0.001252(1\text{-}x) + x \times 0.04978 = 0.04493\ \mathrm{m^3/kg}$$

c. Nitrogen 120 K, 60% quality in Table B.6.1

$$v = 0.001915 + x \times 0.00608 = 0.005563\ \mathrm{m^3/kg}$$

3.41

Give the phase and P, x for nitrogen at:

a) $T = 120$ K, $v = 0.006$ m^3/kg

b) $T = 140$ K, $v = 0.002$ m^3/kg

Solution:

a) Table B.6.1 $v < v_g = 0.00799$ m^3/kg so state is two-phase L + V

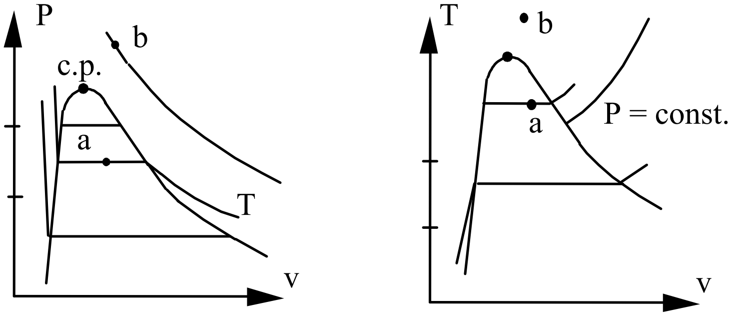
$P = P_{sat} =$ **2513 kPa**

$$x = \frac{v - v_f}{v_{fg}} = (0.006 - 0.001915)/0.00608 = \mathbf{0.6719}$$

b) Table B.6.1 $T > T_c = 126.2$ K so go to B.6.2 superheated vapor.

Look at any entry for 140 K then you see a too large v meaning P is higher. Last one

P = 10 000 kPa, **x = undefined**



3.42

You want a pot of water to boil at 105°C. How heavy a lid should you put on the 15 cm diameter pot when $P_{atm} = 101$ kPa?

Solution:

Table B.1.1 at 105°C : $P_{sat} = 120.8$ kPa

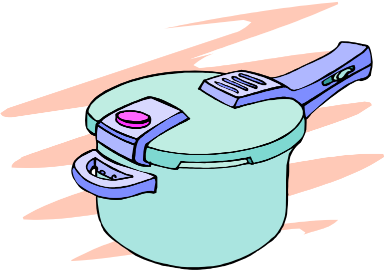
$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} 0.15^2 = 0.01767 \text{ m}^2$$

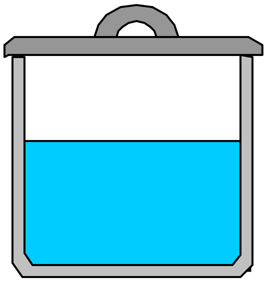
$$F_{net} = (P_{sat} - P_{atm})\, A = (120.8 - 101) \text{ kPa} \times 0.01767 \text{ m}^2$$
$$= 0.3498 \text{ kN} = 350 \text{ N}$$

$$F_{net} = m_{lid}\, g$$

$$m_{lid} = F_{net}/g = \frac{350}{9.807} = \mathbf{35.7\ kg}$$

Some lids are clamped on, the problem deals with one that stays on due to its weight.





3.43

Water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process. What is the new quality and pressure?

Solution:

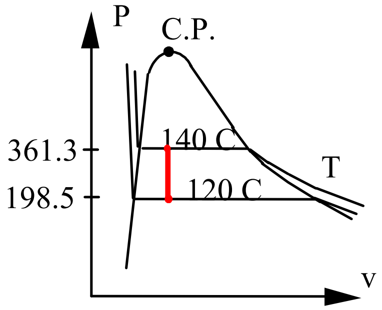
State 1 from Table B.1.1 at 120°C

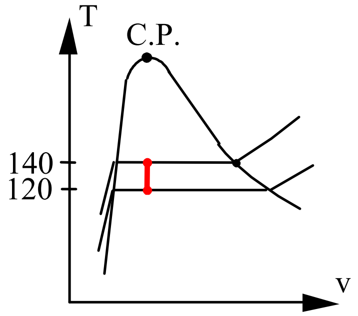
$$v = v_f + x\, v_{fg} = 0.001060 + 0.25 \times 0.8908 = 0.22376 \text{ m}^3/\text{kg}$$

State 2 has same v at 140°C also from Table B.1.1

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.22376 - 0.00108}{0.50777} = \mathbf{0.4385}$$

$$P = P_{sat} = \mathbf{361.3\ kPa}$$





3.44

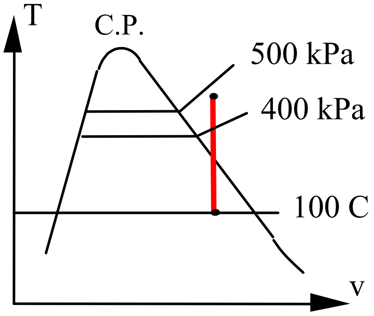
A sealed rigid vessel has volume of 1 m^3 and contains 2 kg of water at 100°C. The vessel is now heated. If a safety pressure valve is installed, at what pressure should the valve be set to have a maximum temperature of 200°C?

Solution:

Process: $v = V/m = \text{constant}$

State 1: $v_1 = 1/2 = 0.5\ m^3/kg$
from Table B.1.1
it is 2-phase

State 2: 200°C, 0.5 m^3/kg
Table B.1.3 between 400
and 500 kPa so interpolate



$$P \cong 400 + \frac{0.5 - 0.53422}{0.42492 - 0.53422} \times (500 - 400) = \mathbf{431.3\ kPa}$$

3.45

Saturated water vapor at 200 kPa is in a constant pressure piston cylinder. At this state the piston is 0.1 m from the cylinder bottom. How much is this distance and the temperature if the water is cooled to occupy half the original volume?

Solution:

State 1: B 1.2 $v_1 = v_g\,(200\text{ kPa}) = 0.8857\ \text{m}^3/\text{kg}$, $T_1 = 120.2°\text{C}$

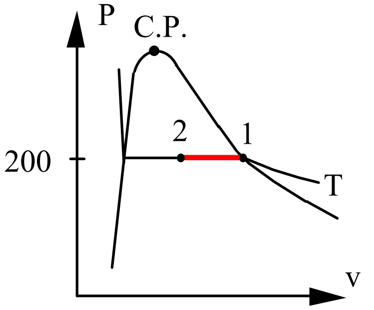
Process: $P = \text{constant} = 200\text{ kPa}$

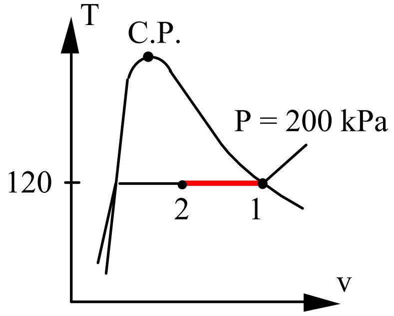
State 2: $P,\ v_2 = v_1/2 = 0.44285\ \text{m}^3/\text{kg}$

Table B.1.2 $v_2 < v_g$ so two phase $T_2 = T_{sat} = 120.2°\text{C}$

Height is proportional to volume

$h_2 = h_1 \times\ v_2/v_1 = 0.1 \times 0.5 = \mathbf{0.05m}$





3.46

Saturated liquid water at 60°C is put under pressure to decrease the volume by 1% keeping the temperature constant. To what pressure should it be compressed?

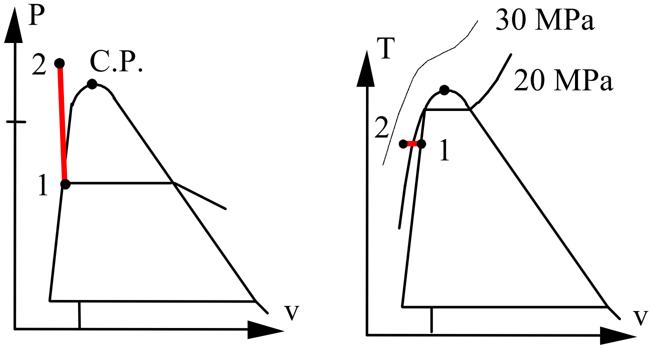
Solution:

State 1: T = 60°C , x = 0.0; Table B.1.1: $v = 0.001017 \text{ m}^3/\text{kg}$

Process: T = constant = 60°C

State 2: T, $v = 0.99 \times v_{f\,(60°C)} = 0.99 \times 0.001017 = 0.0010068 \text{ m}^3/\text{kg}$

Between 20 & 30 MPa in Table B.1.4, P ≅ **23.8 MPa**



3.47

Water at 200 kPa with a quality of 25% has its temperature raised 20°C in a constant pressure process. What is the new quality and volume?

Solution:

State 1 from Table B.1.2 at 200 kPa

$$v = v_f + x\, v_{fg} = 0.001061 + 0.25 \times 0.88467 = 0.22223 \text{ m}^3\text{/kg}$$

State 2 has same P from Table B.1.2 at 200 kPa

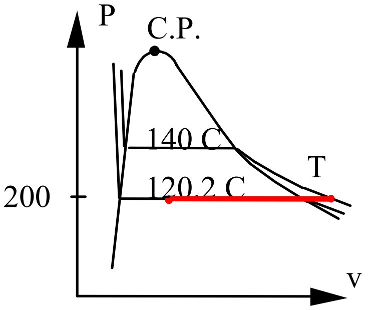
$$T_2 = T_{sat} + 20 = 120.23 + 20 = 140.23^{\circ}\text{C}$$

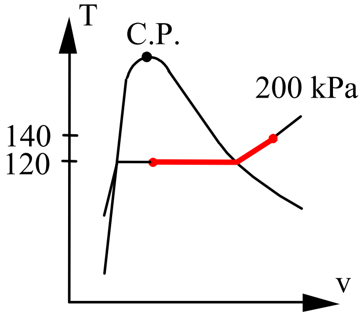
so state is superheated vapor

$$x = \textbf{undefined}$$

$$v_2 = 0.88573 + (0.95964 - 0.88573)\frac{20}{150 - 120.23} = \mathbf{0.9354\ m^3/kg}$$

Interpolate between the saturated vapor state (the first entry in table B.1.3 for 200 kPa) and the 150°C superheated vapor state also at 200 kPa.





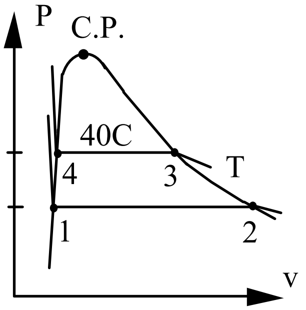
3.48

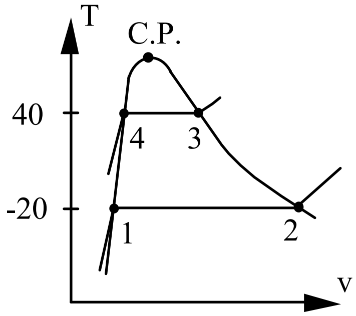
In your refrigerator the working substance evaporates from liquid to vapor at -20 ^oC inside a pipe around the cold section. Outside (on the back or below) is a black grille inside which the working substance condenses from vapor to liquid at +40 ^oC. For each location find the pressure and the change in specific volume (v) if the substance is ammonia.

Solution:

The properties come from the saturated tables where each phase change takes place at constant pressure and constant temperature.

Substance	TABLE	T	P_{sat} , kPa	$\Delta v = v_{fg}$, m^3/kg
Ammonia	B.2.1	40 ^oC	1555	0.0814
Ammonia	B.2.1	-20 ^oC	190	0.622





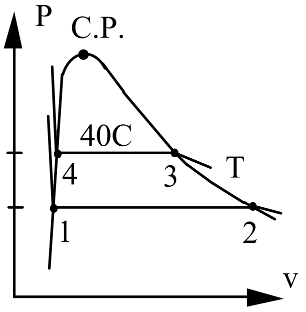
3.49

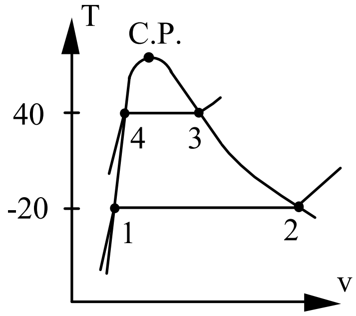
Repeat the previous problem with the substances
a) R-134a b) R-410a

In your refrigerator, the working substance evaporates from liquid to vapor at -20 ^oC inside a pipe around the cold section. Outside (on the back or below) is a black grille inside which the working substance condenses from vapor to liquid at +40 ^oC. For each location find the pressure and the change in specific volume (v).

Solution:
The properties come from the saturated tables where each phase change takes place at constant pressure and constant temperature.

Substance	TABLE	T	P_{sat} , kPa	$\Delta v = v_{fg}$, m^3/kg
R-134a	B.5.1	40 ^oC	1017	0.019
R-134a	B.5.1	-20 ^oC	134	0.146
R-410a	B.4.1	40 ^oC	2421	0.00865
R-410a	B.4.1	-20 ^oC	400	0.064



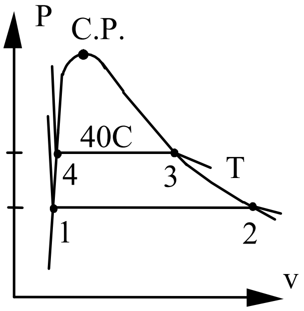


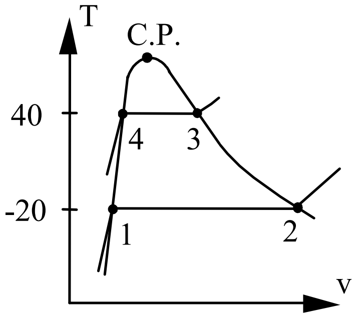
3.50

Repeat Problem 3.48 with CO_2, condenser at + 20°C and evaporator at -30°C.

Solution:
The properties come from the saturated tables where each phase change takes place at constant pressure and constant temperature.

Substance	TABLE	T	P_{sat} , kPa	$\Delta v = v_{fg}$
CO_2	B.3.1	20 °C	5729	0.00386
CO_2	B.3.1	-30 °C	1428	0.026





3.51

A glass jar is filled with saturated water at 500 kPa, quality 25%, and a tight lid is put on. Now it is cooled to −10°C. What is the mass fraction of solid at this temperature?

Solution:

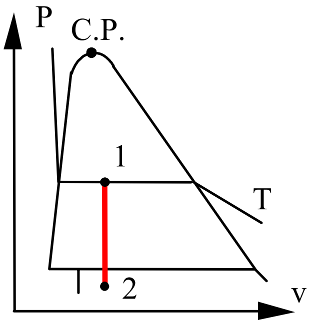
Constant volume and mass $\Rightarrow$ $v_1 = v_2 = V/m$

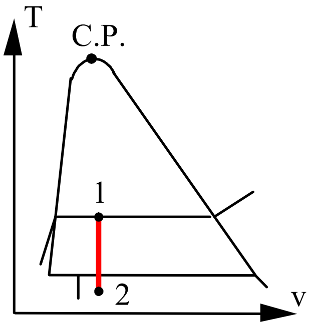
From Table B.1.2: $v_1 = 0.001093 + 0.25 \times 0.3738 = 0.094543$

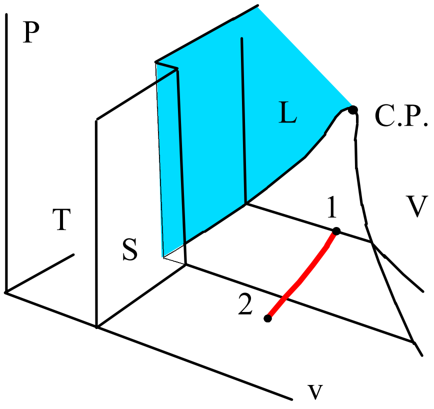
From Table B.1.5: $v_2 = 0.0010891 + x_2 \times 446.756 = v_1 = 0.094543$

$\Rightarrow$ $x_2 = 0.0002$ mass fraction vapor

$x_{solid} = 1 - x_2 = 0.9998$ or **99.98 %**





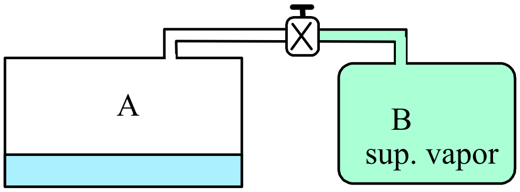


3.52

Two tanks are connected as shown in Fig. P3.52, both containing water. Tank A is at 200 kPa, $v = 0.5$ m^3/kg, $V_A = 1$ m^3 and tank B contains 3.5 kg at 0.5 MPa, 400°C. The valve is now opened and the two come to a uniform state. Find the final specific volume.

Solution:

Control volume: both tanks. Constant total volume and mass process.



State A1: (P, v) $m_A = V_A/v_A = 1/0.5 = 2$ kg

State B1: (P, T) Table B.1.3 $v_B = 0.6173$ m^3/kg

$$\Rightarrow V_B = m_B v_B = 3.5 \times 0.6173 = 2.1606 \text{ m}^3$$

Final state: $m_{tot} = m_A + m_B = 5.5$ kg

$$V_{tot} = V_A + V_B = 3.1606 \text{ m}^3$$

$$v_2 = V_{tot}/m_{tot} = \mathbf{0.5746\ m^3/kg}$$

3.53

Saturated vapor R-134a at 50^oC changes volume at constant temperature. Find the new pressure, and quality if saturated, if the volume doubles. Repeat the question for the case the volume is reduced to half the original volume.

Solution:

1: (T, x) B.4.1: $v_1 = v_g = 0.01512 \ m^3/kg$, $P_1 = P_{sat} = 1318$ kPa

2: $v_2 = 2v_1 = 0.03024 \ m^3/kg$ superheated vapor

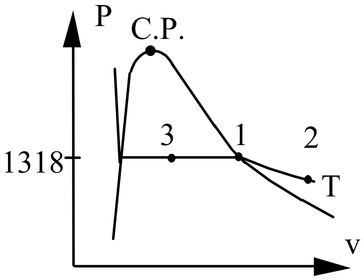
Interpolate between 600 kPa and 800 kPa

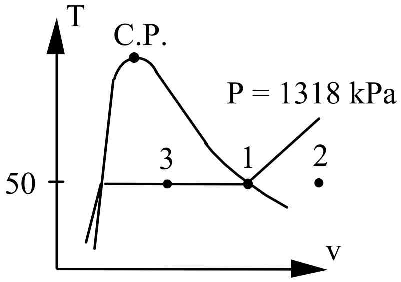
$$P_2 = 600 + 200 \times \frac{0.03024 - 0.03974}{0.02861 - 0.03974} = \mathbf{771 \ kPa}$$

3: $v_3 = v_1/2 = 0.00756 \ m^3/kg < v_g$: two phase

$$x_3 = \frac{v_3 - v_f}{v_{fg}} = \frac{0.00756 - 0.000908}{0.01422} = \mathbf{0.4678}$$

$P_3 = P_{sat} = \mathbf{1318 \ kPa}$



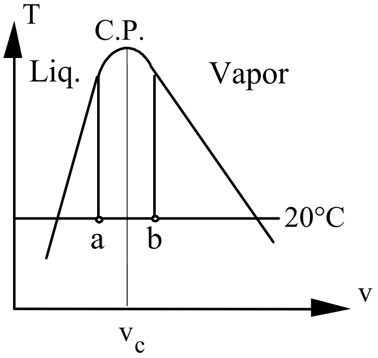


3.54

A steel tank contains 6 kg of propane (liquid + vapor) at 20°C with a volume of 0.015 m^3. The tank is now slowly heated. Will the liquid level inside eventually rise to the top or drop to the bottom of the tank? What if the initial mass is 1 kg instead of 6 kg?

Solution:

Constant volume and mass $v_2 = v_1 = \frac{V}{m} = \frac{0.015 \text{ m}^3}{6 \text{ kg}} = 0.0025 \text{ m}^3\text{/kg}$



A.2: $v_c = 0.00454 \text{ m}^3\text{/kg} > v_1$
eventually reaches sat. liquid.
⇒ **level rises to top**

If m = 1 kg ⇒ $v_1 = 0.015 \text{ m}^3\text{/kg} > v_c$
then it will reach saturated vapor.
⇒ **level falls**

3.55

Saturated water vapor at 60°C has its pressure decreased to increase the volume by 10% keeping the temperature constant. To what pressure should it be expanded?

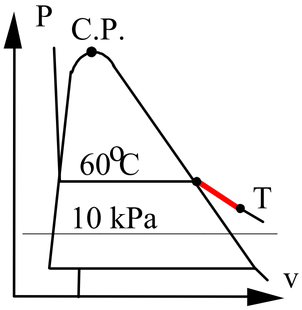
Solution:

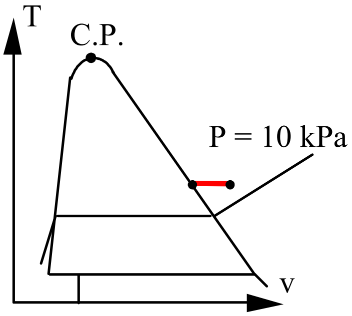
Initial state: $v = 7.6707 \text{ m}^3/\text{kg}$ from table B.1.1

Final state: $v = 1.10 \times v_g = 1.1 \times 7.6707 = 8.4378 \text{ m}^3/\text{kg}$

Interpolate at 60°C between saturated (P = 19.94 kPa) and superheated vapor P = 10 kPa in Tables B.1.1 and B.1.3

$$P \cong 19.941 + (10 - 19.941)\,\frac{8.4378 - 7.6707}{15.3345 - 7.6707} = \mathbf{18.9\ kPa}$$





Comment: T,v ⇒ P = 18 kPa (software) v is not linear in P, more like 1/P, so the linear interpolation in P is not very accurate.

3.56

Ammonia at 20°C with a quality of 50% and total mass 2 kg is in a rigid tank with an outlet valve at the bottom. How much liquid (mass) can you take out through the valve assuming the temperature stays constant?

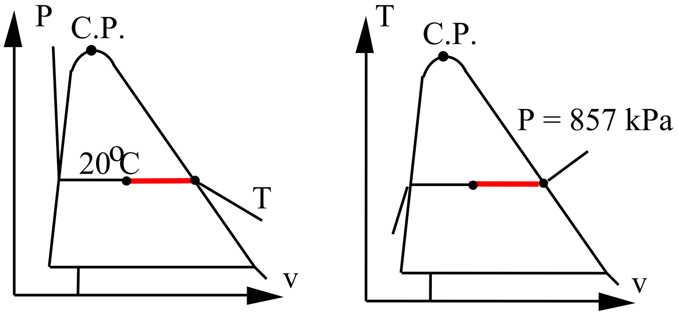
Solution:

The bottom has liquid until the state inside becomes saturated vapor.

$$V = m_1 v_1 = 2 \times 0.5\ (0.001638 + 0.14922) = 0.15086\ \text{m}^3$$

$$m_2 = V/v_2 = 0.15086\ \text{m}^3/\ 0.14922\ \text{m}^3/\text{kg} = 1.0198\ \text{kg}$$

$$m = m_1 - m_2 = 2 - 1.0198 = \mathbf{0.989\ kg}$$

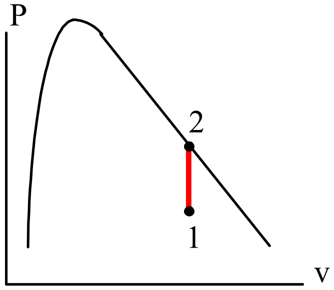


3.57

A sealed rigid vessel of 2 m^3 contains a saturated mixture of liquid and vapor R-134a at 10°C. If it is heated to 50°C, the liquid phase disappears. Find the pressure at 50°C and the initial mass of the liquid.

Solution:

Process: constant volume and constant mass.



State 2 is saturated vapor, from table B.5.1

$P_2 = P_{sat}(50°C) = \mathbf{1.318\ MPa}$

State 1: same specific volume as state 2

$v_1 = v_2 = 0.015124\ m^3/kg$

$v_1 = 0.000794 + x_1 \times 0.048658$

$\Rightarrow\ x_1 = 0.2945$

$m = V/v_1 = 2/0.015124 = 132.24$ kg; $m_{liq} = (1 - x_1)m = \mathbf{93.295\ kg}$

3.58

A storage tank holds methane at 120 K, with a quality of 25 %, and it warms up by 5°C per hour due to a failure in the refrigeration system. How long time will it take before the methane becomes single phase and what is the pressure then?

Solution: Use Table B.7.1

Assume rigid tank $v = \text{constant} = v_1$

$$v_1 = 0.002439 + 0.25 \times 0.30367 = 0.078366 \;\; \mathrm{m^3/kg}$$

We then also see that $v_1 > v_c = 0.00615 \;\; \mathrm{m^3/kg}$

All single phase when $v = v_g \;\; => \;\; T \cong 145 \text{ K}$

$$\Delta t = \frac{\Delta T}{(5°\mathrm{C/h})} \cong \frac{145 - 120}{5} = \mathbf{5\ hours} \qquad P = P_{sat} = \mathbf{824\ kPa}$$

3.59

Ammonia at 10°C and mass 0.1 kg is in a piston cylinder with an initial volume of 1 m^3. The piston initially resting on the stops has a mass such that a pressure of 900 kPa will float it. Now the ammonia is slowly heated to 50°C. Find the final pressure and volume.

Solution:

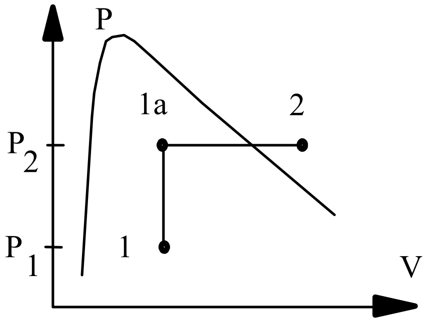
C.V. Ammonia, constant mass.

Process: $V = \text{constant}$ unless $P = P_{float}$

State 1: $T = 10\ ^oC,\quad v_1 = \frac{V}{m} = \frac{1}{10} = 0.1\ m^3/kg$

From Table B.2.1 $v_f < v < v_g$

$$x_1 = \frac{v - v_f}{v_{fg}} = \frac{0.1 - 0.0016}{0.20381} = 0.4828$$



State 1a: $P = 900$ kPa, $v = v_1 = 0.1\ m^3/kg < v_g$ at 900 kPa

This state is two-phase $T_{1a} = 21.52^oC$

Since $T_2 > T_{1a}$ then $v_2 > v_{1a}$

State 2: 50°C and on line(s) means

$P_2 =$ **900 kPa** which is superheated vapor.

Table B.2.2 : $v_2 = 0.16263\ m^3/kg$

$V_2 = mv_2 = \mathbf{1.6263\ m^3}$

3.60

A 400-m^3 storage tank is being constructed to hold LNG, liquified natural gas, which may be assumed to be essentially pure methane. If the tank is to contain 90% liquid and 10% vapor, by volume, at 100 kPa, what mass of LNG (kg) will the tank hold? What is the quality in the tank?

Solution:

CH_4 is in the section B tables.

From Table B.7.1: $v_f \cong 0.002366\ m^3/kg$, (interpolated)

From Table B.7.2: $v_g \cong 0.55665\ m^3/kg$ (first entry 100 kPa)

$$m_{liq} = \frac{V_{liq}}{v_f} = \frac{0.9 \times 400}{0.002366} = 152\ 155.5\ \text{kg}; \quad m_{vap} = \frac{V_{vap}}{v_g} = \frac{0.1 \times 400}{0.55665} = 71.86\ \text{kg}$$

$$m_{tot} = \mathbf{152\ 227\ kg}, \qquad x = m_{vap} / m_{tot} = \mathbf{4.72 \times 10^{-4}}$$

3.61

A boiler feed pump delivers 0.05 m^3/s of water at 240°C, 20 MPa. What is the mass flowrate (kg/s)? What would be the percent error if the properties of saturated liquid at 240°C were used in the calculation? What if the properties of saturated liquid at 20 MPa were used?

Solution:

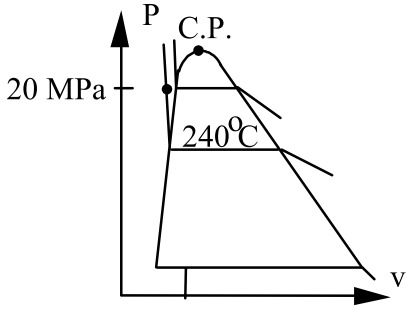
State 1: (T, P) compressed liquid seen in B.1.4: $v = 0.001205\ m^3/kg$

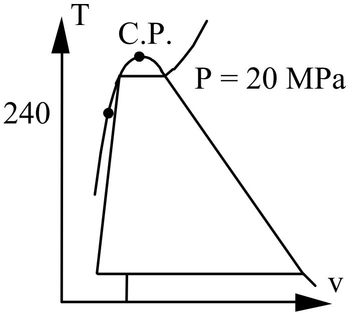
$\dot{m} = \dot{V}/v = 0.05/0.001205 =$ **41.5 kg/s**

$v_{f\,(240°C)} = 0.001229\ m^3/kg \Rightarrow \dot{m} = 40.68$ kg/s **error 2%**

$v_{f\,(20\ MPa)} = 0.002036\ m^3/kg \Rightarrow \dot{m} = 24.56$ kg/s **error 41%**

$v_{f\,(20\ MPa)} = 0.002036\ m^3/kg \Rightarrow \dot{m} = 24.56$ kg/s **error 41%**



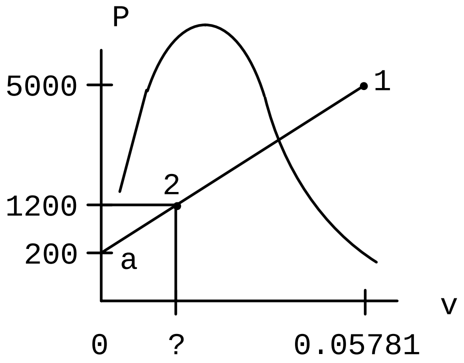


The constant T line is nearly vertical for the liquid phase in the P-v diagram. The state is at so high P, T that the saturated liquid line is not extremely steep.

3.62

A piston/cylinder arrangement is loaded with a linear spring and the outside atmosphere. It contains water at 5 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that P_{lift} = 200 kPa. The system now cools until the pressure reaches 1200 kPa. Find the mass of water, the final state (T_2, v_2) and plot the P–v diagram for the process.

Solution:



1: Table B.1.3 $\Rightarrow$ $v_1 = 0.05781\ m^3/kg$

$m = V/v_1 = 0.1/0.05781 = 1.73\ kg$

Straight line: $P = P_a + C \times v$

$$v_2 = v_1 \frac{P_2 - P_a}{P_1 - P_a} = \mathbf{0.01204\ m^3/kg}$$

$v_2 < v_g(1200\ kPa)$ so two-phase T_2 = **188°C**

$\Rightarrow x_2 = (v_2 - 0.001139)/0.1622 = 0.0672$

3.63

A pressure cooker (closed tank) contains water at 100°C with the liquid volume being 1/10 of the vapor volume. It is heated until the pressure reaches 2.0 MPa. Find the final temperature. Has the final state more or less vapor than the initial state?

Solution:

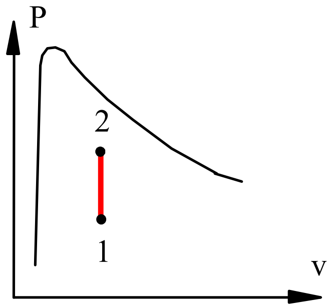
State 1: $V_f = m_f v_f = V_g/10 = m_g v_g/10$;

Table B.1.1: $v_f = 0.001044 \text{ m}^3/\text{kg}$, $v_g = 1.6729 \text{ m}^3/\text{kg}$

$$x_1 = \frac{m_g}{m_g + m_f} = \frac{10\ m_f v_f / v_g}{m_f + 10\ m_f v_f / v_g} = \frac{10\ v_f}{10\ v_f + v_g} = \frac{0.01044}{0.01044 + 1.6729} = 0.0062$$

$v_1 = 0.001044 + 0.0062 \times 1.67185 = 0.01141 \text{ m}^3/\text{kg}$

State 2: $v_2 = v_1 = 0.01141 \text{ m}^3/\text{kg} < v_g(2\text{MPa})$ from B.1.2 so two-phase



At state 2: $v_2 = v_f + x_2\ v_{fg}$

$0.01141 = 0.001177 + x_2 \times 0.09845$

$\Rightarrow \quad x_2 = 0.104$

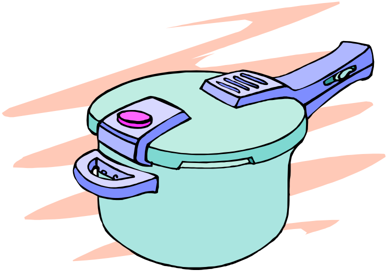
More vapor at final state

$T_2 = T_{sat}(2\text{MPa}) =$ **212.4°C**

3.64

A pressure cooker has the lid screwed on tight. A small opening with $A = 5\ \mathrm{mm}^2$ is covered with a petcock that can be lifted to let steam escape. How much mass should the petcock have to allow boiling at 120°C with an outside atmosphere at 101.3 kPa?

Table B.1.1.: $P_{sat} = 198.5\ \mathrm{kPa}$



$$F = mg = \Delta P \times A$$

$$m = \Delta P \times A/g$$

$$= \frac{(198.5\text{-}101.3)\times 1000\times 5\times 10^{-6}}{9.807}$$

$$= 0.0496\ \mathrm{kg} = \mathbf{50\ g}$$

Ideal Gas Law

3.65

What is the relative (%) change in P if we double the absolute temperature of an ideal gas keeping mass and volume constant? What will it be if we double V having m, T constant.

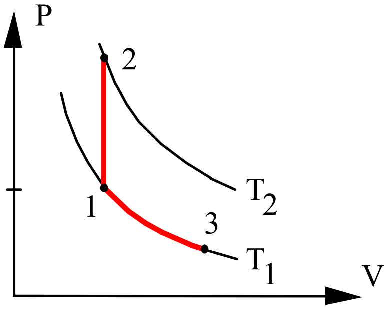
Ideal gas law: $PV = mRT$

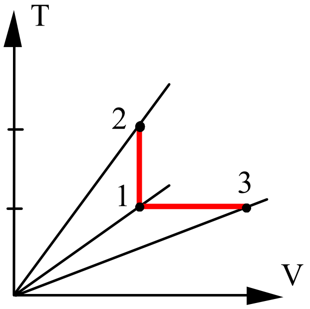
State 2: $P_2V = mRT_2 = mR2T_1 = 2P_1V \Rightarrow P_2 = 2P_1$

Relative change = $\Delta P/P_1 = P_1/P_1 = 1 =$ **100%**

State 3: $P_3V_3 = mRT_1 = P_1V_1 \Rightarrow P_3 = P_1V_1/V_3 = P_1/2$

Relative change = $\Delta P/P_1 = -P_1/2P_1 = -0.5 =$ **-50%**





3.66

A 1-m^3 tank is filled with a gas at room temperature 20°C and pressure 100 kPa. How much mass is there if the gas is a) air, b) neon or c) propane ?

Solution:

Use Table A.2 to compare T and P to the critical T and P with

$T = 20°C = 293.15$ K ; $P = 100$ kPa $<< P_c$ for all

Air : $T >> T_{C,N2}$; $T_{C,O2} = 154.6$ K so ideal gas; R= 0.287 kJ/kg K

Neon: $T >> T_c = 44.4$ K so ideal gas; R = 0.41195 kJ/kg K

Propane: $T < T_c = 370$ K, but $P << P_c = 4.25$ MPa

so gas R = 0.18855 kJ/kg K

All states are ideal gas states so the ideal gas law applies

$$PV = mRT$$

$$\text{a)}\quad m = \frac{PV}{RT} = \frac{100 \times 1}{0.287 \times 293.15} = \mathbf{1.189\ kg}$$

$$\text{b)}\quad m = \frac{100 \times 1}{0.41195 \times 293.15} = \mathbf{0.828\ kg}$$

$$\text{c)}\quad m = \frac{100 \times 1}{0.18855 \times 293.15} = \mathbf{1.809\ kg}$$

3.67

Calculate the ideal gas constant for argon and hydrogen based on table A.2 and verify the value with Table A.5

argon: $R = \bar{R} / M = 8.3145 / 39.948 = 0.2081$ same as Table A.5

hydrogen: $R = \bar{R} / M = 8.3145 / 2.016 = 4.124256$ same as Table A.5

3.68

A pneumatic cylinder (a piston cylinder with air) must close a door with a force of 500 N. The cylinder cross-sectional area is 5 cm^2 and its volume is 50 cm^3. What is the air pressure and its mass?

$$F = PA - P_o A \Rightarrow$$

$$P = P_o + F/A = 100 \text{ kPa} + \frac{500 \text{ N}}{0.0005 \text{ m}^2 \times 1000 \text{ N/kN}} = \mathbf{1100\ kPa}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{1100 \text{ kPa} \times 0.00005 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 298 \text{ K}} = \mathbf{0.00064\ kg = 0.64\ g}$$

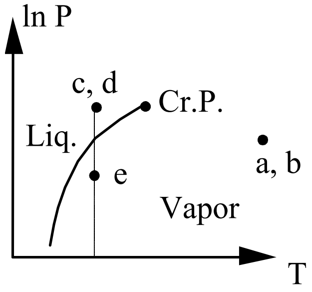
Comment: Dependent upon your understanding of the problem you could also have neglected the atmospheric pressure to get 1000 kPa and 0.58 g for answers.

3.69

Is it reasonable to assume that at the given states the substance behaves as an ideal gas?

Solution:

a) Oxygen, O_2	at	30°C, 3 MPa	**Ideal Gas** ($T \gg T_c = 155$ K from A.2)
b) Methane, CH_4	at	30°C, 3 MPa	**Ideal Gas** ($T \gg T_c = 190$ K from A.2)
c) Water, H_2O	at	30°C, 3 MPa	**NO** compressed liquid $P > P_{sat}$ (B.1.1)
d) R-134a	at	30°C, 3 MPa	**NO** compressed liquid $P > P_{sat}$ (B.5.1)
e) R-134a	at	30°C, 100 kPa	**Ideal Gas** P is low $< P_{sat}$ (B.5.1)



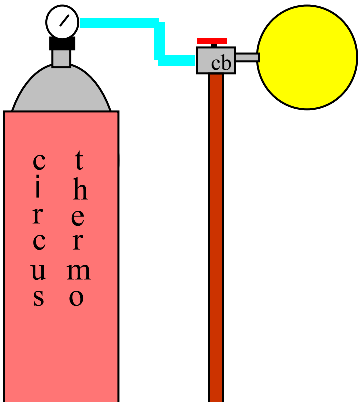
3.70

Helium in a steel tank is at 250 kPa, 300 K with a volume of 0.1 m^3. It is used to fill a balloon. When the pressure drops to 150 kPa, the flow of helium stops by itself. If all the helium still is at 300 K, how big a balloon did I get?

Solution:

State 1: $m = V/v$ assume ideal gas so

$$m = \frac{P_1V_1}{RT_1} = \frac{250 \times 0.1}{2.0771 \times 300} = 0.0401 \text{ kg}$$



State 2: Same mass so then ($T_2 = T_1$)

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1V_1\,RT_2}{RT_1\,P_2} = V_1\,\frac{P_1}{P_2}$$

$$= 0.1\,\frac{250}{150} = 0.16667 \text{ m}^3$$

The balloon volume is

$$V_{balloon} = V_2 - V_1 = 0.16667 - 0.1 = \mathbf{0.06667\ m^3}$$

3.71

A hollow metal sphere of 150-mm inside diameter is weighed on a precision beam balance when evacuated and again after being filled to 875 kPa with an unknown gas. The difference in mass is 0.0025 kg, and the temperature is 25°C. What is the gas, assuming it is a pure substance listed in Table A.5 ?

Solution:

Assume an ideal gas with total volume: $V = \frac{\pi}{6}(0.15)^3 = 0.001767 \text{ m}^3$

$$M = \frac{m\bar{R}T}{PV} = \frac{0.0025 \times 8.3145 \times 298.2}{875 \times 0.001767} = \mathbf{4.009} \approx M_{\text{He}}$$

=> **Helium Gas**

3.72

A spherical helium balloon of 10 m in diameter is at ambient T and P, 15^{o}C and 100 kPa. How much helium does it contain? It can lift a total mass that equals the mass of displaced atmospheric air. How much mass of the balloon fabric and cage can then be lifted?

We need to find the masses and the balloon volume

$$V = \frac{\pi}{6} D^3 = \frac{\pi}{6} 10^3 = 523.6 \text{ m}^3$$

$$m_{He} = \rho V = \frac{V}{v} = \frac{PV}{RT} = \frac{100 \text{ kPa} \times 523.6 \text{ m}^3}{2.0771 \text{ kJ/kgK} \times 288 \text{ K}} = \mathbf{87.5\ kg}$$

$$m_{air} = \frac{PV}{RT} = \frac{100 \text{ kPa} \times 523.6 \text{ m}^3}{0.287 \text{ kJ/kgK} \times 288 \text{ K}} = 633 \text{ kg}$$

$$m_{lift} = m_{air} - m_{He} = 633\text{-}87.5 = \mathbf{545.5\ kg}$$

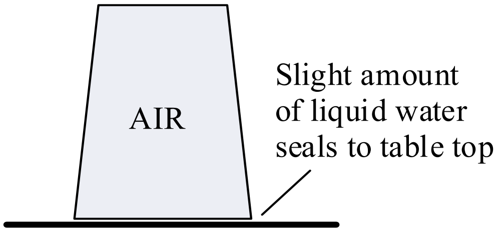
3.73

A glass is cleaned in 45^{o}C hot water and placed on the table bottom up. The room air at 20^{o}C that was trapped in the glass gets heated up to 40^{o}C and some of it leaks out so the net resulting pressure inside is 2 kPa above ambient pressure of 101 kPa. Now the glass and the air inside cools down to room temperature. What is the pressure inside the glass?

Solution:

1 air: 40^{o}C, 103 kPa
2 air: 20^{o}C, ?

Constant Volume: $V_1 = V_2$,



Constant Mass $m_1 = m_2$

Ideal Gas $P_1V_1 = m_1RT_1$ and $P_2V_2 = m_1RT_2$

Take Ratio

$$P_2 = P_1 \frac{T_1}{T_2} = 103 \times \frac{20 + 273}{40 + 273} = \mathbf{96.4\ kPa}$$

This is a vacuum relative to atm pressure so the glass is pressed against table.

3.74

Air in an internal combustion engine has 227^{o}C, 1000 kPa with a volume of 0.1 m^3. Now combustion heats it to 1500 K in a constant volume process. What is the mass of air and how high does the pressure become?

The mass comes from knowledge of state 1 and ideal gas law

$$m = \frac{P_1V_1}{RT_1} = \frac{1000 \text{ kPa} \times 0.1 \text{ m}^3}{0.287 \text{ kJ/kgK} \times (227 + 273) \text{ K}} = \mathbf{0.697\ kg}$$

The final pressure is found from the ideal gas law written for state 1 and state 2 and then eliminate the mass, gas constant and volume ($V_2 = V_1$) between the equations

$$P_1 V_1 = m RT_1 \qquad \text{and} \qquad P_2 V_2 = m RT_2$$

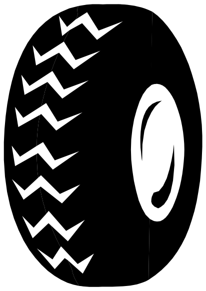
$$P_2 = P_1 \times T_2/T_1 = 1000 \times \frac{1500}{500} = \mathbf{3000\ kPa}$$

3.75

Air in an automobile tire is initially at −10°C and 190 kPa. After the automobile is driven awhile, the temperature gets up to 10°C. Find the new pressure. You must make one assumption on your own.

Solution:

Assume constant volume and that air is an ideal gas



$$P_2 = P_1 \times T_2/T_1$$

$$= 190 \times \frac{283.15}{263.15} = \mathbf{204.4\ kPa}$$

3.76

A rigid tank of 1 m^3 contains nitrogen gas at 600 kPa, 400 K. By mistake someone lets 0.5 kg flow out. If the final temperature is 375 K what is then the final pressure?

Solution:

$$m = \frac{PV}{RT} = \frac{600 \times 1}{0.2968 \times 400} = 5.054 \text{ kg}$$

$$m_2 = m - 0.5 = 4.554 \text{ kg}$$

$$P_2 = \frac{m_2RT_2}{V} = \frac{4.554 \times 0.2968 \times 375}{1} = \mathbf{506.9\ kPa}$$

3.77

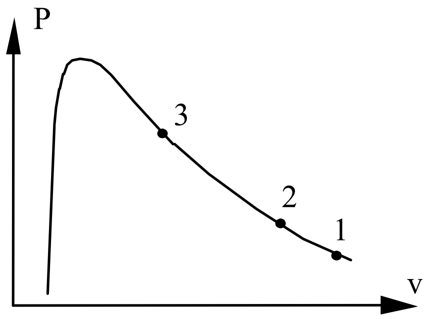
Assume we have 3 states of saturated vapor R-134a at +40 ^oC, 0 ^oC and -40 ^oC. Calculate the specific volume at the set of temperatures and corresponding saturated pressure assuming ideal gas behavior. Find the percent relative error = $100(v - v_g)/v_g$ with v_g from the saturated R-134a table.

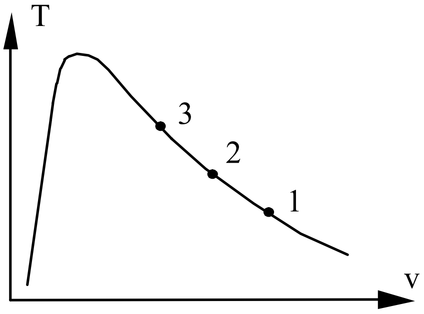
Solution:

R-134a. Table values from Table B.5.1 P_{sat}, $v_g(T)$

Ideal gas constant from Table A.5: $R_{R\text{-}134a}$ = 0.08149 kJ/kg K

T	P_{sat} , kPa	v_g	$v_{ID.G.}$ = RT / P_{sat}	error %
-40 ^oC	51.8	0.35696	0.36678	2.75
0 ^oC	294	0.06919	0.07571	9.4
40 ^oC	1017	0.02002	0.02509	25.3





3.78

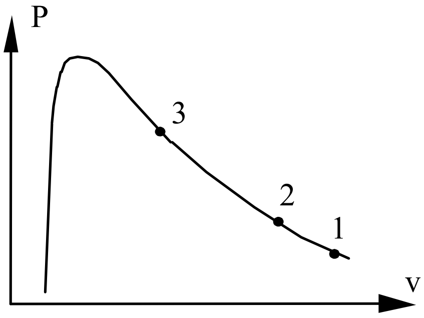
Do Problem 3.77, but for the substance R-410a.

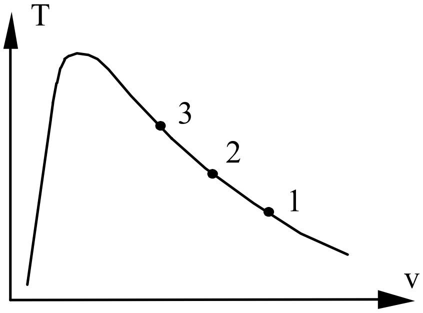
Solution:

R-410a. Table values from Table B.4.1 P_{sat}, $v_g(T)$

Ideal gas constant from Table A.5: $R_{R\text{-}410a} = 0.1146$ kJ/kg K

T	P_{sat} , kPa	v_g	$v_{ID.G.} = RT / P_{sat}$	error %
-40 °C	175.0	0.14291	0.1527	6.8
0 °C	798.7	0.03267	0.03919	20
40 °C	2420.7	0.00967	0.01483	53.3





3.79

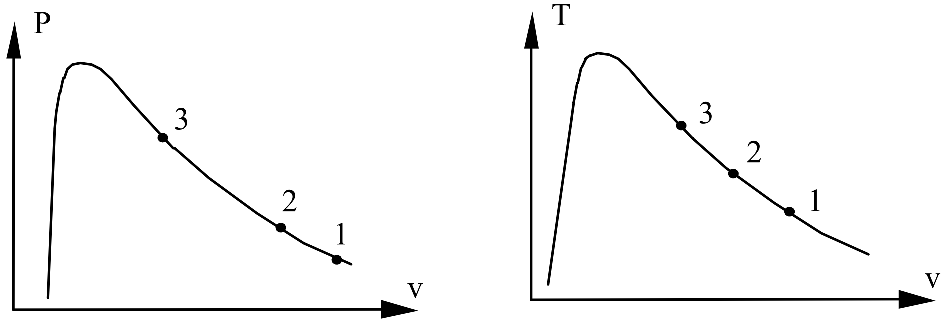
Do Problem 3.77, but for the substance ammonia.

Solution:

NH3. Table values from Table B.2.1 P_{sat}, $v_g(T)$

Ideal gas constant from Table A.5: $R_{ammonia} = 0.4882$ kJ/kg K

T	P_{sat} , kPa	v_g	$v_{ID.G.} = RT / P_{sat}$	error %
-40 °C	71.7	1.5526	1.5875	2.25
0 °C	429.6	0.28929	0.3104	7.3
40 °C	1555	0.08313	0.09832	18.3



3.80

A 1 m^3 rigid tank has propane at 100 kPa, 300 K and connected by a valve to another tank of 0.5 m^3 with propane at 250 kPa, 400 K. The valve is opened and the two tanks come to a uniform state at 325 K. What is the final pressure?

Solution:

Propane is an ideal gas ($P << P_c$) with R = 0.1886 kJ/kgK from Tbl. A.5

$$m_A = \frac{P_A V_A}{R T_A} = \frac{100 \times 1}{0.1886 \times 300} = 1.7674 \text{ kg}$$

$$m_B = \frac{P_B V_B}{R T_B} = \frac{250 \times 0.5}{0.1886 \times 400} = 1.6564 \text{ kg}$$

$$V_2 = V_A + V_B = 1.5 \text{ m}^3$$

$$m_2 = m_A + m_B = 3.4243 \text{ kg}$$

$$P_2 = \frac{m_2 R T_2}{V_2} = \frac{3.4243 \times 0.1886 \times 325}{1.5} = \mathbf{139.9 \text{ kPa}}$$

3.81

A vacuum pump is used to evacuate a chamber where some specimens are dried at 50°C. The pump rate of volume displacement is 0.5 m^3/s with an inlet pressure of 0.1 kPa and temperature 50°C. How much water vapor has been removed over a 30-min period?

Solution:

Use ideal gas since P << lowest P in steam tables.

From table A.5 we get R = 0.46152 kJ/kg K

$m = \dot{m}\,\Delta t$ with mass flow rate as: $\dot{m} = \dot{V}/v = P\dot{V}/RT$ (ideal gas)

$$\Rightarrow \quad m = P\dot{V}\Delta t/RT = \frac{0.1 \times 0.5 \times 30 \times 60}{(0.46152 \times 323.15)} = \mathbf{0.603\ kg}$$

3.82

A 1-m^3 rigid tank with air at 1 MPa, 400 K is connected to an air line as shown in Fig. P3.82. The valve is opened and air flows into the tank until the pressure reaches 5 MPa, at which point the valve is closed and the temperature inside is 450K.

a. What is the mass of air in the tank before and after the process?

b. The tank eventually cools to room temperature, 300 K. What is the pressure inside the tank then?

Solution:

P, T known at both states and assume the air behaves as an ideal gas.

$$m_{air1} = \frac{P_1V}{RT_1} = \frac{1000 \times 1}{0.287 \times 400} = \mathbf{8.711\ kg}$$

$$m_{air2} = \frac{P_2V}{RT_2} = \frac{5000 \times 1}{0.287 \times 450} = \mathbf{38.715\ kg}$$

Process 2 → 3 is constant V, constant mass cooling to T_3

$$P_3 = P_2 \times (T_3/T_2) = 5000 \times (300/450) = \mathbf{3.33\ MPa}$$

3.83

A cylindrical gas tank 1 m long, inside diameter of 20 cm, is evacuated and then filled with carbon dioxide gas at 20°C. To what pressure should it be charged if there should be 1.2 kg of carbon dioxide?

Solution:

Assume CO_2 is an ideal gas, table A.5: $R = 0.1889 \text{ kJ/kg K}$

$$V_{cyl} = A \times L = \frac{\pi}{4}(0.2)^2 \times 1 = 0.031416 \text{ m}^3$$

$$P V = mRT \qquad => \qquad P = \frac{mRT}{V}$$

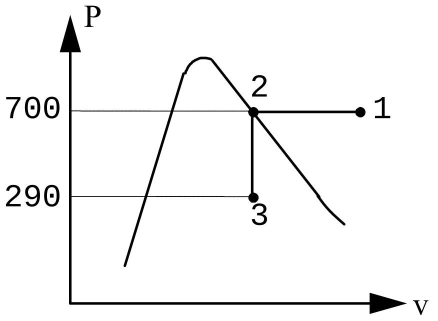
$$\Rightarrow P = \frac{1.2 \text{ kg} \times 0.1889 \text{ kJ/kg K} \times (273.15 + 20) \text{ K}}{0.031416 \text{ m}^3} = \mathbf{2115\ kPa}$$

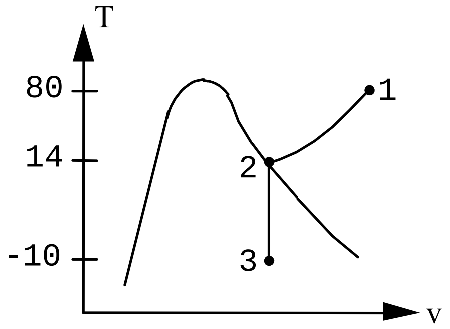
3.84

Ammonia in a piston/cylinder arrangement is at 700 kPa, 80°C. It is now cooled at constant pressure to saturated vapor (state 2) at which point the piston is locked with a pin. The cooling continues to −10°C (state 3). Show the processes 1 to 2 and 2 to 3 on both a *P–v* and *T–v* diagram.

Solution:

State 1: T, P from table B.2.2 this is superheated vapor.
State 2: T, x from table B.2.1
State 3: T, v two-phase





Compressibility Factor

3.85

How close to ideal gas behavior (find Z) is ammonia at saturated vapor, 100 kPa? How about saturated vapor at 2000 kPa?

Table B.2.2: $v_1 = 1.1381 \text{ m}^3\text{/kg}$, $T_1 = -33.6^{o}\text{C}$, $P_1 = 100 \text{ kPa}$

$v_2 = 0.06444 \text{ m}^3\text{/kg}$, $T_2 = 49.37^{o}\text{C}$, $P_2 = 2000 \text{ kPa}$

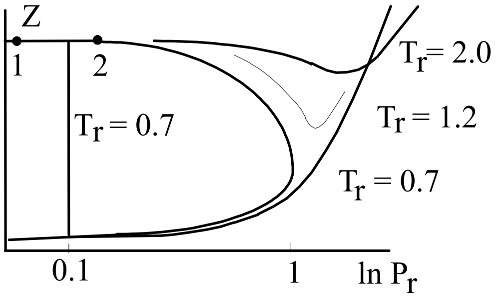
Table A.5: $R = 0.4882 \text{ kJ/kg K}$

Extended gas law: $Pv = ZRT$ so we can calculate Z from this

$$Z_1 = \frac{P_1 v_1}{RT_1} = \frac{100 \times 1.1381}{0.4882 \times (273.15 - 33.6)} = 0.973$$

$$Z_2 = \frac{P_2 v_2}{RT_2} = \frac{2000 \times 0.06444}{0.4882 \times (273.15 + 49.37)} = 0.8185$$

So state 1 is close to ideal gas and state 2 is not so close.



3.86

Carbon dioxide at 60$^{\mathrm{o}}$C is pumped at a very high pressure 10 MPa into an oil well to reduce the oil viscosity for better oil flow. What is its compressibility?

Table B.3.2: $v = 0.00345\ \mathrm{m^3/kg}$

$$Z = Pv/RT = \frac{10\,000\ \mathrm{kPa} \times 0.00345\ \mathrm{m^3/kg}}{0.1889\ \mathrm{kJ/kg\text{-}K} \times 333\ \mathrm{K}} = \mathbf{0.55}$$

3.87

Find the compressibility for carbon dioxide at 60^{o}C and 10 MPa using Fig. D.1

Solution:

Table A.2 CO2: $T_c = 304.1$ K $P_c = 7.38$ MPa

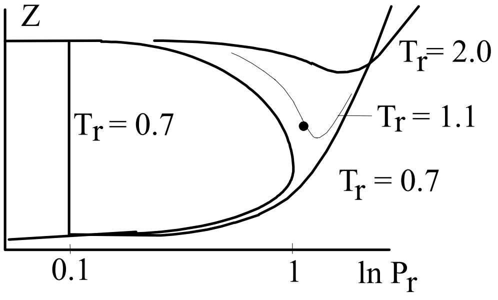
$T_r = T/T_c = 333/304.1 = 1.095$

$P_r = P/P_c = 10/7.38 = 1.355$

From Figure D.1: $\mathbf{Z \approx 0.45}$

Compare with table B.3.2: $v = 0.00345$ m^3/kg

$$Z = Pv/RT = \frac{10\ 000 \text{ kPa} \times 0.00345 \text{ m}^3\text{/kg}}{0.1889 \text{ kJ/kg-K} \times 333 \text{ K}} = 0.55$$



3.88

What is the percent error in specific volume if the ideal gas model is used to represent the behavior of superheated ammonia at 40°C, 500 kPa? What if the generalized compressibility chart, Fig. D.1, is used instead?

Solution:

NH_3 $T = 40°C = 313.15\text{ K},\ T_c = 405.5\text{ K},\ P_c = 11.35\text{ MPa}$ from Table A.1

Table B.2.2: $v = \mathbf{0.2923\ m^3/kg}$

Ideal gas: $v = \frac{RT}{P} = \frac{0.48819 \times 313}{500} = \mathbf{0.3056\ m^3/kg} \quad \Rightarrow \ \mathbf{4.5\%\ error}$

Figure D.1: $T_r = \frac{313.15}{405.5} = 0.772, \quad P_r = \frac{0.5}{11.35} = 0.044 \quad \Rightarrow\ Z = \mathbf{0.97}$

$$v = \frac{ZRT}{P} = \mathbf{0.2964\ m^3/kg} \quad \Rightarrow\ \mathbf{1.4\%\ error}$$

3.89

A cylinder fitted with a frictionless piston contains butane at 25°C, 500 kPa. Can the butane reasonably be assumed to behave as an ideal gas at this state ?

Solution

Butane 25°C, 500 kPa, Table A.2: $T_c = 425$ K; $P_c = 3.8$ MPa

$$T_r = \frac{25 + 273}{425} = 0.701; \qquad P_r = \frac{0.5}{3.8} = 0.13$$

Look at generalized chart in Figure D.1

Actual $P_r > P_{r,\,sat} = 0.1$ => **liquid!! not a gas**

The pressure should be less than 380 kPa to have a gas at that T.

3.90

Estimate the saturation pressure of chlorine at 300 K.

Solution:

We do not have a table in the B section for Chlorine so we must use the generalized chart.

Table A.2: $P_c = 7.98$ MPa, $T_c = 416.9$ K

$T_r = T/T_c = 300 / 416.9 = 0.7196$

Figure D.1: $P_{r\ sat} = 0.13$ (same estimation from Table D.4)

$P = P_c\ P_{r\ sat} = 7.98 \times 0.13 =$ **1.04 MPa**

If you use the CATT3 program then you will find $P_{r\ sat} = 0.122$ and $P = 973$ kPa

3.91

A bottle with a volume of 0.1 m^3 contains butane with a quality of 75% and a temperature of 300 K. Estimate the total butane mass in the bottle using the generalized compressibility chart.

Solution:

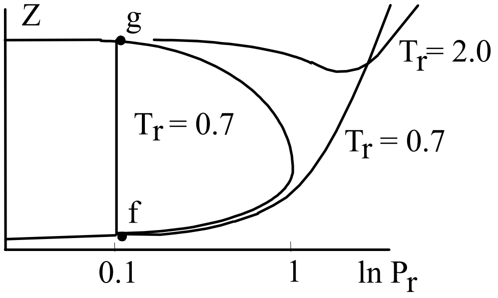
We need to find the property v the mass is: $m = V/v$

so find v given T_1 and x as : $v = v_f + x\, v_{fg}$

Table A.2: Butane $T_c = 425.2$ K $P_c = 3.8$ MPa = 3800 kPa

$T_r = 300/425.2 = 0.705$ =>

From Fig. D.1 or table D.4: $Z_f \approx 0.02$; $Z_g \approx 0.9$; $P_{r\,sat} = 0.1$



$P = P_{sat} = P_{r\,sat} \times P_c = 0.1 \times 3.80 \times 1000 = 380$ kPa

$v_f = Z_f RT/P = 0.02 \times 0.14304 \times 300/380 = 0.00226\ m^3/kg$

$v_g = Z_g RT/P = 0.9 \times 0.14304 \times 300/380 = 0.1016\ m^3/kg$

$v = 0.00226 + 0.75 \times (0.1016 - 0.00226) = 0.076765\ m^3/kg$

$m = \frac{V}{v} = \frac{0.1}{0.076765} = \mathbf{1.303\ kg}$

3.92

Find the volume of 2 kg of ethylene at 270 K, 2500 kPa using Z from Fig. D.1

Ethylene Table A.2: $T_c = 282.4$ K, $P_c = 5.04$ MPa

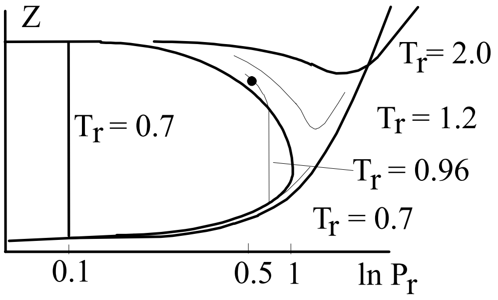
Table A.5: $R = 0.2964$ kJ/kg K

The reduced temperature and pressure are:

$$T_r = \frac{T}{T_c} = \frac{270}{282.4} = 0.956, \quad P_r = \frac{P}{P_c} = \frac{2.5}{5.04} = 0.496$$

Enter the chart with these coordinates and read: $Z = 0.76$

$$V = \frac{mZRT}{P} = \frac{2 \text{ kg} \times 0.76 \times 0.2964 \text{ kJ/kg-K} \times 270 \text{ K}}{2500 \text{ kPa}} = \mathbf{0.0487\ m^3}$$



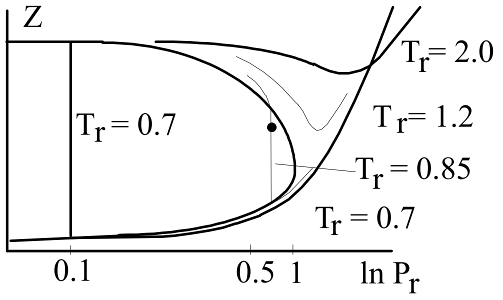
3.93

With $T_r = 0.85$ and a quality of 0.6, find the compressibility factor using Fig. D.1

For the saturated states we will use Table D.4 instead of the figure. There we can see at $T_r = 0.85$

$$Z_f = 0.062, \qquad Z_g = 0.747$$

$$Z = (1 - x)\, Z_f + xZ_g = (1 - 0.6)\, 0.062 + 0.6 \times 0.747 = \mathbf{0.473}$$



3.94

Argon is kept in a rigid 5 m^3 tank at −30°C, 3 MPa. Determine the mass using the compressibility factor. What is the error (%) if the ideal gas model is used?

Solution:

No Argon table, so we use generalized chart Fig. D.1

$T_r = 243.15/150.8 = 1.612,\quad P_r = 3000/4870 = 0.616 \quad => \quad Z \cong 0.96$

$$m = \frac{PV}{ZRT} = \frac{3000 \times 5}{0.96 \times 0.2081 \times 243.2} = \mathbf{308.75\ kg}$$

Ideal gas $Z = 1$

$$m = \frac{PV}{RT} = 296.4\ \text{kg} \qquad \mathbf{4\%\ error}$$

3.95

Refrigerant R-32 is at -10 ^oC with a quality of 15%. Find the pressure and specific volume.

Solution:

For R-32 there is no section B table printed. We will use compressibility chart.

From Table A.2: $T_c = 351.3$ K ; $P_c = 5.78$ MPa ;

From Table A.5: $R = 0.1598$ kJ/kg K

$T_r = T/T_c = 263/351.3 = 0.749$

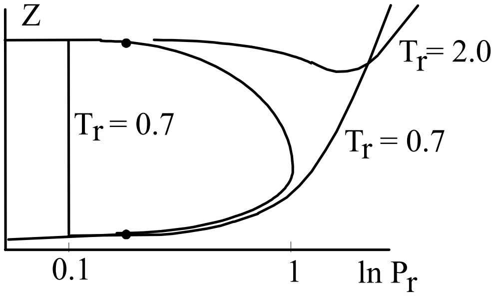
From Table D.4 or Figure D.1, $Z_f \approx 0.029$; $Z_g \approx 0.86$; $P_{r\ sat} \approx 0.16$

$P = P_{r\ sat}\ P_c = 0.16 \times 5780 =$ **925 kPa**

$v = v_f + x\ v_{fg} = (Z_f + x \times Z_{fg})\ RT/P$

$= [0.029 + 0.15 \times (0.86 - 0.029)] \times 0.1598$ kJ/kg-K $\times$ 263 K/ 925 kPa

$=$ **0.007 m^3/kg**



3.96

To plan a commercial refrigeration system using R-123 we would like to know how much more volume saturated vapor R-123 occupies per kg at -30 ^{o}C compared to the saturated liquid state.

Solution:

For R-123 there is no section B table printed. We will use compressibility chart.

From Table A.2 $T_c = 456.9$ K ; $P_c = 3.66$ MPa ; M = 152.93

$T_r = T/T_c = 243/456.9 = 0.53$

$R = \bar{R}/M = 8.31451 / 152.93 = 0.0544$

The value of T_r is below the range in Fig. D.1 so use the table D.4

Table D.4, $Z_g = 0.979$ $Z_f = 0.00222$

$Z_{fg} = 0.979 - 0.0022 = 0.9768$; $P_r = P_{r\,sat} = 0.0116$

$P = P_r \times P_c = 42.5$ kPa

$v_{fg} = Z_{fg}\,RT/P = 0.9768 \times 0.0544 \times 243 / 42.5 = \mathbf{0.304\ m^3/kg}$

Comment: If you check with the software the solution is off by a factor of 6. The linear interpolation is poor and so is the approximation for $P_{r\,sat}$ so the real saturation pressure should be 6.75 kPa. Also the very small value of Z_f is inaccurate by itself, minute changes in the curve gives large relative variations.

3.97

A new refrigerant R-125 is stored as a liquid at -20 ^oC with a small amount of vapor. For a total of 1.5 kg R-125 find the pressure and the volume.

Solution:

As there is no section B table use compressibility chart.

Table A.2: R-125 $T_c = 339.2$ K $P_c = 3.62$ MPa

$$T_r = T / T_c = 253.15 / 339.2 = 0.746$$

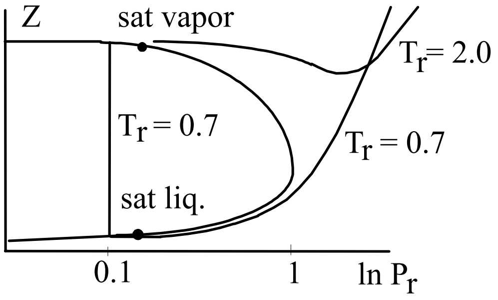
We can read from Figure D.1 or a little more accurately interpolate from table D.4 entries:

$$P_{r\,sat} = 0.16 \;;\quad Z_g = 0.86 \;;\quad Z_f = 0.029$$

$P = P_{r\,sat} P_c = 0.16 \times 3620 \text{ kPa} =$ **579 kPa**

$V_{liq} = Z_f m_{liq} RT/P = 0.029 \times 1.5 \text{ kg} \times 0.06927 \text{ kJ/kgK} \times 253.15 \text{ K}/ 579 \text{ kPa}$

$= \mathbf{0.0013\ m^3}$



Equations of State

For these problems see appendix D for the equation of state (EOS) and chapter 14.

3.98

Determine the pressure of nitrogen at 160 K, v = 0.00291 m^3/kg using ideal gas, van der Waal Equation of State and the nitrogen table.

Nitrogen from table A.2: $T_c = 126.2$ K, $P_c = 3390$ kPa,

Ideal gas: $$P = \frac{RT}{v} = \frac{0.2968 \times 160}{0.00291} \frac{\text{kJ/kg-K} \times \text{K}}{\text{m}^3\text{/kg}} = \mathbf{16\ 319\ kPa}$$

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8}\frac{RT_c}{P_c} = 0.125 \times \frac{0.2968 \times 126.2}{3390} = 0.001\ 381\ \text{m}^3\text{/kg},$$

$$a = 27\ b^2\ P_c = 27 \times (0.001\ 381)^2 \times 3390 = 0.174\ 562\ \text{kPa}\ (\text{m}^3\text{/kg})^2$$

The EOS is: $$P = \frac{RT}{v-b} - \frac{a}{v^2} = \frac{0.2968 \times 160}{0.00291 - 0.001\ 381} - \frac{0.174\ 562}{0.00291^2} = \mathbf{10\ 444\ kPa}$$

Table B.6.2: P = **10 000 kPa**.

3.99

Determine the pressure of nitrogen at 160 K, v = 0.00291 m^3/kg using Redlich-Kwong Equation of State and the nitrogen table.

Nitrogen from table A.2: $T_c = 126.2$ K, $P_c = 3390$ kPa,

$$T_r = T/T_c = 160/126.2 = 1.26783$$

For **Redlich-Kwong EOS** we have the parameters from Table D.1

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.2968 \times 126.2}{3390} = 0.000\,957\,3 \text{ m}^3/\text{kg},$$

$$a = 0.42748\, T_r^{-1/2} \frac{R^2T_c^2}{P_c} = 0.42748 \times \frac{0.2968^2 \times 126.2^2}{1.2678^{1/2} \times 3390} = 0.157\,122 \text{ kPa } (\text{m}^3/\text{kg})^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$= \frac{0.2968 \times 160}{0.00291 - 0.000\,9573} - \frac{0.157122}{0.00291^2 + 0.000\,9573 \times 0.00291}$$

$$= \mathbf{10\,357 \text{ kPa}}$$

Table B.6.2: P = **10 000 kPa**.

3.100

Determine the pressure of nitrogen at 160 K, v = 0.00291 m^3/kg using Soave Equation of State and the nitrogen table.

Nitrogen from table A.2: $T_c = 126.2$ K, $P_c = 3390$ kPa,

$T_r = T/T_c = 160/126.2 = 1.26783$

For **Soave EOS** see Appendix D (very close to Redlich-Kwong)

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

where the parameters are from Table D.1 and D.4

$$\omega = 0.039$$

$$f = 0.48 + 1.574\omega - 0.176\omega^2 = 0.54112$$

$$a_o = 0.42748\ [1 + f\,(1 - T_r^{1/2})\ ]^{\,2} = 0.371184$$

$$b = 0.08664\,\frac{RT_c}{P_c} = 0.08664 \times \frac{0.2968 \times 126.2}{3390} = 0.000\,957\,3\ \mathrm{m^3/kg},$$

$$a = 0.371184\,\frac{R^2T_c^2}{P_c} = 0.371184 \times \frac{0.2968^2 \times 126.2^2}{3390} = 0.153\,616\ \mathrm{kPa\ (m^3/kg)^2}$$

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$= \frac{0.2968 \times 160}{0.00291 - 0.000\,9573} - \frac{0.153\,616}{0.00291^2 + 0.000\,9573 \times 0.00291}$$

= **10 669 kPa**

Nitrogen Table B.6.2: P = **10 000 kPa**.

3.101

Carbon dioxide at 60$^{\text{o}}$C is pumped at a very high pressure 10 MPa into an oil well to reduce the oil viscosity for better oil flow. We want to find its specific volume from the CO_2 table, ideal gas and van der Waals equation of state by iteration.

Table B.3.2: v = **0.00345 m^3/kg**.

Ideal gas: $v = \frac{RT}{P} = \frac{0.1889 \times (60 + 273.15)}{10\,000} \; \frac{\text{kJ/kg-K} \times \text{K}}{\text{kPa}} = \mathbf{0.006\,293\ m^3/kg}$

Carbon dioxide from table A.2: $T_c = 304.1$ K, $P_c = 7380$ kPa,

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8}\frac{RT_c}{P_c} = 0.125 \times \frac{0.1889 \times 304.1}{7380} = 0.000\,972\,98 \text{ m}^3\text{/kg},$$

$$a = 27\, b^2\, P_c = 27 \times (0.000\,972\,98)^2 \times 7380 = 0.188\,6375 \text{ kPa (m}^3\text{/kg)}^2$$

The EOS is: $P = \frac{RT}{v - b} - \frac{a}{v^2}$

Since it is nonlinear in v we use trial and error starting with the Table entry.

$$v = 0.00345: \quad P = \frac{0.1889 \times 333.15}{0.00345 - 0.000\,972\,98} - \frac{0.188\,6375}{0.00345^2} = 9557.8 \text{ kPa} \quad \text{low}$$

$$v = 0.003 \quad P = \frac{0.1889 \times 333.15}{0.003 - 0.000\,972\,98} - \frac{0.188\,6375}{0.003^2} = 10\,086.8 \text{ kPa} \quad \text{high}$$

$$v = 0.00307: \quad P = \frac{0.1889 \times 333.15}{0.00307 - 0.000\,972\,98} - \frac{0.188\,6375}{0.00307^2} = 9995.4 \quad \text{OK.}$$

v = **0.00307 m^3/kg**

3.102

Solve the previous problem using the Redlich-Kwong equation of state. Notice this becomes trial and error.

Carbon dioxide from table A.2: $T_c = 304.1$ K, $P_c = 7380$ kPa,

$T_r = T/T_c = 333.15/304.1 = 1.09553$

For **Redlich-Kwong EOS** we have the parameters from Table D.1

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.1889 \times 304.1}{7380} = 0.000\,6744 \text{ m}^3\text{/kg},$$

$$a = 0.42748\, T_r^{-1/2} \frac{R^2T_c^2}{P_c} = 0.42748 \times \frac{0.1889^2 \times 304.1^2}{1.09553^{1/2} \times 7380} = 0.18262 \text{ kPa (m}^3\text{/kg)}^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$10\,000 = \frac{0.1889 \times 333.15}{v - 0.000\,6744} - \frac{0.18262}{v^2 + 0.000\,6744 \times v}$$

Trial and error on v (start guided by Table B.3.2):

$v = 0.0035$ m^3/kg ⇨ $P = 9772.8$ kPa so v smaller

$v = 0.0033$ m^3/kg ⇨ $P = 10044.7$ kPa so v larger

$v = 0.0034$ m^3/kg ⇨ $P = 9906.5$ kPa

linear interpolation gives

$v =$ **0.00333 m^3/kg** ⇨ $P = 10002.7$ kPa OK.

3.103

Solve Problem 3.101 using the Soave equation of state, ω = 0.239. Notice this becomes trial and error.

Carbon dioxide from table A.2: $T_c = 304.1$ K, $P_c = 7380$ kPa,

$$T_r = T/T_c = 333.15/304.1 = 1.09553$$

For **Soave EOS** see Appendix D (very close to Redlich-Kwong)

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

where the parameters are from Table D.1 and D.4

$$\omega = 0.239$$

$$f = 0.48 + 1.574\omega - 0.176\omega^2 = 0.84613$$

$$a_o = 0.42748\ [1 + f\,(1 - T_r^{1/2})\,]^2 = 0.394381$$

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.1889 \times 304.1}{7380} = 0.000\ 6744\ \text{m}^3/\text{kg},$$

$$a = 0.394381 \frac{R^2T_c^2}{P_c} = 0.394381 \times \frac{0.1889^2 \times 304.1^2}{7380} = 0.176\ 342\ \text{kPa}\ (\text{m}^3/\text{kg})^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$10\ 000 = \frac{0.1889 \times 333.15}{v - 0.000\ 6744} - \frac{0.176\ 342}{v^2 + 0.000\ 6744 \times v}$$

Trial and error on v (start guided by Table B.3.2):

$v = 0.0035\ \text{m}^3/\text{kg}$ ⇨ $P = 10\ 202$ kPa so v larger

$v = 0.0036\ \text{m}^3/\text{kg}$ ⇨ $P = 10\ 051$ kPa so v close

$v = 0.00363\ \text{m}^3/\text{kg}$ ⇨ $P = 10\ 006$ kPa OK

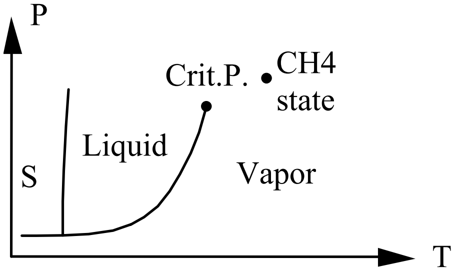
3.104

A tank contains 8.35 kg methane in 0.1 m^3 at 250 K. Find the pressure using ideal gas, van der Waal EOS and the methane table.

The state is given by (T = 250 K, v = V/m = 0.1/8.35 = 0.011976 m^3/kg)

Table A.2 or B.7.2:
T_c = 190.4 K,
P_c = 4600 kPa

$$T_r = \frac{T}{T_c} = \frac{250}{190.4} = 1.313$$



Ideal gas model: $P = \frac{RT}{v} = \frac{0.5183 \times 250}{0.011976} = 10\,820 \text{ kPa}$

For **van der Waal equation of state** from Table D.1 we have

$$b = \frac{1}{8}\frac{RT_c}{P_c} = 0.125 \times \frac{0.5183 \times 190.4}{4600} = 0.002\,681\,64 \text{ m}^3/\text{kg},$$

$$a = 27\, b^2\, P_c = 27 \times (0.002\,681\,64)^2 \times 4600 = 0.893\,15 \text{ kPa } (\text{m}^3/\text{kg})^2$$

The EOS is: $P = \frac{RT}{v-b} - \frac{a}{v^2} = \frac{0.5183 \times 250}{0.011976 - 0.002\,681\,64} - \frac{0.89315}{0.011976^2} = 7714 \text{ kPa}$

Locating the state in **Table B.7.2:** P = 8000 kPa, very close

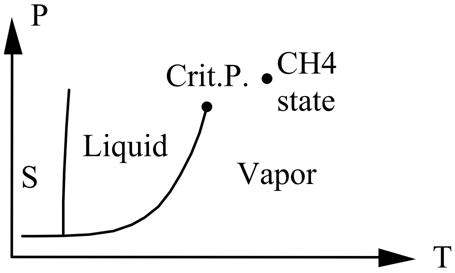
3.105

Do the previous problem using the Redlich-Kwong equation of state.

The state is given by (T = 250 K, v = V/m = 0.1/8.35 = 0.011976 m^3/kg)

Table A.2 or B.7.2:
T_c = 190.4 K,
P_c = 4600 kPa

$$T_r = \frac{T}{T_c} = \frac{250}{190.4} = 1.313$$



Ideal gas model: $P = \frac{RT}{v} = \frac{0.5183 \times 250}{0.011976} = 10\ 820 \text{ kPa}$

For **Redlich-Kwong equation of state** we have the parameters from Table D.1

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.5183 \times 190.4}{4600} = 0.001\ 858\ 7 \text{ m}^3\text{/kg},$$

$$a = 0.42748\ T_r^{-1/2} \frac{R^2T_c^2}{P_c} = 0.42748 \times \frac{0.5183^2 \times 190.4^2}{1.313^{1/2} \times 4600} = 0.789809 \text{ kPa } (\text{m}^3\text{/kg})^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$= \frac{0.5183 \times 250}{0.011976 - 0.0018587} - \frac{0.789809}{0.011976^2 + 0.0018587 \times 0.011976}$$

$$= 8040 \text{ kPa}$$

Locating the state in **Table B.7.2:** P = 8000 kPa, very close

3.106

Do Problem 3.104 using the Soave EOS.

A tank contains 8.35 kg methane in 0.1 m^3 at 250 K. Find the pressure using ideal gas, van der Waal EOS and the methane table.

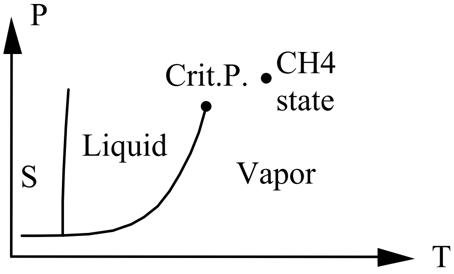
The state is given by (T = 250 K, v = V/m = 0.1/8.35 = 0.011976 m^3/kg)

Table A.2 or B.7.2:

$T_c = 190.4$ K,

$P_c = 4600$ kPa

$$T_r = \frac{T}{T_c} = \frac{250}{190.4} = 1.313$$



Ideal gas model: $P = \frac{RT}{v} = \frac{0.5183 \times 250}{0.011976} = 10\ 820$ kPa

For **Soave EOS** we have the parameters from Table D.1 and D.4

$$\omega = 0.011$$

$$f = 0.48 + 1.574\omega - 0.176\omega^2 = 0.49729$$

$$a_o = 0.42748\ [1 + f\,(1 - T_r^{1/2})\ ]^{\,2} = 0.367714$$

$$b = 0.08664\,\frac{RT_c}{P_c} = 0.08664 \times \frac{0.5183 \times 190.4}{4600} = 0.001\ 8587 \text{ m}^3\text{/kg},$$

$$a = 0.367714\,\frac{R^2T_c^2}{P_c} = 0.367714 \times \frac{0.5183^2 \times 190.4^2}{4600} = 0.778\ 482 \text{ kPa (m}^3\text{/kg)}^2$$

The equation is:

$$P = \frac{RT}{v - b} - \frac{a}{v^2 + bv}$$

$$= \frac{0.5183 \times 250}{0.011976 - 0.001\ 8587} - \frac{0.778\ 482}{0.011976^2 + 0.0018587 \times 0.011976}$$

= **8108.7 kPa**

Locating the state in **Table B.7.2:** P = 8000 kPa

Review Problems

3.107

Determine the quality (if saturated) or temperature (if superheated) of the following substances at the given two states:

Solution:

a) Water, H_2O, use Table B.1.1 or B.1.2

1) 120°C, 1 m^3/kg $v > v_g$ superheated vapor, **T = 120 °C**

2) 10 MPa, 0.01 m^3/kg => two-phase $v < v_g$

$x = (0.01 – 0.001452) / 0.01657 = \mathbf{0.516}$

b) Nitrogen, N_2, table B.6

1) 1 MPa, 0.03 m^3/kg => superheated vapor since $v > v_g$

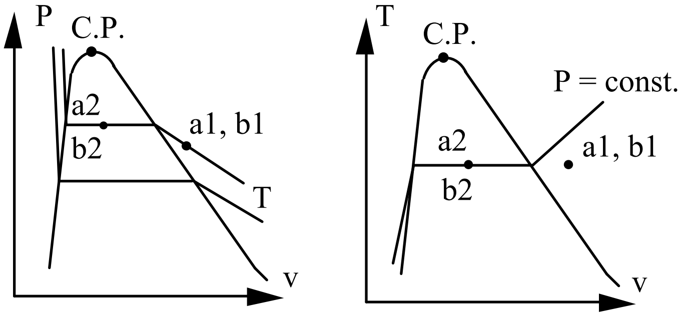
Interpolate between sat. vapor and superheated vapor B.6.2:

$$T \cong 103.73 + (120\text{-}103.73) \times \frac{0.03-0.02416}{0.03117-0.02416} = \mathbf{117\ K}$$

2) 100 K, 0.03 m^3/kg => sat. liquid + vapor as two-phase $v < v_g$

$v = 0.03 = 0.001452 + x \times 0.029764 \Rightarrow \mathbf{x = 0.959}$

States shown are placed relative to the two-phase region, not to each other.



3.108

Give the phase and the missing properties of P, T, v and x.

Solution:

a. R-410a $T = 10^\circ C$ $v = 0.01\ m^3/kg$

Table B.4.1 $v < v_g = 0.02383\ m^3/kg$

sat. liquid + vapor. $P = P_{sat} = 1085.7$ kPa,

$x = (v - v_f)/v_{fg} = (0.01 - 0.000886)/0.02295 = 0.2713$

b. H_2O $T = 350^\circ C$ $v = 0.2\ m^3/kg$

Table B.1.1 at given T: $v > v_g = 0.00881\ m^3/kg$

sup. vapor $P \cong 1.40$ MPa, x = undefined

c. R-410a $T = -5\ ^\circ C$ $P = 600$ kPa

sup. vapor ($P < P_g = 678{,}9$ kPa at $-5^\circ C$)

Table B.4.2:

$v = 0.04351\ m^3/kg$ at $-8.67^\circ C$

$v = 0.04595\ m^3/kg$ at $0^\circ C$

=> $v = 0.04454\ m^3/kg$ at $-5^\circ C$

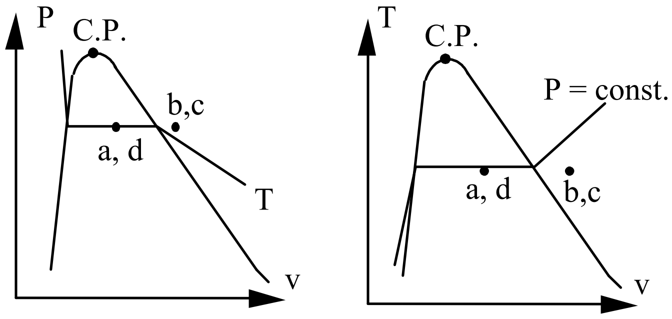
d. R-134a $P = 294$ kPa, $v = 0.05\ m^3/kg$

Table B.5.1: $v < v_g = 0.06919\ m^3/kg$

two-phase $T = T_{sat} = 0^\circ C$

$x = (v - v_f)/v_{fg} = (0.05 - 0.000773)/0.06842 = 0.7195$

States shown are placed relative to the two-phase region, not to each other.



3.109

Find the phase, quality *x* if applicable and the missing property *P* or *T*.

Solution:

a. H_2O $T = 120°C$ $v = 0.5\ m^3/kg$

Table B.1.1 at given T: $v < v_g = 0.89186$

sat. liq. + vap. $P = P_{sat} =$ **198.5 kPa,**

$x = (v - v_f)/v_{fg} = (0.5 - 0.00106)/0.8908 =$ **0.56**

b. H_2O $P = 100$ kPa $v = 1.8\ m^3/kg$

Table B.1.2 at given P: $v > v_g = 1.694$

sup. vap., interpolate in Table B.1.3

$$T = \frac{1.8 - 1.694}{1.93636 - 1.694}(150 - 99.62) + 99.62 = \mathbf{121.65\ °C}$$

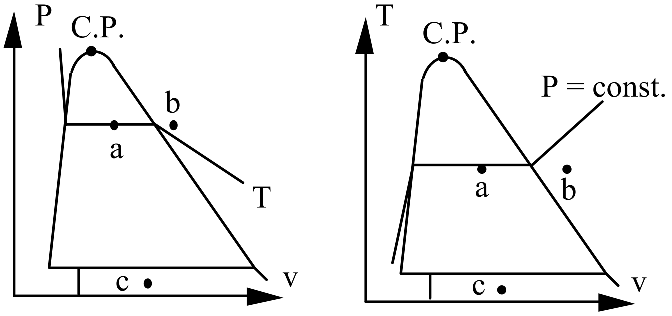
c. H_2O $T = 263$ K $v = 0.2\ m^3/kg$

Table B.1.5 at given $T = -10\ °C$: $v < v_g = 466.757$

sat. solid + vap., $P = P_{sat} =$ **0.26 kPa**,

$x = (v - v_i)/v_{ig} = (200 - 0.001)/466.756 =$ **0.4285**

States shown are placed relative to the two-phase region, not to each other.



3.110

Find the phase, quality *x* if applicable and the missing property *P* or *T*.

Solution:

a. NH_3 $P = 800$ kPa $v = 0.2\ m^3/kg$;

Superheated Vapor ($v > v_g$ at 800 kPa)

Table B 2.2 interpolate between 70°C and 80°C

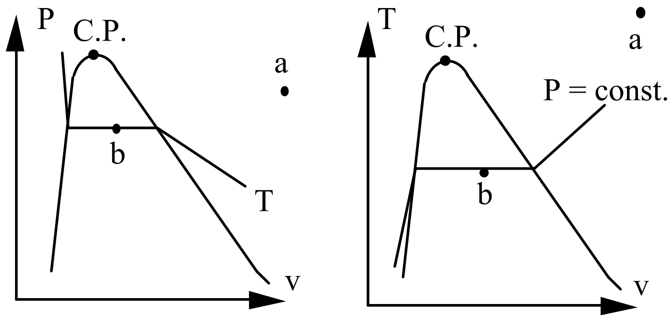
T = **71.4°C**

b. NH_3 $T = 20°C$ $v = 0.1\ m^3/kg$

Table B.2.1 at given T: $v < v_g = 0.14922$

sat. liq. + vap. , $P = P_{sat} =$ **857.5 kPa,**

$x = (v - v_f)/v_{fg} = (0.1 - 0.00164)/0.14758 =$ **0.666**



3.111

Give the phase and the missing properties of P, T, v and x. These may be a little more difficult if the appendix tables are used instead of the software.

a) R-410a at $T = 10°C$, $v = 0.02$ m^3/kg: Table B.4.1 $v > v_g$ at 10°C

=> **sup. vap.** Table B.4.2 interpolate between sat. and sup. both at 10°C

$$P = 680.7 + (600 - 680.7)\frac{0.036-0.03471}{0.04018-0.03471} = \mathbf{661.7\ kPa}$$

b) H_2O $v = 0.2$ m^3/kg , $x = 0.5$: Table B.1.1

sat. liq. + vap. $v = (1-x)\,v_f + x\,v_g$ => $v_f + v_g = 0.4$ m^3/kg

since v_f is so small we find it approximately where $v_g = 0.4$ m^3/kg.

$v_f + v_g = 0.39387$ at 150°C, $v_f + v_g = 0.4474$ at 145°C.

An interpolation gives $T \cong$ **149.4°C,** $P \cong$ **468.2 kPa**

c) H_2O $T = 60°C$, $v = 0.001016$ m^3/kg: Table B.1.1 $v < v_f = 0.001017$

=> **compr. liq.** see Table B.1.4

$v = 0.001015$ at 5 MPa so $P \cong 0.5(5000 + 19.9) =$ **2.51 MPa**

d) NH_3 $T = 30°C$, $P = 60$ kPa : Table B.2.1 $P < P_{sat}$

=> **sup. vapor** interpolate in Table B.2.2

$$v = 2.94578 + (1.95906 - 2.94578)\frac{60 - 50}{75 - 50} = \mathbf{2.551\ m^3/kg}$$

v is not linearly proportional to P (more like 1/P) so the computer table gives a more accurate value of 2.45 m^3/kg

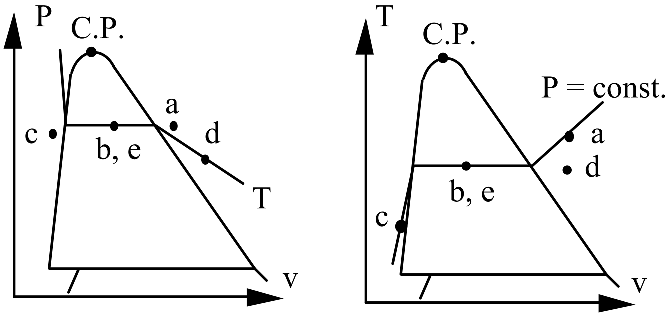
e) R-134a $v = 0.005$ m^3/kg , $x = 0.5$: **sat. liq. + vap.** Table B.5.1

$v = (1-x)\,v_f + x\,v_g$ => $v_f + v_g = 0.01$ m^3/kg

$v_f + v_g = 0.010946$ at 65°C, $v_f + v_g = 0.009665$ at 70°C.

An interpolation gives: $T \cong$ **68.7°C**, $P =$ **2.06 MPa**

States shown are placed relative to the two-phase region, not to each other.



3.112

Refrigerant-410a in a piston/cylinder arrangement is initially at 50°C, $x = 1$. It is then expanded in a process so that $P = Cv^{-1}$ to a pressure of 100 kPa. Find the final temperature and specific volume.

Solution:

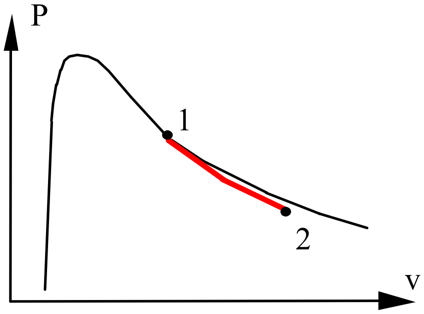
State 1: 50°C, x = 1 Table B.4.1: $P_1 = 3065.2$ kPa, $v_1 = 0.00707$ m^3/kg

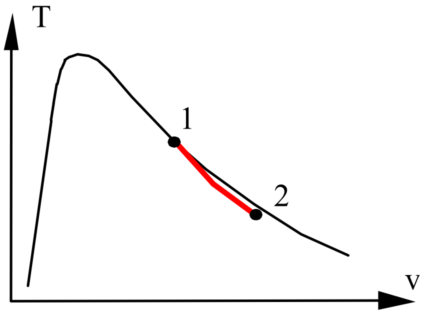
Process: $Pv = C = P_1v_1$; => $P_2 = C/v_2 = P_1v_1/v_2$

State 2: 100 kPa and $v_2 = v_1P_1/P_2 =$ **0.2167 m^3/kg**

$v_2 < v_g$ at 100 kPa, $T_2 \cong$ **-51.65°C** from Table B.3.2

Notice T is **not** constant



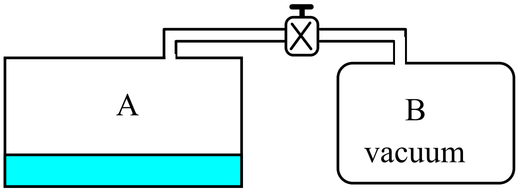


The first part of the process may be in the sup. vapor region.

3.113

Consider two tanks, A and B, connected by a valve, as shown in Fig. P3.113. Each has a volume of 200 L and tank A has R-410a at 25°C, 10% liquid and 90% vapor by volume, while tank B is evacuated. The valve is now opened and saturated vapor flows from A to B until the pressure in B has reached that in A, at which point the valve is closed. This process occurs slowly such that all temperatures stay at 25°C throughout the process. How much has the quality changed in tank A during the process?

Solution:



State A1: Table B.4.1 $v_f = 0.000944\ m^3/kg$, $v_g = 0.01514\ m^3/kg$

$$m_{A1} = \frac{V_{liq1}}{v_{f\,25°C}} + \frac{V_{vap1}}{v_{g\,25°C}} = \frac{0.1 \times 0.2}{0.000944} + \frac{0.9 \times 0.2}{0.01514}$$

$$= 21.186 + 11.889 = 33.075\ kg$$

$$x_{A1} = \frac{11.889}{33.075} = 0.3594\ ;$$

State B2: Assume A still two-phase so saturated P for given T

$$m_{B2} = \frac{V_B}{v_{g\,25°C}} = \frac{0.2}{0.01514} = 13.210\ kg$$

State A2: mass left is $m_{A2} = 33.075 - 13.210 = 19.865\ kg$

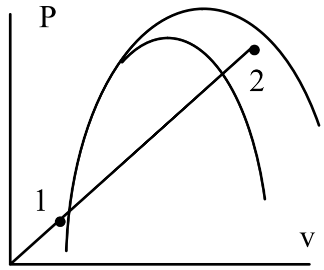
$$v_{A2} = \frac{0.2}{19.865} = 0.010068 = 0.000944 + x_{A2} \times 0.01420$$

$x_{A2} = 0.6425$ $\Delta x = \mathbf{0.283}$

3.114

Water in a piston/cylinder is at 90°C, 100 kPa, and the piston loading is such that pressure is proportional to volume, *P* = *CV*. Heat is now added until the temperature reaches 200°C. Find the final pressure and also the quality if in the two-phase region.

Solution:



Final state: 200°C , on process line $P = CV$

State 1: Table B.1.1: $v_1 = 0.001036\ \text{m}^3/\text{kg}$

$P_2 = P_1 v_2/v_1$ from process equation

Check state 2 in Table B.1.1

$v_g(T_2) = 0.12736$; $P_g(T_2) = 1.5538$ MPa

If $v_2 = v_g(T_2)$ $\Rightarrow$ $P_2 = 12.3$ MPa $> P_g$ not OK

If sat. $P_2 = P_g(T_2) = 1553.8$ kPa $\Rightarrow$ $v_2 = 0.0161\ \text{m}^3\text{kg} < v_g$ sat. OK,

$\mathbf{P_2 = 1553.8}$ **kPa**, $x_2 = (0.0161 - 0.001156) / 0.1262 = \mathbf{0.118}$

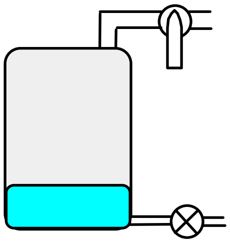
3.115

A tank contains 2 kg of nitrogen at 100 K with a quality of 50%. Through a volume flowmeter and valve, 0.5 kg is now removed while the temperature remains constant. Find the final state inside the tank and the volume of nitrogen removed if the valve/meter is located at

a. The top of the tank
b. The bottom of the tank

Solution

Table B.6.1:



$v_1 = 0.001452 + x_1 \times 0.029764 = 0.016334 \text{ m}^3/\text{kg}$

$V_{tank} = m_1 v_1 = 0.0327 \text{ m}^3$

$m_2 = m_1 - 0.5 = 1.5 \text{ kg}$

$v_2 = V_{tank}/m_2 = 0.0218 \; < \; v_g(T)$

$$x_2 = \frac{0.0218-0.001452}{0.031216-0.001452} = \mathbf{0.6836}$$

Top: flow out is sat. vap. $v_g = 0.031216 \text{ m}^3/\text{kg}$, $V_{out} = m_{out} v_g = \mathbf{0.0156 \; m^3}$

Bottom: flow out is sat. liq. $v_f = 0.001452$ $V_{out} = m_{out} v_f = \mathbf{0.000726 \; m^3}$

3.116

A spring-loaded piston/cylinder contains water at 500°C, 3 MPa. The setup is such that pressure is proportional to volume, *P* = *CV*. It is now cooled until the water becomes saturated vapor. Sketch the *P-v* diagram and find the final pressure.

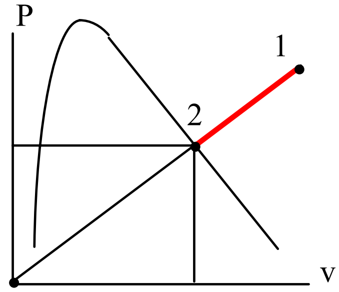
Solution:

State 1: Table B.1.3: $v_1 = 0.11619 \text{ m}^3/\text{kg}$

Process: m is constant and $P = C_0V = C_0m\ v = C\ v$

$$P = Cv \Rightarrow C = P_1/v_1 = 3000/0.11619 = 25820 \text{ kPa kg/m}^3$$

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)



Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \Rightarrow C = 20074$ (low)

2.5 MPa $v_g = 0.07998 \Rightarrow C = 31258$ (high)

2.25 MPa $v_g = 0.08875 \Rightarrow C = 25352$ (low)

Interpolate to get the right C $\Rightarrow$ $\mathbf{P_2 = 2270 \text{ kPa}}$

3.117

A container with liquid nitrogen at 100 K has a cross sectional area of 0.5 m^2. Due to heat transfer, some of the liquid evaporates and in one hour the liquid level drops 30 mm. The vapor leaving the container passes through a valve and a heater and exits at 500 kPa, 260 K. Calculate the volume rate of flow of nitrogen gas exiting the heater.

Solution:

Properties from table B.6.1 for volume change, exit flow from table B.6.2:

$$\Delta V = A \times \Delta h = 0.5 \times 0.03 = 0.015 \text{ m}^3$$

$$\Delta m_{liq} = -\Delta V/v_f \qquad = -0.015/0.001452 = -10.3306 \text{ kg}$$

$$\Delta m_{vap} = \Delta V/v_g \qquad = 0.015/0.0312 = 0.4808 \text{ kg}$$

$$m_{out} = 10.3306 - 0.4808 = 9.85 \text{ kg}$$

$$v_{exit} = 0.15385 \text{ m}^3\text{/kg}$$

$$\dot{V} = \dot{m}v_{exit} = (9.85 / 1\ h) \times 0.15385 \text{ m}^3\text{/kg}$$

$$= 1.5015 \text{ m}^3\text{/h} = \mathbf{0.02526\ m^3/min}$$

3.118

For a certain experiment, R-410a vapor is contained in a sealed glass tube at 20°C. It is desired to know the pressure at this condition, but there is no means of measuring it, since the tube is sealed. However, if the tube is cooled to –20°C small droplets of liquid are observed on the glass walls. What is the initial pressure?

Solution:

Control volume: R-410a fixed volume (V) & mass (m) at 20°C

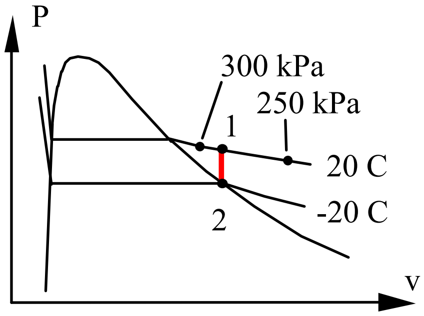
Process: cool to -20°C at constant v, so we assume saturated vapor

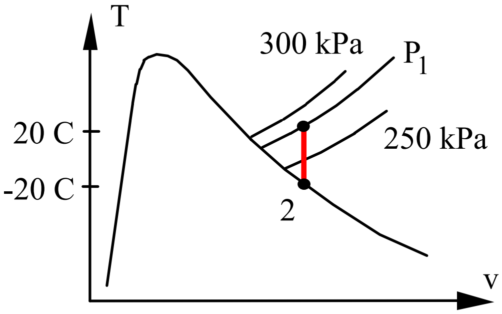
State 2: $v_2 = v_{g \text{ at } -20°C} = 0.06480 \text{ m}^3/\text{kg}$

State 1: 20°C, $v_1 = v_2 = 0.06480 \text{ m}^3/\text{kg}$

interpolate between 400 and 500 kPa in Table B.4.2

=> $\mathbf{P_1 = 485 \text{ kPa}}$





3.119

A cylinder/piston arrangement contains water at 105°C, 85% quality with a volume of 1 L. The system is heated, causing the piston to rise and encounter a linear spring as shown in Fig. P3.119. At this point the volume is 1.5 L, piston diameter is 150 mm, and the spring constant is 100 N/mm. The heating continues, so the piston compresses the spring. What is the cylinder temperature when the pressure reaches 200 kPa?

Solution:

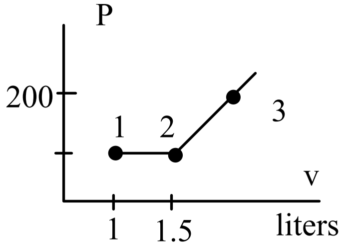
$P_1 = 120.8$ kPa, $v_1 = v_f + x\, v_{fg} = 0.001047 + 0.85*1.41831 = 1.20661$

$m = V_1 / v_1 = \frac{0.001}{1.20661} = 8.288\times10^{-4}$ kg

$v_2 = v_1\,(V_2 / V_1) = 1.20661\times 1.5 = 1.8099$

& $P = P_1 = 120.8$ kPa ($T_2 = 203.5$°C)

$P_3 = P_2 + (k_s/A_p^2)\, m(v_3 - v_2)$ linear spring



$A_p = (\pi/4) \times 0.15^2 = 0.01767\ m^2$; $k_s = 100$ kN/m (matches P in kPa)

$200 = 120.8 + (100/0.01767^{\,2}) \times 8.288\times10^{-4}(v_3 - 1.8099)$

$200 = 120.8 + 265.446\ (v_3 - 1.8099) \Rightarrow \quad v_3 = 2.1083\ m^3/kg$

$T_3 \cong 600 + 100 \times (2.1083 - 2.01297)/(2.2443 - 2.01297) \cong$ **641°C**

3.120

Determine the mass of methane gas stored in a 2 m^3 tank at −30°C, 2 MPa. Estimate the percent error in the mass determination if the ideal gas model is used.
Solution:

Table B.7

Methane Table B.7.1 at −30°C = 243.15 K > T_c = 190.6 K, so superheated vapor in Table B.7.2. Linear interpolation between 225 and 250 K at 2 MPa.

$$\Rightarrow \quad v \cong 0.05289 + \frac{243.15\text{-}225}{250\text{-}225} \times (0.06059 - 0.05289) = 0.05848 \text{ m}^3\text{/kg}$$

$$m = V/v = 2/0.05848 = \mathbf{34.2\ kg}$$

Ideal gas assumption

$$v = \frac{RT}{P} = \frac{0.51835 \times 243.15}{2000} = 0.06302 \text{ m}^3\text{/kg}$$

$$m = \frac{V}{v} = \frac{2}{0.06302} = 31.74 \text{ kg}$$

Error:

$\Delta m = 2.46$ kg ; **7.2% too large**

Comment: The compressibility of the methane Z = 0.93.

3.121

A cylinder containing ammonia is fitted with a piston restrained by an external force that is proportional to cylinder volume squared. Initial conditions are 10°C, 90% quality and a volume of 5 L. A valve on the cylinder is opened and additional ammonia flows into the cylinder until the mass inside has doubled. If at this point the pressure is 1.2 MPa, what is the final temperature?

Solution:

State 1 Table B.2.1: $v_1 = 0.0016 + 0.9(0.205525 - 0.0016) = 0.18513 \text{ m}^3\text{/kg}$

$P_1 = 615 \text{ kPa}; \quad V_1 = 5 \text{ L} = 0.005 \text{ m}^3$

$m_1 = V/v = 0.005/0.18513 = 0.027 \text{ kg}$

State 2: $P_2 = 1.2$ MPa, Flow in so: $m_2 = 2\ m_1 = 0.054$ kg

Process: Piston $F_{ext} = KV^2 = PA \quad => \quad P = CV^2 \quad => P_2 = P_1\ (V_2/V_1)^2$

From the process equation we then get:

$$V_2 = V_1\ (P_2/P_1)^{1/2} = 0.005\ \left(\frac{1200}{615}\right)^{1/2} = 0.006984 \text{ m}^3$$

$$v_2 = V/m = \frac{0.006984}{0.054} = 0.12934 \text{ m}^3\text{/kg}$$

At P_2, v_2: $\quad T_2 =$ **70.9°C**

3.122

A cylinder has a thick piston initially held by a pin as shown in Fig. P3.122. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m^3 and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas returns to ambient temperature. Is the piston against the stops?

Solution:

Force balance on piston determines equilibrium float pressure.

Piston $m_p = A_p \times l \times \rho$ $\qquad \rho_{piston} = 8000 \text{ kg/m}^3$

$$P_{ext\ on\ CO_2} = P_0 + \frac{m_p g}{A_p} = 101 + \frac{A_p \times 0.1 \times 9.807 \times 8000}{A_p \times 1000} = 108.8 \text{ kPa}$$

Pin released, as $P_1 > P_{ext}$ piston moves up, $T_2 = T_o$ & if piston at stops,

then $V_2 = V_1 \times H_2/H_1 = V_1 \times 150 / 100$

Ideal gas with $T_2 = T_1$ then gives

$$\Rightarrow \; P_2 = P_1 \times V_1 / V_2 = 200 \times \frac{100}{150} = 133 \text{ kPa} \; > \; P_{ext}$$

$\Rightarrow$ **piston is at stops, and P_2 = 133 kPa**

3.123

What is the percent error in pressure if the ideal gas model is used to represent the behavior of superheated vapor R-410a at 60°C, 0.03470 m^3/kg? What if the generalized compressibility chart, Fig. D.1, is used instead (iterations needed)?

Solution:

Real gas behavior: $P = 1000$ kPa from Table B.4.2

Ideal gas constant: $R = \bar{R}/M = 8.31451/72.585 = 0.1146$ kJ/kg K

$P = RT/v = 0.1146 \times (273.15 + 60) / 0.0347$

$= \mathbf{1100\ kPa}$ **which is 10% too high**

Generalized chart Fig D.1 and critical properties from A.2:

$T_r = 333.2/(273.15 + 71.3) = 0.967;\quad P_c = 4901$ kPa

Assume $P = 1000$ kPa $\Rightarrow$ $P_r = 0.204$ $\Rightarrow$ $Z \cong 0.92$

$v = ZRT/P = 0.92 \times 0.1146 \times 333.15 /1000 = 0.03512$ too high

Assume $P = 1050$ kPa $\Rightarrow$ $P_r = 0.214$ $\Rightarrow$ $Z \cong 0.915$

$v = ZRT/P = 0.915 \times 0.1146 \times 333.15 / 1050 = 0.03327$ too low

$$P \cong 1000 + (1050 - 1000) \times \frac{0.03470 - 0.03512}{0.03327 - 0.03512} = \mathbf{1011\ kPa\ \ 1.1\ \%\ high}$$

3.124

An initially deflated and flat balloon is connected by a valve to a 12 m^3 storage tank containing helium gas at 2 MPa and ambient temperature, 20°C. The valve is opened and the balloon is inflated at constant pressure, P_o = 100 kPa, equal to ambient pressure, until it becomes spherical at D_1 = 1 m. If the balloon is larger than this, the balloon material is stretched giving a pressure inside as

$$P = P_0 + C\left(1 - \frac{D_1}{D}\right)\frac{D_1}{D}$$

The balloon is inflated to a final diameter of 4 m, at which point the pressure inside is 400 kPa. The temperature remains constant at 20°C. What is the maximum pressure inside the balloon at any time during this inflation process? What is the pressure inside the helium storage tank at this time?

Solution:

At the end of the process we have D = 4 m so we can get the constant C as

$$P = 400 = P_0 + C\ (\ 1 - \frac{1}{4})\ \frac{1}{4} = 100 + C \times 3/16 \quad \Rightarrow \quad C = 1600 \text{ kPa}$$

The pressure is: $P = 100 + 1600\ (\ 1 - X^{-1})\ X^{-1}$; $\quad X = D / D_1$

Differentiate to find max: $\frac{dP}{dD} = C\ (\ - X^{-2} + 2\ X^{-3}\ )\ /\ D_1 = 0$

$$\Rightarrow\ - X^{-2} + 2\ X^{-3} = 0 \ \Rightarrow\ X = 2$$

at max P => $D = 2D_1 = 2$ m; $\quad V = \frac{\pi}{6} D^3 = 4.18\ m^3$

$$\text{Pmax} = 100 + 1600\ (\ 1 - \frac{1}{2})\ \frac{1}{2} = \mathbf{500\ kPa}$$

Helium is ideal gas A.5: $m = \frac{PV}{RT} = \frac{500 \times 4.189}{2.0771 \times 293.15} = 3.44$ kg

$$m_{TANK,\,1} = \frac{PV}{RT} = \frac{2000 \times 12}{2.0771 \times 293.15} = 39.416 \text{ kg}$$

$m_{TANK,\,2} = 39.416 - 3.44 = 35.976$ kg

$P_{T2} = m_{TANK,\,2}\,RT/V = (\ m_{TANK,\,1} /\ m_{TANK,\,2}\ ) \times P_1$ = **1825.5 kPa**

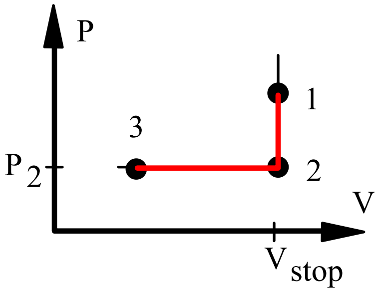
3.125

A piston/cylinder arrangement, shown in Fig. P3.125, contains air at 250 kPa, 300°C. The 50-kg piston has a diameter of 0.1 m and initially pushes against the stops. The atmosphere is at 100 kPa and 20°C. The cylinder now cools as heat is transferred to the ambient.

a. At what temperature does the piston begin to move down?

b. How far has the piston dropped when the temperature reaches ambient?

Solution:



Piston $A_p = \frac{\pi}{4} \times 0.1^2 = 0.00785\ \text{m}^2$

Balance forces when piston floats:

$$P_{float} = P_o + \frac{m_p g}{A_p} = 100 + \frac{50 \times 9.807}{0.00785 \times 1000}$$

$$= 162.5\ \text{kPa} = P_2 = P_3$$

To find temperature at 2 assume ideal gas:

$$T_2 = T_1 \times \frac{P_2}{P_1} = 573.15 \times \frac{162.5}{250} = \mathbf{372.5\ K}$$

b) Process 2 -> 3 is constant pressure as piston floats to $T_3 = T_o = 293.15$ K

$$V_2 = V_1 = A_p \times H = 0.00785 \times 0.25 = 0.00196\ \text{m}^3 = 1.96\ \text{L}$$

Ideal gas and $P_2 = P_3$ => $V_3 = V_2 \times \frac{T_3}{T_2} = 1.96 \times \frac{293.15}{372.5} = \mathbf{1.54\ L}$

$$\Delta H = (V_2 - V_3)/A = (1.96-1.54) \times 0.001/0.00785 = \mathbf{0.053\ m = 5.3\ cm}$$

Linear Interpolation

3.126

Find the pressure and temperature for saturated vapor R-410a with v = 0.1 m^3/kg

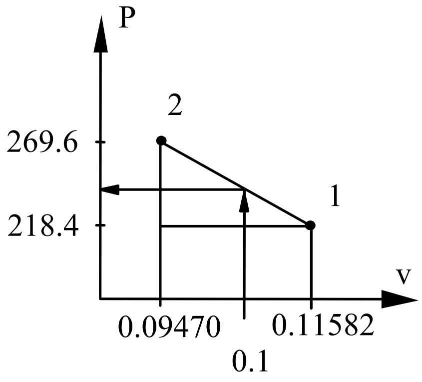
Solution:

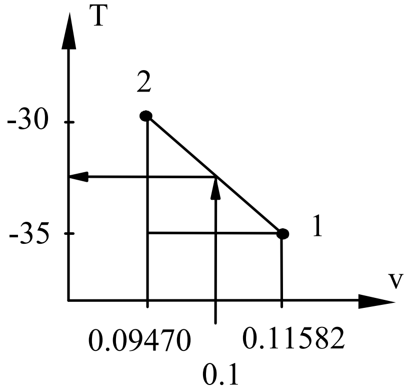
Table B.4.1 Look at the saturated vapor column v_g and it is found between −35°C and −30°C. We must then do a linear interpolation between these values.

$$T = -35 + [\,-30 - (-35)]\,\frac{0.1 - 0.11582}{0.09470 - 0.11582}$$

$$= -35 + 5 \times 0.749 = \mathbf{-31.3^{\circ}C}$$

$$P = 218.4 + (269.6 - 218.4) \times 0.749 = \mathbf{256.7\ kPa}$$





To understand the interpolation equation look at the smaller and larger triangles formed in the figure. The ratio of the side of the small triangle in v as (0.11582 - 0.1) to the side of the large triangle (0.11582 – 0.09470) is equal to 0.749. This fraction of the total ΔP = 269.6 – 218.4 or ΔT = –30 – (–35) is added to the lower value to get the desired interpolated result.

3.127

Use a linear interpolation to estimate properties of ammonia to fill out the table below

	P [kPa]	T [°C]	v [m^3/kg]	x
a)	550			0.75
b)	80	20		
c)		10	0.4	

Solution:

a) Find the pressures in Table B.2.1 that brackets the given pressure.

$$T = 5 + (10 - 5)\,\frac{550 - 515.9}{615.2 - 515.9} = 5 + 5 \times 0.341 = \mathbf{6.7\ ^\circ C}$$

$$v_f = 0.001583 + (0.0016 - 0.001583)\ 0.341 = 0.001589\ \mathrm{m^3/kg}$$

$$v_g = 0.24299 + (0.20541 - 0.24299)\ 0.341 = 0.230175\ \mathrm{m^3/kg}$$

$$v = v_f + x v_{fg} = 0.001589 + 0.75(0.230175 - 0.001589)$$

$$= \mathbf{0.1729\ m^3/kg}$$

b) Interpolate between 50 and 100 kPa to get properties at 80 kPa

$$v = 2.8466 + (1.4153 - 2.8466)\,\frac{80 - 50}{100 - 50}$$

$$= 2.8466 + (-1.4313) \times 0.6 = \mathbf{1.9878\ m^3/kg}$$

x: Undefined

c) Table B.2.1: $v > v_g$ so the state is superheated vapor.

Table B.2.2 locate state between 300 and 400 kPa.

$$P = 300 + (400 - 300)\,\frac{0.4 - 0.44251}{0.32701 - 0.44251}$$

$$= 300 + 100 \times 0.368 = \mathbf{336.8\ kPa}$$

x: Undefined

3.128

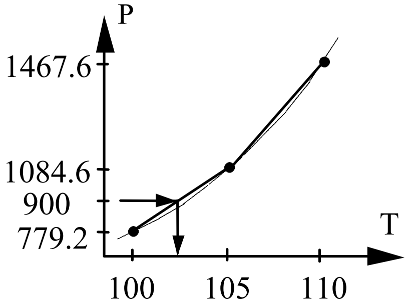
Use a linear interpolation to estimate Tsat at 900 kPa for nitrogen. Sketch by hand the curve Psat(T) by using a few table entries around 900 kPa from table B.6.1. Is your linear interpolation over or below the actual curve?

Solution:

The 900 kPa in Table B.6.1 is located between 100 and 105 K.

$$T = 100 + (105 - 100)\,\frac{900 - 779.2}{1084.6 - 779.2}$$
$$= 100 + 5 \times 0.3955 = \mathbf{102\ K}$$

The actual curve has a positive second derivative (it curves up) so T is slightly underestimated by use of the chord between the 100 K and the 105 K points, as the chord is above the curve.



3.129

Use a double linear interpolation to find the pressure for superheated R-134a at 13°C with v = 0.3 m^3/kg.

Solution:

Table B.5.2: Superheated vapor

At 10°C, 0.3 m^3/kg

$$P = 50 + (100 - 50) \times \frac{0.3 - 0.45608}{0.22527 - 0.45608} = 83.8 \text{ kPa}$$

At 20°C, 0.3 m^3/kg

$$P = 50 + (100 - 50) \times \frac{0.3 - 0.47287}{0.23392 - 0.47287} = 86.2 \text{ kPa}$$

Interpolating at 13°C,

$P = 83.8 + (3/10) \times (86.2 - 83.8) = \mathbf{84.5\ kPa}$

This could also be interpolated as following:

At 13°C, 50 kPa, $v = 0.45608 + (3/10) \times 0.0168 = 0.4611\ m^3/kg$

At 13°C, 100 kPa, $v = 0.22527 + (3/10) \times 0.0087 = 0.2279\ m^3/kg$

Interpolating at 0.3 m^3/kg.

$$P = 50 + (100 - 50) \times \frac{0.3 - 0.4611}{0.2279 - 0.4611} = \mathbf{84.5\ kPa}$$

3.130

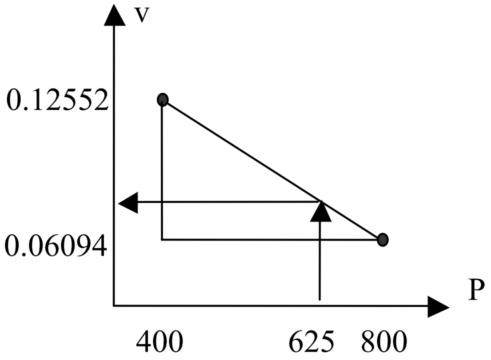
Find the specific volume for CO_2 at 0^oC and 625 kPa.

Solution:

The state is superheated vapor in Table B.3.2 between 400 and 800 kPa.

$$v = 0.12552 + (0.06094 - 0.12552)\,\frac{625 - 400}{800 - 400}$$

$$= 0.12552 + (-0.06458) \times 0.5625 = \mathbf{0.0892\ m^3/kg}$$



Computer Tables

3.131

Use the computer software to find the properties for water at the 4 states in Problem 3.35

Start the software, click the tab for water as the substance, and click the small calculator icon. Select the proper CASE for the given properties.

	CASE	RESULT
a)	1 (T, P)	Compressed liquid, x = undefined, v = 0.001002 m^3/kg
b)	5 (P, v)	Two-phase, T = 151.9°C, x = 0.5321
c)	1 (T, P)	Sup. vapor, x = undefined, v = 0.143 m^3/kg
d)	4 (T, x)	$P = P_{sat}$ = 8581 kPa, v = 0.01762 m^3/kg

3.132

Use the computer software to find the properties for ammonia at the 2 states listed in Problem 3.32

Start the software, click the tab for cryogenic substances, and click the tab for the substance ammonia. Then click the small calculator icon and select the proper CASE for the given properties.

	CASE	RESULT
a)	2 (T, v)	Sup. vapor, x = undefined, P = 1200 kPa
b)	4 (T, x)	Two-phase, P = 2033 kPa, v = 0.03257 m^3/kg

3.133

Use the computer software to find the properties for ammonia at the 3 states listed in Problem 3.127

Start the software, click the tab for cryogenic substances, select ammonia and click the small calculator icon. Select the proper CASE for the given properties.

	CASE	RESULT
a)	8 (P, x)	T = 6.795°C, v = 0.1719 m^3/kg
b)	1 (T, P)	Sup. vapor, x = undefined, v = 1.773 m^3/kg
c)	2 (T, v)	Sup. vapor, x = undefined, P = 330.4 kPa

3.134

Find the value of the saturated temperature for nitrogen by linear interpolation in table B.6.1 for a pressure of 900 kPa. Compare this to the value given by the computer software.

The 900 kPa in Table B.6.1 is located between 100 and 105 K.

$$T = 100 + (105 - 100)\,\frac{900 - 779.2}{1084.6 - 779.2}$$

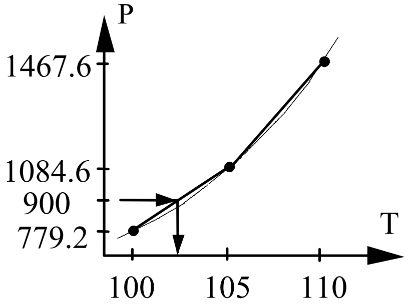
$$= 100 + 5 \times 0.3955 = 101.98 \text{ K}$$

The actual curve has a positive second derivative (it curves up) so T is slightly underestimated by use of the chord between the 100 K and the 105 K points, as the chord is above the curve.

From the computer software:

CASE: 8 (P,x) $T = -171^\circ\text{C} = 102.15$ K

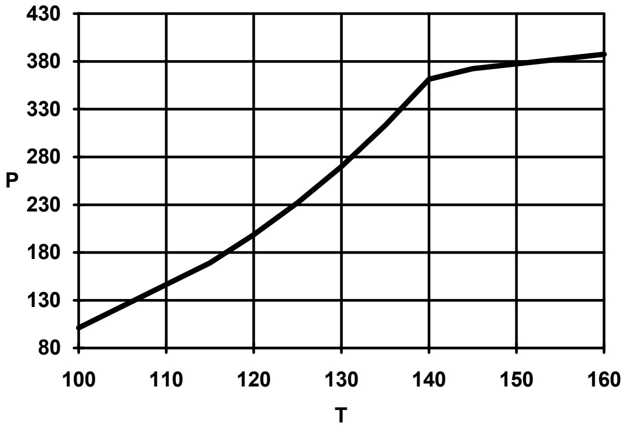
So we notice that the curvature has only a minor effect.

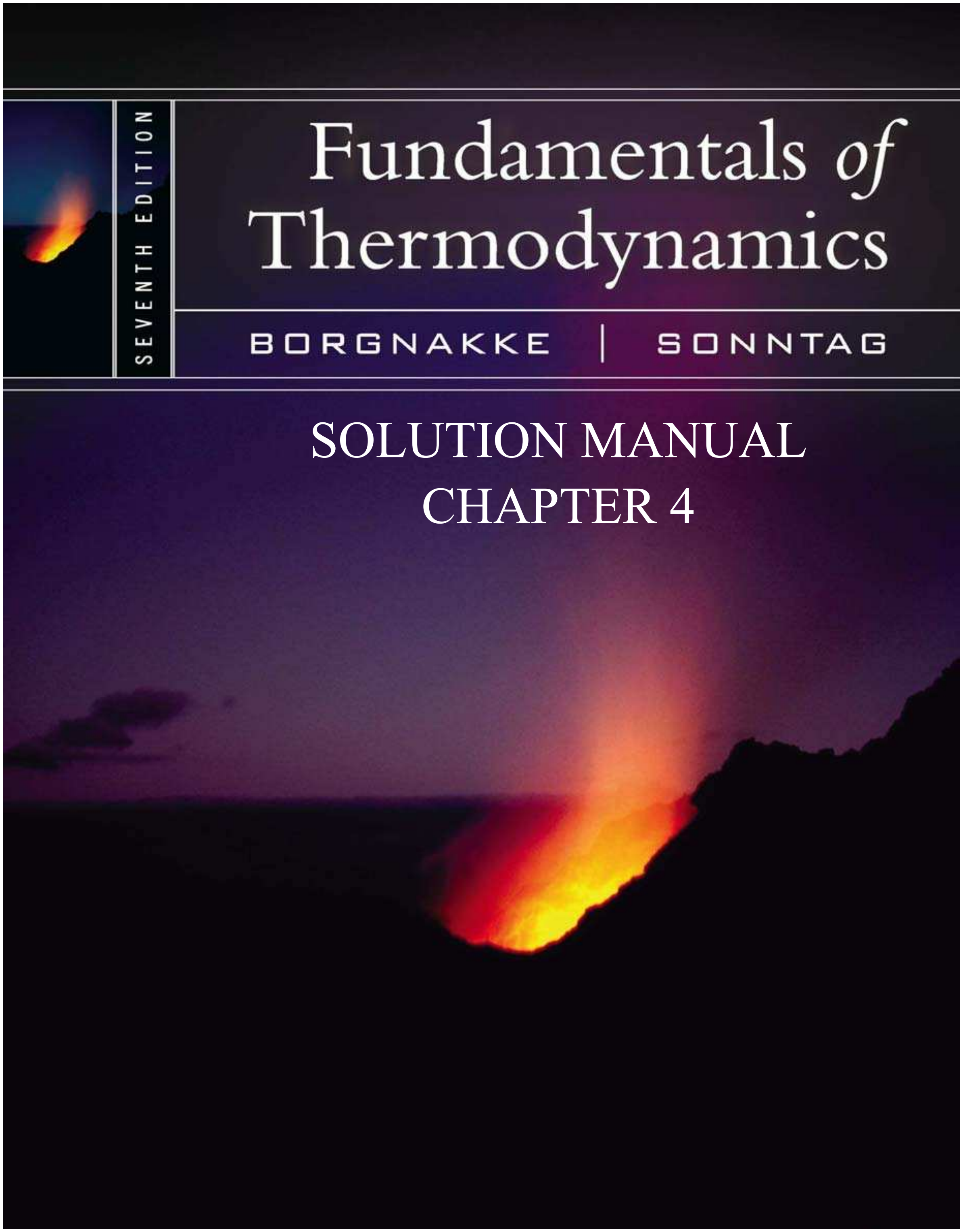


3.135

Use the computer software to sketch the variation of pressure with temperature in Problem 3.44. Extend the curve a little into the single-phase region.

P was found for a number of temperatures. A small table of (P, T) values were entered into a spreadsheet and a graph made as shown below. The superheated vapor region is reached at about 140°C and the graph shows a small kink at that point.





SOLUTION MANUAL
CHAPTER 4

CONTENT

In-Text Concept Questions

4.a

The electric company charges the customers per kW-hour. What is that in SI units?

Solution:

The unit kW-hour is a rate multiplied with time. For the standard SI units the rate of energy is in W and the time is in seconds. The integration in Eq.4.21 becomes

$$1 \text{ kW- hour} = 1000 \text{ W} \times 60 \frac{\text{min}}{\text{hour}} \text{hour} \times 60 \frac{\text{s}}{\text{min}} = 3\ 600\ 000 \text{ Ws}$$

$$= 3\ 600\ 000 \text{ J} = \mathbf{3.6\ MJ}$$

4.b

Torque and energy and work have the same units (N m). Explain the difference.

Solution:

Work = force × displacement, so units are N × m. Energy in transfer

Energy is stored, could be from work input 1 J = 1 N m

Torque = force × arm static, no displacement needed

4.c

What is roughly the relative magnitude of the work in the process 1-2c versus the process 1-2a shown in figure 4.8?

By visual inspection the area below the curve 1-2c is roughly 50% of the rectangular area below the curve 1-2a. To see this better draw a straight line from state 1 to point f on the axis. This curve has exactly 50% of the area below it.

4.d

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. Is the work positive, negative or zero?

The boundary work is: $W = \int P \, dV$

P drops but does V go up or down?

The process equation is: $PV^n = C$

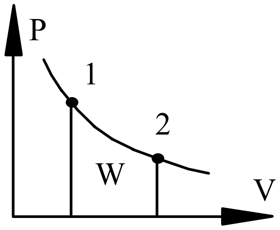
so we can solve for P to show it in a P-V diagram

$$P = CV^{-n}$$

as n = 1.667 the curve drops as V goes up we see

$$V_2 > V_1 \quad \text{giving} \quad dV > 0$$

and the work is then positive.

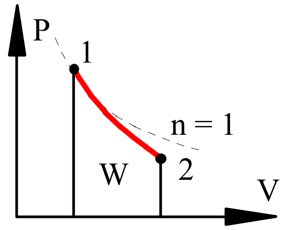


4.e

An ideal gas goes through an expansion process where the volume doubles. Which process will lead to the larger work output: an isothermal process or a polytropic process with $n = 1.25$?

The process equation is: $PV^n = C$

The polytropic process with $n = 1.25$ drops the pressure faster than the isothermal process with $n = 1$ and the area below the curve is then smaller.



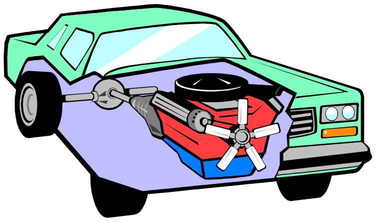
Concept Problems

4.1

A car engine is rated at 160 hp. What is the power in SI units?

Solution:

The horsepower is an older unit for power usually used for car engines. The conversion to standard SI units is given in Table A.1



1 hp = 0.7355 kW = 735.5 W

1 hp = 0.7457 kW for the UK horsepower

$$160 \text{ hp} = 160 \times 745.7 \text{ W} = 119\,312 \text{ W} = \mathbf{119.3\ kW}$$

4.2

Two engines provide the same amount of work to lift a hoist. One engine can provide 3 F in a cable and the other 1 F, What can you say about the motion of the point where the force F acts in the two engines?

Since the two work terms are the same we get

$$W = \int F\, dx = 3\ F\ x_1 = 1\ F\ x_2$$

$$x_2 = 3\ x_1$$

so the lower force has a larger displacement.

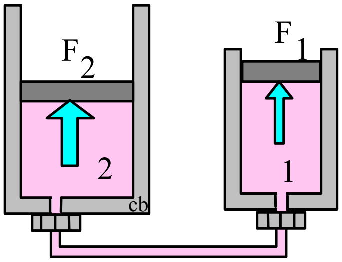
4.3

Two hydraulic piston/cylinders are connected through a hydraulic line so they have roughly the same pressure. If they have diameters of D_1 and $D_2 = 2D_1$ respectively, what can you say about the piston forces F_1 and F_2?

For each cylinder we have the total force as: $F = PA_{cyl} = P\,\pi\,D^2/4$

$$F_1 = PA_{cyl\ 1} = P\,\pi\,D_1^2/4$$

$$F_2 = PA_{cyl\ 2} = P\,\pi\,D_2^2/4 = P\,\pi\,4\,D_1^2/4 = 4\,F_1$$



The forces are the total force acting up due to the cylinder pressure. There must be other forces on each piston to have a force balance so the pistons do not move.

4.4

Normally pistons have a flat head, but in diesel engines pistons can have bowls in them and protruding ridges. Does this geometry influence the work term?

The shape of the surface does not influence the displacement

$$dV = A_n \, dx$$

where A_n is the area projected to the plane normal to the direction of motion.

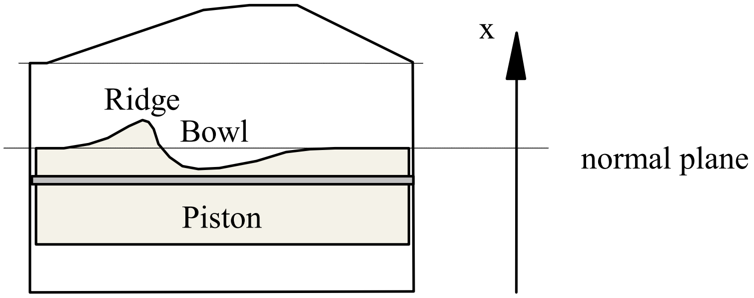
$$A_n = A_{cyl} = \pi D^2/4$$

Work is

$$dW = F \, dx = P \, dV = P \, A_n \, dx = P \, A_{cyl} \, dx$$

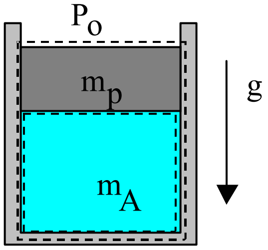
and thus unaffected by the surface shape.

Semi-spherical head is made to make room for larger valves.



4.5

CV A is the mass inside a piston-cylinder, CV B is that plus the piston, outside which is the standard atmosphere. Write the process equation and the work term for the two CVs.



Solution:

C.V. A: Process: $P = P_o + m_p g/A_{cyl} = C$,

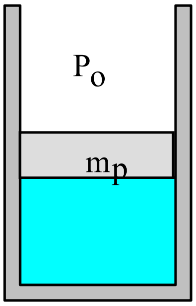
$$ {}_1W_2 = \int P\, dV = P_1 \int m\, dv = P_1 m(v_2 - v_1) $$

C.V. B: Process: $P = P_o = C$,

$$ {}_1W_2 = \int P_o\, dV = P_o \int m\, dv = P_o m(v_2 - v_1) $$

4.6

Assume a physical set-up as in Fig. P4.5. We now heat the cylinder. What happens to P, T and v (up, down or constant)? What transfers do we have for Q and W (pos., neg., or zero)?



Solution:

Process: $P = P_o + m_pg/A_{cyl} = C$

Heat in so T increases, v increases and Q is positive.

As the volume increases the work is positive. ${}_1W_2 = \int P\, dV$

4.7

For a buffer storage of natural gas (CH_4) a large bell in a container can move up and down keeping a pressure of 105 kPa inside. The sun then heats the container and the gas from 280 K to 300 K during 4 hours. What happens to the volume and what is the sign of the work term?

Solution

The process has constant pressure

Ideal gas: $PV = mRT$ as T increases then V **increases**

$_1W_2 = \int P\, dV > 0$ so **positive**.

4.8

A drag force on an object moving through a medium (like a car through air or a submarine through water) is $F_d = 0.225 \; A \; \rho \mathbf{V}^2$. Verify the unit becomes Newton.

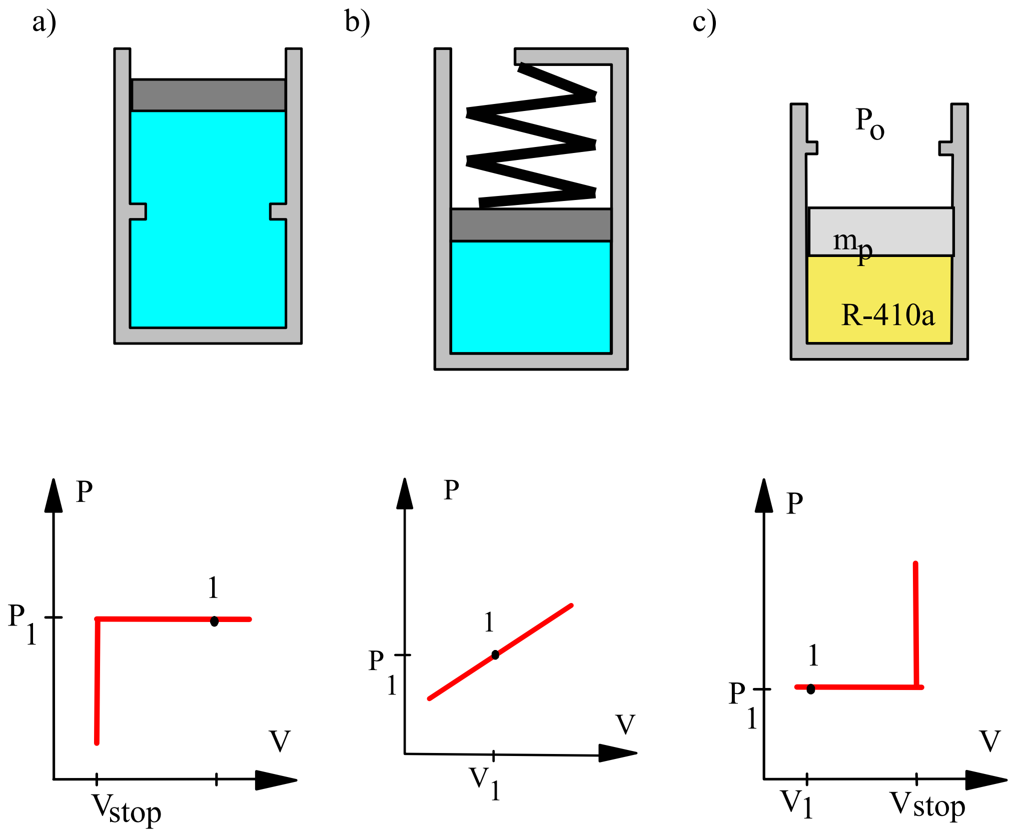
Solution:

$$F_d = 0.225 \; A \; \rho \mathbf{V}^2$$

$$\text{Units} = \; m^2 \; \times (\; kg/m^3 \;) \times (\; m^2/ \; s^2 \;) = kg \; m \; / \; s^2 = N$$

4.9

The sketch shows a physical situation, show the possible process in a P-v diagram.



4.10

For the indicated physical set-up in a-b and c above write a process equation and the expression for work.

a) $P = P_1$ and $V \geq V_{stop}$ or $V = V_{stop}$ and $P \leq P_1$

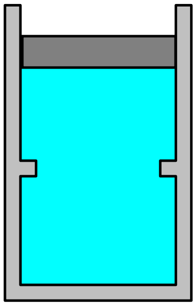
$${}_1W_2 = P_1(V_2 - V_1) \quad [\, P_1 = P_{float} \,]$$

b) $P = A + BV$; $\quad {}_1W_2 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

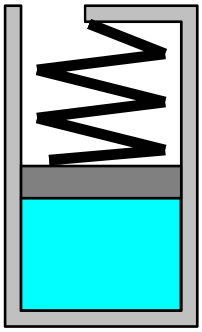
c) $P = P_1$ and $V \leq V_{stop}$ or $V = V_{stop}$ and $P \geq P_1$

$${}_1W_2 = P_1(V_2 - V_1) \quad [\, P_1 = P_{float} \,]$$

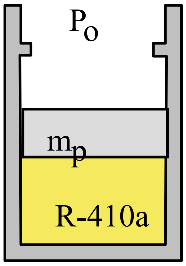
a)

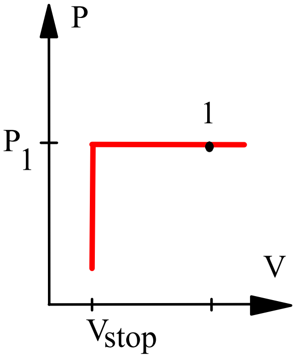


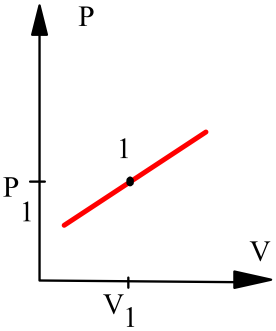
b)

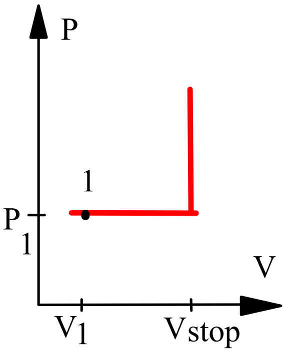


c)



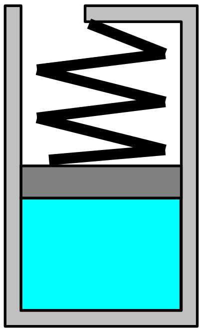






4.11

Assume the physical situation as in Fig. P4.9b; what is the work term a, b, c or d?



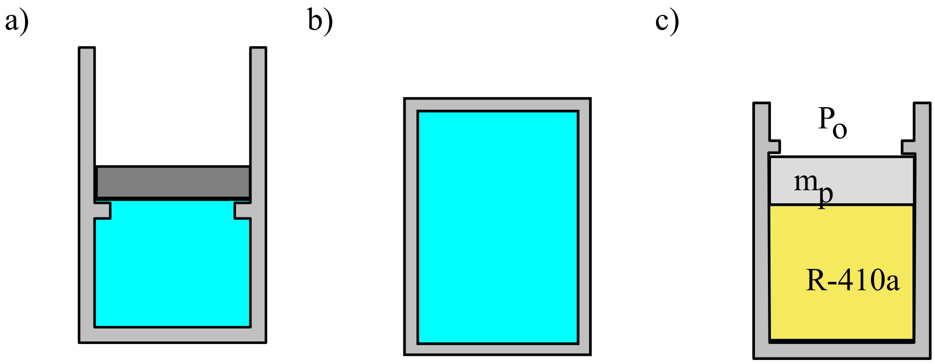
a: $_1w_2 = P_1(v_2 - v_1)$ b: $_1w_2 = v_1(P_2 - P_1)$

c: $_1w_2 = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$ d: $_1w_2 = \frac{1}{2}(P_1 - P_2)(v_2 + v_1)$

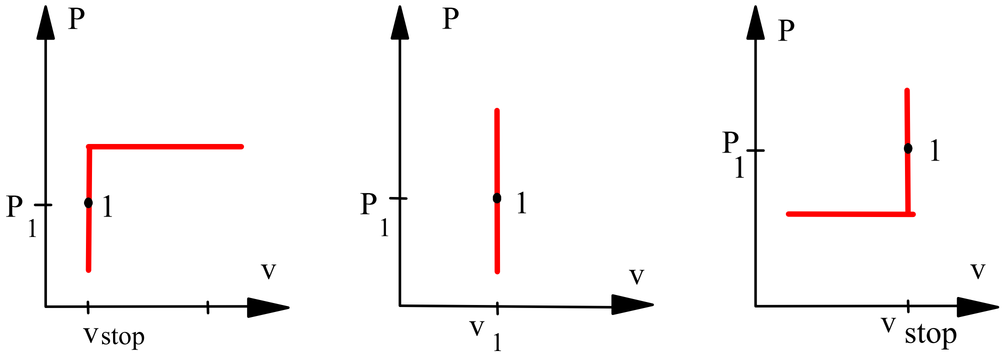
Solution: work term is formula c the area under the process curve in a P-v diagram.

4.12

The sketch in Fig. P4.12 shows a physical situation; show the possible process in a P-v diagram.



Solution:



4.13

What can you say about the beginning state of the R-410a in Fig. P4.9 versus the case in Fig. P4.12 for the same piston-cylinder?

For the case where the piston floats as in Fig. P4.9 the pressure of the R-410a must equal the equilibrium pressure that floats (balance forces on) the piston.

The situation in Fig. P4.12 is possible if the R-410a pressure equals or exceeds the float pressure.

4.14

Show how the polytropic exponent *n* can be evaluated if you know the end state properties, (P_1, V_1) and (P_2, V_2).

Polytropic process: $PV^n = C$

Both states must be on the process line: $P_2V_2^n = C = P_1V_1^n$

Take the ratio to get: $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$

and then take ln of the ratio

$$\ln\left(\frac{P_1}{P_2}\right) = \ln\left(\frac{V_2}{V_1}\right)^n = n \ln\left(\frac{V_2}{V_1}\right)$$

now solve for the exponent n

$$n = \ln\left(\frac{P_1}{P_2}\right) \Big/ \ln\left(\frac{V_2}{V_1}\right)$$

4.15

A piece of steel has a conductivity of k = 15 W/mK and a brick has k = 1 W/mK. How thick a steel wall will provide the same insulation as a 10 cm thick brick?

The heat transfer due to conduction is from Eq. 4.18

$$\dot{Q} = -\,kA\,\frac{dT}{dx} \approx kA\,\frac{\Delta T}{\Delta x}$$

For the same area and temperature difference the heat transfers become the same for equal values of (k / Δx) so

$$\left(\frac{k}{\Delta x}\right)_{brick} = \left(\frac{k}{\Delta x}\right)_{steel} \qquad =>$$

$$\Delta x_{steel} = \Delta x_{brick}\,\frac{k_{steel}}{k_{brick}} = 0.1\text{ m} \times \frac{15}{1} = \mathbf{1.5\ m}$$

4.16

A thermopane window, see Fig. 4.27, traps some gas between the two glass panes. Why is this beneficial?

The gas has a very low conductivity relative to a liquid or solid so the heat transfer for a given thickness becomes smaller. The gap is furthermore made so small that possible natural convection motion is reduced to a minimum. It becomes a trade off to minimize the overall heat transfer due to conduction and convection. Typically these windows can be manufactured with an E-glaze to reduce radiation loss (winter) or gain (summer).

4.17

On a chilly 10°C fall day a house, 20°C inside, loses 6 kW by heat transfer. What transfer happens on a 30°C warm summer day assuming everything else is the same?

The heat transfer is $\dot{Q} = CA\,\Delta T$ where the details of the heat transfer is in the factor C. Assuming those details are the same then it is the temperature difference that changes the heat transfer so

$$\dot{Q} = CA\,\Delta T = 6\text{ kW} = CA\,(20 - 10) \quad => \quad CA = 0.6\,\frac{\text{kW}}{\text{K}}$$

Then

$$\dot{Q} = CA\,\Delta T = 0.6\,\frac{\text{kW}}{\text{K}} \times (20 - 30)\text{ K} = -\,6\text{ kW} \quad \text{(it goes in)}$$

Force displacement work

4.18

A piston of mass 2 kg is lowered 0.5 m in the standard gravitational field. Find the required force and work involved in the process.

Solution:

$$F = ma = 2 \text{ kg} \times 9.80665 \text{ m/s}^2 = \mathbf{19.61\ N}$$

$$W = \int F\, dx = F \int dx = F\, \Delta x = 19.61 \text{ N} \times 0.5 \text{ m} = \mathbf{9.805\ J}$$

4.19

A hydraulic cylinder of area 0.01 m^2 must push a 1000 kg arm and shovel 0.5 m straight up. What pressure is needed and how much work is done?

$F = mg = 1000 \text{ kg} \times 9.81 \text{ m/s}^2$
$= 9810 \text{ N} = PA$

$P = F/A = 9810 \text{ N}/ 0.01 \text{ m}^2$
$= 981\,000 \text{ Pa} = \mathbf{981\ kPa}$

$W = \int F\, dx = F\, \Delta x = 9810 \text{ N} \times 0.5 \text{ m} = \mathbf{4905\ J}$

4.20

An escalator raises a 100 kg bucket of sand 10 m in 1 minute. Determine the total amount of work done during the process.

Solution:

The work is a force with a displacement and force is constant: F = mg

$$W = \int F\,dx = F \int dx = F\,\Delta x = 100\text{ kg} \times 9.80665\text{ m/s}^2 \times 10\text{ m} = \mathbf{9807\ J}$$

4.21

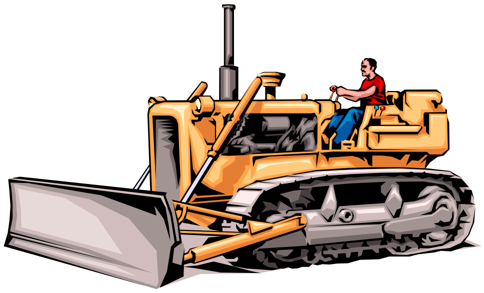
A bulldozer pushes 500 kg of dirt 100 m with a force of 1500 N. It then lifts the dirt 3 m up to put it in a dump truck. How much work did it do in each situation?

Solution:

$W = \int F\,dx = F\,\Delta x$

$= 1500\text{ N} \times 100\text{ m}$

$= 150\,000\text{ J} = \mathbf{150\ kJ}$



$W = \int F\,dz = \int mg\,dz = mg\,\Delta Z$

$= 500\text{ kg} \times\ 9.807\text{ m/s}^2 \times 3\text{ m}$

$= 14\,710\text{ J} = \mathbf{14.7\ kJ}$

4.22

A hydraulic cylinder has a piston of cross sectional area 15 cm^2 and a fluid pressure of 2 MPa. If the piston is moved 0.25 m how much work is done?

Solution:

The work is a force with a displacement and force is constant: F = PA

$$W = \int F\,dx = \int PA\,dx = PA\,\Delta x$$

$$= 2000 \text{ kPa} \times 15 \times 10^{-4} \text{ m}^2 \times 0.25 \text{ m} = \mathbf{0.75\ kJ}$$

Units: $\text{kPa m}^2 \text{ m} = \text{kN m}^{-2} \text{ m}^2 \text{ m} = \text{kN m} = \text{kJ}$

4.23

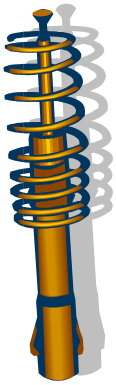
A linear spring, $F = k_s(x - x_0)$, with spring constant $k_s = 500$ N/m, is stretched until it is 100 mm longer. Find the required force and work input.

Solution:

$$F = k_s(x - x_0) = 500 \times 0.1 = \mathbf{50\ N}$$

$$W = \int F\,dx = \int k_s(x - x_0)d(x - x_0) = k_s(x - x_0)^2/2$$

$$= 500\ \frac{N}{m} \times (0.1^2/2)\ m^2 = \mathbf{2.5\ J}$$



4.24

Two hydraulic cylinders maintain a pressure of 1200 kPa. One has a cross sectional area of 0.01 m^2 the other 0.03 m^2. To deliver a work of 1 kJ to the piston how large a displacement (V) and piston motion H is needed for each cylinder? Neglect P_{atm}.

Solution:

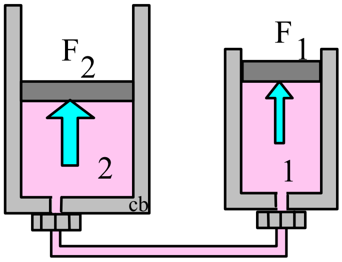
$$W = \int F\,dx = \int P\,dV = \int PA\,dx = PA^*\,H = P\Delta V$$

$$\Delta V = \frac{W}{P} = \frac{1\text{ kJ}}{1200\text{ kPa}} = \mathbf{0.000\,833\ m^3}$$

Both cases the height is $H = \Delta V/A$

$$H_1 = \frac{0.000833}{0.01} = \mathbf{0.0833\ m}$$

$$H_2 = \frac{0.000833}{0.03} = \mathbf{0.0278\ m}$$



4.25

Two hydraulic piston/cylinders are connected with a line. The master cylinder has an area of 5 cm^2 creating a pressure of 1000 kPa. The slave cylinder has an area of 3 cm^2. If 25 J is the work input to the master cylinder what is the force and displacement of each piston and the work out put of the slave cylinder piston?

Solution:

$$W = \int F_x \, dx = \int P \, dv = \int P \, A \, dx = P \, A \, \Delta x$$

$$\Delta x_{master} = \frac{W}{PA} = \frac{25}{1000 \times 5 \times 10^{-4}} = 0.05 \text{ m}$$

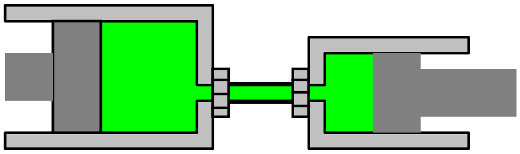
$$A\Delta x = \Delta V = 5 \times 10^{-4} \times 0.05 = 2.5 \times 10^{-5} \text{ m} = \Delta V_{slave} = A \, \Delta x \rightarrow$$

$$\Delta x_{slave} = \Delta V/A = 2.5 \times 10^{-5} / 3 \times 10^{-4} = 0.0083\,33 \text{ m}$$

$$F_{master} = P \, A = 1000 \times 5 \times 10^{-4} \times 10^{3} = \mathbf{500 \text{ N}}$$

$$F_{slave} = P \, A = 1000 \times 10^{3} \times 3 \times 10^{-4} = \mathbf{300 \text{ N}}$$

$$W_{slave} = F \, \Delta x = 300 \times 0.08333 = \mathbf{25 \text{ J}}$$



4.26

The rolling resistance of a car depends on its weight as: F = 0.006 m g. How long will a car of 1400 kg drive for a work input of 25 kJ?

Solution:

Work is force times distance so assuming a constant force we get

$$W = \int F\,dx = F\,x = 0.006\ mgx$$

Solve for x

$$x = \frac{W}{0.006\ mg} = \frac{25\ \text{kJ}}{0.006 \times 1400\ \text{kg} \times 9.807\ \text{m/s}^2} = \mathbf{303.5\ m}$$

4.27

The air drag force on a car is 0.225 A $\rho \mathbf{V}^2$. Assume air at 290 K, 100 kPa and a car frontal area of 4 m^2 driving at 90 km/h. How much energy is used to overcome the air drag driving for 30 minutes?

The formula involves density and velocity and work involves distance so:

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{100}{0.287 \times 290} = 1.2015 \frac{kg}{m^3}$$

$$\mathbf{V} = 90 \frac{km}{h} = 90 \times \frac{1000}{3600} \frac{m}{s} = 25 \text{ m/s}$$

$$\Delta x = \mathbf{V}\, \Delta t = 25 \text{ m/s} \times 30 \text{ min} \times 60 \text{ s/min} = 45\,000 \text{ m}$$

Now

$$F = 0.225\, A\, \rho\, \mathbf{V}^2 = 0.225 \times 4 \times 1.2015 \times 25^2$$

$$= 675.8 \text{ m}^2 \frac{kg}{m^3} \times \frac{m^2}{s^2} = \mathbf{676\ N}$$

$$W = F\, \Delta x = 676 \text{ N} \times 45\,000 \text{ m} = 30\,420\,000 \text{ J} = \mathbf{30.42\ MJ}$$

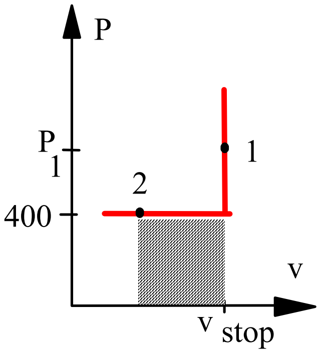
Boundary work simple 1 step process

4.28

The R-410a in Problem 4.12 c is at 1000 kPa, 50°C with mass 0.1 kg. It is cooled so the volume is reduced to half the initial volume . The piston mass and gravitation is such that a pressure of 400 kPa will float the piston. Find the work in the process.

If the volume is reduced the piston must drop and thus float with P = 400 kPa. The process therefore follows a process curve shown in the P-V diagram.

Table B.4.2: $v_1 = 0.03320\ \text{m}^3/\text{kg}$



$$
\begin{aligned}
{}_1W_2 &= \int P dV = \text{area} \\
&= P_{float}\,(V_2 - V_1) = -P_{float}\,V_1/2 \\
&= -400\ \text{kPa} \times 0.1\ \text{kg} \times 0.0332\ \text{m}^3/\text{kg}\ /2 \\
&= \mathbf{-\ 0.664\ kJ}
\end{aligned}
$$

4.29

A steam radiator in a room at 25°C has saturated water vapor at 110 kPa flowing through it, when the inlet and exit valves are closed. What is the pressure and the quality of the water, when it has cooled to 25^{o}C? How much work is done?

Solution: Control volume radiator.

After the valve is closed no more flow, constant volume and mass.

1: $x_1 = 1$, $P_1 = 110$ kPa $\Rightarrow$ $v_1 = v_g = 1.566\ m^3/kg$ from Table B.1.2

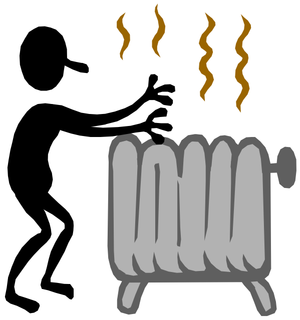
2: $T_2 = 25^{o}C$, ?

Process: $v_2 = v_1 = 1.566\ m^3/kg = [0.001003 + x_2 \times 43.359]\ m^3/kg$

$$x_2 = \frac{1.566 - 0.001003}{43.359} = \mathbf{0.0361}$$

State 2 : T_2 , x_2 From Table B.1.1 $\mathbf{P_2 = Psat = 3.169\ kPa}$

$${}_1W_2 = \int PdV = \mathbf{0}$$



4.30

A constant pressure piston cylinder contains 0.2 kg water as saturated vapor at 400 kPa. It is now cooled so the water occupies half the original volume. Find the work in the process.

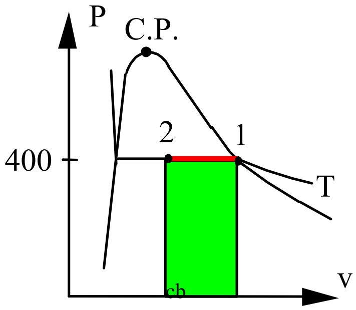
Solution:

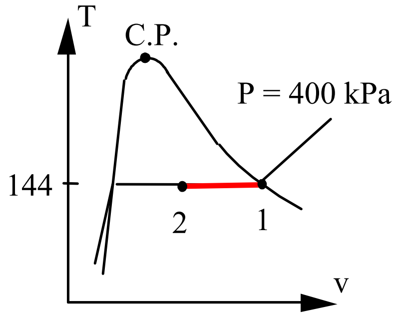
Table B.1.2 $v_1 = 0.4625\ \mathrm{m^3/kg}$ $V_1 = mv_1 = 0.0925\ \mathrm{m^3}$

$v_2 = v_1 / 2 = 0.23125\ \mathrm{m^3/kg}$ $V_2 = V_1 / 2 = 0.04625\ \mathrm{m^3}$

Process: $P = C$ so the work term integral is

$W = \int P dV = P(V_2 - V_1) = 400\ \mathrm{kPa} \times (0.04625 - 0.0925)\ \mathrm{m^3} = \mathbf{-18.5\ kJ}$





4.31

Find the specific work in Problem 3.47.
Solution:

State 1 from Table B.1.2 at 200 kPa

$$v = v_f + x\, v_{fg} = 0.001061 + 0.25 \times 0.88467 = 0.22223 \text{ m}^3/\text{kg}$$

State 2 has same P from Table B.1.2 at 200 kPa

$$T_2 = T_{sat} + 20 = 120.23 + 20 = 140.23^{\text{o}}\text{C}$$

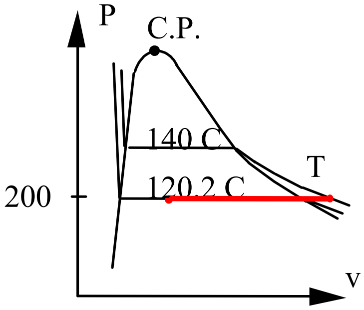
so state is superheated vapor

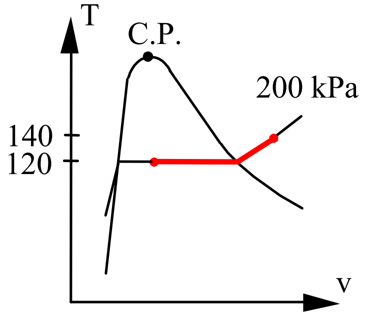
$$v_2 = 0.88573 + (0.95964 - 0.88573)\frac{20}{150 - 120.23} = 0.9354 \text{ m}^3/\text{kg}$$

Process P = C: $\quad {}_1w_2 = \int P\, dv = P_1(v_2 - v_1)$

$= 200\ (0.9354 - 0.22223)$

$=$ **142.6 kJ/kg**





4.32

A 400-L tank A, see figure P4.32, contains argon gas at 250 kPa, 30°C. Cylinder B, having a frictionless piston of such mass that a pressure of 150 kPa will float it, is initially empty. The valve is opened and argon flows into B and eventually reaches a uniform state of 150 kPa, 30°C throughout. What is the work done by the argon?

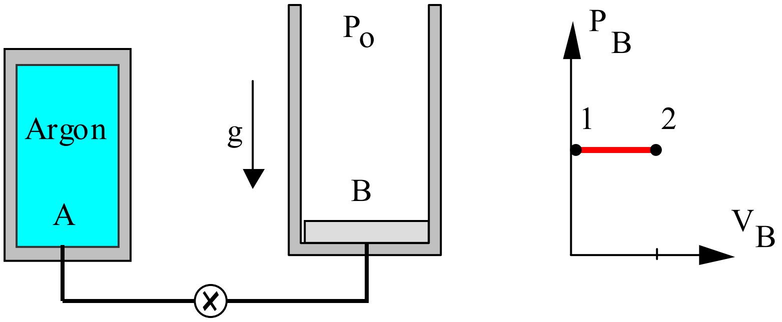
Solution:

Take C.V. as all the argon in both A and B. Boundary movement work done in cylinder B against constant external pressure of 150 kPa. Argon is an ideal gas, so write out that the mass and temperature at state 1 and 2 are the same

$$P_{A1}V_A = m_ART_{A1} = m_ART_2 = P_2(V_A + V_{B2})$$

$$\Rightarrow \quad V_{B2} = \frac{250 \times 0.4}{150} - 0.4 = 0.2667 \text{ m}^3$$

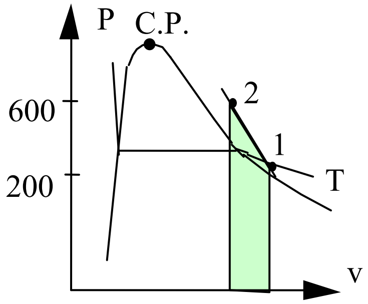
$${}_1W_2 = \int_1^2 P_{ext}dV = P_{ext}(V_{B2} - V_{B1}) = 150 \text{ kPa } (0.2667 - 0) \text{ m}^3 = \mathbf{40\ kJ}$$



Notice there is a pressure loss in the valve so the pressure in B is always 150 kPa while the piston floats.

4.33

A piston cylinder contains 1.5 kg water at 200 kPa, 150°C. It is now heated in a process where pressure is linearly related to volume to a state of 600 kPa, 350°C. Find the final volume and the work in the process.



Take as CV the 1.5 kg of water.

$m_2 = m_1 = m$;

Process Eq.: $P = A + BV$ (linearly in V)

State 1: (P, T) => $v_1 = 0.95964\ m^3/kg$,

State 2: (P, T) => $v_2 = 0.47424\ m^3/kg$, $V_2 = mv_2 = 0.7114\ m^3$

From process eq.:

$$_1W_2 = \int P\, dV = \text{area} = \frac{m}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1.5}{2}\text{ kg } (200 + 600)\text{ kPa } (0.47424 - 0.95964)\ m^3/kg = \mathbf{-291.24\ kJ}$$

Notice volume is reduced so work is negative.

4.34

A cylinder fitted with a frictionless piston contains 5 kg of superheated refrigerant R-134a vapor at 1000 kPa, 140°C. The setup is cooled at constant pressure until the R-134a reaches a quality of 25%. Calculate the work done in the process.

Solution:

Constant pressure process boundary work. State properties from Table B.5.2

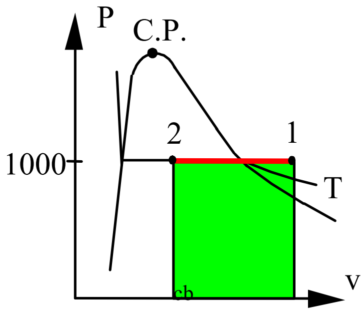
State 1: $v = 0.03150 \text{ m}^3\text{/kg}$,

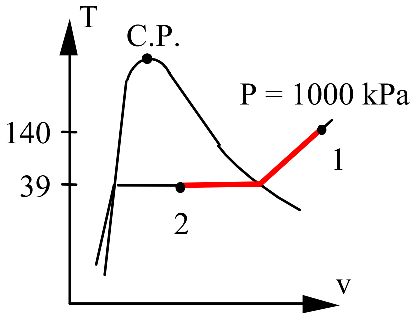
State 2: $v = 0.000871 + 0.25 \times 0.01956 = 0.00576 \text{ m}^3\text{/kg}$

Interpolated to be at 1000 kPa, numbers at 1017 kPa could have been used in which case: $v = 0.00566 \text{ m}^3\text{/kg}$

$${}_1W_2 = \int P\, dV = P\,(V_2 - V_1) = mP\,(v_2 - v_1)$$

$$= 5 \times 1000\,(0.00576 - 0.03150) = \mathbf{-128.7\ kJ}$$





4.35

A piston cylinder contains air at 600 kPa, 290 K and a volume of 0.01 m^3. A constant pressure process gives 54 kJ of work out. Find the final volume and temperature of the air.

Solution:

$$W = \int P\ dV = P\,\Delta V$$

$$\Delta V = W/P = \frac{54}{600} = 0.09\ m^3$$

$$V_2 = V_1 + \Delta V = 0.01 + 0.09 = 0.1\ m^3$$

Assuming ideal gas, PV = mRT, then we have

$$T_2 = \frac{P_2\,V_2}{mR} = \frac{P_2\,V_2}{P_1\,V_1}T_1 = \frac{V_2}{V_1}T_1 = \frac{0.1}{0.01} \times 290 = \mathbf{2900\ K}$$

4.36

A piston/cylinder has 5 m of liquid 20°C water on top of the piston (m = 0) with cross-sectional area of 0.1 m^2, see Fig. P2.56. Air is let in under the piston that rises and pushes the water out over the top edge. Find the necessary work to push all the water out and plot the process in a P-V diagram.

Solution:

$$P_1 = P_o + \rho g H$$

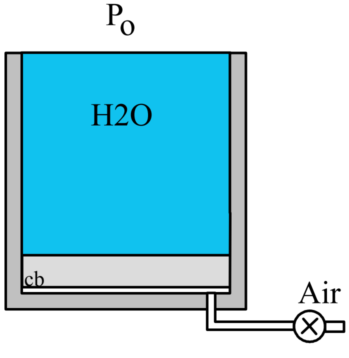
$$= 101.32 + 997 \times 9.807 \times 5 / 1000 = 150.2 \text{ kPa}$$

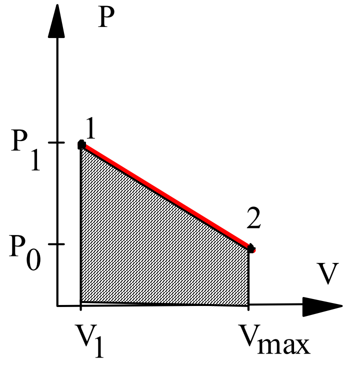
$$\Delta V = H \times A = 5 \times 0.1 = 0.5 \text{ m}^3$$

$${}_1W_2 = \text{AREA} = \int P \, dV = ½ (P_1 + P_o)(V_{max} - V_1)$$

$$= ½ (150.2 + 101.32) \text{ kPa} \times 0.5 \text{ m}^3$$

$$\mathbf{= 62.88 \text{ kJ}}$$





4.37

Saturated water vapor at 200 kPa is in a constant pressure piston cylinder. At this state the piston is 0.1 m from the cylinder bottom and cylinder area is 0.25 m^2. The temperature is then changed to 200°C. Find the work in the process.

Solution:

State 1 from B.1.2 (P, x): $v_1 = v_g = 0.8857\ m^3/kg$ (also in B.1.3)

State 2 from B.1.3 (P, T): $v_2 = 1.0803\ m^3/kg$

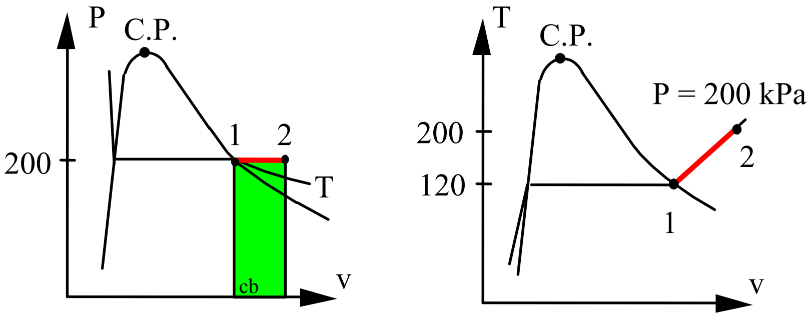
Since the mass and the cross sectional area is the same we get

$$h_2 = \frac{v_2}{v_1} \times h_1 = \frac{1.0803}{0.8857} \times 0.1 = 0.122\ \text{m}$$

Process: P = C so the work integral is

$$W = \int PdV = P(V_2 - V_1) = PA\ (h_2 - h_1)$$

$$W = 200\ \text{kPa} \times 0.25\ \text{m}^2 \times (0.122 - 0.1)\ \text{m} = \mathbf{1.1\ kJ}$$



4.38

A piston cylinder contains 1 kg of liquid water at 20^{o}C and 300 kPa, as shown in Fig. P4.38. There is a linear spring mounted on the piston such that when the water is heated the pressure reaches 3 MPa with a volume of 0.1 m^3.

a) Find the final temperature

b) Plot the process in a P-v diagram.

c) Find the work in the process.

Solution:

Take CV as the water. This is a constant mass:

$$m_2 = m_1 = m \;;$$

State 1: Compressed liquid, take saturated liquid at same temperature.

$$\text{B.1.1:}\;\; v_1 = v_f(20) = 0.001002 \text{ m}^3\text{/kg},$$

State 2: $v_2 = V_2/m = 0.1/1 = 0.1$ m^3/kg and P = 3000 kPa from B.1.3

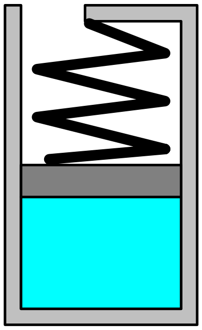
=> Superheated vapor close to T = 400^{o}C

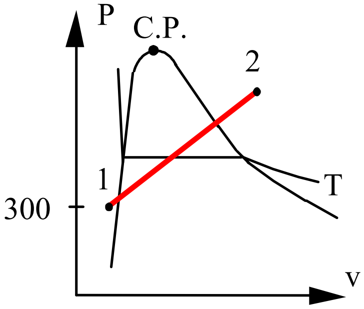
Interpolate: $T_2 =$ **404^{o}C**

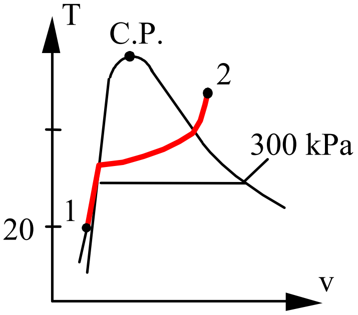
Work is done while piston moves at linearly varying pressure, so we get:

$$_1W_2 = \int P\, dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}\,(P_1 + P_2)(V_2 - V_1)$$

$$= 0.5\;(300 + 3000)(0.1 - 0.001) = \mathbf{163.35\ kJ}$$







4.39

Find the specific work in Problem 3.53 for the case the volume is reduced.

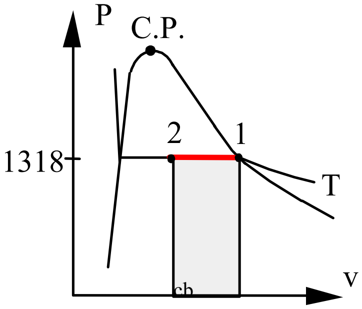
Saturated vapor R-134a at 50°C changes volume at constant temperature. Find the new pressure, and quality if saturated, if the volume doubles. Repeat the question for the case the volume is reduced to half the original volume.

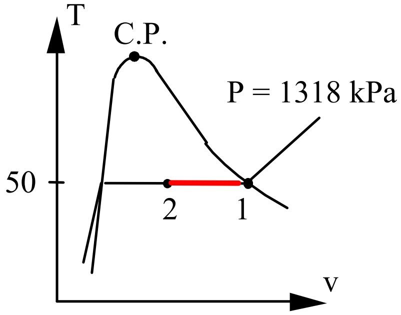
Solution:

R-134a 50°C

Table B.4.1: $v_1 = v_g = 0.01512\ m^3/kg$, $v_2 = v_1 / 2 = 0.00756\ m^3/kg$

$_1W_2 = \int PdV = 1318.1\ kPa\ (0.00756 - 0.01512)\ m^3/kg = \mathbf{-9.96\ kJ/kg}$





4.40

A piston/cylinder contains 1 kg water at 20°C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature, volume and the process work.

Solution

1: $v_1 = V/m = 0.1\ m^3/1\ kg = 0.1\ m^3/kg$ (two-phase state)

2: Constant volume: $v_2 = v_g = v_1$

$$V_2 = V_1 = \mathbf{0.1\ m^3}$$

$$_1W_2 = \int P\, dV = \mathbf{0}$$

State 2: $(v_2, x_2 = 1)$

$$T_2 = T_{sat} = 210 + 5\,\frac{0.1 - 0.10324}{0.09361 - 0.10324} = \mathbf{211.7°C}$$

4.41

Ammonia (0.5 kg) is in a piston cylinder at 200 kPa, -10^{o}C is heated in a process where the pressure varies linear with the volume to a state of 120^{o}C, 300 kPa. Find the work the ammonia gives out in the process.

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Process: $P = A + BV$ (linear in V)

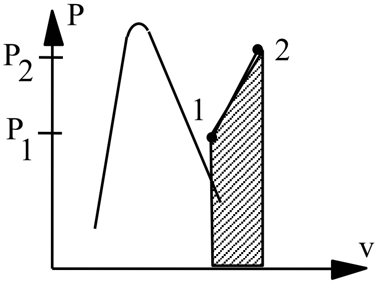
State 1: Superheated vapor $v_1 = 0.6193$ m^3/kg

State 2: Superheated vapor $v_2 = 0.63276$ m^3/kg

Work is done while piston moves at increasing pressure, so we get

$$_1W_2 = \int P\, dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

$$= ½(200 + 300) \times 0.5\,(0.63276 - 0.6193) = \mathbf{1.683\ kJ}$$



4.42

Air in a spring loaded piston/cylinder has a pressure that is linear with volume, $P = A + BV$. With an initial state of P = 150 kPa, V = 1 L and a final state of 800 kPa and volume 1.5 L it is similar to the setup in Problem 4.38. Find the work done by the air.

Solution:

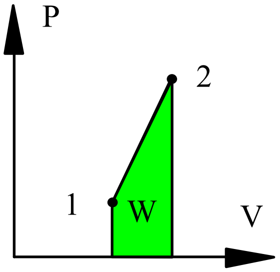
Knowing the process equation: $P = A + BV$ giving a linear variation of pressure versus volume the straight line in the P-V diagram is fixed by the two points as state 1 and state 2. The work as the integral of PdV equals the area under the process curve in the P-V diagram.

State 1: $P_1 = 150$ kPa $\quad V_1 = 1 \text{ L} = 0.001 \text{ m}^3$

State 2: $P_2 = 800$ kPa $\quad V_2 = 1.5 \text{ L} = 0.0015 \text{ m}^3$

Process: $P = A + BV$ $\quad$ linear in V

$$\Rightarrow \quad {}_1W_2 = \int_1^2 PdV = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1)$$



$$= \frac{1}{2}(150 + 800) \text{ kPa } (1.5 - 1) \times 0.001 \text{ m}^3 = \mathbf{0.2375\ kJ}$$

4.43

Air, 3 kg, is in a piston cylinder similar to Fig. P.4.5 at 27^{o}C, 300 kPa. It is now heated to 500 K. Plot the process path in a P-v diagram, and find the work in the process.

Solution:

Ideal gas $PV = mRT$

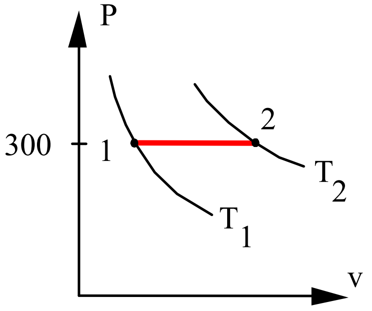
State 1: T_1, P_1 ideal gas so $P_1V_1 = mRT_1$

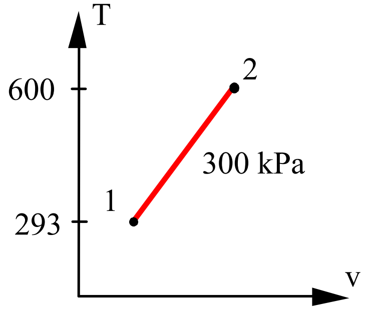
$V_1 = mR\,T_1 / P_1 = 3 \times 0.287 \times 293.15/300 = 0.8413\ m^3$

State 2: $T_2, P_2 = P_1$ and ideal gas so $P_2V_2 = mRT_2$

$V_2 = mR\,T_2 / P_2 = 3 \times 0.287 \times 600/300 = \mathbf{1.722\ m^3}$

${}_1W_2 = \int PdV = P\,(V_2 - V_1) = 300\,(1.722 - 0.8413) = \mathbf{264.2\ kJ}$



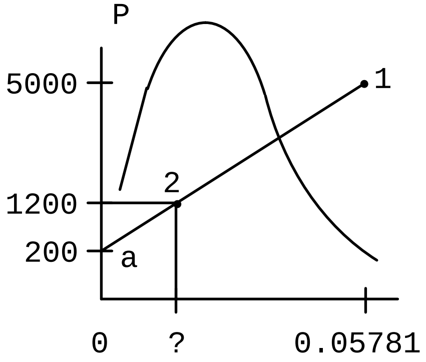


4.44

Find the work for Problem 3.62.

A piston/cylinder arrangement is loaded with a linear spring and the outside atmosphere. It contains water at 5 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that P_{lift} = 200 kPa. The system now cools until the pressure reaches 1200 kPa. Find the mass of water, the final state (T_2, v_2) and plot the P–v diagram for the process.

Solution :



1: 5 MPa, 400°C ⇒ v_1= 0.05781 m^3/kg

$m = V/v_1 = 0.1/0.05781 = 1.73$ kg

Straight line: $P = P_a + Cv$

$$v_2 = v_1 \frac{P_2 - P_a}{P_1 - P_a} = \mathbf{0.01204\ m^3/kg}$$

$v_2 < v_g$(1200 kPa) so two-phase T_2 = **188°C**

$$\Rightarrow\ x_2 = \frac{v_2 - 0.001139}{0.1622} = 0.0672$$

The P-V coordinates for the two states are then:

(P_1 = 5 MPa, V_1 = 0.1 m^3), (P_2 = 1200 kPa, $V_2 = mv_2$ = 0.02083 m^3)

P vs. V is linear so $_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

$$= \frac{1}{2}(5000 + 1200)(0.02083 - 0.1)\ \text{kPa-m}^3$$

$$= \mathbf{-245.4\ kJ}$$

4.45

Heat transfer to a block of 1.5 kg ice at -10^{o}C melts it to liquid at 10^{o}C in a kitchen. How much work does the water gives out?

Work is done against the atmosphere due to volume change in the process.

State 1: Compressed solid, B.1.5, $v_1 = 0.0010891$ m^3/kg

State 2: Compressed liquid B.1.1 $v_2 = 0.001000$ m^3/kg

$$_1W_2 = \int PdV = P_o (V_2 - V_1) = P_o\, m\, (v_2 - v_1)$$

$$= 101.325 \text{ kPa} \times 1.5 \text{ kg} \times (0.001 - 0.0010891) \text{ m}^3\text{/kg}$$

$$= \mathbf{-\ 0.0135\ kJ}$$

Notice the work is negative, the volume is reduced!

4.46

A piston cylinder contains 0.5 kg air at 500 kPa, 500 K. The air expands in a process so P is linearly decreasing with volume to a final state of 100 kPa, 300 K. Find the work in the process.

Solution:

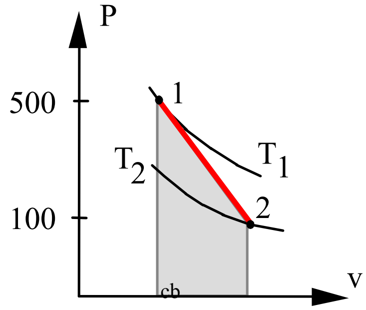
Process: $P = A + BV$ (linear in V, decreasing means B is negative)

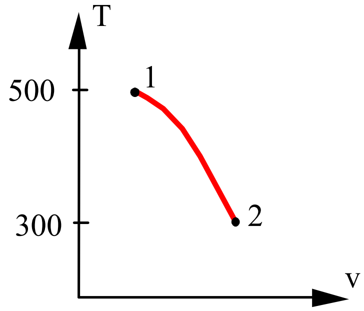
From the process: ${}_1W_2 = \int P dV = \text{AREA} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

$$V_1 = mR\,T_1/P_1 = 0.5 \times 0.287 \times (500/500) = 0.1435 \text{ m}^3$$

$$V_2 = mR\,T_2/P_2 = 0.5 \times 0.287 \times (300/100) = 0.4305 \text{ m}^3$$

$${}_1W_2 = \frac{1}{2} \times (500 + 100) \text{ kPa} \times (0.4305 - 0.1435) \text{ m}^3 = \mathbf{86.1\ kJ}$$





Polytropic process

4.47

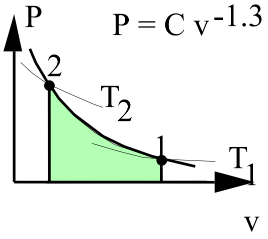
A nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder. It starts out at 600 K, 600 kPa and ends at 800 K. Is the work positive, negative or zero?

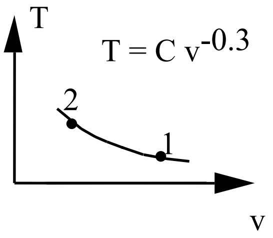
The work is a boundary work so it is

$$W = \int P dV = \int Pm\, dv = \text{AREA}$$

so the sign depends on the sign for dV (or dv). The process looks like the following

The actual process is on a steeper curve than n = 1.





As the temperature increases we notice the volume decreases so

$$dv < 0 \quad \Rightarrow \quad W < 0$$

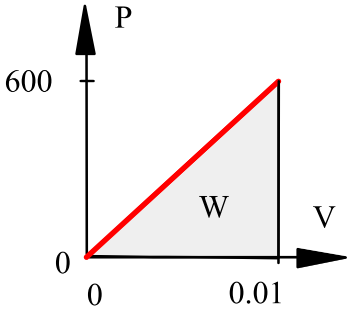
Work is **negative** and goes into the nitrogen gas.

4.48

Consider a mass going through a polytropic process where pressure is directly proportional to volume (n = – 1). The process start with $P = 0$, $V = 0$ and ends with $P = 600$ kPa, $V = 0.01$ m^3. Find the boundary work done by the mass.

Solution:

The setup has a pressure that varies linear with volume going through the initial and the final state points. The work is the area below the process curve.



$$W = \int P dV = \text{AREA}$$

$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= \frac{1}{2}(P_2 + 0)(V_2 - 0)$$

$$= \frac{1}{2} P_2 V_2 = \frac{1}{2} \times 600 \times 0.01 = \mathbf{3\ kJ}$$

4.49

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much work does it give out?

Solution:

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

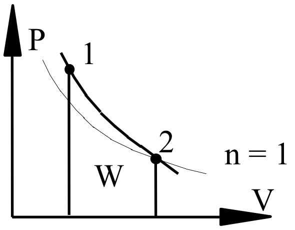
Solve for the volume at state 2

$$V_2 = V_1\ (P_1/P_2)^{1/n} = 0.25 \times \left(\frac{125}{100}\right)^{0.6} = 0.2852\ m^3$$

Work from Eq.4.4

$$_1W_2 = \frac{P_2V_2 - P_1\ V_1}{1-n} = \frac{100\times 0.2852 - 125\times 0.25}{1 - 1.667}\ \text{kPa m}^3 = \mathbf{4.09\ kJ}$$

The actual process is on a steeper curve than n = 1.



4.50

Air at 1500 K, 1000 kPa expands in a polytropic process, n = 1.5, to a pressure of 200 kPa. How cold does the air become and what is the specific work out?

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Solve for the temperature at state 2 by using ideal gas (PV = mRT)

$$\frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1} = \frac{P_2}{P_1} \times \left(\frac{P_1}{P_2}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n}$$

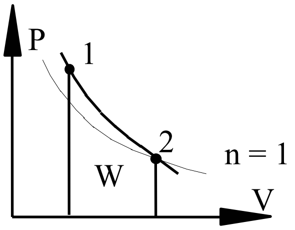
$$T_2 = T_1\ (P_2/P_1)^{(n-1)/n} = 1500 \times \left(\frac{200}{1000}\right)^{(1.5-1)/1.5} = \mathbf{877.2\ K}$$

Work from Eq.4.5

$${}_1w_2 = \frac{P_2\, v_2 - P_1\, v_1}{1\text{-}n} = \frac{R(P_2 - T_1)}{1\text{-}n}$$

$$= \frac{0.287(877.2 - 1500)}{1 - 1.5} = \mathbf{357.5\ kJ/kg}$$

The actual process is on a steeper curve than n = 1.



4.51

The piston/cylinder shown in Fig. P4.51 contains carbon dioxide at 300 kPa, 100°C with a volume of 0.2 m^3. Mass is added at such a rate that the gas compresses according to the relation $PV^{1.2}$ = constant to a final temperature of 200°C. Determine the work done during the process.

Solution:

From Eq. 4.4 for the polytopic process $PV^n = \text{const}\ (n \neq 1)$

$$ {}_1W_2 = \int_1^2 PdV = \frac{P_2V_2 - P_1V_1}{1 - n} $$

Assuming ideal gas, PV = mRT

$$ {}_1W_2 = \frac{mR(T_2 - T_1)}{1 - n}, $$

But $\quad mR = \frac{P_1V_1}{T_1} = \frac{300 \times 0.2}{373.15} \frac{\text{kPa m}^3}{\text{K}} = 0.1608 \text{ kJ/K}$

$$ {}_1W_2 = \frac{0.1608(473.2 - 373.2)}{1 - 1.2} \frac{\text{kJ K}}{\text{K}} = \mathbf{-80.4\ kJ} $$

4.52

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Process: $Pv^n = \text{Const} = P_1 v_1^n = P_2\, v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3\text{/kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3\text{/kg}$$

From the process equation

$$(P_2/ P_1) = (v_1/ v_2)^n \quad => \quad \ln(P_2/ P_1) = n \ln(v_1/ v_2)$$

$$n = \ln(P_2/ P_1) \,/\, \ln(v_1/ v_2) = \frac{\ln 2.4}{\ln 1.56} = \mathbf{1.969}$$

The work is now from Eq.4.4 per unit mass

$$_1w_2 = \frac{P_2v_2\text{-}P_1v_1}{1\text{-}n} = \frac{R(T_2 - T_1)}{1\text{-}n} = \frac{0.287(500 - 325)}{1\text{-}1.969} = \mathbf{-51.8\ kJ/kg}$$

4.53

A gas initially at 1 MPa, 500°C is contained in a piston and cylinder arrangement with an initial volume of 0.1 m^3. The gas is then slowly expanded according to the relation *PV* = constant until a final pressure of 100 kPa is reached. Determine the work for this process.

Solution:

By knowing the process and the states 1 and 2 we can find the relation between the pressure and the volume so the work integral can be performed.

Process: $PV = C \quad \Rightarrow \quad V_2 = P_1V_1/P_2 = 1000 \times 0.1/100 = 1\ m^3$

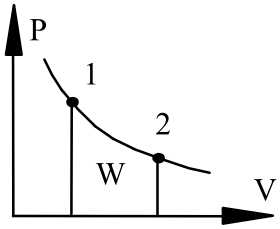
For this process work is integrated to Eq.4.5

$$_1W_2 = \int P\ dV = \int CV^{-1}dV = C\ \ln(V_2/V_1)$$

$$_1W_2 = P_1V_1 \ln \frac{V_2}{V_1}$$

$$= 1000\ \text{kPa} \times 0.1\ m^3 \times \ln(1/0.1)$$

$$= \mathbf{230.3\ kJ}$$



4.54

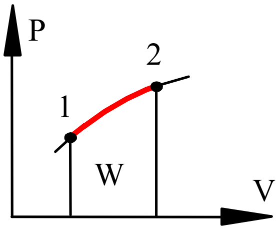
A balloon behaves so the pressure is $P = C_2 V^{1/3}$, $C_2 = 100$ kPa/m. The balloon is blown up with air from a starting volume of 1 m^3 to a volume of 3 m^3. Find the final mass of air assuming it is at 25°C and the work done by the air.

Solution:

The process is polytropic with exponent $n = -1/3$.

$$P_1 = C_2 V^{1/3} = 100 \times 1^{1/3} = 100 \text{ kPa}$$

$$P_2 = C_2 V^{1/3} = 100 \times 3^{1/3} = 144.22 \text{ kPa}$$



$$_1W_2 = \int P\, dV = \frac{P_2V_2 - P_1V_1}{1 - n} \qquad \text{(Equation 4.4)}$$

$$= \frac{144.22 \times 3 - 100 \times 1}{1 - (-1/3)} \text{ kPa-m}^3 = \mathbf{249.5\ kJ}$$

$$m_2 = \frac{P_2V_2}{RT_2} = \frac{144.22 \times 3}{0.287 \times 298} \frac{\text{kPa-m}^3}{\text{kJ/kg}} = \mathbf{5.056\ kg}$$

4.55

A piston cylinder contains 0.1 kg nitrogen at 100 kPa, 27°C and it is now compressed in a polytropic process with n = 1.25 to a pressure of 250 kPa. What is the work involved?

Take CV as the nitrogen. $m_2 = m_1 = m$;

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

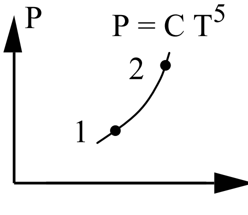
From the ideal gas law and the process equation we can get:

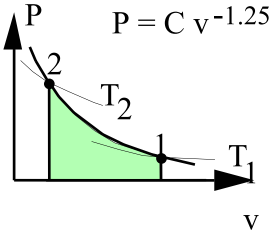
State 2: $T_2 = T_1 \left(P_2 / P_1\right)^{\frac{n-1}{n}} = 300.15 \left(\frac{250}{100}\right)^{\frac{0.25}{1.25}} = 360.5 \text{ K}$

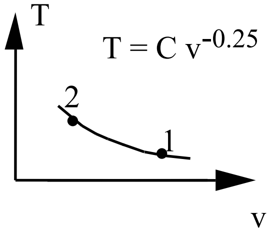
From process eq.:

$$_1W_2 = \int P\, dV = \text{area} = \frac{m}{1-n}(P_2v_2 - P_1v_1) = \frac{mR}{1-n}(T_2 - T_1)$$

$$= \frac{0.1 \times 0.2968}{1 - 1.25}(360.5 - 300.15) = \mathbf{-7.165\ kJ}$$







4.56

A piston cylinder contains 0.1 kg air at 100 kPa, 400 K which goes through a polytropic compression process with n = 1.3 to a pressure of 300 kPa. How much work has the air done in the process?

Solution:

Process: $Pv^n = \text{Const.}$

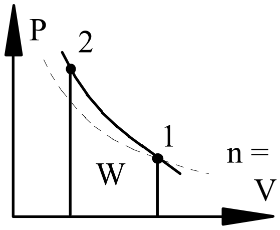
$$T_2 = T_1 \; (P_2 \, V_2 \,/\, P_1V_1) = T_1 \; (P_2 \,/\, P_1)(P_1 \,/\, P_2 \,)^{1/n}$$

$$= 400 \times (300/100)^{(1 \text{ - } 1/1.3)} = 515.4 \text{ K}$$

Work term is already integrated giving Eq.4.4

$${}_1W_2 \quad = \frac{1}{1-n}(P_2 \, V_2 - \; P_1V_1) = \frac{mR}{1-n}\,(\, T_2 - T_1) \qquad \text{Since Ideal gas,}$$

$$= \frac{0.1 \times 0.287}{1 - 1.3} \times (515.4 - 400) = \mathbf{-11 \; kJ}$$



4.57

A balloon behaves such that the pressure inside is proportional to the diameter squared. It contains 2 kg of ammonia at 0°C, 60% quality. The balloon and ammonia are now heated so that a final pressure of 600 kPa is reached. Considering the ammonia as a control mass, find the amount of work done in the process.

Solution:

Process : $P \propto D^2$, with $V \propto D^3$ this implies $P \propto D^2 \propto V^{2/3}$ so

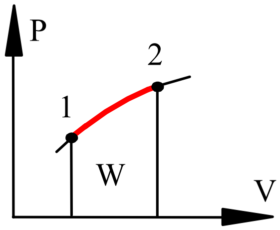
$PV^{-2/3}$ = constant, which is a polytropic process, $n = -2/3$

From table B.2.1: $V_1 = mv_1 = 2(0.001566 + 0.6 \times 0.28783) = 0.3485 \text{ m}^3$

$$V_2 = V_1 \left(\frac{P_2}{P_1}\right)^{3/2} = 0.3485 \left(\frac{600}{429.3}\right)^{3/2} = 0.5758 \text{ m}^3$$

$$_1W_2 = \int P\, dV = \frac{P_2V_2 - P_1V_1}{1 - n} \qquad \text{(Equation 4.4)}$$

$$= \frac{600 \times 0.5758 - 429.3 \times 0.3485}{1 - (-2/3)} \text{ kPa-m}^3 = \mathbf{117.5\ kJ}$$



4.58

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass. $m_2 = m_1 = m$

Process: $Pv^{1.5} = \text{constant}$ until P = 500 kPa

1: (T, x) $v_1 = 0.09921\ \text{m}^3/\text{kg}$, $P = P_{sat} = 201.7$ kPa from Table B.5.1

2: (P, process) $v_2 = v_1\ (P_1/P_2)^{(1/1.5)}$

$$= 0.09921 \times (201.7/500)^{2/3} = \mathbf{0.05416}\ \text{m}^3/\text{kg}$$

Given (P, v) at state 2 from B.5.2 it is superheated vapor at $\mathbf{T_2 = 79°C}$

Process gives $P = C\ v^{-1.5}$, which is integrated for the work term, Eq.(4.4)

$$_1W_2 = \int P\ dV = \frac{m}{1 - 1.5}\ (P_2v_2 - P_1v_1)$$

$$= \frac{2}{-0.5}\ \text{kg} \times (500 \times 0.05416 - 201.7 \times 0.09921)\ \text{kPa-m}^3/\text{kg}$$

$$= \mathbf{-7.07\ kJ}$$

4.59

A piston/cylinder contains water at 500°C, 3 MPa. It is cooled in a polytropic process to 200°C, 1 MPa. Find the polytropic exponent and the specific work in the process.

Solution:

Polytropic process: $Pv^n = C$

Both states must be on the process line: $P_2v_2^n = C = P_1v_1^n$

Take the ratio to get: $\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^n$

and then take ln of the ratio: $\ln\left(\frac{P_1}{P_2}\right) = \ln\left(\frac{v_2}{v_1}\right)^n = n \ln\left(\frac{v_2}{v_1}\right)$

now solve for the exponent n

$$n = \ln\left(\frac{P_1}{P_2}\right) \Big/ \ln\left(\frac{v_2}{v_1}\right) = \frac{1.0986}{0.57246} = \mathbf{1.919}$$

Table B.1.3 $v_1 = 0.11619\ \mathrm{m^3/kg}$, $v_2 = 0.20596\ \mathrm{m^3/kg}$

$$_1w_2 = \int P\, dv = \frac{P_2v_2 - P_1v_1}{1 - n} \qquad \text{(Equation 4.4)}$$

$$= \frac{1000 \times 0.20596 - 3000 \times 0.11619}{1 - 1.919}\ \mathrm{kPa\text{-}m^3/kg}$$

$= \mathbf{155.2\ kJ/kg}$

4.60

A spring loaded piston/cylinder assembly contains 1 kg water at 500^{o}C, 3 MPa. The setup is such that the pressure is proportional to volume, P = CV. It is now cooled until the water becomes saturated vapor. Sketch the P-v diagram and find the final state by iterations and the work in the process.

Solution :

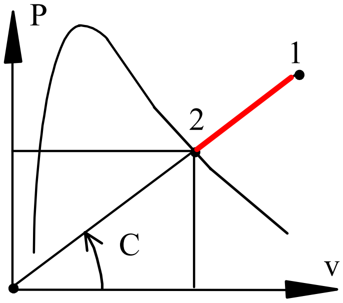
State 1: Table B.1.3: $v_1 = 0.11619 \text{ m}^3/\text{kg}$

Process: m is constant and $P = C_0V = C_0m\ v = C\ v$

polytropic process with n = –1

$P = Cv \quad \Rightarrow \quad C = P_1/v_1 = 3000/0.11619 = 25820 \text{ kPa kg/m}^3$

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)



Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \Rightarrow C = P/v_g = 20074$ (low)

2.5 MPa $v_g = 0.07998 \Rightarrow C = P/v_g = 31258$ (high)

2.25 MPa $v_g = 0.08875 \Rightarrow C = P/v_g = 25352$ (low)

Now interpolate to match the right slope C:

$$P_2 = 2250 + 250\,\frac{25\,820 - 25\,352}{31\,258 - 25\,352} = 2270 \text{ kPa},$$

$$v_2 = P_2/C = 2270/25820 = 0.0879 \text{ m}^3/\text{kg}$$

P is linear in V so the work becomes (area in P-v diagram)

$$_1W_2 = \int P\,dv = m\,\frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= 1 \text{ kg} \times \frac{1}{2}(3000 + 2270)(0.0879 - 0.11619) \text{ kPa-m}^3$$

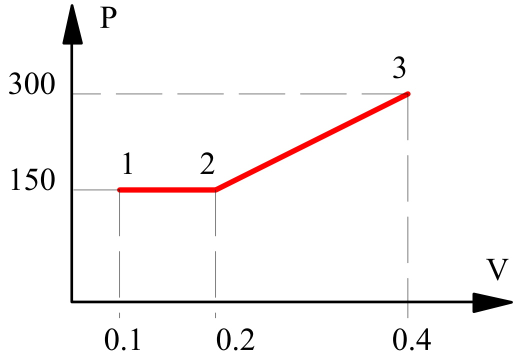
$$= \mathbf{-\ 74.5\ kJ}$$

4.61

Consider a two-part process with an expansion from 0.1 to 0.2 m^3 at a constant pressure of 150 kPa followed by an expansion from 0.2 to 0.4 m^3 with a linearly rising pressure from 150 kPa ending at 300 kPa. Show the process in a P-V diagram and find the boundary work.

Solution:

By knowing the pressure versus volume variation the work is found. If we plot the pressure versus the volume we see the work as the area below the process curve.



$$ {}_1W_3 = {}_1W_2 + {}_2W_3 = \int_1^2 PdV + \int_2^3 PdV $$

$$ = P_1 (V_2 - V_1) + \frac{1}{2} (P_2 + P_3)(V_3\text{-}V_2) $$

$$ = 150 \text{ kPa } (0.2 - 1.0) \text{ m}^3 + \frac{1}{2} (150 + 300) \text{ kPa } (0.4 - 0.2) \text{ m}^3 $$

$$ = (15 + 45) \text{ kJ} = \mathbf{60 \text{ kJ}} $$

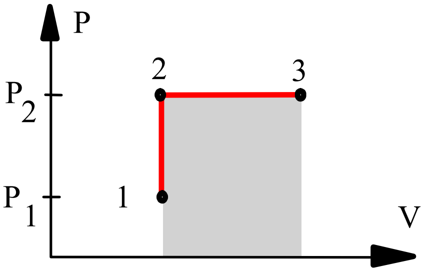
4.62

A helium gas is heated at constant volume from a state of 100 kPa, 300 K to 500 K. A following process expands the gas at constant pressure to three times the initial volume. What is the specific work in the combined process?

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for helium.

State 1: T, P => $v_1 = RT_1/P_1$

State 2: $V_2 = V_1$ => $P_2 = P_1\ (T_2/T_1)$

State 3: $P_3 = P_2$ => $V_3 = 3V_2$; $T_3 = T_2\ v_3/v_2 = 500 \times 3 = 1500$ K

We find the work by summing along the process path.

$$ {}_1w_3 = {}_1w_2 + {}_2w_3 = {}_2w_3 = P_3(v_3 - v_2) = R(T_3 - T_2) $$

$$ = 2.0771 \text{ kJ/kg-K} \times (1500 - 500) \text{ K} = \mathbf{2077\ kJ/kg} $$

4.63

Find the work in Problem 3.59.

Ammonia at 10^{o}C with a mass of 10 kg is in a piston cylinder arrangement with an initial volume of 1 m^3. The piston initially resting on the stops has a mass such that a pressure of 900 kPa will float it. The ammonia is now slowly heated to 50^{o}C. Find the work in the process.

C.V. Ammonia, constant mass.

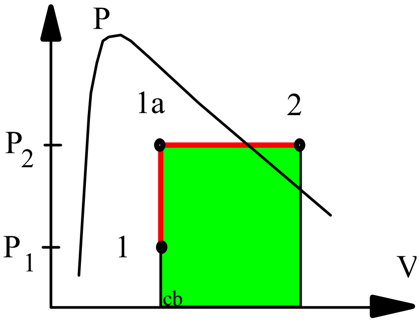
Process: V = constant unless $P = P_{float}$

State 1: T = 10^{o}C, $v_1 = \frac{V}{m} = \frac{1}{10} = 0.1$ m^3/kg

From Table B.2.1 $v_f < v < v_g$

$x_1 = (v - v_f)/v_{fg} = (0.1 - 0.0016)/0.20381$

$= 0.4828$



State 1a: P = 900 kPa, $v = v_1 = 0.1 < v_g$ at 900 kPa

This state is two-phase $T_{1a} = 21.52^{o}$C

Since $T_2 > T_{1a}$ then $v_2 > v_{1a}$

State 2: 50^{o}C and on line(s) means 900 kPa which is superheated vapor.

From Table B.2.2 linear interpolation between 800 and 1000 kPa:

$v_2 = 0.1648$ m^3/kg, $V_2 = mv_2 = 1.648$ m^3

${}_1W_2 = \int P\, dV = P_{float}\,(V_2 - V_1) = 900 \text{ kPa } (1.648 - 1.0) \text{ m}^3$

= **583.2 kJ**

4.64

A piston/cylinder arrangement shown in Fig. P4.64 initially contains air at 150 kPa, 400°C. The setup is allowed to cool to the ambient temperature of 20°C.

a. Is the piston resting on the stops in the final state? What is the final pressure in the cylinder?

b. What is the specific work done by the air during this process?

Solution:

State 1: $P_1 = 150 \text{ kPa}, \quad T_1 = 400°\text{C} = 673.2 \text{ K}$

State 2: $T_2 = T_0 = 20°\text{C} = 293.2 \text{ K}$

For all states air behave as an ideal gas.

a) If piston at stops at 2, $V_2 = V_1/2$ and pressure less than $P_{lift} = P_1$

$$\Rightarrow P_2 = P_1 \times \frac{V_1}{V_2} \times \frac{T_2}{T_1} = 150 \text{ kPa} \times 2 \times \frac{293.2}{673.2} = 130.7 \text{ kPa} \ < P_1$$

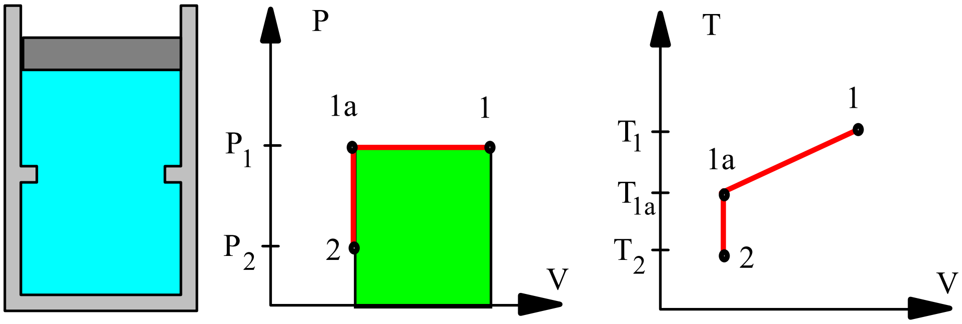
$\Rightarrow$ Piston is resting on stops at state 2.

b) Work done while piston is moving at constant $P_{ext} = P_1$.

$$_1W_2 = \int P_{ext}\, dV = P_1 (V_2 - V_1) \ ; \quad V_2 = \frac{1}{2} V_1 = \frac{1}{2} m \ RT_1/P_1$$

$$_1w_2 = {}_1W_2/m = RT_1 \left(\frac{1}{2} - 1\right) = -\frac{1}{2} \times 0.287 \text{ kJ/kg-K} \times 673.2 \text{ K}$$

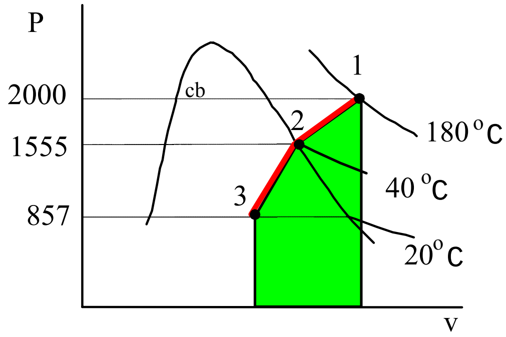
$$= \mathbf{-96.6 \ kJ/kg}$$



4.65

A cylinder containing 1 kg of ammonia has an externally loaded piston. Initially the ammonia is at 2 MPa, 180°C and is now cooled to saturated vapor at 40°C, and then further cooled to 20°C, at which point the quality is 50%. Find the total work for the process, assuming a piecewise linear variation of *P* versus *V*.

Solution:



State 1: (T, P) Table B.2.2

$v_1 = 0.10571\ \text{m}^3/\text{kg}$

State 2: (T, x) Table B.2.1 sat. vap.

$P_2 = 1555$ kPa,

$v_2 = 0.08313\ \text{m}^3/\text{kg}$

State 3: (T, x) $P_3 = 857$ kPa, $v_3 = (0.001638 + 0.14922)/2 = 0.07543\ \text{m}^3/\text{kg}$

Sum the work as two integrals each evaluated by the area in the P-v diagram.

$$_1W_3 = \int_1^3 P dV \approx \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1) + \left(\frac{P_2 + P_3}{2}\right) m(v_3 - v_2)$$

$$= \frac{2000 + 1555}{2}\ \text{kPa} \times 1\ \text{kg} \times (0.08313 - 0.10571)\ \text{m}^3/\text{kg}$$

$$+ \frac{1555 + 857}{2}\ \text{kPa} \times 1\ \text{kg} \times (0.07543 - 0.08313)\ \text{m}^3/\text{kg}$$

= **-49.4 kJ**

4.66

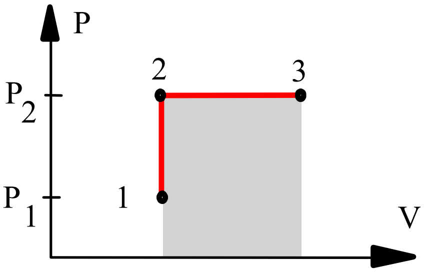
A piston cylinder has 1.5 kg of air at 300 K and 150 kPa. It is now heated up in a two step process. First constant volume to 1000 K (state 2) then followed by a constant pressure process to 1500 K, state 3. Find the final volume and the work in the process.

Solution:

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for air.

State 1: T, P => $V_1 = mRT_1/P_1 = 1.5\times0.287\times300/150 = 0.861\ m^3$

State 2: $V_2 = V_1$ => $P_2 = P_1\ (T_2/T_1) = 150\times1000/300 = 500$ kPa

State 3: $P_3 = P_2$ => $V_3 = V_2\ (T_3/T_2) = 0.861\times1500/1000 = \mathbf{1.2915\ m^3}$

We find the work by summing along the process path.

$$ {}_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = P_3(V_3 - V_2) $$

$$ = 500 \text{ kPa} \times (1.2915 - 0.861)\ m^3 = \mathbf{215.3\ kJ} $$

4.67

The refrigerant R-410a is contained in a piston/cylinder as shown in Fig. P4.67, where the volume is 11 L when the piston hits the stops. The initial state is –30°C, 150 kPa with a volume of 10 L. This system is brought indoors and warms up to 15°C.

a. Is the piston at the stops in the final state?

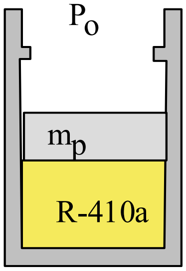
b. Find the work done by the R-410a during this process.

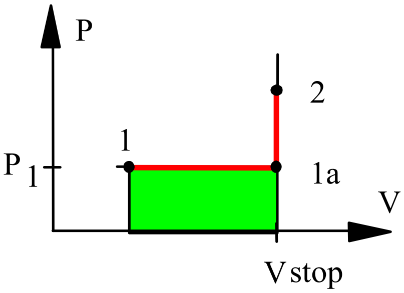
Solution:

Initially piston floats, $V < V_{stop}$ so the piston moves at constant $P_{ext} = P_1$ until it reaches the stops or 15°C, whichever is first.

a) From Table B.4.2: $v_1 = 0.17732\ m^3/kg$,

$$m = V/v = \frac{0.010}{0.17732} = 0.056395 \text{ kg}$$





Check the temperature at state 1a: $P_{1a} = 150$ kPa, $v = V_{stop}/m$.

$$v_{1a} = V/m = \frac{0.011}{0.056395} = 0.19505\ m^3/kg \quad => \quad T_{1a} = -9.4°C \ \& \ T_2 = 15°C$$

Since $T_2 > T_{1a}$ then it follows that $P_2 > P_1$ and the piston is against stop.

b) Work done at constant $P_{ext} = P_1$.

$${}_1W_2 = \int P_{ext}\, dV = P_{ext}(V_2 - V_1) = 150(0.011 - 0.010) = \mathbf{0.15\ kJ}$$

4.68

A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 4.67, with a maximum enclosed volume of 0.002 m^3 if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the work in the process.

Solution:

Take CV as the water which is a control mass: $m_2 = m_1 = m$;

Table B.1.1: 20°C => $P_{sat} = 2.34$ kPa

State 1: Compressed liquid $v = v_f(20) = 0.001002\ m^3/kg$

State 1a: $v_{stop} = 0.002\ m^3/kg$, 300 kPa

State 2: Since $P_2 = 600\ kPa > P_{lift}$ then piston is pressed against the stops

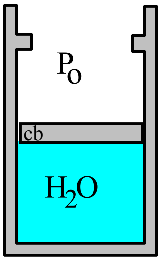
$$v_2 = v_{stop} = 0.002\ m^3/kg \text{ and } V = 0.002\ m^3$$

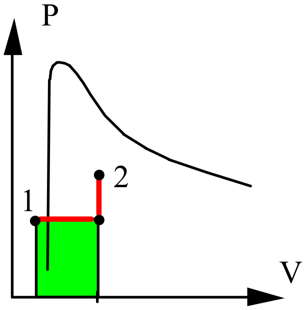
For the given P : $v_f < v < v_g$ so 2-phase $T = T_{sat} = 158.85$ °C

Work is done while piston moves at P_{lift} = constant = 300 kPa so we get

$$_1W_2 = \int P\, dV = m\, P_{lift}(v_2 - v_1) = 1\ kg \times 300\ kPa \times (0.002 - 0.001002)\ m^3$$

$$= \mathbf{0.30\ kJ}$$

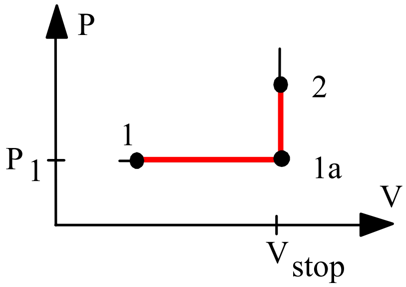




4.69

A piston/cylinder contains 50 kg of water at 200 kPa with a volume of 0.1 m^3. Stops in the cylinder restricts the enclosed volume to 0.5 m^3, similar to the setup in Problem 4.67. The water is now heated to 200°C. Find the final pressure, volume and the work done by the water.

Solution:



Initially the piston floats so the equilibrium lift pressure is 200 kPa

1: 200 kPa, $v_1 = 0.1/50 = 0.002 \text{ m}^3/\text{kg}$,

2: 200°C, on line

Check state 1a:

$$v_{stop} = 0.5/50 = 0.01 \text{ m}^3/\text{kg}$$

=>

Table B.1.2: 200 kPa , $v_f < v_{stop} < v_g$

State 1a is two phase at 200 kPa and $T_{stop} \approx 120.2$ °C so as $T_2 > T_{stop}$ the state is higher up in the P-V diagram with

$$v_2 = v_{stop} < v_g = 0.127 \text{ m}^3/\text{kg} \text{ (at 200°C)}$$

State 2 two phase => $P_2 = P_{sat}(T_2) =$ **1.554 MPa**, $V_2 = V_{stop} =$ **0.5 m^3**

$${}_1W_2 = {}_1W_{stop} = 200 \text{ kPa} \times (0.5 - 0.1) \text{ m}^3 = \mathbf{80 \text{ kJ}}$$

4.70

A piston/cylinder assembly (Fig. P4.70) has 1 kg of R-134a at state 1 with 110°C, 600 kPa, and is then brought to saturated vapor, state 2, by cooling while the piston is locked with a pin. Now the piston is balanced with an additional constant force and the pin is removed. The cooling continues to a state 3 where the R-134a is saturated liquid. Show the processes in a P-V diagram and find the work in each of the two steps, 1 to 2 and 2 to 3.

Solution :

CV R-134a This is a control mass.

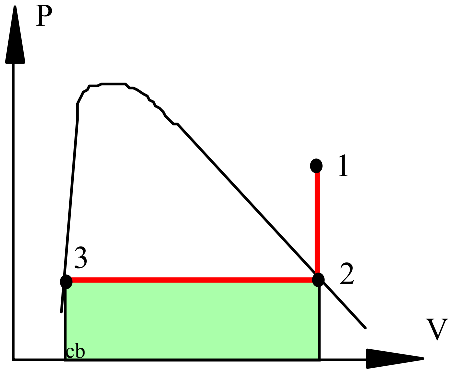
Properties from table B.5.1 and 5.2

State 1: (T,P) B.5.2 => $v = 0.04943\ m^3/kg$

State 2: given by fixed volume $v_2 = v_1$ and $x_2 = 1.0$ so from B.5.1

$$v_2 = v_1 = v_g = 0.04943\ m^3/kg \quad => \quad T = 10°C$$

State 3 reached at constant P (F = constant) $v_3 = v_f = 0.000794\ m^3/kg$



Since no volume change from 1 to 2 => $\mathbf{{}_1W_2 = 0}$

$${}_2W_3 = \int P\ dV = P(V_3 - V_2) = mP(v_3 - v_2) \qquad \text{Constant pressure}$$

$$= 415.8\ kPa \times (0.000794 - 0.04943)\ m^3/kg \times 1\ kg$$

$$= \mathbf{-20.22\ kJ}$$

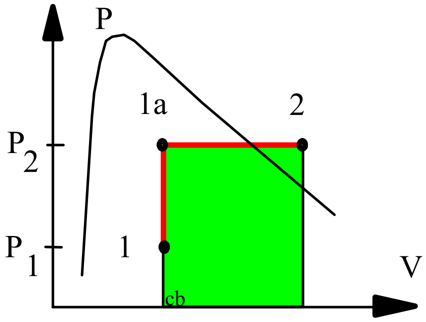
4.71

Find the work in Problem 3.84.

Ammonia at 10°C with a mass of 10 kg is in a piston cylinder arrangement with an initial volume of 1 m^3. The piston initially resting on the stops has a mass such that a pressure of 900 kPa will float it. The ammonia is now slowly heated to 50°C. Find the work in the process.

C.V. Ammonia, constant mass.

Process: $V = \text{constant}$ unless $P = P_{float}$



State 1: $T = 10^oC$, $v_1 = \frac{V}{m} = \frac{1}{10} = 0.1\ m^3/kg$

From Table B.2.1 $v_f < v < v_g$

$x_1 = (v - v_f)/v_{fg} = (0.1 - 0.0016)/0.20381$

$= 0.4828$

State 1a: $P = 900$ kPa, $v = v_1 = 0.1 < v_g$ at 900 kPa

This state is two-phase $T_{1a} = 21.52^oC$

Since $T_2 > T_{1a}$ then $v_2 > v_{1a}$

State 2: 50°C and on line(s) means 900 kPa which is superheated vapor.

From Table B.2.2 linear interpolation between 800 and 1000 kPa:

$v_2 = 0.1648\ m^3/kg$, $V_2 = mv_2 = 1.648\ m^3$

${}_1W_2 = \int P\, dV = P_{float}\,(V_2 - V_1) = 900\ \text{kPa} \times (1.648 - 1.0)\ m^3$

$=$ **583.2 kJ**

4.72

10 kg of water in a piston cylinder arrangement exists as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it, see Fig. P4.72.

a) Find the final temperature and volume of the water.

b) Find the work given out by the water.

Solution:

Take CV as the water $m_2 = m_1 = m$;

Process: v = constant until $P = P_{lift}$ then P is constant.

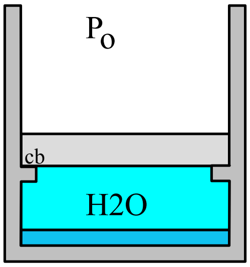
State 1: $v_1 = v_f + x\,v_{fg} = 0.001043 + 0.5 \times 1.69296 = 0.8475\ m^3/kg$

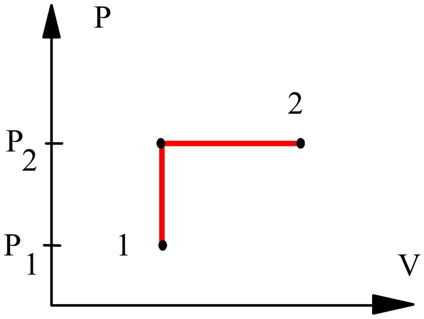
State 2: $v_2,\ P_2 \le P_{lift}$ => $v_2 = 3 \times 0.8475 = 2.5425\ m^3/kg$;

$T_2 = \mathbf{829°C}$ **;** $V_2 = m\,v_2 = \mathbf{25.425\ m^3}$

$${}_1W_2 = \int P\,dV = P_{lift} \times (V_2 - V_1)$$

$$= 200\ kPa \times 10\ kg \times (2.5425 - 0.8475)\ m^3/kg = \mathbf{3390\ kJ}$$





4.73

A piston cylinder setup similar to Problem 4.72 contains 0.1 kg saturated liquid and vapor water at 100 kPa with quality 25%. The mass of the piston is such that a pressure of 500 kPa will float it. The water is heated to 300°C. Find the final pressure, volume and the work, ${}_1W_2$.

Solution:

Take CV as the water: $m_2 = m_1 = m$

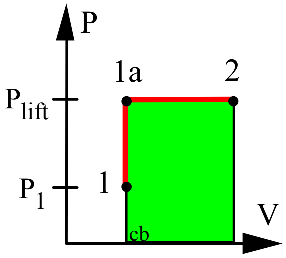
Process: v = constant until $P = P_{lift}$

To locate state 1: Table B.1.2

$v_1 = 0.001043 + 0.25 \times 1.69296 = 0.42428 \text{ m}^3/\text{kg}$

1a: $v_{1a} = v_1 = 0.42428 \text{ m}^3/\text{kg} > v_g$ at 500 kPa

so state 1a is superheated vapor $T_{1a} = 200°\text{C}$



State 2 is 300°C so heating continues after state 1a to 2 at constant P =>

2: T_2, $P_2 = P_{lift}$ = **500 kPa** => Table B.1.3 $v_2 = 0.52256 \text{ m}^3/\text{kg}$;

$V_2 = mv_2 = \mathbf{0.05226 \ m^3}$

${}_1W_2 = P_{lift} (V_2 - V_1) = 500 \text{ kPa} \times (0.05226 - 0.04243) \text{ m}^3 = \mathbf{4.91 \ kJ}$

4.74

A piston cylinder contains air at 1000 kPa, 800 K with a volume of 0.05 m^3. The piston is pressed against the upper stops and it will float at a pressure of 750 kPa. Then the air is cooled to 400 K. What is the process work?

We need to find state 2. Let us see if we proceed past state 1a during the cooling.

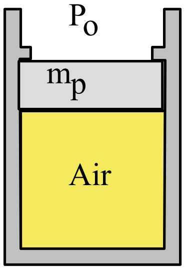
$$T_{1a} = T_1\ P_{float} / P_1 = 800 \times 750 / 100 = 600 \text{ K}$$

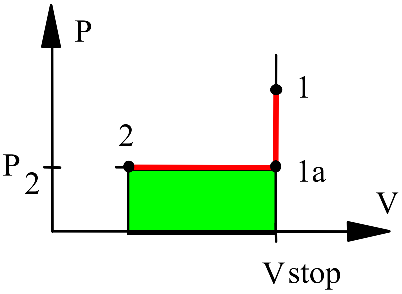
so we do cool below T_{1a}. That means the piston is floating. Write the ideal gas law for state 1 and 2 to get

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1V_1T_2}{P_2T_1} = \frac{1000 \times 0.05 \times 400}{750 \times\ 800} = 0.0333 \text{ m}^3$$

$$_1W_2 = {}_{1a}W_2 = \int P\ dV = P_2\ (V_2 - V_1)$$

$$= 750 \text{ kPa} \times (0.0333 - 0.05) \text{ m}^3 = \mathbf{-12.5\ kJ}$$





Other types of work and general concepts

4.75

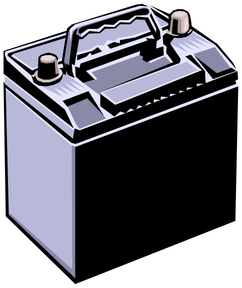
Electric power is volts times ampere (P = V i). When a car battery at 12 V is charged with 6 amp for 3 hours how much energy is delivered?

Solution:

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t = V\, i\, \Delta t$$

$$= 12\ \text{V} \times 6\ \text{Amp} \times 3 \times 3600\ \text{s}$$

$$= 777\ 600\ \text{J} = \mathbf{777.6\ kJ}$$



Remark: Volt times ampere is also watts, 1 W = 1 V × 1 Amp. = 1 J/s

4.76

A copper wire of diameter 2 mm is 10 m long and stretched out between two posts. The normal stress (pressure) $\sigma = E(L - L_o)/L_o$, depends on the length L versus the un-stretched length L_o and Young's modulus $E = 1.1 \times 10^6$ kPa. The force is $F = A\sigma$ and measured to be 110 N. How much longer is the wire and how much work was put in?

Solution:

$F = As = AE\,\Delta L/L_o$ and $\Delta L = FL_o/AE$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.002^2 = 3.142 \times 10^{-6}\ \text{m}^2$$

$$\Delta L = \frac{110 \times 10}{3.142 \times 10^{-6} \times 1.1 \times 10^6 \times 10^3} = \mathbf{0.318\ m}$$

$$_1W_2 = \int F\,dx = \int A\,s\,dx = \int AE\frac{x}{L_o}\,dx$$

$$= \frac{AE}{L_o}\,½\,x^2 \qquad \text{where } x = L - L_o$$

$$= \frac{3.142 \times 10^{-6} \times 1.1 \times 10^6 \times 10^3}{10} \times ½ \times 0.318^2 = \mathbf{17.47\ J}$$

4.77

A 0.5-m-long steel rod with a 1-cm diameter is stretched in a tensile test. What is the required work to obtain a relative strain of 0.1%? The modulus of elasticity of steel is 2 × 10^8 kPa.

Solution :

$$-{}_1W_2 = \frac{AEL_0}{2}(e)^2, \quad A = \frac{\pi}{4}(0.01)^2 = 78.54 \times 10^{-6}\ \mathrm{m}^2$$

$$-{}_1W_2 = \frac{78.54\times10^{-6} \times 2\times10^8 \times 0.5}{2}\ \mathrm{m}^2\text{-kPa-m}\ (10^{-3})^2$$

$$= \mathbf{3.93\ J}$$

4.78

A soap bubble has a surface tension of S = 3 × 10^{-4} N/cm as it sits flat on a rigid ring of diameter 5 cm. You now blow on the film to create a half sphere surface of diameter 5 cm. How much work was done?

$$ {}_1W_2 = \int F\ dx = \int S\ dA = S\,\Delta A $$

$$ = 2 \times S \times \left(\frac{\pi}{2}D^2 - \frac{\pi}{4}D^2\right) $$

$$ = 2 \times 3 \times 10^{-4}\ \text{N/cm} \times 100\ \text{cm/m} \times \frac{\pi}{2}\,0.05^2\ \text{m}^2\ (1 - 0.5) $$

$$ = \mathbf{1.18 \times 10^{-4}\ \ J} $$

Notice the bubble has 2 surfaces.

$A_1 = \frac{\pi}{4} D^2$,

$A_2 = ½\, \pi\, D^2$

4.79

A film of ethanol at 20°C has a surface tension of 22.3 mN/m and is maintained on a wire frame as shown in Fig. P4.79. Consider the film with two surfaces as a control mass and find the work done when the wire is moved 10 mm to make the film 20 × 40 mm.

Solution :

Assume a free surface on both sides of the frame, i.e., there are two surfaces 20 × 30 mm

$$W = -\int \mathcal{S}\, dA = -22.3\times10^{-3}\ \text{N/m} \times 2\ (800 - 600)\times10^{-6}\ \text{m}^2$$

$$= -8.92\times10^{-6}\ \text{J} = \mathbf{-8.92\ \mu J}$$

4.80

Assume we fill a spherical balloon from a bottle of helium gas. The helium gas provides work $\int PdV$ that stretches the balloon material $\int S\,dA$ and pushes back the atmosphere $\int P_o\,dV$. Write the incremental balance for $dW_{helium} = dW_{stretch} + dW_{atm}$ to establish the connection between the helium pressure, the surface tension S and P_o as a function of radius.

$$W_{He} = \int P\,dV = \int S\,dA + \int P_o\,dV$$

$$dW_{He} = P\,dV = S\,dA + P_o\,dV$$

$$dV = d\,(\tfrac{\pi}{6} D^3) = \tfrac{\pi}{6} \times 3D^2\,dD$$

$$dA = d\,(2 \times \pi \times D^2) = 2\pi\,(2D)\,dD$$

$$P\,\frac{\pi}{2} D^2\,dD = S\,(4\pi) D\,dD + P_o\,\frac{\pi}{2} D^2\,dD$$

Divide by $\frac{\pi}{2} D^2$ to recognize

$$\mathbf{P_{He} = P_o + 8\,\frac{S}{D}}$$

4.81

Assume a balloon material with a constant surface tension of S = 2 N/m. What is the work required to stretch a spherical balloon up to a radius of r = 0.5 m? Neglect any effect from atmospheric pressure.

Assume the initial area is small, and that we have 2 surfaces inside and out

$$W = -\int S\, dA = -S\,(A_2 - A_1)$$

$$= -\,S(A_2) \;= -S(\,2\times \pi\, \mathbf{D}_2^2\,)$$

$$= -2\text{ N/m} \times 2 \times \pi \times 1\text{ m}^2 \;= -12.57\text{ J}$$

$$W_{in} = -W = \mathbf{12.57\ J}$$

4.82

A sheet of rubber is stretched out over a ring of radius 0.25 m. I pour liquid water at 20°C on it, as in Fig. P4.82, so the rubber forms a half sphere (cup). Neglect the rubber mass and find the surface tension near the ring?

Solution:

$F\uparrow = F\downarrow \;\; ; \;\; F\uparrow = SL$

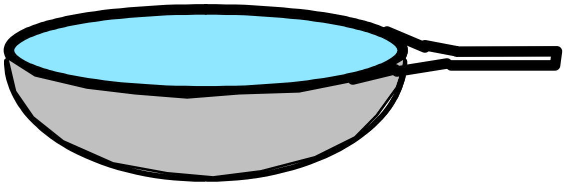
The length is the perimeter, 2πr, and there is two surfaces

$$S \times 2 \times 2\pi r = m_{H2o}\, g = \rho_{H2o}\, Vg = \rho_{H2o}\times \frac{1}{12}\,\pi\,(2r)^{3} g = \rho_{H2o}\times \pi\,\frac{2}{3}\, r^{3}$$

$$S = \rho_{H2o}\,\frac{1}{6}\, r^{2}\, g$$

$$= 997\ \text{kg/m}^{3} \times \frac{1}{6} \times 0.25^{2}\ \text{m}^{2} \times 9.81\ \text{m/s}^{2}$$

$$= \mathbf{101.9\ N/m}$$

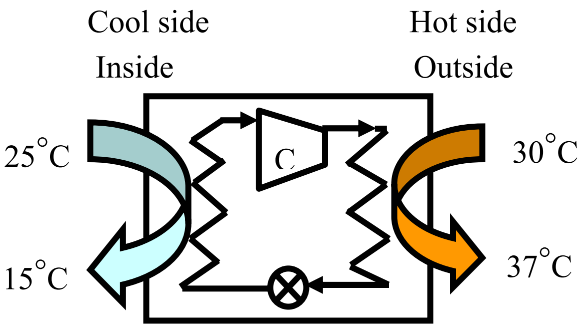


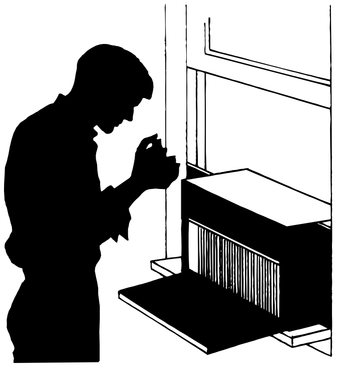
4.83

Consider a window-mounted air conditioning unit used in the summer to cool incoming air. Examine the system boundaries for rates of work and heat transfer, including signs.

Solution :

Air-conditioner unit, steady operation with no change of temperature of AC unit.





- electrical work (power) input operates unit,

+Q rate of heat transfer from the room which actually is room air being cooled,

a larger -Q rate of heat transfer (sum of the other two energy rates) out to the outside air.

4.84

Consider a light bulb that is on. Explain where we have rates of work and heat transfer (include modes) that moves energy.

Solution:

Electrical power comes in, that is rate of work.

In the wire filament this electrical work is converted to internal energy so the wire becomes hot and radiates energy out. There is also some heat transfer by conduction-convection to the gas inside the bulb so that become warm.

The gas in turn heats the glass by conduction/convection and some of the radiation may be absorbed by the glass (often the glass is coated white)

All the energy leaves the bulb as a combination of radiation over a range of wavelengths some of which you cannot see and conduction convection heat transfer to the air around the bulb.

4.85

Consider a household refrigerator that has just been filled up with room-temperature food. Define a control volume (mass) and examine its boundaries for rates of work and heat transfer, including sign.

a. Immediately after the food is placed in the refrigerator

b. After a long period of time has elapsed and the food is cold

Solution:

I. C.V. Food.

a) short term.: -Q from warm food to cold refrigerator air. Food cools.

b) Long term: -Q goes to zero after food has reached refrigerator T.

II. C.V. refrigerator space, not food, not refrigerator system

a) short term: +Q from the warm food, +Q from heat leak from room into cold space. -Q (sum of both) to refrigeration system. If not equal the refrigerator space initially warms slightly and then cools down to preset T.

b) long term: small -Q heat leak balanced by -Q to refrigeration system.

Note: For refrigeration system CV any Q in from refrigerator space plus electrical W input to operate system, sum of which is Q rejected to the room.

4.86

A room is heated with an electric space heater on a winter day. Examine the following control volumes, regarding heat transfer and work , including sign.

a) The space heater.
b) Room
c) The space heater and the room together

Solution:

a) The space heater.

Electrical work (power) input, and equal (after system warm up) Q out to the room.

b) Room

Q input from the heater balances Q loss to the outside, for steady (no temperature change) operation.

c) The space heater and the room together

Electrical work input balances Q loss to the outside, for steady operation.

Rates of work

4.87

A 100 hp car engine has a drive shaft rotating at 2000 RPM. How much torque is on the shaft for 25% of full power?

Solution:

Power = 0.25 × 100 hp = 0.25 × 73.5 kW (if SI hp) = 18.375 kW = $T\omega$

ω = angular velocity (rad/s) = RPM 2 π /60

$$T = \text{Power}/\omega = \frac{\text{power} \times 60}{\text{RPM} \times 2\pi} = \frac{18.375 \text{ kW} \times 60 \text{ s/min}}{2000 \times 2\pi \text{ rad/min}} = \mathbf{87.73\ Nm}$$

We could also have used UK hp to get 0.25 × 74.6 kW. then T = 89 Nm.

4.88

A car uses 25 hp to drive at a horizontal level at constant 100 km/h. What is the traction force between the tires and the road?

Solution:

We need to relate the rate of work to the force and velocity

$$dW = F\,dx \quad \Rightarrow \quad \frac{dW}{dt} = \dot{W} = F\,\frac{dx}{dt} = F\mathbf{V}$$

$$F = \dot{W} / \mathbf{V}$$

$$\dot{W} = 25 \text{ hp} = 25 \times 0.7355 \text{ kW} = 18.39 \text{ kW}$$

$$\mathbf{V} = 100 \times \frac{1000}{3600} = 27.78 \text{ m/s}$$

$$F = \dot{W} / \mathbf{V} = \frac{18.39 \text{ kW}}{27.78 \text{ m/s}} = \mathbf{0.66\ kN}$$

Units: $\text{kW} / (\text{ms}^{-1}) = \text{kW s m}^{-1} = \text{kJ s}^{-1}\text{s m}^{-1} = \text{kN m m}^{-1} = \text{kN}$

4.89

An escalator raises a 100 kg bucket of sand 10 m in 1 minute. Determine the rate of work done during the process.

Solution:

The work is a force with a displacement and force is constant: F = mg

$$W = \int F\,dx = F\int dx = F\,\Delta x = 100\text{ kg} \times 9.80665\text{ m/s}^2 \times 10\text{ m} = 9807\text{ J}$$

The rate of work is work per unit time

$$\dot{W} = \frac{W}{\Delta t} = \frac{9807\text{ J}}{60\text{ s}} = \mathbf{163\ W}$$

4.90

A crane lifts a bucket of cement with a total mass of 450 kg vertically up with a constant velocity of 2 m/s. Find the rate of work needed to do that.

Solution:

Rate of work is force times rate of displacement. The force is due to gravity (a = 0) alone.

$$\dot{W} = F\mathbf{V} = mg \times \mathbf{V} = 450 \text{ kg} \times 9.807 \text{ ms}^{-2} \times 2 \text{ ms}^{-1} = 8826 \text{ J/s}$$

$$\dot{W} = \mathbf{8.83 \text{ kW}}$$

4.91

A force of 1.2 kN moves a truck with 60 km/h up a hill. What is the power?

Solution:

$$\dot{W} = F\,\mathbf{V} = 1.2\text{ kN} \times 60\text{ (km/h)}$$

$$= 1.2 \times 10^3 \times 60 \times \frac{10^3}{3600}\,\frac{\text{Nm}}{\text{s}}$$

$$= 20\,000\text{ W} = \mathbf{20\ kW}$$

4.92

A piston/cylinder of cross sectional area 0.01 m^2 maintains constant pressure. It contains 1 kg water with a quality of 5% at 150^{o}C. If we heat so 1 g/s liquid turns into vapor what is the rate of work out?

$V_{vapor} = m_{vapor}\, v_g\,, \quad V_{liq} = m_{liq}\, v_f$

$m_{tot} = \text{constant} = m_{vapor}\, m_{liq}$

$V_{tot} = V_{vapor} + V_{liq}$

$\dot{m}_{tot} = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$

$$\dot{V}_{tot} = \dot{V}_{vapor} + \dot{V}_{liq} = \dot{m}_{vapor} v_g + \dot{m}_{liq} v_f$$
$$= \dot{m}_{vapor}\ (v_g - v_f) = \dot{m}_{vapor}\, v_{fg}$$

$\dot{W} = P\dot{V} \ = P\ \dot{m}_{vapor}\, v_{fg}$

$= 475.9$ kPa × 0.001 kg/s × 0.39169 m^3/kg **= 0.1864 kW**
= 186 W

4.93

Consider the car with the rolling resistance as in problem 4.26. How fast can it drive using 30 hp?

$$F = 0.006\ mg$$

$$\text{Power} \;= F \times \mathbf{V} \;= 30\ \text{hp} = \dot{W}$$

$$\mathbf{V} = \dot{W} / F = \frac{\dot{W}}{0.006\ mg} = \frac{30 \times 0.7457 \times 1000}{0.006 \times 1200 \times 9.81} = \mathbf{271.5\ m/s}$$

Comment: This is a very high velocity, the rolling resistance is low relative to the air resistance.

4.94

Consider the car with the air drag force as in problem 4.27. How fast can it drive using 30 hp?

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{100}{0.287 \times 290} = 1.2015\ \frac{\text{kg}}{\text{m}^3} \quad \text{and} \quad A = 4\ \text{m}^2$$

Drag force: $F_{drag} = 0.225\ A\ \rho\ \mathbf{V}^2$

Power for drag force: $\dot{W}_{drag} = 30\ \text{hp} \times 0.7457 = 22.371\ \text{kW}$

$$\dot{W}_{drag} = F_{drag}\ \mathbf{V} = 0.225 \times 4 \times 1.2015 \times \mathbf{V}^3$$

$$\mathbf{V}^3 = \dot{W}_{drag} / (0.225 \times 4 \times 1.2015) = 20\ 688$$

$$\mathbf{V} = 27.452\ \text{m/s} = 27.452 \times \frac{3600}{1000} = \mathbf{98.8\ km/h}$$

4.95

Consider a 1400 kg car having the rolling resistance as in problem 4.26 and air resistance as in problem 4.27. How fast can it drive using 30 hp?

$$F_{tot} = F_{rolling} + F_{air} = 0.006\ mg + 0.225\ A\rho\mathbf{V}^2$$

$$m = 1400\text{ kg}, \ A = 4\text{ m}^2$$

$$\rho = P/RT = 1.2015\text{ kg/m}^3$$

$$\dot{W} = F\mathbf{V} = 0.006\ mg\mathbf{V} + 0.225\ \rho A\mathbf{V}^3$$

Nonlinear in V so solve by trial and error.

$$\dot{W} = 30\text{ hp} = 30 \times 0.7355\text{ kW} = 22.06\text{ kW}$$

$$= 0.0006 \times 1400 \times 9.807\ \mathbf{V} + 0.225 \times 1.2015 \times 4\ \mathbf{V}^3$$

$$= 82.379V + 1.08135\ \mathbf{V}^3$$

$\mathbf{V} = 25$ m/s $\Rightarrow$ $\dot{W} = 18\ 956$ W

$\mathbf{V} = 26$ m/s $\quad \dot{W} = 21\ 148$ W

$\mathbf{V} = 27$ m/s $\quad \dot{W} = 23\ 508$ W

Linear interpolation

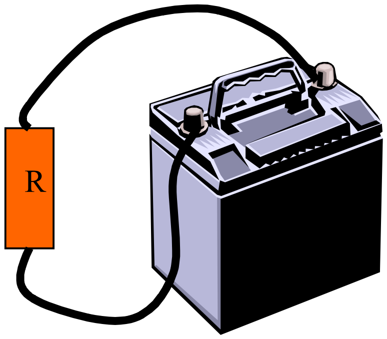
$\mathbf{V} = 26.4$ m/s **= 95 km/h**

4.96

A current of 10 amp runs through a resistor with a resistance of 15 ohms. Find the rate of work that heats the resistor up.

Solution:

$$\dot{W} = \text{power} = E\ i = R\ i^2 = 15 \times 10 \times 10 = \mathbf{1500\ W}$$



4.97

A battery is well insulated while being charged by 12.3 V at a current of 6 A. Take the battery as a control mass and find the instantaneous rate of work and the total work done over 4 hours.

Solution :

Battery thermally insulated ⇒ $Q = 0$

For constant voltage E and current i,

Power = E i = 12.3 × 6 = **73.8 W** [Units V × A = W]

$$W = \int \text{power } dt = \text{power } \Delta t$$

$$= 73.8 \times 4 \times 60 \times 60 = 1\ 062\ 720 \text{ J} = \mathbf{1062.7\ kJ}$$

4.98

A torque of 650 Nm rotates a shaft of diameter 0.25 m with $\omega = 50$ rad/s. What are the shaft surface speed and the transmitted power?

Solution:

$\mathbf{V} = \omega r = \omega D/2 = 50 \times 0.25 / 2 = $ **6.25 m/s**

Power $= T\omega = 650 \times 50$ Nm/s $= 32\,500$ W $=$ **32.5 kW**

Recall also Power $= FV = (T/r)\,V = T\,V/r = T\omega$

4.99

Air at a constant pressure in a piston cylinder is at 300 kPa, 300 K and a volume of 0.1 m^3. It is heated to 600 K over 30 seconds in a process with constant piston velocity. Find the power delivered to the piston.

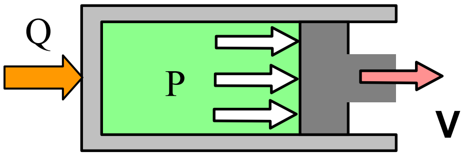
Solution:

Process: P = constant :

Boundary work: $dW = P\,dV \quad \Rightarrow \quad \dot{W} = P\dot{V}$

$V_2 = V_1 \times (T_2/T_1) = 0.1 \times (600/300) = 0.2\ m^3$

$$\dot{W} = P \frac{\Delta V}{\Delta t} = 300 \times \frac{0.2 - 0.1}{30}\ \text{kPa}\ \frac{m^3}{s} = \mathbf{1\ kW}$$



Remark: Since we do not know the area we do not know the velocity

4.100

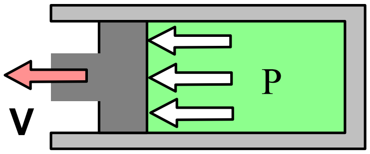
A pressure of 650 kPa pushes a piston of diameter 0.25 m with **V** = 5 m/s. What is the volume displacement rate, the force and the transmitted power?

$$A = \frac{\pi}{4} D^2 = 0.049087 \text{ m}^2$$

$$\dot{V} = A\mathbf{V} = 0049087 \text{ m}^2 \times 5 \text{ m/s} = \mathbf{0.2454\ m^3/s}$$

$$F = P\ A = 650 \text{ kPa} \times 0.049087 \text{ m}^2 = \mathbf{31.9\ kN}$$

$$\dot{W} = \text{power} = F\ \mathbf{V} = P\ \dot{V} = 650 \text{ kPa} \times 0.2454 \text{ m}^3\text{/s} = \mathbf{159.5\ kW}$$



4.101

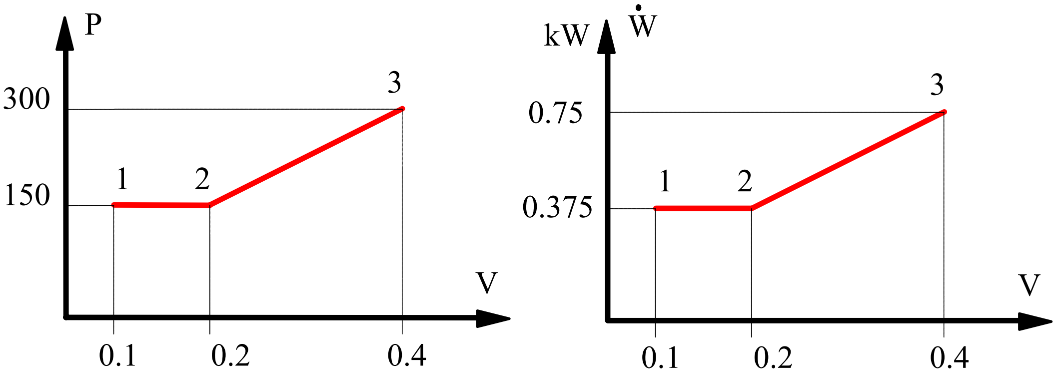
Assume the process in Problem 4.61 takes place with a constant rate of change in volume over 2 minutes. Show the power (rate of work) as a function of time.

Solution:

$W = \int P\, dV$ since 2 min = 120 sec

$\dot{W} = P\ (\Delta V / \Delta t)$

$(\Delta V / \Delta t) = 0.3\ m^3 / 120\ s = 0.0025\ m^3/s$



Heat Transfer rates

4.102

Find the rate of conduction heat transfer through a 1.5 cm thick hardwood board, k = 0.16 W/m K, with a temperature difference between the two sides of 20^{o}C.

One dimensional heat transfer by conduction, we do not know the area so we can find the flux (heat transfer per unit area W/m^2).

$$\dot{q} = \dot{Q}/A = k\ \frac{\Delta T}{\Delta x} = 0.16 \frac{W}{m\ K} \times \frac{20}{0.015} \frac{K}{m} = \mathbf{213\ W/m^2}$$

4.103

A pot of steel, conductivity 50 W/m K, with a 5 mm thick bottom is filled with 15°C liquid water. The pot has a diameter of 20 cm and is now placed on an electric stove that delivers 250 W as heat transfer. Find the temperature on the outer pot bottom surface assuming the inner surface is at 15°C.

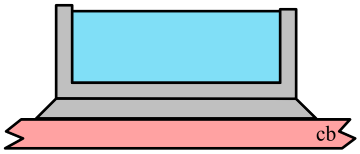
Solution :

Steady conduction through the bottom of the steel pot. Assume the inside surface is at the liquid water temperature.

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x} \;\Rightarrow\; \Delta T = \dot{Q}\ \Delta x / kA$$

$$\Delta T = 250\ \text{W} \times 0.005\ \text{m} /(50\ \text{W/m-K} \times \frac{\pi}{4} \times 0.2^2\ \text{m}^2) = 0.796\ \text{K}$$

$$T = 15 + 0.796 \cong \mathbf{15.8°C}$$



4104

The sun shines on a 150 m^2 road surface so it is at 45°C. Below the 5 cm thick asphalt, average conductivity of 0.06 W/m K, is a layer of compacted rubbles at a temperature of 15°C. Find the rate of heat transfer to the rubbles.

Solution :

This is steady one dimensional conduction through the asphalt layer.

$$\dot{Q} = k\,A\,\frac{\Delta T}{\Delta x}$$

$$= 0.06\,\frac{W}{m\text{-}K} \times 150\ m^2 \times \frac{45\text{-}15}{0.05}\,\frac{K}{m}$$

$$= \mathbf{5400\ W}$$

4.105

A water-heater is covered up with insulation boards over a total surface area of 3 m^2. The inside board surface is at 75°C and the outside surface is at 18°C and the board material has a conductivity of 0.08 W/m K. How thick a board should it be to limit the heat transfer loss to 200 W ?

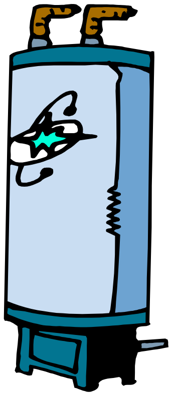
Solution :

Steady state conduction through a single layer board.

$$\dot{Q}_{cond} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta x = k\ A\ \Delta T/\dot{Q}$$

$$\Delta x = 0.08\ \frac{W}{m\ K} \times 3 m^2 \times \frac{75 - 18}{200}\ \frac{K}{W}$$

$$= \mathbf{0.068\ m}$$



4.106

A large condenser (heat exchanger) in a power plant must transfer a total of 100 MW from steam running in a pipe to sea water being pumped through the heat exchanger. Assume the wall separating the steam and seawater is 4 mm of steel, conductivity 15 W/m K and that a maximum of 5°C difference between the two fluids is allowed in the design. Find the required minimum area for the heat transfer neglecting any convective heat transfer in the flows.

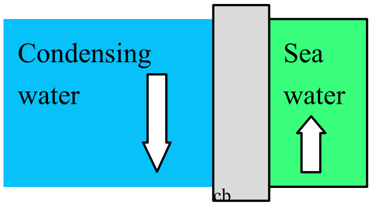
Solution :

Steady conduction through the 4 mm steel wall.

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad A = \dot{Q}\ \Delta x / k\Delta T$$

$$A = 100 \times 10^6\ \text{W} \times 0.004\ \text{m} / (15\ \text{W/mK} \times 5\ \text{K})$$

$$= \mathbf{480\ m^2}$$



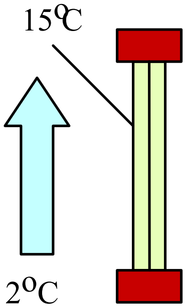
4.107

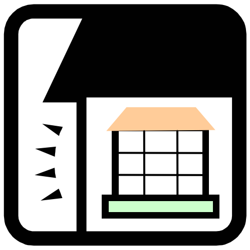
A 2 m^2 window has a surface temperature of 15^{o}C and the outside wind is blowing air at 2^{o}C across it with a convection heat transfer coefficient of h = 125 W/m^2K. What is the total heat transfer loss?

Solution:

$$\dot{Q} = h\,A\,\Delta T = 125\ \text{W/m}^2\text{K} \times 2\ \text{m}^2 \times (15 - 2)\ \text{K} = \mathbf{3250\ W}$$

as a rate of heat transfer out.





4.108

You drive a car on a winter day with the atmospheric air at −15°C and you keep the outside front windshield surface temperature at +2°C by blowing hot air on the inside surface. If the windshield is 0.5 m^2 and the outside convection coefficient is 250 W/m^2K find the rate of energy loos through the front windshield. For that heat transfer rate and a 5 mm thick glass with k = 1.25 W/m K what is then the inside windshield surface temperature?

Solution :

The heat transfer from the inside must match the loss on the outer surface to give a steady state (frost free) outside surface temperature.

$$\dot{Q}_{conv} = h\ A\ \Delta T = 250 \times 0.5 \times [2 - (-15)]$$

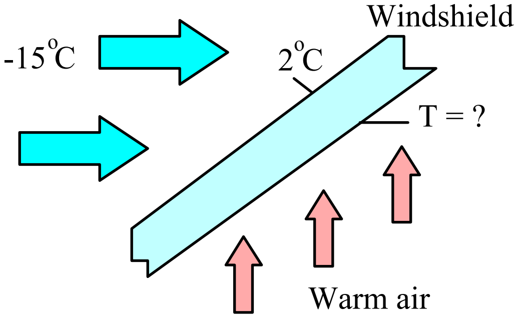
$$= 250 \times 0.5 \times 17 = \mathbf{2125\ W}$$

This is a substantial amount of power.

$$\dot{Q}_{cond} = k\ \ A\ \frac{\Delta T}{\Delta x} \qquad \Rightarrow \qquad \Delta T = \frac{\dot{Q}}{kA}\,\Delta x$$

$$\Delta T = \frac{2125\ \ W}{1.25\ W/mK \times 0.5\ m^2} \times 0.005\ m = 17\ K$$

$$T_{in} = T_{out} + \Delta T = 2 + 17 = \mathbf{19°C}$$



4.109

The brake shoe and steel drum on a car continuously absorbs 25 W as the car slows down. Assume a total outside surface area of 0.1 m^2 with a convective heat transfer coefficient of 10 W/m^2 K to the air at 20°C. How hot does the outside brake and drum surface become when steady conditions are reached?

Solution :

$$\dot{Q} = hA\Delta T \quad \Rightarrow \quad \Delta T = \frac{\dot{Q}}{hA}$$

$$\Delta T = (T_{BRAKE} - 20) = \frac{25}{10 \times 0.1} = \mathbf{25\ °C}$$

$$T_{BRAKE} = 20 + 25 = \mathbf{45°C}$$

4.110

Due to a faulty door contact the small light bulb (25 W) inside a refrigerator is kept on and limited insulation lets 50 W of energy from the outside seep into the refrigerated space. How much of a temperature difference to the ambient at 20°C must the refrigerator have in its heat exchanger with an area of 1 m^2 and an average heat transfer coefficient of 15 W/m^2 K to reject the leaks of energy.

Solution :

$$\dot{Q}_{tot} = 25 + 50 = 75 \text{ W to go out}$$

$$\dot{Q} = hA\,\Delta T = 15 \times 1 \times \Delta T = 75 \text{ W}$$

$$\Delta T = \frac{\dot{Q}}{hA} = \frac{75}{15 \times 1} = 5\ ^\circ\text{C}$$

OR T must be at least **25 °C**

4.111

The black grille on the back of a refrigerator has a surface temperature of 35°C with a total surface area of 1 m^2. Heat transfer to the room air at 20°C takes place with an average convective heat transfer coefficient of 15 W/m^2 K. How much energy can be removed during 15 minutes of operation?

Solution :

$$\dot{Q} = hA\ \Delta T; \qquad Q = \dot{Q}\ \Delta t = hA\ \Delta T\ \Delta t$$

$$Q = 15\ W/m^2\ K \times 1\ m^2 \times (35\text{-}20)\ K \times 15\ min \times 60\ s/min$$

$$= 202\ 500\ J = \mathbf{202.5\ kJ}$$

4.112

A wall surface on a house is at 30°C with an emissivity of $\varepsilon = 0.7$. The surrounding ambient to the house is at 15°C, average emissivity of 0.9. Find the rate of radiation energy from each of those surfaces per unit area.

Solution :

$$\dot{Q}/A = \varepsilon\sigma A T^4, \qquad \sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}$$

a) $\dot{Q}/A = 0.7 \times 5.67 \times 10^{-8} \times (273.15 + 30)^4 = \mathbf{335\ W/m^2}$

b) $\dot{Q}/A = 0.9 \times 5.67 \times 10^{-8} \times 288.15^4 = \mathbf{352\ W/m^2}$

4.113

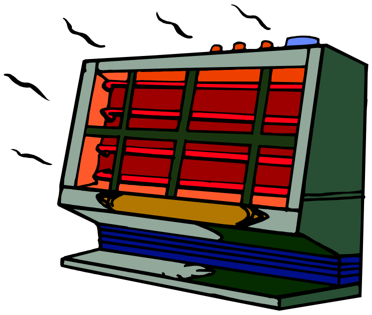
A radiant heating lamp has a surface temperature of 1000 K with $\varepsilon = 0.8$. How large a surface area is needed to provide 250 W of radiation heat transfer?

Radiation heat transfer. We do not know the ambient so let us find the area for an emitted radiation of 250 W from the surface

$$\dot{Q} = \varepsilon \sigma A T^4$$

$$A = \frac{\dot{Q}}{\varepsilon \sigma T^4} = \frac{250}{0.8 \times 5.67 \times 10^{-8} \times 1000^4} \; \frac{\text{W}}{\text{W/m}^2}$$

$$= \mathbf{0.0055 \;\; m^2}$$



4.114

A log of burning wood in the fireplace has a surface temperature of 450°C. Assume the emissivity is 1 (perfect black body) and find the radiant emission of energy per unit surface area.

Solution :

$$\begin{aligned}\dot{Q}/A &= 1 \times \sigma\ T^4 \\ &= 5.67 \times 10^{-8} \times (273.15 + 450)^4 \\ &= 15\,505\ \mathrm{W/m^2} \\ &= \mathbf{15.5\ kW/m^2}\end{aligned}$$

4.115

A radiant heat lamp is a rod, 0.5 m long and 0.5 cm in diameter, through which 400 W of electric energy is deposited. Assume the surface has an emissivity of 0.9 and neglect incoming radiation. What will the rod surface temperature be ?

Solution :

For constant surface temperature outgoing power equals electric power.

$\dot{Q}_{rad} = \varepsilon\sigma A T^4 = \dot{Q}_{el} \Rightarrow$

$T^4 = \dot{Q}_{el} / \varepsilon\sigma A = 400 / (0.9 \times 5.67 \times 10^{-8} \times 0.5 \times \pi \times 0.005)$

$= 9.9803 \times 10^{11}\ K^4 \quad \Rightarrow \quad T \cong$ **1000 K OR 725 °C**

Review Problems

4.116

A nonlinear spring has the force versus displacement relation of $F = k_{ns}(x - x_0)^n$. If the spring end is moved to x_1 from the relaxed state, determine the formula for the required work.

Solution:

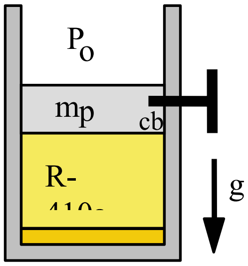
In this case we know F as a function of x and can integrate

$$W = \int F dx = \int k_{ns}(x - x_o)^n \, d(x - x_o) = \frac{k_{ns}}{n + 1} (x_1 - x_o)^{n+1}$$

4.117

A vertical cylinder (Fig. P4.117) has a 61.18-kg piston locked with a pin trapping 10 L of R-410a at 10°C, 90% quality inside. Atmospheric pressure is 100 kPa, and the cylinder cross-sectional area is 0.006 m^2. The pin is removed, allowing the piston to move and come to rest with a final temperature of 10°C for the R-410a. Find the final pressure, final volume and the work done by the R-410a.

Solution:



State 1: (T, x) from table B.4.1

$$v_1 = 0.0008 + 0.9 \times 0.03391 = 0.03132 \text{ m}^3\text{/kg}$$

$$m = V_1/v_1 = 0.010/0.03132 = 0.319 \text{ kg}$$

Force balance on piston gives the equilibrium pressure

$$P_2 = P_0 + m_P g/ A_P = 100 + \frac{61.18 \times 9.807}{0.006 \times 1000} = \mathbf{200\ kPa}$$

State 2: (T,P) in Table B.4.2 $v_2 = 0.13129 \text{ m}^3\text{/kg}$

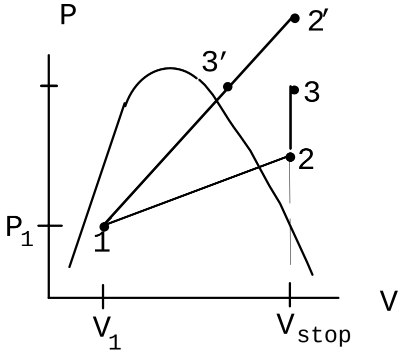
$$V_2 = mv_2 = 0.319 \text{ kg} \times 0.13129 \text{ m}^3\text{/kg} = 0.04188 \text{ m}^3 = \mathbf{41.88\ L}$$

$${}_1W_2 = \int P_{equil}\, dV = P_2(V_2\text{-}V_1) = 200 \text{ kPa } (0.04188\text{- } 0.010) \text{ m}^3 = \mathbf{6.38\ kJ}$$

4.118

Two kilograms of water is contained in a piston/cylinder (Fig. P4.110) with a massless piston loaded with a linear spring and the outside atmosphere. Initially the spring force is zero and $P_1 = P_o = 100$ kPa with a volume of 0.2 m^3. If the piston just hits the upper stops the volume is 0.8 m^3 and $T = 600°C$. Heat is now added until the pressure reaches 1.2 MPa. Find the final temperature, show the *P–V* diagram and find the work done during the process.

Solution:



State 1: $v_1 = V/m = 0.2 / 2 = 0.1\ m^3/kg$

Process: $1 \rightarrow 2 \rightarrow 3$ or $1 \rightarrow 3'$

State at stops: 2 or 2'

$v_2 = V_{stop}/m = 0.4\ m^3/kg$ & $T_2 = 600°C$

Table B.1.3 $\Rightarrow$ $P_{stop} = 1\ MPa < P_3$

since $P_{stop} < P_3$ the process is as $1 \rightarrow 2 \rightarrow 3$

State 3: $P_3 = 1.2$ MPa, $v_3 = v_2 = 0.4\ m^3/kg$ $\Rightarrow$ $T_3 \cong$ **770°C**

$$W_{13} = W_{12} + W_{23} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + 0$$

$$= \frac{1}{2}(100 + 1000)\ kPa \times (0.8 - 0.2)\ m^3$$

= **330 kJ**

4.119

A piston/cylinder contains butane, C_4H_{10}, at 300°C, 100 kPa with a volume of 0.02 m^3. The gas is now compressed slowly in an isothermal process to 300 kPa.

a. Show that it is reasonable to assume that butane behaves as an ideal gas during this process.

b. Determine the work done by the butane during the process.

Solution:

a) $$T_{r1} = \frac{T}{T_c} = \frac{573.15}{425.2} = 1.35; \qquad P_{r1} = \frac{P}{P_c} = \frac{100}{3800} = 0.026$$

From the generalized chart in figure D.1 $\quad Z_1 = 0.99$

$$T_{r2} = \frac{T}{T_c} = \frac{573.15}{425.2} = 1.35; \qquad P_{r2} = \frac{P}{P_c} = \frac{300}{3800} = 0.079$$

From the generalized chart in figure D.1 $\quad Z_2 = 0.98$

Ideal gas model is adequate for both states.

b) Ideal gas $\quad T = \text{constant} \Rightarrow PV = mRT = \text{constant}$

$$W = \int P\, dV = P_1V_1 \ln \frac{P_1}{P_2} = 100 \text{kPa} \times 0.02 \text{ m}^3 \times \ln \frac{100}{300}$$

$= \mathbf{-2.2\ kJ}$

4.120

Consider the process described in Problem 3.116. With 1 kg water as a control mass, determine the boundary work during the process.

A spring-loaded piston/cylinder contains water at 500°C, 3 MPa. The setup is such that pressure is proportional to volume, *P* = *CV*. It is now cooled until the water becomes saturated vapor. Sketch the *P-v* diagram and find the final pressure.

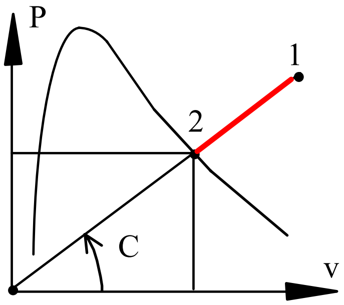
Solution :

State 1: Table B.1.3: $v_1 = 0.11619 \text{ m}^3/\text{kg}$

Process: m is constant and $P = C_0V = C_0m\,v = C\,v$

$$P = Cv \quad \Rightarrow \quad C = P_1/v_1 = 3000/0.11619 = 25820 \text{ kPa kg/m}^3$$

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)



Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \Rightarrow C = P/v_g = 20074$ (low)

2.5 MPa $v_g = 0.07998 \Rightarrow C = P/v_g = 31258$ (high)

2.25 MPa $v_g = 0.08875 \Rightarrow C = P/v_g = 25352$ (low)

Now interpolate to match the right slope C:

$P_2 = 2270$ kPa, $v_2 = P_2/C = 2270/25820 = 0.0879 \text{ m}^3/\text{kg}$

P is linear in V so the work becomes (area in P-v diagram)

$$_1w_2 = \int P\,dv = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1}{2}(3000 + 2270)(0.0879 - 0.11619) = \mathbf{-\ 74.5\ kJ/kg}$$

4.121

A cylinder having an initial volume of 3 m^3 contains 0.1 kg of water at 40°C. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process splitting it into two steps. Assume the water vapor is an ideal gas during the first step of the process.

Solution: C.V. Water

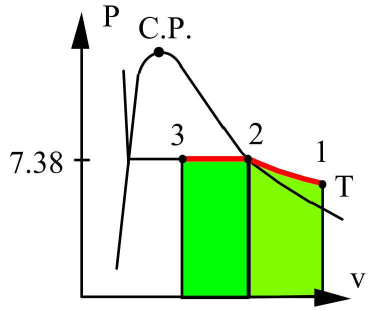
State 2: (40°C, x = 1) Tbl B.1.1 => $P_G = 7.384$ kPa, $v_G = 19.52$ m^3/kg

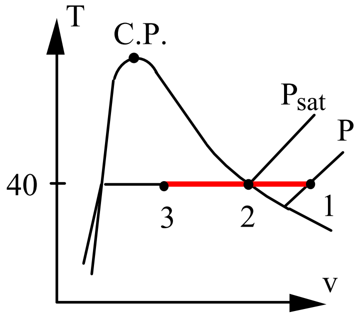
State 1: $v_1 = V_1/m = 3 / 0.1 = 30$ m^3/kg $(> v_G)$

so $H_2O \sim$ ideal gas from 1-2 so since constant T

$$P_1 = P_G \frac{v_G}{v_1} = 7.384 \times \frac{19.52}{30} = 4.8 \text{ kPa}$$

$$V_2 = mv_2 = 0.1 \times 19.52 = 1.952 \text{ m}^3$$





Process T = C: and ideal gas gives work from Eq.4.5

$${}_1W_2 = \int_1^2 PdV = P_1V_1 \ln\frac{V_2}{V_1} = 4.8 \times 3.0 \times \ln\frac{1.952}{3} = -6.19 \text{ kJ}$$

$v_3 = 0.001008 + 0.5 \times 19.519 = 9.7605$ => $V_3 = mv_3 = 0.976$ m^3

$P = C = P_g$: This gives a work term as

$${}_2W_3 = \int_2^3 PdV = P_g (V_3 - V_2) = 7.384 \text{ kPa } (0.976 - 1.952) \text{ m}^3 = -7.21 \text{ kJ}$$

Total work:

$${}_1W_3 = {}_1W_2 + {}_2W_3 = -6.19 - 7.21 = \mathbf{-13.4\ kJ}$$

4.122

A piston/cylinder (Fig. P4.72) contains 1 kg of water at 20°C with a volume of 0.1 m^3. Initially the piston rests on some stops with the top surface open to the atmosphere, P_o and a mass so a water pressure of 400 kPa will lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume and the work, ${}_1W_2$.

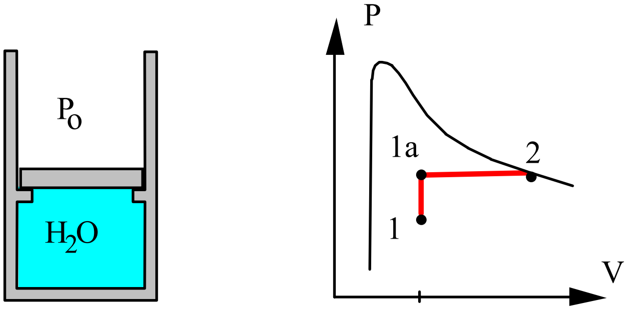
Solution:

(a) State to reach lift pressure of $P = 400$ kPa, $v = V/m = 0.1$ m^3/kg

Table B.1.2: $v_f < v < v_g = 0.4625$ m^3/kg

=> $T = T_{sat}$ = **143.63°C**

(b) State 2 is saturated vapor at 400 kPa since state 1a is two-phase.



$v_2 = v_g = 0.4625$ m^3/kg , $V_2 = m v_2 = 0.4625$ m^3,

Pressure is constant as volume increase beyond initial volume.

$${}_1W_2 = \int P\,dV = P(V_2 - V_1) = P_{lift}(V_2 - V_1)$$

$= 400$ kPa $(0.4625 - 0.1)$ m^3 = **145 kJ**

4.123

Find the work for Problem 3.112.

Refrigerant-410a in a piston/cylinder arrangement is initially at 50°C, $x = 1$. It is then expanded in a process so that $P = Cv^{-1}$ to a pressure of 100 kPa. Find the final temperature and specific volume.

Solution:

Knowing the process (P versus V) and states 1 and 2 allows calculation of W.

State 1: 50°C, x = 1 Table B.4.1: $P_1 = 3065.2$ kPa, $v_1 = 0.00707$ m^3/kg

Process: $Pv = C = P_1v_1$; => $P_2 = C/v_2 = P_1v_1/v_2$

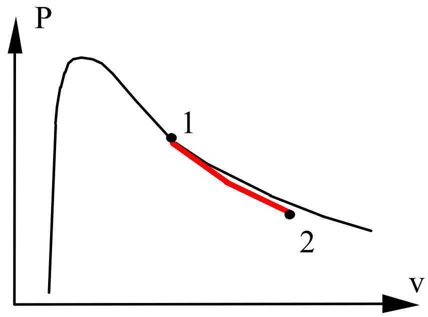
State 2: 100 kPa and $v_2 = v_1P_1/P_2 = 0.2167$ m^3/kg

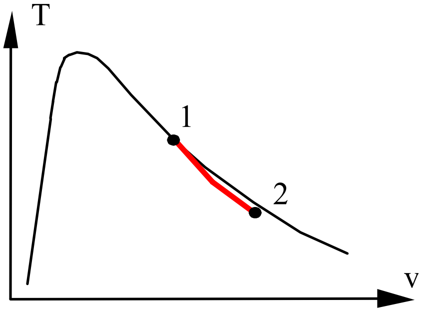
$v_2 < v_g$ at 100 kPa, $T_2 \cong$ **-51.65°C** from Table B.3.2

Notice T is **not** constant. It is not an ideal gas in this range

The constant C for the work term is P_1v_1 so per unit mass we get

$$_1w_2 = P_1v_1 \ln\frac{v_2}{v_1} = 3065.2 \times 0.0707 \times \ln\frac{0.2167}{0.0707} = \mathbf{242.7\ kJ/kg}$$





4.124

A cylinder fitted with a piston contains propane gas at 100 kPa, 300 K with a volume of 0.2 m^3. The gas is now slowly compressed according to the relation $PV^{1.1}$ = constant to a final temperature of 340 K. Justify the use of the ideal gas model. Find the final pressure and the work done during the process.

Solution:

The process equation and T determines state 2. Use ideal gas law to say

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = 100 \left(\frac{340}{300}\right)^{\frac{1.1}{0.1}} = \mathbf{396\ kPa}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/n} = 0.2 \left(\frac{100}{396}\right)^{1/1.1} = 0.0572\ \mathrm{m}^3$$

For propane Table A.2: $T_c = 370$ K, $P_c = 4260$ kPa, Figure D.1 gives Z.

$T_{r1} = 0.81,\ P_{r1} = 0.023 \quad => \quad Z_1 = 0.98$

$T_{r2} = 0.92,\ P_{r2} = 0.093 \quad => \quad Z_2 = 0.95$

Ideal gas model **OK** for both states, minor corrections could be used. The work is integrated to give Eq.4.4

$$_1W_2 = \int P\, dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{(396 \times 0.0572) - (100 \times 0.2)}{1 - 1.1}\ \mathrm{kPa\ m^3}$$

$$= \mathbf{-26.7\ kJ}$$

4.125

Consider the nonequilibrium process described in Problem 3.122. Determine the work done by the carbon dioxide in the cylinder during the process.

A cylinder has a thick piston initially held by a pin as shown in Fig. P3.122. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m^3 and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas returns to ambient temperature. Is the piston against the stops?

Solution:

Knowing the process (P vs. V) and the states 1 and 2 we can find W.

If piston floats or moves:

$$P = P_{\text{lift}} = P_o + \rho H g = 101.3 + 8000 \times 0.1 \times 9.807 / 1000 = 108.8 \text{ kPa}$$

Assume the piston is at the stops (since $P_1 > P_{\text{lift}}$ piston would move)

$V_2 = V_1 \times 150 / 100 = (\pi/4)\, 0.1^2 \times 0.1 \times 1.5 = 0.000785 \times 1.5 = 0.001\ 1775 \text{ m}^3$

For max volume we must have $P > P_{\text{lift}}$ so check using ideal gas and constant T process: $P_2 = P_1 V_1 / V_2 = 200/1.5 = 133 \text{ kPa} > P_{\text{lift}}$ and piston is at stops.

$${}_1W_2 = \int P_{\text{lift}}\, dV = P_{\text{lift}} (V_2 - V_1) = 108.8 \text{ kPa } (0.0011775 - 0.000785) \text{ m}^3$$
$$= \mathbf{0.0427\ kJ}$$

Remark: The work is determined by the equilibrium pressure, P_{lift}, and not the instantaneous pressure that will accelerate the piston (give it kinetic energy). We need to consider the quasi-equilibrium process to get W.

4.126

The gas space above the water in a closed storage tank contains nitrogen at 25°C, 100 kPa. Total tank volume is 4 m^3, and there is 500 kg of water at 25°C. An additional 500 kg water is now forced into the tank. Assuming constant temperature throughout, find the final pressure of the nitrogen and the work done on the nitrogen in this process.

Solution:

The water is compressed liquid and in the process the pressure goes up so the water stays as liquid. Incompressible so the specific volume does not change. The nitrogen is an ideal gas and thus highly compressible.

State 1: $V_{H_2O\,1} = 500 \times 0.001003 = 0.5015\ m^3$

$V_{N_2\,1} = 4.0 - 0.5015 = 3.4985\ m^3$

State 2: $V_{N_2\,2} = 4.0 - 2 \times 0.5015 = 2.997\ m^3$

Process: T = C gives $P_1V_1 = mRT = P_2V_2$

$$P_{N_2\,2} = 100 \times \frac{3.4985}{2.997} = \mathbf{116.7\ kPa}$$

Constant temperature gives $P = mRT/V$ i.e. pressure inverse in V for which the work term is integrated to give Eq.4.5

$$W_{by\,N_2} = \int_1^2 P_{N_2} dV_{N_2} = P_1V_1 \ln(V_2/V_1)$$

$$= 100 \times 3.4985 \times \ln\frac{2.997}{3.4985} = \mathbf{-54.1\ kJ}$$

4.127

Consider the problem of inflating the helium balloon, as described in problem 3.124. For a control volume that consists of the helium inside the balloon determine the work done during the filling process when the diameter changes from 1 m to 4 m.

Solution :

Inflation at constant $P = P_0 = 100$ kPa to $D_1 = 1$ m, then

$$P = P_0 + C\,(D^{*\,-1} - D^{*\,-2}), \qquad D^* = D / D_1,$$

to $D_2 = 4$ m, $P_2 = 400$ kPa, from which we find the constant C as:

$$400 = 100 + C[\,(1/4) - (1/4)^2\,] \quad => \quad C = 1600 \text{ kPa}$$

The volumes are: $V = \frac{\pi}{6} D^3 \;\; => \;\; V_1 = 0.5236 \text{ m}^3; \quad V_2 = 33.51 \text{ m}^3$

$$W_{CV} = \int_1^2 PdV$$

$$= P_0(V_2 - V_1) + \int_1^2 C(D^{*\,-1} - D^{*\,-2})dV$$

$$V = \frac{\pi}{6} D^3, \qquad dV = \frac{\pi}{2} D^2\, dD \;=\; \frac{\pi}{2} D_1^{\,3}\, D^{*\,2}\, dD^*$$

$$\Rightarrow W_{CV} = P_0(V_2 - V_1) + 3CV_1 \int_{D_1^*=1}^{D_2^*=4} (D^*-1)dD^*$$

$$= P_0(V_2 - V_1) + 3CV_1\Big[\frac{D_2^{*\,2} - D_1^{*\,2}}{2} - (D_2^* - D_1^*)\Big]_1^4$$

$$= 100 \times (33.51 - 0.5236) + 3 \times 1600 \times 0.5236\,[\frac{16-1}{2} - (4-1)]$$

= **14 608 kJ**

4.128

Air at 200 kPa, 30°C is contained in a cylinder/piston arrangement with initial volume 0.1 m^3 . The inside pressure balances ambient pressure of 100 kPa plus an externally imposed force that is proportional to $V^{0.5}$. Now heat is transferred to the system to a final pressure of 225 kPa. Find the final temperature and the work done in the process.

Solution:

C.V. Air. This is a control mass. Use initial state and process to find T_2

$$P_1 = P_0 + CV^{1/2}; \quad 200 = 100 + C(0.1)^{1/2}, \quad C = 316.23 \Rightarrow$$

$$225 = 100 + CV_2^{1/2} \Rightarrow V_2 = 0.156 \text{ m}^3$$

$$P_2V_2 = mRT_2 = \frac{P_1V_1}{T_1} T_2 \Rightarrow$$

$$T_2 = (P_2V_2 / P_1V_1)\, T_1 = 225 \times 0.156 \times 303.15 / (200 \times 0.1) = 532 \text{ K} = 258.9°\text{C}$$

$$W_{12} = \int P\, dV = \int \left(P_0 + CV^{1/2}\right) dV$$

$$= P_0 (V_2 - V_1) + C \times \frac{2}{3} \times (V_2^{3/2} - V_1^{3/2})$$

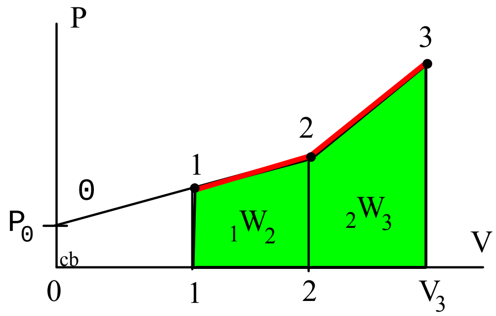
$$= 100\,(0.156 - 0.1) + 316.23 \times \frac{2}{3} \times (0.156^{3/2} - 0.1^{3/2})$$

$$= 5.6 + 6.32 = \mathbf{11.9\ kJ}$$

4.129

Two springs with same spring constant are installed in a massless piston/cylinder with the outside air at 100 kPa. If the piston is at the bottom, both springs are relaxed and the second spring comes in contact with the piston at $V = 2\ \text{m}^3$. The cylinder (Fig. P4.129) contains ammonia initially at –2°C, $x = 0.13$, $V = 1\ \text{m}^3$, which is then heated until the pressure finally reaches 1200 kPa. At what pressure will the piston touch the second spring? Find the final temperature and the total work done by the ammonia.

Solution :



State 1: P = 399.7 kPa   Table B.2.1

$v = 0.00156 + 0.13 \times 0.3106 = 0.0419$

At bottom state 0: $0\ \text{m}^3$, 100 kPa

State 2: $V = 2\ \text{m}^3$ and on line   0-1-2

Final state 3: 1200 kPa, on line segment 2.

Slope of line 0-1-2: $\Delta P/\Delta V = (P_1 - P_0)/\Delta V = (399.7\text{-}100)/1 = 299.7\ \text{kPa/m}^3$

$$P_2 = P_1 + (V_2 - V_1)\Delta P/\Delta V = 399.7 + (2\text{-}1)\times 299.7 = \mathbf{699.4\ kPa}$$

State 3: Last line segment has twice the slope.

$$P_3 = P_2 + (V_3 - V_2)2\Delta P/\Delta V \quad \Rightarrow \quad V_3 = V_2 + (P_3 - P_2)/(2\Delta P/\Delta V)$$

$$V_3 = 2 + (1200\text{-}699.4)/599.4 = 2.835\ \text{m}^3$$

$$v_3 = v_1 V_3/V_1 = 0.0419 \times 2.835/1 = 0.1188 \quad \Rightarrow \quad T = \mathbf{51°C}$$

$${}_1W_3 = {}_1W_2 + {}_2W_3 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + \frac{1}{2}(P_3 + P_2)(V_3 - V_2)$$

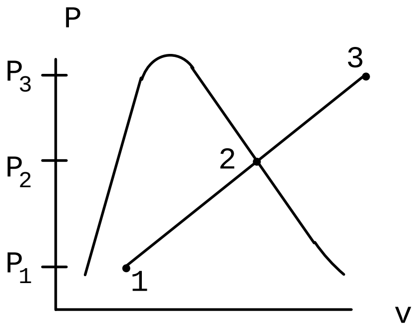
$$= 549.6 + 793.0 = \mathbf{1342.6\ kJ}$$

4.130

A spring-loaded piston/cylinder arrangement contains R-134a at 20°C, 24% quality with a volume 50 L. The setup is heated and thus expands, moving the piston. It is noted that when the last drop of liquid disappears the temperature is 40°C. The heating is stopped when $T = 130$°C. Verify the final pressure is about 1200 kPa by iteration and find the work done in the process.

Solution:

C.V. R-134a. This is a control mass.



State 1: Table B.5.1 =>

$v_1 = 0.000817 + 0.24*0.03524 = 0.009274$

$P_1 = 572.8$ kPa,

$m = V/ v_1 = 0.050 / 0.009274 = 5.391$ kg

Process: Linear Spring

$P = A + Bv$

State 2: $x_2 = 1, T_2 \Rightarrow P_2 = 1.017$ MPa, $v_2 = 0.02002$ m^3/kg

Now we have fixed two points on the process line so for final state 3:

$$P_3 = P_1 + \frac{P_2 - P_1}{v_2 - v_1}(v_3 - v_1) = \text{RHS} \qquad \text{Relation between } P_3 \text{ and } v_3$$

State 3: T_3 and on process line $\Rightarrow$ iterate on P_3 given T_3

at $P_3 = 1.2$ MPa => $v_3 = 0.02504$ => P_3 - RHS = -0.0247

at $P_3 = 1.4$ MPa => $v_3 = 0.02112$ => P_3 - RHS = 0.3376

Linear interpolation gives :

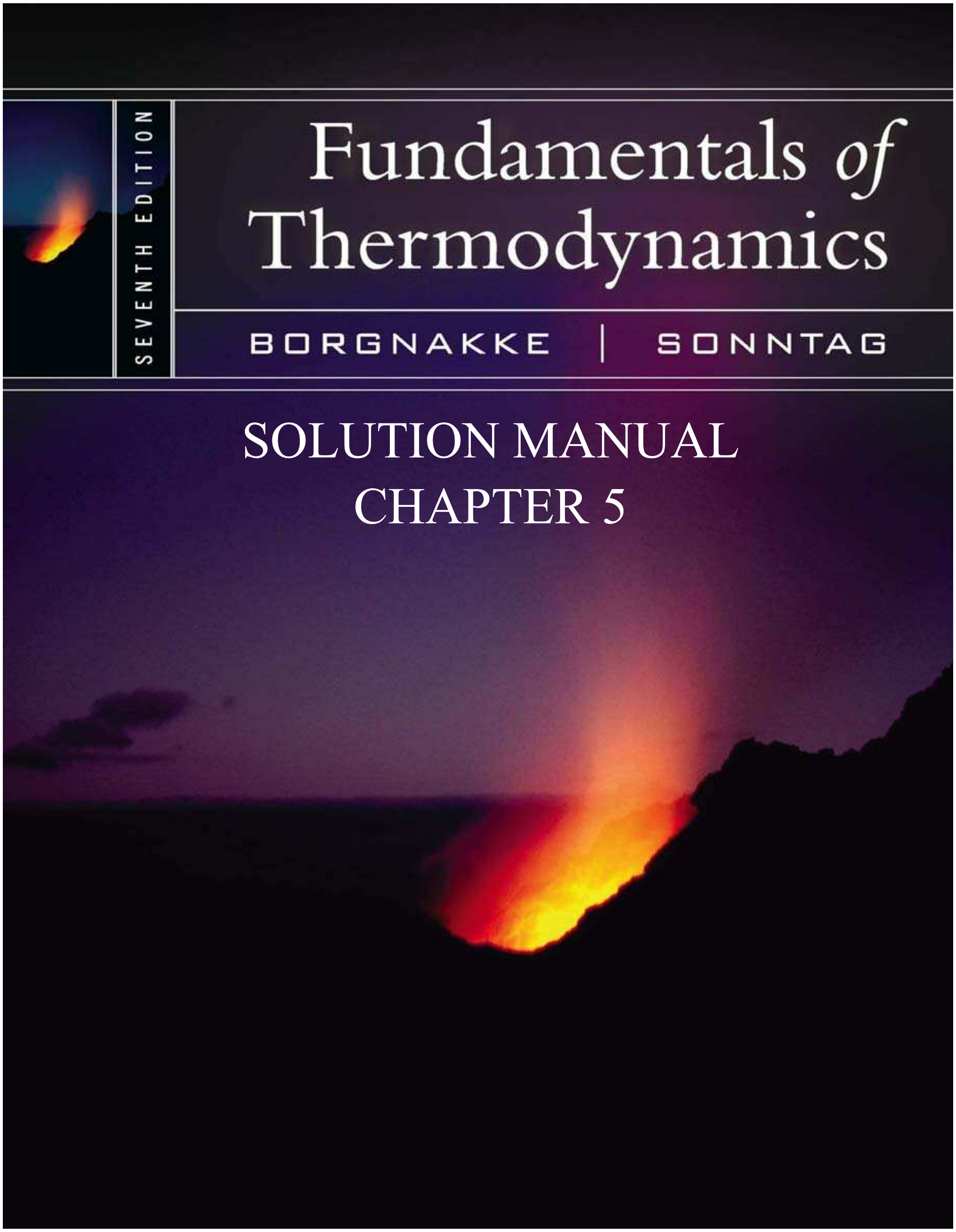
$$P_3 \cong 1200 + \frac{0.0247}{0.3376 + 0.0247}(1400\text{-}1200) = 1214 \text{ kPa}$$

$$v_3 = 0.02504 + \frac{0.0247}{0.3376 + 0.0247}(0.02112\text{-}0.02504) = 0.02478 \text{ m}^3/\text{kg}$$

$$W_{13} = \int P\, dV = \frac{1}{2}(P_1 + P_3)(V_3 - V_1) = \frac{1}{2}(P_1 + P_3)\, m\, (v_3 - v_1)$$

$$= \frac{1}{2}\; 5.391 \text{ kg } (572.8 + 1214) \text{ kPa } (0.02478 - 0.009274) \text{ m}^3/\text{kg}$$

= **74.7 kJ**



SEVENTH EDITION

Fundamentals of Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 5

CONTENT

In-Text Concept Questions

5.a

In a complete cycle what is the net change in energy and in volume?

For a complete cycle the substance has no change in energy and therefore no storage, so the net change in energy is zero.

For a complete cycle the substance returns to its beginning state, so it has no change in specific volume and therefore no change in total volume.

5.b

Explain in words what happens with the energy terms for the stone in Example 5.2. What would happen if it were a bouncing ball falling to a hard surface?

In the beginning all the energy is potential energy associated with the gravitational force. As the stone falls the potential energy is turned into kinetic energy and in the impact the kinetic energy is turned into internal energy of the stone and the water. Finally the higher temperature of the stone and water causes a heat transfer to the ambient until ambient temperature is reached.

With a hard ball instead of the stone the impact would be close to elastic transforming the kinetic energy into potential energy (the material acts as a spring) that is then turned into kinetic energy again as the ball bounces back up. Then the ball rises up transforming the kinetic energy into potential energy (mgZ) until zero velocity is reached and it starts to fall down again. The collision with the floor is not perfectly elastic so the ball does not rise exactly up to the original height losing a little energy into internal energy (higher temperature due to internal friction) with every bounce and finally the motion will die out. All the energy eventually is lost by heat transfer to the ambient or sits in lasting deformation (internal energy) of the substance.

5.c

Make a list of at least 5 systems that store energy, explaining which form of energy.

A spring that is compressed. Potential energy $(1/2)kx^2$
A battery that is charged. Electrical potential energy. V Amp h
A raised mass (could be water pumped up higher) Potential energy mgH
A cylinder with compressed air. Potential (internal) energy like a spring.
A tank with hot water. Internal energy mu
A fly-wheel. Kinetic energy (rotation) $(1/2)I\omega^2$
A mass in motion. Kinetic energy $(1/2)m\mathbf{V}^2$

5.d

A constant mass goes through a process where 100 J of heat transfer comes in and 100 J of work leaves. Does the mass change state?

Yes it does.

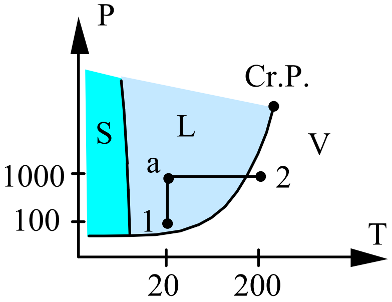
As work leaves a control mass its volume must go up, v increases

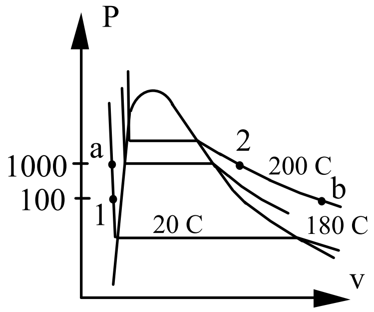
As heat transfer comes in an amount equal to the work out means u is constant if there are no changes in kinetic or potential energy.

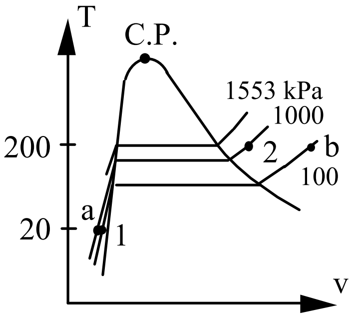
5.e

Water is heated from 100 kPa, 20^{o}C to 1000 kPa, 200^{o}C. In one case pressure is raised at T = C, then T is raised at P = C. In a second case the opposite order is done. Does that make a difference for $_1Q_2$ and $_1W_2$?

Yes it does. Both $_1Q_2$ and $_1W_2$ are process dependent. We can illustrate the work term in a P-v diagram.







In one case the process proceeds from 1 to state “a” along constant T then from “a” to state 2 along constant P.

The other case proceeds from 1 to state “b” along constant P and then from “b” to state 2 along constant T.

5.f

A rigid insulated tank A contains water at 400 kPa, 800C. A pipe and valve connect this to another rigid insulated tank B of equal volume having saturated water vapor at 100 kPa. The valve is opened and stays open while the water in the two tanks comes to a uniform final state. Which two properties determine the final state?

Continuity eq.: $m_2 - m_{1A} - m_{1B} = 0 \;\Rightarrow\; m_2 = m_{1A} + m_{1B}$

Energy eq.: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = 0 - 0$

Process: Insulated: ${}_1Q_2 = 0$,

Rigid: $V_2 = C = V_A + V_B \;\Rightarrow\; {}_1W_2 = 0$

From continuity eq. and process: $v_2 = V_2/m_2 = \frac{m_{1A}}{m_2} v_{1A} + \frac{m_{1B}}{m_2} v_{1B}$

From energy eq.: $u_2 = \frac{m_{1A}}{m_2} u_{1A} + \frac{m_{1B}}{m_2} u_{1B}$

Final state 2: (v_2, u_2) both are the mass weighted average of the initial values.

5.g

To determine v or u for some liquid or solid, is it more important that I know P or T?

T is more important, v and u are nearly independent of P.

5.h

To determine v or u for an ideal gas, is it more important that I know P or T?

For v they are equally important ($v = RT/P$), but for u only T is important. For an ideal gas u is a function of T only (independent of P).

5.i

I heat 1 kg of substance at a constant pressure (200 kPa) 1 degree. How much heat is needed if the substance is water at 10^{o}C, steel at 25^{o}C, air at 325 K, or ice at –10^{o}C.

Heating at constant pressure gives (recall the analysis in Section 5.5, page 141)

$${}_1Q_2 = H_2 - H_1 = m(h_2 - h_1) \approx m\, C_p\, (T_2 - T_1)$$

For all cases: ${}_1Q_2$ = 1 kg × C × 1 K

Water 10^{o}C, 200 kPa (liquid)	so A.4:	C = 4.18 kJ/kg-K,	${}_1Q_2$ = 4.18 kJ
Steel 25^{o}C, 200 kPa (solid)	so A.3:	C = 0.46 kJ/kg-K	${}_1Q_2$ = 0.46 kJ
Air 325 K, 200 kPa (gas)	so A.5:	C_p = 1.004 kJ/kg-K	${}_1Q_2$ = 1.004 kJ
Ice –10^{o}C, 200 kPa (solid)	so A.3:	C = 2.04 kJ/kg-K	${}_1Q_2$ = 2.04 kJ

Comment: For liquid water we could have interpolated $h_2 - h_1$ from Table B.1.1 and for ice we could have used Table B.1.5. For air we could have used Table A.7.

Concept Problems

5.1

What is 1 cal in SI units and what is the name given to 1 N-m?

Look in the conversion factor table A.1 under energy:

$$1 \text{ cal (Int.)} = 4.1868 \text{ J} = 4.1868 \text{ Nm} = 4.1868 \text{ kg m}^2/\text{s}^2$$

This was historically defined as the heat transfer needed to bring 1 g of liquid water from 14.5°C to 15.5°C, notice the value of the heat capacity of water in Table A.4

1 N-m = 1 J or Force times displacement = energy = Joule

5.2

Why do we write ΔE or $E_2 - E_1$ whereas we write ${}_1Q_2$ and ${}_1W_2$?

ΔE or $E_2 - E_1$ is the **change** in the stored energy from state 1 to state 2 and depends only on states 1 and 2 not upon the process between 1 and 2.

${}_1Q_2$ and ${}_1W_2$ are amounts of energy **transferred during the process** between 1 and 2 and depend on the process path. The quantities are associated with the process and they are not state properties.

5.3

If a process in a control mass increases energy $E_2 - E_1 > 0$ can you say anything about the sign for ${}_1Q_2$ and ${}_1W_2$?

No.

The net balance of the heat transfer and work terms from the energy equation is

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 > 0$$

but that does not separate the effect of the two terms.

5.4

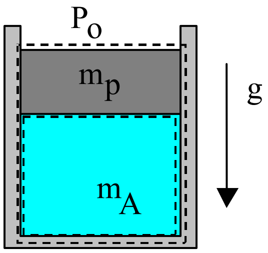
When you windup a spring in a toy or stretch a rubber band what happens in terms of work, energy, and heat transfer? Later, when they are released, what happens then?

In both processes work is put into the device and the energy is stored as potential energy. If the spring or rubber is inelastic some of the work input goes into internal energy (it becomes warmer or permanently deformed) and not into its potential energy. Being warmer than the ambient air it cools slowly to ambient temperature.

When the spring or rubber band is released the potential energy is transferred back into work given to the system connected to the end of the spring or rubber band. If nothing is connected the energy goes into kinetic energy and the motion is then dampened as the energy is transformed into internal energy.

5.5

CV A is the mass inside a piston-cylinder, CV B is that plus the piston outside, which is the standard atmosphere. Write the energy equation and work term for the two CVs assuming we have a non-zero Q between state 1 and state 2.



CV A: $E_2 - E_1 = m_A(e_2 - e_1) = m_A(u_2 - u_1) = {}_1Q_2 - {}_1WA_2$

$${}_1WA_2 = \int P\, dV = P(V_2 - V_1)$$

CV B: $E_2 - E_1 = m_A(e_2 - e_1) + m_{pist}(e_2 - e_1) = m_A(u_2 - u_1) + m_{pist}(gZ_2 - gZ_1)$

$$= {}_1Q_2 - {}_1WB_2$$

$${}_1WB_2 = \int P_o\, dV = P_o(V_2 - V_1)$$

Notice how the P inside CV A is $P = P_o + m_{pist}g / A_{cyl}$ i.e. the first work term is larger than the second. The difference between the work terms is exactly equal to the potential energy of the piston sitting on the left hand side in the CV B energy Eq. The two equations are mathematically identical.

$${}_1WA_2 = P(V_2 - V_1) = [P_o + m_{pist}g / A_{cyl}]\, (V_2 - V_1) = {}_1WB_2 + m_{pist}g(V_2 - V_1)/A_{cyl}$$

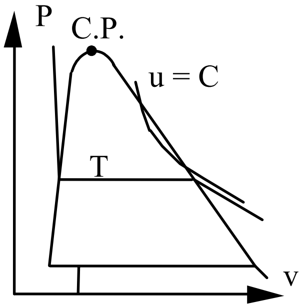
$$= {}_1WB_2 + m_{pist}g(Z_2 - Z_1)$$

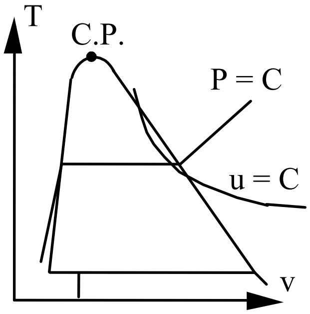
5.6

Saturated water vapor has a maximum for u and h at around 235^{o}C. Is it similar for other substances?

Look at the various substances listed in appendix B. Everyone has a maximum u and h somewhere along the saturated vapor line at different T for each substance. This means the constant u and h curves are different from the constant T curves and some of them cross over the saturated vapor line twice, see sketch below.

Constant h lines are similar to the constant u line shown.





Notice the constant u (or h) line becomes parallel to the constant T lines in the superheated vapor region for low P where it is an ideal gas. In the T-v diagram the constant u (or h) line becomes horizontal in the ideal gas region.

5.7

Some liquid water is heated so it becomes superheated vapor. Do I use u or h in the energy equation? Explain.

The energy equation for a control mass is: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

The storage of energy is a change in u (when we neglect kinetic and potential energy changes) and that is always so. To solve for the heat transfer we must know the work in the process and it is for a certain process (P = C) that the work term combines with the change in u to give a change in h. To avoid confusion you should always write the energy equation as shown above and substitute the appropriate expression for the work term when you know the process equation that allows you to evaluate work.

5.8

Some liquid water is heated so it becomes superheated vapor. Can I use specific heat to find the heat transfer? Explain.

NO.

The specific heat can not give any information about the energy required to do the phase change. The specific heat is useful for single phase state changes only.

5.9

Look at the R-410a value for u_f at -50°C. Can the energy really be negative? Explain.

The absolute value of u and h are arbitrary. A constant can be added to all u and h values and the table is still valid. It is customary to select the reference such that u for saturated liquid water at the triple point is zero. The standard for refrigerants like R-410a is that h is set to zero as saturated liquid at -40°C, other substances like cryogenic substances like nitrogen, methane etc. may have different states at which h is set to zero. The ideal gas tables use a zero point for h as 25°C or at absolute zero, 0 K.

5.10

A rigid tank with pressurized air is used to a) increase the volume of a linear spring loaded piston cylinder (cylindrical geometry) arrangement and b) to blow up a spherical balloon. Assume that in both cases $P = A + BV$ with the same A and B. What is the expression for the work term in each situation?

The expression is exactly the same; the geometry does not matter as long as we have the same relation between P and V then

$$
\begin{aligned}
{}_1W_2 &= \int P\, dV = \int (A + BV)\, dV \\
&= A(V_2 - V_1) + 0.5\, B\, (V_2^2 - V_1^2) \\
&= A(V_2 - V_1) + 0.5\, B\, (V_2 + V_1)\, (V_2 - V_1) \\
&= 0.5\, [\, A + B\, V_2 + A + B\, V_1]\, (V_2 - V_1) \\
&= 0.5\, (P_1 + P_2)\, (V_2 - V_1)
\end{aligned}
$$

Notice the last expression directly gives the area below the curve in the P-V diagram.

5.11

An ideal gas in a piston-cylinder is heated with 2 kJ during an isothermal process. How much work is involved?

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = 0$ since $u_2 = u_1$ (isothermal)

Then

$${}_1W_2 = m\ {}_1w_2 = {}_1Q_2 = m\ {}_1Q_2 = \mathbf{2\ kJ}$$

5.12

An ideal gas in a piston-cylinder is heated with 2 kJ during an isobaric process. Is the work pos., neg., or zero?

As the gas is heated u and T increase and since PV = mRT it follows that the volume increase and thus work goes out.

$$\mathbf{w > 0}$$

5.13

You heat a gas 10 K at P = C. Which one in Table A.5 requires most energy? Why?

A constant pressure process in a control mass gives (recall Eq.5.29)

$$ {}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1 \approx C_p \,\Delta T $$

The one with the highest specific heat is hydrogen, H_2. The hydrogen has the smallest mass, but the same kinetic energy per mol as other molecules and thus the most energy per unit mass is needed to increase the temperature.

5.14

A 500 W electric space heater with a small fan inside heats air by blowing it over a hot electrical wire. For each control volume: a) wire only b) all the room air and c) total room plus the heater, specify the storage, work and heat transfer terms as + 500W or -500W or 0 W, neglect any $\dot{Q}$ through the room walls or windows.

	Storage	Work	Heat transfer
Wire	0 W	-500 W	-500 W
Room air	500 W	0 W	500 W
Tot room	500 W	-500 W	0 W

Kinetic and Potential Energy

5.15

A piston motion moves a 25 kg hammerhead vertically down 1 m from rest to a velocity of 50 m/s in a stamping machine. What is the change in total energy of the hammerhead?

Solution: C.V. Hammerhead

The hammerhead does not change internal energy (i.e. same P, T), but it does have a change in kinetic and potential energy.

$$E_2 - E_1 = m(u_2 - u_1) + m[(1/2)\mathbf{V}_2{}^2 - 0] + mg\,(Z_2\ - 0)$$

$$= 0 + 25\text{ kg} \times (1/2) \times (50\text{ m/s})^2 + 25\text{ kg} \times 9.80665\text{ m/s}^2 \times (-1)\text{ m}$$

$$= 31250\text{ J} - 245.17\text{ J} = 31005\text{ J} = \mathbf{31\ kJ}$$

5.16

A steel ball weighing 5 kg rolls horizontal with 10 m/s. If it rolls up an incline how high up will it be when it comes to rest assuming standard gravitation.

C.V. Steel ball.

Energy Eq.: $E_2 - E_1 = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

$$E_1 = mu_1 + mgZ_1 + 0.5\ mV^2$$

$$E_2 = mu_2 + mgZ_2 + 0$$

We assume the steel ball does not change temperature ($u_2 = u_1$) so then the energy equation gives

$$mu_2 + mgZ_2 - mu_1 - mgZ_1 - 0.5\ mV^2 = 0$$

$$mg\ (Z_2 - Z_1) = 0.5\ mV^2$$

$$Z_2 - Z_1 = 0.5\ V^2/g = 0.5 \times 10^2\ (m^2/s^2)\ /\ (9.81\ m/s^2) = \mathbf{5.1\ m}$$

5.17

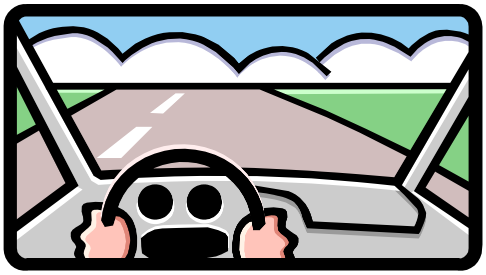
A 1200 kg car accelerates from zero to 100 km/h over a distance of 400 m. The road at the end of the 400 m is at 10 m higher elevation. What is the total increase in the car kinetic and potential energy?

Solution:

$$\Delta KE = ½\ m\ (\mathbf{V}_2^2 - \mathbf{V}_1^2)$$

$$\mathbf{V}_2 = 100\ \text{km/h} = \frac{100 \times 1000}{3600}\ \text{m/s}$$

$$= 27.78\ \text{m/s}$$



$\Delta KE = ½ \times 1200\ \text{kg} \times (27.78^2 - 0^2)\ (\text{m/s})^2 = 463\ 037\ \ \text{J} = \mathbf{463\ kJ}$

$\Delta PE = mg(Z_2 - Z_1) = 1200\ \text{kg} \times 9.807\ \text{m/s}^2\ (\ 10 - 0\ )\ \text{m} = 117684\ \text{J} \mathbf{= 117.7\ kJ}$

5.18

A hydraulic hoist raises a 1750 kg car 1.8 m in an auto repair shop. The hydraulic pump has a constant pressure of 800 kPa on its piston. What is the increase in potential energy of the car and how much volume should the pump displace to deliver that amount of work?

Solution: C.V. Car.

No change in kinetic or internal energy of the car, neglect hoist mass.

$$E_2 - E_1 = PE_2 - PE_1 = mg\,(Z_2 - Z_1)$$

$$= 1750 \times 9.80665 \times 1.8 = \mathbf{30\ 891\ \ J}$$

The increase in potential energy is work into car from pump at constant P.

$$W = E_2 - E_1 = \int P\,dV = P\,\Delta V \qquad \Rightarrow$$

$$\Delta V = \frac{E_2 - E_1}{P} = \frac{30891}{800 \times 1000} = \mathbf{0.0386\ m^3}$$

5.19

The rolling resistance of a car depends on its weight as: F = 0.006 mg. How far will a car of 1200 kg roll if the gear is put in neutral when it drives at 90 km/h on a level road without air resistance?

Solution:

The car decreases its kinetic energy to zero due to the force (constant) acting over the distance.

$$m\,(1/2\mathbf{V}_2^2 - 1/2\mathbf{V}_1^2) = -{}_1W_2 = -\int F\,dx = -FL$$

$$\mathbf{V}_2 = 0, \quad \mathbf{V}_1 = 90\,\frac{\text{km}}{\text{h}} = \frac{90 \times 1000}{3600}\ \text{ms}^{-1} = 25\ \text{ms}^{-1}$$

$$-1/2\ m\mathbf{V}_1^2 = -FL = -\ 0.006\ mgL$$

$$\rightarrow \qquad L = \frac{0.5\,\mathbf{V}_1^2}{0.0006g} = \frac{0.5 \times 25^2}{0.006 \times 9.807}\ \frac{\text{m}^2/\text{s}^2}{\text{m/s}^2} = 5311\ \text{m}$$

Remark: Over 5 km! The air resistance is much higher than the rolling resistance so this is not a realistic number by itself.

5.20

A 1200 kg car is accelerated from 30 to 50 km/h in 5 s. How much work is that? If you continue from 50 to 70 km/h in 5 s; is that the same?

The work input is the increase in kinetic energy.

$$E_2 – E_1 = (1/2)m\left[\mathbf{V}_2^2 - \mathbf{V}_1^2\right] = {}_1W_2$$

$$= 0.5 \times 1200 \text{ kg } [50^2 – 30^2] \left(\frac{\text{km}}{\text{h}}\right)^2$$

$$= 600 \text{ [} 2500 – 900 \text{] kg} \left(\frac{1000 \text{ m}}{3600 \text{ s}}\right)^2 = 74\ 074 \text{ J} = \mathbf{74.1\ kJ}$$

The second set of conditions does not become the same

$$E_2 – E_1 = (1/2)m\left[\mathbf{V}_2^2 - \mathbf{V}_1^2\right] = 600 \text{ [} 70^2 – 50^2 \text{] kg} \left(\frac{1000 \text{ m}}{3600 \text{ s}}\right)^2 = \mathbf{111\ kJ}$$

5.21

Airplane takeoff from an aircraft carrier is assisted by a steam driven piston/cylinder device with an average pressure of 1250 kPa. A 17500 kg airplane should be accelerated from zero to a speed of 30 m/s with 30% of the energy coming from the steam piston. Find the needed piston displacement volume.

Solution: C.V. Airplane.

No change in internal or potential energy; only kinetic energy is changed.

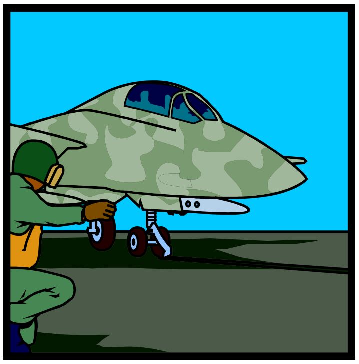
$$E_2 - E_1 = m\,(1/2)\,(\mathbf{V}_2^2 - 0) = 17500 \text{ kg} \times (1/2) \times 30^2 \text{ (m/s)}^2$$

$$= 7875\,000 \text{ J} = 7875 \text{ kJ}$$

The work supplied by the piston is 30% of the energy increase.

$$W = \int P\,dV = P_{avg}\;\Delta V = 0.30\,(E_2 - E_1)$$

$$= 0.30 \times 7875 = 2362.5 \text{ kJ}$$



$$\Delta V = \frac{W}{P_{avg}} = \frac{2362.5}{1250}\,\frac{\text{kJ}}{\text{kPa}} = \mathbf{1.89\ m^3}$$

5.22

Solve Problem 5.21, but assume the steam pressure in the cylinder starts at 1000 kPa, dropping linearly with volume to reach 100 kPa at the end of the process.

Solution: C.V. Airplane.

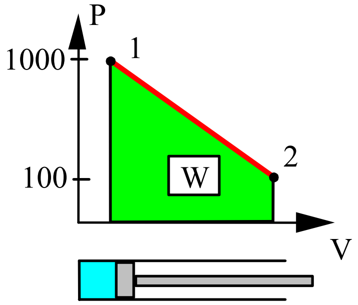
$$E_2 – E_1 = m\,(1/2)\,(\mathbf{V}_2^2\ - 0)$$

$$= 17\,500\ \text{kg} \times (1/2) \times 30^2\ (\text{m/s})^2$$

$$= 7875\,000\ \text{J} = 7875\ \text{kJ}$$

$$W = 0.30(E_2 – E_1) = 0.30 \times 7875 = 2362.5\ \text{kJ}$$

$$W = \int P\,dV = (1/2)(P_{beg} + P_{end})\,\Delta V$$



$$\Delta V = \frac{W}{P_{avg}} = \frac{2362.5\ \text{kJ}}{1/2(1000 + 100)\ \text{kPa}} = \mathbf{4.29\ m^3}$$

5.23

A 25 kg piston is above a gas in a long vertical cylinder. Now the piston is released from rest and accelerates up in the cylinder reaching the end 5 m higher at a velocity of 25 m/s. The gas pressure drops during the process so the average is 600 kPa with an outside atmosphere at 100 kPa. Neglect the change in gas kinetic and potential energy, and find the needed change in the gas volume.

Solution:

C.V. Piston

$$(E_2 - E_1)_{PIST.} = m(u_2 - u_1) + m[(1/2)V_2^2 - 0] + mg\,(H_2 - 0)$$

$$= 0 + 25 \times (1/2) \times 25^2 + 25 \times 9.80665 \times 5$$

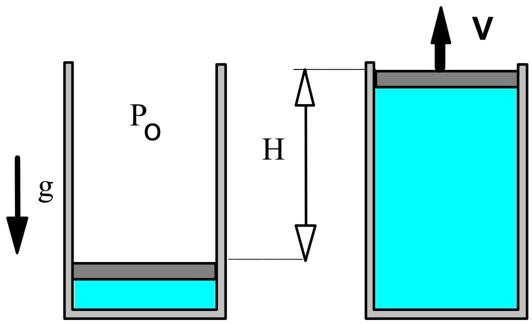
$$= 7812.5 + 1225.8 = 9038.3 \text{ J} = 9.038 \text{ kJ}$$

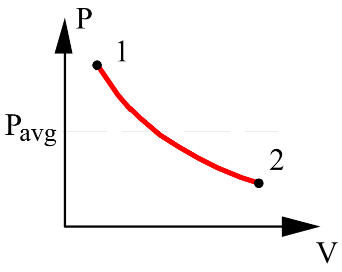
Energy equation for the piston is:

$$E_2 - E_1 = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} - P_o\ \Delta V_{gas}$$

(remark $\Delta V_{atm} = -\Delta V_{gas}$ so the two work terms are of opposite sign)

$$\Delta V_{gas} = \frac{9.038}{600 - 100}\ \frac{\text{kJ}}{\text{kPa}} = \mathbf{0.018\ m^3}$$





5.24

A piston of 2 kg is accelerated to 20 m/s from rest. What constant gas pressure is required if the area is 10 cm^2, the travel 10 cm and the outside pressure is 100 kPa?

C.V. Piston

$$(E_2 – E_1)_{PIST.} = m(u_2 – u_1) + m[(1/2)V_2^2 – 0] + mg (0 – 0)$$

$$= (1/2)\ m\ V_2^2 = 0.5 \times 2 \text{ kg} \times 20^2 \text{ (m/s)}^2 = 400 \text{ J}$$

Energy equation for the piston is:

$$(E_2 – E_1)_{PIST.} = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} – P_o\ \Delta V_{gas}$$

$$\Delta V_{gas} = A\ L = 10 \text{ cm}^2 \times 10 \text{ cm} = 0.0001 \text{ m}^3$$

$$P_{avg}\ \Delta V_{gas} = (E_2 – E_1)_{PIST.} + P_o\ \Delta V_{gas}$$

$$P_{avg} = (E_2 – E_1)_{PIST.} / \Delta V_{gas} + P_o$$

$$= 400 \text{ J} / 0.0001 \text{ m}^3 + 100 \text{ kPa}$$

$$= 4000 \text{ kPa} + 100 \text{ kPa} = \mathbf{4100\ kPa}$$

Properties (u, h) from General Tables

5.25

Find the phase and the missing properties of T, P, v, u and x for water at:

a. 500 kPa, 100^oC b. 5000 kPa, u = 800 kJ/kg

c. 5000 kPa, v = 0.06 m^3/kg d. -6^oC, v = 1 m^3/kg

Solution:

a) Look in Table B.1.2 at 500 kPa

$T < T_{sat} = 151^oC$ => compressed liquid

Table B.1.4: $v = 0.001043\ m^3/kg$, $u = 418.8$ kJ/kg

b) Look in Table B.1.2 at 5000 kPa

$u < u_f = 1147.78$ kJ/kg => compressed liquid

Table B.1.4: between 180^oC and 200^oC

$$T = 180 + (200 - 180)\frac{800 - 759.62}{848.08 - 759.62} = 180 + 20\times0.4567 = 189.1^oC$$

$$v = 0.001124 + 0.4567\ (0.001153 - 0.001124) = 0.001137\ m^3/kg$$

c) Look in Table B.1.2 at 5000 kPa

$v > v_g = 0.03944\ m^3/kg$ => superheated vapor

Table B.1.3: between 400^oC and 450^oC.

$$T = 400 + 50\times\frac{0.06 - 0.05781}{0.0633 - 0.05781} = 400 + 50\times0.3989 = 419.95^oC$$

$$u = 2906.58 + 0.3989 \times (2999.64 - 2906.58) = 2943.7\ kJ/kg$$

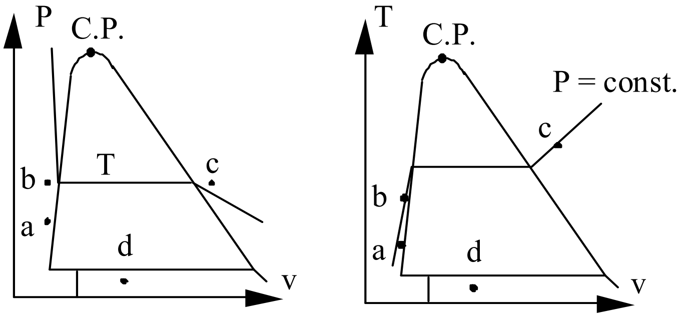
d) B.1.5: $v_i < v < v_g = 334.14\ m^3/kg$ => 2-phase, $P = P_{sat} =$ 887.6 kPa,

$$x = (v - v_i) / v_{fg} = (0.01 - 0.000857)/0.02224 = 0.4111$$

$$u = u_i + x\ u_{fg} = 248.34 + 0.4111\times148.68 = 309.46\ kJ/kg$$

5.26

States shown are placed relative to the two-phase region, not to each other.



5.27

Find the phase and the missing properties of P, T, v, u and x

a. Water at 5000 kPa, u = 3000 kJ/kg

b. Ammonia at 50°C, v = 0.08506 m^3/kg

c. Ammonia at 28°C, 1200 kPa

d. R-134a at 20°C, u = 350 kJ/kg

a) Check in Table B.1.2 at 5000 kPa: $u > u_g = 2597$ kJ/kg

Goto B.1.3 it is found very close to **450°C,** x = undefined, v = 0.0633 m^3/kg

b) Table B.2.1 at 50°C: $v > v_g = 0.06337\ m^3/kg$, so superheated vapor

Table B.2.2: close to 1600 kPa, u = 1364.9 kJ/kg, x = undefined

c) Table B.2.1 between 25 and 30°C: We see $P > P_{sat} = 1167$ kPa (30°C)

We conclude compressed liquid without any interpolation.

$$v = v_f = 0.001658 + \frac{28 - 25}{5}(0.00168 - 0.001658) = 0.00167\ m^3/kg$$

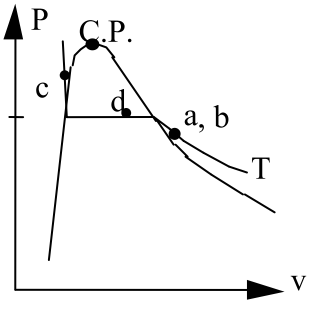
$$u = u_f = 296 + \frac{28 - 25}{5}(320.46 - 296.59) = 310.91\ kJ/kg$$

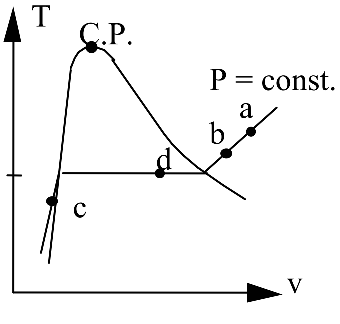
d) Table B.5.1 at 20°C: $227.03 = u_f < u < u_g = 389.19$ kJ/kg so two-phase

$$x = \frac{u - u_f}{u_{fg}} = \frac{350 - 227.03}{162.16} = 0.7583, \quad P = P_{sat} = 572.8\ kPa$$

$$v = v_f + x\ v_{fg} = 0.000817 + x \times 0.03524 = 0.02754\ m^3/kg$$

all states are relative to the two-phase region, not to each other





5.28

Find the missing properties and give the phase of the ammonia, NH_3.

a. $T = 65^{o}C$, $P = 600$ kPa $\quad u = ? \quad v = ?$

b. $T = 20^{o}C$, $P = 100$ kPa $\quad u = ? \quad v = ? \quad x = ?$

c. $T = 50^{o}C$, $v = 0.1185\ m^3/kg$ $\quad u = ? \quad P = ? \quad x = ?$

Solution:

a) Table B.2.1 $P < Psat$ => superheated vapor Table B.2.2:

$$v = 0.5 \times 0.25981 + 0.5 \times 0.26888 = \mathbf{0.2645\ m^3/kg}$$

$$u = 0.5 \times 1425.7 + 0.5 \times 1444.3 = \mathbf{1435\ kJ/kg}$$

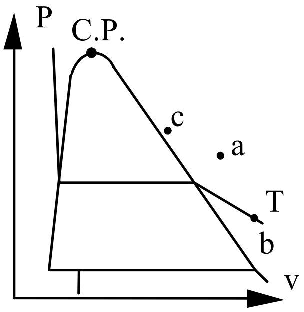
b) Table B.2.1: $P < Psat$ => x = **undefined**, **superheated vapor**, from B.2.2:

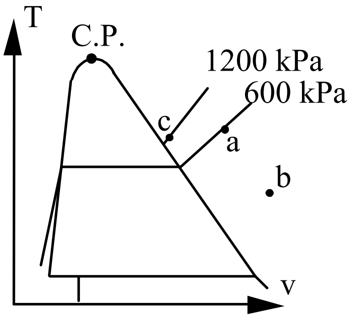
v = **1.4153 m^3/kg** ; u = **1374.5 kJ/kg**

c) Sup. vap. ($v > v_g$) Table B.2.2. P = **1200 kPa**, x = **undefined**

u = **1383 kJ/kg**

States shown are placed relative to the two-phase region, not to each other.





5.29

Find the missing properties of (u, h, and x)

a. H_2O $T = 120°C$, $v = 0.5\ m^3/kg$

b. H_2O $T = 100°C$, $P = 10$ MPa

c. N_2 $T = 100$ K, $x = 0.75$

d. N_2 $T = 200$ K, $P = 200$ kPa

e. NH_3 $T = 100°C$, $v = 0.1\ m^3/kg$

Solution:

a) Table B.1.1: $v_f < v < v_g$ => L+V mixture, P = **198.5 kPa**,

x = (0.5 - 0.00106)/0.8908 = **0.56**,

u = 503.48 + 0.56 × 2025.76 = **1637.9 kJ/kg**

b) Table B.1.4: compressed liquid, v = 0**.001039 m^3/kg**, u = **416.1 kJ/kg**

c) Table B.6.1: 100 K, x = 0.75

v = 0.001452 + 0.75 × 0.02975 = **0.023765 m^3/kg**

u = -74.33 + 0.75 ×137.5 = **28.8 kJ/kg**

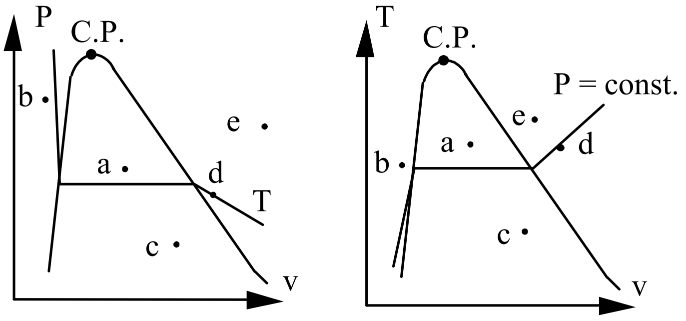
d) Table B.6.2: 200 K, 200 kPa

v = **0.29551 m^3/kg** ; u = **147.37 kJ/kg**

e) Table B.2.1: $v > v_g$ => superheated vapor, x = **undefined**

$$\text{B.2.2:}\quad P = 1600 + 400 \times \frac{0.1 - 0.10539}{0.08248-0.10539} = \mathbf{1694\ kPa}$$

States shown are placed relative to the two-phase region, not to each other.



5.30

Find the missing properties among (T, P, v, u, h and x if applicable) and indicate the states in a P-v and a T-v diagram for

a.	R-410a	P = 500 kPa, h = 300 kJ/kg
b.	R-410a	$T = 10^{o}C$, u = 200 kJ/kg
c.	R-134a	$T = 40^{o}C$, h = 400 kJ/kg

Solution:

a) Table B.4.1: $h > h_g$ => **superheated vapor**, look in section 500 kPa and interpolate

$$T = 0 + 20 \times \frac{300 - 287.84}{306.18 - 287.84} = 20 \times 0.66303 = \mathbf{13.26°C},$$

$$v = 0.05651 + 0.66303 \times (0.06231\text{-}0.05651) = \mathbf{0.06036\ m^3/kg},$$

$$u = 259.59 + 0.66303 \times (275.02 - 259.59) = \mathbf{269.82\ kJ/kg}$$

b) Table B.4.1: $u < u_g = 255.9$ kJ/kg => L+V mixture, P = **1085.7 kPa**

$$x = \frac{u - u_f}{u_{fg}} = \frac{200 - 72.24}{183.66} = \mathbf{0.6956},$$

$$v = 0.000886 + 0.6956 \times 0.02295 = \mathbf{0.01685\ m^3/kg},$$

$$h = 73.21 + 0.6956 \times 208.57 = \mathbf{218.3\ kJ/kg}$$

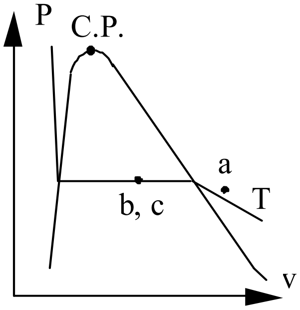
c) Table B.5.1: $h < h_g$ => **two-phase L + V,** look in B.5.1 at 40°C:

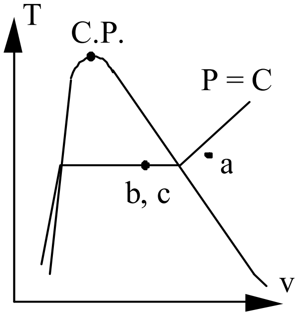
$$x = \frac{h - h_f}{h_{fg}} = \frac{400 - 256.5}{163.3} = 0.87875, \qquad P = P_{sat} = \mathbf{1017\ kPa},$$

$$v = 0.000\ 873 + 0.87875 \times 0.01915 = \mathbf{0.0177\ m^3/kg}$$

$$u = 255.7 + 0.87875 \times 143.8 = \mathbf{382.1\ kJ/kg}$$

States shown are placed relative to the two-phase region, not to each other.





5.31

Find the missing properties.

a. H_2O $T = 250°C$, $v = 0.02\ m^3/kg$ $P = ?$ $u = ?$

b. N_2 $T = 120\ K$, $P = 0.8\ MPa$ $x = ?$ $h = ?$

c. H_2O $T = -2°C$, $P = 100\ kPa$ $u = ?$ $v = ?$

d. R-134a $P = 200\ kPa$, $v = 0.12\ m^3/kg$ $u = ?$ $T = ?$

Solution:

a) Table B.1.1 at 250°C: $v_f < v < v_g$ $\Rightarrow$ P = Psat = **3973 kPa**

$x = (v - v_f)/ v_{fg} = (0.02 – 0.001251)/0.04887 = 0.38365$

$u = u_f + x\ u_{fg} = 1080.37 + 0.38365 \times 1522.0 =$ **1664.28 kJ/kg**

b) Table B.6.1 P is lower than P_{sat} so it is super heated vapor

=> x = **undefined** and we find the state in Table B.6.2

Table B.6.2: h = **114.02 kJ/kg**

c) Table B.1.1 : $T < T_{triple\ point}$ => B.1.5: $P > P_{sat}$ so compressed solid

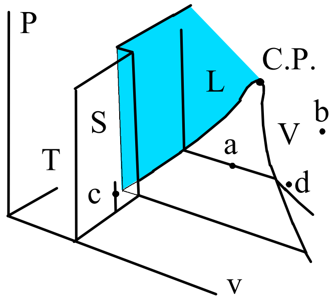
$u \cong u_i =$ **-337.62 kJ/kg** $v \cong v_i =$ $\mathbf{1.09 \times 10^{-3}\ m^3/kg}$

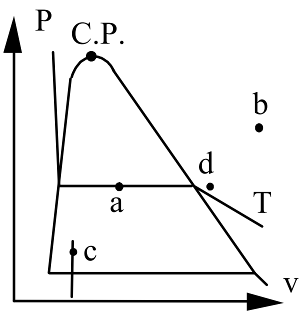
approximate compressed solid with saturated solid properties at same T.

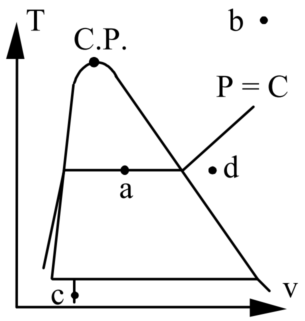
d) Table B.5.1 $v > v_g$ superheated vapor => Table B.5.2.

T ~ **32.5°C** = 30 + (40 – 30) × (0.12 – 0.11889)/(0.12335 - 0.11889)

u = 403.1 + (411.04 – 403.1) × 0.24888 = **405.07 kJ/kg**







5.32

Find the missing properties of (P, T, v, u, h and x) and indicate the states in a P-v and T-v diagram for

a. Water at 5000 kPa, u = 1000 kJ/kg (Table B.1 reference)

b. R-134a at 20^oC, u = 300 kJ/kg

c. Nitrogen at 250 K, 200 kPa

Solution:

a) Compressed liquid: B.1.4 interpolate between 220^oC and 240^oC.

$T = \mathbf{233.3^oC}$, $v = \mathbf{0.001213\ m^3/kg}$, x = **undefined**

b) Table B.5.1: $u < u_g$ => two-phase liquid and vapor

$x = (u - u_f)/u_{fg} = (300 - 227.03)/162.16 = 0.449988 = \mathbf{0.45}$

$v = 0.000817 + 0.45*0.03524 = \mathbf{0.01667\ m^3/kg}$

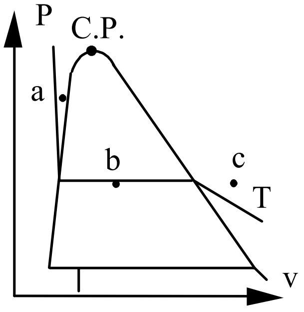
c) Table B.6.1: $T > T_{sat}$ (200 kPa) so superheated vapor in Table B.6.2

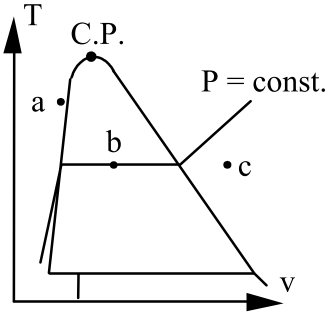
x = **undefined**

$v = 0.5(0.35546 + 0.38535) = \mathbf{0.3704\ m^3/kg}$,

$u = 0.5(177.23 + 192.14) = \mathbf{184.7\ kJ/kg}$

States shown are placed relative to the two-phase region, not to each other.





5.33

Find the missing properties for CO_2 at:

a) $20^{o}C$, 2 MPa $v = ?$ and $h = ?$

b) $10^{o}C$, $x = 0.5$ $P = ?$, $u = ?$

c) 1 MPa, $v = 0.05\ m^3/kg$, $T = ?$, $h = ?$

Solution:

a) Table B.3.1 $P < P_{sat} = 5729$ kPa so superheated vapor.

Table B.3.2: $v = 0.0245\ m^3/kg$, $h = 368.42$ kJ/kg

b) Table B.3.1 $P = P_{sat} = 4502$ kPa

$u = u_f + x\ u_{fg} = 107.6 + 0.5 \times 169.07 = 192.14$ kJ/kg

c) Table B.3.1 $v > v_g \approx 0.0383\ m^3/kg$ so superheated vapor

Table B.3.2: Between 0 and $20^{o}C$ so interpolate.

$$T = 0 + 20 \times \frac{0.05 - 0.048}{0.0524 - 0.048} = 20 \times 0.4545 = 9.09^{o}C$$

$$h = 361.14 + (379.63 - 361.14) \times 0.4545 = 369.54 \text{ kJ/kg}$$

5.34

Saturated liquid water at 20^{o}C is compressed to a higher pressure with constant temperature. Find the changes in u and h from the initial state when the final pressure is a) 500 kPa, b) 2000 kPa

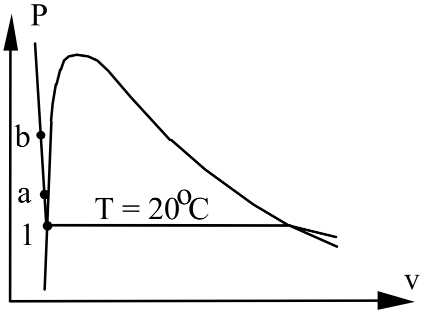
Solution:

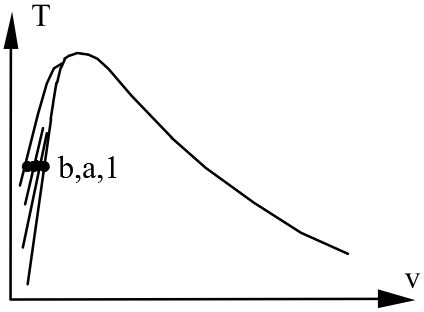
State 1 is located in Table B.1.1 and the states a-c are from Table B.1.4

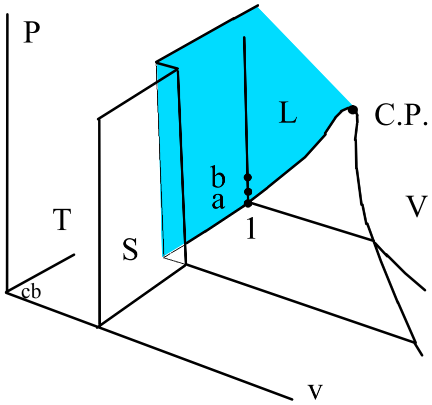
State	u [kJ/kg]	h [kJ/kg]	$\Delta u = u - u_1$	$\Delta h = h - h_1$	$\Delta(Pv)$
1	83.94	83.94			
a	83.91	84.41	-0.03	0.47	0.5
b	83.82	85.82	-0.12	1.88	2

For these states u stays nearly constant, dropping slightly as P goes up.

h varies with Pv changes.







Energy Equation: Simple Process

5.35

Saturated vapor R-410a at 0^{o}C in a rigid tank is cooled to -20^{o}C. Find the specific heat transfer.

Solution:

C.V.: R-410a in tank. $m_2 = m_1$;

Energy Eq.5.11: $(u_2 - u_1) = {}_1q_2 - {}_1w_2$

Process: V = constant, $v_2 = v_1 = V/m$ => $\mathbf{{}_1w_2 = 0}$

Table B.4.1: State 1: $u_1 = 253.0$ kJ/kg

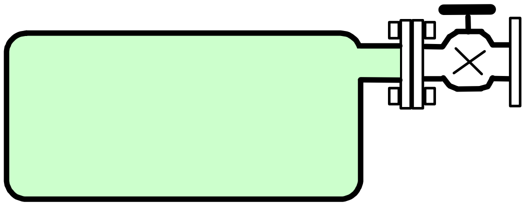
State 2: -20^{o}C, $v_2 = v_1 = V/m$, look in Table B.4.1 at -20^{o}C

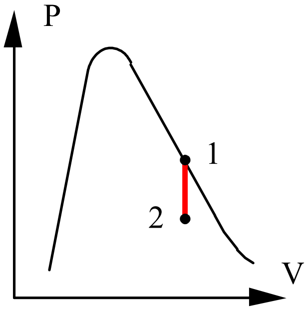
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.03267 - 0.000803}{0.06400} = 0.4979$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 27.92 + x_2 \times 218.07 = 136.5 \text{ kJ/kg}$$

From the energy equation

$${}_1q_2 = (u_2 - u_1) = (\ 136.5 - 253.0\ ) = \mathbf{-116.5\ kJ/kg}$$





5.36

A 100-L rigid tank contains nitrogen (N_2) at 900 K, 3 MPa. The tank is now cooled to 100 K. What are the work and heat transfer for this process?

Solution:

C.V.: Nitrogen in tank. $m_2 = m_1$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $v_2 = v_1 = V/m$ => $\mathbf{{}_1W_2 = 0}$

Table B.6.2: State 1: $v_1 = 0.0900\ m^3/kg$ => $m = V/v_1 = 1.111$ kg

$u_1 = 691.7$ kJ/kg

State 2: 100 K, $v_2 = v_1 = V/m$, look in Table B.6.2 at 100 K

200 kPa: $v = 0.1425\ m^3/kg$; $u = 71.7$ kJ/kg

400 kPa: $v = 0.0681\ m^3/kg$; $u = 69.3$ kJ/kg

so a linear interpolation gives:

$$P_2 = 200 + 200\ (0.09 - 0.1425)/(0.0681 - 0.1425) = 341 \text{ kPa}$$

$$u_2 = 71.7 + (69.3 - 71.7)\frac{0.09 - 0.1425}{0.0681 - 0.1425} = 70.0 \text{ kJ/kg},$$

$${}_1Q_2 = m(u_2 - u_1) = 1.111 \text{ kg } (70.0 - 691.7) \text{ kJ/kg} = \mathbf{-690.7\ kJ}$$

5.37

Saturated vapor carbon dioxide at 2 MPa in a constant pressure piston cylinder is heated to 20°C. Find the specific heat transfer.

Solution:

C.V. CO2: $m_2 = m_1 = m$;

Energy Eq.5.11 $(u_2 - u_1) = {}_1q_2 - {}_1w_2$

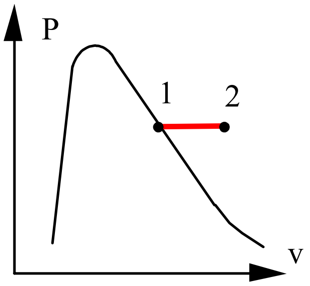
Process: P = const. $\Rightarrow {}_1w_2 = \int P dv = P\Delta v = P(v_2 - v_1)$

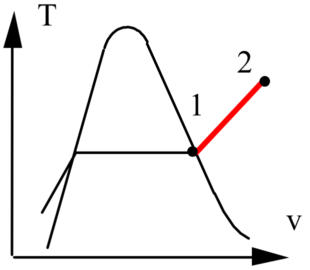
State 1: Table B3.2 (or B3.1) $h_1 = 323.95$ kJ/kg

State 2: Table B.3.2 $h_2 = 368.42$ kJ/kg

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 = (u_2 - u_1) + P(v_2 - v_1) = (h_2 - h_1)$$

$${}_1q_2 = 368.42 - 323.95 = \mathbf{44.47\ kJ/kg}$$



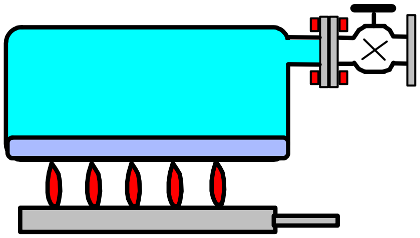


5.38

Two kg water at 120^{o}C with a quality of 25% has its temperature raised 20^{o}C in a constant volume process as in Fig. P5.38. What are the heat transfer and work in the process?

Solution:

C.V. Water. This is a control mass



Energy Eq.: $m\,(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $V = \text{constant}$

$\rightarrow\ {}_1W_2 = \int P\,dV = \mathbf{0}$

State 1: T, x_1 from Table B.1.1

$$v_1 = v_f + x_1\, v_{fg} = 0.00106 + 0.25 \times 0.8908 = 0.22376\ \text{m}^3\text{/kg}$$

$$u_1 = u_f + x_1\, u_{fg} = 503.48 + 0.25 \times 2025.76 = 1009.92\ \text{kJ/kg}$$

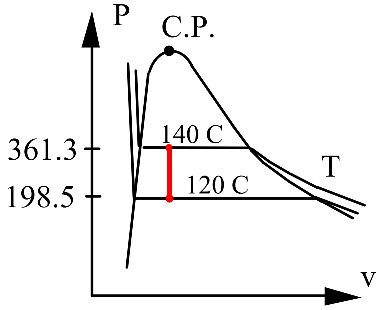
State 2: $T_2,\ v_2 = v_1 < v_{g2} = 0.50885\ \text{m}^3\text{/kg}$ so two-phase

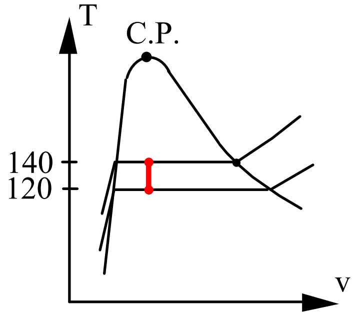
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.22376 - 0.00108}{0.50777} = 0.43855$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 588.72 + x_2 \times 1961.3 = 1448.84\ \text{kJ/kg}$$

From the energy equation

$$_1Q_2 = m(u_2 - u_1) = 2\ (\ 1448.84 - 1009.92\ ) = \mathbf{877.8\ kJ}$$



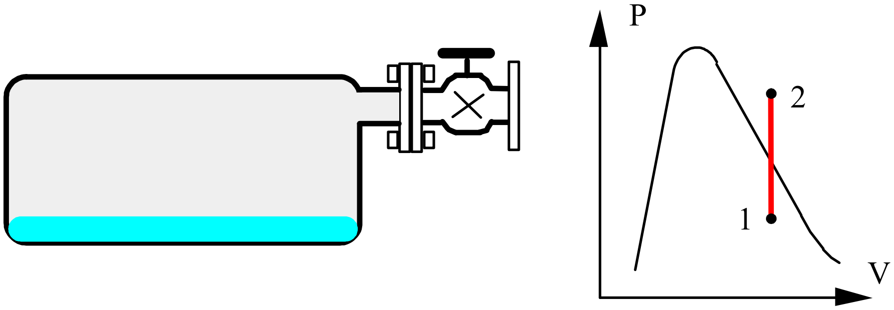


5.39

Ammonia at 0°C, quality 60% is contained in a rigid 200-L tank. The tank and ammonia is now heated to a final pressure of 1 MPa. Determine the heat transfer for the process.

Solution:

C.V.: NH_3



Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant volume ⇒ $v_2 = v_1$ & ${}_1W_2 = 0$

State 1: Table B.2.1 two-phase state.

$v_1 = 0.001566 + x_1 \times 0.28783 = 0.17426 \text{ m}^3/\text{kg}$

$u_1 = 179.69 + 0.6 \times 1138.3 = 862.67 \text{ kJ/kg}$

$m = V/v_1 = 0.2/0.17426 = 1.148 \text{ kg}$

State 2: P_2 , $v_2 = v_1$ superheated vapor Table B.2.2

$\Rightarrow T_2 \cong 100°C, \quad u_2 \cong 1490.5 \text{ kJ/kg}$

So solve for heat transfer in the energy equation

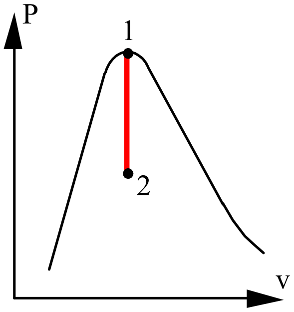
${}_1Q_2 = m(u_2 - u_1) = 1.148(1490.5 - 862.67) =$ **720.75 kJ**

5.40

A test cylinder with constant volume of 0.1 L contains water at the critical point. It now cools down to room temperature of 20°C. Calculate the heat transfer from the water.

Solution:

C.V.: Water



$m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant volume $\Rightarrow v_2 = v_1$

Properties from Table B.1.1

State 1: $v_1 = v_c = 0.003155\ m^3/kg$,

$u_1 = 2029.6$ kJ/kg

$m = V/v_1 = 0.0317$ kg

State 2: T_2, $v_2 = v_1 = 0.001002 + x_2 \times 57.79$

$x_2 = 3.7\times10^{-5}$, $u_2 = 83.95 + x_2 \times 2319 = 84.04$ kJ/kg

Constant volume => ${}_1W_2 = \emptyset$

${}_1Q_2 = m(u_2 - u_1) = 0.0317(84.04 - 2029.6) =$ **-61.7 kJ**

5.41

A rigid tank holds 0.75 kg ammonia at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

C.V. The ammonia, this is a control mass.

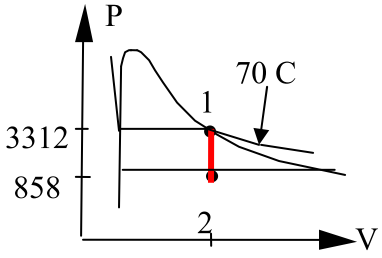
Process: Rigid tank V = C => v = constant & $_1W_2 = \int_1^2 PdV = \mathbf{0}$

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$,

State 1: $v_1 = 0.03787\ m^3/kg$,

$u_1 = 1338.9\ kJ/kg$

State 2: **T, v** => two-phase (straight down in P-v diagram from state 1)



$x_2 = (v - v_f)/v_{fg} = (0.03787 - 0.001638)/0.14758 = 0.2455$

$u_2 = u_f + x_2\ u_{fg} = 272.89 + 0.2455 \times 1059.3 = 532.95\ kJ/kg$

$_1Q_2 = m(u_2 - u_1) = 0.75\ (532.95 - 1338.9) = \mathbf{-604.5\ kJ}$

5.42

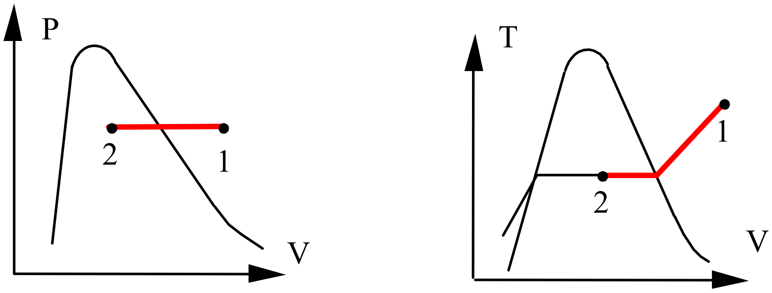
A cylinder fitted with a frictionless piston contains 2 kg of superheated refrigerant R-134a vapor at 350 kPa, 100^oC. The cylinder is now cooled so the R-134a remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

Solution:

C.V.: R-134a $\quad m_2 = m_1 = m$;

Energy Eq.5.11 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$



State 1: Table B.5.2 $\quad h_1 = (490.48 + 489.52)/2 = 490$ kJ/kg

State 2: Table B.5.1 $\quad h_2 = 206.75 + 0.75 \times 194.57 = 352.7$ kJ/kg (350.9 kPa)

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1)$$

$${}_1Q_2 = 2 \times (352.7 - 490) = \mathbf{-274.6\ kJ}$$

5.43

Water in a 150-L closed, rigid tank is at 100°C, 90% quality. The tank is then cooled to −10°C. Calculate the heat transfer during the process.

Solution:

C.V.: Water in tank. $m_2 = m_1$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $v_2 = v_1$, ${}_1W_2 = 0$

State 1: Two-phase L + V look in Table B.1.1

$v_1 = 0.001044 + 0.9 \times 1.6719 = 1.5057\ \text{m}^3/\text{kg}$

$u_1 = 418.94 + 0.9 \times 2087.6 = 2297.8\ \text{kJ/kg}$

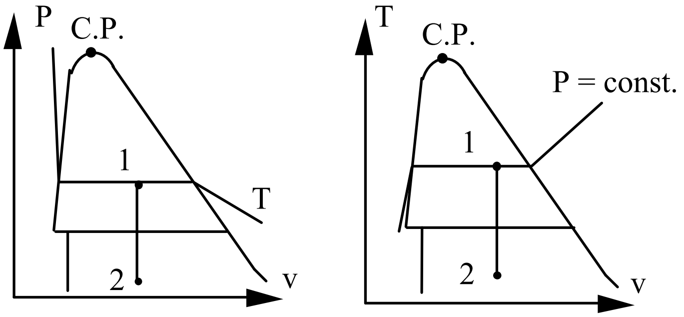
State 2: T_2, $v_2 = v_1$ ⇒ mix of saturated solid + vapor Table B.1.5

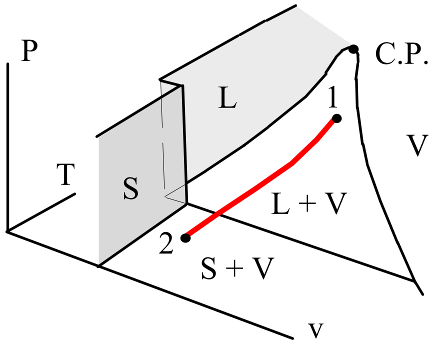
$v_2 = 1.5057 = 0.0010891 + x_2 \times 466.7 \quad \Rightarrow \quad x_2 = 0.003224$

$u_2 = -354.09 + 0.003224 \times 2715.5 = -345.34\ \text{kJ/kg}$

$m = V/v_1 = 0.15/1.5057 = 0.09962\ \text{kg}$

${}_1Q_2 = m(u_2 - u_1) = 0.09962(-345.34 - 2297.8) = \mathbf{-263.3\ kJ}$





5.44

A piston/cylinder contains 50 kg of water at 200 kPa with a volume of 0.1 m^3. Stops in the cylinder are placed to restrict the enclosed volume to a maximum of 0.5 m^3. The water is now heated until the piston reaches the stops. Find the necessary heat transfer.

Solution:

C.V. H_2O m = constant

Energy Eq.5.11: $m(e_2 - e_1) = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : P = constant (forces on piston constant)

$$\Rightarrow {}_1W_2 = \int P\, dV = P_1 (V_2 - V_1)$$

Properties from Table B.1.1

State 1: $v_1 = 0.1/50 = 0.002\ m^3/kg$ => 2-phase as $v_1 < v_g$

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.002 - 0.001061}{0.88467} = 0.001061$$

$$h_1 = 504.68 + 0.001061 \times 2201.96 = 507.02\ kJ/kg$$

State 2: $v_2 = 0.5/50 = 0.01\ m^3/kg$ also 2-phase same P

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.01 - 0.001061}{0.88467} = 0.01010$$

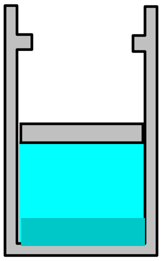
$$h_2 = 504.68 + 0.01010 \times 2201.96 = 526.92\ kJ/kg$$

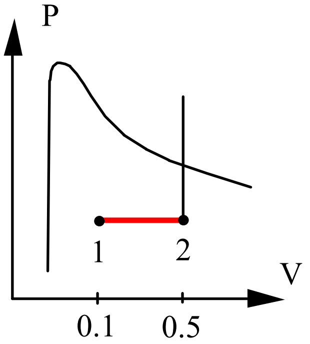
Find the heat transfer from the energy equation as

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

$${}_1Q_2 = 50\ kg \times (526.92 - 507.02)\ kJ/kg = \mathbf{995\ kJ}$$

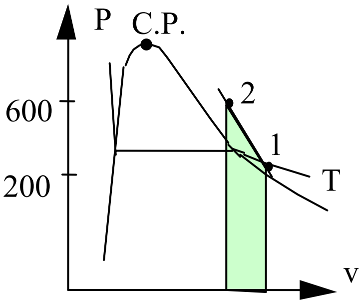
[Notice that ${}_1W_2 = P_1 (V_2 - V_1) = 200 \times (0.5 - 0.1) = 80$ kJ]





5.45

Find the heat transfer for the process in Problem 4.33



Take as CV the 1.5 kg of water. $m_2 = m_1 = m$;

Energy Eq.5.11 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $\quad P = A + BV$ (linearly in V)

State 1: (P, T) => $v_1 = 0.95964\ \text{m}^3/\text{kg}$,

$u_1 = 2576.87\ \text{kJ/kg}$

State 2: (P, T) => $v_2 = 0.47424\ \text{m}^3/\text{kg}$, $u_2 = 2881.12\ \text{kJ/kg}$

From process eq.: $\quad {}_1W_2 = \int P\,dV = \text{area} = \frac{m}{2}(P_1 + P_2)(v_2 - v_1)$

$$= \frac{1.5}{2}\ \text{kg}\ (200 + 600)\ \text{kPa}\ (0.47424 - 0.95964)\ \text{m}^3/\text{kg}$$

$$= -291.24\ \text{kJ}$$

From energy eq.: $\quad {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.5(2881.12 - 2576.87) - 291.24$

$= \mathbf{165.14\ kJ}$

5.46

A 10-L rigid tank contains R-410a at −10°C, 80% quality. A 10-A electric current (from a 6-V battery) is passed through a resistor inside the tank for 10 min, after which the R-410a temperature is 40°C. What was the heat transfer to or from the tank during this process?

Solution:

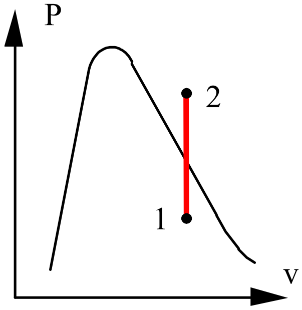
C.V. R-410a in tank. Control mass at constant V.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant V $\Rightarrow$ $v_2 = v_1$

=> no boundary work, but electrical work



State 1 from table B.4.1

$v_1 = 0.000827 + 0.8 \times 0.04470 = 0.03659 \text{ m}^3\text{/kg}$

$u_1 = 42.32 + 0.8 \times 207.36 = 208.21 \text{ kJ/kg}$

$m = V/v = 0.010/0.03659 = 0.2733 \text{ kg}$

State 2: Table B.4.2 at 40°C and $v_2 = v_1 = 0.03659 \text{ m}^3\text{/kg}$

=> superheated vapor, so use linear interpolation to get

$P_2 = 800 + 200 \times (0.03659 - 0.04074)/(0.03170 - 0.04074)$

$= 800 + 200 \times 0.45907 = 892 \text{ kPa}$

$u_2 = 286.83 + 0.45907\times (284.35 - 286.83) = 285.69 \text{ kJ/kg}$

$${}_1W_{2\ \text{elec}} = -\text{power} \times \Delta t = -\text{Amp} \times \text{volts} \times \Delta t = -\frac{10 \times 6 \times 10 \times 60}{1000} = -36 \text{ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.2733\ (285.69 - 208.21) - 36 = \mathbf{-14.8\ kJ}$$

5.47

A piston/cylinder contains 1 kg water at 20°C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature and the amount of heat transfer in the process.

Solution:

C.V. Water. This is a control mass

Energy Eq.: $m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : V = constant → ${}_1W_2 = 0$

State 1: $T, v_1 = V_1/m = 0.1 \text{ m}^3/\text{kg} > v_f$ so two-phase

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.1-0.001002}{57.7887} = 0.0017131$$

$$u_1 = u_f + x_1 \, u_{fg} = 83.94 + x_1 \times 2318.98 = 87.913 \text{ kJ/kg}$$

State 2: $v_2 = v_1 = 0.1$ & $x_2 = 1$

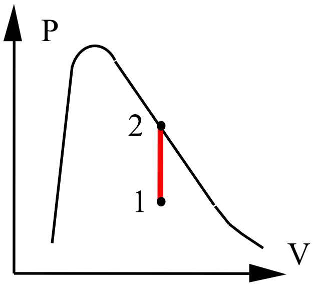
→ found in Table B.1.1 between 210°C and 215° C

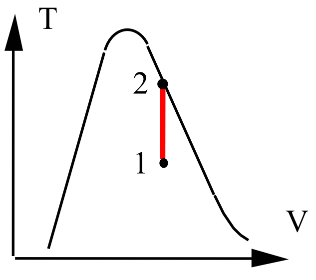
$$T_2 = 210 + 5 \times \frac{0.1-0.10441}{0.09479-0.10441} = 210 + 5 \times 0.4584 = 212.3°\text{C}$$

$$u_2 = 2599.44 + 0.4584 \, (2601.06 - 2599.44) = 2600.2 \text{ kJ/kg}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) = 1(2600.2 - 87.913) = \mathbf{2512.3 \ kJ}$$





5.48

A piston cylinder contains 1.5 kg water at 600 kPa, 350°C. It is now cooled in a process where pressure is linearly related to volume to a state of 200 kPa, 150°C. Plot the P-v diagram for the process and find both the work and the heat transfer in the process.

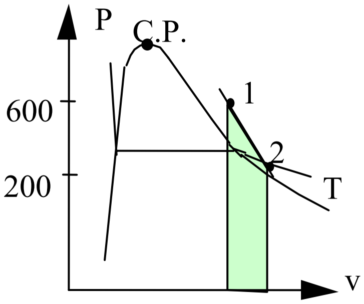
Take as CV the 1.5 kg of water.

$m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = A + BV$ (linearly in V)

State 1: (P, T) => $v_1 = 0.47424\ \text{m}^3\text{/kg}$, $u_1 = 2881.12\ \text{kJ/kg}$



State 2: (P, T) => $v_2 = 0.95964\ \text{m}^3\text{/kg}$, $u_2 = 2576.87\ \text{kJ/kg}$

From process eq.: $${}_1W_2 = \int P\,dV = \text{area} = \frac{m}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1.5}{2}\ \text{kg}\ (200 + 600)\ \text{kPa}\ (0.95964 - 0.47424)\ \text{m}^3\text{/kg}$$

$$= \mathbf{291.24\ kJ}$$

From energy eq.: $${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.5(2576.87 - 2881.12) + 291.24$$

$$= \mathbf{-165.14\ kJ}$$

5.49

Two kg water at 200 kPa with a quality of 25% has its temperature raised 20^{o}C in a constant pressure process. What are the heat transfer and work in the process?

C.V. Water. This is a control mass

Energy Eq.: $m\ (u_2 - u_1\ ) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} \rightarrow {}_1W_2 = \int P\ dV = mP\ (v_2 - v_1)$

State 1: Two-phase given P,x so use Table B.1.2

$v_1 = 0.001061 + 0.25 \times 0.88467 = 0.22223\ m^3/kg$

$u_1 = 504047 + 0.25 \times 2025.02 = 1010.725\ kJ/kg$

$T\ = T + 20 = 120.23 + 20 = 140.23$

State 2 is superheated vapor

$$v_2 = 0.88573 + \frac{20}{150\text{-}120.23} \times (0.95964 - 0.88573\ ) = 0.9354\ m^3/kg$$

$$u_2 = 2529.49 + \frac{20}{150\text{-}120.23} (2576.87\text{-}\ 2529.49) = 2561.32\ kJ/kg$$

From the process equation we get

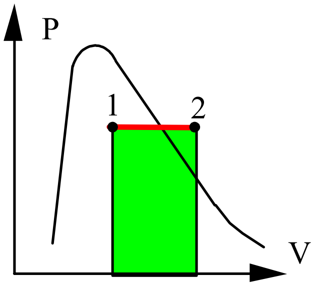
${}_1W_2 = mP\ (v_2 - v_1) = 2 \times 200\ (\ 0.9354\ \text{-}\ 0.22223) =$ **285.3 kJ**

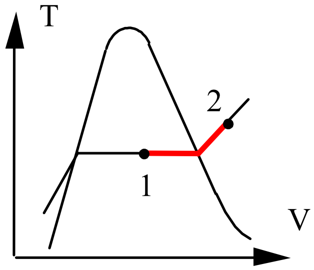
From the energy equation

${}_1Q_2 = m\ (u_2 - u_1) + {}_1W_2$

$= 2\ (\ 2561.32 - 1010.725\ ) + 285.3$

$= 3101.2 + 285.27 =$ **3386.5 kJ**





5.50

A water-filled reactor with volume of 1 m^3 is at 20 MPa, 360°C and placed inside a containment room as shown in Fig. P5.50. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 200 kPa.

Solution:

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

State 1: Table B.1.4 $v_1 = 0.001823$ m^3/kg, $u_1 = 1702.8$ kJ/kg

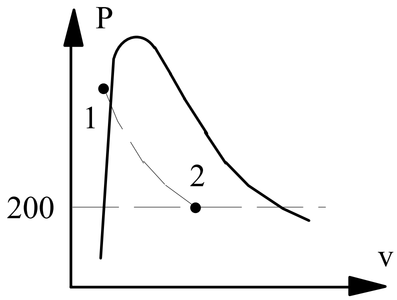
Energy equation then gives $u_2 = u_1 = 1702.8$ kJ/kg

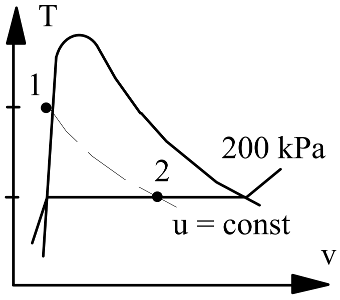
State 2: $P_2 = 200$ kPa, $u_2 < u_g$ => Two-phase Table B.1.2

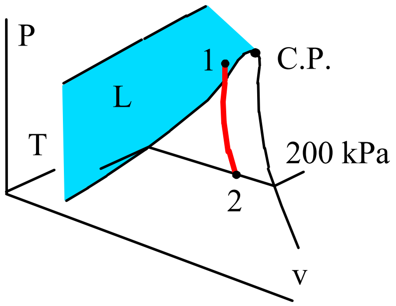
$x_2 = (u_2 - u_f)/ u_{fg} = (1702.8 – 504.47)/2025.02 = 0.59176$

$v_2 = 0.001061 + 0.59176 \times 0.88467 = 0.52457$ m^3/kg

$V_2 = m_2 v_2 = 548.5 \times 0.52457 =$ **287.7 m^3**







5.51

A 25 kg mass moves with 25 m/s. Now a brake system brings the mass to a complete stop with a constant deceleration over a period of 5 seconds. The brake energy is absorbed by 0.5 kg water initially at 20°C, 100 kPa. Assume the mass is at constant P and T. Find the energy the brake removes from the mass and the temperature increase of the water, assuming P = C.

Solution:

C.V. The mass in motion.

$$E_2 - E_1 = \Delta E = 0.5\, m\mathbf{V}^2 = 0.5 \times 25 \times 25^2/1000 = \mathbf{7.8125\ kJ}$$

C.V. The mass of water.

$m(u_2 - u_1)_{H_2O} = \Delta E = 7.8125$ kJ $\quad \Rightarrow \quad u_2 - u_1 = 7.8125 / 0.5 = 15.63$ kJ/kg

$u_2 = u_1 + 15.63 = 83.94 + 15.63 = 99.565$ kJ/kg

Assume $u_2 = u_f$ then from Table B.1.1: $T_2 \cong 23.7°C$, $\Delta T = \mathbf{3.7°C}$

We could have used $u_2 - u_1 = C\Delta T$ with C from Table A.4: C = 4.18 kJ/kg K giving $\Delta T = 15.63/4.18 = \mathbf{3.7°C}$.

5.52

Find the heat transfer for the process in Problem 4.41

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$ (linear in V)

State 1: Superheated vapor $v_1 = 0.6193$ m^3/kg, $u_1 = 1316.7$ kJ/kg

State 2: Superheated vapor $v_2 = 0.63276$ m^3/kg, $u_2 = 1542.0$ kJ/kg

Work is done while piston moves at increasing pressure, so we get

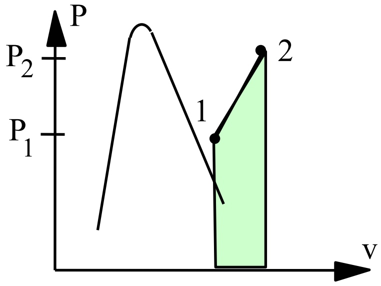
$$ {}_1W_2 = \int P\,dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1) $$

$$ = ½(200 + 300)\text{ kPa} \times 0.5\text{ kg }(0.63276 - 0.6193)\text{ m}^3\text{/kg} = 1.683\text{ kJ} $$

Heat transfer is found from the energy equation

$$ {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5\text{ kg }(1542.0 - 1316.7)\text{ kJ/kg} + 1.683\text{ kJ} $$

$$ = 112.65 + 1.683 = \mathbf{114.3\ kJ} $$



5.53

A piston/cylinder arrangement has the piston loaded with outside atmospheric pressure and the piston mass to a pressure of 150 kPa, shown in Fig. P5.53. It contains water at −2°C, which is then heated until the water becomes saturated vapor. Find the final temperature and specific work and heat transfer for the process.

Solution:

C.V. Water in the piston cylinder.

Continuity: $m_2 = m_1$,

Energy Eq. per unit mass: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $P = \text{constant} = P_1$, $\Rightarrow$ ${}_1w_2 = \int_1^2 P\,dv = P_1(v_2 - v_1)$

State 1: T_1 , P_1 => Table B.1.5 compressed solid, take as saturated solid.

$$v_1 = 1.09\times10^{-3}\ \text{m}^3\text{/kg}, \quad u_1 = -337.62\ \text{kJ/kg}$$

State 2: $x = 1$, $P_2 = P_1 = 150$ kPa due to process => Table B.1.2

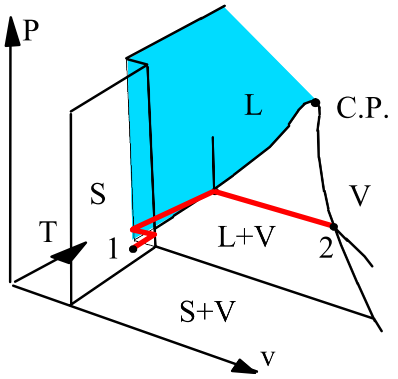
$$v_2 = v_g(P_2) = 1.1593\ \text{m}^3\text{/kg}, \quad T_2 = \mathbf{111.4°C}\ ; \quad u_2 = 2519.7\ \text{kJ/kg}$$

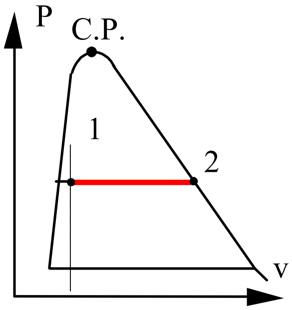
From the process equation

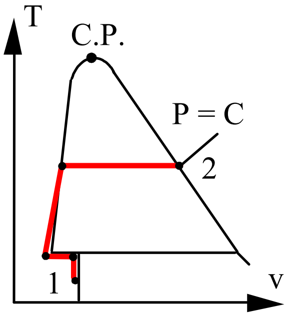
$${}_1w_2 = P_1(v_2 - v_1) = 150(1.1593 - 1.09\times10^{-3}) = \mathbf{173.7\ kJ/kg}$$

From the energy equation

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 2519.7 - (-337.62) + 173.7 = \mathbf{3031\ kJ/kg}$$







5.54

A constant pressure piston/cylinder assembly contains 0.2 kg water as saturated vapor at 400 kPa. It is now cooled so the water occupies half the original volume. Find the heat transfer in the process.

Solution:

C.V. Water. This is a control mass.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant => ${}_1W_2 = Pm(v_2 - v_1)$

So solve for the heat transfer:

$$ {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1) $$

State 1: Table B.1.2 $v_1 = 0.46246\ m^3/kg$; $h_1 = 2738.53\ kJ/kg$

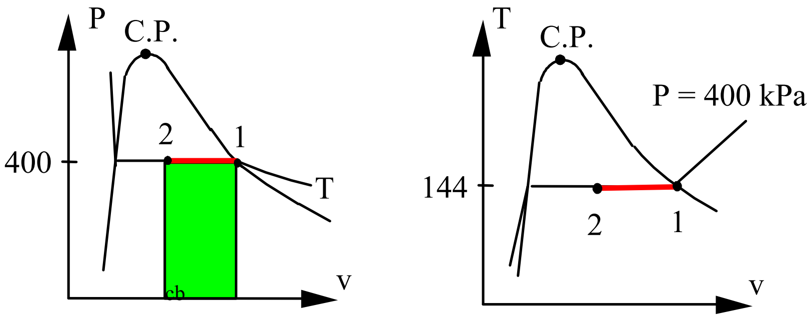
State 2: $v_2 = v_1 / 2 = 0.23123 = v_f + x\ v_{fg}$ from Table B.1.2

$$ x_2 = (v_2 - v_f) / v_{fg} = (0.23123 - 0.001084) / 0.46138 = 0.4988 $$

$$ h_2 = h_f + x_2\ h_{fg} = 604.73 + 0.4988 \times 2133.81 = 1669.07\ kJ/kg $$

Now the heat transfer becomes

$$ {}_1Q_2 = 0.2\ (1669.07 - 2738.53) = \mathbf{-213.9\ kJ} $$



5.55

A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapor water at 120°C, as shown in Fig. P5.55. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m^2, and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.

Solution:

C.V. Water in cylinder.

Continuity: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: (T, x) Table B.1.1 => $v_1 = 0.89186 \text{ m}^3/\text{kg}$, $u_1 = 2529.2 \text{ kJ/kg}$

Process: $$P_2 = P_1 + \frac{k_s m}{A_p{}^2}(v_2 - v_1) = 198.5 + \frac{15 \times 0.5}{(0.05)^2}(v_2 - 0.89186)$$

State 2: $P_2 = 500$ kPa and on the process curve (see above equation).

$$=> \quad v_2 = 0.89186 + (500 - 198.5) \times (0.05^2/7.5) = 0.9924 \text{ m}^3/\text{kg}$$

(P, v) Table B.1.3 => $T_2 =$ **803°C**; $u_2 = 3668$ kJ/kg

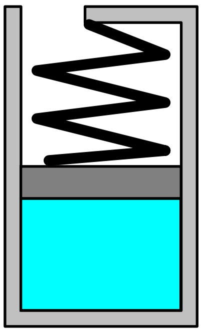
The process equation allows us to evaluate the work

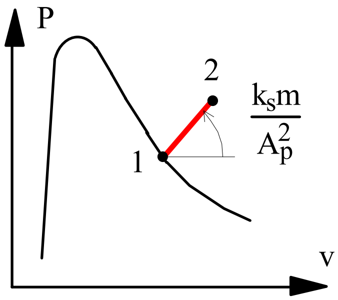
$$ {}_1W_2 = \int PdV = \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1)$$

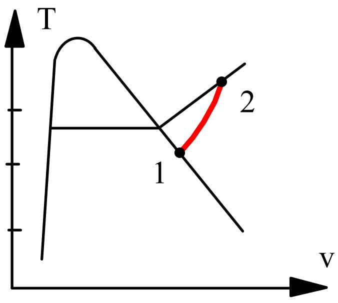
$$= \left(\frac{198.5 + 500}{2}\right) \times 0.5 \times (0.9924 - 0.89186) = 17.56 \text{ kJ}$$

Substitute the work into the energy equation and solve for the heat transfer

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5 \times (3668 - 2529.2) + 17.56 = \mathbf{587\ kJ}$$







5.56

A piston cylinder arrangement with a linear spring similar to Fig. P5.55 contains R-134a at 15^{o}C, x = 0.6 and a volume of 0.02 m^3. It is heated to 60^{o}C at which point the specific volume is 0.03002 m^3/kg. Find the final pressure, the work and the heat transfer in the process.

Take CV as the R-134a.

$$m_2 = m_1 = m \quad ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: T_1, x_1 => Two phase so Table B.5.1: $P_1 = P_{sat} = 489.5$ kPa

$$v_1 = v_f + x_1 v_{fg} = 0.000805 + 0.6 \times 0.04133 = 0.0256 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 220.1 + 0.6 \times 166.35 = 319.91 \text{ kJ/kg}$$

$$m = V_1/v_1 = 0.02 \text{ m}^3 / 0.0256 \text{ m}^3/\text{kg} = 0.78125 \text{ kg}$$

State 2: (T, v) Superheated vapor, Table B.5.2.

$$P_2 = 800 \text{ kPa}, \; v_2 = 0.03002 \text{ m}^3/\text{kg}, \; u_2 = 421.2 \text{ kJ/kg}$$

$$V_2 = m v_2 = 0.78125 \times 0.03002 = 0.02345 \text{ m}^3$$

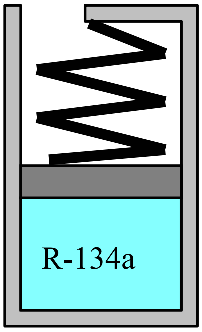
Work is done while piston moves at linearly varying pressure, so we get

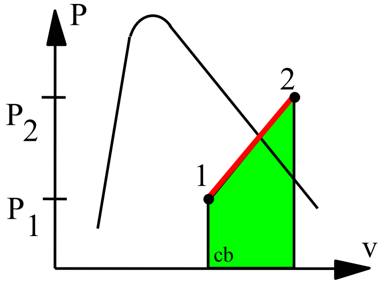
$$_1W_2 = \int P\, dV = \text{area} = P_{avg} (V_2 - V_1) = 0.5(P_2 + P_1)(V_2 - V_1)$$

$$= 0.5 \times (489.5 + 800) \text{ kPa } (0.02345 - 0.02) \text{ m}^3 = \mathbf{2.22\ kJ}$$

Heat transfer is found from the energy equation

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.78125 \times (421.2 - 319.91) + 2.22 = \mathbf{81.36\ kJ}$$





5.57

A closed steel bottle contains CO_2 at −20°C, *x* = 20% and the volume is 0.05 m^3. It has a safety valve that opens at a pressure of 6 MPa. By accident, the bottle is heated until the safety valve opens. Find the temperature and heat transfer when the valve first opens.

Solution:

C.V.: CO_2 : $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

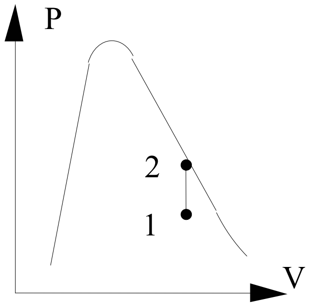
Process: constant volume process $\Rightarrow {}_1W_2 = 0$

State 1: (T, x) Table B.3.1

$v_1 = 0.000969 + 0.2 \times 0.01837 = 0.004643 \text{ m}^3/\text{kg}$

=> $m = V/v_1 = 0.05/0.004643 = 10.769 \text{ kg}$

$u_1 = 39.64 + 0.2 \times 246.25 = 88.89 \text{ kJ/kg}$



State 2: P_2 , $v_2 = v_1$ => very close to saturated vapor, use 6003 kPa in Table B.3.1: **T ≅ 22°C**, $x_2 = (0.004643 - 0.001332)/0.00341 = 0.971$

$u_2 = 142.03 + 0.971 \times 119.89 = 258.44 \text{ kJ/kg}$

${}_1Q_2 = m(u_2 - u_1) = 10.769\ (258.44 - 88.89) = \mathbf{1825.9\ kJ}$

5.58

Superheated refrigerant R-134a at 20°C, 0.5 MPa is cooled in a piston/cylinder arrangement at constant temperature to a final two-phase state with quality of 50%. The refrigerant mass is 5 kg, and during this process 500 kJ of heat is removed. Find the initial and final volumes and the necessary work.

Solution:

C.V. R-134a, this is a control mass.

Continuity: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -500 - {}_1W_2$

State 1: T_1 , P_1 Table B.5.2, $v_1 = 0.04226$ m^3/kg ; $u_1 = 390.52$ kJ/kg

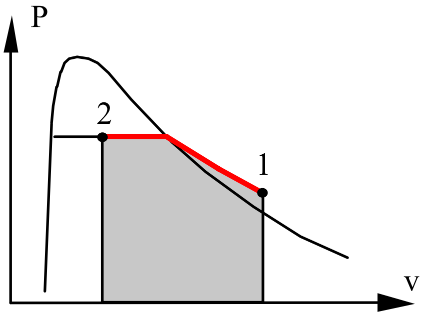
=> $V_1 = mv_1 =$ **0.211 m^3**

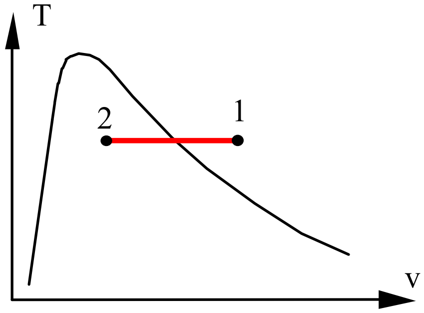
State 2: T_2 , x_2 ⇒ Table B.5.1

$u_2 = 227.03 + 0.5 \times 162.16 = 308.11$ kJ/kg,

$v_2 = 0.000817 + 0.5 \times 0.03524 = 0.018437$ m^3/kg => $V_2 = mv_2 =$ **0.0922 m^3**

${}_1W_2 = -500 - m(u_2 - u_1) = -500 - 5 \times (308.11 - 390.52) =$ **-87.9 kJ**





5.59

A 1-L capsule of water at 700 kPa, 150°C is placed in a larger insulated and otherwise evacuated vessel. The capsule breaks and its contents fill the entire volume. If the final pressure should not exceed 125 kPa, what should the vessel volume be?

Solution:

C.V. Larger vessel.

Continuity: $m_2 = m_1 = m = V/v_1 = 0.916$ kg

Process: expansion with ${}_1Q_2 = 0$, ${}_1W_2 = 0$

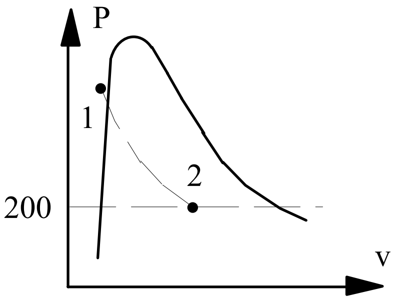
Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 \Rightarrow u_2 = u_1$

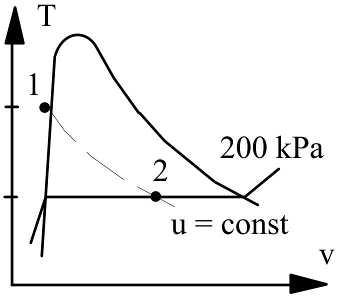
State 1: $v_1 \cong v_f = 0.001091$ m^3/kg; $u_1 \cong u_f = 631.66$ kJ/kg

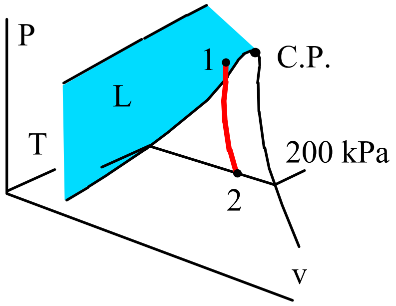
State 2: P_2 , u_2 $\Rightarrow$ $x_2 = \frac{631.66 - 444.16}{2069.3} = 0.09061$

$v_2 = 0.001048 + 0.09061 \times 1.37385 = 0.1255$ m^3/kg

$V_2 = mv_2 = 0.916 \times 0.1255 =$ **0.115 m^3** = **115 L**







5.60

A piston cylinder contains carbon dioxide at -20°C and quality 75%. It is compressed in a process where pressure is linear in volume to a state of 3 MPa and 20°C. Find the specific heat transfer.

CV Carbon dioxide out to the source, both ${}_1Q_2$ and ${}_1W_2$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$ => ${}_1W_2 = \int P\, dV = ½\, m(P_1 + P_2)\,(v_2 - v_1)$

State 1: Table B.3.1 $P = 1969.6$ kPa

$v_1 = 0.000969 + 0.75 \times 0.01837 = 0.01475$ m³/kg,

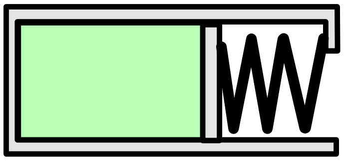
$u_1 = 39.64 + 0.75 \times 246.25 = 224.33$ kJ/kg,

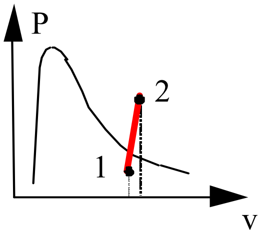
State 2: Table B.3 $v_2 = 0.01512$ m³/kg, $u_2 = 310.21$ kJ/kg,

${}_1w_2 = ½\,(P_1 + P_2)(v_2 - v_1) = ½ \times (1969.6 + 3000)(0.01512 - 0.01475)$

$= \mathbf{0.92\ kJ/kg}$

${}_1q_2 = u_2 - u_1 + {}_1w_2 = 310.21 - 224.33 + 0.92 = \mathbf{86.8\ kJ/kg}$



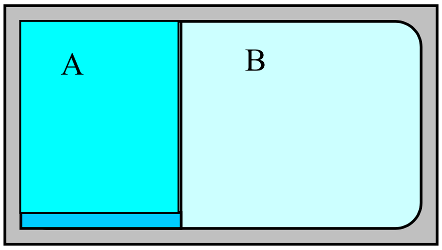


5.61

A rigid tank is divided into two rooms by a membrane, both containing water, shown in Fig. P5.61. Room A is at 200 kPa, $v = 0.5\ \mathrm{m^3/kg}$, $V_A = 1\ \mathrm{m^3}$, and room B contains 3.5 kg at 0.5 MPa, 400°C. The membrane now ruptures and heat transfer takes place so the water comes to a uniform state at 100°C. Find the heat transfer during the process.

Solution:

C.V.: Both rooms A and B in tank.



Continuity Eq.: $m_2 = m_{A1} + m_{B1}$;

Energy Eq.: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

State 1A: (P, v) Table B.1.2, $m_{A1} = V_A/v_{A1} = 1/0.5 = 2$ kg

$$x_{A1} = \frac{v - v_f}{v_{fg}} = \frac{0.5 - 0.001061}{0.88467} = 0.564$$

$$u_{A1} = u_f + x\ u_{fg} = 504.47 + 0.564 \times 2025.02 = 1646.6\ \mathrm{kJ/kg}$$

State 1B: Table B.1.3, $v_{B1} = 0.6173$, $u_{B1} = 2963.2$, $V_B = m_{B1}v_{B1} = 2.16\ \mathrm{m^3}$

Process constant total volume: $V_{tot} = V_A + V_B = 3.16\ \mathrm{m^3}$ and ${}_1W_2 = 0$

$$m_2 = m_{A1} + m_{B1} = 5.5\ \mathrm{kg} \quad \Rightarrow \quad v_2 = V_{tot}/m_2 = 0.5746\ \mathrm{m^3/kg}$$

State 2: T_2 , v_2 $\Rightarrow$ Table B.1.1 two-phase as $v_2 < v_g$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.5746 - 0.001044}{1.67185} = 0.343 ,$$

$$u_2 = u_f + x\ u_{fg} = 418.91 + 0.343 \times 2087.58 = 1134.95\ \mathrm{kJ/kg}$$

Heat transfer is from the energy equation

$${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = \mathbf{-7421\ kJ}$$

5.62

Two kilograms of nitrogen at 100 K, $x = 0.5$ is heated in a constant pressure process to 300 K in a piston/cylinder arrangement. Find the initial and final volumes and the total heat transfer required.

Solution:

Take CV as the nitrogen.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant ⇒ ${}_1W_2 = \int P dV = Pm(v_2 - v_1)$

State 1: Table B.6.1

$v_1 = 0.001452 + 0.5 \times 0.02975 = 0.01633\ m^3/kg$, $V_1 = \mathbf{0.0327\ m^3}$

$h_1 = -73.20 + 0.5 \times 160.68 = 7.14\ kJ/kg$

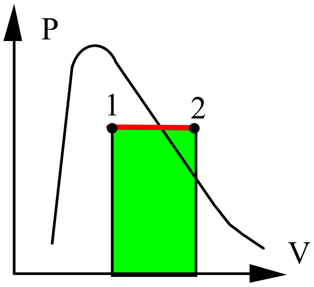
State 2: (P = 779.2 kPa , 300 K) => sup. vapor interpolate in Table B.6.2

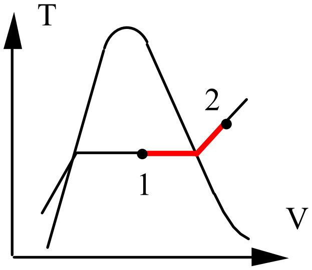
$v_2 = 0.14824 + (0.11115-0.14824) \times 179.2/200 = 0.115\ m^3/kg$, $V_2 = \mathbf{0.23\ m^3}$

$h_2 = 310.06 + (309.62-310.06) \times 179.2/200 = 309.66\ kJ/kg$

Now solve for the heat transfer from the energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2 \times (309.66 - 7.14) = \mathbf{605\ kJ}$





5.63

Water in a tank A is at 250 kPa with a quality of 10% and mass 0.5 kg. It is connected to a piston cylinder holding constant pressure of 200 kPa initially with 0.5 kg water at 400°C. The valve is opened and enough heat transfer takes place to have a final uniform temperature of 150°C. Find the final P and V, the process work and the process heat transfer.

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0 \quad \Rightarrow \quad m_2 = m_{A1} + m_{B1} = 0.5 + 0.5 = 1$ kg

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

State A1: $v_{A1} = 0.001067 + x_{A1} \times 0.71765 = 0.072832$; $V_{A1} = mv = 0.036416\ \mathrm{m^3}$

$u_{A1} = 535.08 + 0.1 \times 2002.14 = 735.22$ kJ/kg

State B1: $v_{B1} = 1.5493\ \mathrm{m^3/kg}$; $u_{B1} = 2966.69$ kJ/kg

$\Rightarrow \quad V_{B1} = m_{B1}v_{B1} = 0.77465\ \mathrm{m^3}$

State 2: If $V_2 > V_{A1}$ then $P_2 =$ **200 kPa** that is the piston floats.

For $(T_2, P_2) = (150°C, 200\ \mathrm{kPa})$ => superheated vapor $u_2 = 2576.87$ kJ/kg

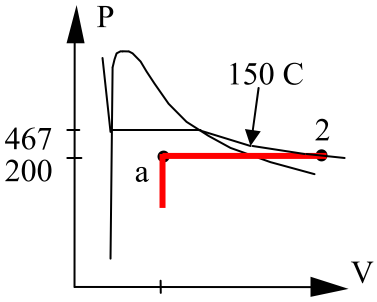
$v_2 = 0.95964\ \mathrm{m^3/kg} \qquad V_2 = m_2v_2 = \mathbf{0.95964\ m^3} > V_{A1}$ checks OK.

The possible state 2 (P,V) combinations are shown. State a is

$$200\ \mathrm{kPa},\ v_a = \frac{V_{A1}}{m_2} = 0.036\ \mathrm{m^3/kg}$$

and thus two-phase

$T_a = 120°C < T_2$



Process: ${}_1W_2 = P_2 (V_2 - V_1) = 200 (0.95964 - 0.77465 - 0.036416) =$ **29.715 kJ**

From the energy Eq.:

$${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$$

$$= 1 \times 2576.87 - 0.5 \times 735.222 - 0.5 \times 2966.69 + 29.715$$

$$= \mathbf{755.63\ kJ}$$

5.64

A 10-m high open cylinder, $A_{cyl} = 0.1$ m^2, contains 20°C water above and 2 kg of 20°C water below a 198.5-kg thin insulated floating piston, shown in Fig. P5.64. Assume standard *g*, *Po*. Now heat is added to the water below the piston so that it expands, pushing the piston up, causing the water on top to spill over the edge. This process continues until the piston reaches the top of the cylinder. Find the final state of the water below the piston (*T*, *P*, *v*) and the heat added during the process.

Solution:

C.V. Water below the piston.

Piston force balance at initial state: $F\uparrow = F\downarrow = P_A A = m_p g + m_B g + P_0 A$

State $1_{A,B}$: Comp. Liq. $\Rightarrow$ $v \cong v_f = 0.001002$ m^3/kg; $u_{1A} = 83.95$ kJ/kg

$V_{A1} = m_A v_{A1} = 0.002$ m^3; $m_{tot} = V_{tot}/v = 1/0.001002 = 998$ kg

mass above the piston $m_{B1} = m_{tot} - m_A =$ **996 kg**

$$P_{A1} = P_0 + (m_p + m_B)g/A = 101.325 + \frac{(198.5 + 996) \times 9.807}{0.1 \times 1000} = \mathbf{218.5\ kPa}$$

State 2_A: $P_{A2} = P_0 + \frac{m_p g}{A} = \mathbf{120.8\ kPa}$; $v_{A2} = V_{tot}/ m_A = \mathbf{0.5\ m^3/kg}$

$x_{A2} = (0.5 - 0.001047)/1.4183 = 0.352$; $T_2 =$ **105°C**

$u_{A2} = 440.0 + 0.352 \times 2072.34 = 1169.5$ kJ/kg

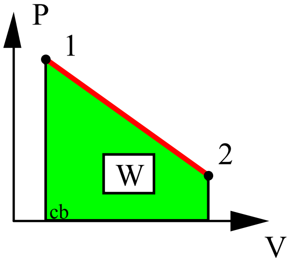
Continuity eq. in A: $m_{A2} = m_{A1}$

Energy: $m_A(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P linear in V as m_B is linear with V

$${}_1W_2 = \int P dV = \frac{1}{2}(218.5 + 120.82)(1 - 0.002)$$

$$= \mathbf{169.32\ kJ}$$



${}_1Q_2 = m_A(u_2 - u_1) + {}_1W_2 = 2170.1 + 169.3 =$ **2340.4 kJ**

5.65

Assume the same setup as in Problem 5.50, but the room has a volume of 100 m^3. Show that the final state is two-phase and find the final pressure by trial and error.

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0 \Rightarrow u_2 = u_1 = 1702.8$ kJ/kg

Total volume and mass => $v_2 = V_{room}/m_2 = 0.1823$ m^3/kg

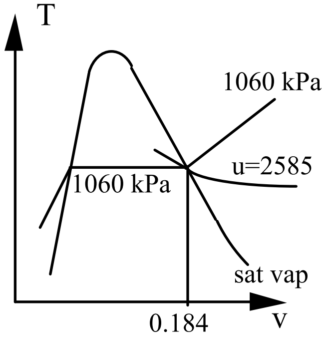
State 2: u_2 , v_2 Table B.1.1 see Figure.

Note that in the vicinity of $v = 0.1823$ m^3/kg crossing the saturated vapor line the internal energy is about 2585 kJ/kg. However, at the actual state 2, u = 1702.8 kJ/kg. Therefore state 2 must be in the two-phase region.

Trial & error $v = v_f + xv_{fg}$; $u = u_f + xu_{fg}$

$$\Rightarrow u_2 = 1702.8 = u_f + \frac{v_2 - v_f}{v_{fg}} u_{fg}$$

Compute RHS for a guessed pressure P_2:



$$P_2 = 600 \text{ kPa: RHS} = 669.88 + \frac{0.1823\text{-}0.001101}{0.31457} \times 1897.52 = 1762.9 \quad \text{too large}$$

$$P_2 = 550 \text{ kPa: RHS} = 655.30 + \frac{0.1823\text{-}0.001097}{0.34159} \times 1909.17 = 1668.1 \quad \text{too small}$$

Linear interpolation to match u = 1702.8 gives $P_2 \cong$ **568.5 kPa**

5.66

A piston cylinder has a water volume separated in $V_A = 0.2\ m^3$ and $V_B = 0.3\ m^3$ by a stiff membrane. The initial state in A is 1000 kPa, x = 0.75 and in B it is 1600 kPa and 250°C. Now the membrane ruptures and the water comes to a uniform state at 200°C. What is the final pressure? Find the work and the heat transfer in the process.

Take the water in A and B as CV.

Continuity: $m_2 - m_{1A} - m_{1B} = 0$

Energy: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = {}_1Q_2 - {}_1W_2$

Process: $P_2 = P_{eq} = \text{constant} = P_{1A}$ as piston floats and m_p, P_o do not change

State 1A: Two phase. Table B.1.2

$$v_{1A} = 0.001127 + 0.75 \times 0.19332 = 0.146117\ m^3/kg,$$

$$u_{1A} = 761.67 + 0.75 \times 1821.97 = 2128.15\ kJ/kg$$

State 1B: Table B.1.3 $v_{1B} = 0.14184\ m^3/kg$, $u_{1B} = 2692.26\ kJ/kg$

$$\Rightarrow \quad m_{1A} = V_{1A}/v_{1A} = 1.3688\ kg, \quad m_{1B} = V_{1B}/v_{1B} = 2.115\ kg$$

State 2: 1000 kPa, 200°C sup. vapor => $v_2 = 0.20596\ m^3/kg$, $u_2 = 2621.9\ kJ/kg$

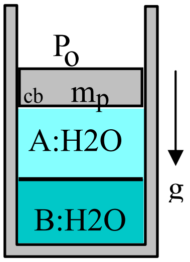
$$m_2 = m_{1A} + m_{1B} = 3.4838\ kg \quad \Rightarrow \quad V_2 = m_2v_2 = 3.4838 \times 0.20596 = 0.7175\ m^3$$

So now

$${}_1W_2 = \int P\,dV = P_{eq}(V_2 - V_1) = 1000\,(0.7175 - 0.5) = \mathbf{217.5\ kJ}$$

$${}_1Q_2 = m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} + {}_1W_2$$

$$= 3.4838 \times 2621.9 - 1.3688 \times 2128.15 - 2.115 \times 2692.26 + 217.5 = \mathbf{744\ kJ}$$



5.67

Two rigid tanks are filled with water. Tank A is 0.2 m^3 at 100 kPa, 150°C and tank B is 0.3 m^3 at saturated vapor 300 kPa. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 300 kPa. Give the two property values that determine the final state and find the heat transfer.

State A1: $u = 2582.75$ kJ/kg, $v = 1.93636\ m^3/kg$

$=> m_{A1} = V/v = 0.2/1.93636 =$ **0.1033 kg**

State B1: $u = 2543.55$ kJ/kg, $v = 0.60582\ m^3/kg$

$=> m_{B1} = V/v = 0.3 / 0.60582 =$ **0.4952 kg**

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$$m_2 = m_{A1} + m_{B1} = 0.1033 + 0.4952 = 0.5985 \text{ kg},$$

$$V_2 = V_{A1} + V_{B1} = 0.2 + 0.3 = 0.5\ m^3$$

$$=> v_2 = V_2/m_2 = 0.5 /0.5985 = \mathbf{0.8354\ m^3/kg}$$

State 2: $[P_2, v_2] = [300 \text{ kPa}, 0.8354\ m^3/kg]$

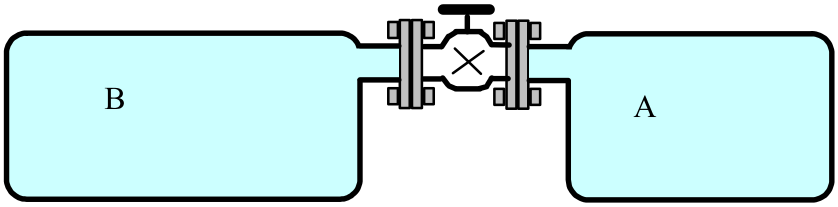
$$=> T_2 = 274.76°C \text{ and } u_2 = 2767.32 \text{ kJ/kg}$$

The energy equation is (neglecting kinetic and potential energy)

$$m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$$

$${}_1Q_2 = 0.5985 \times 2767.32 - 0.1033 \times 2582.75 - 0.4952 \times 2543.55$$

$$= \mathbf{129.9\ kJ}$$



Energy Equation: Multistep Solution

5.68

A piston cylinder shown in Fig. P5.68 contains 0.5 m^3 of R-410a at 2 MPa, 150^oC. The piston mass and atmosphere gives a pressure of 450 kPa that will float the piston. The whole setup cools in a freezer maintained at -20^oC. Find the heat transfer and show the P-v diagram for the process when T_2 = -20^oC.

C.V.: R-410a. Control mass.

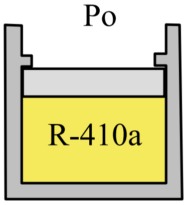
Continuity: m = constant,

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $F\downarrow = F\uparrow = P\,A = P_{air}A + F_{stop}$

$\text{if } V < V_{stop} \Rightarrow F_{stop} = 0$

This is illustrated in the P-v diagram shown below.



State 1: $v_1 = 0.02247\ m^3/kg$, $u_1 = 373.49\ kJ/kg$

$\Rightarrow$ m = V/v = 22.252 kg

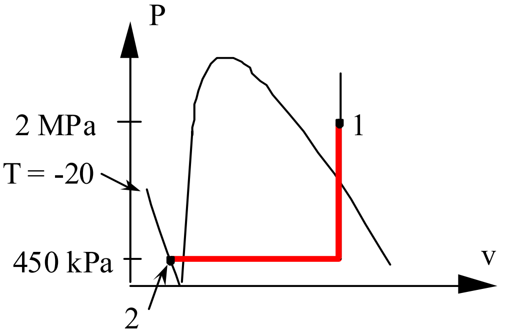
State 2: T_2 and on line $\Rightarrow$ compressed liquid, see figure below.

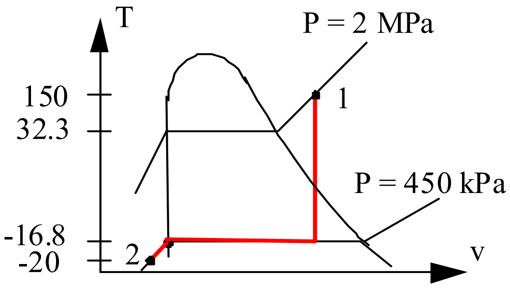
$v_2 \cong v_f = 0.000\,803\ m^3/kg \Rightarrow V_2 = 0.01787\ m^3$; $u_2 = u_f = 27.92\ kJ/kg$

${}_1W_2 = \int PdV = P_{lift}(V_2 - V_1) = 450\ (0.01787 - 0.5) = -217.0\ kJ$;

Energy eq. $\Rightarrow$

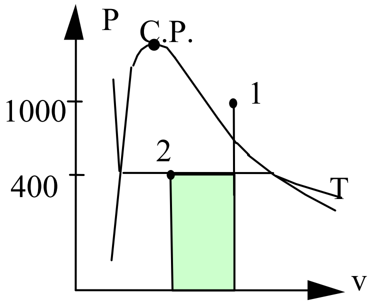
${}_1Q_2 = 22.252\ (27.92 - 373.49) - 217.9 =$ **-7906.6 kJ**





5.69

A setup as in Fig. P5.68 has the R-410a initially at 1000 kPa, 50^{o}C of mass 0.1 kg. The balancing equilibrium pressure is 400 kPa and it is now cooled so the volume is reduced to half the starting volume. Find the work and heat transfer for the process.



Take as CV the 0.1 kg of R-410a.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1: (P, T) => $v_1 = 0.0332 \text{ m}^3/\text{kg}$,

$u_1 = 292.695 \text{ kJ/kg}$

State 2: (P, v) => $v_2 = v_1/2 = 0.0166 \text{ m}^3/\text{kg} < v_g$, so it is two-phase.

$x_2 = (v_2 - v_f) / v_{fg} = (0.0166 - 0.000803)/0.064 = 0.2468$

$u_2 = u_f + x_2 u_{fg} = 27.92 + x_2\ 218.07 = 81.746 \text{ kJ/kg}$

From process eq.: ${}_1W_2 = \int P\, dV = \text{area} = mP_2 (v_2 - v_1)$

$= 0.1 \times 400 (0.0166 - 0.0332) = $ -0.**664 kJ**

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1 \times (81.746 - 292.695) - 0.664$

= **-21.8 kJ**

5.70

A vertical cylinder fitted with a piston contains 5 kg of R-410a at 10°C, shown in Fig. P5.70. Heat is transferred to the system, causing the piston to rise until it reaches a set of stops at which point the volume has doubled. Additional heat is transferred until the temperature inside reaches 50°C, at which point the pressure inside the cylinder is 1.4 MPa.

a. What is the quality at the initial state?

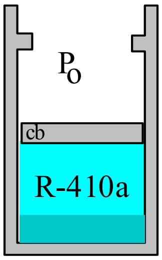
b. Calculate the heat transfer for the overall process.

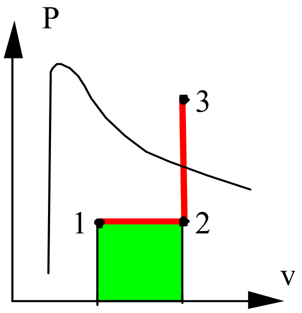
Solution:

C.V. R-410a. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3). So we have: $v_3 = v_2 = 2 \times v_1$

State 3: Table B.4.2 (P,T) $v_3 = 0.02249\ \text{m}^3/\text{kg}$, $u_3 = 287.91$ kJ/kg





So we can then determine state 1 and 2 Table B.4.1:

$v_1 = 0.011245 = 0.000886 + x_1 \times 0.02295 \quad \Rightarrow \quad x_1 = \mathbf{0.4514}$

b) $u_1 = 72.24 + 0.4514 \times 183.66 = 155.14$ kJ/kg

State 2: $v_2 = 0.02249\ \text{m}^3/\text{kg}$, $P_2 = P_1 = 1086$ kPa this is still 2-phase.

We get the work from the process equation (see P-V diagram)

$$_1W_3 = {_1W_2} = \int_1^2 P dV = P_1(V_2 - V_1) = 1086 \times 5\,(0.011245) = 61.1 \text{ kJ}$$

The heat transfer from the energy equation becomes

$$_1Q_3 = m(u_3 - u_1) + {_1W_3} = 5(287.91 - 155.14) + 61.1 = \mathbf{725.0\ kJ}$$

5.71

Find the heat transfer in Problem 4.68.

A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 4.64, with a maximum enclosed volume of 0.002 m^3 if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the work in the process.

Solution:

Take CV as the water. Properties from table B.1

$$m_2 = m_1 = m ; \quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: Compressed liq. $v = v_f(20) = 0.001002 \text{ m}^3\text{/kg}$, $u = u_f = 83.94$ kJ/kg

State 2: Since $P > P_{lift}$ then $v = v_{stop} = 0.002$ and $P = 600$ kPa

For the given P : $v_f < v < v_g$ so 2-phase $T = T_{sat} = 158.85$ °C

$$v = 0.002 = 0.001101 + x \times (0.3157-0.001101) \Rightarrow x = 0.002858$$

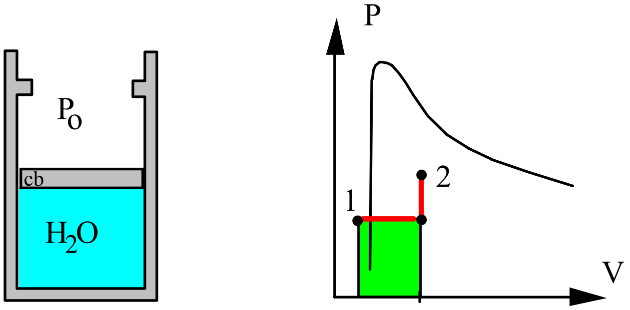
$$u = 669.88 + 0.002858 \times 1897.5 = 675.3 \text{ kJ/kg}$$

Work is done while piston moves at P_{lift}= constant = 300 kPa so we get

$${}_1W_2 = \int P\, dV = m\, P_{lift}\, (v_2 - v_1) = 1 \times 300(0.002 - 0.001002) = 0.299 \text{ kJ}$$

Heat transfer is found from energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1(675.3 - 83.94) + 0.299 = \mathbf{591.66\ kJ}$$



5.72

10 kg of water in a piston cylinder arrangement exists as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it, as in Fig. 5.72. Find the final temperature and the heat transfer in the process.

Solution:

Take CV as the water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: v = constant until $P = P_{lift}$, then P is constant.

State 1: Two-phase so look in Table B.1.2 at 100 kPa

$$u_1 = 417.33 + 0.5 \times 2088.72 = 1461.7 \text{ kJ/kg},$$

$$v_1 = 0.001043 + 0.5 \times 1.69296 = 0.8475 \text{ m}^3\text{/kg}$$

State 2: v_2, $\mathbf{P_2 \le P_{lift}}$ => $v_2 = 3 \times 0.8475 = 2.5425 \text{ m}^3\text{/kg}$;

Interpolate: $\mathbf{T_2 = 829°C}$, $u_2 = 3718.76$ kJ/kg

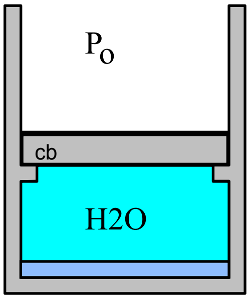
=> $V_2 = mv_2 = 25.425 \text{ m}^3$

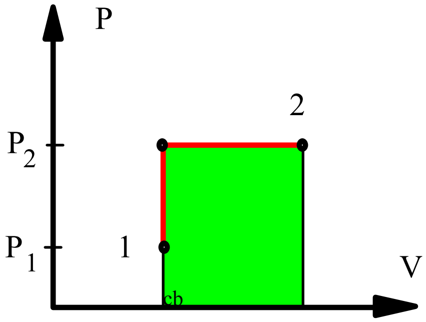
From the process equation (see P-V diagram) we get the work as

$${}_1W_2 = P_{lift}(V_2 - V_1) = 200 \times 10\ (2.5425 - 0.8475) = 3390 \text{ kJ}$$

From the energy equation we solve for the heat transfer

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 10 \times (3718.76 - 1461.7) + 3390 = \mathbf{25\,961\ kJ}$$





5.73

The cylinder volume below the constant loaded piston has two compartments A and B filled with water. A has 0.5 kg at 200 kPa, 150°C and B has 400 kPa with a quality of 50% and a volume of 0.1 m^3. The valve is opened and heat is transferred so the water comes to a uniform state with a total volume of 1.006 m^3.

a) Find the total mass of water and the total initial volume.

b) Find the work in the process

c) Find the process heat transfer.

Solution:

Take the water in A and B as CV.

Continuity: $m_2 - m_{1A} - m_{1B} = 0$

Energy: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = {}_1Q_2 - {}_1W_2$

Process: $P = \text{constant} = P_{1A}$ if piston floats

(V_A positive) i.e. if $V_2 > V_B = 0.1\ m^3$

State A1: Sup. vap. Table B.1.3 $v = 0.95964\ m^3/kg$, $u = 2576.9$ kJ/kg

$\Rightarrow V = mv = 0.5 \times 0.95964 = 0.47982$

State B1: Table B.1.2 $v = (1-x) \times 0.001084 + x \times 0.4625 = 0.2318\ m^3/kg$

$\Rightarrow m = V/v = 0.4314$ kg

$u = 604.29 + 0.5 \times 1949.3 = 1578.9$ kJ/kg

State 2: 200 kPa, $v_2 = V_2/m = 1.006/0.9314 = 1.0801\ m^3/kg$

Table B.1.3 => close to $T_2 = 200°C$ and $u_2 = 2654.4$ kJ/kg

So now

$V_1 = 0.47982 + 0.1 = \mathbf{0.5798\ m^3}$, $m_1 = 0.5 + 0.4314 = \mathbf{0.9314\ kg}$

Since volume at state 2 is larger than initial volume piston goes up and the pressure then is constant (200 kPa which floats piston).

$${}_1W_2 = \int P\,dV = P_{lift}(V_2 - V_1) = 200\,(1.006 - 0.57982) = \mathbf{85.24\ kJ}$$

$${}_1Q_2 = m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} + {}_1W_2$$
$$= 0.9314 \times 2654.4 - 0.5 \times 2576.9 - 0.4314 \times 1578.9 + 85.24 = \mathbf{588\ kJ}$$

5.74

Calculate the heat transfer for the process described in Problem 4.65.

A cylinder containing 1 kg of ammonia has an externally loaded piston. Initially the ammonia is at 2 MPa, 180°C and is now cooled to saturated vapor at 40°C, and then further cooled to 20°C, at which point the quality is 50%. Find the total work for the process, assuming a piecewise linear variation of *P* versus *V*.

Solution:

C.V. Ammonia going through process 1 - 2 - 3. Control mass.

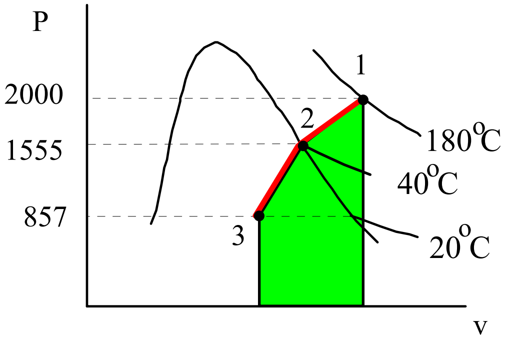
Continuity: m = constant,

Energy Eq.5.11: $m(u_3 - u_1) = {}_1Q_3 - {}_1W_3$

Process: P is piecewise linear in V

State 1: (T, P) Table B.2.2: $v_1 = 0.10571\ m^3/kg$, $u_1 = 1630.7$ kJ/kg

State 2: (T, x) Table B.2.1 sat. vap. $P_2 = 1555$ kPa, $v_2 = 0.08313\ m^3/kg$



State 3: (T, x) $P_3 = 857$ kPa,

$v_3 = (0.001638+0.14922)/2 = 0.07543$ $u_3 = (272.89 + 1332.2)/2 = 802.7$ kJ/kg

Process: piecewise linear P versus V, see diagram. Work is area as:

$$W_{13} = \int_1^3 PdV \approx \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1) + \left(\frac{P_2 + P_3}{2}\right) m(v_3 - v_2)$$

$$= \frac{2000 + 1555}{2} 1(0.08313 - 0.10571) + \frac{1555 + 857}{2} 1(0.07543 - 0.08313)$$

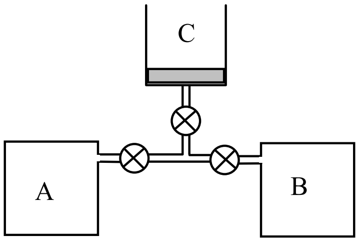
$= \mathbf{-49.4\ kJ}$

From the energy equation, we get the heat transfer as:

${}_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 1\times(802.7 - 1630.7) - 49.4 = \mathbf{-877.4\ kJ}$

5.75

A rigid tank A of volume 0.6 m^3 contains 3 kg water at 120°C and the rigid tank B is 0.4 m^3 with water at 600 kPa, 200°C. They are connected to a piston cylinder initially empty with closed valves. The pressure in the cylinder should be 800 kPa to float the piston. Now the valves are slowly opened and heat is transferred so the water reaches a uniform state at 250°C with the valves open. Find the final volume and pressure and the work and heat transfer in the process.



C.V.: A + B + C.

Only work in C, total mass constant.

$m_2 - m_1 = 0 \quad \Rightarrow \quad m_2 = m_{A1} + m_{B1}$

$U_2 - U_1 = {}_1Q_2 - {}_1W_2$;

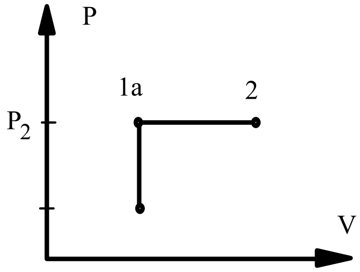
${}_1W_2 = \int PdV = P_{lift}(V_2 - V_1)$

1A: $v = 0.6/3 = 0.2\ m^3/kg \Rightarrow x_{A1} = (0.2 - 0.00106)/0.8908 = 0.223327$

$u = 503.48 + 0.223327 \times 2025.76 = 955.89$ kJ/kg

1B: $v = 0.35202\ m^3/kg \Rightarrow m_{B1} = 0.4/0.35202 = 1.1363$ kg ; $u = 2638.91$ kJ/kg

$m_2 = 3 + 1.1363 = 4.1363$ kg and



$V_2 = V_A + V_B + V_C = 1 + V_C$

Locate state 2: Must be on P-V lines shown

State 1a: 800 kPa,

$$v_{1a} = \frac{V_A + V_B}{m} = 0.24176\ m^3/kg$$

800 kPa, $v_{1a} \Rightarrow T = 173°C$ too low.

Assume 800 kPa: 250°C $\Rightarrow$ $v = 0.29314\ m^3/kg > v_{1a}$ OK

$V_2 = m_2v_2 = 4.1363\ kg \times 0.29314\ m^3/kg = \mathbf{1.21\ m^3}$

Final state is : **800 kPa**; 250°C $\Rightarrow u_2 = 2715.46$ kJ/kg

$W = 800(0.29314 - 0.24176) \times 4.1363 = 800 \times (1.2125 - 1) = \mathbf{170\ kJ}$

$Q = m_2u_2 - m_1u_1 + {}_1W_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$

$= 4.1363 \times 2715.46 - 3 \times 955.89 - 1.1363 \times 2638.91 + 170$

$= 11\ 232 - 2867.7 - 2998.6 + 170 = \mathbf{5536\ kJ}$

5.76

Calculate the heat transfer for the process described in Problem 4.73.

A piston cylinder setup similar to Problem 4.?? contains 0.1 kg saturated liquid and vapor water at 100 kPa with quality 25%. The mass of the piston is such that a pressure of 500 kPa will float it. The water is heated to 300°C. Find the final pressure, volume and the work, ${}_1W_2$.

Solution:

Take CV as the water: $m_2 = m_1 = m$

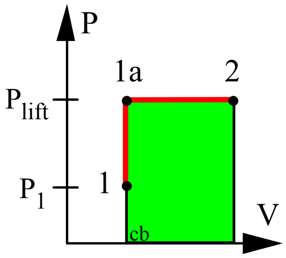
Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: v = constant until $P = P_{lift}$

To locate state 1: Table B.1.2

$v_1 = 0.001043 + 0.25 \times 1.69296 = 0.42428 \text{ m}^3/\text{kg}$

$u_1 = 417.33 + 0.25 \times 2088.7 = 939.5 \text{ kJ/kg}$



State 1a: 500 kPa, $v_{1a} = v_1 = 0.42428 > v_g$ at 500 kPa,

so state 1a is superheated vapor Table B.1.3 $T_{1a} = 200°\text{C}$

State 2 is 300°C so heating continues after state 1a to 2 at constant P = 500 kPa.

2: T_2, $P_2 = P_{lift}$ => Tbl B.1.3 $v_2 = 0.52256 \text{ m}^3/\text{kg}$; $u_2 = 2802.9 \text{ kJ/kg}$

From the process, see also area in P-V diagram

$${}_1W_2 = P_{lift}\ m(v_2 - v_1) = 500 \times 0.1\ (0.5226 - 0.4243) = 4.91 \text{ kJ}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1(2802.9 - 939.5) + 4.91 = \mathbf{191.25\ kJ}$$

5.77

A cylinder/piston arrangement contains 5 kg of water at 100°C with x = 20% and the piston, m_P = 75 kg, resting on some stops, similar to Fig. P5.72. The outside pressure is 100 kPa, and the cylinder area is A = 24.5 cm^2. Heat is now added until the water reaches a saturated vapor state. Find the initial volume, final pressure, work, and heat transfer terms and show the P–v diagram.

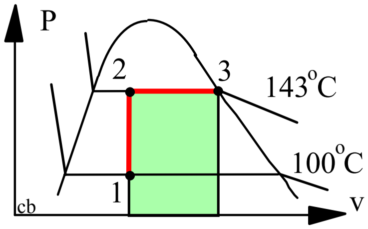
Solution:

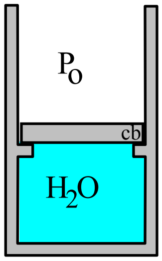
C.V. The 5 kg water.

Continuty: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

$$P_3 = P_2 = P_{lift} = P_0 + m_p\, g / A_p = 100 + \frac{75 \times 9.807}{0.00245 \times 1000} = \mathbf{400\ kPa}$$





State 1: (T,x) Table B.1.1

$v_1 = 0.001044 + 0.2 \times 1.6719$, $V_1 = mv_1 = 5 \times 0.3354 = \mathbf{1.677\ m^3}$

$u_1 = 418.91 + 0.2 \times 2087.58 = 836.4$ kJ/kg

State 3: (P, x = 1) Table B.1.2 => $v_3 = 0.4625 > v_1$, $u_3 = 2553.6$ kJ/kg

Work is seen in the P-V diagram (if volume changes then $P = P_{lift}$)

$${}_1W_3 = {}_2W_3 = P_{ext}m(v_3 - v_2) = 400 \times 5(0.46246 - 0.3354) = \mathbf{254.1\ kJ}$$

Heat transfer is from the energy equation

$${}_1Q_3 = 5\ (2553.6 - 836.4) + 254.1 = \mathbf{8840\ kJ}$$

Energy Equation: Solids and Liquids

5.78

I have 2 kg of liquid water at 20°C, 100 kPa. I now add 20 kJ of energy at a constant pressure. How hot does it get if it is heated? How fast does it move if it is pushed by a constant horizontal force? How high does it go if it is raised straight up?

a) Heat at 100 kPa.

Energy equation:

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1) = H_2 - H_1 = m(h_2 - h_1)$$

$$h_2 = h_1 + {}_1Q_2/m = 83.94 + 20/2 = 94.04 \text{ kJ/kg}$$

Back interpolate in Table B.1.1: $T_2 = \mathbf{22.5^oC}$

[We could also have used $\Delta T = {}_1Q_2/mC = 20 / (2*4.18) = 2.4^oC$]

b) Push at constant P. It gains kinetic energy.

$$0.5 \text{ m } \mathbf{V}_2^2 = {}_1W_2$$

$$\mathbf{V}_2 = \sqrt{2\ {}_1W_2/m} = \sqrt{2 \times 20 \times 1000 \text{ J}/2 \text{ kg}} = \mathbf{141.4 \text{ m/s}}$$

c) Raised in gravitational field

$$m\ g\ Z_2 = {}_1W_2$$

$$Z_2 = {}_1W_2/m\ g = \frac{20\ 000 \text{ J}}{2 \text{ kg} \times 9.807 \text{ m/s}^2} = \mathbf{1019 \text{ m}}$$

Comment: Notice how fast (500 km/h) and how high it should by to have the same energy as raising the temperature just 2 degrees. I.e. in most applications we can disregard the kinetic and potential energies unless we have very high **V** or Z.

5.79

A copper block of volume 1 L is heat treated at 500°C and now cooled in a 200-L oil bath initially at 20°C, shown in Fig. P5.79. Assuming no heat transfer with the surroundings, what is the final temperature?

Solution:

C.V. Copper block and the oil bath.

Also assume no change in volume so the work will be zero.

Energy Eq.: $U_2 - U_1 = m_{met}(u_2 - u_1)_{met} + m_{oil}(u_2 - u_1)_{oil} = {}_1Q_2 - {}_1W_2 = 0$

Properties from Table A.3 and A.4

$$m_{met} = V\rho = 0.001 \text{ m}^3 \times 8300 \text{ kg/m}^3 = 8.3 \text{ kg},$$

$$m_{oil} = V\rho = 0.2 \text{ m}^3 \times 910 \text{ kg/m}^3 = 182 \text{ kg}$$

Solid and liquid Eq.5.17: $\Delta u \cong C_V \Delta T$,

Table A.3 and A.4: $C_{V\ met} = 0.42 \frac{\text{kJ}}{\text{kg K}}$, $C_{V\ oil} = 1.8 \frac{\text{kJ}}{\text{kg K}}$

The energy equation for the C.V. becomes

$$m_{met}C_{V\ met}(T_2 - T_{1,met}) + m_{oil}C_{V\ oil}(T_2 - T_{1,oil}) = 0$$

$$8.3 \times 0.42(T_2 - 500) + 182 \times 1.8\ (T_2 - 20) = 0$$

$$331.09\ T_2 - 1743 - 6552 = 0$$

$$\Rightarrow\ T_2 = \mathbf{25\ °C}$$

5.80

Because a hot water supply must also heat some pipe mass as it is turned on so it does not come out hot right away. Assume 80°C liquid water at 100 kPa is cooled to 45°C as it heats 15 kg of copper pipe from 20 to 45°C. How much mass (kg) of water is needed?

Solution:

C.V. Water and copper pipe. No external heat transfer, no work.

Energy Eq.5.11: $U_2 - U_1 = \Delta U_{cu} + \Delta U_{H_2O} = 0 - 0$

From Eq.5.18 and Table A.3:

$$\Delta U_{cu} = mC\ \Delta T = 15\text{ kg} \times 0.42\ \frac{\text{kJ}}{\text{kg K}} \times (45 - 20)\text{ K} = 157.5\text{ kJ}$$

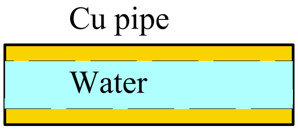
From the energy equation

$$m_{H_2O} = -\ \Delta U_{cu}\ /\ \Delta u_{H_2O}$$

$$m_{H_2O} = \ \Delta U_{cu}\ /\ C_{H_2O}(-\ \Delta T_{H_2O}) = \frac{157.5}{4.18 \times 35}\ = \mathbf{1.076\ kg}$$

or using Table B.1.1 for water

$$m_{H_2O} = \ \Delta U_{cu}\ /\ (\ u_1 - u_2) = \frac{157.5}{334.84 - 188.41} = \mathbf{1.076\ kg}$$



The real problem involves a flow and is not analyzed by this simple process.

5.81

In a sink 5 liters of water at 70°C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20°C. What is the final uniform temperature neglecting any heat loss and work?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the water: $v_f = 0.001023\ m^3/kg$, $V = 5\ L = 0.005\ m^3$; $m = V/v = 4.8876\ kg$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}\,(T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59\ kJ/K$$

Energy Eq.: $22.59\ T_2 = 4.8876 \times 4.18 \times 70 + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times 20$

$= 1430.11 + 43.2$

$T_2 = \mathbf{65.2^oC}$

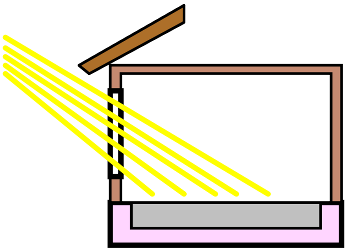
5.82

A house is being designed to use a thick concrete floor mass as thermal storage material for solar energy heating. The concrete is 30 cm thick and the area exposed to the sun during the daytime is 4 m × 6 m. It is expected that this mass will undergo an average temperature rise of about 3°C during the day. How much energy will be available for heating during the nighttime hours?

Solution:

C.V.: Control mass concrete.

$$V = 4 \times 6 \times 0.3 = 7.2 \text{ m}^3$$



Concrete is a solid with some properties listed in Table A.3

$$m = \rho V = 2200 \text{ kg/m}^3 \times 7.2 \text{ m}^3 = 15\ 840 \text{ kg}$$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

The available heat transfer is the change in U. From Eq.5.18 and C from table A.3

$$\Delta U = m\ C\ \Delta T = 15\ 840 \text{ kg} \times 0.88 \frac{\text{kJ}}{\text{kg K}} \times 3 \text{ K} = 41818 \text{ kJ} = \mathbf{41.82\ MJ}$$

5.83

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and air pressure.

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_iC_{v\,i}\,(T_2 - T_1)_i = T_2\sum m_iC_{v\,i} - \sum m_iC_{v\,i}T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_iC_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

Energy Eq.: $7.282\ T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$

$+ 0.1 \times 0.717 \times (400\text{-}273.15) = 372.745 \text{ kJ}$

$T_2 = \mathbf{51.2^oC}$

The volume of the air is constant so from PV = mRT it follows that P varies with T

$P_2 = P_1\ T_2/T_{1\ air} = 100 \times 324.34 / 400 =$ **81 kPa**

5.84

A car with mass 1275 kg drives at 60 km/h when the brakes are applied quickly to decrease its speed to 20 km/h. Assume the brake pads are 0.5 kg mass with heat capacity of 1.1 kJ/kg K and the brake discs/drums are 4.0 kg steel. Further assume both masses are heated uniformly. Find the temperature increase in the brake assembly.

Solution:

C.V. Car. Car loses kinetic energy and brake system gains internal u.

No heat transfer (short time) and no work term.

m = constant;

Energy Eq.5.11: $E_2 - E_1 = 0 - 0 = m_{car} \frac{1}{2}(V_2^2 - V_1^2) + m_{brake}(u_2 - u_1)$

The brake system mass is two different kinds so split it, also use C_v from Table A.3 since we do not have a u table for steel or brake pad material.

$$m_{steel}\, C_v\, \Delta T \;+ m_{pad}\, C_v\, \Delta T \;=\; m_{car}\, 0.5\, (60^2 - 20^2) \left(\frac{1000}{3600}\right)^2 \text{ m}^2/\text{s}^2$$

$$(4 \times 0.46 + 0.5 \times 1.1) \frac{\text{kJ}}{\text{K}} \Delta T \;= 1275 \text{ kg} \times 0.5 \times (3200 \times 0.077\,16) \text{ m}^2/\text{s}^2$$

$$= 157\,406 \text{ J} = 157.4 \text{ kJ}$$

$$\Rightarrow \Delta T \;= \mathbf{65.9\ °C}$$

5.85

A computer CPU chip consists of 50 g silicon, 20 g copper, 50 g polyvinyl chloride (plastic). It heats from 15°C to 70°C as the computer is turned on. How much energy does the heating require?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\sum m_i(u_2 - u_1)_i = \sum m_iC_{v\,i}\,(T_2 - T_1)_i = (T_2 - T_1)\sum m_iC_{v\,i}$$

$$\sum m_iC_{v\,i} = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$$U_2 - U_1 = 0.0914 \times (70 - 15) = \mathbf{5.03\ kJ}$$

5.86

A 25 kg steel tank initially at –10°C is filled up with 100 kg of milk (assume properties as water) at 30°C. The milk and the steel come to a uniform temperature of +5 °C in a storage room. How much heat transfer is needed for this process?

Solution:

C.V. Steel + Milk. This is a control mass.

Energy Eq.5.11: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process: V = constant, so there is no work

$${}_1W_2 = 0.$$

Use Eq.5.18 and values from A.3 and A.4 to evaluate changes in u

$$ {}_1Q_2 = m_{steel}\,(u_2 - u_1)_{steel} + m_{milk}(u_2 - u_1)_{milk}$$

$$= 25\text{ kg} \times 0.466\,\frac{\text{kJ}}{\text{kg K}} \times [5 - (-10)]\text{ K} + 100\text{ kg} \times 4.18\,\frac{\text{kJ}}{\text{kg K}} \times (5 - 30)\text{ K}$$

$$= 172.5 - 10450 = \mathbf{-10277\ kJ}$$

5.87

A 1 kg steel pot contains 1 kg liquid water both at 15°C. It is now put on the stove where it is heated to the boiling point of the water. Neglect any air being heated and find the total amount of energy needed.

Solution:

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

State 2: $T_2 = T_{sat}$ (1atm) = 100°C

Tbl B.1.1 : $u_1 = 62.98$ kJ/kg, $u_2 = 418.91$ kJ/kg

Tbl A.3 : $C_{st} = 0.46$ kJ/kg K

Solve for the heat transfer from the energy equation

$$ {}_1Q_2 = U_2 - U_1 = m_{st}\,(u_2 - u_1)_{st} + m_{H2O}\,(u_2 - u_1)_{H2O} $$
$$ = m_{st}C_{st}\;(T_2 - T_1) + m_{H2O}\,(u_2 - u_1)_{H2O} $$

$$ {}_1Q_2 = 1\text{ kg} \times 0.46\,\frac{\text{kJ}}{\text{kg K}} \times (100 - 15)\text{ K} + 1\text{ kg} \times (418.91 - 62.98)\text{ kJ/kg} $$
$$ = 39.1 + 355.93 = \mathbf{395\ kJ} $$

5.88

A piston cylinder (0.5 kg steel altogether) maintaining a constant pressure has 0.2 kg R-134a as saturated vapor at 150 kPa. It is heated to 40^{o}C and the steel is at the same temperature as the R-134a at any time. Find the work and heat transfer for the process.

C.V. The R-134a plus the steel. Constant total mass

$m_2 = m_1 = m$;

$U_2 - U_1 = m_{R134a}(u_2 - u_1)_{R134a} + m_{steel}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: B.5.2 sat. vapor $v_1 = 0.13139$ m^3/kg, $u_1 = 368.06$ kJ/kg

State 2: B.5.2 sup. vapor $v_2 = 0.16592$ m^3/kg, $u_2 = 411.59$ kJ/kg

$V_1 = mv_1 = 0.2 \times 0.13139 = 0.02628$ m^3

$V_2 = mv_2 = 0.2 \times 0.16592 = 0.03318$ m^3

Steel: A.3, $C_{steel} = 0.46$ kJ/kg-K

Process: P = C for the R134a and constant volume for the steel =>

${}_1W_2 = \int P\, dV = P_1(V_2 - V_1) = 150$ kPa $(0.03318 - 0.02628)$ m^3

= **1.035 kJ**

${}_1Q_2 = m_{R134a}\,(u_2 - u_1) + m_{steel}\,(u_2 - u_1) + {}_1W_2$

$= m_{R134a}\,(u_2 - u_1) + m_{steel}C_{steel}\,(T_2 - T_1) + {}_1W_2$

$= 0.2 \times (411.59 - 368.06) + 0.5 \times 0.46 \times [40 - (-17.29)] + 1.035$

$= 8.706 + 13.177 + 1.035 =$ **22.92 kJ**

5.89

An engine consists of a 100 kg cast iron block with a 20 kg aluminum head, 20 kg steel parts, 5 kg engine oil and 6 kg glycerine (antifreeze). Everything begins at 5^{o}C and as the engine starts we want to know how hot it becomes if it absorbs a net of 7000 kJ before it reaches a steady uniform temperature.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

So sum over the various parts of the left hand side in the energy equation

$$m_{Fe}\,(u_2 - u_1) + m_{Al}\,(u_2 - u_1)_{Al} + m_{st}\,(u - u_1)_{st}$$
$$+ m_{oil}\,(u_2 - u_1)_{oil} + m_{gly}\,(u_2 - u_1)_{gly} = {}_1Q_2$$

Table A.3 : $C_{Fe} = 0.42$, $C_{Al} = 0.9$, $C_{st} = 0.46$ all units of kJ/kg K

Table A.4 : $C_{oil} = 1.9$, $C_{gly} = 2.42$ all units of kJ/kg K

So now we factor out $T_2 - T_1$ as $u_2 - u_1 = C(T_2 - T_1)$ for each term

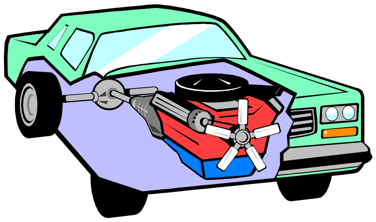
$$[\, m_{Fe}C_{Fe} + m_{Al}C_{Al} + m_{st}C_{st} + m_{oil}C_{oil} + m_{gly}C_{gly} \,]\,(T_2 - T_1) = {}_1Q_2$$

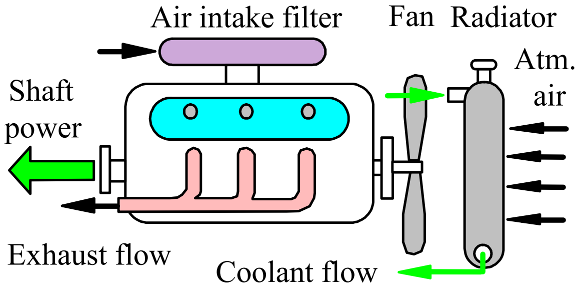
$$T_2 - T_1 = {}_1Q_2 / \Sigma m_i\, C_i$$

$$= \frac{7000}{100\times 0.42 + 20\times 0.9 + 20\times 0.46 + 5\times 1.9 + 6\times 2.42}$$

$$= \frac{7000}{93.22} = 75 \text{ K}$$

$$T_2 = T_1 + 75 = 5 + 75 = \mathbf{80^oC}$$





Properties (u, h, C_v and C_p), Ideal Gas

5.90

Use the ideal gas air table A.7 to evaluate the heat capacity C_p at 300 K as a slope of the curve h(T) by Δh/ΔT. How much larger is it at 1000 K and 1500 K.

Solution :

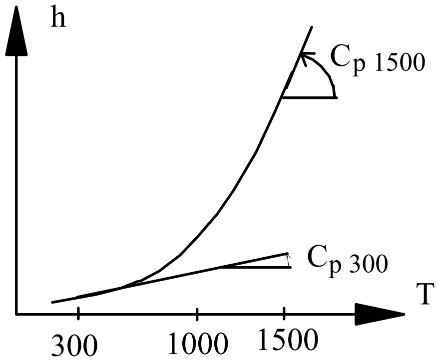
From Eq.5.24:

$$C_p = \frac{dh}{dT} = \frac{\Delta h}{\Delta T} = \frac{h_{320} - h_{290}}{320 - 290} = \mathbf{1.005\ kJ/kg\ K}$$

$$1000\text{K} \quad C_p = \frac{\Delta h}{\Delta T} = \frac{h_{1050} - h_{950}}{1050 - 950} = \frac{1103.48 - 989.44}{100} = \mathbf{1.140\ kJ/kg\ K}$$

$$1500\text{K} \quad C_p = \frac{\Delta h}{\Delta T} = \frac{h_{1550} - h_{1450}}{1550 - 1450} = \frac{1696.45 - 1575.4}{100} = \mathbf{1.21\ kJ/kg\ K}$$

Notice an increase of 14%, 21% respectively.



5.91

We want to find the change in u for carbon dioxide between 600 K and 1200 K.

a) Find it from a constant C_{vo} from table A.5

b) Find it from a C_{vo} evaluated from equation in A.6 at the average T.

c) Find it from the values of u listed in table A.8

Solution :

a) $\Delta u \cong C_{vo}\, \Delta T = 0.653 \times (1200 - 600) = \mathbf{391.8\ kJ/kg}$

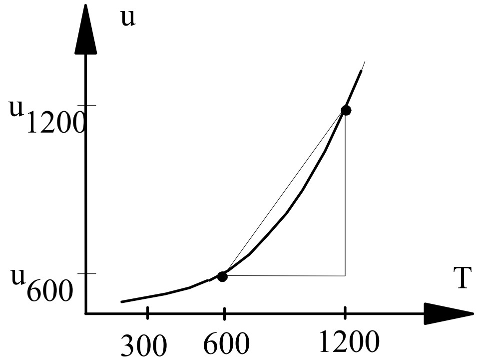
b) $T_{avg} = \frac{1}{2}(1200 + 600) = 900, \qquad \theta = \frac{T}{1000} = \frac{900}{1000} = 0.9$

$C_{po} = 0.45 + 1.67 \times 0.9 - 1.27 \times 0.9^2 + 0.39 \times 0.9^3 = 1.2086 \text{ kJ/kg K}$

$C_{vo} = C_{po} - R = 1.2086 - 0.1889 = 1.0197 \text{ kJ/kg K}$

$\Delta u = 1.0197 \times (1200 - 600) = \mathbf{611.8\ kJ/kg}$

c) $\Delta u = 996.64 - 392.72 = \mathbf{603.92\ kJ/kg}$



5.92

We want to find the change in u for carbon dioxide between 50°C and 200°C at a pressure of 10 MPa. Find it using ideal gas and Table A.5 and repeat using the B section table.

Solution:

Using the value of C_{vo} for CO_2 from Table A.5,

$\Delta u = C_{vo}\,\Delta T = 0{,}653 \times (200 - 50) =$ **97.95 kJ/kg**

Using values of u from Table B3.2 at 10 000 kPa, with linear interpolation between 40°C and 60°C for the 50°C value,

$\Delta u = u_{200} - u_{50} = 437.6 - 230.9 =$ **206.7 kJ/kg**

Note: Since the state 50°C, 10 000 kPa is in the dense-fluid supercritical region, a linear interpolation is quite inaccurate. The proper value for u at this state is found from the CATT software to be 245.1 instead of 230.9. This results is

$\Delta u = u_{200} - u_{50} = 437.6 - 245.1 =$ **192.5 kJ/kg**

5.93

We want to find the change in u for oxygen gas between 600 K and 1200 K.

a) Find it from a constant C_{vo} from table A.5

b) Find it from a C_{vo} evaluated from equation in A.6 at the average T.

c) Find it from the values of u listed in table A.8

Solution:

a) $\Delta u \cong C_{vo}\,\Delta T = 0.662 \times (1200 - 600) = \mathbf{397.2\ kJ/kg}$

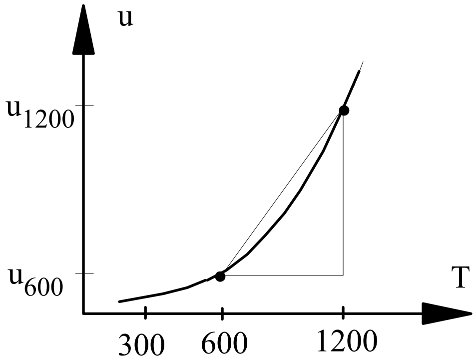
b) $T_{avg} = \frac{1}{2}(1200 + 600) = 900\text{ K}, \qquad \theta = \frac{T}{1000} = \frac{900}{1000} = 0.9$

$C_{po} = 0.88 - 0.0001 \times 0.9 + 0.54 \times 0.9^2 - 0.33 \times 0.9^3 = 1.0767$

$C_{vo} = C_{po} - R = 1.0767 - 0.2598 = 0.8169\text{ kJ/kg K}$

$\Delta u = 0.8169 \times (1200 - 600) = \mathbf{490.1\ kJ/kg}$

c) $\Delta u = 889.72 - 404.46 = \mathbf{485.3\ kJ/kg}$



5.94

Estimate the constant specific heats for R-134a from Table B.5.2 at 100 kPa and 125^{o} C. Compare this to table A.5 and explain the difference.

Solution:

Using values at 100 kPa for h and u at 120^{o}C and 130^{o}C from Table B5.2, the approximate specific heats at 125^{o}C are

$$C_p \approx \frac{\Delta h}{\Delta T} = \frac{521.98 - 511.95}{130 - 120} = 1.003 \text{ kJ/kg K}$$

compared with 0.852 kJ/kg K for the ideal-gas value at 25^{o}C from Table A.5.

$$C_v \approx \frac{\Delta u}{\Delta T} = \frac{489.36 - 480.16}{130 - 120} = 0.920 \text{ kJ/kg K}$$

compared with 0.771 kJ/kg K for the ideal-gas value at 25^{o}C from Table A.5.

There are two reasons for the differences. First, R-134a is not exactly an ideal gas at the given state, 125^{o}C and 100 kPa. Second and by far the biggest reason for the differences is that R-134a, chemically CF_3CH_2 , is a polyatomic molecule with multiple vibrational mode contributions to the specific heats (see Appendix C), such that they are strongly dependent on temperature. Note that if we repeat the above approximation for C_p in Table B.5.2 at 25^{o}C, the resulting value is 0.851 kJ/kg K.

5.95

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific internal energy, using a) the steam tables, b) the ideal gas water table A.8, and c) ≈the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3 Superheated vapor $u_1 = 2564.48$ kJ/kg

State 2: Table B.1.3 $u_2 = 4467.23$ kJ/kg

$u_2 - u_1 = 4467.23 - 2564.48 = \mathbf{1902.75\ kJ/kg}$

b)

Table A.8 at 423.15 K: $u_1 = 591.41$ kJ/kg

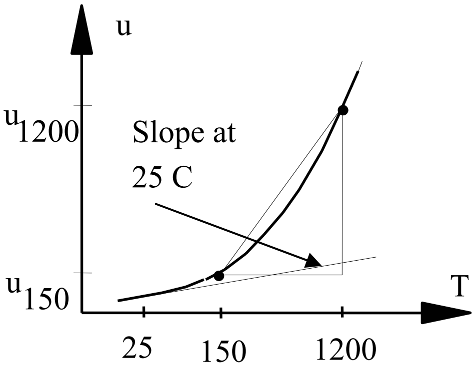
Table A.8 at 1473.15 K: $u_2 = 2474.25$ kJ/kg

$u_2 - u_1 = 2474.25 - 591.41 = \mathbf{1882.8\ kJ/kg}$

c) Table A.5 : $C_{vo} = 1.41$ kJ/kgK

$u_2 - u_1 = 1.41$ kJ/kgK $(1200 – 150)$ K = $\mathbf{1480.5\ kJ/kg}$

Notice how the average slope from 150 C to 1200 C is higher than the one at 25 C (= C_{vo})



5.96

Nitrogen at 300 K, 3 MPa is heated to 500 K. Find the change in enthalpy using a) Table B.6, b) Table A.8, and c) Table A.5.

B.6.2 $h_2 - h_1 = 519.29 - 304.94 = 214.35$ kJ/kg

A.8 $h_2 - h_1 = 520.75 - 311.67 = 209.08$ kJ/kg

A.5 $h_2 - h_1 = C_{po} (T_2 - T_1) = 1.042$ kJ/kg-K $(500 - 300)$ K $= 208.4$ kJ/kg

Comment: The results are listed in order of accuracy (B.6.2 best).

5.97

For a special application we need to evaluate the change in enthalpy for carbon dioxide from 30^{o}C to 1500^{o}C at 100 kPa. Do this using constant specific heat from Table A.5 and repeat using Table A.8. Which is the more accurate one?

Solution:

Using constant specific heat:

$$\Delta h = C_{po}\Delta T = 0.842\ (1500 - 30) = \mathbf{1237.7\ kJ/kg}$$

Using Table A.8:

$$30°C = 303.15\ K\ \ =>\ \ h = 214.38 + \frac{3.15}{50}(257.9 - 214.38) = 217.12\ kJ/kg$$

$$1500°C = 1773.15\ K\ \ =>$$

$$h = 1882.43 + \frac{73.15}{100}(2017.67 - 1882.43) = 1981.36\ kJ/kg$$

$$\Delta h = 1981.36 - 217.12 = \mathbf{1764.2\ kJ/kg}$$

The result from A.8 is best. For large ΔT or small ΔT at high T_{avg}, constant specfic heat is poor approximation.

5.98

Repeat the previous problem but use a constant specific heat at the average temperature from equation in Table A.6 and also integrate the equation in Table A.6 to get the change in enthalpy.

$$T_{ave} = \frac{1}{2}(30 + 1500) + 273.15 = 1038.15 \text{ K}; \quad \theta = T/1000 = 1.0382$$

$$\text{Table A.6} \quad \Rightarrow \quad C_{po} = 1.2513 \text{ kJ/kg-K}$$

$$\Delta h = C_{po,ave}\, \Delta T = 1.2513 \times 1470 = \mathbf{1839\ kJ/kg}$$

For the entry to Table A.6:

$$30°\text{C} = 303.15 \text{ K} \;\; => \;\; \theta_1 = 0.30315$$

$$1500°\text{C} = 1773.15 \text{ K} \;\; => \;\; \theta_2 = 1.77315$$

$$\Delta h = h_2 - h_1 = \int C_{po}\, dT$$

$$= [0.45\,(\theta_2 - \theta_1) + 1.67 \times \frac{1}{2}(\theta_2^2 - \theta_1^2)$$

$$-1.27 \times \frac{1}{3}(\theta_2^3 - \theta_1^3) + 0.39 \times \frac{1}{4}(\theta_2^4 - \theta_1^4)] = \mathbf{1762.76\ kJ/kg}$$

5.99

Reconsider Problem 5.97 and examine if also using Table B.3 would be more accurate and explain.

Table B.3 does include non-ideal gas effects, however at 100 kPa these effects are extremely small so the answer from Table A.8 is accurate.

Table B.3. does not cover the 100 kPa superheated vapor states as the saturation pressure is below the triple point pressure. Secondly Table B.3 does not go up to the high temperatures covered by Table A.8 and A.9 at which states you do have ideal gas behavior. Table B.3 covers the region of states where the carbon dioxide is close to the two-phase region and above the critical point (dense fluid) which are all states where you cannot assume ideal gas.

5.100

Water at 20°C, 100 kPa, is brought to 200 kPa, 1500°C. Find the change in the specific internal energy, using the water table and the ideal gas water table in combination.

Solution:

State 1: Table B.1.1 $u_1 \cong u_f = 83.95$ kJ/kg

State 2: Highest T in Table B.1.3 is 1300°C

Using a Δu from the ideal gas tables, A.8, we get

$u_{1500} = 3139$ kJ/kg $\qquad u_{1300} = 2690.72$ kJ/kg

$u_{1500} - u_{1300} = 448.26$ kJ/kg

We now add the ideal gas change at low P to the steam tables, B.1.3, $u_x = 4683.23$ kJ/kg as the reference.

$$u_2 - u_1 = (u_2 - u_x)_{\text{ID.G.}} + (u_x - u_1)$$

$$= 448.28 + 4683.23 - 83.95 = \mathbf{5048\ kJ/kg}$$

5.101

An ideal gas is heated from 500 to 1500 K. Find the change in enthalpy using constant specific heat from Table A.5 (room temperature value) and discuss the accuracy of the result if the gas is

a. Argon b. Oxygen c. Carbon dioxide

Solution:

$T_1 = 500$ K, $T_2 = 1500$ K, $\Delta h = C_{P0}(T_2-T_1)$

a) Ar : $\Delta h = 0.520(1500-500) = 520$ kJ/kg

Monatomic inert gas very good approximation.

b) O_2 : $\Delta h = 0.922(1500-500) = 922$ kJ/kg

Diatomic gas approximation is OK with some error.

c) CO_2: $\Delta h = 0.842(1500-500) = 842$ kJ/kg

Polyatomic gas heat capacity changes, see figure 5.11

See also appendix C for more explanation.

Energy Equation: Ideal Gas

5.102

Air is heated from 300 to 350 K at V = C. Find ${}_1q_2$. What if from 1300 to 1350 K?

Process: V = C $\rightarrow$ ${}_1W_2 = Ø$

Energy Eq.: $u_2 - u_1 = {}_1q_2 - 0 \rightarrow$ ${}_1q_2 = u_2 - u_1$

Read the u-values from Table A.7.1

a) ${}_1q_2 = u_2 - u_1 = 250.32 - 214.36 =$ **36.0 kJ/kg**

b) ${}_1q_2 = u_2 - u_1 = 1067.94 - 1022.75 =$ **45.2 kJ/kg**

case a) $C_v \approx 36/50 = 0.72$ kJ/kg K , see A.5

case b) $C_v \approx 45.2/50 = 0.904$ kJ/kg K (25 % higher)

5.103

A 250 L rigid tank contains methane at 500 K, 1500 kPa. It is now cooled down to 300 K. Find the mass of methane and the heat transfer using ideal gas.

Solution:

Ideal gas, constant volume

$$P_2 = P_1 \times (T_2 / T_1) = 1500 \times 300 / 500 = 900 \text{ kPa}$$

$$m = P_1V/RT_1 = \frac{1500 \times 0.25}{0.5183 \times 500} = \mathbf{1.447\ kg}$$

Use specific heat from Table A.5

$$u_2 - u_1 = C_v\,(T_2 - T_1) = 1.736\,(300 - 500) = -347.2 \text{ kJ/kg}$$

$${}_1Q_2 = m(u_2 - u_1) = 1.447(-347.2) = \mathbf{-502.4\ kJ}$$

5.104

A rigid tank has 1 kg air at 300 K, 120 kPa and it is heated by an external heater. Use Table A.7 to find the work and the heat transfer for the process.

CV Air in tank, this is a C.M. at constant volume.
This process is a constant volume process so $\mathbf{{}_1W_2 = 0}$ and ${}_1Q_2$ comes in.

Energy Eq: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process & ideal gas: $V_2 = V_1$; $P_1V_1 = mRT_1$, $P_2V_2 = mRT_2$

$$P_2 = P_1\, T_2 / T_1 = 120 \times 1500 / 300 = \mathbf{600\ kPa}$$

Solving using Table A.7 gives:

$${}_1Q_2 = m(u_2 - u_1) = 1(1205.25 - 214.36) = \mathbf{990.89\ kJ}$$

5.105

A rigid container has 2 kg of carbon dioxide gas at 100 kPa, 1200 K that is heated to 1400 K. Solve for the heat transfer using a. the heat capacity from Table A.5 and b. properties from Table A.8

Solution:

C.V. Carbon dioxide, which is a control mass.

Energy Eq.5.11: $U_2 - U_1 = m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

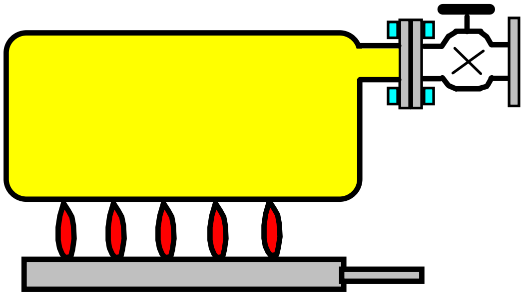
Process: $\Delta V = 0 \Rightarrow {}_1W_2 = 0$

a) For constant heat capacity we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

$${}_1Q_2 \cong mC_{vo} (T_2 - T_1) = 2 \times 0.653 \times (1400 - 1200) = \mathbf{261.2\ kJ}$$

b) Taking the u values from Table A.8 we get

$${}_1Q_2 = m (u_2 - u_1) = 2 \times (1218.38 - 996.64) = \mathbf{443.5\ kJ}$$



5.106

Do the previous problem for nitrogen, N_2, gas.

A rigid container has 2 kg of carbon dioxide gas at 100 kPa, 1200 K that is heated to 1400 K. Solve for the heat transfer using a. the heat capacity from Table A.5 and b. properties from Table A.8

Solution:

C.V. Nitrogen gas, which is a control mass.

Energy Eq.5.11: $U_2 - U_1 = m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

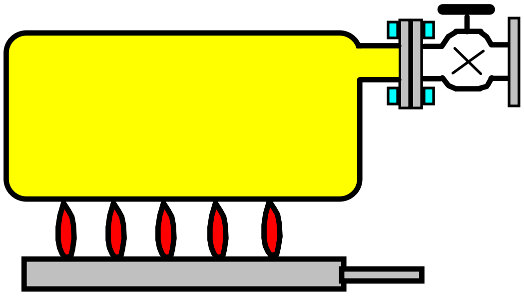
Process: $\Delta V = 0 \Rightarrow {}_1W_2 = 0$

a) For constant heat capacity we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

$${}_1Q_2 \cong mC_{vo} (T_2 - T_1) = 2 \times 0.745 \times (1400 - 1200) = \mathbf{298\ kJ}$$

b) Taking the u values from Table A.8, we get

$${}_1Q_2 = m (u_2 - u_1) = 2 \times (1141.35 - 957) = \mathbf{368.7\ kJ}$$



5.107

A tank has a volume of 1 m^3 with oxygen at 15^{o}C, 300 kPa. Another tank contains 4 kg oxygen at 60^{o}C, 500 kPa. The two tanks are connected by a pipe and valve which is opened allowing the whole system to come to a single equilibrium state with the ambient at 20^{o}C. Find the final pressure and the heat transfer.

C.V. Both tanks of constant volume.

Continuity Eq.: $m_2 - m_{1A} - m_{1B} = 0$

Energy Eq.: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = {}_1Q_2 - {}_1W_2$

Process Eq.: $V_2 = V_A + V_B = \text{constant}, \quad {}_1W_2 = 0$

State 1A: $m_{1A} = \dfrac{P_{1A}V_A}{RT_{1A}} = \dfrac{300 \text{ kPa} \times 1 \text{ m}^3}{0.2598 \text{ kJ/kg-K} \times 288.15 \text{ K}} = 4.007 \text{ kg}$

State 1B: $V_B = \dfrac{m_{1B}RT_{1B}}{P_{1B}} = \dfrac{4 \text{ kg} \times 0.2598 \text{ kJ/kg-K} \times 333.15 \text{ K}}{500 \text{ kPa}} = 0.6924 \text{ m}^3$

State 2: $(T_2, v_2 = V_2/m_2) \quad V_2 = V_A + V_B = 1 + 0.6924 = 1.6924 \text{ m}^3$

$m_2 = m_{1A} + m_{1B} = 4.007 + 4 = 8.007 \text{ kg}$

$$P_2 = \frac{m_2RT_2}{V_2} = \frac{8.007 \text{ kg} \times 0.2598 \text{ kJ/kg-K} \times 293.15 \text{ K}}{1.6924 \text{ m}^3} = \mathbf{360.3 \text{ kPa}}$$

Heat transfer from energy equation

$${}_1Q_2 = m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = m_{1A}(u_2 - u_{1A}) + m_{1B}(u_2 - u_{1B})$$

$$= m_{1A}C_v (T_2 - T_{1A}) + m_{1B} C_v (T_2 - T_{1B})$$

$$= 4.007 \text{ kg}\times 0.662 \text{ kJ/kg-K}\times(20 - 15) \text{ K} + 4 \text{ kg}\times 0.662 \text{ kJ/kg-K} \times(20 - 60) \text{ K}$$

$$= \mathbf{- 92.65 \text{ kJ}}$$

5.108

Find the heat transfer in Problem 4.43.

A piston cylinder contains 3 kg of air at 20^{o}C and 300 kPa. It is now heated up in a constant pressure process to 600 K.

Solution:

Ideal gas $PV = mRT$

State 1: T_1, P_1

State 2: $T_2, P_2 = P_1$

$P_2V_2 = mRT_2$ $\quad V_2 = mR\,T_2 / P_2 = 3\times0.287\times600 / 300 = 1.722\ m^3$

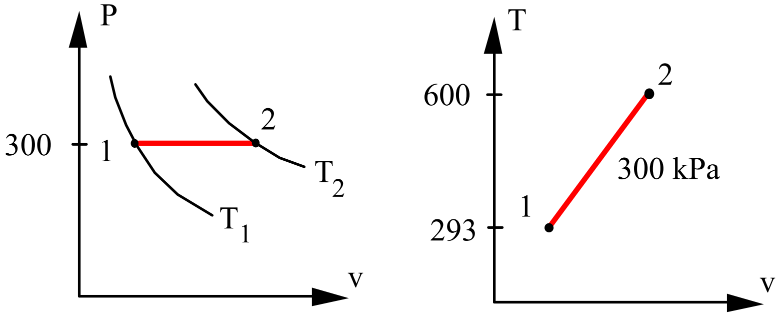
Process: $P = \text{constant}$,

$${}_1W_2 = \int PdV = P\,(V_2 - V_1) = 300\,(1.722 - 0.8413) = 264.2\ kJ$$

Energy equation becomes

$$U_2 - U_1 = {}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

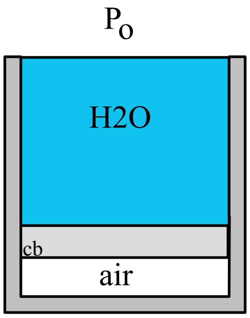
$${}_1Q_2 = U_2 - U_1 + {}_1W_2 = 3(435.097 - 209.45) + 264.2 = \mathbf{941\ kJ}$$

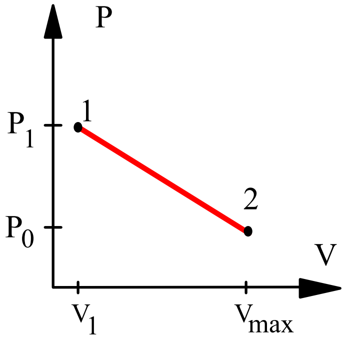


5.109

A 10-m high cylinder, cross-sectional area 0.1 m^2, has a massless piston at the bottom with water at 20°C on top of it, shown in Fig. P5.109. Air at 300 K, volume 0.3 m^3, under the piston is heated so that the piston moves up, spilling the water out over the side. Find the total heat transfer to the air when all the water has been pushed out.

Solution:





The water on top is compressed liquid and has volume and mass

$$V_{H_2O} = V_{tot} - V_{air} = 10 \times 0.1 - 0.3 = 0.7 \text{ m}^3$$

$$m_{H_2O} = V_{H_2O}/v_f = 0.7 / 0.001002 = 698.6 \text{ kg}$$

The initial air pressure is then

$$P_1 = P_0 + m_{H_2O}g/A = 101.325 + \frac{698.6 \times 9.807}{0.1 \times 1000} = \mathbf{169.84\ kPa}$$

and then $m_{air} = PV/RT = \dfrac{169.84 \times 0.3}{0.287 \times 300} = 0.592 \text{ kg}$

State 2: No liquid over piston: $P_2 = P_0 = 101.325$ kPa, $V_2 = 10 \times 0.1 = 1 \text{ m}^3$

State 2: P_2, V_2 $\Rightarrow$ $T_2 = \dfrac{T_1P_2V_2}{P_1V_1} = \dfrac{300 \times 101.325 \times 1}{169.84 \times 0.3} = 596.59 \text{ K}$

The process line shows the work as an area

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(169.84 + 101.325)(1 - 0.3) = 94.91 \text{ kJ}$$

The energy equation solved for the heat transfer becomes

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 \cong mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.592 \times 0.717 \times (596.59 - 300) + 94.91 = \mathbf{220.7\ kJ}$$

Remark: we could have used u values from Table A.7:

$u_2 - u_1 = 432.5 - 214.36 = 218.14$ kJ/kg versus 212.5 kJ/kg with C_v.

5.110

A piston cylinder contains air at 600 kPa, 290 K and a volume of 0.01 m^3. A constant pressure process gives 18 kJ of work out. Find the final temperature of the air and the heat transfer input.

Solution:

C.V AIR control mass

Continuity Eq.: $m_2 - m_1 = 0$

Energy Eq.: $m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = C$ so ${}_1W_2 = \int P\, dV = P(V_2 - V_1)$

1 : P_1 , T_1,V_1 2 : $P_1 = P_2$, ?

$m_1 = P_1V_1/RT_1 = 600 \times 0.01 / 0.287 \times 290 = 0.0721$ kg

${}_1W_2 = P(V_2 - V_1) = 18 \text{ kJ} \rightarrow$

$V_2 - V_1 = {}_1W_2 / P = 18 \text{ kJ} / 600 \text{ kPa} = 0.03 \text{ m}^3$

$V_2 = V_1 + {}_1W_2 / P = 0.01 + 0.03 = 0.04 \text{ m}^3$

Ideal gas law : $P_2V_2 = mRT_2$

$$T_2 = P_2V_2 / mR = \frac{P_2V_2}{P_1V_1} T_1 = \frac{0.04}{0.01} \times 290 = \mathbf{1160\ K}$$

Energy equation with u's from table A.7.1

$${}_1Q_2 = m (u_2 - u_1) + {}_1W_2$$

$$= 0.0721 (898.04 - 207.19) + 18$$

$$= \mathbf{67.81\ kJ}$$

5.111

An insulated cylinder is divided into two parts of 1 m^3 each by an initially locked piston, as shown in Fig. P5.111. Side A has air at 200 kPa, 300 K, and side B has air at 1.0 MPa, 1000 K. The piston is now unlocked so it is free to move, and it conducts heat so the air comes to a uniform temperature $T_A = T_B$. Find the mass in both A and B, and the final T and P.

C.V. A + B Force balance on piston: $P_A A = P_B A$

So the final state in A and B is the same.

State 1A: Table A.7 $u_{A1} = 214.364$ kJ/kg,

$m_A = P_{A1}V_{A1}/RT_{A1} = 200 \times 1/(0.287 \times 300) = \mathbf{2.323\ kg}$

State 1B: Table A.7 $u_{B1} = 759.189$ kJ/kg,

$m_B = P_{B1}V_{B1}/RT_{B1} = 1000 \times 1/(0.287 \times 1000) = \mathbf{3.484\ kg}$

For chosen C.V. ${}_1Q_2 = 0$, ${}_1W_2 = 0$ so the energy equation becomes

$$m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0$$

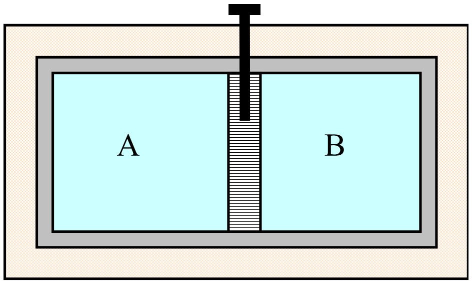
$$(m_A + m_B)u_2 = m_A u_{A1} + m_B u_{B1}$$

$$= 2.323 \times 214.364 + 3.484 \times 759.189 = 3143 \text{ kJ}$$

$$u_2 = 3143/(3.484 + 2.323) = 541.24 \text{ kJ/kg}$$

From interpolation in Table A.7: $\Rightarrow$ $\mathbf{T_2 = 736\ K}$

$$P = (m_A + m_B)RT_2/V_{tot} = 5.807 \text{ kg} \times 0.287 \frac{\text{kJ}}{\text{kg K}} \times 736 \text{ K}/ 2 \text{ m}^3 = \mathbf{613\ kPa}$$



5.112

Find the specific heat transfer for the helium in Problem 4.62

A helium gas is heated at constant volume from a state of 100 kPa, 300 K to 500 K. A following process expands the gas at constant pressure to three times the initial volume. What is the specific work in the combined process?

Solution :

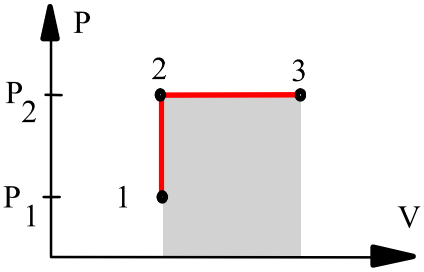
C.V. Helium. This is a control mass.

Energy Eq.5.11: $u_3 - u_1 = {}_1q_3 - {}_1w_3$

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for helium.

State 1: T, P => $v_1 = RT_1/P_1$

State 2: $V_2 = V_1$ => $P_2 = P_1 (T_2/T_1)$

State 3: $P_3 = P_2$ => $V_3 = 3V_2$; $T_3 = T_2\, v_3/v_2 = 500 \times 3 = 1500$ K

We find the work by summing along the process path.

$$ {}_1w_3 = {}_1w_2 + {}_2w_3 = {}_2w_3 = P_3(v_3 - v_2) = R(T_3 - T_2) $$
$$ = 2.0771(1500 - 500) = 2077 \text{ kJ/kg} $$

The heat transfer is from the energy equation

$$ {}_1q_3 = u_3 - u_1 + {}_1w_3 = C_{vo}(T_3 - T_1) + {}_1w_3 $$
$$ = 3.116\,(1500 - 300) + 2077 = \mathbf{5816\ kJ/kg} $$

5.113

A rigid insulated tank is separated into two rooms by a stiff plate. Room A of 0.5 m^3 contains air at 250 kPa, 300 K and room B of 1 m^3 has air at 500 kPa, 1000 K. The plate is removed and the air comes to a uniform state without any heat transfer. Find the final pressure and temperature.

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B$; $V = V_A + V_B = 1.5\ m^3$

Energy Eq.: $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = Q - W = 0$

Process Eq.: $V = \text{constant} \Rightarrow W = 0$; Insulated $\Rightarrow Q = 0$

Ideal gas at 1: $m_A = P_{A1}V_A/RT_{A1} = 250 \times 0.5/(0.287 \times 300) = 1.452$ kg

$u_{A1} = 214.364$ kJ/kg from Table A.7

Ideal gas at 2: $m_B = P_{B1}V_B/RT_{B1} = 500 \times 1/(0.287 \times 1000) = 1.742$ kg

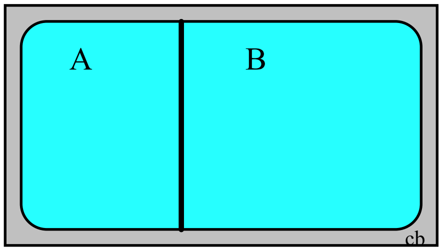
$u_{B1} = 759.189$ kJ/kg from Table A.7

$m_2 = m_A + m_B = 3.194$ kg

$$u_2 = \frac{m_A u_{A1} + m_B u_{B1}}{m_2} = \frac{1.452 \times 214.364 + 1.742 \times 759.189}{3.194} = 511.51 \text{ kJ/kg}$$

=> Table A.7.1: $\mathbf{T_2 = 698.6\ K}$

$P_2 = m_2 RT_2 /V = 3.194 \times 0.287 \times 698.6/1.5 =$ **426.9 kPa**



5.114

A cylinder with a piston restrained by a linear spring contains 2 kg of carbon dioxide at 500 kPa, 400°C. It is cooled to 40°C, at which point the pressure is 300 kPa. Calculate the heat transfer for the process.

Solution:

C.V. The carbon dioxide, which is a control mass.

Continuity Eq.: $m_2 - m_1 = 0$

Energy Eq.: $m\,(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = A + BV$ (linear spring)

$${}_1W_2 = \int P dV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

Equation of state: $PV = mRT$ (ideal gas)

State 1: $V_1 = mRT_1/P_1 = 2 \times 0.18892 \times 673.15 /500 = 0.5087 \text{ m}^3$

State 2: $V_2 = mRT_2/P_2 = 2 \times 0.18892 \times 313.15 /300 = 0.3944 \text{ m}^3$

$${}_1W_2 = \frac{1}{2}(500 + 300)(0.3944 - 0.5087) = -45.72 \text{ kJ}$$

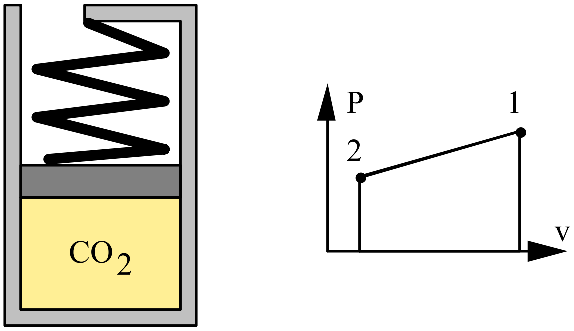
To evaluate $u_2 - u_1$ we will use the specific heat at the average temperature.

From Figure 5.11: $C_{po}(T_{avg}) = 45/44 = 1.023 \Rightarrow C_{vo} = 0.83 = C_{po} - R$

For comparison the value from Table A.5 at 300 K is $C_{vo} = 0.653$ kJ/kg K

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_{vo}(T_2 - T_1) + {}_1W_2$$

$$= 2 \times 0.83(40 - 400) - 45.72 = \mathbf{-643.3\ kJ}$$



Remark: We could also have used the ideal gas table in A.8 to get $u_2 - u_1$.

5.115

A piston/cylinder has 0.5 kg air at 2000 kPa, 1000 K as shown. The cylinder has stops so $V_{min} = 0.03\ m^3$. The air now cools to 400 K by heat transfer to the ambient. Find the final volume and pressure of the air (does it hit the stops?) and the work and heat transfer in the process.

We recognize this is a possible two-step process, one of constant P and one of constant V. This behavior is dictated by the construction of the device.

Continuity Eq.: $m_2 - m_1 = 0$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = \text{constant} = F/A = P_1 \quad \text{if} \quad V > V\text{min}$

$V = \text{constant} = V_{1a} = V_{min} \quad \text{if} \quad P < P_1$

State 1: (P, T) $V_1 = mRT_1/P_1 = 0.5 \times 0.287 \times 1000/2000 = 0.07175\ m^3$

The only possible P-V combinations for this system is shown in the diagram so both state 1 and 2 must be on the two lines. For state 2 we need to know if it is on the horizontal P line segment or the vertical V segment. Let us check state 1a:

State 1a: $P_{1a} = P_1,\ V_{1a} = V_{min}$

$$\text{Ideal gas so } T_{1a} = T_1 \frac{V_{1a}}{V_1} = 1000 \times \frac{0.03}{0.07175} = 418\ K$$

We see that $T_2 < T_{1a}$ and state 2 must have $V_2 = V_{1a} = V_{min} = 0.03\ m^3$.

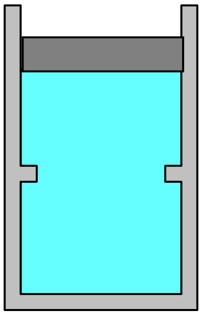
$$P_2 = P_1 \times \frac{T_2}{T_1} \times \frac{V_1}{V_2} = 2000 \times \frac{400}{1000} \times \frac{0.07175}{0.03} = 1913.3\ kPa$$

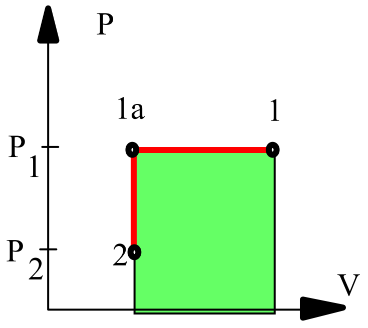
The work is the area under the process curve in the P-V diagram

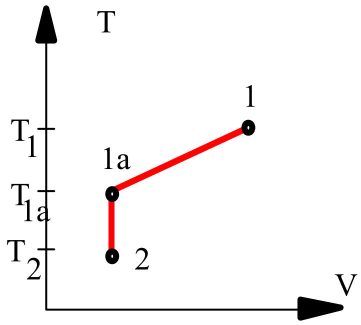
$${}_1W_2 = \int_1^2 P\, dV = P_1 (V_{1a} - V_1) = 2000\ kPa\ (0.03 - 0.07175)\ m^3 = \mathbf{-\ 83.5\ kJ}$$

Now the heat transfer is found from the energy equation, u's from Table A.7.1,

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5\ (286.49 - 759.19) - 83.5 = \mathbf{-319.85\ kJ}$$







5.116

A piston/cylinder contains 1.5 kg of air at 300 K and 150 kPa. It is now heated up in a two-step process. First constant volume to 1000 K (state 2) and then followed by a constant pressure process to 1500 K, state 3. Find the heat transfer for the process.

Solution:

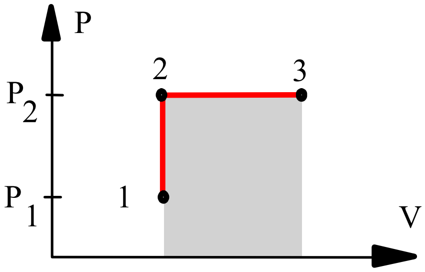
C.V. Helium. This is a control mass.

Energy Eq.5.11: $U_3 - U_1 = {}_1Q_3 - {}_1W_3$

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for air.

State 1: T, P => $V_1 = mRT_1/P_1 = 1.5 \times 0.287 \times 300/150 = 0.861 \text{ m}^3$

State 2: $V_2 = V_1$ => $P_2 = P_1 (T_2/T_1) = 150 \times 1000/300 = 500$ kPa

State 3: $P_3 = P_2$ => $V_3 = V_2 (T_3/T_2) = 0.861 \times 1500/1000 = 1.2915 \text{ m}^3$

We find the work by summing along the process path.

$$ {}_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = P_3(V_3 - V_2) $$

$$ = 500 \text{ kPa } (1.2915 - 0.861) \text{ m}^3 = 215.3 \text{ kJ} $$

The heat transfer is from the energy equation and we will use Table A.7 for u

$$ {}_1Q_3 = U_3 - U_1 + {}_1W_3 = m (u_3 - u_1) + {}_1W_3 $$

$$ = 1.5 (1205.25 - 214.36) + 215.3 = \mathbf{1701.6 \text{ kJ}} $$

Comment: We used Table A.7 due to the large temperature differences.

5.117

Air in a rigid tank is at 100 kPa, 300 K with a volume of 0.75 m^3. The tank is heated 400 K, state 2. Now one side of the tank acts as a piston letting the air expand slowly at constant temperature to state 3 with a volume of 1.5 m^3. Find the pressures at states 2 and 3, Find the total work and total heat transfer.

State 1: $m = P_1V_1 / RT_1 = \frac{100 \times 0.75}{0.287 \times 300} \frac{\text{kPa m}^3}{\text{kJ/kg}} = 0.871 \text{ kg}$

Process 1 to 2: Constant volume heating, $dV = 0 \Rightarrow {}_1W_2 = 0$

$$P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = \mathbf{133.3 \text{ kPa}}$$

Process 2 to 3: Isothermal expansion, $dT = 0 \Rightarrow u_3 = u_2$ and

$$P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = \mathbf{66.67 \text{ kPa}}$$

$$_2W_3 = \int_2^3 P\, dV = P_2V_2 \ln \left(\frac{V_3}{V_2}\right) = 133.3 \times 0.75 \ln(2) = 69.3 \text{ kJ}$$

The overall process:

$$_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = \mathbf{69.3 \text{ kJ}}$$

From the energy equation

$$_1Q_3 = m(u_3 - u_1) + {}_1W_3 = m\, C_v\, (T_3 - T_1) + {}_1W_3$$

$$= 0.871 \times 0.717\, (400 - 300) + 69.3 = \mathbf{131.8 \text{ kJ}}$$

5.118

Water at 100 kPa, 400 K is heated electrically adding 700 kJ/kg in a constant pressure process. Find the final temperature using

a) The water tables B.1 b) The ideal gas tables A.8

c) Constant specific heat from A.5

Solution :

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $P = \text{constant} \quad \Rightarrow \quad {}_1w_2 = P (v_2 - v_1)$

Substitute this into the energy equation to get

$${}_1q_2 = h_2 - h_1$$

Table B.1:

$$h_1 \cong 2675.46 + \frac{126.85 - 99.62}{150 - 99.62} \times (2776.38 - 2675.46) = 2730.0 \text{ kJ/kg}$$

$$h_2 = h_1 + {}_1q_2 = 2730 + 700 = 3430 \text{ kJ/kg}$$

$$T_2 = 400 + (500 - 400) \times \frac{3430 - 3278.11}{3488.09 - 3278.11} = \mathbf{472.3°C}$$

Table A.8:

$$h_2 = h_1 + {}_1q_2 = 742.4 + 700 = 1442.4 \text{ kJ/kg}$$

$$T_2 = 700 + (750 - 700) \times \frac{1442.4 - 1338.56}{1443.43 - 1338.56} = 749.5 \text{ K} = \mathbf{476.3°C}$$

Table A.5

$$h_2 - h_1 \cong C_{po} (T_2 - T_1)$$

$$T_2 = T_1 + {}_1q_2 / C_{po} = 400 + 700 / 1.872 = 773.9\text{K} = \mathbf{500.8°C}$$

5.119

Air in a piston/cylinder at 200 kPa, 600 K, is expanded in a constant-pressure process to twice the initial volume (state 2), shown in Fig. P5.101. The piston is then locked with a pin and heat is transferred to a final temperature of 600 K. Find *P*, *T*, and *h* for states 2 and 3, and find the work and heat transfer in both processes.

Solution:

C.V. Air. Control mass $m_2 = m_3 = m_1$

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$;

Process 1 to 2: P = constant => ${}_1w_2 = \int P\,dv = P_1(v_2 - v_1) = R(T_2 - T_1)$

Ideal gas $Pv = RT \Rightarrow T_2 = T_1 v_2/v_1 = 2T_1 = \mathbf{1200\ K}$

$P_2 = P_1 = 200$ kPa, ${}_1w_2 = RT_1 = \mathbf{172.2\ kJ/kg}$

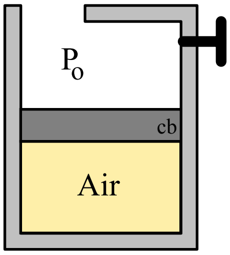
Table A.7 $\mathbf{h_2 = 1277.8\ kJ/kg}$, $\mathbf{h_3 = h_1 = 607.3\ kJ/kg}$

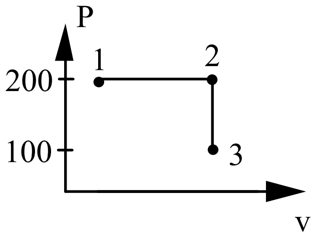
${}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1 = 1277.8 - 607.3 = \mathbf{670.5\ kJ/kg}$

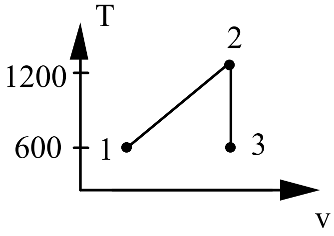
Process 2→3: $v_3 = v_2 = 2v_1 \Rightarrow \mathbf{{}_2w_3 = 0,}$

$P_3 = P_2T_3/T_2 = P_1T_1/2T_1 = P_1/2 = \mathbf{100\ kPa}$

${}_2q_3 = u_3 - u_2 = 435.1 - 933.4 = \mathbf{-498.3\ kJ/kg}$







5.120

A spring loaded piston/cylinder contains 1.5 kg of air at 27C and 160 kPa. It is now heated to 900 K in a process where the pressure is linear in volume to a final volume of twice the initial volume. Plot the process in a P-v diagram and find the work and heat transfer.

Take CV as the air.

$m_2 = m_1 = m$; $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$ => ${}_1W_2 = \int P\,dV$ = area $= 0.5(P_1 + P_2)(V_2 - V_1)$

State 1: Ideal gas. $V_1 = mRT_1/P_1 = 1.5 \times 0.287 \times 300/160 = 0.8072\ m^3$

Table A.7 $u_1 = u(300) = 214.36$ kJ/kg

State 2: $P_2V_2 = mRT_2$ so ratio it to the initial state properties

$P_2V_2 / P_1V_1 = P_2 2 / P_1 = mRT_2 / mRT_1 = T_2 / T_1$ =>

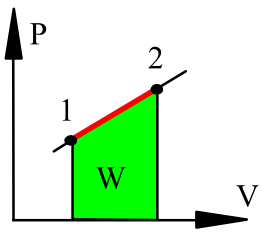
$P_2 = P_1 (T_2 / T_1)(1/2) = 160 \times (900/300) \times (1/2) = 240$ kPa

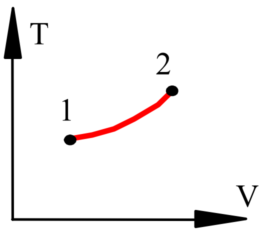
Work is done while piston moves at linearly varying pressure, so we get

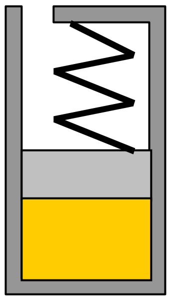
${}_1W_2 = 0.5(P_1 + P_2)(V_2 - V_1) = 0.5 \times (160 + 240)$ kPa $\times\ 0.8072\ m^3$ = **161.4 kJ**

Heat transfer is found from energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.5 \times (674.824 - 214.36) + 161.4$ = **852.1 kJ**







Energy Equation: Polytropic Process

5.121

A helium gas in a piston cylinder is compressed from 100 kPa, 300 K to 200 kPa in a polytropic process with n = 1.5. Find the specific work and specific heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

We can calculate the actual specific work from Eq.4.5 and heat transfer from the energy equation be first finding T_2 as:

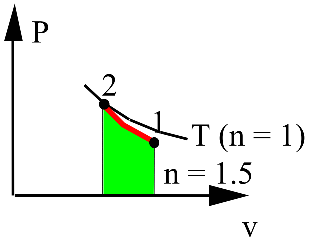
$$\text{Process:} \quad T_2 = T_1\,(P_2/P_1)^{\frac{n-1}{n}} = 300\,(2)^{\frac{0.5}{1.5}} = 377.98 \text{ K}$$

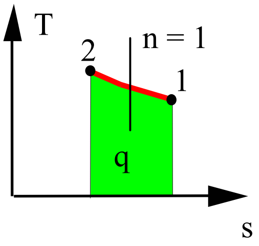
$${}_1w_2 = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{2.0771}{1-1.5}(377.98 - 300) = -323.9 \text{ kJ/kg}$$

$u_2 - u_1 = C_v(T_2 - T_1) = 242.99$ kJ/kg,

Energy Eq.: ${}_1q_2 = u_2 - u_1 + {}_1w_2 = -80.9$ kJ/kg





Helium Table A.5

$k = \gamma = 1.667$ so $n < k$

$C_v = 3.116$ kJ/kgK,

$R = 2.0771$ kJ/kgK

5.122

Oxygen at 300 kPa, 100°C is in a piston/cylinder arrangement with a volume of 0.1 m^3. It is now compressed in a polytropic process with exponent, $n = 1.2$, to a final temperature of 200°C. Calculate the heat transfer for the process.

Solution:

Continuty: $m_2 = m_1$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: T_1 , P_1 & ideal gas, small change in T, so use Table A.5

$$\Rightarrow m = \frac{P_1V_1}{RT_1} = \frac{300 \times 0.1 \text{ m}^3}{0.25983 \times 373.15} = 0.309 \text{ kg}$$

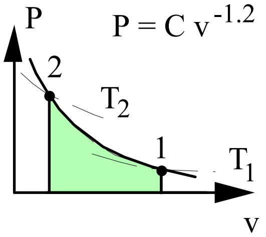
Process: PV^n = constant

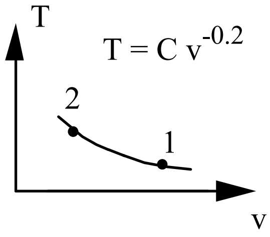
$${}_1W_2 = \frac{1}{1\text{-}n}(P_2V_2 - P_1V_1) = \frac{mR}{1\text{-}n}(T_2 - T_1) = \frac{0.309 \times 0.25983}{1 - 1.2}(200 - 100)$$

$$= -40.2 \text{ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 \cong mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.3094 \times 0.662\ (200 - 100) - 40.2 = \mathbf{-19.7\ kJ}$$





5.123

A piston cylinder contains 0.1 kg air at 300 K and 100 kPa. The air is now slowly compressed in an isothermal (T = C) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution :

Process: T = C & ideal gas $\Rightarrow$ PV = mRT = constant

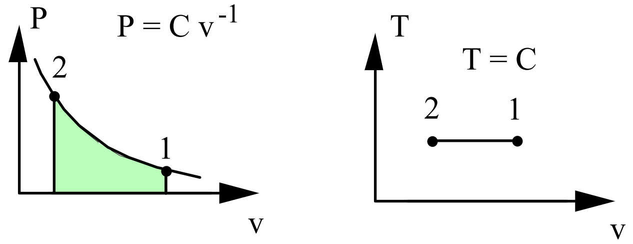
$$_1W_2 = \int P dV = \int \frac{mRT}{V} dV = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

$$= 0.1 \times 0.287 \times 300 \ln (100 / 250) = \mathbf{-7.89\ kJ}$$

since $T_1 = T_2 \Rightarrow \quad u_2 = u_1$

The energy equation thus becomes

$$_1Q_2 = m \times (u_2 - u_1) + {_1W_2} = {_1W_2} = \mathbf{-7.89\ kJ}$$



5.124

A piston cylinder contains 0.1 kg nitrogen at 100 kPa, 27^{o}C and it is now compressed in a polytropic process with n = 1.25 to a pressure of 250 kPa. Find the heat transfer.

Take CV as the nitrogen. $m_2 = m_1 = m$;

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $$T_2 = T_1 \left(P_2 / P_1\right)^{\frac{n-1}{n}} = 300.15 \left(\frac{250}{100}\right)^{\frac{0.25}{1.25}} = 360.5 \text{ K}$$

From process eq.:

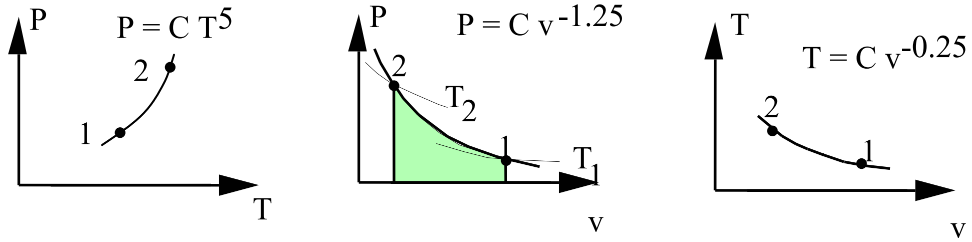
$$_1W_2 = \int P\, dV = \text{area} = \frac{m}{1-n}(P_2v_2 - P_1v_1) = \frac{mR}{1-n}(T_2 - T_1)$$

$$= \frac{0.1 \times 0.2968}{1 - 1.25}(360.5 - 300.15) = -7.165 \text{ kJ}$$

From energy eq.:

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.1 \times 0.745(360.5 - 300.15) - 7.165 = \mathbf{-2.67\ kJ}$$



5.125

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much heat transfer is involved?

Solution:

C.V. Helium gas, this is a control mass.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Ideal gas (A.5): $m = PV/RT = \dfrac{125 \times 0.25}{2.0771 \times 350} = 0.043 \text{ kg}$

Solve for the volume at state 2

$$V_2 = V_1\,(P_1/P_2)^{1/n} = 0.25 \times \left(\frac{125}{100}\right)^{0.6} = 0.2852 \text{ m}^3$$

$$T_2 = T_1\,P_2V_2/(P_1V_1) = 350\,\frac{100 \times 0.2852}{125 \times 0.25} = 319.4 \text{ K}$$

Work from Eq.4.4

$${}_1W_2 = \frac{P_2V_2 - P_1\,V_1}{1\text{-}n} = \frac{100\times 0.2852 - 125\times 0.25}{1 - 1.667}\text{ kPa m}^3 = 4.09 \text{ kJ}$$

Use specific heat from Table A.5 to evaluate $u_2 - u_1$**,** $C_v = 3.116$ kJ/kg K

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m\,C_v\,(T_2 - T_1) + {}_1W_2$$
$$= 0.043 \times 3.116 \times (319.4 - 350) + 4.09 = \mathbf{-0.01\ kJ}$$

5.126

Find the specific heat transfer for problem 4.52

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Process: $Pv^n = \text{Const} = P_1 v_1^n = P_2\ v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3\text{/kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3\text{/kg}$$

From the process equation

$$(P_2/\ P_1) = (v_1/\ v_2)^n \quad \Rightarrow \quad \ln(P_2/\ P_1) = n \ln(v_1/\ v_2)$$

$$n = \ln(P_2/\ P_1) \Big/ \ln(v_1/\ v_2) = \frac{\ln 2.4}{\ln 1.56} = 1.969$$

The work is now from Eq.4.4 per unit mass

$${}_1w_2 = \frac{P_2v_2 - P_1v_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} = \frac{0.287(500 - 325)}{1-1.969} = -51.8 \text{ kJ/kg}$$

The energy equation (per unit mass) gives

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 \cong C_v(T_2 - T_1) + {}_1w_2$$

$$= 0.717\ (500 - 325) - 51.8 = \mathbf{73.67\ kJ/kg}$$

5.127

A piston/cylinder has nitrogen gas at 750 K and 1500 kPa. Now it is expanded in a polytropic process with n = 1.2 to P = 750 kPa. Find the final temperature, the specific work and specific heat transfer in the process.

C.V. Nitrogen. This is a control mass going through a polytropic process.

Continuty: $m_2 = m_1$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = \text{constant}$

Substance ideal gas: $Pv = RT$

$$T_2 = T_1\,(P_2/P_1)^{\frac{n-1}{n}} = 750\left(\frac{750}{1500}\right)^{\frac{0.2}{1.2}} = 750 \times 0.8909 = \mathbf{668\ K}$$

The work is integrated as in Eq.4.4

$${}_1w_2 = \int P dv = \frac{1}{1-n}\,(P_2v_2 - P_1v_1) = \frac{R}{1-n}\,(T_2 - T_1)$$

$$= \frac{0.2968}{1-1.2}(668 - 750) = \mathbf{121.7\ kJ/kg}$$

The energy equation with values of u from Table A.8 is

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 502.8 - 568.45 + 121.7 = \mathbf{56.0\ kJ/kg}$$

If constant specific heat is used from Table A.5

$${}_1q_2 = C_v(T_2 - T_1) + {}_1w_2 = 0.745(668 - 750) + 121.7 = \mathbf{60.6\ kJ/kg}$$

5.128

A gasoline engine has a piston/cylinder with 0.1 kg air at 4 MPa, 1527^{o}C after combustion and this is expanded in a polytropic process with n = 1.5 to a volume 10 times larger. Find the expansion work and heat transfer using Table A.5 heat capacity.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: Pv^n = Constant (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5 \text{ kPa}$

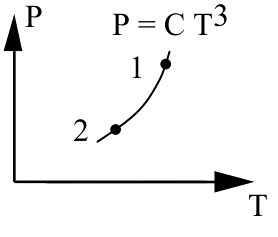
$$T_2 = T_1 \ (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

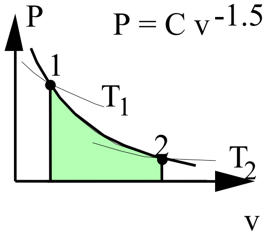
From process eq.: $${}_1W_2 = \int P\, dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$$

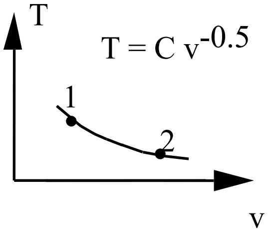
$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = \mathbf{70.64 \ kJ}$$

From energy eq.: $${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.1 \times 0.717(569.3 - 1800) + 70.64 = \mathbf{-17.6 \ kJ}$$







5.129

Solve the previous problem using Table A.7

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $$P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5 \text{ kPa}$$

$$T_2 = T_1 \ (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

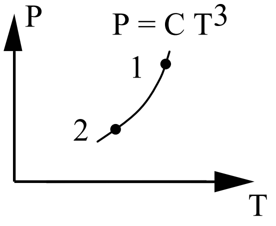
From process eq.: $${}_1W_2 = \int P\, dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$$

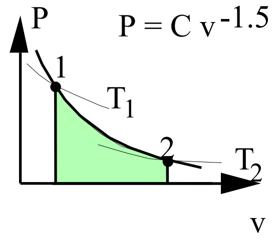
$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = \mathbf{70.64\ kJ}$$

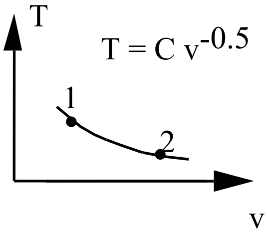
From energy eq.: $${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 0.1\ (411.78 - 1486.33) + 70.64 = \mathbf{-36.8\ kJ}$$

The only place where Table A.7 comes in is for values of u_1 and u_2







5.130

A piston/cylinder arrangement of initial volume 0.025 m^3 contains saturated water vapor at 180°C. The steam now expands in a polytropic process with exponent $n = 1$ to a final pressure of 200 kPa, while it does work against the piston. Determine the heat transfer in this process.

Solution:

C.V. Water. This is a control mass.

State 1: Table B.1.1 $P = 1002.2$ kPa, $v_1 = 0.19405\ m^3/kg$, $u_1 = 2583.7$ kJ/kg ,

$$m = V/v_1 = 0.025/0.19405 = 0.129 \text{ kg}$$

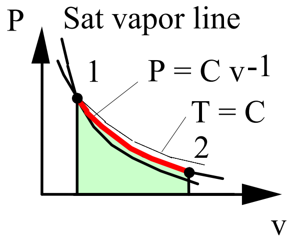
Process: $Pv = \text{const.} = P_1v_1 = P_2v_2$; polytropic process $n = 1$.

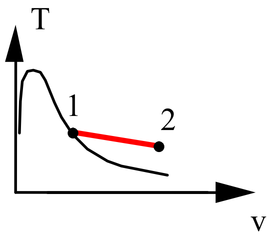
$$\Rightarrow v_2 = v_1P_1/P_2 = 0.19405 \times 1002.1/200 = 0.9723\ m^3/kg$$

State 2: $P_2, v_2 \Rightarrow$ Table B.1.3 $T_2 \cong 155°C$, $u_2 = 2585$ kJ/kg

$$_1W_2 = \int PdV = P_1V_1 \ln\frac{v_2}{v_1} = 1002.2 \text{ kPa} \times 0.025\ m^3 \ln\frac{0.9723}{0.19405} = 40.37 \text{ kJ}$$

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 0.129(2585 - 2583.7) + 40.37 = \mathbf{40.54\ kJ}$$





Notice T drops, it is not an ideal gas.

5.131

A piston/cylinder in a car contains 0.2 L of air at 90 kPa, 20°C, shown in Fig. P5.131. The air is compressed in a quasi-equilibrium polytropic process with polytropic exponent $n = 1.25$ to a final volume six times smaller. Determine the final pressure, temperature, and the heat transfer for the process.

Solution:

C.V. Air. This is a control mass going through a polytropic process.

Continuty: $m_2 = m_1$

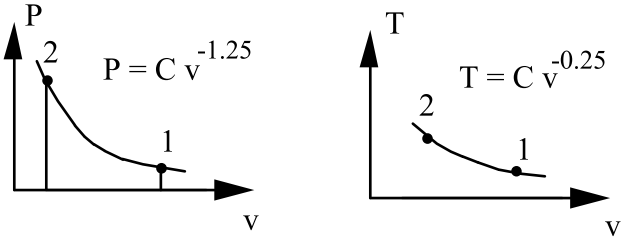
Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = \text{const.}$

$$P_1v_1^n = P_2v_2^n \Rightarrow P_2 = P_1(v_1/v_2)^n = 90 \times 6^{1.25} = \mathbf{845.15\ kPa}$$

Substance ideal gas: $Pv = RT$

$$T_2 = T_1(P_2v_2/P_1v_1) = 293.15(845.15/90 \times 6) = \mathbf{458.8\ K}$$



$$m = \frac{PV}{RT} = \frac{90 \times 0.2\times10^{-3}}{0.287 \times 293.15} = 2.14\times10^{-4}\ \text{kg}$$

The work is integrated as in Eq.4.4

$${}_1w_2 = \int Pdv = \frac{1}{1 - n}(P_2v_2 - P_1v_1) = \frac{R}{1 - n}(T_2 - T_1)$$

$$= \frac{0.287}{1 - 1.25}(458.8 - 293.15) = -190.17\ \text{kJ/kg}$$

The energy equation with values of u from Table A.7 is

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 329.4 - 208.03 - 190.17 = -68.8\ \text{kJ/kg}$$

$${}_1Q_2 = m\ {}_1q_2 = \mathbf{-0.0147\ kJ} \quad \text{(i.e a heat loss)}$$

5.132

A piston/cylinder has 1 kg propane gas at 700 kPa, 40°C. The piston cross-sectional area is 0.5 m^2, and the total external force restraining the piston is directly proportional to the cylinder volume squared. Heat is transferred to the propane until its temperature reaches 700°C. Determine the final pressure inside the cylinder, the work done by the propane, and the heat transfer during the process.

Solution:

C.V. The 1 kg of propane.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = P_{ext} = CV^2 \Rightarrow \quad PV^{-2} = \text{constant}$, polytropic $n = -2$

Ideal gas: PV = mRT, and process yields

$$P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 700 \left(\frac{700+273.15}{40+273.15}\right)^{2/3} = \mathbf{1490.7\ kPa}$$

The work is integrated as Eq.4.4

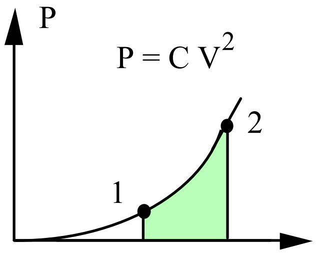
$${}_1W_2 = \int_1^2 PdV = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{mR(T_2 - T_1)}{1 - n}$$

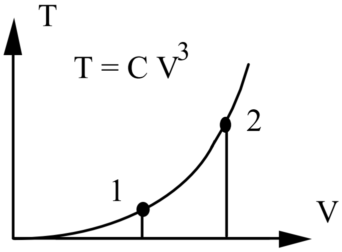
$$= \frac{1\times 0.18855 \times (700-40)}{1-(-2)} = \mathbf{41.48\ kJ}$$

The energy equation with specific heat from Table A.5 becomes

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$$

$$= 1 \times 1.490 \times (700 - 40) + 41.48 = \mathbf{1024.9\ kJ}$$





5.133

A piston cylinder contains pure oxygen at ambient conditions 20°C, 100 kPa. The piston is moved to a volume that is 7 times smaller than the initial volume in a polytropic process with exponent n = 1.25. Use constant heat capacity to find the final pressure and temperature, the specific work and the specific heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process Eq: $Pv^n = C$; $P_2 = P_1\,(v_1/v_2)^n = 100\,(7)^{1.25} = 1138.6\text{ kPa}$

From the ideal gas law and state 2 (P, v) we get

$$T_2 = T_1(\,P_2/P_1)(v_1/v_2) = 293 \times \frac{1138.6}{100} \times (1/7) = 476.8\text{ K}$$

We could also combine process eq. and gas law to give: $T_2 = T_1\,(v_1/v_2)^{n-1}$

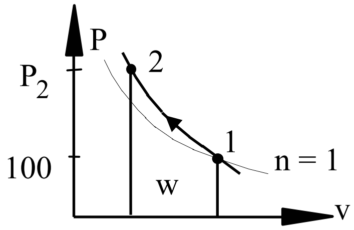
Polytropic work Eq. 4.5: ${}_1w_2 = \frac{1}{1\text{-n}}\,(P_2v_2 - P_1v_1) = \frac{R}{1\text{-n}}\,(T_2 - T_1)$

$${}_1w_2 = \frac{0.2598}{1 - 1.25}\,\frac{\text{kJ}}{\text{kg K}} \times (476.8 - 293.2)\text{ K} = \mathbf{-190.88\ kJ/kg}$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v\,(T_2 - T_1) + {}_1w_2$$

$$= 0.662\,(476.8 - 293.2) - 190.88 = \mathbf{-69.3\ kJ/kg}$$

The actual process is on a steeper curve than n = 1.



5.134

An air pistol contains compressed air in a small cylinder, shown in Fig. P5.134. Assume that the volume is 1 cm^3, pressure is 1 MPa, and the temperature is 27°C when armed. A bullet, $m = 15$ g, acts as a piston initially held by a pin (trigger); when released, the air expands in an isothermal process (T = constant). If the air pressure is 0.1 MPa in the cylinder as the bullet leaves the gun, find

a. The final volume and the mass of air.

b. The work done by the air and work done on the atmosphere.

c. The work to the bullet and the bullet exit velocity.

Solution:

C.V. Air.

Air ideal gas: $m_{air} = P_1V_1/RT_1 = 1000 \times 10^{-6}/(0.287 \times 300) = \mathbf{1.17{\times}10^{-5}\ kg}$

Process: $PV = \text{const} = P_1V_1 = P_2V_2 \ \Rightarrow\ V_2 = V_1P_1/P_2 = \mathbf{10\ cm^3}$

$$_1W_2 = \int PdV = \int \frac{P_1V_1}{V}\, dV = P_1V_1 \ln (V_2/V_1) = \mathbf{2.303\ J}$$

$$_1W_{2,ATM} = P_0(V_2 - V_1) = 101 \times (10 - 1) \times 10^{-6} \text{ kJ} = \mathbf{0.909\ J}$$

$$W_{bullet} = {}_1W_2 - {}_1W_{2,ATM} = 1.394 \text{ J} = \frac{1}{2}\, m_{bullet}(V_{exit})^2$$

$$V_{exit} = (2W_{bullet}/m_B)^{1/2} = (2 \times 1.394/0.015)^{1/2} = \mathbf{13.63\ m/s}$$

5.135

Calculate the heat transfer for the process described in Problem 4.58.

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass

Continuity: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^{1.5} = \text{constant}$. Polytropic process with $n = 1.5$

1: (T, x) P = Psat = 201.7 kPa from Table B.5.1

$v_1 = 0.09921\ \text{m}^3/\text{kg}$, $u_1 = 372.27\ \text{kJ/kg}$

2: (P, process)

$$v_2 = v_1\ (P_1/P_2)^{(1/1.5)} = 0.09921\times\ (201.7/500)^{0.667} = 0.05416\ \text{m}^3/\text{kg}$$

=> Table B.5.2 superheated vapor, $T_2 = 79°\text{C}$, $V_2 = mv_2 = 0.027\ \text{m}^3$

$u_2 = 440.9\ \text{kJ/kg}$

Process gives $P = C\ v^{(-1.5)}$, which is integrated for the work term, Eq.4.4

$${}_1W_2 = \int P\ dV = m(P_2v_2 - P_1v_1)/(1-1.5)$$

$$= -2\times0.5\times\ (500\times0.05416 - 201.7\times0.09921) = -7.07\ \text{kJ}$$

$${}_1Q_2 = m(u_2 - u_1)\ +\ {}_1W_2 = 0.5(440.9 - 372.27) + (-7.07) = \mathbf{27.25\ kJ}$$

Energy Equation in Rate Form

5.136

A crane use 2 kW to raise a 100 kg box 20 m. How much time does it take?

$$\text{Power} = \dot{W} = F\mathbf{V} = mg\mathbf{V} = mg\frac{L}{t}$$

$$t = \frac{mgL}{\dot{W}} = \frac{100\text{ kg } 9.807\text{ m/s}^2\ 20\text{ m}}{2000\text{ W}} = \mathbf{9.81\ s}$$

5.137

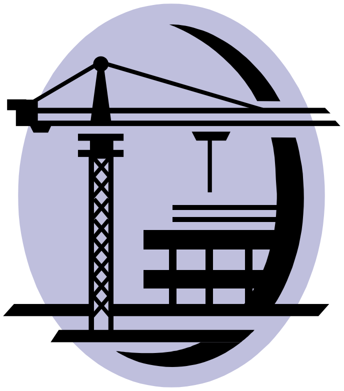
A crane lifts a load of 450 kg vertically up with a power input of 1 kW. How fast can the crane lift the load?

Solution :

Power is force times rate of displacement

$$\dot{W} = F \cdot \mathbf{V} = mg \cdot \mathbf{V}$$

$$\mathbf{V} = \frac{\dot{W}}{mg} = \frac{1000}{450 \times 9.806} \frac{W}{N} = \mathbf{0.227\ m/s}$$



5.138

A pot of 1.2 kg water at 20$^{\text{o}}$C is put on a stove supplying 1250 W to the water. What is the rate of temperature increase (K/s)?

Energy Equation on a rate form: $\frac{dE_{water}}{dt} = \frac{dU_{water}}{dt} = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V}$

$$\dot{Q} = \frac{dU_{water}}{dt} + P\dot{V} = \frac{dH_{water}}{dt} = m_{water}C_p \frac{dT_{water}}{dt}$$

$$\frac{dT_{water}}{dt} = \dot{Q} \,/\, m_{water}C_p = 1.250 \,/\, (1.2 \times 4.18\,) = \mathbf{0.2492\ K/s}$$

5.139

The rate of heat transfer to the surroundings from a person at rest is about 400 kJ/h. Suppose that the ventilation system fails in an auditorium containing 100 people. Assume the energy goes into the air of volume 1500 m^3 initially at 300 K and 101 kPa. Find the rate (degrees per minute) of the air temperature change.

Solution:

$$\dot{Q} = n\,\dot{q} = 100 \times 400 = \mathbf{40\,000\ kJ/h} = \mathbf{666.7\ kJ/min}$$

$$\frac{dE_{air}}{dt} = \dot{Q} = m_{air}C_v \frac{dT_{air}}{dt}$$

$$m_{air} = PV/RT = 101 \times 1500 / 0.287 \times 300 = 1759.6 \text{ kg}$$

$$\frac{dT_{air}}{dt} = \dot{Q}\,/mC_v = 666.7 / (1759.6 \times 0.717) = \mathbf{0.53°C/min}$$

5.140

A pot of water is boiling on a stove supplying 325 W to the water. What is the rate of mass (kg/s) vaporizing assuming a constant pressure process?

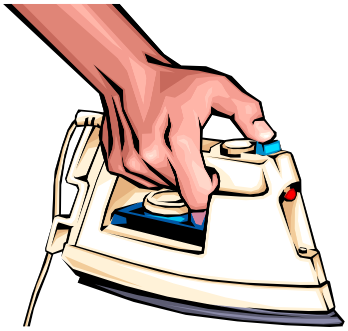
To answer this we must assume all the power goes into the water and that the process takes place at atmospheric pressure 101 kPa, so T = 100°C.

Energy equation

$$dQ = dE + dW = dU + PdV = dH = h_{fg}\, dm$$

$$\frac{dQ}{dt} = h_{fg} \frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{\dot{Q}}{h_{fg}} = \frac{325 \text{ W}}{2257 \text{ kJ/kg}} = \mathbf{0.144\ g/s}$$



The volume rate of increase is

$$\frac{dV}{dt} = \frac{dm}{dt} v_{fg} = 0.144 \text{ g/s} \times 1.67185 \text{ m}^3\text{/kg}$$

$$= 0.24 \times 10^{-3} \text{ m}^3\text{/s} = 0.24 \text{ L/s}$$

5.141

A pot of 1.2 kg water at $20^{o}C$ is put on a stove supplying 1250 W to the water. After how long time can I expect it to come to a boil ($100^{o}C$)?

Energy Equation on a rate form: $\frac{dE_{water}}{dt} = \frac{dU_{water}}{dt} = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V}$

$$\dot{Q} = \frac{dU_{water}}{dt} + P\dot{V} = \frac{dH_{water}}{dt} = m_{water}C_p \frac{dT_{water}}{dt}$$

Integrate over time

$$Q = \dot{Q}\,\Delta t = \Delta H = m_{water}\,(h_2 - h_1) \approx m_{water}C_p\,(T_2 - T_1)$$

$$\Delta t = m_{water}\,(h_2 - h_1)/\dot{Q} \approx m_{water}C_p\,(T_2 - T_1)/\dot{Q}$$

$$= 1.2\,(419.02 - 83.94)/1.25 \;\approx\; 1.2 \times 4.18\,(100 - 20)/1.25$$

$$= \mathbf{321.7\ s} \;\approx\; 5.5\text{ min}$$

Comment: Notice how close the two results are, i.e. use of constant C_p is OK.

5.142

A mass of 3 kg nitrogen gas at 2000 K, V = C, cools with 500 W. What is dT/dt?

Process: $V = C \rightarrow {}_1W_2 = 0$

$$\frac{dE}{dt} = \frac{dU}{dt} = m\frac{dU}{dt} = mC_v\frac{dT}{dt} = \dot{Q} - W = \dot{Q} = -500\text{ W}$$

$$C_{v\,2000} = \frac{du}{dT} = \frac{\Delta u}{\Delta T} = \frac{u_{2100} - u_{1900}}{2100\text{-}1900} = \frac{1819.08 - 1621.66}{200} = 0.987\text{ kJ/kg K}$$

$$\frac{dT}{dt} = \frac{\dot{Q}}{mC_V} = \frac{-500\text{ W}}{3 \times 0.987\text{ kJ/K}} = \mathbf{-0.17}\,\frac{\mathbf{K}}{\mathbf{s}}$$

Remark: Specific heat from Table A.5 has $C_{v\,300} = 0.745$ kJ/kg K which is nearly 25% lower and thus would over-estimate the rate with 25%.

5.143

A computer in a closed room of volume 200 m^3 dissipates energy at a rate of 10 kW. The room has 50 kg wood, 25 kg steel and air, with all material at 300 K, 100 kPa. Assuming all the mass heats up uniformly, how long will it take to increase the temperature 10°C?

Solution:

C.V. Air, wood and steel. $m_2 = m_1$; no work

Energy Eq.5.11: $U_2 - U_1 = {}_1Q_2 = \dot{Q}\Delta t$

The total volume is nearly all air, but we can find volume of the solids.

$V_{wood} = m/\rho = 50/510 = 0.098\ m^3$; $V_{steel} = 25/7820 = 0.003\ m^3$

$V_{air} = 200 - 0.098 - 0.003 = 199.899\ m^3$

$$m_{air} = PV/RT = 101.325 \times 199.899/(0.287 \times 300) = 235.25\ kg$$

We do not have a u table for steel or wood so use heat capacity from A.3.

$$\Delta U = [m_{air}\, C_v + m_{wood}\, C_v + m_{steel}\, C_v\,]\Delta T$$
$$= (235.25 \times 0.717 + 50 \times 1.38 + 25 \times 0.46)\ 10$$
$$= 1686.7 + 690 + 115 = 2492\ kJ = \dot{Q} \times \Delta t = 10\ kW \times \Delta t$$

=> $\Delta t = 2492/10 =$ **249.2 sec = 4.2 minutes**

5.144

A drag force on a car, with frontal area A = 2 m^2, driving at 80 km/h in air at 20^{o}C is $F_d = 0.225\ A\ \rho_{air}\mathbf{V}^2$. How much power is needed and what is the traction force?

$$\dot{W} = F\mathbf{V}$$

$$\mathbf{V} = 80\ \frac{km}{h} = 80 \times \frac{1000}{3600}\ ms^{-1} = 22.22\ ms^{-1}$$

$$\rho_{AIR} = \frac{P}{RT} = \frac{101}{0.287 \times 293} = 1.20\ kg/m^3$$

$$F_d = 0.225\ A\rho\mathbf{V}^2 = 0.225 \times 2 \times 1.2 \times 22.22^2 = \mathbf{266.61\ N}$$

$$\dot{W} = F\mathbf{V} = 266.61\ N \times 22.22\ m/s = 5924\ W = \mathbf{5.92\ kW}$$

5.145

A piston/cylinder of cross sectional area 0.01 m^2 maintains constant pressure. It contains 1 kg water with a quality of 5% at 150^{o}C. If we heat so 1 g/s liquid turns into vapor what is the rate of heat transfer needed?

Solution:

Control volume the water.

Continuity Eq.: $m_{tot} = \text{constant} = m_{vapor} + m_{liq}$

on a rate form: $\dot{m}_{tot} = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$

Energy Eq.: $\frac{dE}{dt} = \frac{d}{dt}(m_{vapor}u_g + m_{liq}u_f) = \dot{m}_{vapor}(u_g - u_f) = \dot{m}_{vapor}u_{fg} = \dot{Q} - \dot{W}$

$V_{vapor} = m_{vapor}\, v_g\,, \quad V_{liq} = m_{liq}\, v_f\,; \qquad V_{tot} = V_{vapor} + V_{liq}$

$\dot{V}_{tot} = \dot{V}_{vapor} + \dot{V}_{liq} = \dot{m}_{vapor}v_g + \dot{m}_{liq}v_f = \dot{m}_{vapor}\,(v_g - v_f) = \dot{m}_{vapor}\, v_{fg}$

$\dot{W} = P\dot{V} = P\, \dot{m}_{vapor}\, v_{fg}$

Substitute the rate of work into the energy equation and solve for the heat transfer

$\dot{Q} = \dot{m}_{vapor}u_{fg} + \dot{W} = \dot{m}_{vapor}u_{fg} + P\, \dot{m}_{vapor}\, v_{fg} = \dot{m}_{vapor}\, h_{fg}$

$= 0.001 \times 2114.26 =$ **2.114 kW**

5.146

A small elevator is being designed for a construction site. It is expected to carry four 75-kg workers to the top of a 100-m tall building in less than 2 min. The elevator cage will have a counterweight to balance its mass. What is the smallest size (power) electric motor that can drive this unit?

Solution:

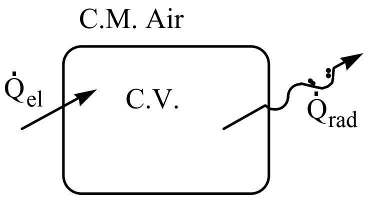
$m = 4 \times 75 = 300$ kg ; $\Delta Z = 100$ m ; $\Delta t = 2$ minutes

$$-\dot{W} = \Delta\dot{PE} = mg\,\frac{\Delta Z}{\Delta t} = \frac{300 \times 9.807 \times 100}{1000 \times 2 \times 60} = \mathbf{2.45\ kW}$$

5.147

The heaters in a spacecraft suddenly fail. Heat is lost by radiation at the rate of 100 kJ/h, and the electric instruments generate 75 kJ/h. Initially, the air is at 100 kPa, 25°C with a volume of 10 m^3. How long will it take to reach an air temperature of –20°C?

Solution:



Continuity Eq: $\frac{dM}{dt} = 0$

Energy Eq: $\frac{dE}{dt} = \dot{Q}_{el} - \dot{Q}_{rad}$

$\dot{W} = 0$
$\dot{KE} = 0$
$\dot{PE} = 0$

$$\dot{E} = \dot{U} = \dot{Q}_{el} - \dot{Q}_{rad} = \dot{Q}_{net} \;\Rightarrow\; U_2 - U_1 = m(u_2 - u_1) = \dot{Q}_{net}(t_2 - t_1)$$

$$\text{Ideal gas: } m = \frac{P_1V_1}{RT_1} = \frac{100 \times 10}{0.287 \times 298.15} = 11.688 \text{ kg}$$

$$m(u_2 - u_1) = m\,C_{v0}(T_2 - T_1) = 11.688 \times 0.717\,(-20 - 25) = -377.1 \text{ kJ}$$

$$t_2 - t_1 = m\,(u_2 - u_1)/\dot{Q}_{net} = (-377.1 \text{ kJ})\,/(-25 \text{ kJ/h}) = \mathbf{15.08\ h}$$

5.148

A steam generating unit heats saturated liquid water at constant pressure of 800 kPa in a piston cylinder. If 1.5 kW of power is added by heat transfer find the rate (kg/s) of saturated vapor that is made.

Solution:

Energy equation on a rate form making saturated vapor from saturated liquid

$$\dot{U} = (\dot{m}u) = \dot{m}\Delta u = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V} = \dot{Q} – P\ \dot{m}\Delta v$$

Rearrange to solve for heat transfer rate

$$\dot{Q} = \dot{m}(\Delta u + \Delta vP) = \dot{m}\ \Delta h = \dot{m}\ h_{fg}$$

So now

$$\dot{m} = \dot{Q}/\ h_{fg} = 1500\ /\ 2048.04 = \mathbf{0.732\ kg/s}$$

5.149

As fresh poured concrete hardens, the chemical transformation releases energy at a rate of 2 W/kg. Assume the center of a poured layer does not have any heat loss and that it has an average heat capacity of 0.9 kJ/kg K. Find the temperature rise during 1 hour of the hardening (curing) process.

Solution:

$\dot{U} = (m\dot{u}) = mC_V\dot{T} = \dot{Q} = m\dot{q}$

$\dot{T} = \dot{q}/C_V = 2\times10^{-3} / 0.9$

$= 2.222 \times 10^{-3}$ °C/sec

$\Delta T = \dot{T}\Delta t = 2.222 \times 10^{-3} \times 3600 = \mathbf{8\ °C}$

5.150

Water is in a piston cylinder maintaining constant P at 700 kPa, quality 90% with a volume of 0.1 m^3. A heater is turned on heating the water with 2.5 kW. What is the rate of mass (kg/s) vaporizing?

Solution:

Control volume water.

Continuity Eq.: $m_{tot} = \text{constant} = m_{vapor} + m_{liq}$

on a rate form: $\dot{m}_{tot} = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$

Energy equation: $\dot{U} = \dot{Q} - \dot{W} = \dot{m}_{vapor}\, u_{fg} = \dot{Q} - P\, \dot{m}_{vapor}\, v_{fg}$

Rearrange to solve for $\dot{m}_{vapor}$

$$\dot{m}_{vapor}\,(u_{fg} + Pv_{fg}) = \dot{m}_{vapor}\, h_{fg} = \dot{Q}$$

$$\dot{m}_{vapor} = \dot{Q}/h_{fg} = \frac{2.5}{2066.3}\,\frac{\text{kW}}{\text{kJ/kg}} = \mathbf{0.0012\ kg/s}$$

5.151

A 500 Watt heater is used to melt 2 kg of solid ice at -10°C to liquid at $+5^{\circ}$C at a constant pressure of 150 kPa.

a) Find the change in the total volume of the water.

b) Find the energy the heater must provide to the water.

c) Find the time the process will take assuming uniform T in the water.

Solution:

Take CV as the 2 kg of water. $m_2 = m_1 = m$;

Energy Eq.5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: Compressed solid, take saturated solid at same temperature.

$v = v_i(-10) = 0.0010891\ \text{m}^3/\text{kg}, \ \ h = h_i = -354.09\ \text{kJ/kg}$

State 2: Compressed liquid, take saturated liquid at same temperature

$v = v_f = 0.001, \ \ h = h_f = 20.98\ \text{kJ/kg}$

Change in volume:

$$V_2 - V_1 = m(v_2 - v_1) = 2(0.001 - 0.0010891) = \mathbf{0.000178\ m^3}$$

Work is done while piston moves at constant pressure, so we get

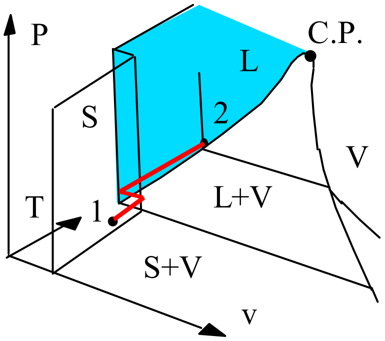
$${}_1W_2 = \int P\, dV = \text{area} = P(V_2 - V_1) = -150 \times 0.000178 = -0.027\ \text{kJ} = -27\ \text{J}$$

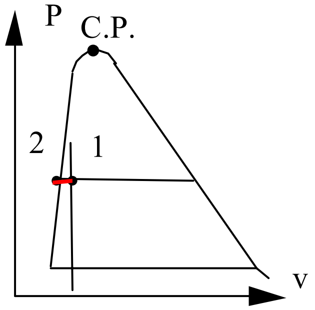
Heat transfer is found from energy equation

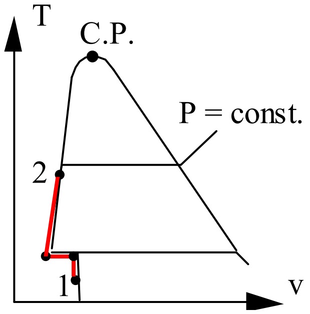
$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2 \times [20.98 - (-354.09)] = \mathbf{750\ kJ}$$

The elapsed time is found from the heat transfer and the rate of heat transfer

$$t = {}_1Q_2/\dot{Q} = (750\ \text{kJ}\ /500\ \text{W}) \times 1000\ \text{J/kJ} = 1500\ \text{s} = \mathbf{25\ min}$$







Problem Analysis (no numbers required)

5.152

Consider Problem 5.57 with the steel bottle as C.V. Write the process equation that is valid until the valve opens and plot the P-v diagram for the process.

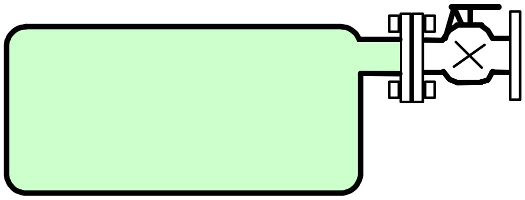
Process: constant volume process

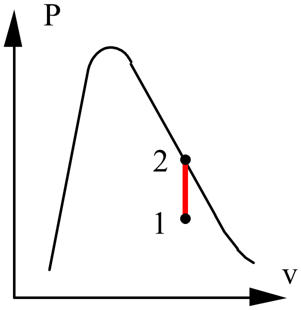
constant mass

$V = mv = C \quad \Rightarrow \quad v_2 = v_1$

${}_1W_2 = \int P \, dV = 0$

State 1: (T, x) so two-phase in Table B.3.1





5.153

Consider Problem 5.50. Take the whole room as a C.V. and write both conservation of mass and energy equations. Write some equations for the process (two are needed) and use those in the conservation equations. Now specify the four properties that determines initial (2) and final state (2), do you have them all? Count unknowns and match with equations to determine those.

C.V.: Containment room and reactor.

Mass: $m_2 - m_1 = 0$; $m_2 = m_1 = V_{reactor}/v_1$

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Room volume constant $V = C$ ⇨ ${}_1W_2 = 0$

Room insulated ⇨ ${}_1Q_2 = 0$

Using these in the equation for mass and energy gives:

$m_2 = V_2/v_2 = m_1$; $m(u_2 – u_1) = 0 – 0 = 0$

State 1: P_1, T_1 so Table B.1.4 gives v_1, u_1 ⇨ m_1

State 2: P_2, ?

We do not know **one** state 2 property and the total room volume

Energy equation then gives $u_2 = u_1$ (a state 2 property)

State 2: P_2, u_2 ⇨ v_2

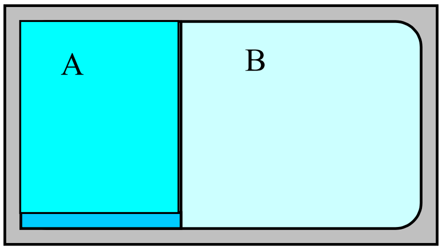
Now we have the room volume as

Continuity Eq.: $m_2 = V/v_2 = m_1$ so $V = m_1 v_2$

5.154

Take problem 5.61 and write the left hand side (storage change) of the conservation equations for mass and energy. How do you write m_1 and Eq. 5.5?

C.V.: Both rooms A and B in tank.



Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$;

Energy Eq.: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

Notice how the state 1 term split into two terms

$$m_1 = \int \rho\, dV = \int (1/v)\, dV = V_A/v_{A1} + V_B/v_{B1} = m_{A1} + m_{B1}$$

and for energy as

$$m_1u_1 = \int \rho u\, dV = \int (u/v)\, dV = (u_{A1}/v_{A1})V_A + (u_{B1}/v_{B1})V_B$$
$$= m_{A1}u_{A1} + m_{B1}u_{B1}$$

Formulation continues as:

Process constant total volume: $V_{tot} = V_A + V_B$ and ${}_1W_2 = 0$

$m_2 = m_{A1} + m_{B1}$ => $v_2 = V_{tot}/m_2$

etc.

5.155

Consider Problem 5.70 with the final state given but that you were not told the piston hits the stops and only told $V_{stop} = 2\ V_1$. Sketch the possible P-v diagram for the process and determine which number(s) you need to uniquely place state 2 in the diagram. There is a kink in the process curve what are the coordinates for that state? Write an expression for the work term.

C.V. R-410a. Control mass goes through process: 1 -> 2 -> 3

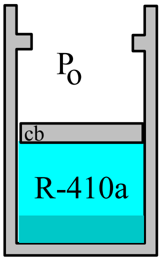
As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3). So we have: $v_3 = v_2 = 2 \times v_1$

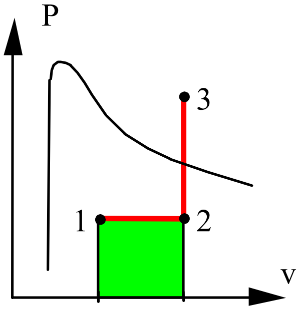
State 2: $V_2 = V_{stop} \Rightarrow v_2 = 2 \times v_1 = v_3$ and $P_2 = P_1 \Rightarrow T_2 = \dots$

State 3: Table B.4.2 (P, T) Compare $P_3 > P_2$ and $T_3 > T_2$

$$v_3 = 0.02015\ \text{m}^3/\text{kg},\quad u_3 = 248.4\ \text{kJ/kg}$$

$$W = \int P\, dV = P(V_2 - V_1) = Pm\,(v_2 - v_1)$$





5.156

Look at problem 5.115 and plot the P-v diagram for the process. Only T_2 is given, how do you determine the 2^{nd} property of the final state? What do you need to check and does it have an influence on the work term?

Process: $P = \text{constant} = F/A = P_1 \quad \text{if} \quad V > V\text{min}$

$V = \text{constant} = V_{1a} = V_{min} \quad \text{if} \quad P < P_1$

State 1: (P, T) $V_1 = mRT_1/P_1 = 0.5 \times 0.287 \times 1000/2000 = 0.07175 \text{ m}^3$

The only possible P-V combinations for this system are shown in the diagram so both state 1 and 2 must be on the two lines. For state 2 we need to know if it is on the horizontal P line segment or the vertical V segment. Let us check state 1a:

State 1a: $P_{1a} = P_1,\ V_{1a} = V_{min}$, Ideal gas so $T_{1a} = T_1 \dfrac{V_{1a}}{V_1}$

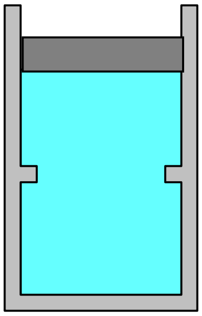
We see if $T_2 < T_{1a}$ then state 2 must have $V_2 = V_{1a} = V_{min} = 0.03 \text{ m}^3$. So state 2 is known by (T_2, v_2) and $P_2 = P_1 \times \dfrac{T_2}{T_1} \times \dfrac{V_1}{V_2}$

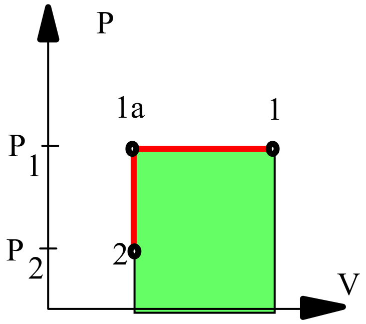
If it was that $T_2 > T_{1a}$ then we know state 2 as: $T_2, P_2 = P_1$ and we then have

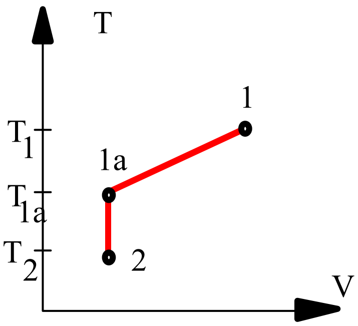
$$V_2 = V_1 \times \frac{T_2}{T_1}$$

The work is the area under the process curve in the P-V diagram and so it does make a difference where state 2 is relative to state 1a. For the part of the process that proceeds along the constant volume V_{min} the work is zero there is only work when the volume changes.

$$_1W_2 = \int_1^2 P\, dV = P_1 (V_{1a} - V_1)$$







Review

5.157

Ten kilograms of water in a piston/cylinder setup with constant pressure is at 450°C and a volume of 0.633 m^3. It is now cooled to 20°C. Show the *P–v* diagram and find the work and heat transfer for the process.

Solution:

C.V. The 10 kg water.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

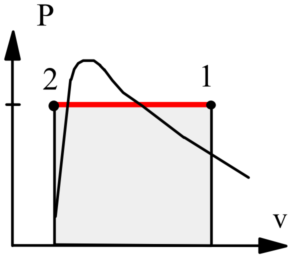
Process: $P = C \qquad \Rightarrow \qquad {}_1W_2 = mP(v_2 - v_1)$

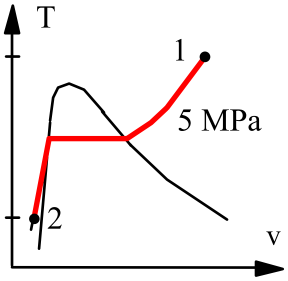
State 1: $(T, v_1 = 0.633/10 = 0.0633\ m^3/kg)$ Table B.1.3

$P_1 = 5\ MPa, \quad h_1 = 3316.2\ kJ/kg$

State 2: $(P = P = 5\ MPa, 20°C) \Rightarrow$ Table B.1.4

$v_2 = 0.000\ 999\ 5\ m^3/kg\ ; \qquad h_2 = 88.65\ kJ/kg$





The work from the process equation is found as

$${}_1W_2 = 10 \times 5000 \times (0.0009995 - 0.0633) = \mathbf{-3115\ kJ}$$

The heat transfer from the energy equation is

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

$${}_1Q_2 = 10 \times (88.65 - 3316.2) = \mathbf{-32276\ kJ}$$

5.158

Ammonia, NH_3, is contained in a sealed rigid tank at 0°C, $x = 50\%$ and is then heated to 100°C. Find the final state P_2, u_2 and the specific work and heat transfer.

Solution:

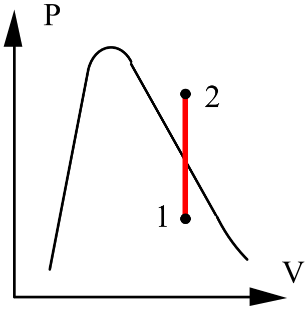
Continuity Eq.: $m_2 = m_1$;

Energy Eq.5.11: $E_2 - E_1 = {}_1Q_2$; $({}_1W_2 = 0)$

Process: $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.001566 + 0.5 \times 0.28783 = 0.14538 \text{ m}^3\text{/kg}$

Table B.2.2: v_2 & $T_2 \Rightarrow$ between 1000 kPa and 1200 kPa

$$P_2 = 1000 + 200 \frac{0.14538 - 0.17389}{0.14347 - 0.17389} = \mathbf{1187\ kPa}$$



$$u_2 = 1490.5 + (1485.8 - 1490.5) \times 0.935$$
$$= 1485.83 \text{ kJ/kg}$$
$$u_1 = 179.69 + 0.5 \times 1138.3 = 748.84 \text{ kJ/kg}$$

Process equation gives no displacement: ${}_1w_2 = \mathbf{0}$;

The energy equation then gives the heat transfer as

$${}_1q_2 = u_2 - u_1 = 1485.83 - 748.84 = \mathbf{737\ kJ/kg}$$

5.159

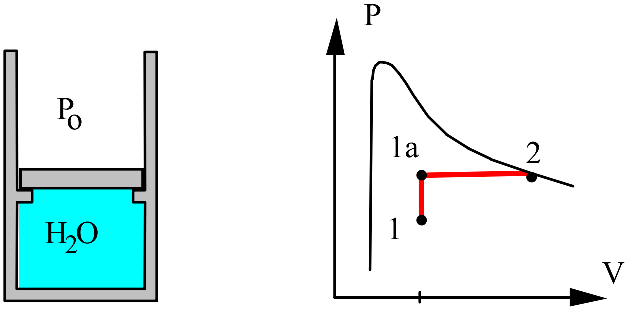
Find the heat transfer in Problem 4.122.

A piston/cylinder (Fig. P4.122) contains 1 kg of water at 20°C with a volume of 0.1 m^3. Initially the piston rests on some stops with the top surface open to the atmosphere, P_o and a mass so a water pressure of 400 kPa will lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume and the work, ${}_1W_2$.

Solution:

C.V. Water. This is a control mass.

$$m_2 = m_1 = m \;; \quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$



State 1: 20 C, $v_1 = V/m = 0.1/1 = 0.1\ m^3/kg$

$$x = (0.1 - 0.001002)/57.789 = 0.001713$$

$$u_1 = 83.94 + 0.001713 \times 2318.98 = 87.92\ kJ/kg$$

To find state 2 check on state 1a:

$P = 400$ kPa, $v = v_1 = 0.1\ m^3/kg$

Table B.1.2: $v_f < v < v_g = 0.4625\ m^3/kg$

State 2 is saturated vapor at 400 kPa since state 1a is two-phase.

$v_2 = v_g = 0.4625\ m^3/kg$, $V_2 = m\,v_2 = 0.4625\ m^3$, $u_2 = u_g = 2553.6$ kJ/kg

Pressure is constant as volume increase beyond initial volume.

$${}_1W_2 = \int P\,dV = P(V_2 - V_1) = P_{lift}(V_2 - V_1) = 400\,(0.4625 - 0.1) = 145\ kJ$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1\,(2553.6 - 87.92) + 145 = \mathbf{2610.7\ kJ}$$

5.160

A piston/cylinder contains 1 kg of ammonia at 20°C with a volume of 0.1 m^3, shown in Fig. P5.160. Initially the piston rests on some stops with the top surface open to the atmosphere, *P*o, so a pressure of 1400 kPa is required to lift it. To what temperature should the ammonia be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume, and the heat transfer.

Solution:

C.V. Ammonia which is a control mass.

$$m_2 = m_1 = m \; ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

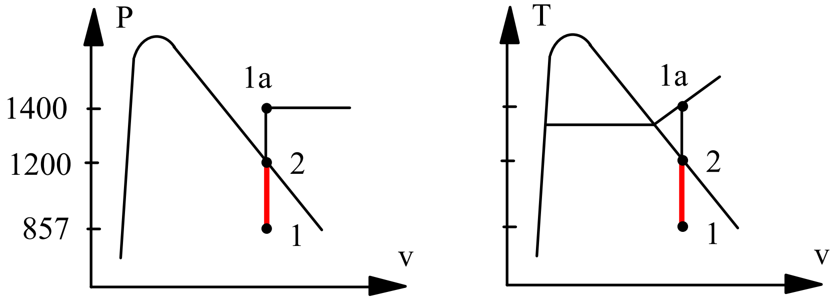
State 1: 20°C; $v_1 = 0.10 < v_g \Rightarrow x_1 = (0.1 - 0.001638)/0.14758 = 0.6665$

$$u_1 = u_f + x_1\, u_{fg} = 272.89 + 0.6665 \times 1059.3 = 978.9 \text{ kJ/kg}$$

Process: Piston starts to lift at state 1a (P_{lift}, v_1)

State 1a: 1400 kPa, v_1 Table B.2.2 (superheated vapor)

$$T_a = 50 + (60 - 50)\,\frac{0.1 - 0.09942}{0.10423 - 0.09942} = 51.2 \text{ °C}$$



State 2: x = 1.0, $v_2 = v_1 \Rightarrow V_2 = mv_2 = \mathbf{0.1\ m^3}$

$$T_2 = 30 + (0.1 - 0.11049) \times 5/(0.09397 - 0.11049) = \mathbf{33.2\ °C}$$

$u_2 = 1338.7$ kJ/kg; ${}_1W_2 = 0$;

$${}_1Q_2 = m_1q_2 = m(u_2 - u_1) = 1\,(1338.7 - 978.9) = \mathbf{359.8\ kJ/kg}$$

5.161

Consider the system shown in Fig. P5.161. Tank A has a volume of 100 L and contains saturated vapor R-134a at 30°C. When the valve is cracked open, R-134a flows slowly into cylinder B. The piston mass requires a pressure of 200 kPa in cylinder B to raise the piston. The process ends when the pressure in tank A has fallen to 200 kPa. During this process heat is exchanged with the surroundings such that the R-134a always remains at 30°C. Calculate the heat transfer for the process.

Solution:

C.V. The R-134a. This is a control mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process in B: If $V_B > 0$ then $P = P_{float}$ (piston must move)

$$\Rightarrow \quad {}_1W_2 = \int P_{float}\, dV = P_{float} m(v_2 - v_1)$$

Work done in B against constant external force (equilibrium P in cyl. B)

State 1: 30°C, x = 1. Table B.5.1: $v_1 = 0.02671\ m^3/kg$, $u_1 = 394.48$ kJ/kg

$$m = V/v_1 = 0.1 / 0.02671 = 3.744 \text{ kg}$$

State 2: 30°C, 200 kPa superheated vapor Table B.5.2

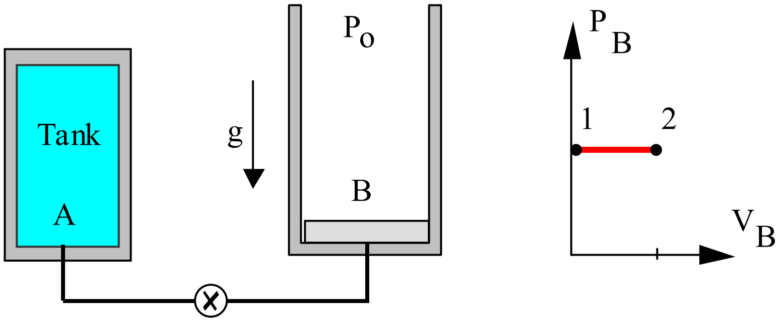
$$v_2 = 0.11889\ m^3/kg, \quad u_2 = 403.1 \text{ kJ/kg}$$

From the process equation

$${}_1W_2 = P_{float} m(v_2 - v_1) = 200 \times 3.744 \times (0.11889 - 0.02671) = 69.02 \text{ kJ}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 3.744 \times (403.1 - 394.48) + 69.02 = \mathbf{101.3\ kJ}$$



5.162

Water in a piston/cylinder, similar to Fig. P5.160, is at 100°C, x = 0.5 with mass 1 kg and the piston rests on the stops. The equilibrium pressure that will float the piston is 300 kPa. The water is heated to 300°C by an electrical heater. At what temperature would all the liquid be gone? Find the final (P,v), the work and heat transfer in the process.

C.V. The 1 kg water.

Continuty: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

State 1: (T,x) Table B.1.1

$$v_1 = 0.001044 + 0.5 \times 1.6719 = 0.83697 \text{ m}^3\text{/kg}$$

$$u_1 = 418.91 + 0.5 \times 2087.58 = 1462.7 \text{ kJ/kg}$$

State 1a: (300 kPa, $v = v_1 > v_{g\ 300\ kPa} = 0.6058$ m³/kg) so superheated vapor

Piston starts to move at state 1a, ${}_1W_{1a} = 0$, $u_{1a} = 2768.82$ kJ/kg

$${}_1Q_{1a} = m(u – u) = 1\ (2768.82 – 1462.7) = 1306.12 \text{ kJ}$$

State 1b: reached before state 1a so $v = v_1 = v_g$ see this in B.1.1

$$T_{1b} = 120 + 5\ (0.83697 – 0.8908)/(0.76953 – 0.8908) = \mathbf{122.2^oC}$$

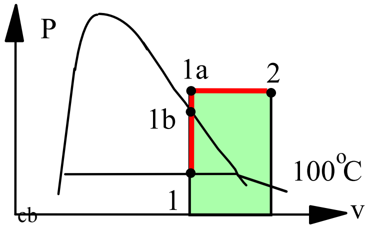
State 2: $(T_2 > T_{1a})$ Table B.1.3 => $v_2 = 0.87529$, $u_2 = 2806.69$ kJ/kg

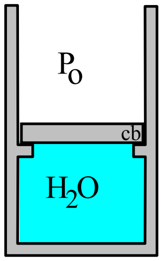
Work is seen in the P-V diagram (when volume changes $P = P_{lift}$)

$${}_1W_2 = {}_{1a}W_2 = P_2\ m(v_2 - v_1) = 300 \times 1(0.87529 - 0.83697) = \mathbf{11.5\ kJ}$$

Heat transfer is from the energy equation

$${}_1Q_2 = 1\ (2806.69 - 1462.7) + 11.5 = \mathbf{1355.5\ kJ}$$





5.163

A rigid container has two rooms filled with water, each 1 m^3 separated by a wall (see Fig. P5.61). Room A has P = 200 kPa with a quality x = 0.80. Room B has P = 2 MPa and T = 400°C. The partition wall is removed and the water comes to a uniform state, which after a while due to heat transfer has a temperature of 200°C. Find the final pressure and the heat transfer in the process.

Solution:

C.V. A + B. Constant total mass and constant total volume.

Continuity: $m_2 - m_{A1} - m_{B1} = 0$; $V_2 = V_A + V_B = 2\ m^3$

Energy Eq.5.11: $U_2 - U_1 = m_2u_2 - m_{A1}u_{A1} - m_{A1}u_{A1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process: $V = V_A + V_B = \text{constant}$ => ${}_1W_2 = 0$

State 1A: Table B.1.2 $u_{A1} = 504.47 + 0.8 \times 2025.02 = 2124.47$ kJ/kg,

$v_{A1} = 0.001061 + 0.8 \times 0.88467 = 0.70877\ m^3/kg$

State 1B: Table B.1.3 $u_{B1} = 2945.2$, $v_{B1} = 0.1512$

$m_{A1} = 1/v_{A1} = 1.411$ kg $m_{B1} = 1/v_{B1} = 6.614$ kg

State 2: T_2, $v_2 = V_2/m_2 = 2/(1.411 + 6.614) = 0.24924\ m^3/kg$

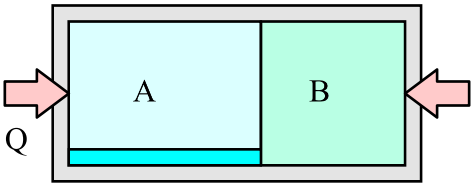
Table B.1.3 superheated vapor. $800\ kPa < P_2 < 1\ MPa$

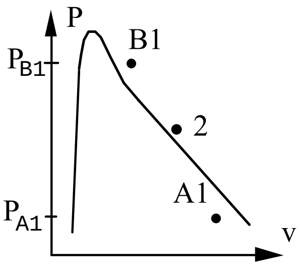
Interpolate to get the proper v_2

$$P_2 \cong 800 + \frac{0.24924-0.2608}{0.20596-0.2608} \times 200 = \mathbf{842\ kPa} \qquad u_2 \cong 2628.8\ kJ/kg$$

From the energy equation

$${}_1Q_2 = 8.025 \times 2628.8 - 1.411 \times 2124.47 - 6.614 \times 2945.2 = \mathbf{-\ 1381\ kJ}$$





5.164

A piston held by a pin in an insulated cylinder, shown in Fig. P5.164, contains 2 kg water at 100°C, quality 98%. The piston has a mass of 102 kg, with cross-sectional area of 100 cm^2, and the ambient pressure is 100 kPa. The pin is released, which allows the piston to move. Determine the final state of the water, assuming the process to be adiabatic.

Solution:

C.V. The water. This is a control mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process in cylinder: $P = P_{float}$ (if piston not supported by pin)

$$P_2 = P_{float} = P_0 + m_p g/A = 100 + \frac{102 \times 9.807}{100 \times 10^{-4} \times 10^3} = 200 \text{ kPa}$$

We thus need one more property for state 2 and we have one equation namely the energy equation. From the equilibrium pressure the work becomes

$$_1W_2 = \int P_{float}\, dV = P_2\, m(v_2 - v_1)$$

With this work the energy equation gives per unit mass

$$u_2 - u_1 = {}_1q_2 - {}_1w_2 = 0 - P_2(v_2 - v_1)$$

or with rearrangement to have the unknowns on the left hand side

$$u_2 + P_2v_2 = h_2 = u_1 + P_2v_1$$

$$h_2 = u_1 + P_2v_1 = 2464.8 + 200 \times 1.6395 = 2792.7 \text{ kJ/kg}$$

State 2: (P_2 , h_2) Table B.1.3 => $T_2 \cong$ **161.75°C**

5.165

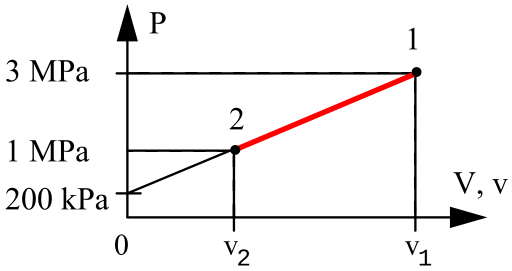
A piston/cylinder arrangement has a linear spring and the outside atmosphere acting on the piston, shown in Fig. P5.165. It contains water at 3 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that a pressure of 200 kPa inside is required to balance the forces. The system now cools until the pressure reaches 1 MPa. Find the heat transfer for the process.

Solution:

C.V. Water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$



State 1: Table B.1.3

$v_1 = 0.09936\ m^3/kg,\ u_1 = 2932.8\ kJ/kg$

$m = V/v_1 = 0.1/0.09936 = 1.006\ kg$

Process: Linear spring so P linear in v.

$P = P_0 + (P_1 - P_0)v/v_1$

$$v_2 = \frac{(P_2 - P_0)v_1}{P_1 - P_0} = \frac{(1000 - 200)0.09936}{3000 - 200} = 0.02839\ m^3/kg$$

State 2: $P_2 , v_2 \Rightarrow x_2 = (v_2 - 0.001127)/0.19332 = 0.141, \quad T_2 = 179.91°C,$

$u_2 = 761.62 + x_2 \times 1821.97 = 1018.58\ kJ/kg$

Process => $_1W_2 = \int PdV = \frac{1}{2} m(P_1 + P_2)(v_2 - v_1)$

$= \frac{1}{2} 1.006\ (3000 + 1000)(0.02839 -0.09936) = -142.79\ kJ$

Heat transfer from the energy equation

$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.006(1018.58 - 2932.8) - 142.79 =$ **-2068.5 kJ**

5.166

A piston/cylinder, shown in Fig. P5.166, contains R-410a at – 20°C, $x = 20\%$. The volume is 0.2 m^3. It is known that $V_{stop} = 0.4$ m^3, and if the piston sits at the bottom, the spring force balances the other loads on the piston. It is now heated up to 20°C. Find the mass of the fluid and show the P–v diagram. Find the work and heat transfer.

Solution:

C.V. R-410a, this is a control mass. Properties in Table B.4.

Continuity Eq.: $m_2 = m_1$

Energy Eq.5.11: $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$, $V < 0.4\ m^3$, $A = 0$ (at $V = 0$, $P = 0$)

State 1: $v_1 = 0.000803 + 0.2 \times 0.0640 = 0.0136\ m^3/kg$

$u_1 = 27.92 + 0.2 \times 218.07 = 71.5\ kJ/kg$

$m = m_1 = V_1/v_1 = 14.706\ kg$

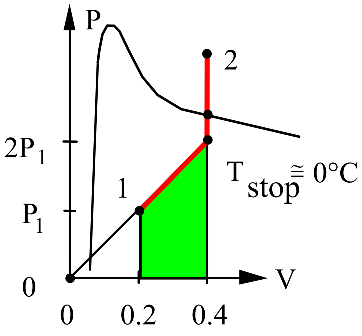
System: on line

$V \le V_{stop}$;

$P_1 = 399.6\ kPa$

$P_{stop} = 2P_1 = 799.2\ kPa$

$v_{stop} = 2v_1 = 0.0272\ m^3/kg$



State stop: $(P,v) \Rightarrow T_{stop} \cong 0°C$ TWO-PHASE STATE

Since $T_2 > T_{stop} \Rightarrow v_2 = v_{stop} = 0.0272\ m^3/kg$

State 2: (T_2, v_2) Table B.4.2: Interpolate between 1000 and 1200 kPa

$P_2 = 1035\ kPa$; $u_2 = 366.5\ kJ/kg$

From the process curve, see also area in P-V diagram, the work is

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_{stop})(V_{stop} - V_1) = \frac{1}{2}(399.6 + 799.2)0.2 = \mathbf{119.8\ kJ}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 14.706(266.5 – 71.5) + 119.8 = \mathbf{2987.5\ kJ}$$

5.167

Consider the piston/cylinder arrangement shown in Fig. P5.167. A frictionless piston is free to move between two sets of stops. When the piston rests on the lower stops, the enclosed volume is 400 L. When the piston reaches the upper stops, the volume is 600 L. The cylinder initially contains water at 100 kPa, 20% quality. It is heated until the water eventually exists as saturated vapor. The mass of the piston requires 300 kPa pressure to move it against the outside ambient pressure. Determine the final pressure in the cylinder, the heat transfer and the work for the overall process.

Solution:

C.V. Water. Check to see if piston reaches upper stops.

Energy Eq.5.11: $m(u_4 - u_1) = {}_1Q_4 - {}_1W_4$

Process: If $P < 300$ kPa then $V = 400$ L, line 2-1 and below
If $P > 300$ kPa then $V = 600$ L, line 3-4 and above
If $P = 300$ kPa then $400 \text{ L} < V < 600 \text{ L}$ line 2-3

These three lines are shown in the P-V diagram below and is dictated by the motion of the piston (force balance).

State 1: $v_1 = 0.001043 + 0.2\times1.693 = 0.33964$; $m = V_1/v_1 = \dfrac{0.4}{0.33964} = 1.178$ kg

$u_1 = 417.36 + 0.2 \times 2088.7 = 835.1$ kJ/kg

State 3: $v_3 = \dfrac{0.6}{1.178} = 0.5095 < v_G = 0.6058$ at $P_3 = 300$ kPa

$\Rightarrow$ Piston does reach upper stops to reach sat. vapor.

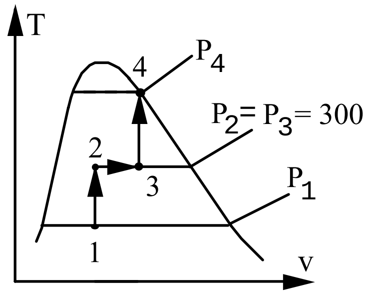
State 4: $v_4 = v_3 = 0.5095$ m^3/kg $= v_G$ at P_4 From Table B.1.2

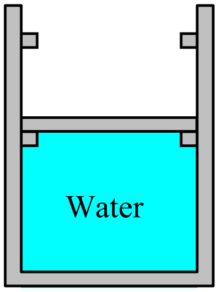
=> $P_4 =$ **361 kPa,** $u_4 = 2550.0$ kJ/kg

${}_1W_4 = {}_1W_2 + {}_2W_3 + {}_3W_4 = 0 + {}_2W_3 + 0$

${}_1W_4 = P_2(V_3 - V_2) = 300 \times (0.6 - 0.4) =$ **60 kJ**

${}_1Q_4 = m(u_4 - u_1) + {}_1W_4 = 1.178(2550.0 - 835.1) + 60 =$ **2080 kJ**





5.168

A spherical balloon contains 2 kg of R-410a at 0°C, 30% quality. This system is heated until the pressure in the balloon reaches 1 MPa. For this process, it can be assumed that the pressure in the balloon is directly proportional to the balloon diameter. How does pressure vary with volume and what is the heat transfer for the process?

Solution:

C.V. R-410a which is a control mass.

$$m_2 = m_1 = m \;;$$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: 0°C, x = 0.3. Table B.4.1 gives $P_1 = 798.7$ kPa

$$v_1 = 0.000855 + 0.3 \times 0.03182 = 0.01040 \text{ m}^3\text{/kg}$$

$$u_1 = 57.07 + 0.3 \times 195.95 = 115.86 \text{ kJ/kg}$$

Process: $P \propto D, \;\; V \propto D^3 \;\; \Rightarrow \;\; PV^{-1/3} = \text{constant}$, polytropic $n = -1/3$.

$$\Rightarrow \;\; V_2 = mv_2 = V_1 \,(P_2 /P_1)^3 = mv_1 \,(P_2 /P_1)^3$$

$$v_2 = v_1 \,(P_2 /P_1)^3 = 0.01040 \times (1000 / 798.7)^3 = 0.02041 \text{ m}^3\text{/kg}$$

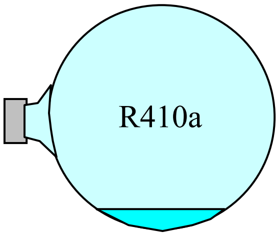
State 2: $P_2 = 1$ MPa, process : $v_2 = 0.02041 \;\rightarrow$ Table B.4.2, $T_2 = 7.25°\text{C}$ (sat)

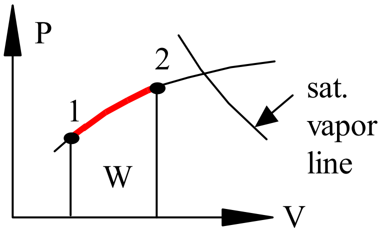
$$v_f = 0.000877,\; v_{fg} = 0.02508 \text{ m}^3\text{/kg},\; u_f = 68.02,\; u_{fg} = 187.18 \text{ kJ/kg}$$

$$x_2 = 0.7787, \quad u_2 = 213.7 \text{ kJ/kg},$$

$${}_1W_2 = \int P\, dV = m\,\frac{P_2v_2 - P_1v_1}{1 - n} = 2\,\frac{1000 \times 0.02041 - 798.7 \times 0.0104}{1 - (-1/3)} = 18.16 \text{ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2\,(213.7 - 115.86) + 18.16 = \mathbf{213.8\ kJ}$$





Notice: The R-410a is not an ideal gas at any state in this problem.

5.169

A 1 m^3 tank containing air at 25°C and 500 kPa is connected through a valve to another tank containing 4 kg of air at 60°C and 200 kPa. Now the valve is opened and the entire system reaches thermal equilibrium with the surroundings at 20°C. Assume constant specific heat at 25°C and determine the final pressure and the heat transfer.

Control volume all the air. Assume air is an ideal gas.

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$

Energy Eq.: $U_2 - U_1 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

Process Eq.: $V = \text{constant} \quad \Rightarrow \quad {}_1W_2 = 0$

State 1:

$$m_{A1} = \frac{P_{A1}V_{A1}}{RT_{A1}} = \frac{(500\text{ kPa})(1\text{m}^3)}{(0.287\text{ kJ/kgK})(298.2\text{ K})} = 5.84\text{ kg}$$

$$V_{B1} = \frac{m_{B1}RT_{B1}}{P_{B1}} = \frac{(4\text{ kg})(0.287\text{ kJ/kgK})(333.2\text{ K})}{(200\text{ kN/m}^2)} = 1.91\text{ m}^3$$

State 2: $T_2 = 20°\text{C}$, $v_2 = V_2/m_2$

$$m_2 = m_{A1} + m_{B1} = 4 + 5.84 = 9.84\text{ kg}$$

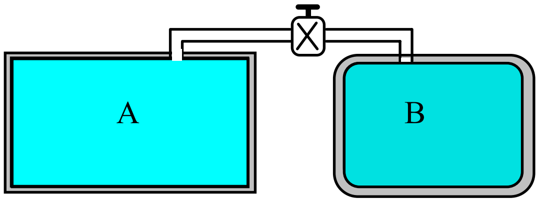
$$V_2 = V_{A1} + V_{B1} = 1 + 1.91 = 2.91\text{ m}^3$$

$$P_2 = \frac{m_2RT_2}{V_2} = \frac{(9.84\text{ kg})(0.287\text{ kJ/kgK})(293.2\text{ K})}{2.91\text{ m}^3} = \mathbf{284.5\text{ kPa}}$$

Energy Eq.5.5 or 5.11:

$$\begin{aligned}{}_1Q_2 &= U_2 - U_1 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} \\ &= m_{A1}(u_2 - u_{A1}) + m_{B1}(u_2 - u_{B1}) \\ &= m_{A1}C_{v0}(T_2 - T_{A1}) + m_{B1}C_{v0}(T_2 - T_{B1}) \\ &= 5.84 \times 0.717\ (20 - 25) + 4 \times 0.717\ (20 - 60) = \mathbf{-135.6\text{ kJ}}\end{aligned}$$

The air gave energy out.



5.170

Ammonia (2 kg) in a piston/cylinder is at 100 kPa, -20°C and is now heated in a polytropic process with n = 1.3 to a pressure of 200 kPa. Do not use ideal gas approximation and find T_2, the work and heat transfer in the process.

Take CV as the Ammonia, constant mass.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = \text{constant} \quad (n = 1.3)$

State 1: Superheated vapor table B.2.2.

$$v_1 = 1.2101 \text{ m}^3\text{/kg}, \quad u_1 = 1307.8 \text{ kJ/kg}$$

Process gives: $v_2 = v_1 (P_1/P_2)^{1/n} = 1.2101\ (100/200)^{1/1.3} = 0.710 \text{ m}^3\text{/kg}$

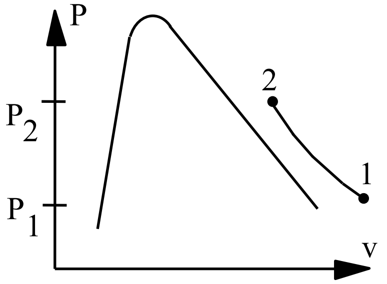
State 2: Table B.2.2 at 200 kPa interpolate: $u_2 = 1376.49$ kJ/kg, $T_2 = 24^{\circ}\text{C}$

Work is done while piston moves at increasing pressure, so we get

$${}_1W_2 = \frac{m}{1-n}(P_2v_2 - P_1v_1) = \frac{2}{1-1.3}(200\times0.71 - 100\times1.2101) = \mathbf{-139.9\ kJ}$$

Heat transfer is found from the energy equation

$$\begin{aligned}{}_1Q_2 &= m(u_2 - u_1) + {}_1W_2 \\ &= 2\ (1376.49 - 1307.8) - 139.9 \\ &= \mathbf{-2.52\ kJ}\end{aligned}$$



5.171

A piston/cylinder arrangement B is connected to a 1-m^3 tank A by a line and valve, shown in Fig. P5.171. Initially both contain water, with A at 100 kPa, saturated vapor and B at 400°C, 300 kPa, 1 m^3. The valve is now opened and, the water in both A and B comes to a uniform state.

a. Find the initial mass in A and B.

b. If the process results in T_2 = 200°C, find the heat transfer and work.

Solution:

C.V.: A + B. This is a control mass.

Continuity equation: $m_2 - (m_{A1} + m_{B1}) = 0$;

Energy: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

System: if $V_B \geq 0$ piston floats $\Rightarrow$ $P_B = P_{B1}$ = const.

if $V_B = 0$ then $P_2 < P_{B1}$ and $v = V_A/m_{tot}$ see P-V diagram

$${}_1W_2 = \int P_B dV_B = P_{B1}(V_2 - V_1)_B = P_{B1}(V_2 - V_1)_{tot}$$

State A1: Table B.1.1, x = 1

v_{A1} = 1.694 m^3/kg, u_{A1} = 2506.1 kJ/kg

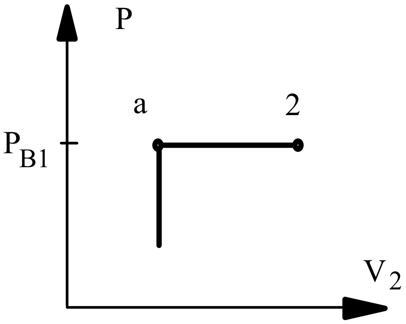
$m_{A1} = V_A/v_{A1}$ = **0.5903 kg**

State B1: Table B.1.2 sup. vapor

v_{B1} = 1.0315 m^3/kg, u_{B1} = 2965.5 kJ/kg

$m_{B1} = V_{B1}/v_{B1}$ = **0.9695 kg**

$m_2 = m_{TOT}$ = 1.56 kg



* At (T_2, P_{B1}) $v_2 = 0.7163 > v_a = V_A/m_{tot} = 0.641$ so $V_{B2} > 0$

so now state 2: $P_2 = P_{B1}$ = 300 kPa, T_2 = 200 °C

=> u_2 = 2650.7 kJ/kg and $V_2 = m_2 v_2 = 1.56 \times 0.7163 = 1.117$ m^3

(we could also have checked T_a at: 300 kPa, 0.641 m^3/kg => T = 155 °C)

${}_1W_2 = P_{B1}(V_2 - V_1)$ = **-264.82 kJ**

${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$ = **-484.7 kJ**

5.172

A small flexible bag contains 0.1 kg ammonia at –10°C and 300 kPa. The bag material is such that the pressure inside varies linear with volume. The bag is left in the sun with with an incident radiation of 75 W, loosing energy with an average 25 W to the ambient ground and air. After a while the bag is heated to 30°C at which time the pressure is 1000 kPa. Find the work and heat transfer in the process and the elapsed time.

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$ (linear in V)

State 1: Compressed liquid $P > P_{sat}$, take saturated liquid at same temperature.

$$v_1 = v_{f\,-10} = 0.001534 \text{ m}^3\text{/kg}, \qquad u_1 = u_f = 133.96 \text{ kJ/kg}$$

State 2: Table B.2.1 at 30°C : $P < P_{sat}$ so superheated vapor

$$v_2 = 0.13206 \text{ m}^3\text{/kg},\ u_2 = 1347.1 \text{ kJ/kg}, \qquad V_2 = mv_2 = \mathbf{0.0132\ m^3}$$

Work is done while piston moves at increacing pressure, so we get

$${}_1W_2 = ½(300 + 1000)*0.1(0.13206 - 0.001534) = \mathbf{8.484\ kJ}$$

Heat transfer is found from the energy equation

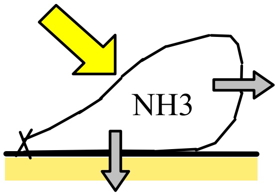
$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1\ (1347.1 - 133.96) + 8.484$$

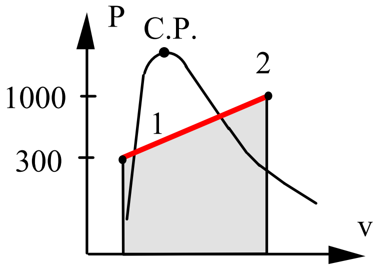
$$= 121.314 + 8.484 = \mathbf{129.8\ kJ}$$

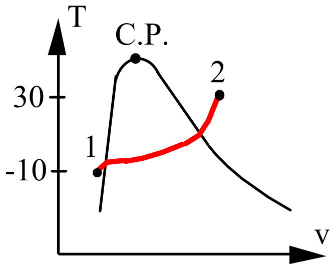
$$\dot{Q}_{net} = 75 - 25 = 50 \text{ Watts}$$

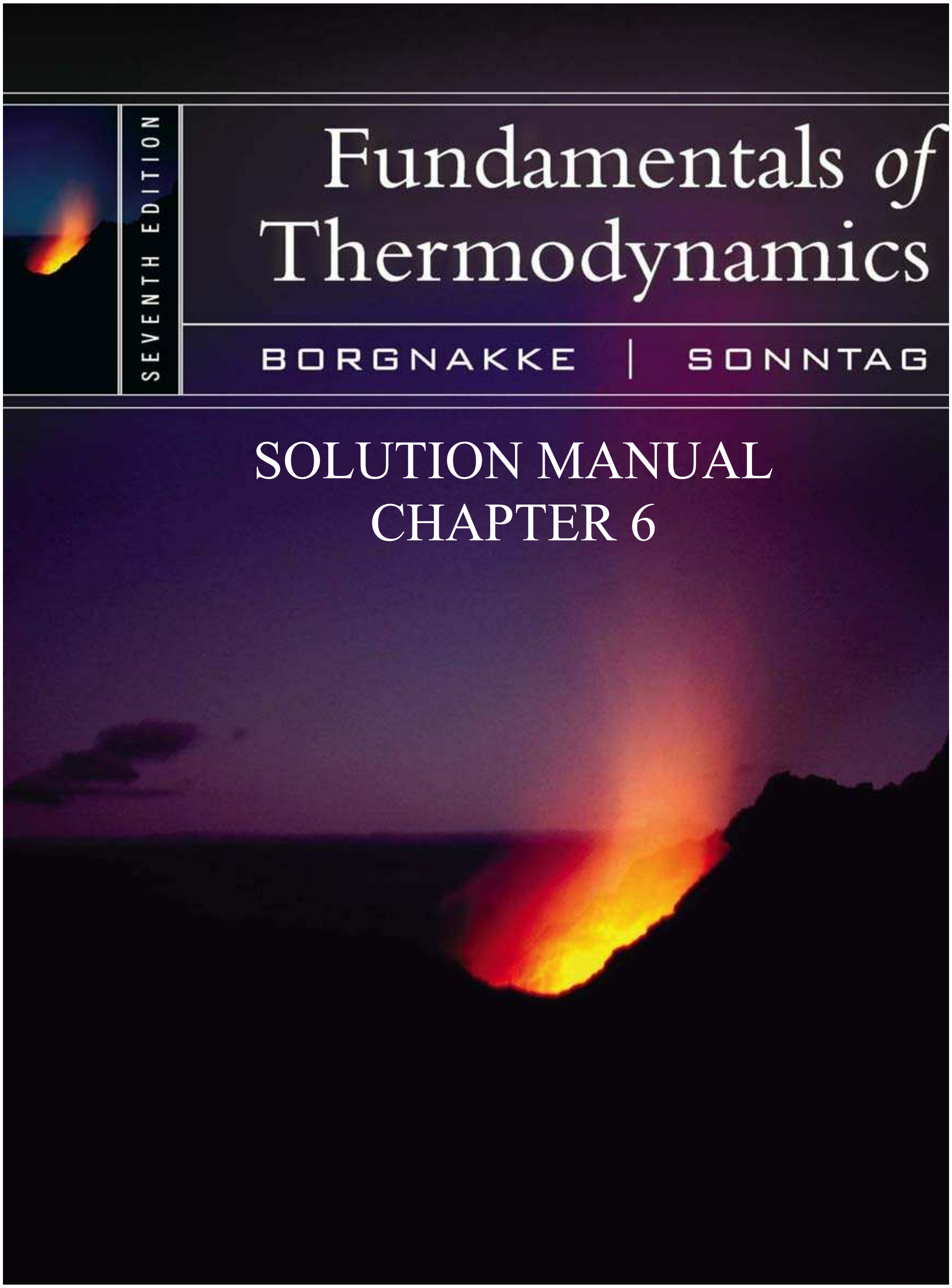
Assume the constant rate $\dot{Q}_{net} = dQ/dt = {}_1Q_2 / t$, so the time becomes

$$t = {}_1Q_2 / \dot{Q}_{net} = \frac{129800}{50} = \mathbf{2596\ s = 43.3\ min}$$









Fundamentals *of* Thermodynamics

SEVENTH EDITION

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 6

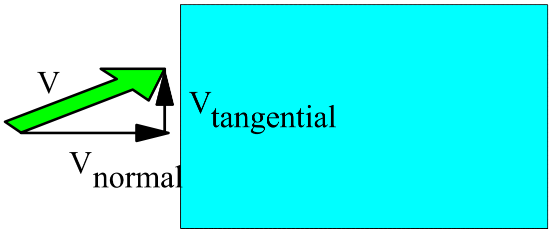
CONTENT

In-Text Concept Questions

6.a

A mass flow rate into a control volume requires a normal velocity component. Why?

The tangential velocity component does not bring any substance across the control volume surface as it flows parallel to it, the normal component of velocity brings substance in or out of the control volume according to its sign. The normal component must be into the control volume to bring mass in, just like when you enter a bus (it does not help that you run parallel with the bus side).



6.b

Can a steady state device have boundary work?

No. Any change in size of the control volume would require either a change in mass inside or a change in state inside, neither of which is possible in a steady-state process.

6.c

Can you say something about changes in $\dot{m}$ and $\dot{V}$ through a steady flow device?

The continuity equation expresses the conservation of mass, so the total amount of $\dot{m}$ entering must be equal to the total amount leaving. For a single flow device the mass flow rate is constant through it, so you have the same mass flow rate across any total cross-section of the device from the inlet to the exit.

The volume flow rate is related to the mass flow rate as

$$\dot{V} = v\,\dot{m}$$

so it can vary if the state changes (then v changes) for a constant mass flow rate.

This also means that the velocity can change (influenced by the area as $\dot{V} = \mathbf{V}A$) and the flow can experience an acceleration (like in a nozzle) or a deceleration (as in a diffuser).

6.d

In a multiple device flow system, I want to determine a state property. Where should I be looking for information—upstream or downstream?

Generally flow is affected more by what happened to it which is upstream than what is in front of it. Only the pressure information can travel upstream and give rise to accelerations (nozzle) or decelerations (stagnation type flow). Heat transfer that can heat or cool a flow can not travel very fast and is easily overpowered by the convection. If the flow velocity exceeds the speed of sound even the pressure information can not travel upstream.

6.e

How does a nozzle or sprayhead generate kinetic energy?

By accelerating the fluid from a high pressure towards the lower pressure, which is outside the nozzle. The higher pressure pushes harder than the lower pressure so there is a net force on any mass element to accelerate it.

6.f

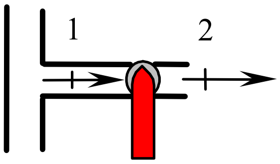
What is the difference between a nozzle flow and a throttle process?

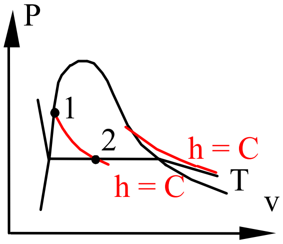
In both processes a flow moves from a higher to a lower pressure. In the nozzle the pressure drop generates kinetic energy, whereas that does not take place in the throttle process. The pressure drop in the throttle is due to a flow restriction and represents a loss.

6.g

If you throttle a saturated liquid what happens to the fluid state? What if this is done to an ideal gas?

The throttle process is approximated as a constant enthalpy process. Changing the state from saturated liquid to a lower pressure with the same h gives a two-phase state so some of the liquid will vaporize and it becomes colder.



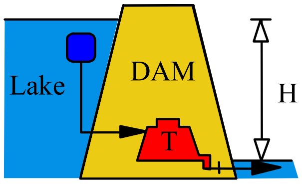


If the same process happens in an ideal gas then same h gives the same temperature (h a function of T only) at the lower pressure.

6.h

A turbine at the bottom of a dam has a flow of liquid water through it. How does that produce power? Which terms in the energy equation are important if the CV is the turbine only? If the CV is the turbine plus the upstream flow up to the top of the lake, which terms in the energy equation are then important?

The water at the bottom of the dam in the turbine inlet is at a high pressure. It runs through a nozzle generating kinetic energy as the pressure drops. This high kinetic energy flow impacts a set of rotating blades or buckets, which converts the kinetic energy to power on the shaft and the flow leaves at low pressure and low velocity.



CV Turbine only.

The high P in and the low P out shows up in the (h = u + Pv) flow terms of the energy equation giving the difference in the flow work terms Pv in and out.

CV Turbine plus upstream flow.

For this CV the pressures in and out are the same (1 atm) so the difference is in the potential energy terms (gz) included in h_{tot}.

6.i

If you compress air the temperature goes up, why? When the hot air, high P flows in long pipes it eventually cools to ambient T. How does that change the flow?

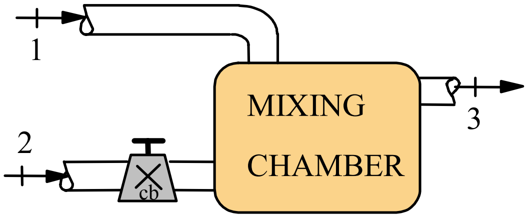
As the air is compressed, volume decreases so work is done on a mass element, its energy and hence temperature goes up. If it flows at nearly constant P and cools its density increases (v decreases) so it slows down for same mass flow rate ($\dot{m} = \rho A\mathbf{V}$) and flow area.

6.j

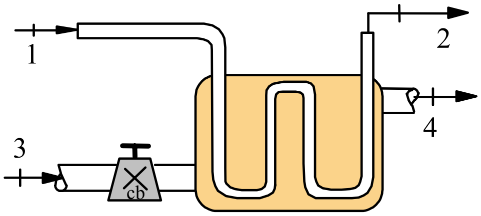
A mixing chamber has all flows at the same P, neglecting losses. A heat exchanger has separate flows exchanging energy, but they do not mix. Why have both kinds?

You might allow mixing when you can use the resulting output mixture, say it is the same substance. You may also allow it if you definitely want the outgoing mixture, like water out of a faucet where you mix hot and cold water. Even if it is different substances it may be desirable, say you add water to dry air to make it more moist, typical for a winter time air-conditioning set-up.

In other cases it is different substances that flow at different pressures with one flow heating or cooling the other flow. This could be hot combustion gases heating a flow of water or a primary fluid flow around a nuclear reactor heating a transfer fluid flow. Here the fluid being heated should stay pure so it does not absorb gases or radioactive particles and becomes contaminated. Even when the two flows have the same substance there may be a reason to keep them at separate pressures.



An open mixing chamber



A closed tube in shell heat exchanger.

6.k

An initially empty cylinder is filled with air from 20°C, 100 kPa until it is full. Assuming no heat transfer is the final temperature larger, equal to or smaller than 20°C? Does the final T depend on the size of the cylinder?

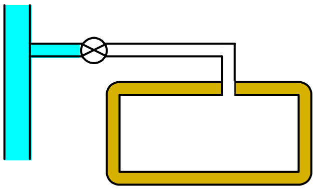
This is a transient problem with no heat transfer and no work. The balance equations for the tank as C.V. become

Continuity Eq.: $m_2 - 0 = m_i$

Energy Eq.: $m_2u_2 - 0 = m_ih_i + Q - W = m_ih_i + 0 - 0$

Final state: $u_2 = h_i = u_i + P_iv_i$ & $P_2 = P_i$

$T_2 > T_i$ and it does not depend on V

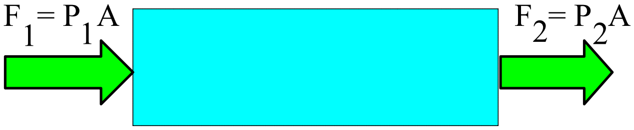


Concept-Study Guide Problems

6.1

A temperature difference drives a heat transfer. Does a similar concept apply to $\dot{m}$?

Yes. A pressure difference drives the flow. The fluid is accelerated in the direction of a lower pressure as it is being pushed harder behind it than in front of it. This also means a higher pressure in front can decelerate the flow to a lower velocity which happens at a stagnation on a wall.



6.2

What kind of effect can be felt upstream in a flow?

Only the pressure can be felt upstream in a subsonic flow. In a supersonic flow no information can travel upstream. The temperature information travels by conduction and even small velocities overpowers the conduction with the convection of energy so the temperature at a given location is mainly given by the upstream conditions and influenced very little by the downstream conditions.

6.3

Which one of the properties (P, v, T) can be controlled in a flow? How?

Since the flow is not contained there is no direct control over the volume and thus no control of v. The pressure can be controlled by installation of a pump or compressor if you want to increase it or use a turbine, nozzle or valve through which the pressure will decrease. The temperature can be controlled by heating or cooling the flow in a heat exchanger.

6.4

Air at 500 kPa is expanded to 100 kPa in two steady flow cases. Case one is a nozzle and case two is a turbine, the exit state is the same for both cases. What can you say about the specific turbine work relative to the specific kinetic energy in the exit flow of the nozzle?

For these single flow devices let us assume they are adiabatic and that the turbine does not have any exit kinetic energy then the energy equations become:

Energy Eq.6.13 nozzle: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Energy Eq.6.13 turbine: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + gZ_2 + w_T$

Comparing the two we get $\frac{1}{2}\mathbf{V}_2^2 = w_T$ so the result is that the nozzle delivers kinetic energy of the same amount as the turbine delivers shaft work.

6.5

Pipes that carry a hot fluid like steam in a power plant, exhaust pipe for a diesel engine in a ship etc. are often insulated. Is that to reduce the heat loss or is there another purpose?

You definitely want to insulate pipes that carry hot steam from the boiler to the turbines in a power plant and pipes that flows hot water from one location to another. Even if the energy in the flow is unwanted the pipes should be insulated for safety. Any place that people could touch a hot surface there is a risk of a burn and that should be avoided.

Pipes for district heating with hot water.

6.6

A windmill takes a fraction of the wind kinetic energy out as power on a shaft. In what manner does the temperature and wind velocity influence the power? Hint: write the power as mass flow rate times specific work.

The work as a fraction f of the flow of kinetic energy becomes

$$\dot{W} = \dot{m}w = \dot{m}\, f \frac{1}{2} \mathbf{V}_{in}^2 = \rho A \mathbf{V}_{in}\, f \frac{1}{2} \mathbf{V}_{in}^2$$

so the power is proportional to the velocity cubed. The temperature enters through the density, so assuming air as ideal gas

$$\rho = 1/v = P/RT$$

and the power is inversely proportional to temperature.

A windmill farm west of Denmark in the North Sea.

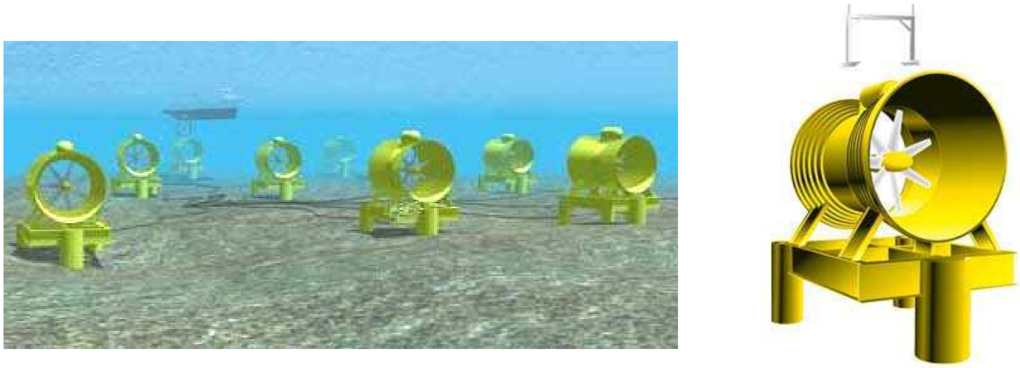
6.7

An underwater turbine extracts a fraction of the kinetic energy from the ocean current. In what manner does the temperature and water velocity influence the power? Hint: write the power as mass flow rate times specific work.

The work as a fraction f of the flow of kinetic energy becomes

$$\dot{W} = \dot{m}w = \dot{m}\, f \frac{1}{2} \mathbf{V}_{in}^2 = \rho A \mathbf{V}_{in}\, f \frac{1}{2} \mathbf{V}_{in}^2$$

so the power is proportional to the velocity cubed. The temperature enters through the density, so assuming water is incompressible density is constant and the power does not vary with the temperature.



A proposed underwater tidal flow turbine farm. Each turbine is for 1 MW with a diameter of 11.5 m mounted on the seafloor.

6.8

A liquid water turbine in the bottom of a dam takes energy out as power on a shaft. Which term(s) in the energy equation are changing and important?

The water at the bottom of the dam in the turbine inlet is at a high pressure. In a standard turbine it runs through blade passages like a propeller shown below to the left. In this case the inlet high pressure is used directly to generate the force on the moving blades.

For the Pelton turbine shown below it runs through a nozzle generating kinetic energy as the pressure drops. The high kinetic energy flow impacts a set of rotating blades or buckets and converts the kinetic energy to power on the shaft so the flow leaves at low pressure and low velocity.

A Pelton turbine above and insert on left. A blade type turbine to the left. The pressure drop and flow rate determine which one is best for the given case.

6.9

You blow a balloon up with air. What kind of work terms including flow work do you see in that case? Where is the energy stored?

As the balloon is blown up mass flow in has flow work associated with it. Also as the balloon grows there is a boundary work done by the inside gas and a smaller boundary work from the outside of the balloon to the atmosphere. The difference between the latter two work terms goes into stretching the balloon material and thus becomes internal energy (or you may call that potential energy) of the balloon material. The work term to the atmosphere is stored in the atmosphere and the part of the flow work that stays in the gas is stored as the gas internal energy.

6.10

A storage tank for natural gas (NG) has a top dome that can move up or down as gas is added or subtracted from the tank maintaining 110 kPa, 290 K inside. A pipeline at 110 kPa, 290 K now supplies some NG to the tank. Does it change state during the filling process? What happens to the flow work?

As the pressure inside the storage tank is the same as in the pipeline the state does not change. However the tank volume goes up and work is done on the moving boundary at the 110 kPa so this work equals the flow work. The net effect is the flow work goes into raising the dome.

Continuity equation and flow rates

6.11

Carbon dioxide at 200 kPa, $10^{o}C$ flows at 1 kg/s in a 0.25 m^2 cross sectional area pipe. Find the velocity and the volume flow rate.

$\dot{m} = AV/v = \dot{V}/v$

$v = RT/P = 0.1889 \text{ kJ/kg-K} \times 283.15 \text{ K}/ 200 \text{ kPa} = 0.267 \text{ m}^3\text{/kg}$

$\dot{V} = v\,\dot{m} = 0.267 \times 1 = 0.267 \text{ m}^3\textbf{/s}$

$V = \dot{V} / A = 0.267 \text{ (m}^3\text{/s)} / 0.25 \text{ m}^2 = 1.07 \text{ m/s}$

6.12

Air at 35°C, 105 kPa, flows in a 100 mm × 150 mm rectangular duct in a heating system. The volumetric flow rate is 0.015 m^3/s. What is the velocity of the air flowing in the duct and what is the mass flow rate?

Solution:

Assume a constant velocity across the duct area with

$$A = 100 \times 150 \times 10^{-6}\ m^2 = 0.015\ m^2$$

and the volumetric flow rate from Eq.6.3,

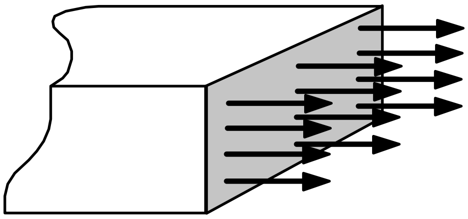
$$\dot{V} = \dot{m}v = A\mathbf{V}$$

$$\mathbf{V} = \frac{\dot{V}}{A} = \frac{0.015\ m^3/s}{0.015\ m^2} = \mathbf{1.0\ m/s}$$

Ideal gas so note:

$$v = \frac{RT}{P} = \frac{0.287 \times 308.2}{105} = 0.8424\ m^3/kg$$

$$\dot{m} = \frac{\dot{V}}{v} = \frac{0.015}{0.8424} = \mathbf{0.0178\ kg/s}$$



6.13

An empty bathtub has its drain closed and is being filled with water from the faucet at a rate of 10 kg/min. After 10 minutes the drain is opened and 4 kg/min flows out and at the same time the inlet flow is reduced to 2 kg/min. Plot the mass of the water in the bathtub versus time and determine the time from the very beginning when the tub will be empty.

Solution:

During the first 10 minutes we have

$$\frac{dm_{cv}}{dt} = \dot{m}_i = 10 \text{ kg/min} , \qquad \Delta m = \dot{m}\, \Delta t_1 = 10 \times 10 = 100 \text{ kg}$$

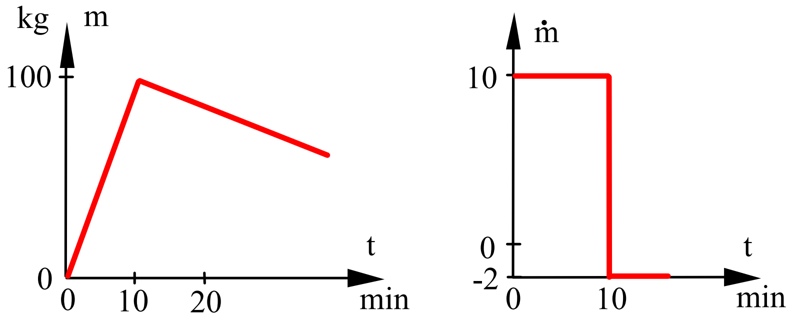
So we end up with 100 kg after 10 min. For the remaining period we have

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e = 2 - 4 = -2 \text{ kg/min}$$

$$\Delta m_2 = \dot{m}_{net}\, \Delta t_2 \rightarrow \quad \Delta t_2 = \frac{\Delta m}{\dot{m}_{net}} = -100/-2 = 50 \text{ min.}$$

So it will take an additional 50 min. to empty

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2 = 10 + 50 = \mathbf{60\ min.}$$



6.14

Saturated vapor R-134a leaves the evaporator in a heat pump system at 10°C, with a steady mass flow rate of 0.1 kg/s. What is the smallest diameter tubing that can be used at this location if the velocity of the refrigerant is not to exceed 7 m/s?

Solution:

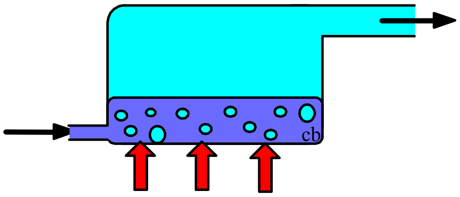
Mass flow rate Eq.6.3: $\dot{m} = \dot{V}/v = A\mathbf{V}/v$

Exit state Table B.5.1: (T = 10°C, x =1) => $v = v_g = 0.04945 \text{ m}^3/\text{kg}$

The minimum area is associated with the maximum velocity for given $\dot{m}$

$$A_{MIN} = \frac{\dot{m}v_g}{\mathbf{V}_{MAX}} = \frac{0.1 \text{ kg/s} \times 0.04945 \text{ m}^3\text{/kg}}{7 \text{ m/s}} = 0.000706 \text{ m}^2 = \frac{\pi}{4} D^2_{MIN}$$

D_{MIN} = **0.03 m** = **30 mm**



6.15

A boiler receives a constant flow of 5000 kg/h liquid water at 5 MPa, 20°C and it heats the flow such that the exit state is 450°C with a pressure of 4.5 MPa. Determine the necessary minimum pipe flow area in both the inlet and exit pipe(s) if there should be no velocities larger than 20 m/s.

Solution:

Mass flow rate from Eq.6.3, both $\mathbf{V} \le 20$ m/s

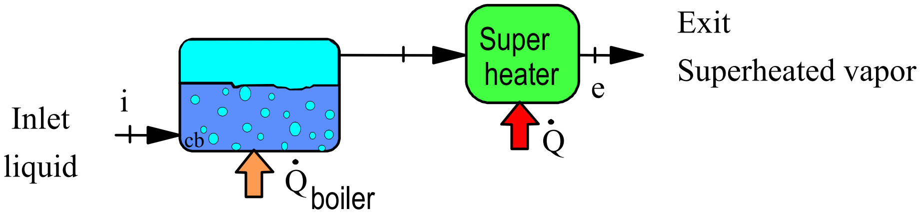
$$\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)_i = (A\mathbf{V}/v)_e = 5000 \frac{1}{3600} \text{ kg/s}$$

Table B.1.4 $\quad v_i = 0.001 \text{ m}^3/\text{kg}$,

Table B.1.3 $\quad v_e = (0.08003 + 0.00633)/2 = 0.07166 \text{ m}^3/\text{kg}$,

$$A_i \ge v_i \dot{m}/\mathbf{V}_i = 0.001 \times \frac{5000}{3600} / 20 = 6.94 \times 10^{-5} \text{ m}^2 = \mathbf{0.69\ cm^2}$$

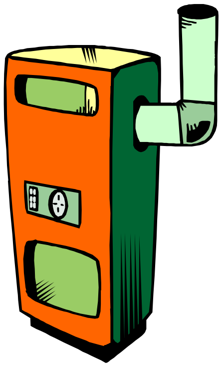
$$A_e \ge v_e \dot{m}/\mathbf{V}_e = 0.07166 \times \frac{5000}{3600} / 20 = 4.98 \times 10^{-3} \text{ m}^2 = \mathbf{50\ cm^2}$$



6.16

A hot air home heating system takes 0.25 m^3/s air at 100 kPa, 17°C into a furnace and heats it to 52°C and delivers the flow to a square duct 0.2 m by 0.2 m at 110 kPa. What is the velocity in the duct?

Solution:



The inflate flow is given by a $\dot{m}_i$

Continuity Eq.: $\dot{m}_i = \dot{V}_i / v_i = \dot{m}_e = A_e\mathbf{V}_e/v_e$

$$\text{Ideal gas: } v_i = \frac{RT_i}{P_i} = \frac{0.287 \times 290}{100} = 0.8323\ \frac{m^3}{kg}$$

$$v_e = \frac{RT_e}{P_e} = \frac{0.287 \times (52 + 273)}{110}$$

$$= 0.8479\ m^3/\ kg$$

$$\dot{m}_i = \dot{V}_i/v_i = 0.25/0.8323 = 0.30\ kg/s$$

$$\mathbf{V}_e = \dot{m}\ v_e/\ A_e = \frac{0.3 \times 0.8479}{0.2 \times 0.2}\ \frac{m^3/s}{m^2} = \mathbf{6.36\ m/s}$$

6.17

A flat channel of depth 1 m has a fully developed laminar flow of air at P_o, T_o with a velocity profile as: $V = 4V_c\,(1 – x/H)x/H$, where V_c is the velocity on the center-line and x is the distance across the channel as shown in Fig. P6.17. Find the total mass flow rate and the average velocity both as functions of V_c and H.

$$\dot{m} = AV/v = \dot{V}/v$$

Since the velocity is distributed we need to integrate over the area. From Eq.6.2

$$\dot{V} = \int V_{local}\, dA = \int V(x)\, W\, dx$$

where W is the depth. Substituting the velocity we get

$$\dot{V} = \int 4V_c\,(x/H)\,(1 - x/H)\,W\,dx$$

$$= 4\,V_c\,WH \int_0^1 z\,(1 - z)\,dz \qquad (z = x/H)$$

$$= 4\,V_c\,WH\,(\tfrac{1}{2}z^2 - \tfrac{1}{3}z^3\,\Big|_0^1 = \frac{2}{3}V_c\,WH = \frac{2}{3}V_c\,A$$

$$V = \dot{V}/A = \frac{2}{3}V_c$$

$$\dot{m} = \dot{V}/\mathbf{v} = \frac{2}{3}V_c\,WH/v$$

6.18

Nitrogen gas flowing in a 50-mm diameter pipe at 15°C, 200 kPa, at the rate of 0.05 kg/s, encounters a partially closed valve. If there is a pressure drop of 30 kPa across the valve and essentially no temperature change, what are the velocities upstream and downstream of the valve?

Solution:

Same inlet and exit area: $A = \frac{\pi}{4}(0.050)^2 = 0.001963 \text{ m}^2$

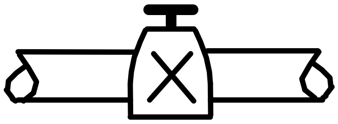
Ideal gas: $v_i = \frac{RT_i}{P_i} = \frac{0.2968 \times 288.2}{200} = 0.4277 \text{ m}^3\text{/kg}$

From Eq.6.3,

$$\mathbf{V}_i = \frac{\dot{m}v_i}{A} = \frac{0.05 \times 0.4277}{0.001963} = \mathbf{10.9\ m/s}$$

Ideal gas: $v_e = \frac{RT_e}{P_e} = \frac{0.2968 \times 288.2}{170} = 0.5032 \text{ m}^3\text{/kg}$

$$\mathbf{V}_e = \frac{\dot{m}v_e}{A} = \frac{0.05 \times 0.5032}{0.001963} = \mathbf{12.8\ m/s}$$



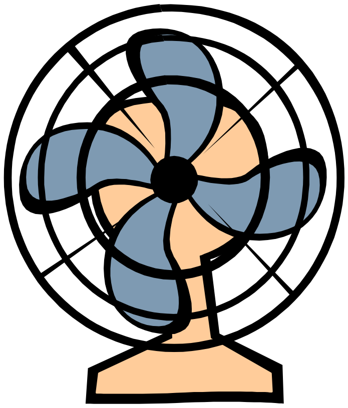
6.19

A household fan of diameter 0.75 m takes air in at 98 kPa, 22°C and delivers it at 105 kPa, 23°C with a velocity of 1.5 m/s. What are the mass flow rate (kg/s), the inlet velocity and the outgoing volume flow rate in m^3/s?

Solution:

Continuity Eq. $\dot{m}_i = \dot{m}_e = A\mathbf{V}/ v$

Ideal gas $v = RT/P$



Area : $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.75^2 = 0.442 \text{ m}^2$

$\dot{V}_e = A\mathbf{V}_e = 0.442 \times 1.5 = \mathbf{0.6627\ m^3/s}$

$$v_e = \frac{RT_e}{P_e} = \frac{0.287 \times (23 + 273)}{105} = 0.8091 \text{ m}^3/\text{kg}$$

$\dot{m}_i = \dot{V}_e/v_e = 0.6627/0.8091 = \mathbf{0.819\ kg/s}$

$A\mathbf{V}_i /v_i = \dot{m}_i = A\mathbf{V}_e / v_e$

$$\mathbf{V}_i = \mathbf{V}_e \times (v_i / v_e) = \mathbf{V}_e \times \frac{RT_i}{P_i v_e} = 1.5 \times \frac{0.287 \times (22 + 273)}{98 \times 0.8091} \mathbf{= 1.6\ m/s}$$

Single flow single device processes

Nozzles, diffusers

6.20

Liquid water at 15°C flows out of a nozzle straight up 15 m. What is nozzle $\mathbf{V}_{exit}$?

Energy Eq.6.13: $h_{exit} + \frac{1}{2}\mathbf{V}_{exit}^2 + gH_{exit} = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gH_2$

If the water can flow 15 m up it has specific potential energy of gH_2 which must equal the specific kinetic energy out of the nozzle $\mathbf{V}_{exit}^2/2$. The water does not change P or T so h is the same.

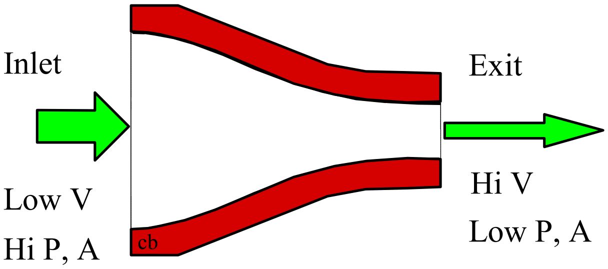
$$\mathbf{V}_{exit}^2/2 = g(H_2 - H_{exit}) = gH \quad =>$$
$$\mathbf{V}_{exit} = \sqrt{2gH} = \sqrt{2 \times 9.807 \times 15 \text{ m}^2/\text{s}^2} = \mathbf{17.15\ m/s}$$

6.21

Nitrogen gas flows into a convergent nozzle at 200 kPa, 400 K and very low velocity. It flows out of the nozzle at 100 kPa, 330 K. If the nozzle is insulated find the exit velocity.

Solution:

C.V. Nozzle steady state one inlet and exit flow, insulated so it is adiabatic.



Energy Eq.6.13: $h_1 + \emptyset = h_2 + \frac{1}{2}\mathbf{V}_2^2$

$$\mathbf{V}_2^2 = 2\,(\,h_1 - h_2\,) \cong 2\,C_{PN_2}\,(T_1 - T_2\,) = 2 \times 1.042\,(400 - 330)$$

$$= 145.88\text{ kJ/kg} = 145\,880\text{ J/kg}$$

$\Rightarrow$ $\mathbf{V}_2 = \mathbf{381.9\ m/s}$

6.22

A nozzle receives 0.1 kg/s steam at 1 MPa, $400^{o}C$ with negligible kinetic energy. The exit is at 500 kPa, $350^{o}C$ and the flow is adiabatic. Find the nozzle exit velocity and the exit area.

Solution:

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$

State 1: $V_1 = 0$, Table B.1.3 $h_1 = 3263.88$ kJ/kg

State 2: Table B.1.3 $h_2 = 3167.65$ kJ/kg

Then from the energy equation

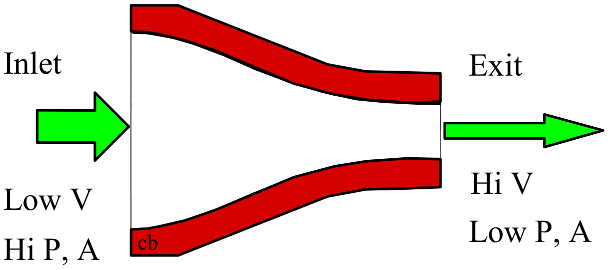
$$\frac{1}{2}\mathbf{V}_2^2 = h_1 - h_2 = 3263.88 - 3167.65 = 96.23 \text{ kJ/kg}$$

$$\mathbf{V}_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2 \times 96.23 \times 1000} = \mathbf{438.7\ m/s}$$

The mass flow rate from Eq.6.3

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v$$

$$A = \dot{m}v/\mathbf{V} = 0.1 \times 0.57012 / 438.7 = 0.00013\ m^2 = \mathbf{1.3\ cm^2}$$



6.23

In a jet engine a flow of air at 1000 K, 200 kPa and 30 m/s enters a nozzle, as shown in Fig. P6.23, where the air exits at 850 K, 90 kPa. What is the exit velocity assuming no heat loss?

Solution:

C.V. nozzle. No work, no heat transfer

Continuity $\dot{m}_i = \dot{m}_e = \dot{m}$

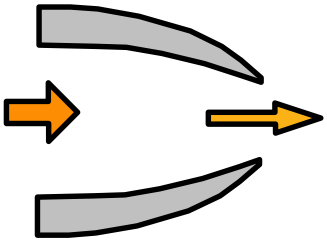
Energy : $\dot{m}\,(h_i + ½\mathbf{V}_i^2) = \dot{m}(h_e + ½\mathbf{V}_e^2)$

Due to high T take h from table A.7.1

$$½\mathbf{V}_e^2 = ½\,\mathbf{V}_i^2 + h_i - h_e$$

$$= \frac{1}{2000}(30)^2 + 1046.22 - 877.4$$

$$= 0.45 + 168.82 = 169.27 \text{ kJ/kg}$$



$$\mathbf{V}_e = (2000 \times 169.27)^{1/2} = \mathbf{581.8\ m/s}$$

6.24

In a jet engine a flow of air at 1000 K, 200 kPa and 40 m/s enters a nozzle where the air exits at 500 m/s, 90 kPa. What is the exit temperature assuming no heat loss?

Solution:

C.V. nozzle, no work, no heat transfer

Continuity $\dot{m}_i = \dot{m}_e = \dot{m}$

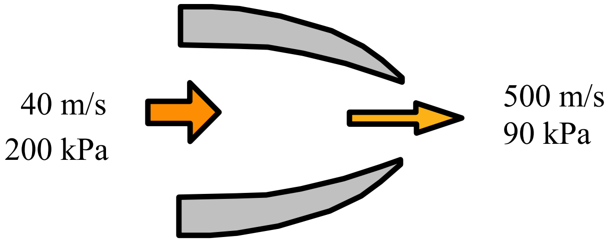
Energy : $\dot{m}\,(h_i + ½\mathbf{V}_i^2) = \dot{m}(h_e + ½\mathbf{V}_e^2)$

Due to the high T we take the h value from Table A.7.1

$$h_e = h_i + ½\,\mathbf{V}_i^2 - ½\mathbf{V}_e^2$$
$$= 1046.22 + 0.5 \times (40^2 – 500^2)\ (1/1000)$$
$$= 1046.22 – 124.2 = 922.02 \text{ kJ/kg}$$

Interpolation in Table A.7.1

$$T_e = 850 + 50\,\frac{922.02 - 877.4}{933.15 - 877.4} = \mathbf{890\ K}$$



6.25

Superheated vapor ammonia enters an insulated nozzle at 20°C, 800 kPa, shown in Fig. P6.25, with a low velocity and at the steady rate of 0.01 kg/s. The ammonia exits at 300 kPa with a velocity of 450 m/s. Determine the temperature (or quality, if saturated) and the exit area of the nozzle.

Solution:

C.V. Nozzle, steady state, 1 inlet and 1 exit flow, insulated so no heat transfer.

Energy Eq.6.13: $q + h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2,$

Process: $q = 0, \quad \mathbf{V}_i = 0$

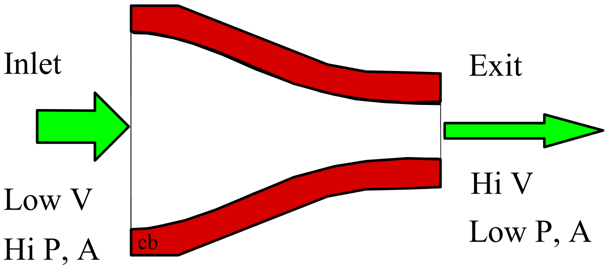
Table B.2.2: $h_i = 1464.9 = h_e + 450^2/(2\times1000) \quad \Rightarrow \quad h_e = 1363.6 \text{ kJ/kg}$

Table B.2.1: $P_e = 300 \text{ kPa}$ Sat. state at −9.2°C :

$h_e = 1363.6 = 138.0 + x_e \times 1293.8,$

$\Rightarrow \quad x_e = \mathbf{0.947}, \qquad v_e = 0.001536 + x_e \times 0.4064 = 0.3864 \text{ m}^3/\text{kg}$

$$A_e = \dot{m}_e v_e/\mathbf{V}_e = 0.01 \times 0.3864 / 450 = \mathbf{8.56 \times 10^{-6}\ m^2}$$



6.26

Air flows into a diffuser at 300 m/s, 300 K and 100 kPa. At the exit the velocity is very small but the pressure is high. Find the exit temperature assuming zero heat transfer.

Solution:

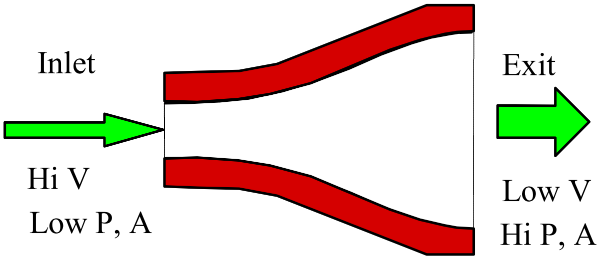
Energy Eq.: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$ and $V_2 = 0$

$$h_2 = h_1 + \frac{1}{2}\mathbf{V}_1^2$$

$$T_2 = T_1 + \frac{1}{2} \times (\mathbf{V}_1^2 / C_p)$$

$$= 300 + \frac{1}{2} \times 300^2 / (1.004 \times 1000) = \mathbf{344.8K}$$



6.27

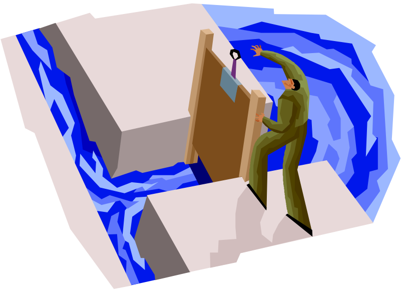
A sluice gate dams water up 5 m. There is a small hole at the bottom of the gate so liquid water at 20^{o}C comes out of a 1 cm diameter hole. Neglect any changes in internal energy and find the exit velocity and mass flow rate.

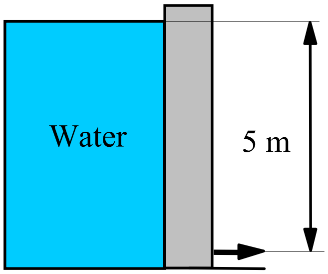
Solution:

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $h_1 = h_2$ both at P = 1 atm

$V_1 = 0$ $Z_1 = Z_2 + 5$ m





$$\frac{1}{2}\mathbf{V}_2^2 = g\,(Z_1 - Z_2)$$

$$\mathbf{V}_2 = \sqrt{2g(Z_1 - Z_2)} = \sqrt{2 \times 9.806 \times 5} = \mathbf{9.902\ m/s}$$

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v = \frac{\pi}{4} D^2 \times (\mathbf{V}_2/v)$$

$$= \frac{\pi}{4} \times (0.01)^2 \times (9.902 / 0.001002) = \mathbf{0.776\ kg/s}$$

6.28

A diffuser, shown in Fig. P6.28, has air entering at 100 kPa, 300 K, with a velocity of 200 m/s. The inlet cross-sectional area of the diffuser is 100 mm^2. At the exit, the area is 860 mm^2, and the exit velocity is 20 m/s. Determine the exit pressure and temperature of the air.

Solution:

Continuity Eq.6.3: $\dot{m}_i = A_i\mathbf{V}_i/v_i = \dot{m}_e = A_e\mathbf{V}_e/v_e$,

Energy Eq.(per unit mass flow)6.13: $h_i + \frac{1}{2}\mathbf{V}_i^2 = h_e + \frac{1}{2}\mathbf{V}_e^2$

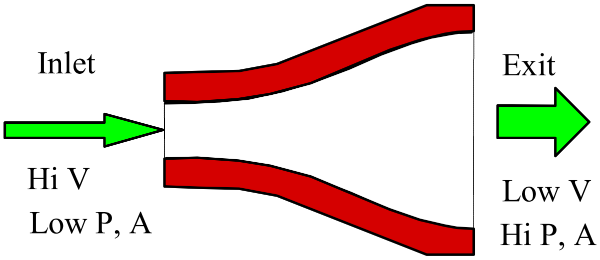
$$h_e - h_i = \frac{1}{2} \times 200^2/1000 - \frac{1}{2} \times 20^2/1000 = 19.8 \text{ kJ/kg}$$

$$T_e = T_i + (h_e - h_i)/C_p = 300 + 19.8/1.004 = \mathbf{319.72\ K}$$

Now use the continuity equation and the ideal gas law

$$v_e = v_i \left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right) = (RT_i/P_i)\left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right) = RT_e/P_e$$

$$P_e = P_i \left(\frac{T_e}{T_i}\right)\left(\frac{A_i\mathbf{V}_i}{A_e\mathbf{V}_e}\right) = 100\left(\frac{319.72}{300}\right)\left(\frac{100 \times 200}{860 \times 20}\right) = \mathbf{123.92\ kPa}$$



6.29

A diffuser receives an ideal gas flow at 100 kPa, 300 K with a velocity of 250 m/s and the exit velocity is 25 m/s. Determine the exit temperature if the gas is argon, helium or nitrogen.

Solution:

C.V. Diffuser: $\dot{m}_i = \dot{m}_e$ & assume no heat transfer ⇒

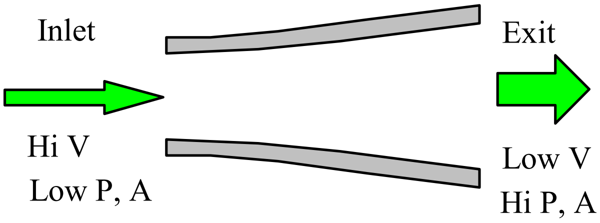
Energy Eq.6.13: $h_i + \frac{1}{2}\mathbf{V}_i^2 = \frac{1}{2}\mathbf{V}_e^2 + h_e \Rightarrow h_e = h_i + \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2$

$$h_e - h_i \approx C_p\,(T_e - T_i) = \frac{1}{2}(\mathbf{V}_i^2 - \mathbf{V}_e^2) = \frac{1}{2}(250^2 - 25^2)$$

$$= 30937.5 \text{ J/kg} = 30.938 \text{ kJ/kg}$$

Specific heats for ideal gases are from table A.5

Argon	$C_p = 0.52$ kJ/kg K;	$\Delta T = \frac{30.938}{0.52} = 59.5$	$\mathbf{T_e = 359.5\ K}$
Helium	$C_p = 5.913$ kJ/kg K;	$\Delta T = \frac{30.938}{5.193} = 5.96$	$\mathbf{T_e = 306\ K}$
Nitrogen	$C_p = 1.042$ kJ/kg K;	$\Delta T = \frac{30.938}{1.042} = 29.7$	$\mathbf{T_e = 330\ K}$



6.30

The front of a jet engine acts as a diffuser receiving air at 900 km/h, -5°C, 50 kPa, bringing it to 80 m/s relative to the engine before entering the compressor. If the flow area is reduced to 80% of the inlet area find the temperature and pressure in the compressor inlet.

Solution:

C.V. Diffuser, Steady state, 1 inlet, 1 exit flow, no q, no w.

Continuity Eq.6.3: $\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)$

Energy Eq.6.12: $\dot{m}\,(h_i + \frac{1}{2}\mathbf{V}_i^2) = \dot{m}\,(\frac{1}{2}\mathbf{V}_e^2 + h_e)$

$$h_e - h_i = C_p (T_e - T_i) = \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2 = \frac{1}{2}\left(\frac{900 \times 1000}{3600}\right)^2 - \frac{1}{2}(80)^2$$

$$= 28050 \text{ J/kg} = 28.05 \text{ kJ/kg}$$

$\Delta T = 28.05/1.004 = 27.9 \;\Rightarrow\; T_e = -5 + 27.9 =$ **22.9°C**

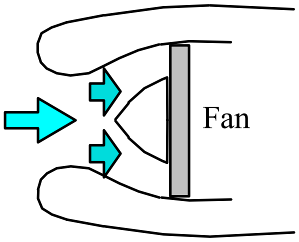
Now use the continuity eq.:

$$A_i\mathbf{V}_i / v_i = A_e\mathbf{V}_e / v_e \;\Rightarrow\; v_e = v_i\left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right)$$

$$v_e = v_i \times \frac{0.8 \times 80}{1 \times 250} = v_i \times 0.256$$

Ideal gas: $Pv = RT \;\Rightarrow\; v_e = RT_e/P_e = RT_i \times 0.256/P_i$

$P_e = P_i\,(T_e/T_i)/0.256 = 50 \times 296/268 \times 0.256 =$ **215.7 kPa**



Throttle flow

6.31

Carbon dioxide used as a natural refrigerant flows out of a cooler at 10 MPa, 40^oC after which it is throttled to 1.4 MPa. Find the state (T, x) for the exit flow.

C.V. Throttle. Steady state,

Process with: $q = w = 0$; and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.6.13: $h_i = h_e$,

Inlet state: Table B.3.2 $h_i = 200.14$ kJ/kg

Exit state: Table B.3.1 since $h < h_g = 323.87$ kJ/kg

Interpolate: $T = \mathbf{-30.6^oC}$, $h_f = 19.2$, $h_{fg} = 304.67$ kJ/kg

$x = (200.14 – 19.2) / 304.67 = \mathbf{0.594}$

6.32

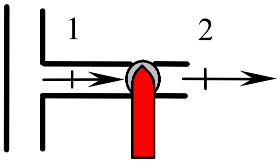
R-134a at 30^{o}C, 800 kPa is throttled so it becomes cold at –10^{o}C. What is exit P?

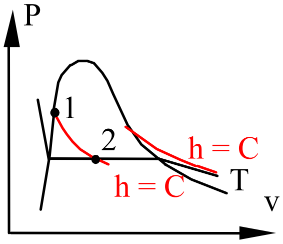
State 1 is slightly compressed liquid so

$$\text{Table B.5.1: } h = h_f = 241.79 \text{ kJ/kg}$$

At the lower temperature it becomes two-phase since the throttle flow has constant h and at –10^{o}C: $h_g = 392.28$ kJ/kg

$$P = P_{sat} = \mathbf{201.7\ kPa}$$





6.33

Helium is throttled from 1.2 MPa, 20°C, to a pressure of 100 kPa. The diameter of the exit pipe is so much larger than the inlet pipe that the inlet and exit velocities are equal. Find the exit temperature of the helium and the ratio of the pipe diameters.

Solution:

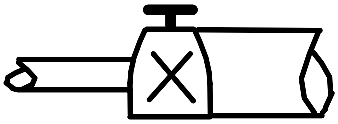
C.V. Throttle. Steady state,

Process with: $q = w = 0$; and $\mathbf{V}_i = \mathbf{V}_e$, $Z_i = Z_e$

Energy Eq.6.13: $h_i = h_e$, Ideal gas => $T_i = T_e = \mathbf{20°C}$

$\dot{m} = \frac{A\mathbf{V}}{RT/P}$ But $\dot{m}$, $\mathbf{V}$, T are constant => $P_iA_i = P_eA_e$

$$\Rightarrow \quad \frac{D_e}{D_i} = \left(\frac{P_i}{P_e}\right)^{1/2} = \left(\frac{1.2}{0.1}\right)^{1/2} = \mathbf{3.464}$$



6.34

Saturated vapor R-134a at 500 kPa is throttled to 200 kPa in a steady flow through a valve. The kinetic energy in the inlet and exit flow is the same. What is the exit temperature?

Solution:

Steady throttle flow

Continuity $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

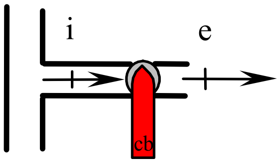
Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = \mathbf{V}_1$

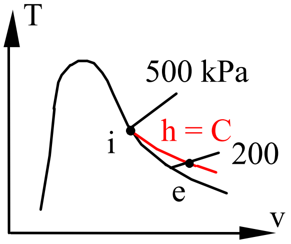
$\Rightarrow \quad h_2 = h_1 = 407.45$ kJ/kg from Table B.5.2

State 2: P_2 & $h_2 \quad \Rightarrow$ superheated vapor

Interpolate between 0°C and 10°C in table B.5.2 in the 200 kPa subtable

$$T_2 = 0 + 10\,\frac{407.45 - 400.91}{409.5 - 400.91} = \mathbf{7.6^oC}$$





6.35

Saturated liquid R-410a at 25°C is throttled to 400 kPa in your refrigerator. What is the exit temperature? Find the percent increase in the volume flow rate.

Solution:

Steady throttle flow. Assume no heat transfer and no change in kinetic or potential energy.

$$h_e = h_i = h_{f\,25^oC} = 97.59 \text{ kJ/kg} = h_{f\,e} + x_e\, h_{fg\,e} \quad \text{at 400 kPa}$$

From table B.3.1 we get $T_e = T_{sat}$ (400 kPa) = **-20°C**

$$x_e = \frac{h_e - h_{f\,e}}{h_{fg\,e}} = \frac{97.59 - 28.24}{243.65} = 0.28463$$

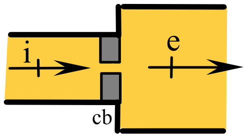
$$v_e = v_f + x_e\, v_{fg} = 0.000803 + x_e\, 0.064 = 0.01902 \text{ m}^3\text{/kg}$$

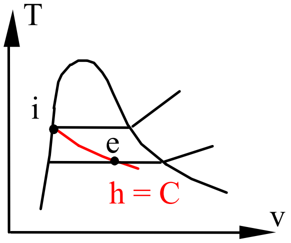
$$v_i = v_{f\,25^oC} = 0.000944 \text{ m}^3\text{/kg}$$

$\dot{V} = \dot{m}v$ so the ratio becomes

$$\frac{\dot{V}_e}{\dot{V}_i} = \frac{\dot{m}v_e}{\dot{m}v_i} = \frac{v_e}{v_i} = \frac{0.01902}{0.000994} = \mathbf{20.23}$$

So the increase is 19.23 times or **1923 %**





6.36

Carbon dioxide is throttled from 20^{o}C, 2 MPa to 800 kPa. Find the exit temperature assuming ideal gas behavior and repeat for real-gas behavior.

C.V. Throttle (valve, restriction), Steady flow, 1 inlet and exit, no q, w

Energy Eq.6.13: $h_i = h_e$

Ideal gas: same h gives $T_i = T_e = 20^{o}C$

Real gas : $\left.\begin{array}{l} h_i = h_e = 368.42 \text{ kJ/kg} \\ P_e = 0.8 \text{ MPa} \end{array}\right\}$ Table B.3.2 $T_e =$ **5.3°C** (= **278 K**)

6.37

Liquid water at 180°C, 2000 kPa is throttled into a flash evaporator chamber having a pressure of 500 kPa. Neglect any change in the kinetic energy. What is the fraction of liquid and vapor in the chamber?

Solution:

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = \mathbf{V}_1$

$\Rightarrow \quad h_2 = h_1 = 763.71$ kJ/kg from Table B.1.4

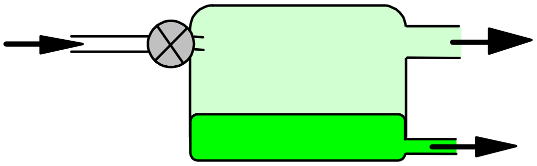
State 2: P_2 & h_2 $\Rightarrow$ 2 – phase

$$h_2 = h_f + x_2\, h_{fg}$$

$$x_2 = (h_2 - h_f) / h_{fg} = \frac{763.71 - 640.21}{2108.47} = 0.0586$$

Fraction of Vapor: $x_2 = 0.0586$ (5.86 %)

Liquid: $1 - x_2 = 0.941$ (94.1 %)



Two-phase out of the valve. The liquid drops to the bottom.

6.38

R-134a is throttled in a line flowing at 25^{o}C, 750 kPa with negligible kinetic energy to a pressure of 165 kPa. Find the exit temperature and the ratio of exit pipe diameter to that of the inlet pipe (D_{ex}/D_{in}) so the velocity stays constant.

Solution:

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = \mathbf{V}_1$

State 1, Table B.5.1: $h_1 = 234.59$ kJ/kg, $v_1 = v_f = 0.000829$ m^3/kg

Use energy eq.: $\Rightarrow$ $h_2 = h_1 = 234.59$ kJ/kg

State 2: P_2 & h_2 $\Rightarrow$ 2 – phase and $T_2 = T_{sat}$ (165 kPa) = **-15°C**

$$h_2 = h_f + x_2\, h_{fg} = 234.59 \text{ kJ/kg}$$

$$x_2 = (h_2 - h_f) / h_{fg} = (234.59 – 180.19) / 209 = 0.2603$$

$$v_2 = v_f + x_2 \times v_{fg} = 0.000746 + 0.2603 \times 0.11932 = 0.0318 \text{ m}^3\text{/kg}$$

Now the continuity equation with $\mathbf{V}_2 = \mathbf{V}_1$ gives, from Eq.6.3,

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v = A_1\mathbf{V}_1/v_1 = (A_2\, \mathbf{V}_1) / v_2$$

$$(A_2 / A_1) = v_2 / v_1 = (D_2 / D_1)^2$$

$$(D_2/D_1) = (v_2 / v_1)^{0.5} = (0.0318 / 0.000829)^{0.5} = \mathbf{6.19}$$

6.39

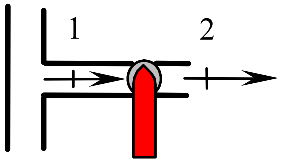
Water flowing in a line at 400 kPa, saturated vapor, is taken out through a valve to 100 kPa. What is the temperature as it leaves the valve assuming no changes in kinetic energy and no heat transfer?

Solution:

C.V. Valve. Steady state, single inlet and exit flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2$

Energy Eq.6.12: $\dot{m}_1 h_1 + \dot{Q} = \dot{m}_2 h_2 + \dot{W}$



Process: Throttling

Small surface area: $\dot{Q} = 0$;

No shaft: $\dot{W} = 0$

Table B.1.2: $h_2 = h_1 = 2738.6$ kJ/kg ⇒ $\mathbf{T_2 = 131.1°C}$

Turbines, Expanders

6.40

A steam turbine has an inlet of 2 kg/s water at 1000 kPa, 350^{o}C and velocity of 15 m/s. The exit is at 100 kPa, 150^{o}C and very low velocity. Find the specific work and the power produced.

Solution:

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2 + w_T$

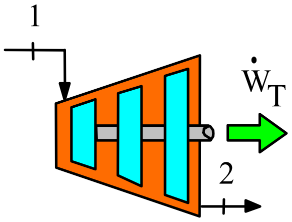
Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = 0$

Table B.1.3: $h_1 = 3157.65$ kJ/kg, $h_2 = 2776.38$ kJ/kg

$$w_T = h_1 + \frac{1}{2}\mathbf{V}_1^2 - h_2 = 3157.65 + \frac{15^2}{2000} - 2776.38 = \mathbf{381.4\ kJ/kg}$$

(remember to convert m^2/s^2 = J/kg to kJ/kg by dividing with 1000)

$$\dot{W}_T = \dot{m} \times w_T = 2 \text{ kg/s} \times 381.4 \text{ kJ/kg}$$
$$= \mathbf{762.8\ kW}$$

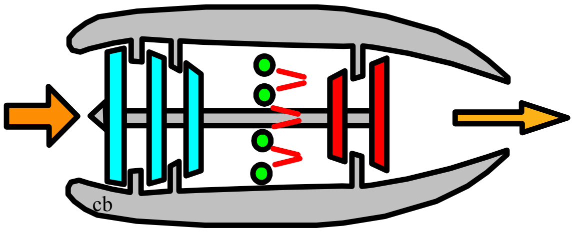


6.41

Air at 20 m/s, 260 K, 75 kPa with 5 kg/s flows into a jet engine and it flows out at 500 m/s, 800 K, 75 kPa. What is the change (power) in flow of kinetic energy?

$$\dot{m}\,\Delta KE = \dot{m}\,\frac{1}{2}\,(\mathbf{V}_e^2 - \mathbf{V}_i^2)$$

$$= 5\ \text{kg/s} \times \frac{1}{2}\,(500^2 - 20^2)\ (\text{m/s})^2\ \frac{1}{1000}\ (\text{kW/W}) = \mathbf{624\ kW}$$



6.42

A liquid water turbine receives 2 kg/s water at 2000 kPa, 20$^{\text{o}}$C and velocity of 15 m/s. The exit is at 100 kPa, 20$^{\text{o}}$C and very low velocity. Find the specific work and the power produced.

Solution:

Energy Eq.6.13: $h_1 + \frac{1}{2}\mathbf{V}_1^2 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + gZ_2 + w_T$

Process: $Z_1 = Z_2$ and $\mathbf{V}_2 = 0$

State 1: Table B.1.4 $h_1 = 85.82$ kJ/kg

State 2: Table B.1.1 $h_2 = 83.94$ (which is at 2.3 kPa so we should add $\Delta Pv = 97.7 \times 0.001$ to this)

$w_T = h_1 + \frac{1}{2}\mathbf{V}_1^2 - h_2 = 85.82 + 15^2/2000 - 83.94 =$ **1.99 kJ/kg**

$\dot{W}_T = \dot{m} \times w_T = 2 \times 1.9925 =$ **3.985 kW**

Notice how insignificant the specific kinetic energy is.

6.43

A windmill with rotor diameter of 30 m takes 40% of the kinetic energy out as shaft work on a day with 20^{o}C and wind speed of 30 km/h. What power is produced?

Solution:

Continuity Eq. $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy $\dot{m}\,(h_i + ½\mathbf{V}_i^2 + gZ_i) = \dot{m}(h_e + ½\mathbf{V}_e^2 + gZ_e) + \dot{W}$

Process information: $\dot{W} = \dot{m}½\mathbf{V}_i^2 \times 0.4$

$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}_i / v_i$

$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} 30^2 = 706.85 \text{ m}^2$

$v_i = RT_i/P_i = \frac{0.287 \times 293}{101.3} = 0.8301 \text{ m}^3\text{/kg}$

$\mathbf{V}_i = 30 \text{ km/h} = \frac{30 \times 1000}{3600} = 8.3333 \text{ m/s}$

$\dot{m} = A\mathbf{V}_i / v_i = \frac{706.85 \times 8.3333}{0.8301} = 7096 \text{ kg/s}$

$½\,\mathbf{V}_i^2 = ½\ 8.3333^2 \text{ m}^2\text{/s}^2 = 34.722 \text{ J/kg}$

$\dot{W} = 0.4\,\dot{m}½\,\mathbf{V}_i^2 = 0.4 \times 7096 \times 34.722 = 98\,555 \text{ W}$

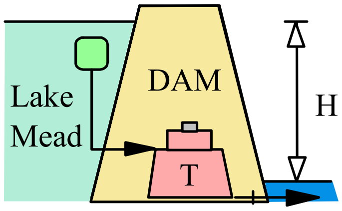
$= \mathbf{98.56\ kW}$

6.44

Hoover Dam across the Colorado River dams up Lake Mead 200 m higher than the river downstream. The electric generators driven by water-powered turbines deliver 1300 MW of power. If the water is 17.5°C, find the minimum amount of water running through the turbines.

Solution:

C.V.: H_2O pipe + turbines,



Continuity: $\dot{m}_{in} = \dot{m}_{ex}$;

Energy Eq.6.13: $(h + \mathbf{V}^2/2 + gz)_{in} = (h + \mathbf{V}^2/2 + gz)_{ex} + w_T$

Water states: $h_{in} \cong h_{ex}$; $v_{in} \cong v_{ex}$

Now the specific turbine work becomes

$$w_T = gz_{in} - gz_{ex} = 9.807 \times 200/1000 = 1.961 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_T/w_T = \frac{1300 \times 10^3 \text{ kW}}{1.961 \text{ kJ/kg}} = 6.63 \times 10^5 \text{ kg/s}$$

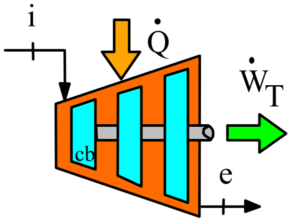
$$\dot{V} = \dot{m}v = 6.63 \times 10^5 \times 0.001001 = \mathbf{664\ m^3/s}$$

6.45

A small expander (a turbine with heat transfer) has 0.05 kg/s helium entering at 1000 kPa, 550 K and it leaves at 250 kPa, 300 K. The power output on the shaft is measured to 55 kW. Find the rate of heat transfer neglecting kinetic energies.

Solution:

C.V. Expander. Steady operation



Cont. $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy $\dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

$$\dot{Q} = \dot{m}\,(h_e - h_i) + \dot{W}$$

Use heat capacity from tbl A.5: $C_{p\ He} = 5.193$ kJ/kg K

$$\dot{Q} = \dot{m}C_p\,(T_e - T_i) + \dot{W}$$
$$= 0.05 \times 5.193\,(300 - 550) + 55$$
$$= -\,64.91 + 55 = \mathbf{-9.9\ kW}$$

6.46

A small turbine, shown in Fig. P 6.46, is operated at part load by throttling a 0.25 kg/s steam supply at 1.4 MPa, 250°C down to 1.1 MPa before it enters the turbine and the exhaust is at 10 kPa. If the turbine produces 110 kW, find the exhaust temperature (and quality if saturated).

Solution:

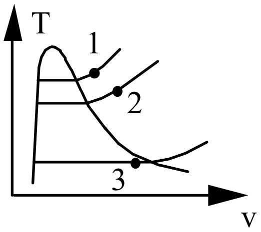
C.V. Throttle, Steady, $q = 0$ and $w = 0$. No change in kinetic or potential energy. The energy equation then reduces to

Energy Eq.6.13: $h_1 = h_2 = 2927.2$ kJ/kg from Table B.1.3

C.V. Turbine, Steady, no heat transfer, specific work: $w = \frac{110}{0.25} = 440$ kJ/kg

Energy Eq.: $h_1 = h_2 = h_3 + w = 2927.2$ kJ/kg

⇒ $h_3 = 2927.2 - 440 = 2487.2$ kJ/kg



State 3: (P, h) Table B.1.2 $h < h_g$

$2487.2 = 191.83 + x_3 \times 2392.8$

⇒ $T =$ **45.8°C** , $x_3 =$ **0.959**

6.47

A small, high-speed turbine operating on compressed air produces a power output of 100 W. The inlet state is 400 kPa, 50°C, and the exit state is 150 kPa, −30°C. Assuming the velocities to be low and the process to be adiabatic, find the required mass flow rate of air through the turbine.

Solution:

C.V. Turbine, no heat transfer, no ΔKE, no ΔPE

Energy Eq.6.13: $h_{in} = h_{ex} + w_T$

Ideal gas so use constant specific heat from Table A.5

$$w_T = h_{in} - h_{ex} \cong C_p(T_{in} - T_{ex})$$

$$= 1.004 \text{ (kJ/kg K) } [50 - (-30)] \text{ K} = 80.3 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w_T \quad \Rightarrow \quad \dot{m} = \dot{W}/w_T = 0.1/80.3 = \mathbf{0.00125\ kg/s}$$

The dentist's drill has a small air flow and is not really adiabatic.

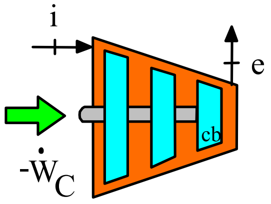
Compressors, fans

6.48

A compressor in a commercial refrigerator receives R-410a at -25°C, $x = 1$. The exit is at 800 kPa, 40°C. Neglect kinetic energies and find the specific work.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$



From Table B.4.1 : $h_i = 269.77$ kJ/kg

From Table B.4.2 : $h_e = 319.42$ kJ/kg

Energy Eq.6.13 reduces to

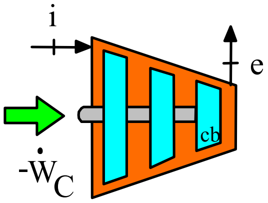
$$w_C = h_i - h_e = (269.77 - 319.42) = \mathbf{-49.65\ kJ/kg}$$

6.49

A refrigerator uses the natural refrigerant carbon dioxide where the compressor brings 0.02 kg/s from 1 MPa, -20^{o}C to 6 MPa using 2 kW of power. Find the compressor exit temperature.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer and $Z_i = Z_e$



From Table B.3.2 : $h_i = 342.31$ kJ/kg

Energy Eq.6.13 reduces to

$$\dot{W} = \dot{m}w_C = \dot{m}\,(h_i - h_e) \Rightarrow \qquad h_e = h_i - \dot{W}/\dot{m}$$

$$h_e = (342.31 - (-2)/0.02) = 442.31 \text{ kJ/kg}$$

From Table B.3.2 :

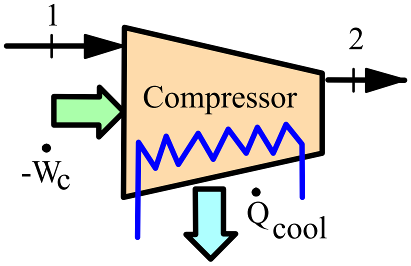
$$T_e = 100 + 20\,(442.31 - 421.69)/(445.02 - 421.69) = \mathbf{117.7\ ^oC}$$

6.50

A compressor brings R-134a from 150 kPa, -10^{o}C to 1200 kPa, 50^{o}C. It is water cooled with a heat loss estimated as 40 kW and the shaft work input is measured to be 150 kW. How much is the mass flow rate through the compressor?

Solution:

C.V Compressor. Steady flow.
Neglect kinetic and potential energies.



Energy : $\dot{m}\, h_i + \dot{Q} = \dot{m} h_e + \dot{W}$

$$\dot{m} = (\dot{Q} - \dot{W})/(h_e - h_i)$$

Look in table B.5.2

$h_i = 393.84$ kJ/kg, $\qquad h_e = 426.84$ kJ/kg

$$\dot{m} = \frac{-40 - (-150)}{426.84 - 393.84} = \mathbf{3.333\ kg/s}$$

6.51

An ordinary portable fan blows 0.2 kg/s room air with a velocity of 18 m/s (see Fig. P6.19). What is the minimum power electric motor that can drive it? Hint: Are there any changes in P or T?

Solution:

C.V. Fan plus space out to near stagnant inlet room air.

Energy Eq.6.13: $q + h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2 + w$

Here $q \cong 0$, $\mathbf{V}_i \cong 0$ and $h_i = h_e$ same P and T

$-w = \mathbf{V}_e^2/2 = 18^2/2000 = 0.162$ kJ/kg

$-\dot{W} = -\dot{m}w = 0.2$ kg/s × 0.162 kJ/kg = **0.032 kW**

6.52

The compressor of a large gas turbine receives air from the ambient at 95 kPa, 20°C, with a low velocity. At the compressor discharge, air exits at 1.52 MPa, 430°C, with velocity of 90 m/s. The power input to the compressor is 5000 kW. Determine the mass flow rate of air through the unit.

Solution:

C.V. Compressor, steady state, single inlet and exit flow.

Energy Eq.6.13: $q + h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2 + w$

Here we assume $q \cong 0$ and $\mathbf{V}_i \cong 0$ so using constant C_{Po} from A.5

$$-w = C_{Po}(T_e - T_i) + \mathbf{V}_e^2/2 = 1.004(430 - 20) + \frac{(90)^2}{2 \times 1000} = 415.5 \text{ kJ/kg}$$

Notice the kinetic energy is 1% of the work and can be neglected in most cases. The mass flow rate is then from the power and the specific work

$$\dot{m} = \frac{\dot{W}_c}{-w} = \frac{5000}{415.5} = \mathbf{12.0\ kg/s}$$

6.53

A compressor in an industrial air-conditioner compresses ammonia from a state of saturated vapor at 150 kPa to a pressure of 800 kPa. At the exit the temperature is measured to be 100$^{\mathrm{o}}$C and the mass flow rate is 0.5 kg/s. What is the required motor size (kW) for this compressor?

Solution:

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w_C = h_1 - h_2$

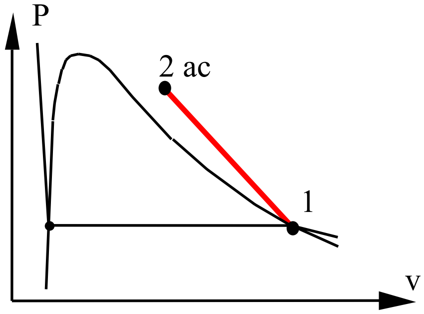
States: 1: B.2.2: $h_1 = 1410.9$ kJ/kg

2: B.1.3 $h_{2,AC} = 1670.6$ kJ/kg

Energy equation:

$$-w_C = h_2 - h_1 = 1670.6 - 1410.9 = 259.7 \text{ kJ/kg}$$

$$\dot{W} = 0.5 \text{ kg/s} \times 259.7 \text{ kJ/kg} = 129.8 \text{ kW}$$



6.54

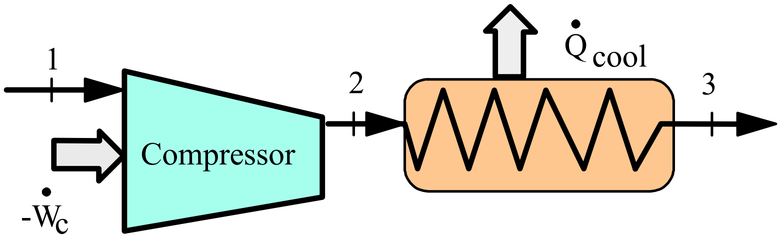
An air compressor takes in air at 100 kPa, 17°C and delivers it at 1 MPa, 600 K to a constant-pressure cooler, which it exits at 300 K. Find the specific compressor work and the specific heat transfer in the cooler.

Solution

C.V. air compressor $q = 0$

Continuity Eq.: $\dot{m}_2 = \dot{m}_1$

Energy Eq.6.13: $h_1 + w_c = h_2$



Compressor section Cooler section

Table A.7:

$w_{c\ in} = h_2 - h_1 = 607.02 - 290.17 = \mathbf{316.85\ kJ/kg}$

C.V. cooler $w = 0$

Continuity Eq.: $\dot{m}_3 = \dot{m}_1$

Energy Eq.6.13: $h_2 = q_{out} + h_3$

$q_{out} = h_2 - h_3 = 607.02 - 300.19 = \mathbf{306.83\ kJ/kg}$

6.55

An exhaust fan in a building should be able to move 2.5 kg/s air at 98 kPa, 20°C through a 0.4 m diameter vent hole. How high a velocity must it generate and how much power is required to do that?

Solution:

C.V. Fan and vent hole. Steady state with uniform velocity out.

Continuity Eq.: $\dot{m} = \text{constant} = \rho A\mathbf{V} = A\mathbf{V} / v = A\mathbf{V}P/RT$

Ideal gas : $Pv = RT$, and area is $A = \frac{\pi}{4} D^2$

Now the velocity is found

$$\mathbf{V} = \dot{m}\, RT/(\frac{\pi}{4} D^2P) = 2.5 \times 0.287 \times 293.15 / (\frac{\pi}{4} \times 0.4^2 \times 98) = \mathbf{17.1\ m/s}$$

The kinetic energy out is

$$\frac{1}{2}\mathbf{V}_2^2 = \frac{1}{2} \times 17.1^2 / 1000 = 0.146 \text{ kJ/kg}$$

which is provided by the work (only two terms in energy equation that does not cancel, we assume $\mathbf{V}_1 = 0$)

$$\dot{W}_{in} = \dot{m}\ \frac{1}{2}\mathbf{V}_2^2 = 2.5 \times 0.146 = \mathbf{0.366\ kW}$$

6.56

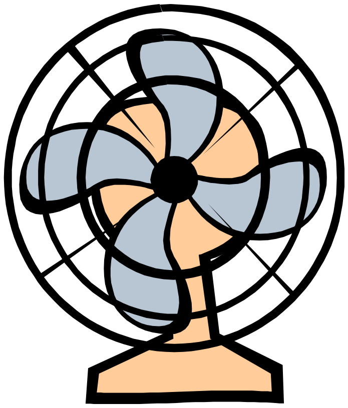
How much power is needed to run the fan in Problem 6.19?

A household fan of diameter 0.75 m takes air in at 98 kPa, 22°C and delivers it at 105 kPa, 23°C with a velocity of 1.5 m/s. What are the mass flow rate (kg/s), the inlet velocity and the outgoing volume flow rate in m^3/s?

Solution:

Continuity Eq. $\dot{m}_i = \dot{m}_e = A\mathbf{V}/v$

Ideal gas $v = RT/P$



Area : $A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.75^2 = 0.442\ m^2$

$\dot{V}_e = A\mathbf{V}_e = 0.442 \times 1.5 = \mathbf{0.6627\ m^3/s}$

$$v_e = \frac{RT_e}{P_e} = \frac{0.287 \times 296}{105} = 0.8091\ m^3/kg$$

$\dot{m}_i = \dot{V}_e/v_e = 0.6627/0.8091 = \mathbf{0.819\ kg/s}$

$A\mathbf{V}_i / v_i = \dot{m}_i = A\mathbf{V}_e / v_e$

$$\mathbf{V}_i = \mathbf{V}_e \times (v_i / v_e) = \mathbf{V}_e \times (RT_i)/(P_i v_e) = 1.5 \times \frac{0.287 \times (22 + 273)}{98 \times 0.8091} = \mathbf{1.6\ m/s}$$

$$\dot{m}\,(h_i + ½\mathbf{V}_i^2) = \dot{m}(h_e + ½\mathbf{V}_e^2) + \dot{W}$$

$$\dot{W} = \dot{m}(h_i + ½V_i^2 - h_e - ½V_e^2) = \dot{m}\,[C_p\,(T_i - T_e) + ½\,V_i^2 - ½V_e^2\,]$$

$$= 0.819\ [\ 1.004\ (-1) + \frac{1.6^2 - 1.5^2}{2000}] = 0.819\ [\ -1.004 + 0.000155]$$

$= \mathbf{-\ 0.81\ kW}$

6.57

A compressor in an air-conditioner receives saturated vapor R-410a at 400 kPa and brings it to 1.8 MPa, 60^{o}C in an adiabatic compression. Find the flow rate for a compressor work of 2 kW?

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w = h_1 - h_2$

States: 1: B.4.2 $h_1 = 271.9$ kJ/kg

2: B.4.2 $h_2 = 323.92$ kJ/kg

$-w_C = h_2 - h_1 = 323.92 - 271.9 = 52.02$ kJ/kg

$\dot{W} = \dot{m}\, w_C \quad \Rightarrow \quad \dot{m} = \dot{W} / w_C = 2 \text{ kW} / 52.02 \text{ kJ/kg} = \mathbf{0.038\ kg/s}$

Heaters/Coolers

6.58

Carbon dioxide enters a steady-state, steady-flow heater at 300 kPa, 300 K, and exits at 275 kPa, 1500 K, as shown in Fig. P6.58. Changes in kinetic and potential energies are negligible. Calculate the required heat transfer per kilogram of carbon dioxide flowing through the heater.

Solution:

C.V. Heater Steady state single inlet and exit flow.

Energy Eq.6.13: $q + h_i = h_e$

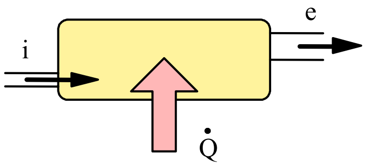


Table A.8: $q = h_e - h_i = 1614.9 – 214.4 =$ **1400.5 kJ/kg**

[If we use C_{p0} from A.5 then $q \cong 0.842(1500 - 300) = 1010.4$ kJ/kg]

Too large ΔT, T_{ave} to use C_{p0} at room temperature.

6.59

A condenser (cooler) receives 0.05 kg/s R-410a at 2000 kPa, 60^oC and cools it to 15^oC. Assume the exit properties are as for saturated liquid same T. What cooling capacity (kW) must the condenser have?

Solution:

C.V. R-410a condenser. Steady state single flow, heat transfer out and no work.

Energy Eq.6.12: $\dot{m}\, h_1 = \dot{m}\, h_2 + \dot{Q}_{out}$

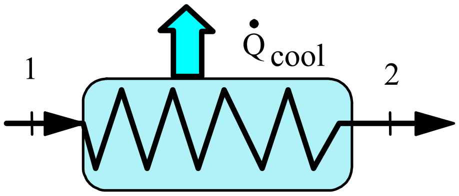
Inlet state: Table B.4.2 $h_1 = 320.62$ kJ/kg,

Exit state: Table B.4.1 $h_2 = 81.15$ kJ/kg (compressed liquid)

Process: Neglect kinetic and potential energy changes.

Cooling capacity is taken as the heat transfer out i.e. positive out so

$$\dot{Q}_{out} = \dot{m}\,(h_1 - h_2) = 0.05 \text{ kg/s } (320.62 - 81.15) \text{ kJ/kg}$$
$$= 11.9735 \text{ kW} = \mathbf{12\ kW}$$



6.60

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.005 kg/s and exits as saturated vapor. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Find the rate of heat transfer in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

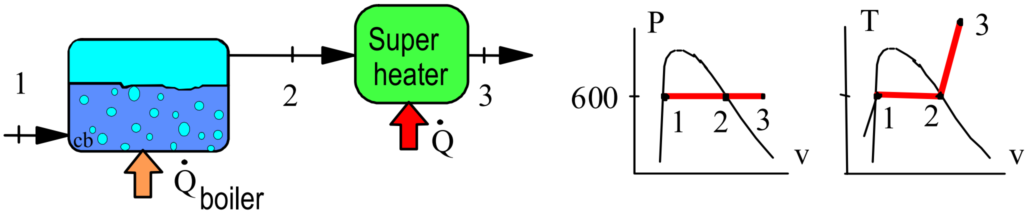


Table B.6.1: $h_1 = -81.469$ kJ/kg, $h_2 = 86.85$ kJ/kg,

Table B.6.2: $h_3 = 289.05$ kJ/kg

Energy Eq.6.13: $q_{boiler} = h_2 - h_1 = 86.85 - (-81.469) = 168.32$ kJ/kg

$\dot{Q}_{boiler} = \dot{m}_1 q_{boiler} = 0.005 \times 168.32 = \mathbf{0.842\ kW}$

C.V. Superheater (same approximations as for boiler)

Energy Eq.6.13: $q_{sup\ heater} = h_3 - h_2 = 289.05 - 86.85 = 202.2$ kJ/kg

$\dot{Q}_{sup\ heater} = \dot{m}_2 q_{sup\ heater} = 0.005 \times 202.2 = \mathbf{1.01\ kW}$

6.61

The air conditioner in a house or a car has a cooler that brings atmospheric air from 30^{o}C to 10^{o}C both states at 101 kPa. If the flow rate is 0.5 kg/s find the rate of heat transfer.

Solution:

CV. Cooler. Steady state single flow with heat transfer.

Neglect changes in kinetic and potential energy and no work term.

Energy Eq.6.13: $q_{out} = h_i - h_e$

Use constant heat capacity from Table A.5 (T is around 300 K)

$$q_{out} = h_i - h_e = C_p \, (T_i - T_e)$$

$$= 1.004 \, \frac{\text{kJ}}{\text{kg K}} \times (30 - 10) \text{ K} = 20.1 \text{ kJ/kg}$$

$$\dot{Q}_{out} = \dot{m} \, q_{out} = 0.5 \times 20.1 = \mathbf{10\ kW}$$

6.62

A chiller cools liquid water for air-conditioning purposes. Assume 2.5 kg/s water at 20^{o}C, 100 kPa is cooled to 5^{o}C in a chiller. How much heat transfer (kW) is needed?

Solution:

C.V. Chiller. Steady state single flow with heat transfer. Neglect changes in kinetic and potential energy and no work term.

Energy Eq.6.13: $q_{out} = h_i - h_e$

Properties from Table B.1.1:

$h_i = 83.94$ kJ/kg and $h_e = 20.98$ kJ/kg

Now the energy equation gives

$$q_{out} = 83.94 - 20.98 = 62.96 \text{ kJ/kg}$$

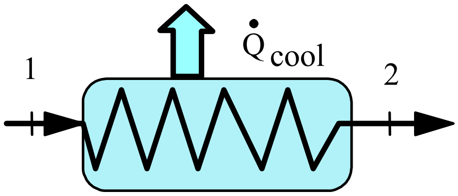
$$\dot{Q}_{out} = \dot{m}\, q_{out} = 2.5 \times 62.96 = \mathbf{157.4\ kW}$$

Alternative property treatment since single phase and small ΔT

If we take constant heat capacity for the liquid from Table A.4

$$q_{out} = h_i - h_e \cong C_p\,(T_i - T_e) \\ = 4.18\,(20 - 5) = 62.7 \text{ kJ/kg}$$

$$\dot{Q}_{out} = \dot{m}\, q_{out} = 2.5 \times 62.7 = \mathbf{156.75\ kW}$$



6.63

Carbon dioxide used as a natural refrigerant flows through a cooler at 10 MPa, which is supercritical so no condensation occurs. The inlet is at 200°C and the exit is at 40°C. Find the specific heat transfer.

C.V. Cooler. Steady state single flow with heat transfer. Neglect changes in kinetic and potential energy and no work term.

Energy Eq.6.13: $0 = h_i - h_e + q$

Properties from Table B.3.2:

$h_i = 519.49$ kJ/kg and $h_e = 200.14$ kJ/kg

Now the energy equation gives

$q = 200.14 - 519.49 =$ **-319.35 kJ/kg**

6.64

A flow of liquid glycerine flows around an engine, cooling it as it absorbs energy. The glycerine enters the engine at 60°C and receives 19 kW of heat transfer. What is the required mass flow rate if the glycerine should come out at maximum 95°C?

Solution:

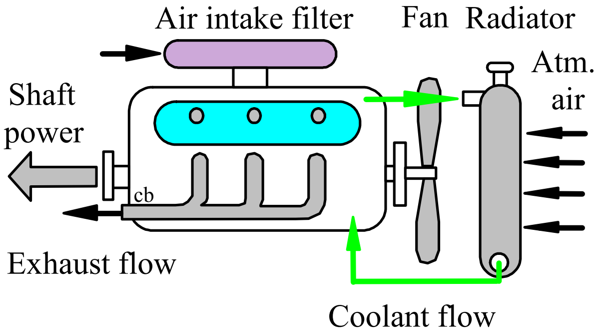
C.V. Liquid flow (glycerine is the coolant), steady flow. no work.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

$$\dot{m} = \dot{Q}/(\,h_e - h_i) = \frac{\dot{Q}}{C_{gly}\,(T_e - T_i)}$$

From table A.4 $\quad C_{gly} = 2.42$ kJ/kg-K

$$\dot{m} = \frac{19}{2.42\,(95 - 60)} = \mathbf{0.224\ kg/s}$$



6.65

In a steam generator, compressed liquid water at 10 MPa, 30°C, enters a 30-mm diameter tube at the rate of 3 L/s. Steam at 9 MPa, 400°C exits the tube. Find the rate of heat transfer to the water.

Solution:

C.V. Steam generator. Steady state single inlet and exit flow.

Constant diameter tube: $A_i = A_e = \frac{\pi}{4}(0.03)^2 = 0.0007068 \text{ m}^2$

Table B.1.4 $\dot{m} = \dot{V}_i/v_i = 0.003/0.0010003 = 3.0 \text{ kg/s}$

$$\mathbf{V}_i = \dot{V}_i/A_i = 0.003/0.0007068 = 4.24 \text{ m/s}$$

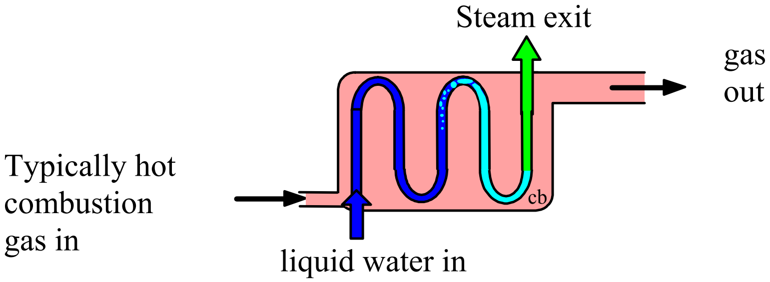
Exit state properties from Table B.1.3

$$\mathbf{V}_e = \mathbf{V}_i \times v_e/v_i = 4.24 \times 0.02993/0.0010003 = 126.86 \text{ m/s}$$

The energy equation Eq.6.12 is solved for the heat transfer as

$$\dot{Q} = \dot{m}\left[(h_e - h_i) + \left(\mathbf{V}_e^2 - \mathbf{V}_i^2\right)/2\right]$$

$$= 3.0\left[3117.8 - 134.86 + \frac{126.86^2 - 4.24^2}{2 \times 1000}\right] = \mathbf{8973\ kW}$$



6.66

In a boiler you vaporize some liquid water at 100 kPa flowing at 1 m/s. What is the velocity of the saturated vapor at 100 kPa if the pipe size is the same? Can the flow then be constant P?

The continuity equation with average values is written

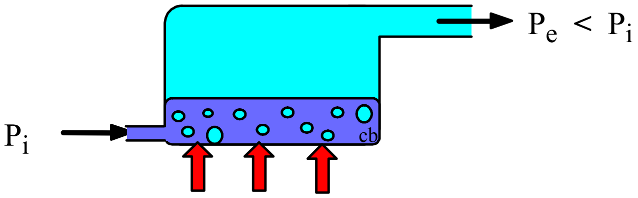
$$\dot{m}_i = \dot{m}_e = \dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v = A\mathbf{V}_i/v_i = A\mathbf{V}_e/v_e$$

From Table B.1.2 at 100 kPa we get

$$v_f = 0.001043 \;\; m^3/kg; \quad v_g = 1.694 \;\; m^3/kg$$

$$\mathbf{V}_e = \mathbf{V}_i \; v_e/v_i = 1 \, \frac{1.694}{0.001043} = \mathbf{1624\ m/s}$$

To accelerate the flow up to that speed you need a large force (ΔPA) so a large pressure drop is needed.



6.67

A cryogenic fluid as liquid nitrogen at 90 K, 400 kPa flows into a probe used in cryogenic surgery. In the return line the nitrogen is then at 160 K, 400 kPa. Find the specific heat transfer to the nitrogen. If the return line has a cross sectional area 100 times larger than the inlet line what is the ratio of the return velocity to the inlet velocity?

Solution:

C.V line with nitrogen. No kinetic or potential energy changes

Continuity Eq.: $\dot{m} = \text{constant} = \dot{m}_e = \dot{m}_i = A_e\mathbf{V}_e/v_e = A_i\mathbf{V}_i/v_i$

Energy Eq.6.13: $q = h_e - h_i$

State i, Table B.6.1: $h_i = -95.58$ kJ/kg, $v_i = 0.001343\ \text{m}^3/\text{kg}$

State e, Table B.6.2: $h_e = 162.96$ kJ/kg, $v_e = 0.11647\ \text{m}^3/\text{kg}$

From the energy equation

$$q = h_e - h_i = 162.96 - (-95.58) = \mathbf{258.5\ kJ/kg}$$

From the continuity equation

$$\mathbf{V}_e/\mathbf{V}_i = A_i/A_e\ (v_e/v_i) = \frac{1}{100}\ \frac{0.11647}{0.001343} = \mathbf{0.867}$$

Pumps, pipe and channel flows

6.68

A steam pipe for a 300-m tall building receives superheated steam at 200 kPa at ground level. At the top floor the pressure is 125 kPa and the heat loss in the pipe is 110 kJ/kg. What should the inlet temperature be so that no water will condense inside the pipe?

Solution:

C.V. Pipe from 0 to 300 m, no ΔKE, steady state, single inlet and exit flow. Neglect any changes in kinetic energy.

Energy Eq.6.13: $q + h_i = h_e + gZ_e$

No condensation means: Table B.1.2, $h_e = h_g$ at 125 kPa = 2685.4 kJ/kg

$$h_i = h_e + gZ_e - q = 2685.4 + \frac{9.807 \times 300}{1000} - (-110) = 2810.1 \text{ kJ/kg}$$

At 200 kPa: T ~ **170°C** Table B.1.3

6.69

A small stream with 20°C water runs out over a cliff creating a 100 m tall waterfall. Estimate the downstream temperature when you neglect the horizontal flow velocities upstream and downstream from the waterfall. How fast was the water dropping just before it splashed into the pool at the bottom of the waterfall?

Solution:

CV. Waterfall, steady state. Assume no $\dot{Q}$ nor $\dot{W}$

Energy Eq.6.13: $h + \frac{1}{2}V^2 + gZ = \text{const.}$

State 1: At the top zero velocity $Z_1 = 100$ m

State 2: At the bottom just before impact, $Z_2 = 0$

State 3: At the bottom after impact in the pool.

$$h_1 + 0 + gZ_1 = h_2 + \frac{1}{2}\mathbf{V}_2^2 + 0 = h_3 + 0 + 0$$

Properties: $h_1 \cong h_2$ same T, P

$$=> \quad \frac{1}{2}\mathbf{V}_2^2 = gZ_1$$

$$\mathbf{V}_2 = \sqrt{2gZ_1} = \sqrt{2 \times 9.806 \times 100} = \mathbf{44.3\ m/s}$$

Energy equation from state 1 to state 3

$$h_3 = h_1 + gZ_1$$

use $\Delta h = C_p\, \Delta T$ with value from Table A.4 (liquid water)

$$T_3 = T_1 + gZ_1 / C_p$$
$$= 20 + 9.806 \times 100 / 4180 = \mathbf{20.23\ °C}$$

6.70

An irrigation pump takes water from a river at 10°C, 100 kPa and pumps it up to an open canal, where it flows out 100 m higher at 10°C. The pipe diameter in and out of the pump is 0.1 m and the motor driving the unit is 5 hp. What is the flow rate neglecting kinetic energy and losses?

C.V. Pump plus pipe to canal.

This is a single steady flow device with atmospheric pressure in and out.

Energy Eq.: $0 = h_i + 0 + gZ_i + w_{p\ in} - h_e - 0 - gZ_e$

$$w_{p\ in} = g(Z_e - Z_i) = 9.81\ \text{m/s}^2 \times 100\ (\text{m/s})^2 = 0.981\ \text{kJ/kg}$$

$$\dot{W}_{p\ in} = 5\ \text{hp} = 5\ \text{hp} \times 0.746\ \text{kW/hp} = 3.73\ \text{kW}$$

Find the flow rate from the work

$$\dot{m} = \dot{W}_{p\ in} / w_{p\ in} = 3.73 / 0.981 = \mathbf{3.8\ kg/s}$$

6.71

Consider a water pump that receives liquid water at 15°C, 100 kPa and delivers it to a same diameter short pipe having a nozzle with exit diameter of 1 cm (0.01 m) to the atmosphere 100 kPa. Neglect the kinetic energy in the pipes and assume constant u for the water. Find the exit velocity and the mass flow rate if the pump draws a power of 1 kW.

Solution:

Continuity Eq.: $\dot{m}_i = \dot{m}_e = A\mathbf{V}/v$; $A = \frac{\pi}{4} D_e^2 = \frac{\pi}{4} \times 0.01^2 = 7.854\times 10^{-5}\ \text{m}^2$

Energy Eq.6.13: $h_i + \frac{1}{2}\mathbf{V}_i^2 + gZ_i = h_e + \frac{1}{2}\mathbf{V}_e^2 + gZ_e + w$

Properties: $h_i = u_i + P_i v_i = h_e = u_e + P_e v_e$; $P_i = P_e$; $v_i = v_e$

$$w = -\frac{1}{2}\mathbf{V}_e^2 \quad\Rightarrow\quad -\dot{W} = \dot{m}\,(\tfrac{1}{2}\mathbf{V}_e^2) = A \times \frac{1}{2}\mathbf{V}_e^3/v_e$$

$$\mathbf{V}_e = \left(\frac{-2\,\dot{W}\,v_e}{A}\right)^{1/3} = \left(\frac{2 \times 1000 \times 0.001001}{7.854\times10^{-5}}\right)^{1/3} = \mathbf{29.43\ m/s}$$

$$\dot{m} = A\mathbf{V}_e/v_e = 7.854\times 10^{-5} \times 29.43 / 0.001001 = \mathbf{2.31\ kg/s}$$

Small water pump for a washing machine. Notice the very simple impeller.

6.72

A cutting tool uses a nozzle that generates a high speed jet of liquid water. Assume an exit velocity of 500 m/s of 20^{o}C liquid water with a jet diameter of 2 mm (0.002 m). How much mass flow rate is this? What size (power) pump is needed to generate this from a steady supply of 20^{o}C liquid water at 200 kPa?

Solution:

C.V. Nozzle. Steady state, single flow.

Continuity equation with a uniform velocity across A

$$\dot{m} = A\mathbf{V}/v = \frac{\pi}{4} D^2\, \mathbf{V} / v = \frac{\pi}{4}\, 0.002^2 \times 500 / 0.001002 = \mathbf{1.568\ kg/s}$$

Assume $Z_i = Z_e = 0$, $u_e = u_i$ and $\mathbf{V}_i = 0$ $P_e = 100$ kPa (atmospheric)

Energy Eq.6.13: $h_i + 0 + 0 = h_e + \frac{1}{2}\mathbf{V}_e^2 + 0 + w$

$$w = h_i - h_e - \frac{1}{2}\mathbf{V}_e^2 = u_i - u_e + P_i v_i - P_e v_e - \frac{1}{2}\mathbf{V}_e^2$$

$$= (P_i - P_e)\, v_i - \frac{1}{2}\mathbf{V}_e^2$$

$$= 0.001002\ \mathrm{m^3/kg} \times (200 - 100)\ \mathrm{kPa} - 0.5 \times \frac{500^2\ \mathrm{m^2/s^2}}{1000\ \mathrm{J/kJ}}$$

$$= 0.1002 - 125 \cong -125\ \mathrm{kJ/kg}$$

$$\dot{W} = \dot{m}w = 1.568\ \mathrm{kg/s}\ (-125\ \mathrm{kJ/kg}) = \mathbf{-196\ kW}$$

6.73

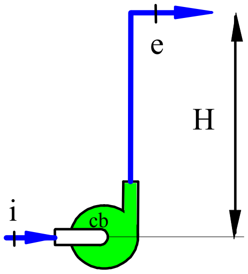
A small water pump is used in an irrigation system. The pump takes water in from a river at 10°C, 100 kPa at a rate of 5 kg/s. The exit line enters a pipe that goes up to an elevation 20 m above the pump and river, where the water runs into an open channel. Assume the process is adiabatic and that the water stays at 10°C. Find the required pump work.

Solution:

C.V. pump + pipe. Steady state , 1 inlet, 1 exit flow. Assume same velocity in and out, no heat transfer.

Continuity Eq.: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m}$

Energy Eq.6.12:



$$\dot{m}(h_{in} + (1/2)\mathbf{V}_{in}^2 + gz_{in}) =$$

$$\dot{m}(h_{ex} + (1/2)\,\mathbf{V}_{ex}^2 + gz_{ex}) + \dot{W}$$

States: $h_{in} = h_{ex}$ same (T, P)

$$\dot{W} = \dot{m}\,g(z_{in} - z_{ex}) = 5 \times 9.807 \times (0 - 20)/1000 = \mathbf{-0.98\ kW}$$

I.E. 0.98 kW required input

6.74

The main waterline into a tall building has a pressure of 600 kPa at 5 m below ground level. A pump brings the pressure up so the water can be delivered at 200 kPa at the top floor 150 m above ground level. Assume a flow rate of 10 kg/s liquid water at 10°C and neglect any difference in kinetic energy and internal energy u. Find the pump work.

Solution:

C.V. Pipe from inlet at -5 m up to exit at +150 m, 200 kPa.

Energy Eq.6.13: $h_i + \frac{1}{2}V_i^2 + gZ_i = h_e + \frac{1}{2}V_e^2 + gZ_e + w$

With the same u the difference in h's are the Pv terms

$$w = h_i - h_e + \frac{1}{2}(V_i^2 - V_e^2) + g\,(Z_i - Z_e)$$

$$= P_iv_i - P_ev_e + g\,(Z_i - Z_e)$$

$$= 600 \times 0.001 - 200 \times 0.001 + 9.806 \times (-5 - 150)/1000$$

$$= 0.4 - 1.52 = -1.12 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 10 \times (-1.12) = \mathbf{-11.2\ kW}$$

6.75

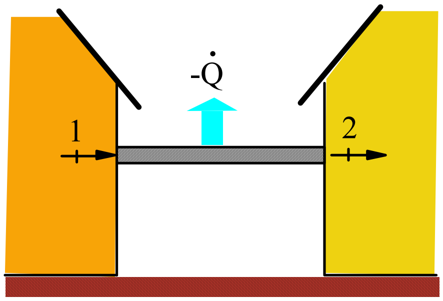
A pipe flows water at 15^{o}C from one building to another. In the winter time the pipe loses an estimated 500 W of heat transfer. What is the minimum required mass flow rate that will ensure that the water does not freeze (i.e. reach 0^{o}C)?

Solution:

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

Assume saturated liquid at given T from table B.1.1

$$\dot{m} = \frac{\dot{Q}}{h_e - h_i} = \frac{-500 \times 10^{-3}}{0 - 62.98} = \frac{0.5}{62.98} = \mathbf{0.007\ 94\ kg/s}$$



Multiple flow single device processes

Turbines, Compressors, Expanders

6.76

A steam turbine receives steam from two boilers. One flow is 5 kg/s at 3 MPa, 700°C and the other flow is 15 kg/s at 800 kPa, 500°C. The exit state is 10 kPa, with a quality of 96%. Find the total power out of the adiabatic turbine.

Solution:

C.V. whole turbine steady, 2 inlets, 1 exit, no heat transfer $\dot{Q} = 0$

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 5 + 15 = 20$ kg/s

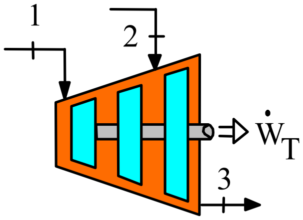
Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{W}_T$

Table B.1.3: $h_1 = 3911.7$ kJ/kg,

$h_2 = 3480.6$ kJ/kg

Table B.1.2: $h_3 = 191.8 + 0.96 \times 2392.8$

$= 2488.9$ kJ/kg



$\dot{W}_T = 5 \times 3911.7 + 15 \times 3480.6 - 20 \times 2488.9 = 21\,990$ kW = **22 MW**

6.77

A compressor receives 0.05 kg/s R-410a at 200 kPa, -20$^{\mathrm{o}}$C and 0.1 kg/s R-410a at 400 kPa, 0$^{\mathrm{o}}$C. The exit flow is at 1000 kPa, 60$^{\mathrm{o}}$C as shown in Fig. P6.77. Assume it is adiabatic, neglect kinetic energies and find the required power input.

C.V. whole compressor steady, 2 inlets, 1 exit, no heat transfer $\dot{Q} = 0$

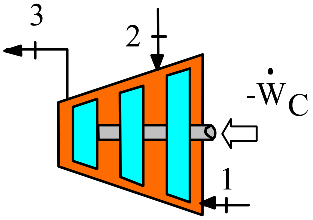
Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 0.05 + 0.1 = 0.15$ kg/s

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{W}_C = \dot{m}_3 h_3$

Table B.3.2: $h_1 = 278.72$ kJ/kg,

$h_2 = 290.42$ kJ/kg

Table B.3.2: $h_3 = 335.75$ kJ/kg



$$\dot{W}_C = 0.05 \times 278.72 + 0.1 \times 290.42 - 0.15 \times 335.75 = -7.385 \text{ kW}$$

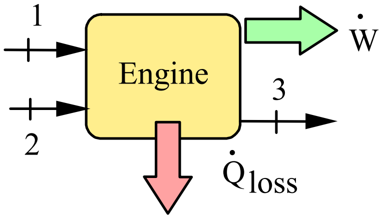
Power input is **7.4 kW**

6.78

Two steady flows of air enters a control volume, shown in Fig. P6.78. One is 0.025 kg/s flow at 350 kPa, 150°C, state 1, and the other enters at 450 kPa, 15°C, state 2. A single flow of air exits at 100 kPa, –40°C, state 3. The control volume rejects 1 kW heat to the surroundings and produces 4 kW of power. Neglect kinetic energies and determine the mass flow rate at state 2.

Solution:

C.V. Steady device with two inlet and one exit flows, we neglect kinetic energies. Notice here the Q is rejected so it goes out.



Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 0.025 + \dot{m}_2$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{W}_{CV} + \dot{Q}_{loss}$

Substitute the work and heat transfer into the energy equation and use constant heat capacity

$$0.025 \times 1.004 \times 423.2 + \dot{m}_2 \times 1.004 \times 288.2$$

$$= (0.025 + \dot{m}_2)\ 1.004 \times 233.2 \ + 4.0 + 1.0$$

Now solve for $\dot{m}_2$.

$$\dot{m}_2 = \frac{4.0 + 1.0 + 0.025 \times 1.004 \times (233.2 - 423.2)}{1.004\ (288.2 - 233.2)} = \mathbf{0.0042\ kg/s}$$

6.79

A steam turbine receives water at 15 MPa, 600°C at a rate of 100 kg/s, shown in Fig. P6.79. In the middle section 20 kg/s is withdrawn at 2 MPa, 350°C, and the rest exits the turbine at 75 kPa, and 95% quality. Assuming no heat transfer and no changes in kinetic energy, find the total turbine power output.

Solution:

C.V. Turbine Steady state, 1 inlet and 2 exit flows.

Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$; => $\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 80$ kg/s

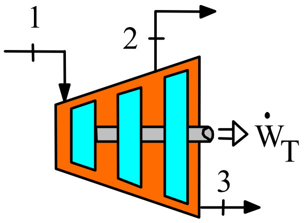
Energy Eq.6.10: $\dot{m}_1 h_1 = \dot{W}_T + \dot{m}_2 h_2 + \dot{m}_3 h_3$

Table B.1.3 $h_1 = 3582.3$ kJ/kg,

$h_2 = 3137$ kJ/kg

Table B.1.2 : $h_3 = h_f + x_3 h_{fg} = 384.3 + 0.95\times2278.6$

$= 2549.1$ kJ/kg



From the energy equation, Eq.6.10

$$=> \quad \dot{W}_T = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = \mathbf{91.565\ MW}$$

6.80

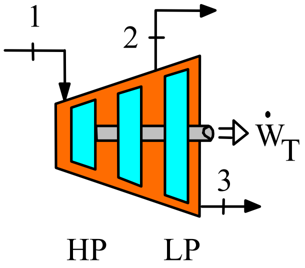
Cogeneration is often used where a steam supply is needed for industrial process energy. Assume a supply of 5 kg/s steam at 0.5 MPa is needed. Rather than generating this from a pump and boiler, the setup in Fig. P6.80 is used so the supply is extracted from the high-pressure turbine. Find the power the turbine now cogenerates in this process.

Solution:

C.V. Turbine, steady state, 1 inlet and 2 exit flows, assume adiabatic, $\dot{Q}_{CV} = 0$

Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Energy Eq.6.10: $\dot{Q}_{CV} + \dot{m}_1h_1 = \dot{m}_2h_2 + \dot{m}_3h_3 + \dot{W}_T$;



Supply state 1: 20 kg/s at 10 MPa, 500°C

Process steam 2: 5 kg/s, 0.5 MPa, 155°C,

Exit state 3: 20 kPa, x = 0.9

Table B.1.3: $h_1 = 3373.7$, $h_2 = 2755.9$ kJ/kg,

Table B.1.2: $h_3 = 251.4 + 0.9 \times 2358.3$

$= 2373.9$ kJ/kg

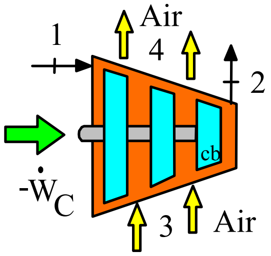
$\dot{W}_T = 20 \times 3373.7 - 5 \times 2755.9 - 15 \times 2373.9 =$ **18.084 MW**

6.81

A compressor receives 0.1 kg/s R-134a at 150 kPa, -10°C and delivers it at 1000 kPa, 40°C. The power input is measured to be 3 kW. The compressor has heat transfer to air at 100 kPa coming in at 20°C and leaving at 25°C. How much is the mass flow rate of air?

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also have an air flow outside the compressor housing no changes in kenetic or potential energy.



Continuity Eq.: $\dot{m}_2 = \dot{m}_1$

Energy Eq. 6.12: $\dot{m}_1 h_1 + \dot{W}_{in} + \dot{m}_{air} h_3 = \dot{m}_2 h_2 + \dot{m}_{air} h_4$

Ideal gas for air and constant heat capacity: $h_4 - h_3 \sim C_{p\ air} (T_4 - T_3)$

$$\dot{m}_{air} = [\dot{m}_1 (h_1 - h_2) + \dot{W}_{in}] / C_{p\ air} (T_4 - T_3)$$

$$= \frac{0.1\ (393.84 - 420.25) + 3}{1.004\ (25\text{-}20)} = \frac{0.359}{5}$$

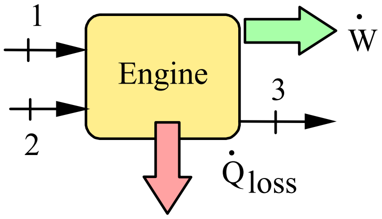
= 0.0715 kg/s

6.82

A large expansion engine has two low velocity flows of water entering. High pressure steam enters at point 1 with 2.0 kg/s at 2 MPa, 500°C and 0.5 kg/s cooling water at 120 kPa, 30°C enters at point 2. A single flow exits at point 3 with 150 kPa, 80% quality, through a 0.15 m diameter exhaust pipe. There is a heat loss of 300 kW. Find the exhaust velocity and the power output of the engine.

Solution:

C.V. : Engine (Steady state)



Constant rates of flow, $\dot{Q}_{loss}$ and $\dot{W}$

State 1: Table B.1.3: $h_1 = 3467.6$ kJ/kg

State 2: Table B.1.1: $h_2 = 125.77$ kJ/kg

$h_3 = 467.1 + 0.8 \times 2226.5 = 2248.3$ kJ/kg

$v_3 = 0.00105 + 0.8 \times 1.15825 = 0.92765 \text{ m}^3/\text{kg}$

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2 + 0.5 = 2.5 \text{ kg/s} = (AV/v) = (\pi/4)D^2V/v$

Energy Eq.6.10: $\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3(h_3 + 0.5\ V^2) + \dot{Q}_{loss} + \dot{W}$

$\mathbf{V} = \dot{m}_3v_3 / [\frac{\pi}{4} D^2] = 2.5 \times 0.92765 / (0.7854 \times 0.15^2) = \mathbf{131.2\ m/s}$

$0.5\ \mathbf{V}^2 = 0.5 \times 131.2^2 /1000 = 8.6$ kJ/kg (remember units factor 1000)

$\dot{W} = 2 \times 3467.6 + 0.5 \times 125.77 - 2.5\ (2248.3 + 8.6) - 300 = \mathbf{1056\ kW}$

Heat Exchangers

6.83

A condenser (heat exchanger) brings 1 kg/s water flow at 10 kPa from 300°C to saturated liquid at 10 kPa, as shown in Fig. P6.84. The cooling is done by lake water at 20°C that returns to the lake at 30°C. For an insulated condenser, find the flow rate of cooling water.

Solution:

C.V. Heat exchanger

Energy Eq.6.10: $\dot{m}_{cool}h_{20} + \dot{m}_{H_2O}h_{300} = \dot{m}_{cool}h_{30} + \dot{m}_{H_2O}h_{f,\ 10\ kPa}$

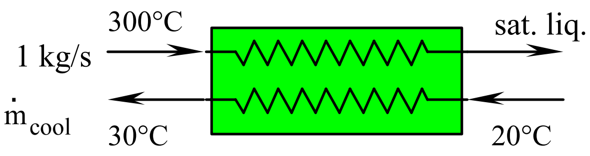


Table B.1.1: $h_{20} = 83.96$ kJ/kg , $h_{30} = 125.79$ kJ/kg

Table B.1.3: $h_{300,\ 10kPa} = 3076.5$ kJ/kg, B.1.2: $h_{f,\ 10\ kPa} = 191.83$ kJ/kg

$$\dot{m}_{cool} = \dot{m}_{H_2O}\frac{h_{300} - h_{f,\ 10kPa}}{h_{30} - h_{20}} = 1 \times \frac{3076.5 - 191.83}{125.79 - 83.96} = \mathbf{69\ kg/s}$$

6.84

In a co-flowing (same direction) heat exchanger 1 kg/s air at 500 K flows into one channel and 2 kg/s air flows into the neighboring channel at 300 K. If it is infinitely long what is the exit temperature? Sketch the variation of T in the two flows.

C.V. mixing section (no $\dot{W}$, $\dot{Q}$)

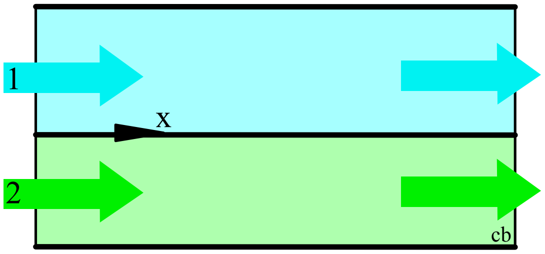
Continuity Eq.: $\dot{m}_1 = \dot{m}_3$ and $\dot{m}_2 = \dot{m}_4$

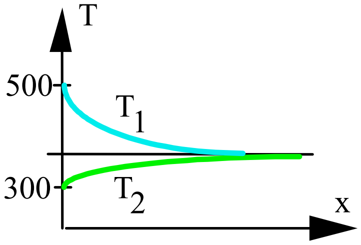
Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_1 h_3 + \dot{m}_2 h_4$

Same exit T: $h_3 = h_4 = [\dot{m}_1 h_1 + \dot{m}_2 h_2] / [\dot{m}_1 + \dot{m}_2]$

Using conctant specific heat

$$T_3 = T_4 = \frac{\dot{m}_1}{\dot{m}_1 + \dot{m}_2} T_1 + \frac{\dot{m}_2}{\dot{m}_1 + \dot{m}_2} T_2 = \frac{1}{3} \times 500 + \frac{2}{3} \times 300 = \mathbf{367\ K}$$



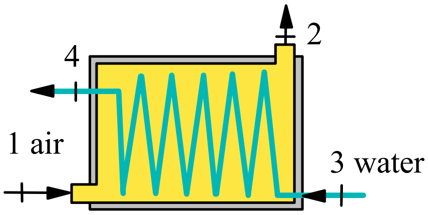


6.85

A heat exchanger, shown in Fig. P6.85, is used to cool an air flow from 800 K to 360 K, both states at 1 MPa. The coolant is a water flow at 15°C, 0.1 MPa. If the water leaves as saturated vapor, find the ratio of the flow rates $\dot{m}_{H_2O}/\dot{m}_{air}$

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.



Continuity Eqs.: Each line has a constant flow rate through it.

Energy Eq.6.10: $\dot{m}_{air}h_1 + \dot{m}_{H_2O}h_3 = \dot{m}_{air}h_2 + \dot{m}_{H_2O}h_4$

Process: Each line has a constant pressure.

Air states, Table A.7.1: $h_1 = 822.20$ kJ/kg, $h_2 = 360.86$ kJ/kg

Water states, Table B.1.1: $h_3 = 62.98$ kJ/kg (at 15°C),

Table B.1.2: $h_4 = 2675.5$ kJ/kg (at 100 kPa)

$$\dot{m}_{H_2O}/\dot{m}_{air} = \frac{h_1 - h_2}{h_4 - h_3} = \frac{822.20 - 360.86}{2675.5 - 62.99} = \mathbf{0.1766}$$

6.86

Air at 600 K flows with 3 kg/s into a heat exchanger and out at 100°C. How much (kg/s) water coming in at 100 kPa, 20°C can the air heat to the boiling point?

C.V. Total heat exchanger. The flows are not mixed so the two flowrates are constant through the device. No external heat transfer and no work.

Energy Eq.6.10: $\dot{m}_{air}h_{air\ in} + \dot{m}_{water}h_{water\ in} = \dot{m}_{air}h_{air\ out} + \dot{m}_{water}h_{water\ out}$

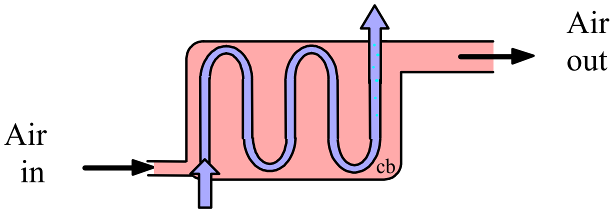
$$\dot{m}_{air}[h_{air\ in} - h_{air\ out}] = \dot{m}_{water}[h_{water\ out} - h_{water\ in}]$$

Table B.1.2: $h_{water\ out} - h_{water\ in} = 2675.46 - 83.94 = 2591.5$ kJ/kg

Table A.7.1: $h_{air\ in} - h_{air\ out} = 607.32 - 374.14 = 233.18$ kJ/kg

Solve for the flow rate of water from the energy equation

$$\dot{m}_{water} = \dot{m}_{air}\frac{h_{air\ in} - h_{air\ out}}{h_{water\ out} - h_{water\ in}} = 3 \times \frac{233.18}{2591.5} = \mathbf{0.27\ kg/s}$$



6.87

An automotive radiator has glycerine at 95^{o}C enter and return at 55^{o}C as shown in Fig. P6.87. Air flows in at 20^{o}C and leaves at 25^{o}C. If the radiator should transfer 25 kW what is the mass flow rate of the glycerine and what is the volume flow rate of air in at 100 kPa?

Solution:

If we take a control volume around the whole radiator then there is no external heat transfer - it is all between the glycerin and the air. So we take a control volume around each flow separately.

Glycerine: $\dot{m}h_i + (-\dot{Q}) = \dot{m}h_e$

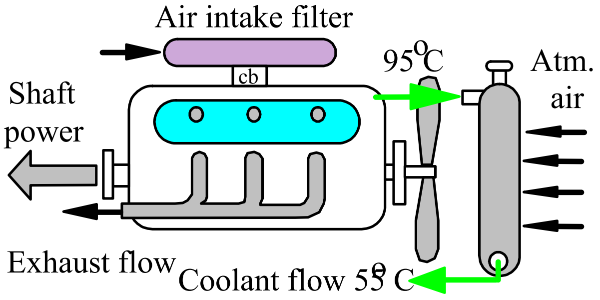
Table A.4: $\dot{m}_{gly} = \frac{-\dot{Q}}{h_e - h_i} = \frac{-\dot{Q}}{C_{gly}(T_e - T_i)} = \frac{-25}{2.42(55 - 95)} = \mathbf{0.258\ kg/s}$

Air $\dot{m}h_i + \dot{Q} = \dot{m}h_e$

Table A.5: $\dot{m}_{air} = \frac{\dot{Q}}{h_e - h_i} = \frac{\dot{Q}}{C_{air}(T_e - T_i)} = \frac{25}{1.004(25 - 20)} = 4.98\text{ kg/s}$

$\dot{V} = \dot{m}v_i\ ;\quad v_i = \frac{RT_i}{P_i} = \frac{0.287 \times 293}{100} = 0.8409\text{ m}^3\text{/kg}$

$\dot{V}_{air} = \dot{m}v_i = 4.98 \times 0.8409 = \mathbf{4.19\ m^3/s}$

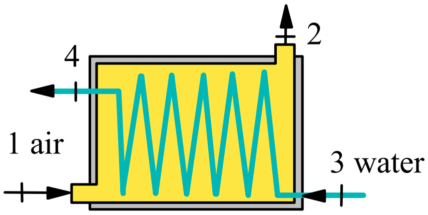


6.88

A superheater brings 2.5 kg/s saturated water vapor at 2 MPa to 450°C. The energy is provided by hot air at 1200 K flowing outside the steam tube in the opposite direction as the water, which is a counter flowing heat exchanger. Find the smallest possible mass flow rate of the air so the air exit temperature is 20°C larger than the incoming water temperature (so it can heat it).

Solution:

C.V. Superheater. Steady state with no external $\dot{Q}$ or any $\dot{W}$ the two flows exchanges energy inside the box. Neglect kinetic and potential energies at all states.



Energy Eq.6.10: $\dot{m}_{H_2O}\, h_3 + \dot{m}_{air}\, h_1 = \dot{m}_{H_2O}\, h_4 + \dot{m}_{air}\, h_2$

Process: Constant pressure in each line.

State 1: Table B.1.2 $T_3 = 212.42°C$, $h_3 = 2799.51$ kJ/kg

State 2: Table B.1.3 $h_4 = 3357.48$ kJ/kg

State 3: Table A.7 $h_1 = 1277.81$ kJ/kg

State 4: $T_2 = T_3 + 20 = 232.42°C = 505.57$ K

$$\text{A.7}: \quad h_2 = 503.36 + \frac{5.57}{20}(523.98 - 503.36) = 509.1 \text{ kJ/kg}$$

From the energy equation we get

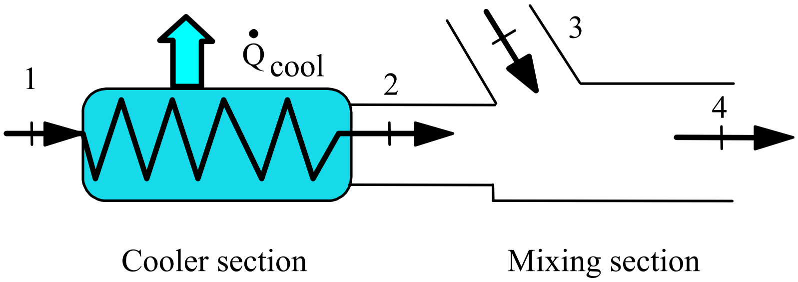
$$\dot{m}_{air} / \dot{m}_{H_2O} = (h_4 - h_3)/(h_1 - h_2)$$

$$= 2.5\,(3357.48 - 2799.51) / (1277.81 - 509.1) = \mathbf{1.815\ kg/s}$$

6.89

A cooler in an air conditioner brings 0.5 kg/s air at 35^{o}C to 5^{o}C, both at 101 kPa and it then mix the output with a flow of 0.25 kg/s air at 20^{o}C, 101 kPa sending the combined flow into a duct. Find the total heat transfer in the cooler and the temperature in the duct flow.

Solution:



C.V. Cooler section (no $\dot{W}$)

Energy Eq.6.12: $\dot{m}h_1 = \dot{m}h_2 + \dot{Q}_{cool}$

$\dot{Q}_{cool} = \dot{m}(h_1 - h_2) = \dot{m}\, C_p\, (T_1 - T_2) = 0.5 \times 1.004 \times (35\text{-}5) = 15.06$ kW

C.V. mixing section (no $\dot{W}$, $\dot{Q}$)

Continuity Eq.: $\dot{m}_2 + \dot{m}_3 = \dot{m}_4$

Energy Eq.6.10: $\dot{m}_2h_2 + \dot{m}_3h_3 = \dot{m}_4h_4$

$\dot{m}_4 = \dot{m}_2 + \dot{m}_3 = 0.5 + 0.25 = 0.75$ kg/s

$\dot{m}_4h_4 = (\dot{m}_2 + \dot{m}_3)h_4 = \dot{m}_2h_2 + \dot{m}_3h_3$

$\dot{m}_2\, (h_4 - h_2) + \dot{m}_3\, (h_4 - h_3) = 0$

$\dot{m}_2\, C_p\, (T_4 - T_2) + \dot{m}_3\, C_p\, (T_4 - T_3) = 0$

$T_4 = (\dot{m}_2 / \dot{m}_4)\, T_2 + (\dot{m}_3 / \dot{m}_4)\, T_3 = 5(0.5/0.75) + 20(0.25/0.75) =$ **10°C**

6.90

Steam at 500 kPa, 300°C is used to heat cold water at 15°C to 75°C for domestic hot water supply. How much steam per kg liquid water is needed if the steam should not condense?

Solution:

C.V. Each line separately. No work but there is heat transfer out of the steam flow and into the liquid water flow.

Water line energy Eq.: $\dot{m}_{liq}h_i + \dot{Q} = \dot{m}_{liq}h_e \quad \Rightarrow \quad \dot{Q} = \dot{m}_{liq}(h_e - h_i)$

For the liquid water look in Table B.1.1

$$\Delta h_{liq} = h_e - h_i = 313.91 - 62.98 = 250.93 \text{ kJ/kg}$$

$$(\cong C_p\, \Delta T = 4.18\ (75 - 15) = 250.8 \text{ kJ/kg})$$

Steam line energy has the same heat transfer but it goes out

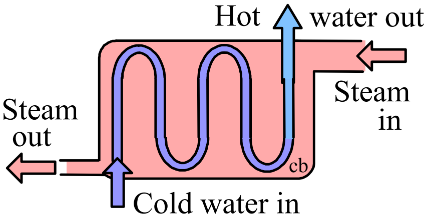
Steam Energy Eq.: $\dot{m}_{steam}h_i = \dot{Q} + \dot{m}_{steam}h_e \quad \Rightarrow \quad \dot{Q} = \dot{m}_{steam}(h_i - h_e)$

For the steam look in Table B.1.3 at 500 kPa

$$\Delta h_{steam} = h_i - h_e = 3064.2 - 2748.67 = 315.53 \text{ kJ/kg}$$

Now the heat transfer for the steam is substituted into the energy equation for the water to give

$$\dot{m}_{steam} / \dot{m}_{liq} = \Delta h_{liq} / \Delta h_{steam} = \frac{250.93}{315.53} = \mathbf{0.795}$$



6.91

A two fluid heat exchanger has 2 kg/s liquid ammonia at 20°C, 1003 kPa entering state 3 and exiting at state 4. It is heated by a flow of 1 kg/s nitrogen at 1500 K, state 1, leaving at 600 K, state 2 similar to Fig. P6.85. Find the total rate of heat transfer inside the heat exchanger. Sketch the temperature versus distance for the ammonia and find state 4 (T, v) of the ammonia.

Solution:

CV: Nitrogen flow line, steady rates of flow, $\dot{Q}$ out and $\dot{W} = 0$

Continiuty: $\dot{m}_1 = \dot{m}_2 = 1$ kg/s ; Energy Eq: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{Q}_{out}$

Tbl. A.8: $h_1 = 1680.7$ kJ/kg; $h_2 = 627.24$ kJ/kg

$\dot{Q}_{out} = \dot{m}_1(h_1 - h_2) = 1\ (1680.7 – 627.24) = \mathbf{1053.5\ kW}$

If Tbl A.5 is used: Cp = 1.042 kJ/kg K

$\dot{Q}_{out} = \dot{m}_1\ Cp\ (T_1 - T_2) = 1×1.042\ (1500 - 600) = \mathbf{937.8\ kW}$

CV The whole heat exchanger: No external $\dot{Q}$, constant pressure in each line.

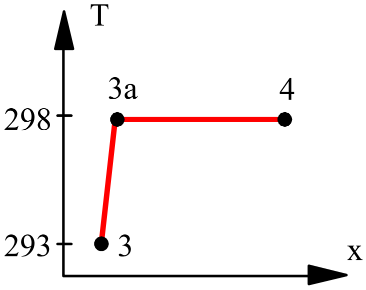
$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_1 h_2 + \dot{m}_3 h_4 \quad => \quad h_4 = h_3 + \dot{m}_1(h_1 - h_2)/\dot{m}_3$

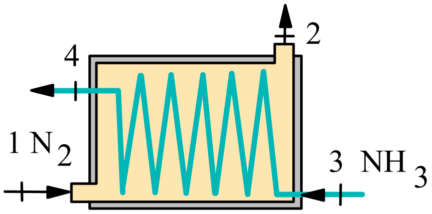
$h_4 = 274.3 + 1053.5 /2 = 801$ kJ/kg $< h_g \quad => \quad$ 2-phase

$x_4 = (h_4 - h_f)/ h_{fg} = (801 - 298.25) / 1165.2 = 0.43147$

$v_4 = v_f + x_4\ v_{fg} = 0.001658 + 0.43147×0.12647 = 0.05623\ m^3/kg$

$T_4 = T_{3a} = 25°C$ This is the boiling temperature for 1003 kPa.





6.92

A copper wire has been heat treated to 1000 K and is now pulled into a cooling chamber that has 1.5 kg/s air coming in at 20^{o}C; the air leaves the other end at 60^{o}C. If the wire moves 0.25 kg/s copper, how hot is the copper as it comes out?

Solution:
C.V. Total chamber, no external heat transfer

Energy eq.: $\dot{m}_{cu}\, h_{icu} + \dot{m}_{air}\, h_{i\,air} = \dot{m}_{cu}\, h_{e\,cu} + \dot{m}_{air}\, h_{e\,air}$

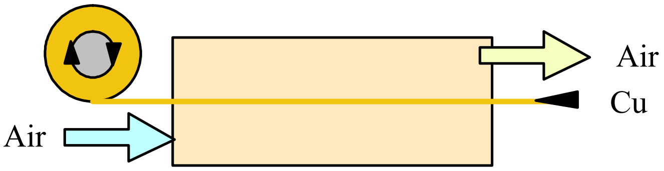
$$\dot{m}_{cu}\,(h_e - h_i)_{cu} = \dot{m}_{air}(h_i - h_e)_{air}$$

$$\dot{m}_{cu}\, C_{cu}\,(T_e - T_i)_{cu} = \dot{m}_{air}\, C_{p\,air}(T_e - T_i)_{air}$$

Heat capacities from A.3 for copper and A.5 for air

$$(T_e - T_i)_{cu} = \frac{\dot{m}_{air} C_{p\,air}}{\dot{m}_{cu} C_{cu}}(T_e - T_i)_{air} = \frac{1.5 \times 1.004}{0.25 \times 0.42}(20 - 60) = -\,573.7\text{ K}$$

$$T_e = T_i - 573.7 = 1000 - 573.7 = \mathbf{426.3\ K}$$

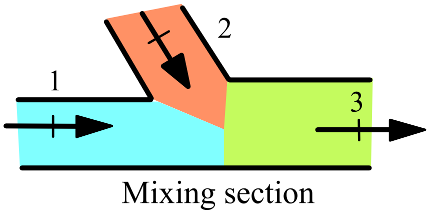


Mixing processes

6.93

Two air flows are combined to a single flow. Flow one is 1 m^3/s at 20°C and the other is 2 m^3/s at 200°C both at 100 kPa. They mix without any heat transfer to produce an exit flow at 100 kPa. Neglect kinetic energies and find the exit temperature and volume flow rate.

Solution:



Mixing section

Cont. $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$= (\dot{m}_1 + \dot{m}_2) h_3$

$$\dot{m}_1 (h_3 - h_1) + \dot{m}_2 (h_3 - h_2) = 0$$

$$\dot{m}_1 C_p (T_3 - T_1) + \dot{m}_2 C_p (T_3 - T_2) = 0$$

$$T_3 = (\dot{m}_i/\dot{m}_3)/T_1 + (\dot{m}_2/\dot{m}_3)T_2$$

We need to find the mass flow rates

$$v_1 = RT_1/P_1 = (0.287 \times 293)/100 = 0.8409 \text{ m}^3/\text{kg}$$

$$v_2 = RT_2/P_2 = (0.287 \times 473)/100 = 1.3575 \text{ m}^3/\text{kg}$$

$$\dot{m}_1 = \frac{\dot{V}_1}{v_1} = \frac{1}{0.8409} = 1.1892 \frac{\text{kg}}{\text{s}}, \quad \dot{m}_2 = \frac{\dot{V}_2}{v_2} = \frac{2}{1.3575} = 1.4733 \frac{\text{kg}}{\text{s}}$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 2.6625 \text{ kg/s}$$

$$T_3 = \frac{1.1892}{2.6625} \times 20 + \frac{1.4733}{2.6625} \times 200 = \mathbf{119.6^o\ C}$$

$$v_3 = \frac{RT_3}{P_3} = \frac{0.287\ (119.6 + 273)}{100} = 1.1268 \text{ m}^3/\text{kg}$$

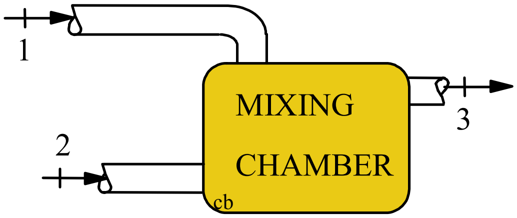
$$\dot{V}_3 = \dot{m}_3\, v_3 = 2.6625 \times 1.1268 = \mathbf{3.0\ m^3/s}$$

6.94

A de-superheater has a flow of ammonia 1.5 kg/s at 1000 kPa, 100^{o}C which is mixed with another flow of ammonia at 25^{o}C and quality 25% in an adiabatic mixing chamber. Find the flow rate of the second flow so the outgoing ammonia is saturated vapor at 1000 kPa.

C.V. Desuperheater.

No external $\dot{Q}$ or $\dot{W}$



Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

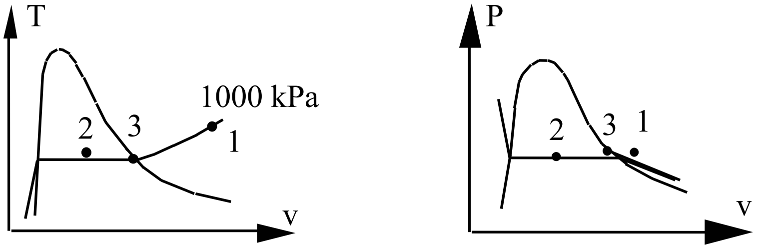
Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

State 1: Table B.2.2 $h_1 = 1664.3$ kJ/kg

State 2: Table B.2.1 $h_2 = 298.25 + 0.25 \times 1165.2 = 589.55$ kJ/kg

State 3: Table B.2.2 $h_3 = 1463.4$ kJ/kg

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 1.5 \times \frac{1664.3 - 1463.4}{1463.4 - 589.55} = \mathbf{0.345\ kg/s}$$



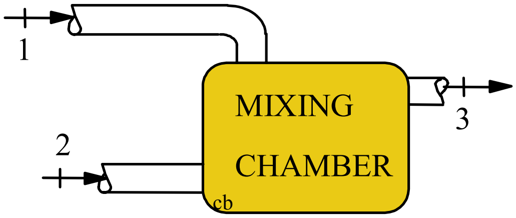
6.95

An open feedwater heater in a powerplant heats 4 kg/s water at 45$^{\mathrm{o}}$C, 100 kPa by mixing it with steam from the turbine at 100 kPa, 250$^{\mathrm{o}}$C. Assume the exit flow is saturated liquid at the given pressure and find the mass flow rate from the turbine.

Solution:

C.V. Feedwater heater.

No external $\dot{Q}$ or $\dot{W}$



Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

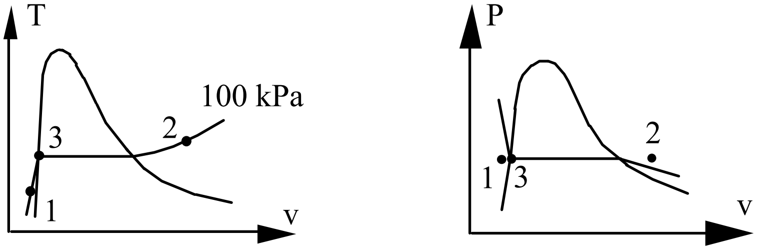
Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

State 1: Table B.1.1 $h = h_f = 188.42$ kJ/kg at 45$^{\mathrm{o}}$C

State 2: Table B.1.3 $h_2 = 2974.33$ kJ/kg

State 3: Table B.1.2 $h_3 = h_f = 417.44$ kJ/kg at 100 kPa

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 4 \times \frac{188.42 - 417.44}{417.44 - 2974.33} = \mathbf{0.358\ kg/s}$$



6.96

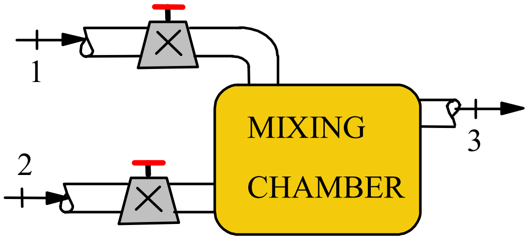
A flow of water at 2000 kPa, 20°C is mixed with a flow of 2 kg/s water at 2000 kPa, 180°C. What should the flowrate of the first flow be to produce an exit state of 200 kPa and 100°C?

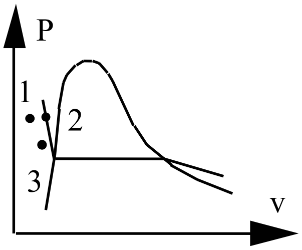
Solution:

C.V. Mixing chamber and valves. Steady state no heat transfer or work terms.

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$





Properties Table B.1.1: $h_1 = 85.8$ kJ/kg; $h_3 = 419.0$ kJ/kg

Table B.1.4: $h_2 = 763.7$ kJ/kg

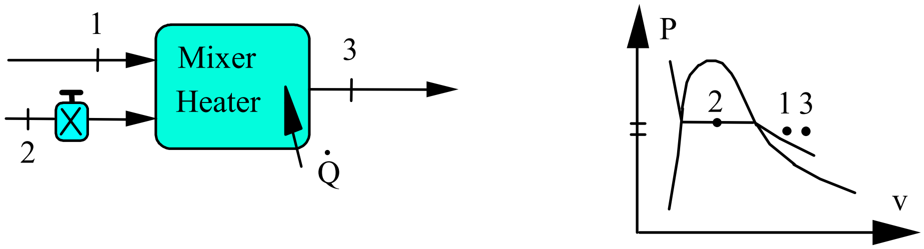
$$\dot{m}_1 = \dot{m}_2 \times \frac{h_2 - h_3}{h_3 - h_1} = 2 \times \frac{763.7 - 419.0}{419.0 - 85.8} = \mathbf{2.069\ kg/s}$$

6.97

A mixing chamber with heat transfer receives 2 kg/s of R-410a at 1 MPa, 40°C in one line and 1 kg/s of R-410a at 15°C, quality 50% in a line with a valve. The outgoing flow is at 1 MPa, 60°C. Find the rate of heat transfer to the mixing chamber.

Solution:

C.V. Mixing chamber. Steady with 2 flows in and 1 out, heat transfer in.



Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$; => $\dot{m}_3 = 2 + 1 = 3$ kg/s

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$

Properties: Table B.4.2: $h_1 = 316.05$ kJ/kg, $h_3 = 335.75$ kJ/kg

Table B.4.1: $h_2 = 81.15 + 0.5 \times 201.64 = 181.97$ kJ/kg

Energy equation then gives the heat transfer as

$\dot{Q} = 3 \times 335.75 - 2 \times 316.05 - 1 \times 181.97 =$ **193.18 kW**

6.98

An insulated mixing chamber receives 2 kg/s R-134a at 1 MPa, 100°C in a line with low velocity. Another line with R-134a as saturated liquid 60°C flows through a valve to the mixing chamber at 1 MPa after the valve, as shown in Fig. P6.97. The exit flow is saturated vapor at 1 MPa flowing at 20 m/s. Find the flow rate for the second line.

Solution:

C.V. Mixing chamber. Steady state, 2 inlets and 1 exit flow.

Insulated q = 0, No shaft or boundary motion w = 0.

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 (h_3 + \frac{1}{2}\mathbf{V}_3^2)$

$$\dot{m}_2 (h_2 - h_3 - \tfrac{1}{2}\mathbf{V}_3^2) = \dot{m}_1 (h_3 + \tfrac{1}{2}\mathbf{V}_3^2 - h_1)$$

1: Table B.5.2: 1 MPa, 100°C, $h_1 = 483.36$ kJ/kg

2: Table B.5.1: $x = \varnothing$, 60°C, $h_2 = 287.79$ kJ/kg

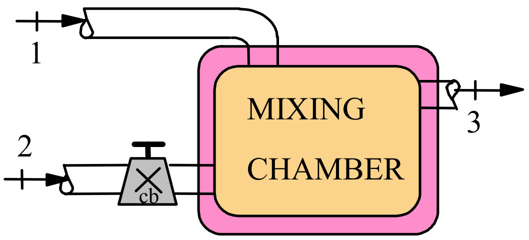
3: Table B.5.1: x = 1, 1 MPa, 20 m/s, $h_3 = 419.54$ kJ/kg

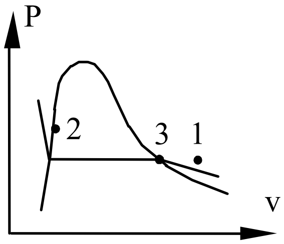
Now solve the energy equation for $\dot{m}_2$

$$\dot{m}_2 = 2 \times [419.54 + \tfrac{1}{2}\, 20^2 \times \frac{1}{1000} - 483.36] \,/\, [287.79 - 419.54 - \tfrac{1}{2}\frac{20^2}{1000}]$$

$$= 2 \times [-63.82 + 0.2] \,/\, [-131.75 - 0.2] = \mathbf{0.964\ kg/s}$$

Notice how kinetic energy was insignificant.





6.99

To keep a jet engine cool some intake air bypasses the combustion chamber. Assume 2 kg/s hot air at 2000 K, 500 kPa is mixed with 1.5 kg/s air 500 K, 500 kPa without any external heat transfer. Find the exit temperature by using constant heat capacity from Table A.5.

Solution:

C.V. Mixing Section

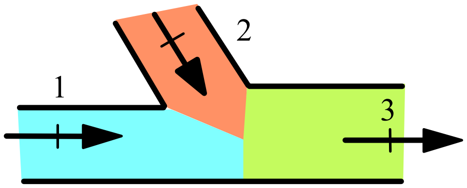
Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \;\; => \;\; \dot{m}_3 = 2 + 1.5 = 3.5$ kg/s

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$h_3 = (\dot{m}_1 h_1 + \dot{m}_2 h_2) / \dot{m}_3 \; ;$$

For a constant specific heat divide the equation for h_3 with C_p to get

$$T_3 = \frac{\dot{m}_1}{\dot{m}_3} T_1 + \frac{\dot{m}_2}{\dot{m}_3} T_2 = \frac{2}{3.5} 2000 + \frac{1.5}{3.5} 500 = \mathbf{1357\ K}$$



Mixing section

6.100

Solve the previous problem using values from Table A.,7
To keep a jet engine cool some intake air bypasses the combustion chamber. Assume 2 kg/s hot air at 2000 K, 500 kPa is mixed with 1.5 kg/s air 500 K, 500 kPa without any external heat transfer. Find the exit temperature by using values from Table A.7.

Solution:

C.V. Mixing Section

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \;\; => \;\; \dot{m}_3 = 2 + 1.5 = 3.5 \text{ kg/s}$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$h_3 = (\dot{m}_1 h_1 + \dot{m}_2 h_2) / \dot{m}_3 \;;$$

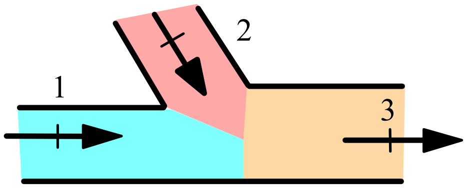
Using A.7 we look up the h at states 1 and 2 to calculate h_3

$$h_3 = \frac{\dot{m}_1}{\dot{m}_3} h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2 = \frac{2}{3.5} 2251.58 + \frac{1.5}{3.5} 503.36 = 1502 \text{ kJ/kg}$$

Now we can backinterpolate to find at what temperature do we have that h

$$T_3 = 1350 + 50 \frac{1502 - 1455.43}{1515.27 - 1455.43} = \mathbf{1389\ K}$$

This procedure is the most accurate.



Mixing section

6.101

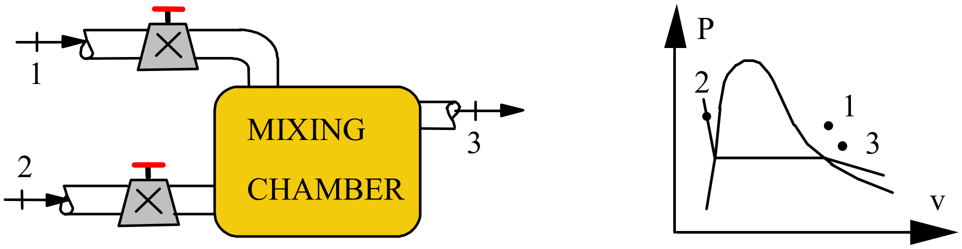
Two flows are mixed to form a single flow. Flow at state 1 is 1.5 kg/s water at 400 kPa, 200°C and flow at state 2 is 500 kPa, 100°C. Which mass flow rate at state 2 will produce an exit T_3 = 150°C if the exit pressure is kept at 300 kPa?

Solution:

C.V. Mixing chamber and valves. Steady state no heat transfer or work terms.

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$



Properties Table B.1.3: h_1 = 2860.51 kJ/kg; h_3 = 2760.95 kJ/kg

Table B.1.4: h_2 = 419.32 kJ/kg

$$\dot{m}_2 = \dot{m}_1 \times \frac{h_1 - h_3}{h_3 - h_2} = 1.5 \times \frac{2860.51 - 2760.95}{2760.95 - 419.32} = \mathbf{0.0638\ kg/s}$$

Multiple Devices, Cycle Processes

6.102

A flow of 5 kg/s water at 100 kPa, 20^{o}C should be delivered as steam at 1000 kPa, 350^{o}C to some application. Consider compressing it to 1000 kPa, 20^{o}C and then heat it at constant 1000 kPa to 350^{o}C. Which devices are needed and find the specific energy transfers in those devices.

To raise the pressure of a liquid flow requires a pump which delivers the difference between the flow work going out and in. Constant pressure heating is a simple heater (or heat exchanger).

Inlet state: B.1.1: $h_1 = 83.94$ kJ/kg, $v_1 = 0.001002$ m^3/kg

State 3: $h_3 = 3157.65$ kJ/kg

The pump delivers the difference between the flow work out and flow work in.

Pump: $-w_p = (P_2 - P_1)v_1 = (1000 - 100)$ kPa × 0.001002 m^3/kg

= **0.9 kJ/kg**.

Heater: $q = h_3 - h_2 = h_3 - (h_1 - w_p)$

$= 3157.65 - (83.94 + 0.9) =$ **3072.8 kJ/kg**

6.103

The following data are for a simple steam power plant as shown in Fig. P6.103.

State	1	2	3	4	5	6	7
P MPa	6.2	6.1	5.9	5.7	5.5	0.01	0.009
T °C		45	175	500	490		40
h kJ/kg	-	194	744	3426	3404	-	168

State 6 has $x_6 = 0.92$, and velocity of 200 m/s. The rate of steam flow is 25 kg/s, with 300 kW power input to the pump. Piping diameters are 200 mm from steam generator to the turbine and 75 mm from the condenser to the steam generator. Determine the velocity at state 5 and the power output of the turbine.

Solution:

Turbine $A_5 = (\pi/4)(0.2)^2 = 0.031\ 42\ \text{m}^2$, $v_5 = 0.06163\ \text{m}^3/\text{kg}$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25\ \text{kg/s} \times 0.061\ 63\ \text{m}^3/\text{kg} / 0.031\ 42\ \text{m}^2 = \mathbf{49\ m/s}$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2\ \text{kJ/kg}$$

$$w_T = h_5 - h_6 + \frac{1}{2}(\mathbf{V}_5^2 - \mathbf{V}_6^2)$$

$$= 3404 - 2393.2 + (49^2 - 200^2)/(2 \times 1000) = 992\ \text{kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 25 \times 992 = \mathbf{24\ 800\ kW}$$

Remark: Notice the kinetic energy change is small relative to enthalpy change.

6.104

For the same steam power plant as shown in Fig. P6.103 and Problem 6.103, assume the cooling water comes from a lake at 15°C and is returned at 25°C. Determine the rate of heat transfer in the condenser and the mass flow rate of cooling water from the lake.

Solution:

Condenser $A_7 = (\pi/4)(0.075)^2 = 0.004\,418\ \text{m}^2$, $v_7 = 0.001\,008\ \text{m}^3/\text{kg}$

$$\mathbf{V}_7 = \dot{m}v_7/A_7 = 25 \times 0.001\,008 \,/\, 0.004\,418 = 5.7\ \text{m/s}$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2\ \text{kJ/kg}$$

$$q_{COND} = h_7 - h_6 + \frac{1}{2}(\mathbf{V}_7^2 - \mathbf{V}_6^2)$$

$$= 168 - 2393.2 + (5.7^2 - 200^2)/(2{\times}1000) = -2245.2\ \text{kJ/kg}$$

$$\dot{Q}_{COND} = 25 \times (-2245.2) = \mathbf{-56\,130\ kW}$$

This rate of heat transfer is carried away by the cooling water so

$$-\dot{Q}_{COND} = \dot{m}_{H_2O}(h_{out} - h_{in})_{H_2O} = 56\,130\ \text{kW}$$

$$=> \qquad \dot{m}_{H_2O} = \frac{56\,130}{104.9 - 63.0} = \mathbf{1339.6\ kg/s}$$

6.105

For the same steam power plant as shown in Fig. P6.103 and Problem 6.103, determine the rate of heat transfer in the economizer, which is a low temperature heat exchanger. Find also the rate of heat transfer needed in the steam generator.

Solution:

Economizer $A_7 = \pi D_7^2/4 = 0.004\ 418\ \text{m}^2$, $v_7 = 0.001\ 008\ \text{m}^3/\text{kg}$

$$\mathbf{V}_2 = \mathbf{V}_7 = \dot{m}v_7/A_7 = 25 \times 0.001\ 008/0.004\ 418 = 5.7\ \text{m/s},$$

$$\mathbf{V}_3 = (v_3/v_2)\mathbf{V}_2 = (0.001\ 118\ /\ 0.001\ 008)\ 5.7 = 6.3\ \text{m/s} \approx \mathbf{V}_2$$

so kinetic energy change unimportant

$q_{ECON} = h_3 - h_2 = 744 - 194 = 550.0$ kJ/kg

$\dot{Q}_{ECON} = \dot{m}q_{ECON} = 25\ (550.0) = $ **13 750 kW**

Generator $A_4 = \pi D_4^2/4 = 0.031\ 42\ \text{m}^2$, $v_4 = 0.060\ 23\ \text{m}^3/\text{kg}$

$\mathbf{V}_4 = \dot{m}v_4/A_4 = 25 \times 0.060\ 23/0.031\ 42 = 47.9$ m/s

$q_{GEN} = 3426 - 744 + (47.9^2 - 6.3^2)/(2\times1000) = 2683$ kJ/kg

$\dot{Q}_{GEN} = \dot{m}q_{GEN} = 25 \times (2683) = $ **67 075 kW**

6.106

A somewhat simplified flow diagram for a nuclear power plant shown in Fig. 1.4 is given in Fig. P6.106. Mass flow rates and the various states in the cycle are shown in the accompanying table. The cycle includes a number of heaters in which heat is transferred from steam, taken out of the turbine at some intermediate pressure, to liquid water pumped from the condenser on its way to the steam drum. The heat exchanger in the reactor supplies 157 MW, and it may be assumed that there is no heat transfer in the turbines.

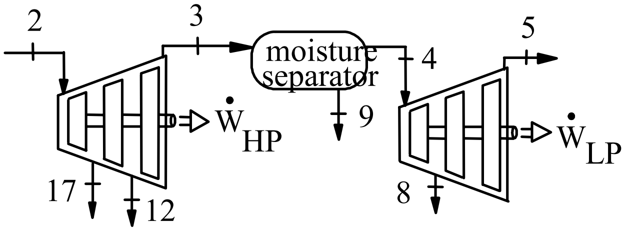
a. Assume the moisture separator has no heat transfer between the two turbinesections, determine the enthalpy and quality (h_4, x_4).

b. Determine the power output of the low-pressure turbine.

c. Determine the power output of the high-pressure turbine.

d. Find the ratio of the total power output of the two turbines to the total power delivered by the reactor.

Solution:



a) Moisture Separator, steady state, no heat transfer, no work

Mass: $\dot{m}_3 = \dot{m}_4 + \dot{m}_9$, Energy: $\dot{m}_3h_3 = \dot{m}_4h_4 + \dot{m}_9h_9$;

$62.874 \times 2517 = 58.212 \times h_4 + 4.662 \times 558 \quad \Rightarrow \quad h_4 = 2673.9$ kJ/kg

$h_4 = 2673.9 = 566.18 + x_4 \times 2160.6 \quad => \quad x_4 = \mathbf{0.9755}$

b) Low Pressure Turbine, steady state no heat transfer

Energy Eq.: $\dot{m}_4h_4 = \dot{m}_5h_5 + \dot{m}_8h_8 + \dot{W}_{CV(LP)}$

$\dot{W}_{CV(LP)} = \dot{m}_4h_4 - \dot{m}_5h_5 - \dot{m}_8h_8$

$= 58.212 \times 2673.9 - 55.44 \times 2279 - 2.772 \times 2459$

$= 22\,489$ kW $=$ **22.489 MW**

c) High Pressure Turbine, steady state no heat transfer

Energy Eq.: $\dot{m}_2h_2 = \dot{m}_3h_3 + \dot{m}_{12}h_{12} + \dot{m}_{17}h_{17} + \dot{W}_{CV(HP)}$

$\dot{W}_{CV(HP)} = \dot{m}_2h_2 - \dot{m}_3h_3 - \dot{m}_{12}h_{12} - \dot{m}_{17}h_{17}$

$= 75.6 \times 2765 - 62.874 \times 2517 - 8.064 \times 2517 - 4.662 \times 2593$

$= 18\,394$ kW $=$ **18.394 MW**

d) $\eta = (\dot{W}_{HP} + \dot{W}_{LP})/\dot{Q}_{REACT} = 40.883/157 = \mathbf{0.26}$

6.107

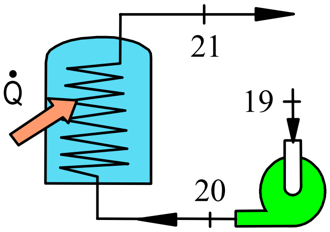
Consider the power plant as described in the previous problem.

a. Determine the quality of the steam leaving the reactor.

b. What is the power to the pump that feeds water to the reactor?

Solution:

a) Reactor: Cont.: $\dot{m}_{20} = \dot{m}_{21}$; $\dot{Q}_{CV} = 157$ MW



Energy Eq.6.12: $\dot{Q}_{CV} + \dot{m}_{20}h_{20} = \dot{m}_{21}h_{21}$

$157\ 000 + 1386 \times 1221 = 1386 \times h_{21}$

$h_{21} = 1334.3 = 1282.4 + x_{21} \times 1458.3$

$\Rightarrow \quad x_{21} = \mathbf{0.0349}$

b) C.V. Reactor feedwater pump

Cont. $\dot{m}_{19} = \dot{m}_{20}$ Energy Eq.6.12: $\dot{m}_{19}h_{19} = \dot{m}_{19}h_{20} + \dot{W}_{Cv,P}$

Table B.1: $h_{19} = h(277°C, 7240\text{ kPa}) = 1220$ kJ/kg, $h_{20} = 1221$ kJ/kg

$\dot{W}_{Cv,P} = \dot{m}_{19}(h_{19} - h_{20}) = 1386(1220 - 1221) = \mathbf{-1386\ kW}$

6.108

A R-410a heat pump cycle shown in Fig. P6.108 has a R-410a flow rate of 0.05 kg/s with 5 kW into the compressor. The following data are given

State	1	2	3	4	5	6
P, kPa	3100	3050	3000	420	400	390
T, °C	120	110	45		-10	-5
h, kJ/kg	377	367	134	-	280	284

Calculate the heat transfer from the compressor, the heat transfer from the R-410a in the condenser and the heat transfer to the R-410a in the evaporator.

Solution:

CV: Compressor

$$\dot{Q}_{COMP} = \dot{m}(h_1 - h_6) + \dot{W}_{COMP} = 0.05\ (377 - 284) - 5.0 = \mathbf{-0.35\ kW}$$

CV: Condenser

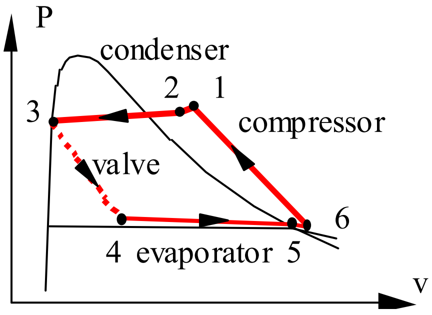
$$\dot{Q}_{COND} = \dot{m}(h_3 - h_2) = 0.05\ \text{kg/s}\ (134 - 367)\ \text{kJ/kg} = \mathbf{-11.65\ kW}$$

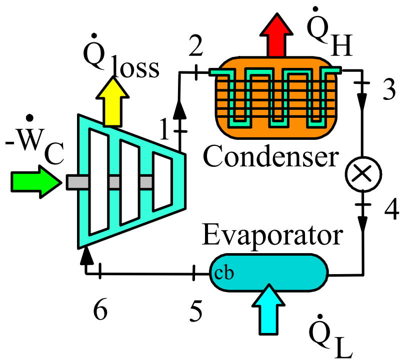
C.V. Valve:

$$h_4 = h_3 = 134\ \text{kJ/kg}$$

CV: Evaporator

$$\dot{Q}_{EVAP} = \dot{m}\ (h_5 - h_4) = 0.05\ \text{kg/s}\ (280 - 134)\ \text{kJ/kg} = \mathbf{7.3\ kW}$$





6.109

A modern jet engine has a temperature after combustion of about 1500 K at 3200 kPa as it enters the turbine setion, see state 3 Fig. P.6.109. The compressor inlet is 80 kPa, 260 K state 1 and outlet state 2 is 3300 kPa, 780 K; the turbine outlet state 4 into the nozzle is 400 kPa, 900 K and nozzle exit state 5 at 80 kPa, 640 K. Neglect any heat transfer and neglect kinetic energy except out of the nozzle. Find the compressor and turbine specific work terms and the nozzle exit velocity.

Solution:

The compressor, turbine and nozzle are all steady state single flow devices and they are adiabatic.

We will use air properties from table A.7.1:

$h_1 = 260.32$, $h_2 = 800.28$, $h_3 = 1635.80$, $h_4 = 933.15$, $h_5 = 649.53$ kJ/kg

Energy equation for the compressor gives

$$w_{c\ in} = h_2 - h_1 = 800.28 - 260.32 = \mathbf{539.36\ kJ/kg}$$

Energy equation for the turbine gives

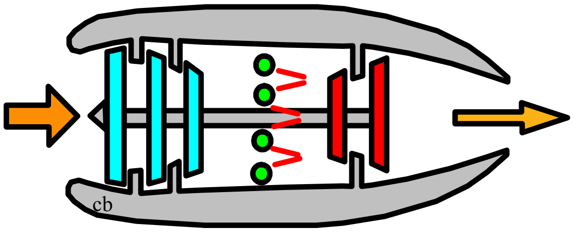
$$w_T = h_3 - h_4 = 1635.80 - 933.15 = \mathbf{702.65\ kJ/kg}$$

Energy equation for the nozzle gives

$$h_4 = h_5 + ½\,\mathbf{V}_5^2$$

$$½\,\mathbf{V}_5^2 = h_4 - h_5 = 933.15 - 649.53 = 283.62 \text{ kJ/kg}$$

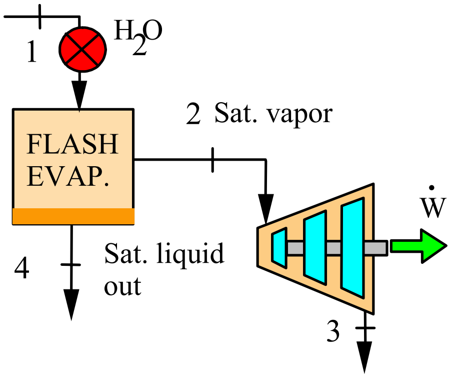
$$\mathbf{V}_5 = [2(h_4 - h_5)]^{1/2} = (2\times 283.62 \times 1000)^{1/2} = \mathbf{753\ m/s}$$



6.110

A proposal is made to use a geothermal supply of hot water to operate a steam turbine, as shown in Fig. P6.110. The high-pressure water at 1.5 MPa, 180°C, is throttled into a flash evaporator chamber, which forms liquid and vapor at a lower pressure of 400 kPa. The liquid is discarded while the saturated vapor feeds the turbine and exits at 10 kPa, 90% quality. If the turbine should produce 1 MW, find the required mass flow rate of hot geothermal water in kilograms per hour.

Solution:



Separation of phases in flash-evaporator constant h in the valve flow so

Table B.1.3: $h_1 = 763.5$ kJ/kg

$h_1 = 763.5 = 604.74 + x \times 2133.8$

$\Rightarrow \quad x = 0.07439 = \dot{m}_2/\dot{m}_1$

Table B.1.2: $h_2 = 2738.6$ kJ/kg;

$h_3 = 191.83 + 0.9 \times 2392.8 = 2345.4$ kJ/kg

Energy Eq.6.12 for the turbine

$$\dot{W} = \dot{m}_2(h_2 - h_3) \qquad => \qquad \dot{m}_2 = \frac{1000}{2738.6 - 2345.4} = 2.543 \text{ kg/s}$$

$$\Rightarrow \quad \dot{m}_1 = \dot{m}_2/x = 34.19 \text{ kg/s} = \mathbf{123\ 075\ kg/h}$$

Transient processes

6.111

An initially empty cylinder is filled with air from 20°C, 100 kPa until it is full. Assuming no heat transfer is the final temperature larger, equal to or smaller than 20°C? Does the final T depend on the size of the cylinder?

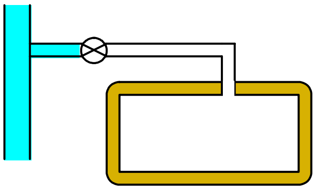
This is a transient problem with no heat transfer and no work. The balance equations for the tank as C.V. become

Continuity Eq.: $m_2 - 0 = m_i$

Energy Eq.: $m_2u_2 - 0 = m_ih_i + Q - W = m_ih_i + 0 - 0$

Final state: $u_2 = h_i$ & $P_2 = P_i$

$T_2 > T_i$ and it does not depend on V



6.112

An evacuated 150-L tank is connected to a line flowing air at room temperature, 25°C, and 8 MPa pressure. The valve is opened allowing air to flow into the tank until the pressure inside is 6 MPa. At this point the valve is closed. This filling process occurs rapidly and is essentially adiabatic. The tank is then placed in storage where it eventually returns to room temperature. What is the final pressure?

Solution:

C.V. Tank:

Continuity Eq.6.15: $m_i = m_2$

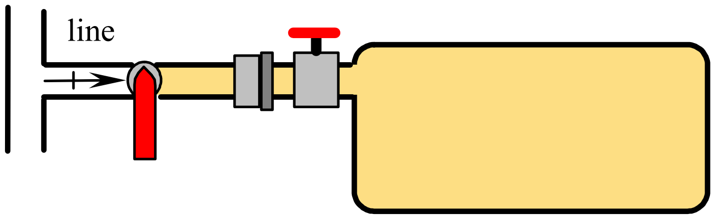
Energy Eq.6.16: $m_i h_i = m_2 u_2 \quad \Rightarrow \quad u_2 = h_i$

Use constant specific heat C_{Po} from table A.5 then energy equation:

$$T_2 = (C_P/C_v)\ T_i = kT_i = 1.4 \times 298.2 = 417.5 \text{ K}$$

Process: constant volume cooling to T_3:

$$P_3 = P_2 \times T_3/T_2 = 6.0 \times 298.15/\ 417.5 = \mathbf{4.29\ MPa}$$



6.113

A 2.5-L tank initially is empty and we want 10 g of ammonia in it. The ammonia comes from a line with saturated vapor at 25°C. To end up with the desired amount we cool the can while we fill it in a slow process keeping the can and content at 30°C. Find the final pressure to reach before closing the valve and the heat transfer?

Solution:

C.V. Tank:

Continuity Eq.6.15: $m_i = m_2$

Energy Eq.6.16: $m_2u_2 - 0 = m_ih_i + {}_1Q_2$

State 2: 30°C, $v_2 = V/m_2 = 0.0025/0.010 = 0.25 \text{ m}^3\text{/kg}$

From Table B.2.2 we locate the state between 500 and 600 kPa.

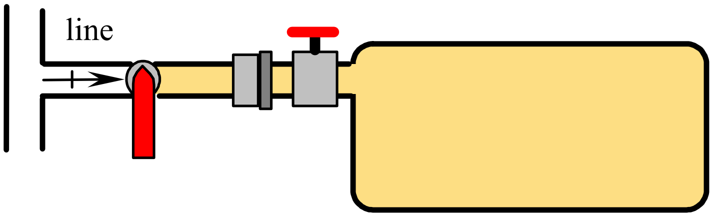
$$P_2 = 500 + (600 - 500)\frac{0.25 - 0.28103}{0.23152 - 0.28103} = \mathbf{562.7\ kPa}$$

$$u_2 = 1370 \text{ kJ/kg},$$

State i Table B.2.2: $h_i = 1463.5$ kJ/kg

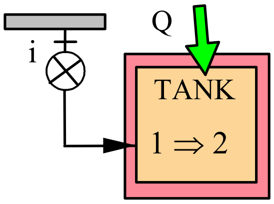
Now use the energy ewquation to solve for the heat transfer

$${}_1Q_2 = m_2u_2 - m_ih_i = m_2(u_2 - h_i)$$
$$= 0.01 \times (1370 - 1463.5\ ) = \mathbf{-0.935\ kJ}$$



6.114

A tank contains 1 m^3 air at 100 kPa, 300 K. A pipe flowing air at 1000 kPa, 300 K is connected to the tank and it is filled slowly to 1000 kPa. Find the heat transfer to reach a final temperature of 300 K.



C.V. The tank volume and the compressor.

This is a transient problem (filling of tank).

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = {}_1Q_2 - {}_1W_2 + m_{in}h_{in}$

Process: Constant volume ${}_1W_2 = 0$,

States: $u_1 = u_2 = u_{in} = u_{300}$; $h_{in} = u_{in} + RT_{in}$

$$m_1 = P_1V_1/RT_1 = 100 \times 1/(0.287 \times 300) = 1.1614 \text{ kg}$$

$$m_2 = P_2V_2/RT_2 = 1000 \times 1/(0.287 \times 300) = 11.6144 \text{ kg}$$

Heat transfer from the energy equation

$$\begin{aligned} {}_1Q_2 &= m_2u_2 - m_1u_1 - m_{in}h_{in} = (m_1 + m_{in})\, u_1 - m_1u_1 - m_{in}u_{in} - m_{in}RT_{in} \\ &= m_1u_1 - m_1u_1 + m_{in}u_1 - m_{in}u_{in} - m_{in}RT_{in} = - m_{in}RT_{in} \\ &= (11.6144 - 1.1614) \text{ kg} \times 0.287 \text{ kJ/kg-K} \times 300 \text{ K} = \mathbf{-900 \text{ kJ}} \end{aligned}$$

6.115

An initially empty canister of volume 0.2 m^3 is filled with carbon dioxide from a line at 800 kPa, 400 K. Assume the process runs until it stops by itself and it is adiabatic. Use constant heat capacity to find the final temperature in the canister.

C.V. Canister and valve, transient process with no heat transfer or work.

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$;

Energy Eq.6.16: $m_2u_2 – m_1u_1 = - m_{in}\, h_{in} + {}_1Q_2 - {}_1W_2$

Process: ${}_1Q_2 = 0$, V= constant so ${}_1W_2 = 0$

State 1: $m_1 = 0$ => $m_2 = m_{in}$

State 2: $P_2 = P_{line} = 800$ kPa, one more property

Energy Eq.: $u_2 = h_{in} = u_{in} + RT_{in} \Rightarrow C_{vo}\, T_2 = C_{vo}\, T_{in} + RT_{in} = C_{Po}T_{in}$

$T_2 = (C_{Po}/C_{vo})\, T_{in} = k \times T_{in} = 1.289 \times 400$ K = **515.6 K**

6.116

Repeat the previous problem but use the ideal gas Tables A.8 to solve it.

C.V. Canister and valve, transient process with no heat transfer or work.

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$;

Energy Eq.6.16: $m_2u_2 – m_1u_1 = - m_{in} h_{in} + {}_1Q_2 - {}_1W_2$

Process: ${}_1Q_2 = 0$, V= constant so ${}_1W_2 = 0$

State 1: $m_1 = 0$ => $m_2 = m_{in}$

State 2: $P_2 = P_{line} = 800$ kPa, one more property

Energy Eq.: $u_2 = h_{in} = 303.76$ kJ/kg from A.8

back interpolate for u_2: $T_2 =$ **495.9 K**

6.117

An initially empty bottle is filled with water from a line at 0.8 MPa, 350$^{\mathrm{o}}$C. Assume no heat transfer and that the bottle is closed when the pressure reaches the line pressure. If the final mass is 0.75 kg find the final temperature and the volume of the bottle.

Solution;

C.V. Bottle, transient process with no heat transfer or work.

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$;

Energy Eq.6.16: $m_2u_2 – m_1u_1 = - m_{in}\, h_{in}$

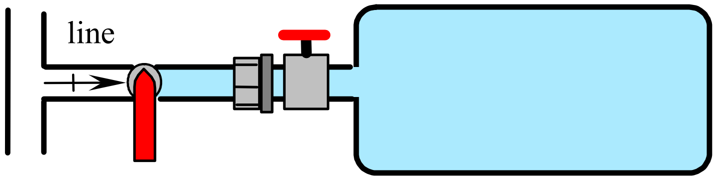
State 1: $m_1 = 0$ => $m_2 = m_{in}$ and $u_2 = h_{in}$

Line state: Table B.1.3: $h_{in} = 3161.68$ kJ/kg

State 2: $P_2 = P_{line} = 800$ kPa, $u_2 = 3161.68$ kJ/kg from Table B.1.3

$T_2 = \mathbf{520^oC}$ and $v_2 = 0.4554\ m^3/kg$

$V_2 = m_2v_2 = 0.75 \times 0.4554 = 0\mathbf{.342\ m^3}$



6.118

A 1-m^3 tank contains ammonia at 150 kPa, 25°C. The tank is attached to a line flowing ammonia at 1200 kPa, 60°C. The valve is opened, and mass flows in until the tank is half full of liquid, by volume at 25°C. Calculate the heat transferred from the tank during this process.

Solution:

C.V. Tank. Transient process as flow comes in.

State 1 Table B.2.2 interpolate between 20 °C and 30°C:

$v_1 = 0.9552\ m^3/kg;\ \ u_1 = 1380.6\ kJ/kg$

$m_1 = V/v_1 = 1/0.9552 = 1.047\ kg$

State 2: 0.5 m^3 liquid and 0.5 m^3 vapor from Table B.2.1 at 25°C

$v_f = 0.001658\ m^3/kg;\ \ v_g = 0.12813\ m^3/kg$

$m_{LIQ2} = 0.5/0.001658 = 301.57\ kg,\quad m_{VAP2} = 0.5/0.12813 = 3.902\ kg$

$m_2 = 305.47\ kg,\quad x_2 = m_{VAP2}/m_2 = 0.01277,$

From continuity equation

$m_i = m_2 - m_1 = 304.42\ kg$

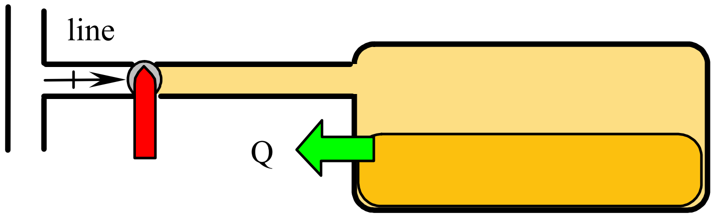
Table B.2.1: $u_2 = 296.6 + 0.01277 \times 1038.4 = 309.9\ kJ/kg$

State inlet: Table B.2.2 $h_i = 1553.3\ kJ/kg$

Energy Eq.6.16:

$Q_{CV} + m_ih_i = m_2u_2 - m_1u_1$

$Q_{CV} = 305.47 \times 309.9 - 1.047 \times 1380.6 - 304.42 \times 1553.3 = \mathbf{-379\ 636\ kJ}$

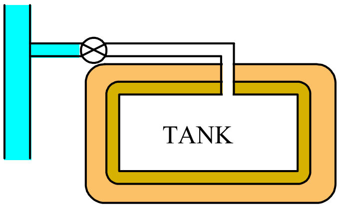


6.119

A 25-L tank, shown in Fig. P6.119, that is initially evacuated is connected by a valve to an air supply line flowing air at 20°C, 800 kPa. The valve is opened, and air flows into the tank until the pressure reaches 600 kPa.Determine the final temperature and mass inside the tank, assuming the process is adiabatic. Develop an expression for the relation between the line temperature and the final temperature using constant specific heats.

Solution:

C.V. Tank:



Continuity Eq.6.15: $m_2 = m_i$

Energy Eq.6.16: $m_2u_2 = m_ih_i$

Table A.7: $u_2 = h_i = 293.64$ kJ/kg

$\Rightarrow T_2 =$ **410.0 K**

$$m_2 = \frac{P_2V}{RT_2} = \frac{600 \times 0.025}{0.287 \times 410} = \mathbf{0.1275\ kg}$$

Assuming constant specific heat,

$$h_i = u_i + RT_i = u_2\,, \quad RT_i = u_2 - u_i = C_{vo}(T_2 - T_i)$$

$$C_{vo}T_2 = (\,C_{vo} + R\,)T_i = C_{Po}T_i\ , \quad T_2 = \left(\frac{C_{Po}}{C_{vo}}\right)T_i = kT_i$$

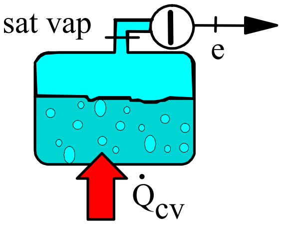
For $T_i = 293.2$K & constant C_{Po}, $T_2 = 1.40 \times 293.2 =$ **410.5 K**

6.120

A 200 liter tank initially contains water at 100 kPa and a quality of 1%. Heat is transferred to the water thereby raising its pressure and temperature. At a pressure of 2 MPa a safety valve opens and saturated vapor at 2 MPa flows out. The process continues, maintaining 2 MPa inside until the quality in the tank is 90%, then stops. Determine the total mass of water that flowed out and the total heat transfer.

Solution:

C.V. Tank, no work but heat transfer in and flow out. Denoting State 1: initial state, State 2: valve opens, State 3: final state.



Continuity Eq.: $m_3 - m_1 = - m_e$

Energy Eq.: $m_3u_3 - m_1u_1 = - m_eh_e + {}_1Q_3$

State 1 Table B.1.2: $v_1 = v_f + x_1v_{fg} = 0.001043 + 0.01\times1.69296$

$= 0.01797 \text{ m}^3/\text{kg}$

$u_1 = u_f + x_1u_{fg} = 417.33 + 0.01\times2088.72 = 438.22 \text{ kJ/kg}$

$m_1 = V/v_1 = 0.2 \text{ m}^3/(0.01797 \text{ m}^3/\text{kg}) = 11.13 \text{ kg}$

State 3 (2MPa): $v_3 = v_f + x_3v_{fg} = 0.001177 + 0.9\times0.09845 = 0.8978 \text{ m}^3/\text{kg}$

$u_3 = u_f + x_3u_{fg} = 906.42 + 0.9\times1693.84 = 2430.88 \text{ kJ/kg}$

$m_3 = V/v_3 = 0.2 \text{ m}^3/(0.08978 \text{ m}^3/\text{kg}) = 2.23 \text{ kg}$

Exit state (2MPa): $h_e = h_g = 2799.51 \text{ kJ/kg}$

Hence: $m_e = m_1 - m_3 = 11.13 \text{ kg} - 2.23 \text{ kg} = \mathbf{8.90 \text{ kg}}$

Applying the 1st law between state 1 and state 3

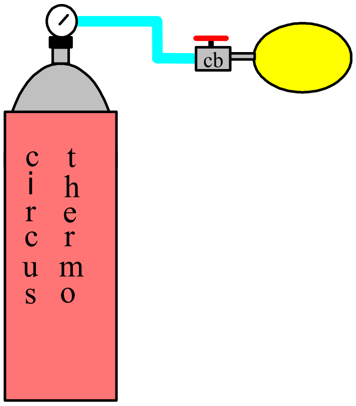
$${}_1Q_3 = m_3u_3 - m_1u_1 + m_eh_e$$
$$= 2.23 \times 2430.88 - 11.13 \times 438.22 + 8.90 \times 2799.51$$
$$= 25\,459 \text{ kJ} = \mathbf{25.46 \text{ MJ}}$$

6.121

Helium in a steel tank is at 250 kPa, 300 K with a volume of 0.1 m^3. It is used to fill a balloon. When the tank pressure drops to 150 kPa the flow of helium stops by itself. If all the helium still is at 300 K how big a balloon did I get? Assume the pressure in the balloon varies linearly with volume from 100 kPa (V = 0) to the final 150 kPa. How much heat transfer did take place?

Solution:

Take a C.V. of all the helium.
This is a control mass, the tank mass changes density and pressure.



Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = 100 + CV$

State 1: P_1, T_1, V_1

State 2: $P_2, T_2, V_2 = ?$

Ideal gas:

$$P_2 V_2 = mRT_2 = mRT_1 = P_1V_1$$

$$V_2 = V_1(P_1/P_2) = 0.1\times (250/150) = 0.16667 \text{ m}^3$$

$$V_{bal} = V_2 - V_1 = 0.16667 - 0.1 = 0.06667 \text{ m}^3$$

$${}_1W_2 = \int P\, dV = \text{AREA} = ½\,(P_1 + P_2)(V_2 - V_1)$$
$$= ½(250 + 150) \times 0.06667 = 13.334 \text{ kJ}$$

$$U_2 - U_1 = {}_1Q_2 - {}_1W_2 = m\,(u_2 - u_1) = mC_v\,(T_2 - T_1) = 0$$

so $\;{}_1Q_2 = {}_1W_2 = \mathbf{13.334\ kJ}$

Remark: The process is transient, but you only see the flow mass if you select the tank or the balloon as a control volume. That analysis leads to more terms that must be elliminated between the tank control volume and the balloon control volume.

6.122

An empty cannister of volume 1 L is filled with R-134a from a line flowing saturated liquid R-134a at 0^{o}C. The filling is done quickly so it is adiabatic. How much mass of R-134a is there after filling? The cannister is placed on a storage shelf where it slowly heats up to room temperature 20^{o}C. What is the final pressure?

C.V. cannister, no work and no heat transfer.

Continuity Eq.6.15: $m_2 = m_i$

Energy Eq.6.16: $m_2u_2 - 0 = m_ih_i = m_ih_{line}$

Table B.5.1: $h_{line} = 200.0$ kJ/kg, $P_{line} = 294$ kPa

From the energy equation we get

$u_2 = h_{line} = 200 \text{ kJ/kg} > u_f = 199.77 \text{ kJ/kg}$

State 2 is two-phase $P_2 = P_{line} = 294$ kPa and $T_2 = 0°C$

$$x_2 = \frac{u_2 - u_f}{u_{fg}} = \frac{200 - 199.77}{178.24} = 0.00129$$

$v_2 = 0.000773 + x_2\ 0.06842 = 0.000\ 861 \text{ m}^3\text{/kg}$

$m_2 = V/v_2 = 0.01/0.000861 = \mathbf{11.61\ kg}$

At 20°C: $v_f = 0.000\ 817 \text{ m}^3\text{/kg} < v_2$ so still two-phase

$P = P_{sat} = \mathbf{572.8\ kPa}$

6.123

A nitrogen line, 300 K and 0.5 MPa, shown in Fig. P6.123, is connected to a turbine that exhausts to a closed initially empty tank of 50 m^3. The turbine operates to a tank pressure of 0.5 MPa, at which point the temperature is 250 K. Assuming the entire process is adiabatic, determine the turbine work.

Solution:

C.V. turbine & tank $\Rightarrow$ Transient process

Conservation of mass Eq.6.15: $m_i = m_2 \Rightarrow m$

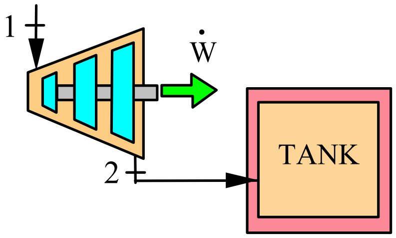
Energy Eq.6.16: $m_i h_i = m_2 u_2 + W_{CV}$; $W_{CV} = m(h_i - u_2)$

Table B.6.2: $P_i = 0.5$ MPa, $T_i = 300$ K, Nitrogen; $h_i = 310.28$ kJ/kg

2: $P_2 = 0.5$ MPa, $T_2 = 250$ K, $u_2 = 183.89$ kJ/kg, $v_2 = 0.154\ m^3/kg$

$m_2 = V/v_2 = 50/0.154 = 324.7$ kg

$W_{CV} = 324.7\ (310.28 - 183.89) = 41\ 039$ kJ = **41.04 MJ**



We could with good accuracy have solved using ideal gas and Table A.5

6.124

A 750-L rigid tank, shown in Fig. P6.124, initially contains water at 250°C, 50% liquid and 50% vapor, by volume. A valve at the bottom of the tank is opened, and liquid is slowly withdrawn. Heat transfer takes place such that the temperature remains constant. Find the amount of heat transfer required to the state where half the initial mass is withdrawn.

Solution:

C.V. vessel

Continuity Eq.6.15: $m_2 - m_1 = - m_e$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = Q_{CV} - m_eh_e$

State 1: $m_{LIQ1} = \frac{0.375}{0.001251} = 299.76$ kg; $m_{VAP1} = \frac{0.375}{0.05013} = 7.48$ kg

$m_1u_1 = 299.76 \times 1080.37 + 7.48 \times 2602.4 = 343\ 318$ kJ

$m_1 = 307.24$ kg; $m_e = m_2 = 153.62$ kg

State 2: $v_2 = \frac{0.75}{153.62} = 0.004882 = 0.001251 + x_2 \times 0.04888$

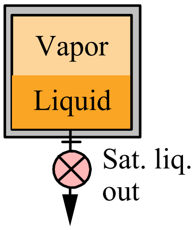
$x_2 = 0.07428$; $u_2 = 1080.37 + 0.07428 \times 1522 = 1193.45$ kJ/kg

Exit state: $h_e = h_f = 1085.34$ kJ/kg

Energy equation now gives the heat transfer as

$$Q_{CV} = m_2u_2 - m_1u_1 + m_eh_e$$

$$= 153.62 \times 1193.45 - 343\ 318 + 153.62 \times 1085.34 = \mathbf{6750\ kJ}$$



6.125

Consider the previous problem but let the line and valve be located in the top of the tank. Now saturated vapor is slowly withdrawn while heat transfer keeps the temperature inside constant. Find the heat transfer required to reach a state where half the original mass is withdrawn.

Solution:

C.V. vessel

Continuity Eq.6.15: $m_2 - m_1 = -m_e$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = Q_{CV} - m_eh_e$

State 1: $m_{LIQ1} = \frac{0.375}{0.001251} = 299.76$ kg; $m_{VAP1} = \frac{0.375}{0.05013} = 7.48$ kg

$m_1u_1 = 299.76 \times 1080.37 + 7.48 \times 2602.4 = 343\,318$ kJ

$m_1 = 307.24$ kg; $m_e = m_2 = 153.62$ kg

State 2: $v_2 = \frac{0.75}{153.62} = 0.004882 = 0.001251 + x_2 \times 0.04888$

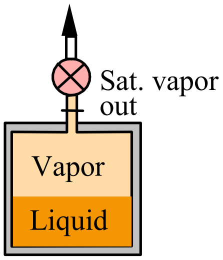
$x_2 = 0.07428$; $u_2 = 1080.37 + 0.07428 \times 1522 = 1193.45$ kJ/kg

Exit state: $h_e = h_g = 2801.52$ kJ/kg

Energy equation now gives the heat transfer as

$Q_{CV} = m_2u_2 - m_1u_1 + m_eh_e$

$= 153.62 \times 1193.45 - 343\,318 + 153.62 \times 2801.52 =$ **270 389 kJ**



6.126

A 2-m^3 insulated vessel, shown in Fig. P6.126, contains saturated vapor steam at 4 MPa. A valve on the top of the tank is opened, and steam is allowed to escape. During the process any liquid formed collects at the bottom of the vessel, so that only saturated vapor exits. Calculate the total mass that has escaped when the pressure inside reaches 1 MPa.

Solution:

C.V. Vessel: Mass flows out.

Continuity Eq.6.15: $m_e = m_1 - m_2$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = -(m_1-m_2)h_e$ or $m_2(h_e-u_2) = m_1(h_e-u_1)$

Average exit enthalpy $h_e \approx (h_{G1}+h_{G2})/2 = (2801.4+2778.1)/2 = 2789.8$

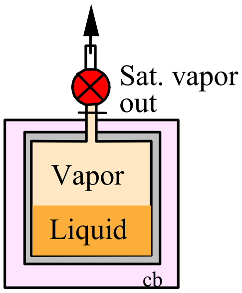
State 1: $m_1 = V/v_1 = 40.177$ kg, $m_2 = V/v_2$

Energy equation $\Rightarrow \frac{2}{v_2}(2789.8-u_2) = 40.177(2789.8-2602.3) = 7533.19$

But $v_2 = .001\,127 + .193\,313\, x_2$ and $u_2 = 761.7 + 1822\, x_2$

Substituting and solving, $x_2 = 0.7936$

$\Rightarrow m_2 = V/v_2 = 12.94$ kg, $m_e =$ **27.24 kg**



6.127

A 2 m tall cylinder has a small hole in the bottom. It is filled with liquid water 1 m high, on top of which is 1 m high air column at atmospheric pressure of 100 kPa. As the liquid water near the hole has a higher P than 100 kPa it runs out. Assume a slow process with constant T. Will the flow ever stop? When?

Solution:

$$P_{bot} = P_{air} + \rho g L_f$$

For the air PV = mRT and the total height is H = 2 m

$$P_{air} = mRT/V_{air} \quad ; V_{air} = A\ L_{air} = A\,(H - L_f)$$

$$P_{bot} = \frac{m_a R_a T_a}{A(H\text{-}L_f)} + \rho_f g\ L_f = \frac{P_{a1}V_{a1}}{A(H\text{-}L_f)} + \rho_f\, gL_f = \frac{P_{a1}L_{a1}}{H\text{-}L_f} + \rho_f\, gL_f \ \geq P_o$$

Solve for L_f ; $\rho_f = 1/(v_f) = 1/0.0021002 = 998\ \text{kg/m}^3$

$$P_{a1}\ L_{a1} + \rho_f\ g\ L_f\,(H - L_f) \geq\ P_o\,(H - L_f)$$

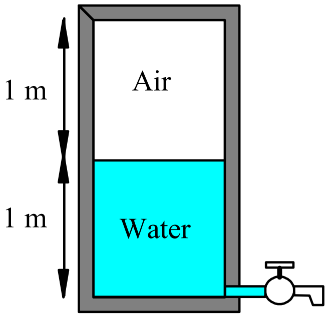
$$(\rho_f gH + P_o)\ L_f - \rho_f g L^2_f = P_o\ H - P_{a1}\ L_{a1} \geq 0$$

Put in numbers and solve quadratic eq.

$$L^2_f\ - [\ H + (P_o/\rho g)\ ]\ L_f\ + \frac{P_oH - P_{a1}L_{a1}}{\rho g} = 0$$

$$(P_o/\rho g) = \frac{100\ \text{kPa m}^3\ \text{s}^3}{998 \times 9.807\ \text{kg m}} = 10.217\ \text{m}$$

$$\frac{P_oH - P_{a1}L_{a1}}{\rho g} = \frac{100\ (2\text{-}1)}{998 \times 9.807} = 10.217\ \text{m}$$



$$L^2_f\ - 12.217\ L_f + 10.217 = 0$$

$$L_f = \frac{12.217}{2} \pm \sqrt{\frac{12.217^2}{4} - \frac{12.217}{4}}\ = 6.1085 \pm 5.2055$$

=> 11.314 or **0.903 m** so the second root is the solution

Verify

$$P_{a2} = P_{a1}\cdot\frac{L_{a1}}{H\text{-}L_f} = 100\,\frac{1}{2 - 0.903} = 91.158\ \text{kPa}$$

$$\rho g L_f = 998 \times 9.807 \times 0.903 = 8838\ \text{Pa} = 8.838\ \text{kPa}$$

$$P_{bot} = P_{a2} + \rho g L_f = 91.158 + 8.838 = 99.996\ \text{kPa} \quad \text{OK}$$

Review Problems

6.128

A pipe of radius R has a fully developed laminar flow of air at P_o, T_o with a velocity profile as: $V = V_c\,[1 – (r/R)^{2}]$, where V_c is the velocity on the center-line and r is the radius as shown in Fig. P6.128. Find the total mass flow rate and the average velocity both as functions of V_c and R.

$$\dot{m} = AV/v = \dot{V}/v$$

Since the velocity is distributed we need to integrate over the area. From Eq.6.2

$$\dot{V} = \int V_{local}\, dA = \int V(r)\, 2\pi r\, dr$$

where W is the depth. Substituting the velocity we get

$$\dot{V} = \int V_c\, 2\pi r\, [1 - (r/R)^2]\, dr$$

$$= 2\pi\, V_c\, R^2 \int_0^1 z\,(1 - z^2)\, dz$$

$$= 2\pi\, V_c\, R^2 \left(\tfrac{1}{2} z^2 - \tfrac{1}{4} z^4 \right|_0^1 = \frac{\pi}{2} V_c\, R^2 = \frac{1}{2} V_c\, A$$

$$V = \dot{V}/A = \frac{1}{2} V_c$$

$$\dot{m} = \dot{V}/\mathbf{v} = \frac{\pi}{2} V_c\, R^2/v$$

6.129

Steam at 3 MPa, 400°C enters a turbine with a volume flow rate of 5 m^3/s. An extraction of 15% of the inlet mass flow rate exits at 600 kPa, 200°C. The rest exits the turbine at 20 kPa with a quality of 90%, and a velocity of 20 m/s. Determine the volume flow rate of the extraction flow and the diameter of the final exit pipe.

Solution:

Inlet flow : $\dot{m}_i = \dot{V}/v = 5/0.09936 = 50.32$ kg/ s (Table B.1.3)

Extraction flow : $\dot{m}_e = 0.15\ \dot{m}_i = 7.55$ kg/ s; $v = 0.35202\ m^3/kg$

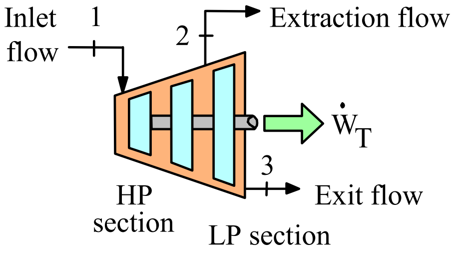
$$\dot{V}_{ex} = \dot{m}_e v = 7.55 \times 0.35202 = \mathbf{2.658\ m^3/\ s}$$

Exit flow : $\dot{m} = 0.85\ \dot{m}_i = 42.77$ kg /s

Table B.1.2 $v = 0.001017 + 0.9 \times 7.64835 = 6.8845\ m^3/kg$

$$\dot{m} = A\mathbf{V}/v \Rightarrow \quad A = (\pi/4)\ D^2 = \dot{m}\ v/\mathbf{V} = 42.77 \times 6.8845/20 = 14.723\ m^2$$

$$\Rightarrow \mathbf{D = 4.33\ m}$$



6.130

In a glass factory a 2 m wide sheet of glass at 1500 K comes out of the final rollers that fix the thickness at 5 mm with a speed of 0.5 m/s. Cooling air in the amount of 20 kg/s comes in at 17°C from a slot 2 m wide and flows parallel with the glass. Suppose this setup is very long so the glass and air comes to nearly the same temperature (a co-flowing heat exchanger) what is the exit temperature?

Solution:

Energy Eq.: $\dot{m}_{glass}h_{glass\ 1} + \dot{m}_{air}h_{air\ 2} = \dot{m}_{glass}h_{glass\ 3} + \dot{m}_{air}h_{air\ 4}$

$\dot{m}_{glass} = \rho\dot{V} = \rho AV = 2500\times 2 \times 0.005\times 0.5 = 12.5$ kg/s

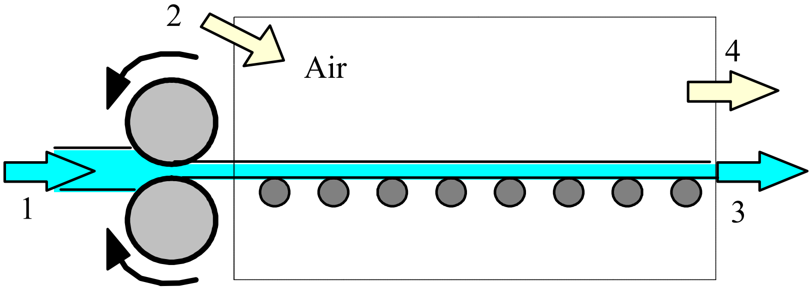
$\dot{m}_{glass}C_{glass}\ (T_3 - T_1) + \dot{m}_{air}\ C_{Pa}\ (T_4 - T_2) = \varnothing$

$T_4 = T_3$, $C_{glass} = 0.80$ kJ/kg K, $C_{Pa} = 1.004$ kJ/kg K

$$T_3 = \frac{\dot{m}_{glass}C_{glass}\ T_1 + \dot{m}_{air}C_{Pa}\ T_2}{\dot{m}_{glass}C_{glass} + \dot{m}_{air}C_{Pa}} = \frac{12.5\times0.80\times1500 + 20\times1.004\times290}{12.5\times0.80 + 20\times1.004}$$

$= \mathbf{692.3\ K}$

We could use table A.7.1 for air, but then it will be trial and error



6.131

Assume a setup similar to the previous problem but the air flows in the opposite direction of the glass, it comes in where the glass goes out. How much air flow at 17^oC is required to cool the glass to 450 K assuming the air must be at least 120 K cooler than the glass at any location?

Solution:

Energy Eq.: $\dot m_1 h_1 + \dot m_4 h_4 = \dot m_3 h_3 + \dot m_2 h_2$

$T_4 = 290$ K and $T_3 = 450$ K

$\dot m_{glass} = \rho \dot V = \rho AV = 2500 \times 2 \times 0.005 \times 0.5 = 12.5$ kg/s

$T_2 \le T_1 - 120 \text{ K} = 1380 \text{ K}$

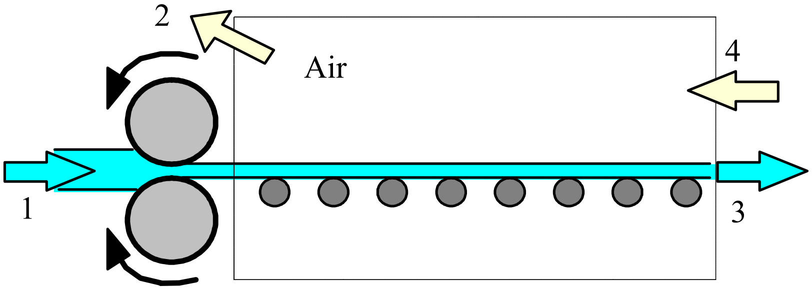
$$\dot m = \dot m_4 = \dot m_2 = \dot m_1 \frac{h_1 - h_3}{h_2 - h_4}$$

Let us check the limit and since T is high use table A.7.1 for air.

$h_4 = 290.43$ kJ/kg, $h_2 = 1491.33$ kJ/kg

$$\dot m = \dot m_4 = \dot m_2 = \dot m_1 \frac{h_1 - h_3}{h_2 - h_4} = \dot m_1 \frac{C_{glass}(T_1 - T_3)}{h_2 - h_4}$$

$$\dot m = 12.5 \frac{0.8\ (1500\text{-}450)}{1491.33 - 290.43} = \mathbf{8.743\ kg/s}$$



6.132

Two kg of water at 500 kPa, 20^{o}C is heated in a constant pressure process to 1700^{o}C. Find the best estimate for the heat transfer.

Solution:

C.V. Heater; steady state 1 inlet and exit, no work term, no ΔKE, ΔPE .

Continuity Eq.: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m}$,

Energy Eq.6.13: $q + h_{in} = h_{ex} \;\Rightarrow\; q = h_{ex} - h_{in}$

steam tables only go up to 1300^{o}C so use an intermediate state at lowest pressure (closest to ideal gas) $h_X(1300^{\mathrm{o}}\mathrm{C}, 10\text{ kPa})$ from Table B.1.3 and table A.8 for the high T change Δh from 1300^{o}C to 1700^{o}C.

$$h_{ex} - h_{in} = (h_{ex} - h_X) + (h_X - h_{in})$$

$$= (4515.55 – 3416.77) + (5409.7 - 83.96) = 6424.5 \text{ kJ/kg}$$

$$Q = m(h_{ex} - h_{in}) = 2 \times 6424.5 = \mathbf{12\ 849\ kJ}$$

6.133

A 500-L insulated tank contains air at 40°C, 2 MPa. A valve on the tank is opened, and air escapes until half the original mass is gone, at which point the valve is closed. What is the pressure inside then?

Solution:

State 1: ideal gas $m_1 = P_1V/RT_1 = \dfrac{2000 \times 0.5}{0.287 \times 313.2} = 11.125$ kg

Continuity eq.6.15: $m_e = m_1 - m_2,\ \ m_2 = m_1/2 \ \Rightarrow\ m_e = m_2 = 5.5625$ kg

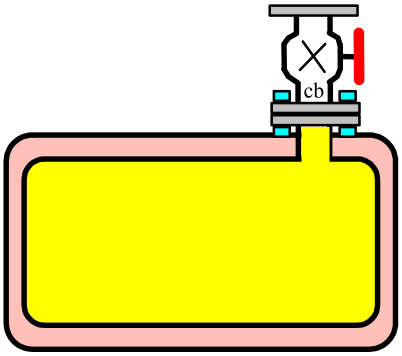
Energy Eq.6.16: $m_2u_2 - m_1u_1 = - m_eh_{e\ AV}$

Substitute constant specific heat from table A.5 and evaluate the exit enthalpy as the average between the beginning and the end values

$$5.5625{\times}0.717\ T_2 - 11.125{\times}0.717{\times}313.2 = - 5.5625{\times}1.004\ (313.2 + T_2)/2$$

Solving, $T_2 = 239.4$ K

$$P_2 = \frac{m_2RT_2}{V} = \frac{5.5625 \times 0.287 \times 239.4}{0.5} = \mathbf{764\ kPa}$$



6.134

Three air flows all at 200 kPa are connected to the same exit duct and mix without external heat transfer. Flow one has 1 kg/s at 400 K, flow two has 3 kg/s at 290 K and flow three has 2 kg/s at 700 K. Neglect kinetic energies and find the volume flow rate in the exit flow.

Solution:

Continuity Eq. $\dot m_1 + \dot m_2 + \dot m_3 = \dot m_4 h_4$

Energy Eq.: $\dot m_1 h_1 + \dot m_2 h_2 = \dot m_3 h_3 + \dot m_4 h_4$

$\dot V_4 = \dot m\ v_4$

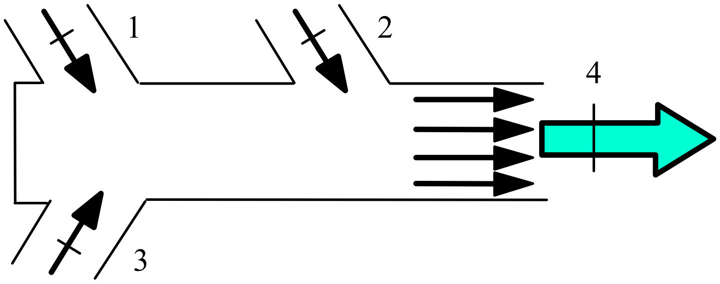
$$h_4 = \frac{\dot m_1}{\dot m_4} h_1 + \frac{\dot m_2}{\dot m_4} h_2 + \frac{\dot m_3}{\dot m_4} h_3 = \frac{1}{6} \times 401.3 + \frac{3}{6} \times 290.43 + \frac{2}{6} \times 713.56$$

$= 449.95$ kJ/kg

$$T_4 = 440 + 20\ \frac{449.95 - 441.93}{462.34 - 441.93} = 447.86 \text{ K}$$

$v_4 = RT_4 / P_4 = 0.287 \times 447.86/200 = \mathbf{0.643\ m^3/kg}$

$\dot V_4 = \dot m_4 v_4 = 6 \times 0.643 = \mathbf{3.858\ m^3/s}$



6.135

Consider the power plant as described in Problem 6.106.

a. Determine the temperature of the water leaving the intermediate pressure heater, T_{13}, assuming no heat transfer to the surroundings.

b. Determine the pump work, between states 13 and 16.

Solution:

a) Intermediate Pressure Heater

Energy Eq.6.10: $\dot{m}_{11}h_{11} + \dot{m}_{12}h_{12} + \dot{m}_{15}h_{15} = \dot{m}_{13}h_{13} + \dot{m}_{14}h_{14}$

$75.6\times284.6 + 8.064\times2517 + 4.662\times584 = 75.6\times h_{13} + 12.726\times349$

$h_{13} = 530.35 \rightarrow T_{13} = \mathbf{126.3°C}$

b) The high pressure pump

Energy Eq.6.12: $\dot{m}_{13}h_{13} = \dot{m}_{16}h_{16} + \dot{W}_{Cv,P}$

$\dot{W}_{Cv,P} = \dot{m}_{13}(h_{13} - h_{16}) = 75.6(530.35 - 565) = \mathbf{-2620\ kW}$

6.136

Consider the powerplant as described in Problem 6.106.

a. Find the power removed in the condenser by the cooling water (not shown).

b. Find the power to the condensate pump.

c. Do the energy terms balance for the low pressure heater or is there a heat transfer not shown?

Solution:

a) Condenser:

Energy Eq.6.10: $\dot{Q}_{CV} + \dot{m}_5 h_5 + \dot{m}_{10} h_{10} = \dot{m}_6 h_6$

$$\dot{Q}_{CV} + 55.44 \times 2279 + 20.16 \times 142.51 = 75.6 \times 138.3$$

$$\dot{Q}_{CV} = -118\ 765 \text{ kW} = \mathbf{-118.77\ MW}$$

b) The condensate pump

$$\dot{W}_{CV,P} = \dot{m}_6 (h_6 - h_7) = 75.6(138.31 - 140) = \mathbf{-127.8\ kW}$$

c) Low pressure heater Assume no heat transfer

$$\dot{m}_{14} h_{14} + \dot{m}_8 h_8 + \dot{m}_7 h_7 + \dot{m}_9 h_9 = \dot{m}_{10} h_{10} + \dot{m}_{11} h_{11}$$

LHS = 12.726×349 + 2.772×2459 + 75.6×140 + 4.662×558 = 24 443 kW

RHS = (12.726 + 2.772 + 4.662) × 142.51 + 75.6 × 284.87 = 24 409 kW

A slight imbalance, but OK.

6.137

A 1-m^3, 40-kg rigid steel tank contains air at 500 kPa, and both tank and air are at 20°C. The tank is connected to a line flowing air at 2 MPa, 20°C. The valve is opened, allowing air to flow into the tank until the pressure reaches 1.5 MPa and is then closed. Assume the air and tank are always at the same temperature and the final temperature is 35°C. Find the final air mass and the heat transfer.

Solution:

Control volume: Air and the steel tank.

Continuity Eq.6.15: $m_2 - m_1 = m_i$

Energy Eq.6.16: $(m_2u_2 - m_1u_1)_{AIR} + m_{ST}(u_2 - u_1)_{ST} = m_ih_i + {}_1Q_2$

$$m_{1\ AIR} = \frac{P_1V}{RT_1} = \frac{500 \times 1}{0.287 \times 293.2} = 5.94\ \text{kg}$$

$$m_{2\ AIR} = \frac{P_2V}{RT_2} = \frac{1500 \times 1}{0.287 \times 308.2} = \mathbf{16.96\ kg}$$

$$m_i = (m_2 - m_1)_{air} = 16.96 - 5.94 = 11.02\ \text{kg}$$

The energy equation now gives

$$\begin{aligned} {}_1Q_2 &= (m_2u_2 - m_1u_1)_{air} + m_{st}(u_2 - u_1)_{st} - m_ih_i \\ &= m_1(u_2 - u_1) + m_i(u_2 - u_i - RT_i) + m_{st}C_{st}(T_2 - T_1) \\ &\cong m_1C_v(T_2 - T_1) + m_i[C_v(T_2 - T_i) - RT_i] + m_{st}C_{st}(T_2 - T_1) \\ &= 5.94 \times 0.717(35 - 20) + 11.02[0.717(35 - 20) - 0.287 \times 293.2] \\ &\quad + 40 \times 0.46(35 - 20) \\ &= 63.885 - 808.795 + 276 \\ &= \mathbf{-\ 468.9\ kJ} \end{aligned}$$

6.138

A steam engine based on a turbine is shown in Fig. P6.138. The boiler tank has a volume of 100 L and initially contains saturated liquid with a very small amount of vapor at 100 kPa. Heat is now added by the burner, and the pressure regulator does not open before the boiler pressure reaches 700 kPa, which it keeps constant. The saturated vapor enters the turbine at 700 kPa and is discharged to the atmosphere as saturated vapor at 100 kPa. The burner is turned off when no more liquid is present in the boiler. Find the total turbine work and the total heat transfer to the boiler for this process.

Solution:

C.V. Boiler tank. Heat transfer, no work and flow out.

Continuity Eq.6.15: $m_2 - m_1 = -\, m_e$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = Q_{CV} - m_eh_e$

State 1: Table B.1.1, 100 kPa => $v_1 = 0.001\,043$, $u_1 = 417.36$ kJ/kg

=> $m_1 = V/v_1 = 0.1/0.001\,043 = 95.877$ kg

State 2: Table B.1.1, 700 kPa => $v_2 = v_g = 0.2729$, $u_2 = 2572.5$ kJ/kg

=> $m_2 = V/v_g = 0.1/0.2729 = 0.366$ kg,

Exit state: Table B.1.1, 700 kPa => $h_e = 2763.5$ kJ/kg

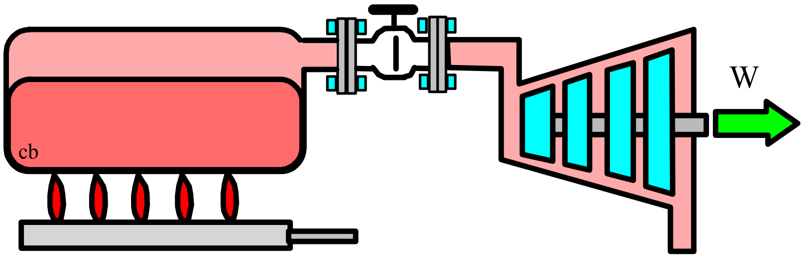
From continuity eq.: $m_e = m_1 - m_2 = 95.511$ kg

$$Q_{CV} = m_2u_2 - m_1u_1 + m_eh_e$$
$$= 0.366 \times 2572.5 - 95.877 \times 417.36 + 95.511 \times 2763.5$$
$$= 224\,871 \text{ kJ} = \mathbf{224.9\ MJ}$$

C.V. Turbine, steady state, inlet state is boiler tank exit state.

Turbine exit state: Table B.1.1, 100 kPa => $h_e = 2675.5$ kJ/kg

$$W_{turb} = m_e\,(h_{in} - h_{ex}) = 95.511 \times (2763.5 - 2675.5) = \mathbf{8405\ kJ}$$



6.139

An insulated spring-loaded piston/cylinder, shown in Fig. P6.139, is connected to an air line flowing air at 600 kPa, 700 K by a valve. Initially the cylinder is empty and the spring force is zero. The valve is then opened until the cylinder pressure reaches 300 kPa. By noting that $u_2 = u_{line} + C_V(T_2 - T_{line})$ and $h_{line} - u_{line} = RT_{line}$ find an expression for T_2 as a function of P_2, P_o, T_{line}. With P = 100 kPa, find T_2.

Solution:

C.V. Air in cylinder, insulated so $_1Q_2 = 0$

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = m_{in}h_{line} - {}_1W_2$

$$m_1 = 0 \;\Rightarrow\; m_{in} = m_2\;;\quad m_2u_2 = m_2h_{line} - \frac{1}{2}(P_0 + P_2)m_2v_2$$

$$\Rightarrow \qquad u_2 + \frac{1}{2}(P_0 + P_2)v_2 = h_{line}$$

Use constant specific heat in the energy equation

$$C_V(T_2 - T_{line}) + u_{line} + \frac{1}{2}(P_0 + P_2)RT_2/P_2 = h_{line}$$

$$\boldsymbol{\left[C_V + \frac{1}{2}\frac{P_0 + P_2}{P_2}R\right]T_2 = (R + C_V)T_{line}}$$

$$\text{with \#'s: } T_2 = \frac{R + C_V}{\frac{2}{3}R + C_V}T_{line}\;;\quad C_V/R = 1/(k-1)\,,\quad k = 1.4$$

$$T_2 = \frac{k - 1 + 1}{\frac{2}{3}k - \frac{2}{3} + 1}T_{line} = \frac{3k}{2k + 1}T_{line} = 1.105\,T_{line} = \mathbf{773.7\ K}$$

6.140

A mass-loaded piston/cylinder, shown in Fig. P6.140, containing air is at 300 kPa, 17°C with a volume of 0.25 m^3, while at the stops $V = 1$ m^3. An air line, 500 kPa, 600 K, is connected by a valve that is then opened until a final inside pressure of 400 kPa is reached, at which point $T = 350$ K. Find the air mass that enters, the work, and heat transfer.

Solution:

C.V. Cylinder volume.

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$

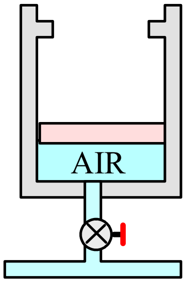
Energy Eq.6.16: $m_2u_2 - m_1u_1 = m_{in}h_{line} + Q_{CV} - {}_1W_2$

Process: P_1 is constant to stops, then constant V to state 2 at P_2

State 1: P_1, T_1 $\quad m_1 = \frac{P_1V}{RT_1} = \frac{300 \times 0.25}{0.287 \times 290.2} = 0.90 \text{ kg}$

State 2:

Open to $P_2 = 400$ kPa, $T_2 = 350$ K



$$m_2 = \frac{400 \times 1}{0.287 \times 350} = 3.982 \text{ kg}$$

$$m_i = 3.982 - 0.90 = \mathbf{3.082\ kg}$$

Only work while constant P

$$_1W_2 = P_1(V_2 - V_1) = 300(1 - 0.25) = \mathbf{225\ kJ}$$

Energy Eq.: $\quad Q_{CV} + m_ih_i = m_2u_2 - m_1u_1 + {}_1W_2$

$$Q_{CV} = 3.982 \times 0.717 \times 350 - 0.90 \times 0.717 \times 290.2 + 225$$
$$- 3.082 \times 1.004 \times 600 = \mathbf{-819.2\ kJ}$$

We could also have used the air tables A.7.1 for the u's and h_i**.**

6.141

A 2-m^3 storage tank contains 95% liquid and 5% vapor by volume of liquified natural gas (LNG) at 160 K, as shown in Fig. P6.62. It may be assumed that LNG has the same properties as pure methane. Heat is transferred to the tank and saturated vapor at 160 K flows into a steady flow heater which it leaves at 300 K. The process continues until all the liquid in the storage tank is gone. Calculate the total amount of heat transfer to the tank and the total amount of heat transferred to the heater.

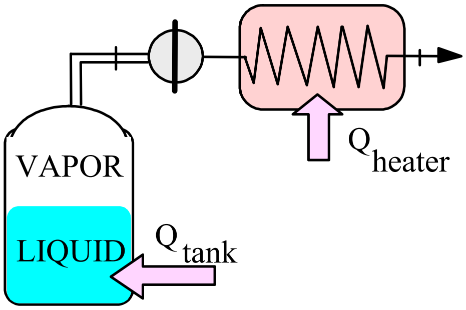
Solution:

CV: Tank, flow out, transient.

Continuity Eq.: $m_2 - m_1 = -m_e$

Energy Eq.:

$$Q_{Tank} = m_2u_2 - m_1u_1 + m_eh_e$$



At 160 K, from Table B.7:

$$m_f = V_f/v_f = \frac{0.95 \times 2}{0.00297} = 639.73 \text{ kg}, \quad m_g = V_g/v_g = \frac{0.05 \times 2}{0.03935} = 2.541 \text{ kg}$$

$$m_1 = 642.271 \text{ kg}, \qquad m_2 = V/v_{g2} = 2/0.03935 = 50.826 \text{ kg}$$

$$m_1u_1 = 639.73(-106.35) + 2.541(207.7) = -67507 \text{ kJ}$$

$$m_e = m_1 - m_2 = 591.445 \text{ kg}$$

$$Q_{Tank} = 50.826 \times 207.7 - (-67\,507) + 591.445 \times 270.3$$

$$= \mathbf{+237\,931\ kJ}$$

CV: Heater, steady flow, $P = P_{G\ 160\ K} = 1593$ kPa

$$Q_{Heater} = m_{e\ Tank}(h_e - h_i)_{Heater}$$

$$= 591.445(612.9 - 270.3) = \mathbf{202\,629\ kJ}$$

Heat transfer problems

6.142

Liquid water at 80^{o}C flows with 0.2 kg/s inside a square duct, side 2 cm insulated with a 1 cm thick layer of foam k = 0.1 W/m K. If the outside foam surface is at 25^{o}C how much has the water temperature dropped for 10 m length of duct? Neglect the duct material and any corner effects (A = 4sL).

Solution:

Conduction heat transfer

$$\dot{Q}_{out} = kA\frac{dT}{dx} = k\ 4\ sL\frac{\Delta T}{\Delta x} = 0.1\times 4\times 0.02\times 10\times(80\text{-}25)/0.01 = 440\text{ W}$$

Energy equation: $\dot{m}_1 h_1 = \dot{m}h_e + \dot{Q}_{out}$

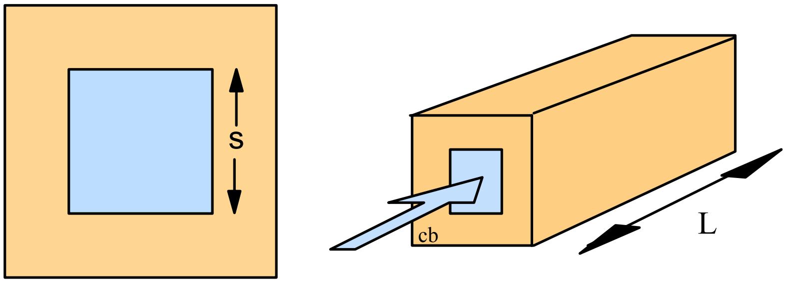
$h_e - h_i = -\dot{Q}/\dot{m} = -\ (440/0.2) = -2200$ J/kg $= -\ 2.2$ kJ/kg

$h_e = h_i$ -2.2 kJ/kg = 334.88 – 2.2 = 332.68 kJ/kg

$$T_e = 80 - \frac{2.2}{334.88\text{-}313.91}\ 5 = 79.48^oC$$

$\Delta T = \mathbf{0.52^oC}$

We could also have used $h_e - h_i = C_p\Delta T$



6.143

Saturated liquid carbon dioxide at 2500 kPa flows at 2 kg/s inside a 10 cm outer diameter steel pipe and outside of the pipe is a flow of air at 22^{o}C with a convection coefficient of h = 150 W/m^2 K. Neglect any ΔT in the steel and any inside convection h and find the length of pipe needed to bring the carbon dioxide to saturated vapor.

Solution:

Energy Eq. water: $\dot{Q} = \dot{m}\,(h_e - h_i) = \dot{m}\, h_{fg}$

Table B.3.1: $h_{fg} = 263.65$ kJ/kg, $T = T_{sat} = -12^oC$ and

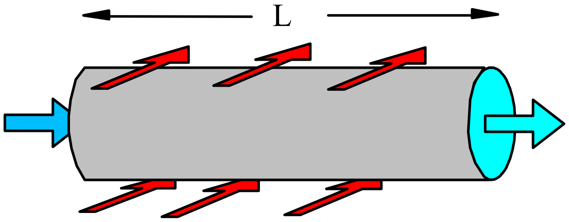
The energy is transferred by heat transfer so

$$\dot{Q} = h\, A\, \Delta T = h\, \pi D\, L\, \Delta T$$

Equate the two expressions for the heat transfer and solve for the length L

$$L = \frac{\dot{Q}}{h\, \pi D\, \Delta T} = \frac{\dot{m} h_{fg}}{h\, \pi D\, \Delta T} = \frac{2 \times 263.65 \times 1000}{150 \times \pi \times 0.1 \times (22 - (-12))}$$

= **329 m**



6.144

A counter-flowing heat exchanger conserves energy by heating cold outside fresh air at 10°C with the outgoing combustion gas (air) at 100°C. Assume both flows are 1 kg/s and the temperature difference between the flows at any point is 50°C. What is the incoming fresh air temperature after the heat exchanger? What is the equivalent (single) convective heat transfer coefficient between the flows if the interface area is 2 m²?

Solution:

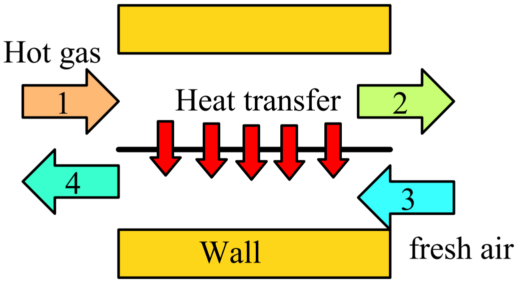
The outside fresh air is heated up to $T_4 = 50^{o}C$ (100 – 50), the heat transfer needed is

$$\dot{Q} = \dot{m}\Delta h = \dot{m}C_p\Delta T = 1 \text{ kg/s} \times 1.004 \frac{\text{kJ}}{\text{kg K}} \times (50 - 10) \text{ K} = 40 \text{ kW}$$

This heat transfer takes place with a temperature difference of 50°C throughout

$$\dot{Q} = h\,A\,\Delta T \quad \Rightarrow \quad h = \frac{\dot{Q}}{A\Delta T} = \frac{40\,000}{2 \times 50}\,\frac{\text{W}}{\text{m}^2\text{ K}} = \mathbf{400}\,\frac{\mathbf{W}}{\mathbf{m^2\ K}}$$

Often the flows may be concentric as a smaller pipe inside a larger pipe.



6.145

A flow of 1000 K, 100 kPa air with 0.5 kg/s in a furnace flows over a steel plate of surface temperature 400 K. The flow is such that the convective heat transfer coefficient is h = 125 W/m^2 K. How much of a surface area does the air have to flow over to exit with a temperature of 800 K? How about 600 K?

Solution:

Convection heat transfer

$\dot{Q} = hA\ \Delta T$

Inlet: $\Delta T_i = 1000 - 400 = 600$ K

a)

Exit: $\Delta T_e = 800 - 400 = 400$ K,

so we can use an average of $\Delta T = 500$ K for heat transfer

$\dot{Q} = \dot{m}_a\ (h_i - h_e) = 0.5(1046.22 - 822.2) = 112$ kW

$$A = \frac{\dot{Q}}{h\ \Delta T} = \frac{112 \times 1000}{125 \times 500} = \mathbf{1.79\ m^2}$$

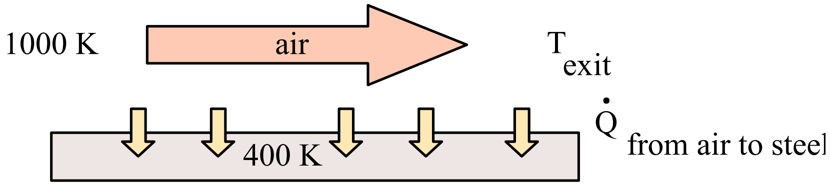
b)

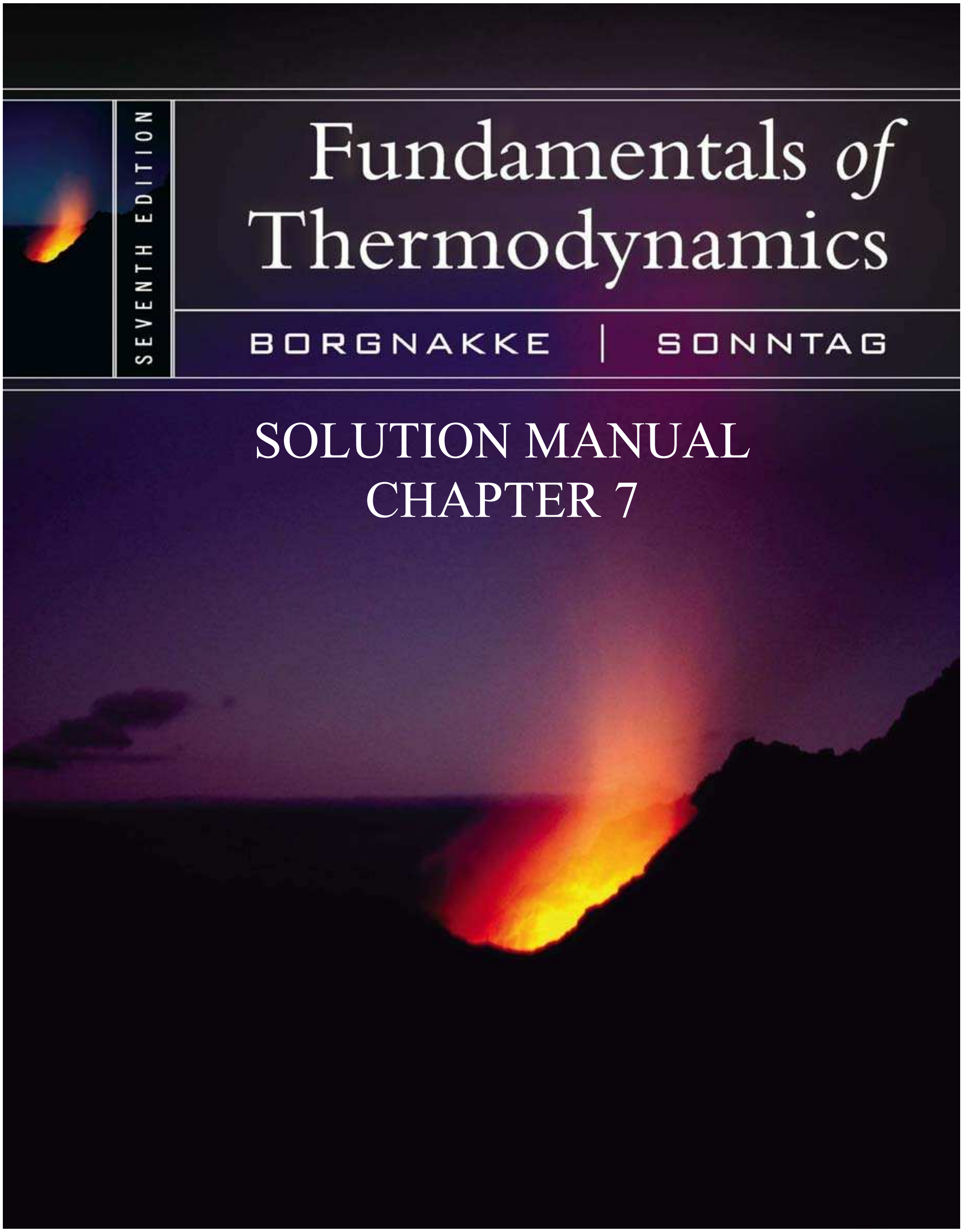
$\dot{Q} = \dot{m}_a\ (h_i - h_e) = 0.5\ (1046.22 - 607.32) = 219.45$ kW

Exit: $\Delta T_{out} = 600 - 400 = 200$ K,

so we have an average of $\Delta T = 400$ K for heat transfer

$$A = \frac{\dot{Q}}{h\Delta T} = \frac{219.45 \times 1000}{125 \times 400} = \mathbf{4.39\ m^2}$$





Fundamentals *of* Thermodynamics

SEVENTH EDITION

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 7

CONTENT CHAPTER 7

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In-Text Concept Questions

7.a

Electrical appliances (TV, stereo) use electric power as input. What happens to the power? Are those heat engines? What does the second law say about those devices?

Most electric appliances such as TV, VCR, stereo and clocks dissipate power in electrical circuits into internal energy (they get warm) some power goes into light and some power into mechanical energy. The light is absorbed by the room walls, furniture etc. and the mechanical energy is dissipated by friction so all the power eventually ends up as internal energy in the room mass of air and other substances.

These are not heat engines, just the opposite happens, namely electrical power is turned into internal energy and redistributed by heat transfer. These are irreversible processes.

7.b

Geothermal underground hot water or steam can be used to generate electric power. Does that violate the second law?

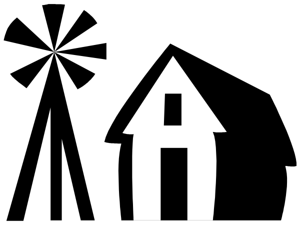
No.
Since the earth is not uniform we consider the hot water or steam supply as coming from one energy source (the high T) and we must reject heat to a low temperature reservoir as the ocean, a lake or the atmosphere which is another energy reservoir.

Iceland uses a significant amount of steam to heat buildings and to generate electricity.

7.c

A windmill produces power on a shaft taking kinetic energy out of the wind. Is it a heat engine? Is it a perpetual machine? Explain.

Since the wind is generated by a complex system driven by solar heat input and radiation out to space it is a kind of heat engine.



Within our lifetime it looks like it is perpetual. However with a different time scale the climate will change, the sun will grow to engulf the earth as it burns out of fuel. There is a storage effect and a non-uniform distribution of states in the system that drives this.

7.d

heat engines and heat pumps (refrigerators) are energy conversion devices altering amounts of energy transfer between Q and W. Which conversion direction (Q → W or W → Q) is limited and which is unlimited according to the second law.

The work output of a heat engine is limited (Q to W).
You can transform W to Q unlimited (a heat pump that does not work well or you may think about heat generated by friction).

7.e

Ice cubes in a glass of liquid water will eventually melt and all the water approach room temperature. Is this a reversible process? Why?

There is heat transfer from the warmer ambient to the water as long as there is a temperature difference. Eventually the temperatures approach each other and there is no more heat transfer. This is irreversible, as we cannot make ice-cubes out of the water unless we run a refrigerator and that requires a work from the surroundings, which does not leave the surroundings unchanged.

7.f

Does a process become more or less reversible with respect to heat transfer if it is fast rather than slow? *Hint*: Recall from Chapter 4 that $\dot{Q} = CA\ \Delta T$.

If the higher heat transfer rate is caused by a larger ΔT then the process is more irreversible so as the process would be slower due to a lower ΔT then it approaches a reversible process. If the rate of heat transfer is altered due to the factor CA with the same ΔT then it is irreversible to the same degree.

7.g

If you generated hydrogen from, say, solar power, which of these would be more efficient: (1) transport it and then burn it in an engine or (2) convert the solar power to electricity and transport that? What else would you need to know in order to give a definite answer?

Case (1):
First there is a certain efficiency when converting solar power to hydrogen. Then the transport and packaging of hydrogen has some energy expenditures associated with it. The hydrogen could be compressed to a high pressure (typically 70 MPa) which is expensive in terms of work input and then stored in a tank. One alternative would be to cool it down to become a liquid to have a much smaller volume but the temperature at which this happens is very low so the cooling and continued cooling under transport requires a significant work input also. Certain materials like metal-hydrides, boron salt slurries and nano-carbon fibers allows storage of hydrogen at more modest pressures and are all currently under investigation as other alternative storage methods. After the hydrogen is transported to an engine then the engine efficiency determines the work output.

Case (2):
If the solar power is located where there is access to electrical transmission lines then it can be used in solar panels, solar heating of water or other substance to run a heat engine cycle like a power plant to generate electricity. To make new transmission lines is costly and has an impact on the environment that must be considered.

You also need to look at the time of day/year at which the power is required and when it is available. The end use also presents some limitations like if the power should be used for a car then the energy must be stored temporarily like in a battery.

Concept Problems

7.1

Two heat engines operate between the same two energy reservoirs and both receives the same Q_H. One engine is reversible and the other is not. What can you say about the two Q_L's?

The reversible heat engine can produce more work (has a higher efficiency) than the irreversible heat engine and due to the energy conservation it then gives out a smaller Q_L compared to the irreversible heat engine.

$$W_{rev} = Q_H - Q_{L\ rev} \quad > \quad W_{irrev} = Q_H - Q_{L\ irrev}$$

$$\Rightarrow \quad Q_{L\ rev} < Q_{L\ irrev}$$

7.2

Compare two domestic heat pumps (A and B) running with the same work input. If A is better than B which one heats the house most?

The statement that A is better means it has a higher COP and since

$$Q_{H\,A} = COP_A\, W \quad > \quad Q_{H\,B} = COP_B\, W$$

it can thus provide more heat to the house. The higher heat comes from the higher Q_L it is able to draw in.

7.3

Suppose we forget the model for heat transfer as $\dot{Q} = CA\ \Delta T$, can we draw some information about direction of Q from the second law?

One of the classical statements of the second law is the Clausius statement saying that you cannot have heat transfer from a lower temperature domain to a higher temperature domain without work input.

The opposite, namely a transfer of heat from a high temperature domain towards a lower temperature domain can happen (which is a heat engine with zero efficiency).

7.4

A combination of two heat engines is shown in Fig. P7.4. Find the overall thermal efficiency as a function of the two individual efficiencies.

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1)$$

7.5

Compare two heat engines receiving the same Q, one at 1200 K and the other at 1800 K; they both reject heat at 500 K. Which one is better?

The maximum efficiency for the engines are given by the Carnot heat engine efficiency as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

Since they have the same low temperature the one with the highest T_H will have a higher efficiency and thus presumably better.

7.6

A car engine takes atmospheric air in at 20°C, no fuel, and exhausts the air at –20°C producing work in the process. What do the first and the second laws say about that?

Energy Eq.: $W = Q_H - Q_L$ = change in energy of air. **OK**

2^{nd} law: Exchange energy with only one reservoir. **NOT OK**.
This is a violation of the statement of Kelvin-Planck.

Remark: You cannot create and maintain your own energy reservoir.

7.7

A combination of two refrigerator cycles is shown in Fig. P7.7. Find the overall COP as a function of COP_1 and COP_2.

The overall COP becomes

$$COP = \beta = \frac{\dot{Q}_L}{\dot{W}_{tot}} = \frac{\dot{Q}_L}{\dot{W}_1}\frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{1}{1 + \dot{W}_2/\dot{W}_1}$$

where we used $\dot{W}_{tot} = \dot{W}_1 + \dot{W}_2$. Use definition of COP_2 and energy equation for refrigerator 1 to eliminate $\dot{Q}_M$ and we have

$$\dot{W}_2 = \dot{Q}_M / COP_2 = (\dot{W}_1 + \dot{Q}_L) / COP_2$$

so then

$$\dot{W}_2 / \dot{W}_1 = (1 + \dot{Q}_L/\dot{W}_1) / COP_2 = (1 + COP_1) / COP_2$$

Finally substitute into the first equation and rearrange a little to get

$$COP = \beta = \frac{COP_1\ COP_2}{COP_1 + COP_2 + 1}$$

7.8

After you have returned from a car trip the car engine has cooled down and is thus back to the state in which it started. What happened to all the energy released in the burning of the gasoline? What happened to all the work the engine gave out?

Solution:

All the energy from the fuel generates heat and work out of the engine. The heat is directly dissipated in the atmosphere and the work is turned into kinetic energy and internal energy by all the frictional forces (wind resistance, rolling resistance, brake action). Eventually the kinetic energy is lost by braking the car so in the end all the energy is absorbed by the environment increasing its internal energy.

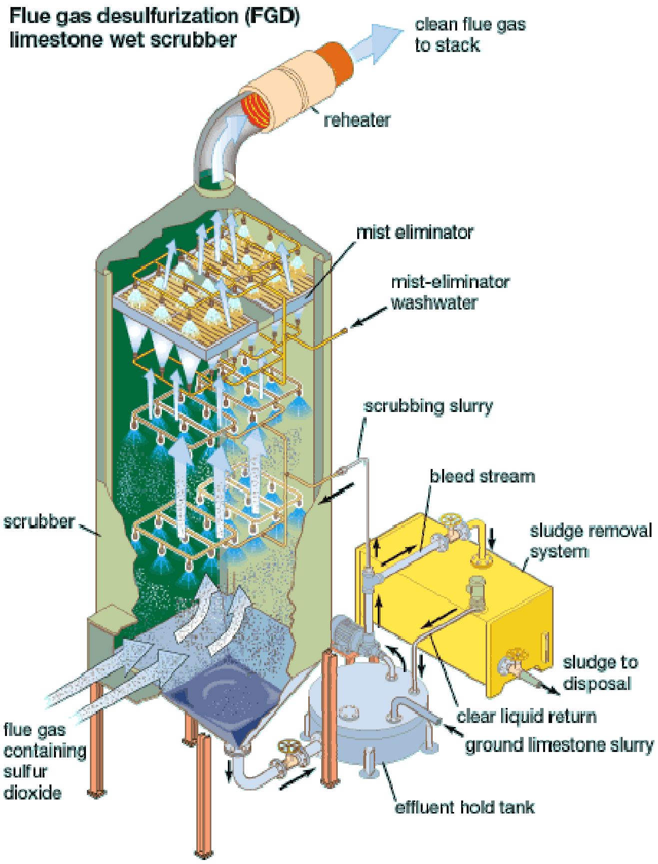
7.9

Does a reversible heat engine burning coal (which, in practice, cannot be done reversibly) have impacts on our world other than depletion of the coal reserve?

Solution:

When you burn coal you form carbon dioxide CO_2 which is a greenhouse gas. It absorbs energy over a wide spectrum of wavelengths and thus traps energy in the atmosphere that otherwise would go out into space.

Coal from various locations also has sulfur and other substances like heavy metals in it. The sulfur generates sulfuric acid (resulting in acid rain) in the atmosphere and can damage the forests.

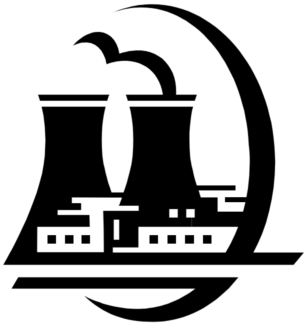


7.10

If the efficiency of a power plant goes up as the low temperature drops, why do power plants not just reject energy at say $-40^{\mathrm{o}}\mathrm{C}$?

In order to reject heat the ambient must be at the low temperature. Only if we moved the plant to the North Pole would we see such a low T.

Remark: You cannot create and maintain your own energy reservoir.



7.11

If the efficiency of a power plant goes up as the low temperature drops why not let the heat rejection go to a refrigerator at, say, –10^{o}C instead of ambient 20^{o}C?

The refrigerator must pump the heat up to 20^{o}C to reject it to the ambient. The refrigerator must then have a work input that will exactly offset the increased work output of the power plant, if they are both ideal. As we can not build ideal devices the actual refrigerator will require more work than the power plant will produce extra.

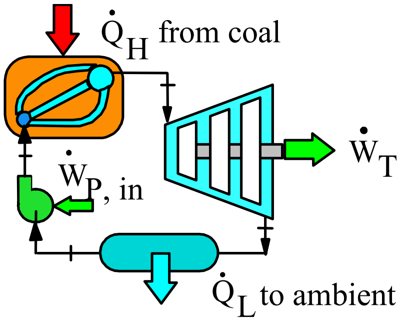
7.12

A coal-fired power plant operates with a high T of 600^{o}C whereas a jet engine has about 1400 K. Does that mean we should replace all power plants with jet engines?

The thermal efficiency is limited by the Carnot heat engine efficiency.

That is, the low temperature is also important. Here the power plant has a much lower T in the condenser than the jet engine has in the exhaust flow so the jet engine does not necessarily have a higher efficiency than the power plant.

Gas-turbines are used in power plants where they can cover peak power demands needed for shorter time periods and their high temperature exhaust can be used to boil additional water for the steam cycle.



7.13

A heat transfer requires a temperature difference, see chapter 4, to push the $\dot{Q}$. What implications do that have for a real heat engine? A refrigerator?

This means that there are temperature differences between the source of energy and the working substance so T_H is smaller than the source temperature. This lowers the maximum possible efficiency. As heat is rejected the working substance must have a higher temperature T_L than the ambient receiving the $\dot{Q}_L$, which lowers the efficiency further.

For a refrigerator the high temperature must be higher than the ambient to which the $\dot{Q}_H$ is moved. Likewise the low temperature must be lower than the cold space temperature in order to have heat transfer from the cold space to the cycle substance. So the net effect is the cycle temperature difference is larger than the reservoir temperature difference and thus the COP is lower than that estimated from the cold space and ambient temperatures.

7.14

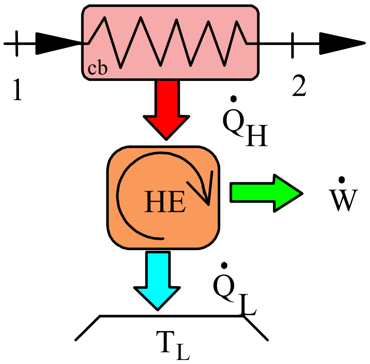
Hot combustion gases (air) at 1500 K are used as heat source in a heat engine where the gas is cooled to 750 K and the ambient is at 300 K. This is not a constant T source. How does that affect the efficiency?

Solution:

If the efficiency is written as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

then T_H is somewhere between 1500 K and 750 K and it is not a linear average.



After studying chapter 8 and 9 we can solve this problem and find the proper average high temperature based on properties at states 1 and 2.

Heat Engines and Refrigerators

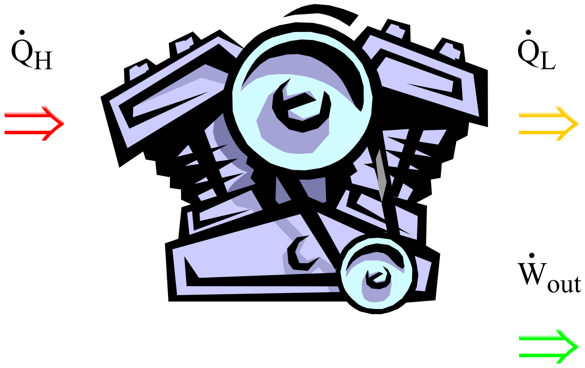
7.15

A gasoline engine produces 20 hp using 35 kW of heat transfer from burning fuel. What is its thermal efficiency and how much power is rejected to the ambient?

Conversion Table A.1: $20 \text{ hp} = 20 \times 0.7457 \text{ kW} = 14.91 \text{ kW}$

Efficiency: $$\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \frac{14.91}{35} = \mathbf{0.43}$$

Energy equation: $$\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 35 - 14.91 = \mathbf{20.1\ kW}$$



7.16

Calculate the thermal efficiency of the steam power plant cycle described in Example 6.9.

Solution:

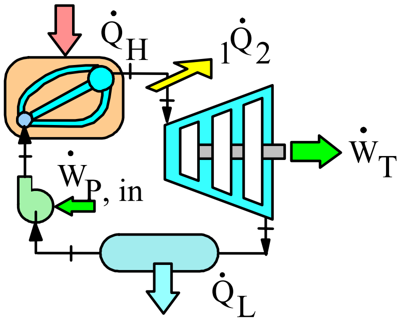
From solution to Example 6.9,

$$w_{net} = w_t + w_p = 640.7 - 4$$

$$= 636.7 \text{ kJ/kg}$$

$$q_H = q_b = 2831 \text{ kJ/kg}$$

$$\eta_{TH} = w_{net}/q_H = \frac{636.7}{2831} = \mathbf{0.225}$$



Notice we cannot write $w_{net} = q_H - q_L$ as there is an extra heat transfer ${}_1\dot{Q}_2$ as a loss in the line. This needs to be accounted for in the overall energy equation.

7.17

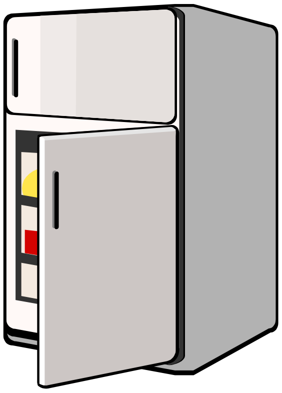
A refrigerator removes 1.5 kJ from the cold space using 1 kJ work input. How much energy goes into the kitchen and what is its coefficient of performance?

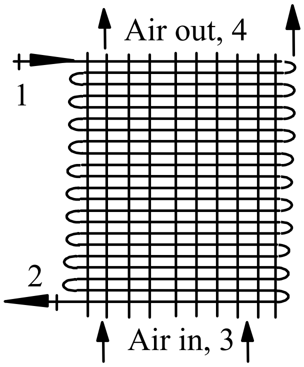
C.V. Refrigerator. The energy Q_H goes into the kitchen air.

Energy Eq.: $Q_H = W + Q_L = 1 + 1.5 = \mathbf{2.5\ kJ}$

COP: $\beta = \dfrac{Q_L}{W} = 1.5 / 1 = \mathbf{1.5}$

The back side of the refrigerator has a black grille that heats the kitchen air. Other models have that at the bottom with a fan to drive the air over it.





7.18

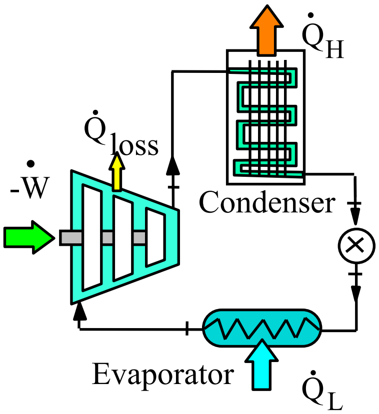
Calculate the coefficient of performance of the R-134a refrigerator given in Example 6.10.

Solution:

From the definition

$$\beta = \dot{Q}_L/\dot{W}_{IN} = \frac{14.54}{5} = 2.91$$

Notice we cannot write $\dot{W}_{IN} = \dot{Q}_H - \dot{Q}_L$ as there is a small Q in the compressor. This needs to be accounted for in the overall energy equation.



7.19

A coal fired power plant has an efficiency of 35% and produces net 500 MW of electricity. Coal releases 25 000 kJ/kg as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta\ \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

then

$$\dot{m} = \frac{\dot{W}}{\eta \times HV} = \frac{500\ \text{MW}}{0.35 \times 25000\ \text{kJ/kg}} = \frac{500 \times 1000\ \text{kJ/s}}{0.35 \times 25000\ \text{kJ/kg}}$$

$$= 57.14\ \text{kg/s} = \mathbf{205\ 714\ kg/h}$$

7.20

Assume we have a refrigerator operating at steady state using 500 W of electric power with a COP of 2.5. What is the net effect on the kitchen air?

Take a C.V. around the whole kitchen. The only energy term that crosses the control surface is the work input $\dot{W}$ apart from energy exchanged with the kitchen surroundings. That is the kitchen is being heated with a rate of $\dot{W}$.

Remark: The two heat transfer rates are both internal to the kitchen. $\dot{Q}_H$ goes into the kitchen air and $\dot{Q}_L$ actually leaks from the kitchen into the refrigerated space, which is the reason we need to drive it out again.

7.21

A room is heated with a 1500 W electric heater. How much power can be saved if a heat pump with a COP of 2.0 is used instead?

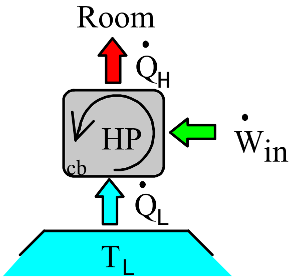
Assume the heat pump has to deliver 1500 W as the $\dot{Q}_H$.

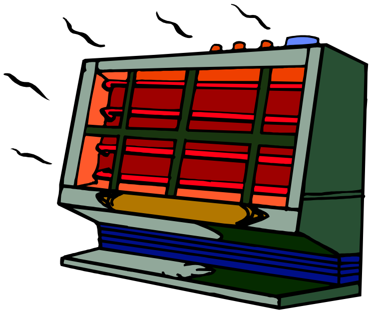
Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H/\beta' = \frac{1500}{2} = 750 \text{ W}$$

So the heat pump requires an input of 750 W thus saving the difference

$$\dot{W}_{saved} = 1500 \text{ W} - 750 \text{ W} = \mathbf{750\ W}$$





7.22

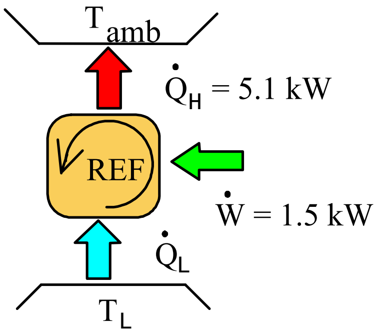
An air-conditioner discards 5.1 kW to the ambient with a power input of 1.5 kW. Find the rate of cooling and the coefficient of performance.

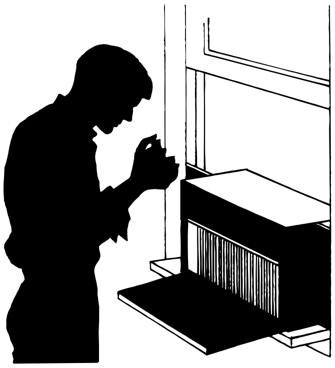
Solution:

In this case $\dot{Q}_H = 5.1$ kW goes to the ambient so

Energy Eq. : $\dot{Q}_L = \dot{Q}_H - \dot{W} = 5.1 - 1.5 = \mathbf{3.6\ kW}$

$$\beta_{\text{REFRIG}} = \frac{\dot{Q}_L}{\dot{W}} = \frac{3.6}{1.5} = \mathbf{2.4}$$





7.23

Calculate the thermal efficiency of the steam power plant cycle described in Problem 6.103.

From solution to Problem 6.103,

Turbine $A_5 = (\pi/4)(0.2)^2 = 0.03142 \text{ m}^2$

$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25 \times 0.06163 / 0.03142 = 49 \text{ m/s}$

$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2 \text{ kJ/kg}$

$w_T = 3404 - 2393.2 - (200^2 - 49^2)/(2 \times 1000) = 992 \text{ kJ/kg}$

$\dot{W}_T = \dot{m}w_T = 25 \times 992 = 24\,800 \text{ kW}$

$\dot{W}_{NET} = 24800 - 300 = 24\,500 \text{ kW}$

From the solution to Problem 6.105

Economizer $A_7 = \pi D_7^2/4 = 0.004\,418 \text{ m}^2$, $v_7 = 0.001\,008 \text{ m}^3/\text{kg}$

$\mathbf{V}_2 = \mathbf{V}_7 = \dot{m}v/A_7 = 25 \times 0.001008 / 0.004418 = 5.7 \text{ m/s}$,

$\mathbf{V}_3 = (v_3/v_2)\mathbf{V}_2 = (0.001\,118 / 0.001\,008)\ 5.7 = 6.3 \text{ m/s} \approx \mathbf{V}_2$

so kinetic energy change is unimportant

$q_{ECON} = h_3 - h_2 = 744 - 194 = 550.0 \text{ kJ/kg}$

$\dot{Q}_{ECON} = \dot{m}q_{ECON} = 25\ (550.0) = 13\,750 \text{ kW}$

Generator $A_4 = \pi D_4^2/4 = 0.031\,42 \text{ m}^2$, $v_4 = 0.060\,23 \text{ m}^3/\text{kg}$

$\mathbf{V}_4 = \dot{m}v_4/A_4 = 25 \times 0.060\,23/0.031\,42 = 47.9 \text{ m/s}$

$q_{GEN} = 3426 - 744 + (47.9^2 - 6.3^2)/(2\times1000) = 2683 \text{ kJ/kg}$

$\dot{Q}_{GEN} = 25 \times (2683) = 67\,075 \text{ kW}$

The total added heat transfer is

$\dot{Q}_H = 13\,758 + 67\,075 = 80\,833 \text{ kW}$

$$\Rightarrow \eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \frac{24500}{80833} = \mathbf{0.303}$$

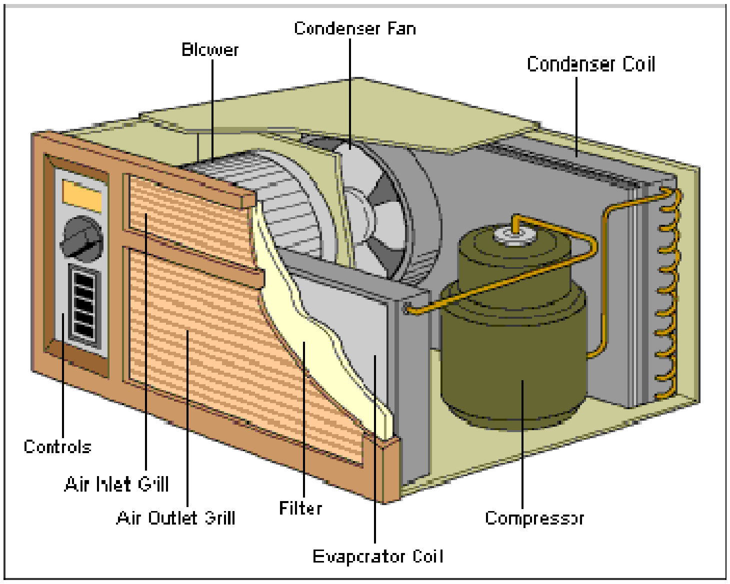
7.24

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 750 W of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

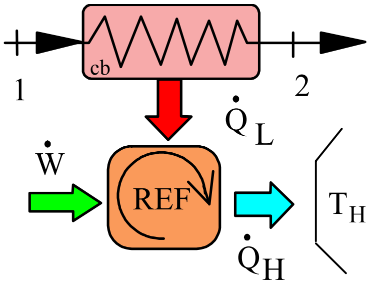
Cooling capacity: $\dot{Q}_L = \beta\, \dot{W} = 1.75 \times 750 = \mathbf{1313\ W}$

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 750$ W, that is heating it.



7.25

A water cooler for drinking water should cool 25 L/h water from 18^{o}C to 10^{o}C using a small refrigeration unit with a COP of 2.5. Find the rate of cooling required and the power input to the unit.



The mass flow rate is

$$\dot{m} = \boldsymbol{\rho}\dot{V} = \frac{25 \times 10^{-3}}{0.001002}\,\frac{1}{3600}\ \text{kg/s} = 6.93\ \text{g/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\ C_P\ (T_1 - T_2)$$
$$= 6.93 \times 10^{-3} \times 4.18 \times (18 - 10) = 0.2318\ \text{kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad ⇨ \quad \dot{W} = \dot{Q}_L / \beta = 0.2318 / 2.5 = \mathbf{0.093\ kW}$$

Comment: The unit does not operate continuously.

7.26

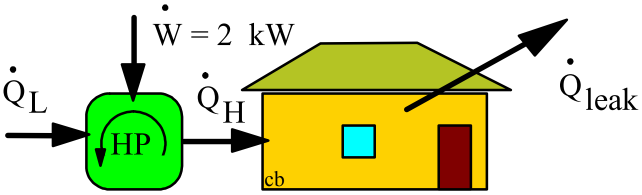
A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 30^{o}C, which loses energy at a rate of 10 kW to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Solution:

Power input: $\dot{W} = 2$ kW

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10$ kW

Definition of COP: $\beta = \text{COP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{10}{2} = \mathbf{5}$



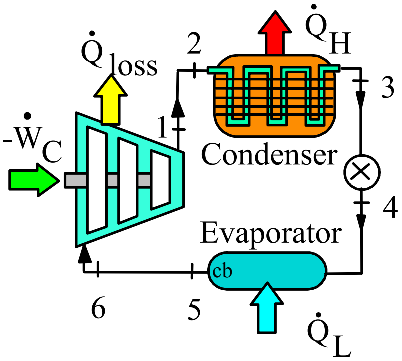
7.27

Calculate the coefficient of performance of the R-410a heat pump cycle described in Problem 6.108.

Solution:

From solution to Problem 6.108,

CV: Condenser



$$\dot{Q}_{COND} = \dot{m}(h_3 - h_2)$$
$$= 0.05 \text{ kg/s } (134 - 367) \text{ kJ/kg}$$
$$= -11.65 \text{ kW}$$

Then with the work as $-\dot{W}_{IN} = 5.0$ kW we have $\dot{Q}_H = -\dot{Q}_{COND}$

$$\text{Heat pump:} \quad \beta' = \dot{Q}_H/\dot{W}_{IN} = \frac{11.65}{5.0} = \mathbf{2.33}$$

7.28

A power plant generates 150 MW of electrical power. It uses a supply of 1000 MW from a geothermal source and rejects energy to the atmosphere. Find the power to the air and how much air should be flowed to the cooling tower (kg/s) if its temperature cannot be increased more than 10°C.

Solution:

C.V. Total power plant.

Energy equation gives the amount of heat rejection to the atmosphere as

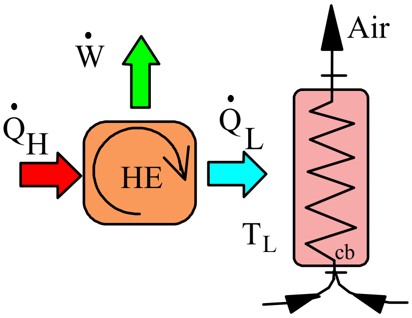
$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1000 - 150 = \mathbf{850\ MW}$$

The energy equation for the air flow that absorbs the energy is

$$\dot{Q}_L = \dot{m}_{air}\,\Delta h = \dot{m}_{air}\,C_p\,\Delta T$$

$$\dot{m}_{air} = \frac{\dot{Q}_L}{C_p\Delta T} = \frac{850 \times 1000}{1.004 \times 10} = \mathbf{84\ 661\ kg/s}$$

Probably too large to make, so some cooling by liquid water or evaporative cooling should be used.

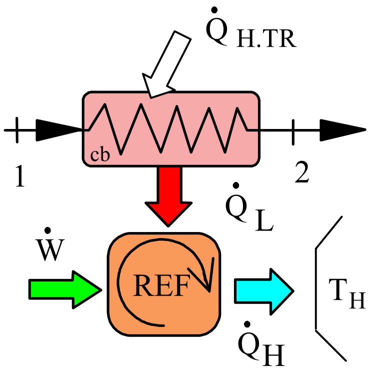


7.29

A water cooler for drinking water should cool 25 L/h water from 18°C to 10°C while the water reservoir also gains 60 W from heat transfer. Assume a small refrigeration unit with a COP of 2.5 does the cooling. Find the total rate of cooling required and the power input to the unit.

The mass flow rate is

$$\dot{m} = \boldsymbol{\rho}\dot{V} = \frac{25 \times 10^{-3}}{0.001002}\,\frac{1}{3600}\ \text{kg/s} = 6.93\ \text{g/s}$$



Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) + \dot{Q}_{H\ TR}$$

$$= \dot{m}\, C_P\, (T_1 - T_2) + \dot{Q}_{H\ TR}$$

$$= 6.93 \times 10^{-3} \times 4.18 \times (18 - 10)\ \text{kW} + 60\ \text{W}$$

$$= 291.8\ \text{W}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 291.8 / 2.5 = \mathbf{116.7\ W}$$

Comment: The unit does not operate continuously.

7.30

A car engine delivers 25 hp to the driveshaft with a thermal efficiency of 30%. The fuel has a heating value of 40 000 kJ/kg. Find the rate of fuel consumption and the combined power rejected through the radiator and exhaust.

Solution:

Heating value (HV): $\dot{Q}_H = \dot{m} \cdot HV$

From the definition of the thermal efficiency

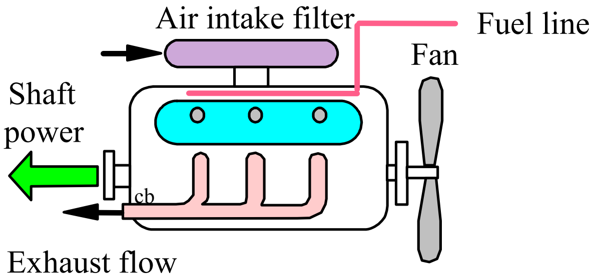
$$\dot{W} = \eta\, \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

$$\dot{m} = \frac{\dot{W}}{\eta \cdot HV} = \frac{25 \times 0.7355}{0.3 \times 40\ 000} = 0.00153 \text{ kg/s} = \mathbf{1.53\ g/s}$$

Conversion of power from hp to kW in Table A.1.

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = (\dot{W}/\eta - \dot{W}) = (\frac{1}{\eta} - 1)\dot{W}$$

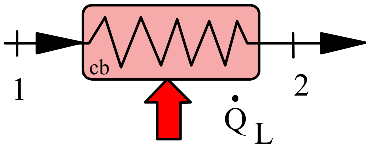
$$= (\frac{1}{0.3} - 1)\ 25 \times 0.7355 = \mathbf{42.9\ kW}$$



7.31

R-410a enters the evaporator (the cold heat exchanger) in an A/C unit at -20°C, x = 28% and leaves at -20°C, x = 1. The COP of the refrigerator is 1.5 and the mass flow rate is 0.003 kg/s. Find the net work input to the cycle.

Energy equation for heat exchanger



$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1\, h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1\, h_{fg}] = \dot{m}\,(1 - x_1)h_{fg}$$

$$= 0.003 \text{ kg/s} \times 0.72 \times 243.65 \text{ kJ/kg} = 0.5263 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.5263 / 1.5 = \mathbf{0.35\ kW}$$

7.32

For each of the cases below determine if the heat engine satisfies the first law (energy equation) and if it violates the second law.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

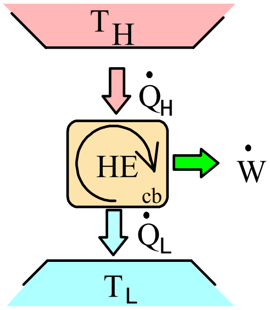
b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck
c	No	Yes, but energy not conserved
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)



7.33

For each of the cases in problem 7.32 determine if a heat pump satisfies the first law (energy equation) and if it violates the second law.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

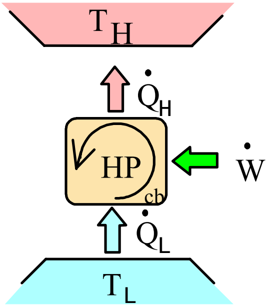
b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1st. law	2nd law
a	Satisfied	Does not violate
b	Satisfied	Does not violate
c	Violated	Does not violate, but 1st law
d	Satisfied	Does violate, Clausius



7.34

A large stationary diesel engine produces 15 MW with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 800 K and the intake air is 290 K. How large a mass flow rate is that, assuming this is the only way we reject heat? Can the exhaust flow energy be used?

Heat engine: $\dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \frac{15}{0.4} = 37.5 \text{ MW}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 37.5 - 15 = 22.5 \text{ kW}$

Exhaust flow: $\dot{Q}_L = \dot{m}_{air}(h_{800} - h_{290})$

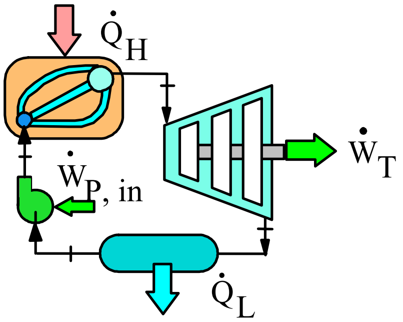
$$\dot{m}_{air} = \frac{\dot{Q}_L}{h_{800} - h_{290}} = \frac{22.5 \times 1000}{822.2 - 290.43} = \mathbf{42.3\ kg/s}$$

The flow of hot gases can be used to heat a building or it can be used to heat water in a steam power plant since that operates at lower temperatures.

7.35

In a steam power plant 1 MW is added in the boiler, 0.58 MW is taken out in the condenser and the pump work is 0.02 MW. Find the plant thermal efficiency. If everything could be reversed find the coefficient of performance as a refrigerator.

Solution:



CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{W}_T = 1 + 0.02 - 0.58 = 0.44 \text{ MW}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H} = \frac{440 - 20}{1000} = \mathbf{0.42}$$

$$\beta = \frac{\dot{Q}_L}{\dot{W}_T - \dot{W}_{P,in}} = \frac{580}{440 - 20} = \mathbf{1.38}$$

7.36

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10^{o}C. Assume the refrigerator has $\beta = 3.5$ and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} = P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = -\ 333.6$ kJ/kg

$${}_1Q_2 = 0.25(-333.4 - 41.99\ ) = -\ 93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -\ {}_1Q_2/\ \beta = 93.848/3.5 = \mathbf{26.81\ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = W/\dot{W} = (26.81 \times 1000)/750 = \mathbf{35.75\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

Second Law and Processes

7.37

Prove that a cyclic device that violates the Kelvin–Planck statement of the second law also violates the Clausius statement of the second law.

Solution: Proof very similar to the proof in section 7.2.

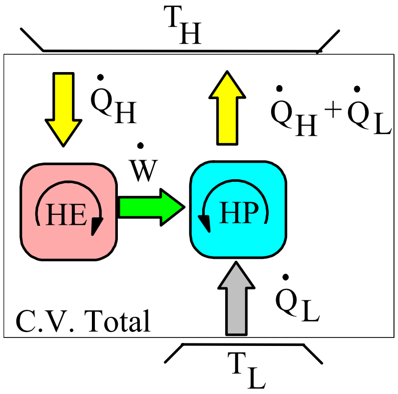
H.E. violating Kelvin receives Q_H from T_H and produces net $W = Q_H$.

This W input to H.P. receiving Q_L from T_L.

H.P. discharges $Q_H + Q_L$ to T_H .

Net Q to T_H is : $-Q_H + Q_H + Q_L = Q_L$.

H.E. + H.P. together transfers Q_L from T_L to T_H with no W thus violates Clausius.



7.38

Assume a cyclic machine that exchanges 6 kW with a 250°C reservoir and has

a. $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

b. $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

and $\dot{Q}_L$ is exchanged with a 30°C ambient. What can you say about the processes in the two cases a and b if the machine is a heat engine? Repeat the question for the case of a heat pump.

Solution:

Heat engine

a. Since $\dot{Q}_L = 0$ impossible Kelvin – Planck

b. Possible, irreversible, $\eta_{\mathbf{eng}} = 0$

Heat pump

a. Possible, irreversible (like an electric heater)

b. Impossible, $\beta \rightarrow \infty$, Clausius

7.39

Discuss the factors that would make the power plant cycle described in Problem 6.103 an irreversible cycle.

Solution:

General discussion, but here are a few of the most significant factors.

1. Combustion process that generates the hot source of energy.
2. Heat transfer over finite temperature difference in boiler.
3. Flow resistance and friction in turbine results in less work out.
4. Flow friction and heat loss to/from ambient in all pipes.
5. Heat transfer over finite temperature difference in condenser.

7.40

Discuss the factors that would make the heat pump described in Problem 6.108 an irreversible cycle.

Solution:

General discussion but here are a few of the most significant factors.

1. Unwanted heat transfer in the compressor.
2. Pressure loss (back flow leak) in compressor
3. Heat transfer and pressure drop in line 1 => 2.
4. Pressure drop in all lines.
5. Throttle process 3 => 4.

7.41

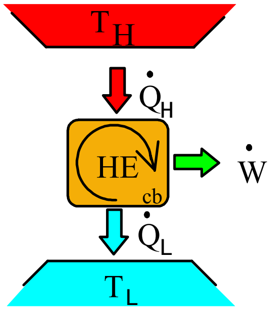
Consider the four cases of a heat engine in problem 7.32 and determine if any of those are perpetual machines of the first or second kind.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW



Solution:

	1^{st}. law	2^{nd} law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck Perpetual machine second kind It violates the 2^{nd} law converts all $\dot{Q}$ to $\dot{W}$
c	No	Yes, but energy not conserved Perpetual machine first kind It generates energy inside
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)

7.42

Consider a heat engine and heat pump connected as shown in figure P7.42. Assume $T_{H1} = T_{H2} > T_{amb}$ and determine for each of the three cases if the setup satisfy the first law and/or violates the 2nd law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	6	4	2	3	2	1
b	6	4	2	5	4	1
c	3	2	1	4	3	1

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, combine Kelvin - Planck
c	Yes	No, combination clausius

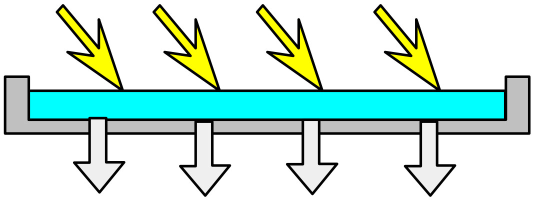
7.43

The water in a shallow pond heats up during the day and cools down during the night. Heat transfer by radiation, conduction and convection with the ambient thus cycles the water temperature. Is such a cyclic process reversible or irreversible?

Solution:

All the heat transfer takes place over a finite ΔT and thus all the heat transfer processes are irreversible.

Conduction and convection have ΔT in the water, which is internally irreversible and ΔT outside the water which is externally irreversible. The radiation is absorbed or given out at the water temperature thus internally (for absorption) and externally (for emission) irreversible.



Carnot Cycles and Absolute Temperature

7.44

Calculate the thermal efficiency of a Carnot cycle heat engine operating between reservoirs at 300°C and 45°C. Compare the result to that of Problem 7.16.

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 1 - \frac{45 + 273}{300 + 273} = \mathbf{0.445} \text{ (Carnot)}$$

$\eta_{7.16} = 0.225$ (efficiency about ½ of the Carnot)

7.45

A Carnot cycle heat engine has an efficiency of 40%. If the high temperature is raised 10% what is the new efficiency keeping the same low temperature?

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 0.4 \quad \Rightarrow \quad \frac{T_L}{T_H} = 0.6$$

so if T_H is raised 10% the new ratio becomes

$$\frac{T_L}{T_{H\ new}} = 0.6\ /1.1 = 0.5454 \quad \Rightarrow \quad \eta_{TH\ new} = 1 - 0.5454 = \mathbf{0.45}$$

7.46

Find the power output and the low T heat rejection rate for a Carnot cycle heat engine that receives 6 kW at 250°C and rejects heat at 30°C as in Problem 7.38.

Solution:

From the definition of the absolute temperature Eq. 7.8

$$\eta_{\text{carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{303}{523} = 0.42$$

Definition of the heat engine efficiency gives the work as

$$\dot{W} = \eta\ \dot{Q}_H = 0.42 \times 6 = \mathbf{2.52\ kW}$$

Apply the energy equation

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 6 - 2.52 = \mathbf{3.48\ kW}$$

7.47

Consider the setup with two stacked (temperature wise) heat engines as in Fig. P7.4. Let T_H = 900 K, T_M = 600 K and T_L = 300 K. Find the two heat engine efficiencies and the combined overall efficiency assuming Carnot cycles.

The individual efficiencies

$$\eta_1 = 1 - \frac{T_M}{T_H} = 1 - \frac{600}{900} = \mathbf{0.333}$$

$$\eta_2 = 1 - \frac{T_L}{T_M} = 1 - \frac{300}{600} = \mathbf{0.5}$$

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \, \dot{Q}_M = \eta_2 \, (1 - \eta_1) \, \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 \, (1 - \eta_1) = 0.333 + 0.5(1 - 0.333) = \mathbf{0.667}$$

Comment: It matches a single heat engine $\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{900} = \frac{2}{3}$

7.48

At a few places where the air is very cold in the winter, like –30°C it is possible to find a temperature of 13°C down below ground. What efficiency will a heat engine have operating between these two thermal reservoirs?

Solution:

$$\eta_{TH} = 1 - \frac{T_L}{T_H}$$

The ground becomes the hot source and the atmosphere becomes the cold side of the heat engine

$$\eta_{TH} = 1 - \frac{273 - 30}{273 + 13} = 1 - \frac{243}{286} = \mathbf{0.15}$$

This is low because of the modest temperature difference.

7.49

Find the maximum coefficient of performance for the refrigerator in your kitchen, assuming it runs in a Carnot cycle.

Solution:

The refrigerator coefficient of performance is

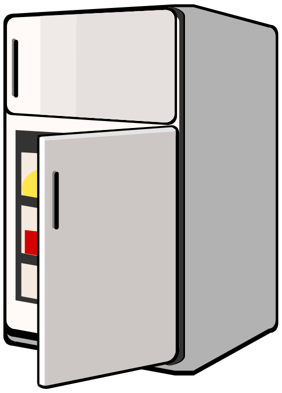
$$\beta = Q_L/W = Q_L/(Q_H - Q_L) = T_L/(T_H - T_L)$$

Assuming $T_L \sim 0°C$, $T_H \sim 35°C$,

$$\beta \le \frac{273.15}{35 - 0} = \mathbf{7.8}$$

Actual working fluid temperatures must be such that

$$T_L < T_{refrigerator} \quad \text{and} \quad T_H > T_{room}$$



A refrigerator does not operate in a Carnot cycle. The actual vapor compression cycle is examined in Chapter 11.

7.50

A refrigerator should remove 500 kJ from some food. Assume the refrigerator works in a Carnot cycle between –10^{o}C and 45^{o}C with a motor-compressor of 500 W. How much time does it take if this is the only cooling load?

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 - 10}{45 - (-10)} = 4.785$$

This gives the relation between the low T heat transfer and the work as

$$\dot{Q}_L = \frac{Q}{t} = 4.785\ \dot{W}$$

$$t = \frac{Q}{\beta\ \dot{W}} = \frac{500 \times 1000}{4.785 \times 500} = \mathbf{209\ s}$$

7.51

A car engine burns 5 kg fuel (equivalent to addition of Q_H) at 1500 K and rejects energy to the radiator and the exhaust at an average temperature of 750 K. If the fuel provides 40 000 kJ/kg what is the maximum amount of work the engine can provide?

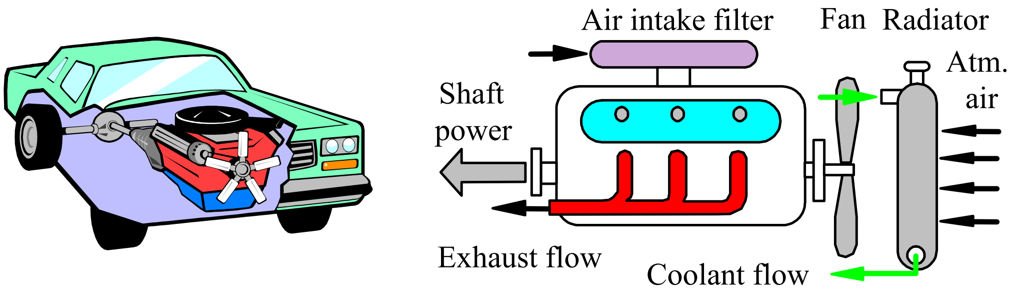
Solution:

A heat engine $Q_H = m\, q_{fuel} = 5 \times 40\,000 = 200\,000$ kJ

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{750}{1500} = 0.5$$

$$W = \eta\, Q_H = \mathbf{100\,000\ kJ}$$



7.52

A large heat pump should upgrade 5 MW of heat at 85°C to be delivered as heat at 150°C. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and $\dot{Q}_L = 5$ MW.

$$\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{273.15 + 150}{150 - 85} = 6.51$$

$$\beta_{REF} = \beta_{HP} - 1 = \frac{\dot{Q}_L}{\dot{W}_{in}} = 5.51$$

Now we can solve for the work

$$\dot{W}_{in} = \dot{Q}_L / \beta_{REF} = 5/5.51 = \mathbf{0.907\ MW}$$

7.53

An air-conditioner provides 1 kg/s of air at 15°C cooled from outside atmospheric air at 35°C. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\ \Delta h \cong \dot{m}\ C_p\Delta T = 1\text{ kg/s} \times 1.004\text{ kJ/kg K} \times 20\text{ K} = 20\text{ kW}$$

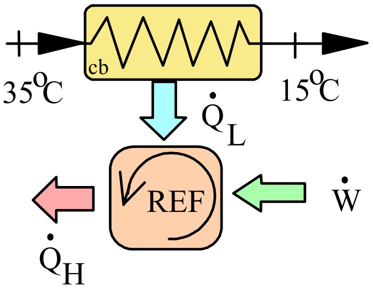
Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 + 15}{35 - 15} = 14.4$$

$$\dot{W} = \dot{Q}_L / \beta = \frac{20.0}{14.4} = \mathbf{1.39\ kW}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 5°C$ and $T_H = 45°C$ are more likely; secondly

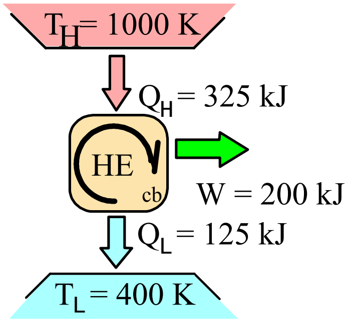
$$\beta < \beta_{carnot}$$



7.54

A cyclic machine, shown in Fig. P7.54, receives 325 kJ from a 1000 K energy reservoir. It rejects 125 kJ to a 400 K energy reservoir and the cycle produces 200 kJ of work as output. Is this cycle reversible, irreversible, or impossible?

Solution:



$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{\text{eng}} = \frac{W}{Q_H} = \frac{200}{325} = 0.615 > \eta_{\text{Carnot}}$$

This is **impossible.**

7.55

A sales person selling refrigerators and deep freezers will guarantee a minimum coefficient of performance of 4.5 year round. How would you evaluate that? Are they all the same?

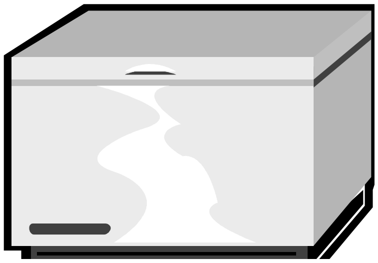
Solution:

Assume a high temperature of 35°C. If a freezer compartment is included $T_L \sim -20°C$ (deep freezer) and fluid temperature is then $T_L \sim -30°C$

$$\beta_{\text{deep freezer}} \le \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{35 - (-30)} = 3.74$$

A hot summer day may require a higher T_H to push Q_H out into the room, so even lower β.

Claim is possible for a refrigerator, but not for a deep freezer.



7.56

A temperature of about 0.01 K can be achieved by magnetic cooling. In this process a strong magnetic field is imposed on a paramagnetic salt, maintained at 1 K by transfer of energy to liquid helium boiling at low pressure. The salt is then thermally isolated from the helium, the magnetic field is removed, and the salt temperature drops. Assume that 1 mJ is removed at an average temperature of 0.1 K to the helium by a Carnot-cycle heat pump. Find the work input to the heat pump and the coefficient of performance with an ambient at 300 K.

Solution:

$$\beta = \dot{Q}_L/\dot{W}_{IN} = \frac{T_L}{T_H - T_L} = \frac{0.1}{299.9} = \mathbf{0.00033}$$

$$\dot{W}_{IN} = \frac{1\times10^{-3}}{0.00033} = \mathbf{3\ J}$$

Remark: This is an extremely large temperature difference for a heat pump.

7-57

The lowest temperature that has been achieved is about 1 × 10^{-6} K. To achieve this an additional stage of cooling is required beyond that described in the previous problem, namely nuclear cooling. This process is similar to magnetic cooling, but it involves the magnetic moment associated with the nucleus rather than that associated with certain ions in the paramagnetic salt. Suppose that 10 μJ is to be removed from a specimen at an average temperature of 10^{-5} K (ten micro-joules is about the potential energy loss of a pin dropping 3 mm). Find the work input to a Carnot heat pump and its coefficient of performance to do this assuming the ambient is at 300 K.

Solution:

$$Q_L = 10\ \mu\text{J} = 10\times10^{-6}\ \text{J} \quad \text{at} \quad T_L = 10^{-5}\ \text{K}$$

$$\Rightarrow Q_H = Q_L \times \frac{T_H}{T_L} = 10\times10^{-6} \times \frac{300}{10^{-5}} = 300\ \text{J}$$

$$W_{in} = Q_H - Q_L = 300 - 10\times10^{-6} \cong \mathbf{300\ J}$$

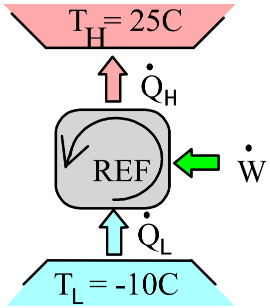
$$\beta = \frac{Q_L}{W_{in}} = \frac{10\times10^{-6}}{300} = \mathbf{3.33\times10^{-8}}$$

7.58

An inventor has developed a refrigeration unit that maintains the cold space at −10°C, while operating in a 25°C room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

$$\beta_{\text{Carnot}} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{263.15}{25 - (-10)} = 7.52$$



$8.5 > \beta_{\text{Carnot}} \Rightarrow$ **impossible claim**

7.59

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10°C. Assume the refrigerator works in a Carnot cycle between –8°C and 35°C with a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} + P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = -\ 333.6$ kJ/kg

$${}_1Q_2 = 0.25(-333.4 - 41.99\ ) = -\ 93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 8}{35 - (-8)} = 6.16$$

$$W = \frac{Q_L}{\beta} = -\frac{{}_1Q_2}{\beta} = \frac{93.848}{6.16} = 15.24 \text{ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = \frac{W}{\dot{W}} = \frac{15.24 \times 1000}{750} = \mathbf{20.3\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

7.60

A heat pump receives energy from a source at 80°C and delivers energy to a boiler that operates at 350 kPa. The boiler input is saturated liquid water and the exit is saturated vapor both at 350 kPa. The heat pump is driven by a 2.5 MW motor and has a COP that is 60% of a Carnot heat pump COP. What is the maximum mass flow rate of water the system can deliver?

$T_H = T_{sat} = 138.88^oC = 412$ K, $\quad h_{fg} = 2148.1$ kJ/kg

$$\beta_{HP\ Carnot} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{412}{138.88 - 80} = 7$$

$\beta_{HP\ ac} = 0.6 \times 7 = 4.2 = \dot{Q}_H/\dot{W}_{in}$

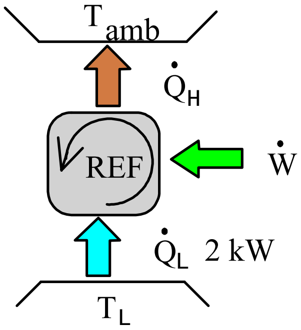
$\dot{Q}_H = 4.2\ \dot{W}_{in} = 4.2 \times 2.5\ \text{MW} = 10.5\ \text{MW} = \dot{m}\ h_{fg}$

$\dot{m} = \dot{Q}_H\ /\ h_{fg} = 10\ 500\ \text{kW}\ /\ 2148.1\ \text{kJ/kg} =$ **4.89 kg/s**

7.61

A household freezer operates in a room at 20°C. Heat must be transferred from the cold space at a rate of 2 kW to maintain its temperature at –30°C. What is the theoretically smallest (power) motor required to operate this freezer?

Solution:



Assume a Carnot cycle between T_L = -30°C and T_H = 20°C:

$$\beta = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{20 - (-30)} = 4.86$$

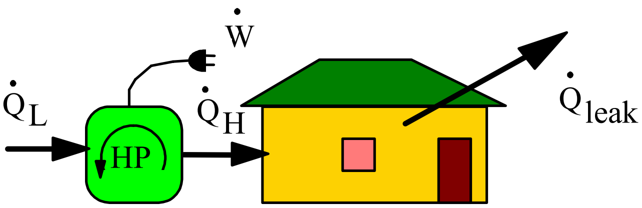
$$\dot{W}_{in} = \dot{Q}_L/\beta = 2/4.86 = \mathbf{0.41\ kW}$$

This is the theoretical minimum power input.
Any actual machine requires a larger input.

7.62

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20°C at all times. When the ambient temperature outside drops to −10°C, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

Solution:



Minimum power if we assume a Carnot cycle

$\dot{Q}_H = \dot{Q}_{leak} = 25$ kW

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.2}{30} = 9.773 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{25}{9.773} = \mathbf{2.56\ kW}$$

7.63

A certain solar-energy collector produces a maximum temperature of 100°C. The energy is used in a cyclic heat engine that operates in a 10°C environment. What is the maximum thermal efficiency? What is it, if the collector is redesigned to focus the incoming light to produce a maximum temperature of 300°C?

Solution:

For $T_H = 100°C = 373.2$ K & $T_L = 283.2$ K

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = \frac{90}{373.2} = \mathbf{0.241}$$

For $T_H = 300°C = 573.2$ K & $T_L = 283.2$ K

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = \frac{290}{573.2} = \mathbf{0.506}$$

7.64

Helium has the lowest normal boiling point of any of the elements at 4.2 K. At this temperature the enthalpy of evaporation is 83.3 kJ/kmol. A Carnot refrigeration cycle is analyzed for the production of 1 kmol of liquid helium at 4.2 K from saturated vapor at the same temperature. What is the work input to the refrigerator and the coefficient of performance for the cycle with an ambient at 300 K?

Solution:

For the Carnot cycle the ratio of the heat transfers is the ratio of temperatures

$$Q_L = n\,\bar{h}_{fg} = 1 \text{ kmol} \times 83.3 \text{ kJ/kmol} = 83.3 \text{ kJ}$$

$$Q_H = Q_L \times \frac{T_H}{T_L} = 83.3 \times \frac{300}{4.2} = 5950 \text{ kJ}$$

$$W_{IN} = Q_H - Q_L = 5950 - 83.3 = \mathbf{5886.7\ kJ}$$

$$\beta = \frac{Q_L}{W_{IN}} = \frac{83.3}{5886.7} = \mathbf{0.0142} \qquad \left[\ = \frac{T_L}{T_H - T_L}\ \right]$$

7.65

A thermal storage is made with a rock (granite) bed of 2 m^3 which is heated to 400 K using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 290 K. The rock bed therefore cools down and as it reaches 290 K the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

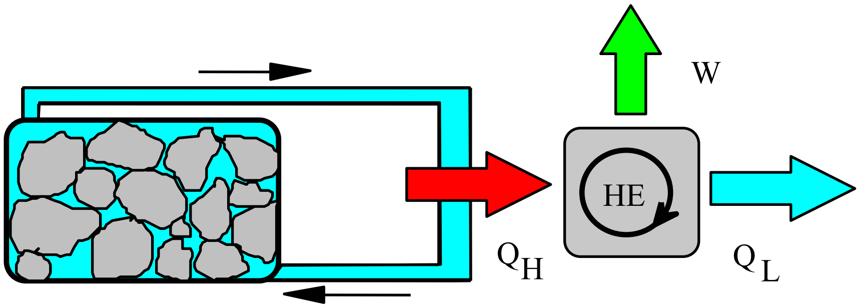
$u_2 - u_1 = q = C\ \Delta T = 0.89\ (400 - 290) = 97.9$ kJ/kg

$m = \rho V = 2750 \times 2 = 5500$ kg , $Q = mq = 5500 \times 97.9 =$ **538 450 kJ**

To get the efficiency use the CARNOT cycle result as

$\eta = 1 - T_o/T_H = 1 - 290/400 =$ **0.275** at the beginning of process

$\eta = 1 - T_o/T_H = 1 - 290/290 =$ **0.0** at the end of process



7.66

In a cryogenic experiment you need to keep a container at −125°C although it gains 100 W due to heat transfer. What is the smallest motor you would need for a heat pump absorbing heat from the container and rejecting heat to the room at 20°C?

Solution:

We do not know the actual device so find the work for a Carnot cycle

$$\beta_{REF} = \dot{Q}_L / \dot{W} = \frac{T_L}{T_H - T_L} = \frac{148.15}{20 - (-125)} = 1.022$$

=> $\dot{W} = \dot{Q}_L / \beta_{REF} = 100/1.022 = \mathbf{97.8\ W}$

7.67

It is proposed to build a 1000-MW electric power plant with steam as the working fluid. The condensers are to be cooled with river water (see Fig. P7.67). The maximum steam temperature is 550°C, and the pressure in the condensers will be 10 kPa. Estimate the temperature rise of the river downstream from the power plant.

Solution:

$$\dot{W}_{NET} = 10^6 \text{ kW}, \quad T_H = 550°\text{C} = 823.3 \text{ K}$$

$$P_{COND} = 10 \text{ kPa} \rightarrow T_L = T_G\ (P = 10 \text{ kPa}) = 45.8°\text{C} = 319 \text{ K}$$

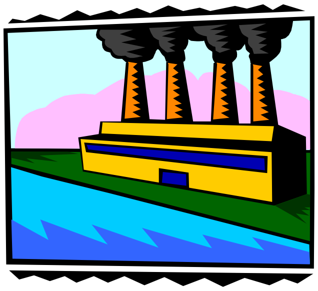
$$\eta_{TH\ CARNOT} = \frac{T_H - T_L}{T_H} = \frac{823.2 - 319}{823.2} = 0.6125$$

$$\Rightarrow \dot{Q}_{L\ MIN} = 10^6 \left(\frac{1 - 0.6125}{0.6125}\right) = 0.6327 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000$ kg/s having an energy flow of

$$\dot{Q}_{L\ MIN} = \dot{m}_{H_2O}\ \Delta h = \dot{m}_{H_2O}\ C_{P\ LIQ\ H_2O}\ \Delta T_{H_2O\ MIN}$$

$$\Rightarrow \Delta T_{H_2O\ MIN} = \frac{\dot{Q}_{L\ MIN}}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{0.6327 \times 10^6}{80000 \times 4.184} = \mathbf{1.9°C}$$



7.68

Repeat the previous problem using a more realistic thermal efficiency of 35%.

$$\dot{W}_{NET} = 10^6 \text{ kW} = \eta_{TH\ ac}\ \dot{Q}_H, \qquad \eta_{TH\ ac} = 0.35$$

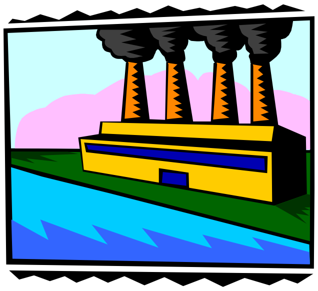
$$\Rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{NET} = \dot{W}_{NET}/\eta_{TH\ ac} - \dot{W}_{NET} = \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$= 10^6 \text{ kW} \left(\frac{1 - 0.35}{0.35}\right) = 1.857 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000$ kg/s having an energy flow of

$$\dot{Q}_L = \dot{m}_{H_2O}\ \Delta h = \dot{m}_{H_2O}\ C_{P\ LIQ\ H_2O}\ \Delta T_{H_2O}$$

$$\Rightarrow \Delta T_{H_2O} = \frac{\dot{Q}_L}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{1.857 \times 10^6}{80\ 000 \times 4.18} = \mathbf{5.6°C}$$



7.69

A steel bottle V = 0.1 m^3 contains R-134a at 20°C, 200 kPa. It is placed in a deep freezer where it is cooled to -20°C. The deep freezer sits in a room with ambient temperature of 20°C and has an inside temperature of -20°C. Find the amount of energy the freezer must remove from the R-134a and the extra amount of work input to the freezer to do the process.

Solution:

C.V. R-134a out to the -20 °C space.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : V = Const $\Rightarrow$ $v_2 = v_1$ $\Rightarrow$ ${}_1W_2 = 0$

Table B.5.2: $v_1 = 0.11436 \text{ m}^3/\text{kg}$, $u_1 = 395.27$ kJ/kg

$m = V/ v_1 = 0.87443$ kg

State 2: $v_2 = v_1 < v_g = 0.14649$ Table B.5.1 => 2 phase

$$\Rightarrow x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.11436 - 0.000738}{0.14576} = 0.77957$$

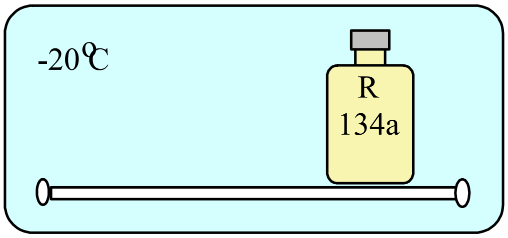
$$u_2 = 173.65 + 0.77957 \times 192.85 = 323.99 \text{ kJ/kg}$$

$${}_1Q_2 = m(u_2 - u_1) = \mathbf{-62.334\ kJ}$$

Consider the freezer and assume Carnot cycle

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 20}{20 - (-20)} = 6.33$$

$$W_{in} = Q_L / \beta = 62.334 / 6.33 = \mathbf{9.85\ kJ}$$

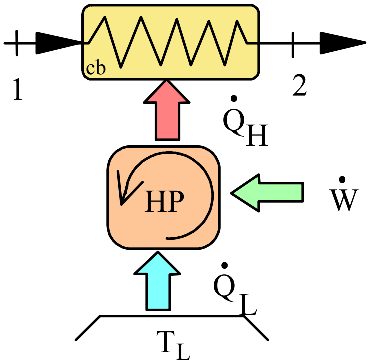


7.70

Sixty kilograms per hour of water runs through a heat exchanger, entering as saturated liquid at 200 kPa and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 16°C. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger



$\dot{m}_1 = \dot{m}_2$; $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.1.2: $h_1 = 504.7$ kJ/kg,

$h_2 = 2706.7$ kJ/kg

$T_H = T_{sat}(P) = 120.93 + 273.15$

$= 394.08$ K

$$\dot{Q}_H = \frac{60}{3600}(2706.7 - 504.7) = 36.7 \text{ kW}$$

Assume a Carnot heat pump.

$$\beta' = \dot{Q}_H/\dot{W} = T_H / (T_H - T_L) = 394.08 / 104.93 = 3.76$$

$$\dot{W} = \dot{Q}_H/\beta' = 36.7/3.76 = \mathbf{9.76\ kW}$$

7.71

A heat engine has a solar collector receiving 0.2 kW per square meter inside which a transfer media is heated to 450 K. The collected energy powers a heat engine which rejects heat at 40$^{\mathrm{o}}$C. If the heat engine should deliver 2.5 kW what is the minimum size (area) solar collector?

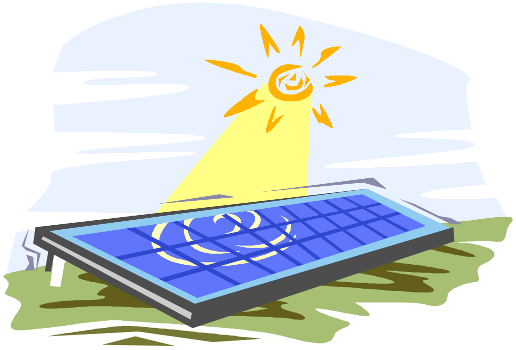
Solution:

$$T_H = 450\text{ K} \qquad T_L = 40^{\mathrm{o}}\text{C} = 313.15\text{ K}$$

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{313.15}{450} = 0.304$$

$$\dot{W} = \eta\, \dot{Q}_H \quad => \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{2.5}{0.304} = 8.224\text{ kW}$$

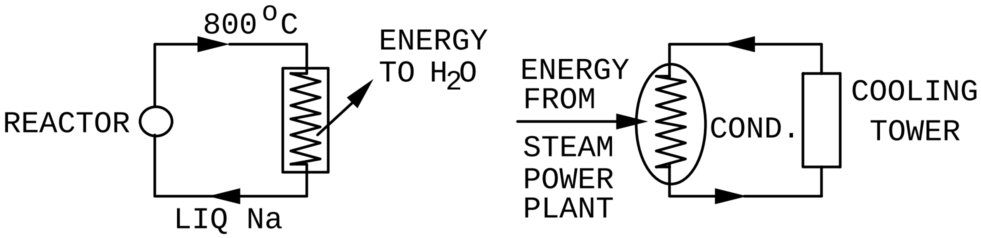
$$\dot{Q}_H = 0.2\text{ A} \quad => \quad \text{A} = \frac{\dot{Q}_H}{0.2} = \mathbf{41\ m^2}$$



7.72

Liquid sodium leaves a nuclear reactor at 800°C and is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 15°C. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:



$$T_H = 800°C = 1073.2 \text{ K}, \quad T_L = 15°C = 288.2 \text{ K}$$

$$\eta_{TH\,MAX} = \frac{T_H - T_L}{T_H} = \frac{1073.2 - 288.2}{1073.2} = \mathbf{0.731}$$

It might be misleading to use 800°C as the value for T_H, since there is not a supply of energy available at a constant temperature of 800°C (liquid Na is cooled to a lower temperature in the heat exchanger).

⇒ The Na cannot be used to boil H_2O at 800°C.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 15°C, and leaves the condenser at some higher temperature.

⇒ The water does not provide for condensing steam at a constant temperature of 15°C.

7.73

A power plant with a thermal efficiency of 40% is located on a river similar to Fig. P7.67. With a total river mass flow rate of 1 × 10^5 kg/s at 15°C find the maximum power production allowed if the river water should not be heated more than 1 degree.

The maximum heating allowed determines the maximum $\dot{Q}_L$ as

$$\dot{Q}_L = \dot{m}_{H_2O}\, \Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O}$$

$$= 1 \times 10^5 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times 1 \text{ K} = 418 \text{ MW}$$

$$= \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$\dot{W}_{NET} = \dot{Q}_L / (1/\eta_{TH\ ac} - 1) = \dot{Q}_L \frac{\eta_{TH\ ac}}{1 - \eta_{TH\ ac}}$$

$$= 418 \text{ MW} \times \frac{0.4}{1 - 0.4} = \mathbf{279\ MW}$$

7.74

A heat pump is driven by the work output of a heat engine as shown in figure P7.74. If we assume ideal devices find the ratio of the total power $\dot{Q}_{L1} + \dot{Q}_{H2}$ that heats the house to the power from the hot energy source $\dot{Q}_{H1}$ in terms of the temperatures.

$$\beta_{HP} = \dot{Q}_{H2}/\dot{W} = \dot{Q}_{H2}/(\dot{Q}_{H2} - \dot{Q}_{L2}) = \frac{T_{room}}{T_{room} - T_{amb}}$$

$$\dot{W} = \eta_{HE} \cdot \dot{Q}_{H1} = (1 - \frac{T_{room}}{T_H})\, \dot{Q}_{H1}$$

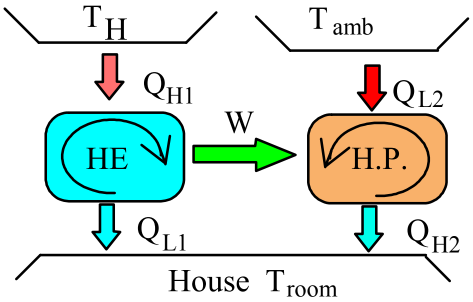
$$\dot{W} = \dot{Q}_{H2}/\beta_{HP} = \frac{T_{room}}{T_{room} - T_{amb}}\, \dot{Q}_{H2}$$

$$\dot{Q}_{L1} = \dot{Q}_{H1} - \dot{W} = [1 - 1 + \frac{T_{room}}{T_H}]\, \dot{Q}_{H1}$$

$$\frac{\dot{Q}_{H2} + \dot{Q}_{L1}}{\dot{Q}_{H1}} = 1 - 1 + \frac{T_{room}}{T_H} + \frac{1 - \frac{T_{room}}{T_H}}{\frac{T_{room} - T_{amb}}{T_{room}}} = \frac{T_{room}}{T_H} + \frac{T_{room} - T^2_{room}/T_H}{T_{room} - T_{amb}}$$

$$= T_{room}\left[\frac{1}{T_H} + \frac{1 - \frac{T_{room}}{T_H}}{T_{room} - T_{amb}}\right] = \frac{T_{room}}{T_H}\left[1 + \frac{T_H - T_{room}}{T_{room} - T_{amb}}\right]$$

$$= \boldsymbol{\frac{T_{room}}{T_H}\left[\frac{T_H - T_{amb}}{T_{room} - T_{amb}}\right]}$$



7.75

A car engine with a thermal efficiency of 33% drives the air-conditioner unit (a refrigerator) besides powering the car and other auxiliary equipment. On a hot (35^oC) summer day the A/C takes outside air in and cools it to 5^oC sending it into a duct using 2 kW of power input and it is assumed to be half as good as a Carnot refrigeration unit. Find the rate of fuel (kW) being burned extra just to drive the A/C unit and its COP. Find the flow rate of cold air the A/C unit can provide.

$$\dot{W}_{extra} = \eta\, \dot{Q}_{H\ extra} \quad \Rightarrow \quad \dot{Q}_{H\ extra} = \dot{W}_{extra} / \eta = 2\ \text{kW} / 0.33 = \mathbf{6\ kW}$$

$$\beta = \frac{Q_L}{W_{IN}} = 0.5\ \beta_{Carnot} = 0.5\ \frac{T_L}{T_H - T_L} = 0.5\ \frac{5 + 273.15}{35 - 5} = 4.636$$

$$\dot{Q}_L = \beta\ \dot{W} = 4.636 \times 2\ \text{kW} = 9.272\ \text{kW} = \dot{m}_{air}\ C_{P\ air}\ \Delta T_{air}$$

$$\dot{m}_{air} = \dot{Q}_L / [C_{P\ air}\ \Delta T_{air}\ ] = \frac{9.272\ \text{kW}}{1.004\ \text{kJ/kg-K} \times (35 - 5)\ \text{K}} = \mathbf{0.308\ kg/s}$$

7.76

Two different fuels can be used in a heat engine operating between the fuel burning temperature and a low temperature of 350 K. Fuel A burns at 2200 K delivering 30 000 kJ/kg and costs $1.50/kg. Fuel B burns at 1200 K, delivering 40 000 kJ/kg and costs $1.30/kg. Which fuel will you buy and why?

Solution:

Fuel A: $\eta_{TH,A} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{2200} = 0.84$

$W_A = \eta_{TH,A} \times Q_A = 0.84 \times 30\,000 = 25\,200$ kJ/kg

$W_A/\$_A = 25\,200/1.5 = 16\,800$ kJ/\$

Fuel B: $\eta_{TH,B} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{1200} = 0.708$

$W_B = \eta_{TH,B} \times Q_B = 0.708 \times 40\,000 = 28\,320$ kJ/kg

$W_B/\$_B = 28\,320/1.3 = 21\,785$ kJ/\$

Select fuel B for more work per dollar though it has a lower thermal efficiency.

7.77

A large heat pump should upgrade 5 MW of heat at 85°C to be delivered as heat at 150°C. Suppose the actual heat pump has a COP of 2.5 how much power is required to drive the unit. For the same COP how high a high temperature would a Carnot heat pump have assuming the same low T?

This is an actual COP for the heat pump as

$$\beta_{HP} = \text{COP} = \dot{Q}_H/\dot{W}_{in} = 2.5 \quad \Rightarrow \quad \dot{Q}_L/\dot{W}_{in} = 1.5$$

$$\dot{W}_{in} = \dot{Q}_L/\ 1.5 = 5\ /\ 1.5 = \mathbf{3.333\ MW}$$

The Carnot heat pump has a COP given by the temperatures as

$$\beta_{HP} = \dot{Q}_H/\dot{W}_{in} = \frac{T_H}{T_H - T_L} = 2.5 \quad \Rightarrow \quad T_H = 2.5\ T_H - 2.5\ T_L$$

$$\Rightarrow \qquad T_H = \frac{2.5}{1.5}\ T_L = \frac{5}{3}\,(85 + 273.15) = \mathbf{597\ K}$$

Finite ΔT Heat Transfer

7.78

The ocean near Havaii has 20^{o}C near the surface and 5^{o}C at some depth. A power plant based on this temperature difference is being planned. How large an efficiency could it have? If the two heat transfer terms (Q_H and Q_L) both require a 2 degree difference to operate what is the maximum efficiency then?

Solution:

$T_H = 20°C = 293.2\text{ K}; \qquad T_L = 5°C = 278.2\text{ K}$

$$\eta_{\text{TH MAX}} = \frac{T_H - T_L}{T_H} = \frac{293.2 - 278.2}{293.2} = \mathbf{0.051}$$

$$\eta_{\text{TH mod}} = \frac{T_{H'} - T_{L'}}{T_{H'}} = \frac{291.2 - 280.2}{291.2} = \mathbf{0.038}$$

This is a very low efficiency so it has to be done on a very large scale to be economically feasible and then it will have some environmental impact.

7.79

A refrigerator keeping 5^oC inside is located in a 30^oC room. It must have a high temperature ΔT above room temperature and a low temperature ΔT below the refrigerated space in the cycle to actually transfer the heat. For a ΔT of 0, 5 and 10^oC respectively calculate the COP assuming a Carnot cycle.

Solution:

From the definition of COP and assuming Carnot cycle

$$\beta = \frac{Q_L}{W_{IN}} = \frac{T_L}{T_H - T_L} \quad \text{when T's are absolute temperatures}$$

	ΔT	T_H ^oC	T_H K	T_L ^oC	T_L K	β
a	0	30	303	5	278	11.1
b	5	35	308	0	273	7.8
c	10	40	313	-5	268	5.96

Notice how the COP drops significantly with the increase in ΔT.

7.80

A house is heated by a heat pump driven by an electric motor using the outside as the low-temperature reservoir. The house loses energy directly proportional to the temperature difference as $\dot{Q}_{loss} = K(T_H - T_L)$. Determine the minimum electric power to drive the heat pump as a function of the two temperatures.

Solution:

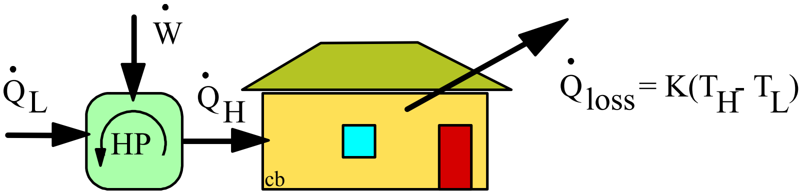
Heat pump COP: $\beta' = \dot{Q}_H/\dot{W}_{in} \le T_H/(T_H - T_L)$;

Heat loss must be added: $\dot{Q}_H = \dot{Q}_{loss} = K(T_H - T_L)$

Solve for required work and substitute in for β'

$$\dot{W}_{in} = \dot{Q}_H/\beta' \ \ge\ K(T_H - T_L) \times (T_H - T_L)/T_H$$

$$\dot{W}_{in} \ge K(T_H - T_L)^2/T_H$$



7.81

A house is heated by an electric heat pump using the outside as the low-temperature reservoir. For several different winter outdoor temperatures, estimate the percent savings in electricity if the house is kept at 20°C instead of 24°C. Assume that the house is losing energy to the outside as in Eq. 7.14.

Solution:

Heat Pump $\dot{Q}_{loss} \propto (T_H - T_L)$

$$\text{Max Perf.} \quad \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{K(T_H - T_L)}{\dot{W}_{IN}}, \quad \dot{W}_{IN} = \frac{K(T_H - T_L)^2}{T_H}$$

A: T_{H_A} = 24°C = 297.2 K B: T_{H_B} = 20°C = 293.2 K

T_L,°C	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
-20	6.514	5.457	16.2 %
-10	3.890	3.070	21.1 %
0	1.938	1.364	29.6 %
10	0.659	0.341	48.3 %

7.82

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance of $\beta = 3$ working as a refrigerator cooling the inside using engine shaft work to drive it. How much fuel energy should be spend extra to remove 1 kJ from the inside?

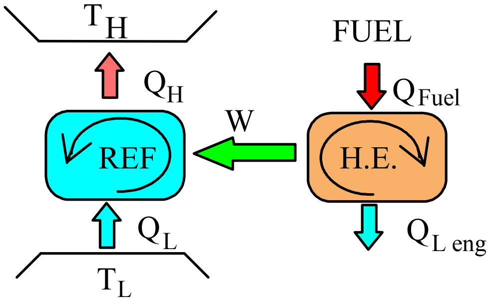
Solution:

Car engine: $W = \eta_{eng} Q_{fuel}$

Air conditioner: $\beta = \dfrac{Q_L}{W}$

$$W = \eta_{eng} Q_{fuel} = \frac{Q_L}{\beta}$$

$$Q_{fuel} = Q_L / (\eta_{eng}\, \beta) = \frac{1}{0.35 \times 3} = \mathbf{0.952\ kJ}$$



7.83

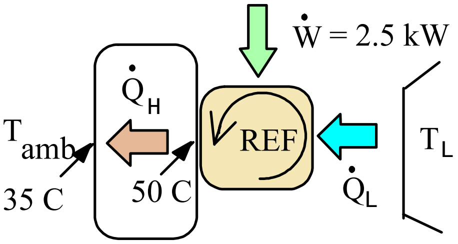
A refrigerator uses a power input of 2.5 kW to cool a 5°C space with the high temperature in the cycle as 50°C. The Q_H is pushed to the ambient air at 35°C in a heat exchanger where the transfer coefficient is 50 W/m^2K. Find the required minimum heat transfer area.

Solution:

$$\dot{W} = 2.5 \text{ kW} = \dot{Q}_H / \beta_{HP}$$

$$\dot{Q}_H = \dot{W} \times \beta_{HP} = 2.5 \times [323 / (50 - 5)] = 17.95 \text{ kW} = h\, A\, \Delta T$$

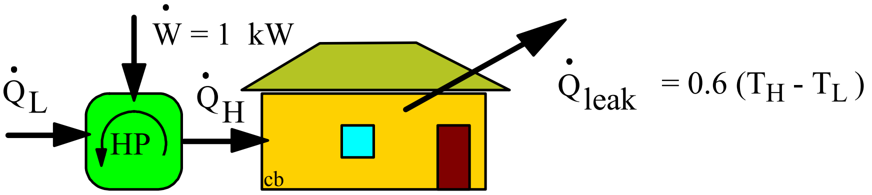
$$A = \frac{\dot{Q}_H}{h\, \Delta T} = \frac{17.95 \times 10^3}{50 \times 15} = \mathbf{23.9\ m^2}$$



7.84

A heat pump has a coefficient of performance that is 50% of the theoretical maximum. It maintains a house at 20°C, which leaks energy of 0.6 kW per degree temperature difference to the ambient. For a maximum of 1.0 kW power input find the minimum outside temperature for which the heat pump is a sufficient heat source.

Solution:



C.V. House. For constant 20°C the heat pump must provide $\dot{Q}_{leak} = 0.6\ \Delta T$

$$\dot{Q}_H = \dot{Q}_{leak} = 0.6 (T_H - T_L) = \beta' \dot{W}$$

C.V. Heat pump. Definition of the coefficient of performance and the fact that the maximum is for a Carnot heat pump.

$$\beta' = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = 0.5\ \beta'_{Carnot} = 0.5 \times \frac{T_H}{T_H - T_L}$$

Substitute into the first equation to get

$$0.6 (T_H - T_L) = [\,0.5 \times T_H / (T_H - T_L)\,]\ 1 \quad =>$$

$$(T_H - T_L)^2 = (0.5 / 0.6)\ T_H \times 1 = 0.5 / 0.6 \times 293.15 = 244.29$$

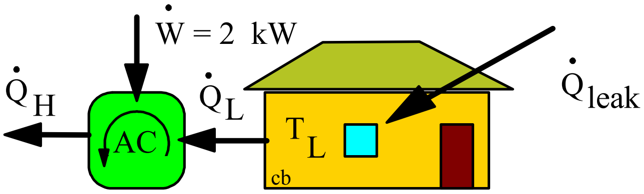
$T_H - T_L = 15.63$ => $T_L = 20 - 15.63 =$ **4.4 °C**

7.85

Consider a room at 20°C that is cooled by an air conditioner with a COP of 3.2 using a power input of 2 kW and the outside is at 35°C. What is the constant in the heat transfer Eq. 7.14 for the heat transfer from the outside into the room?

$$\dot{Q}_L = \beta_{AC}\dot{W} = 3.2 \times 2 \text{ kW} = 6.4 \text{ kW} = \dot{Q}_{\text{leak in}} = CA\ \Delta T$$

$$CA = \dot{Q}_L\ /\ \Delta T = \frac{6.4 \text{ kW}}{(35 - 20) \text{ K}} = \mathbf{0.427\ kW/K}$$



Here:

$T_L = T_{house} = 20^oC$

$T_H = T_{amb} = 35^oC$

7.86

A farmer runs a heat pump with a motor of 2 kW. It should keep a chicken hatchery at 30°C which loses energy at a rate of 0.5 kW per degree difference to the colder ambient. The heat pump has a coefficient of performance that is 50% of a Carnot heat pump. What is the minimum ambient temperature for which the heat pump is sufficient?

Solution:

C.V. Hatchery, steady state.

To have steady state at 30°C for the hatchery

Energy Eq.: $\dot{Q}_H = \dot{Q}_{Loss} = \beta_{AC}\dot{W}$

Process Eq.: $\dot{Q}_{Loss} = 0.5\,(T_H - T_{amb})$; $\beta_{AC} = ½\,\beta_{CARNOT}$

COP for the reference Carnot heat pump

$$\beta_{CARNOT} = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{T_H}{T_H - T_L} = \frac{T_H}{T_H - T_{amb}}$$

Substitute the process equations and this β_{CARNOT} into the energy Eq.

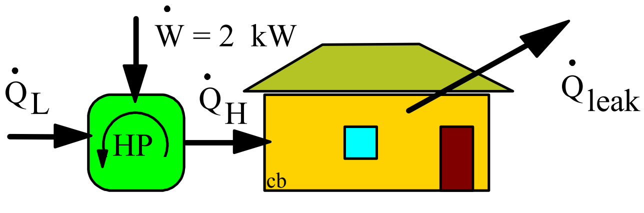
$$0.5\,(T_H - T_{amb}) = ½\,\frac{T_H}{T_H - T_{amb}}\,\dot{W}$$

$$(T_H - T_{amb})^2 = ½\,T_H\dot{W}/0.5 = T_H\dot{W} = (273 + 30) \times 2 = 606\text{ K}^2$$

$$T_H - T_{amb} = 24.62\text{ K}$$

$$T_{amb} = 30 - 24.62 = \mathbf{5.38^oC}$$

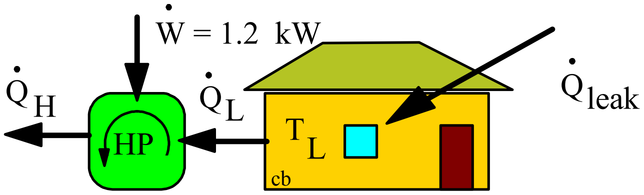
Comment: That of course is not a very low temperature and the size of the system is not adequate for most locations.



7.87

An air conditioner cools a house at $T_L = 20°C$ with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

Solution:



Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$\dot{Q}_{leak} = 0.6\ (T_{amb} - T_{house}) = \dot{Q}_L$ which must be removed by the heat pump.

$$\beta = \dot{Q}_L / \dot{W} = 0.6\ \beta_{carnot} = 0.6\ T_{house} / (T_{amb} - T_{house})$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$$0.6\ (T_{amb} - T_{house})^2 = 0.6\ T_{house}\ \dot{W}$$

Since $T_{house} = 293.15$ K and $\dot{W} = 1.2$ kW it follows

$$(T_{amb} - T_{house})^2 = T_{house}\ \dot{W} = 293.15 \times 1.2 = 351.78\ K^2$$

Solving ⇒ $(T_{amb} - T_{house}) = 18.76$ ⇒ $T_{amb} =$ **311.9 K = 38.8 °C**

7.88

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures, estimate the percent savings in electricity if the house is kept at 25°C instead of 20°C. Assume that the house is gaining energy from the outside directly proportional to the temperature difference as in Eq. 7.14.

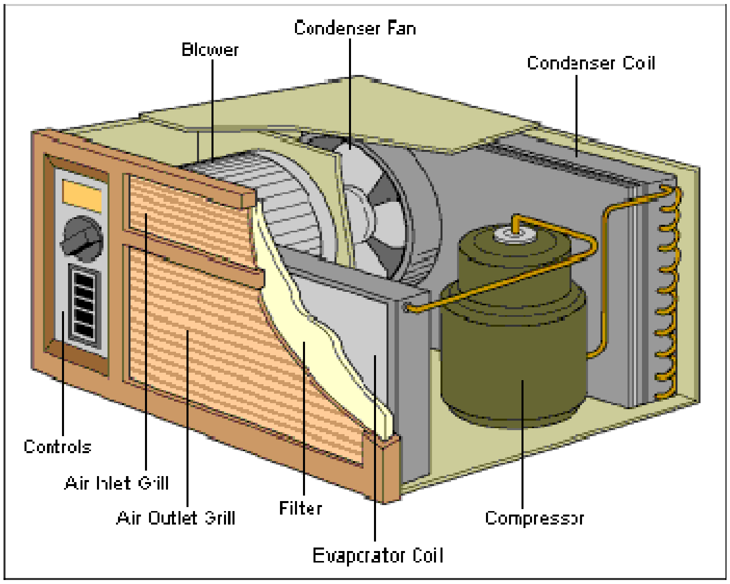
Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

Max Perf. $$\frac{\dot{Q}_L}{\dot{W}_{IN}} = \frac{T_L}{T_H - T_L} = \frac{K(T_H - T_L)}{\dot{W}_{IN}}, \quad \dot{W}_{IN} = \frac{K(T_H - T_L)^2}{T_L}$$

A: T_{L_A} = 20°C = 293.2 K B: T_{L_B} = 25°C = 298.2 K

T_H,°C	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
45	2.132	1.341	37.1 %
40	1.364	0.755	44.6 %
35	0.767	0.335	56.3 %



7.89

A Carnot heat engine, shown in Fig. P7.89, receives energy from a reservoir at T_{res} through a heat exchanger where the heat transferred is proportional to the temperature difference as $\dot{Q}_H = K(T_{res} - T_H)$. It rejects heat at a given low temperature T_L. To design the heat engine for maximum work output show that the high temperature, T_H, in the cycle should be selected as $T_H = \sqrt{T_{res}T_L}$

Solution:

$$W = \eta_{TH}Q_H = \frac{T_H - T_L}{T_H} \times K(T_{res} - T_H) \; ; \quad \text{maximize } W(T_H) \Rightarrow \frac{\delta W}{\delta T_H} = 0$$

$$\frac{\delta W}{\delta T_H} = K(T_{res} - T_H)T_L T_H^{-2} - K(1 - T_L/T_H) = 0$$

$$\Rightarrow \mathbf{T_H = \sqrt{T_{res}T_L}}$$

7.90

Consider a Carnot cycle heat engine operating in outer space. Heat can be rejected from this engine only by thermal radiation, which is proportional to the radiator area and the fourth power of absolute temperature, $\dot{Q}_{rad} \sim KAT^4$. Show that for a given engine work output and given T_H, the radiator area will be minimum when the ratio $T_L/T_H = 3/4$.

Solution:

$$\dot{W}_{NET} = \dot{Q}_H\left(\frac{T_H - T_L}{T_H}\right) = \dot{Q}_L\left(\frac{T_H - T_L}{T_L}\right); \qquad \text{also} \quad \dot{Q}_L = KAT_L^4$$

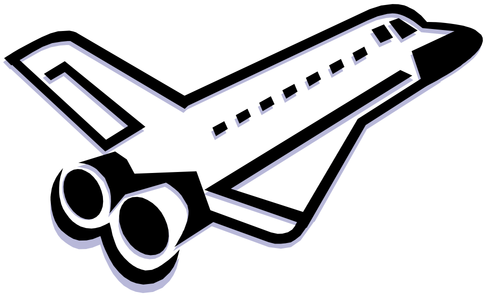
$$\frac{\dot{W}_{NET}}{KT_H^4} = \frac{AT_L^4}{T_H^4}\left(\frac{T_H}{T_L} - 1\right) = A\left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] = \text{const}$$

Differentiating,

$$dA\left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] + A\left[3\left(\frac{T_L}{T_H}\right)^2 - 4\left(\frac{T_L}{T_H}\right)^3\right] d\left(\frac{T_L}{T_H}\right) = 0$$

$$\frac{dA}{d(T_L/T_H)} = -A\left[3\left(\frac{T_L}{T_H}\right)^2 - 4\left(\frac{T_L}{T_H}\right)^3\right] \Big/ \left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] = 0$$

$$\frac{T_L}{T_H} = \frac{3}{4} \qquad \text{for min. A}$$



Check that it is minimum and not maximum with the 2nd derivative > 0.

7.91

On a cold (–10°C) winter day a heat pump provides 20 kW to heat a house maintained at 20°C and it has a $\mathrm{COP_{HP}}$ of 4. How much power does the heat pump require? The next day a winter storm brings the outside to -15°C, assuming the same COP and the same house heat transfer coefficient for the heat loss to the outside air. How much power does the heat pump require then?

If we look at the heat loss for the house we have

$$\dot{Q}_{\text{loss}} = 20\text{ kW} = \text{CA}\,\Delta T \qquad \Rightarrow \qquad \text{CA} = \frac{20\text{ kW}}{20\text{ - (-10) K}} = 0.667\text{ kW/K}$$

So now with the new outdoor temperature we get

$$\dot{Q}_{\text{loss}} = \text{CA}\,\Delta T = 0.667\text{ kW/K} \times [20 - (-15)]\text{ K} = 23.3\text{ kW}$$

$$\dot{Q}_{\text{loss}} = \dot{Q}_H = \text{COP}\;\dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{\text{loss}}/\text{COP} = \frac{23.3\text{ kW}}{4} = \mathbf{5.83\ kW}$$

Ideal Gas Carnot Cycles

7.92

Hydrogen gas is used in a Carnot cycle having an efficiency of 60% with a low temperature of 300 K. During the heat rejection the pressure changes from 90 kPa to 120 kPa. Find the high and low temperature heat transfer and the net cycle work per unit mass of hydrogen.

Solution:

As the efficiency is known, the high temperature is found as

$$\eta = 0.6 = 1 - \frac{T_L}{T_H} \quad => T_H = T_L /(1 - 0.6) = 750 \text{ K}$$

Now the volume ratio needed for the heat transfer, $T_3 = T_4 = T_L$, is

$$v_3 / v_4 = (RT_3 / P_3) / (RT_4 / P_4) = P_4 / P_3 = 120 / 90 = 1.333$$

so from Eq.7.9 we have with R = 4.1243 from Table A.5

$q_L = RT_L \ln (v_3/v_4) =$ **355.95 kJ/kg**

Using the efficiency from Eq.7.4 then

$q_H = q_L / (1 - 0.6) =$ **889.9 kJ/kg**

The net work equals the net heat transfer

$w = q_H - q_L =$ **533.9 kJ/kg**

7.93

Carbon dioxide is used in an ideal gas refrigeration cycle, reverse of Fig. 7.24. Heat absorption is at 250 K and heat rejection is at 325 K where the pressure changes from 1200 kPa to 2400 kPa. Find the refrigeration COP and the specific heat transfer at the low temperature.

The analysis is the same as for the heat engine except the signs are opposite so the heat transfers move in the opposite direction.

$$\beta = \dot{Q}_L / \dot{W} = \beta_{carnot} = T_L / (T_H - T_L) = \frac{250}{325 - 250} = \mathbf{3.33}$$

$$q_H = RT_H \ln(v_2/v_1) = RT_H \ln\left(\frac{P_1}{P_2}\right) = 0.1889 \times 325 \ln\left(\frac{2400}{1200}\right) = 42.55$$

kJ/kg

$$q_L = q_H\, T_L / T_H = 42.55 \times 250 / 325 = \mathbf{32.73\ kJ/kg}$$

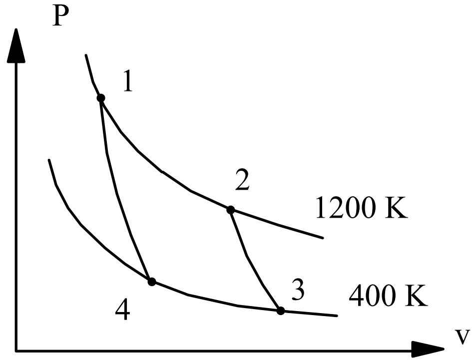
7.94

An ideal gas Carnot cycle with air in a piston cylinder has a high temperature of 1200 K and a heat rejection at 400 K. During the heat addition the volume triples. Find the two specific heat transfers (q) in the cycle and the overall cycle efficiency.

Solution:

The P-v diagram of the cycle is shown to the right.

From the integration along the process curves done in the main text we have Eq.7.7



$$q_H = R\ T_H \ln(v_2/v_1)$$

$$= 0.287 \times 1200 \ln(3)$$

$$= \mathbf{378.4\ kJ/kg}$$

Since it is a Carnot cycle the knowledge of the temperatures gives the cycle efficiency as

$$\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1200} = \mathbf{0.667}$$

from which we can get the other heat transfer from Eq.7.4

$$q_L = q_H\ T_L\ /\ T_H = 378.4\ \ 400\ /\ 1200 = \mathbf{126.1\ kJ/kg}$$

7.95

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 7.24. The high and low temperatures are 600 K and 300 K respectively. The heat added at the high temperature is 250 kJ/kg and the lowest pressure in the cycle is 75 kPa. Find the specific volume and pressure after heat rejection and the net work per unit mass.

Solution:

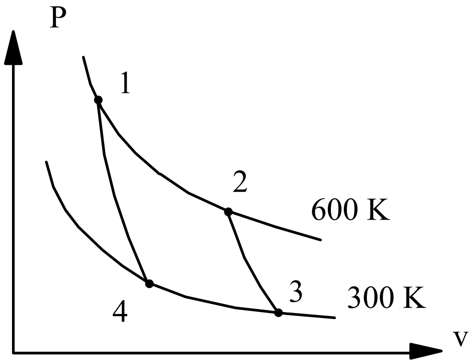
$q_H = 250$ kJ/kg , $T_H = 600$ K, $T_L = 300$ K, $P_3 = 75$ kPa

The states as shown in figure 7.21

1: 600 K , 2: 600 K, 3: 75 kPa, 300 K 4: 300 K

Since this is a Carnot cycle and we know the temperatures the efficiency is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5$$



and the net work becomes

$w_{NET} = \eta q_H = 0.5 \times 250$

$= \mathbf{125\ kJ/kg}$

The heat rejected is

$q_L = q_H - w_{NET} = 125$ kJ/kg

After heat rejection is state 4. From equation 7.9

$3 \rightarrow 4$ Eq.7.9 : $q_L = RT_L \ln (v_3/v_4)$

$v_3 = RT_3 / P_3 = 0.287 \times 300 / 75 = 1.148$ m^3/kg

$v_4 = v_3 \exp(-q_L/RT_L) = 1.148 \exp(-125/0.287 \times 300) = \mathbf{0.2688\ m^3/kg}$

$P_4 = RT_4 / v_4 = 0.287 \times 300 / 0.2688 = \mathbf{320\ kPa}$

Review Problems

7.96

At certain locations geothermal energy in undergound water is available and used as the energy source for a power plant. Consider a supply of saturated liquid water at 150°C. What is the maximum possible thermal efficiency of a cyclic heat engine using this source of energy with the ambient at 20°C? Would it be better to locate a source of saturated vapor at 150°C than use the saturated liquid at 150°C?

Solution:

$$T_{MAX} = 150°C = 423.2\text{ K} = T_H\ ; \quad T_{Min} = 20°C = 293.2\text{ K} = T_L$$

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{130}{423.2} = \mathbf{0.307}$$

Yes. Saturated vapor source at 150°C would remain at 150°C as it condenses to liquid, providing a large energy supply at that temperature.

7.97

A rigid insulated container has two rooms separated by a membrane. Room A contains 1 kg air at 200°C and room B has 1.5 kg air at 20°C, both rooms at 100 kPa. Consider two different cases

1) Heat transfer between A and B creates a final uniform T.
2) The membrane breaks and the air comes to a uniform state.

For both cases find the final temperature. Are the two processes reversible and different? Explain.

Solution:

C.V. Total A+B

1) Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

$$U_2 - U_1 = 0 = m_A (U_2 - U_1)_A + m_B (U_2 - U_1)_B$$

$$\cong m_A \mathbf{C_v} (T_2 - T_{A1}) + m_B \mathbf{C_v} (T_2 - T_{B1})$$

$$\Rightarrow T_2 = \frac{m_A}{m_A + m_B} T_{A1} + \frac{m_B}{m_A + m_B} T_{B1} = \frac{1}{2.5} \times 200 + \frac{1.5}{2.5} \times 20$$

$$= \mathbf{92^oC}$$

$P_{A2} = P_{A1} \times T_2/ T_{A1} = 100 \times (273 + 92) /473 = 77.2$ kPa

$P_{B2} = P_{B1} \times T_2/ T_{B1} = 100 \times (273 + 92) /293 = 124.6$ kPa

2) Same energy eq. Since ideal gas u(T) same $\mathbf{T_2 = 92^oC}$, but now also same P_2

$P_2 = mRT_2 / V_1; \qquad V_1 = V_A + V_B$

$V_1 = m_{A1}RT_{A1}/ P_1 + m_{B1}RT_{B1}/ P_1$

$P_2 = (m_2RT_2 / (m_{A1}RT_{A1}/ P_1 + m_{B1}RT_{B1}/ P_1))$

$$= P_1 (m_2T_2 / (m_{A1}T_{A1} + m_{B1}T_{B1})) = 100 \frac{2.5 (273 + 92)}{1 \times 473 + 1.5 \times 293}$$

$= 100$ kPa

Both cases irreversible 1) Q over a finite ΔT and in 2) mixing of 2 different states (internal u redistribution)

(Case 2) is more irreversible as the final state in 1 could drive a turbine between the two different pressures until equal.

7.98

Consider the combination of the two heat engines as in Fig. P7.4. How should the intermediate temperature be selected so the two heat engines have the same efficiency assuming Carnot cycle heat engines.

Heat engine 1: $\eta_{TH\,1} = 1 - \frac{T_M}{T_H}$

Heat engine 2: $\eta_{TH\,2} = 1 - \frac{T_L}{T_M}$

$$\eta_{TH\,1} = \eta_{TH\,2} \;\Rightarrow\; 1 - \frac{T_M}{T_H} = 1 - \frac{T_L}{T_M} \;\Rightarrow\; \frac{T_M}{T_H} = \frac{T_L}{T_M}$$

$$\Rightarrow\; T_M = \sqrt{T_L T_H}$$

7.99

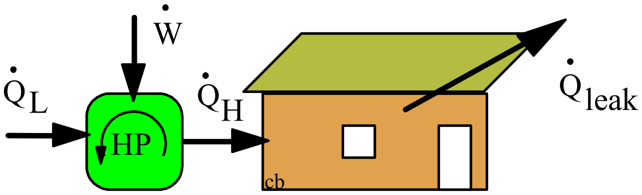
A house should be heated by a heat pump, $\beta' = 2.2$, and maintained at 20°C at all times. It is estimated that it looses 0.8 kW per degree the ambient is lower than the inside. Assume an outside temperature of –10°C and find the needed power to drive the heat pump?

Solution : Ambient $T_L = -10^oC$

Heat pump : $\beta' = \dot{Q}_H/\dot{W}$

House : $\dot{Q}_H = \dot{Q}_{leak} = 0.8 \; (T_H - T_L)$

$$\dot{W} = \dot{Q}_H/\beta' = \dot{Q}_{leak} / \beta' = 0.8 \; (T_H - T_L) / \beta'$$
$$= 0.8[20 - (-10)] /2.2 = \mathbf{10.91 \; kW}$$



7.100

Consider a combination of a gas turbine power plant and a steam power plant as shown in Fig. P7.4. The gas turbine operates at higher temperatures (thus called a topping cycle) than the steam power plant (then called a bottom cycle). Assume both cycles have a thermal efficiency of 32%. What is the efficiency of the overall combination assuming Q_L in the gas turbine equals Q_H to the steam power plant?

Let the gas turbine be heat engine number 1 and the steam power plant the heat engine number 2. Then the overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

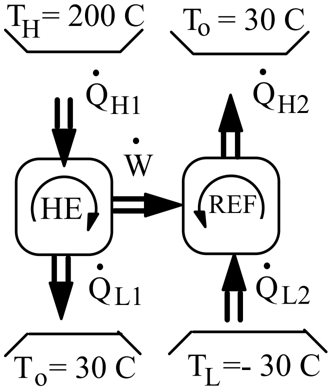
$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1) = 0.32 + 0.32(1 - 0.32) = \mathbf{0.538}$$

7.101

We wish to produce refrigeration at −30°C. A reservoir, shown in Fig. P7.101, is available at 200°C and the ambient temperature is 30°C. Thus, work can be done by a cyclic heat engine operating between the 200°C reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200°C reservoir to the heat transferred from the −30°C reservoir, assuming all processes are reversible.

Solution:

Equate the work from the heat engine to the refrigerator.



$$W = Q_{H1}\left(\frac{T_H - T_0}{T_H}\right)$$

also

$$W = Q_{L2}\left(\frac{T_0 - T_L}{T_L}\right)$$

$$\frac{Q_{H1}}{Q_{L2}} = \left(\frac{T_o - T_L}{T_L}\right)\left(\frac{T_H}{T_H - T_o}\right) = \left(\frac{60}{243.2}\right)\left(\frac{473.2}{170}\right) = \mathbf{0.687}$$

7.102

A 4L jug of milk at 25°C is placed in your refrigerator where it is cooled down to 5°C. The high temperature in the Carnot refrigeration cycle is 45°C and the properties of milk are the same as for liquid water. Find the amount of energy that must be removed from the milk and the additional work needed to drive the refrigerator.

Solution:

C.V milk + out to the 5 °C refrigerator space

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : P = constant = 1 atm => ${}_1W_2 = Pm\,(v_2 - v_1)$

State 1: Table B.1.1, $v_1 \cong v_f = 0.001003\ \mathrm{m^3/kg}$, $h_1 \cong h_f = 104.87$ kJ/kg

$m_2 = m_1 = V_1/v_1 = 0.004 / 0.001003 = 3.988$ kg

State 2: Table B.1.1, $h_2 \cong h_f = 20.98$ kJ/kg

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm\,(v_2 - v_1) = m(h_2 - h_1)$

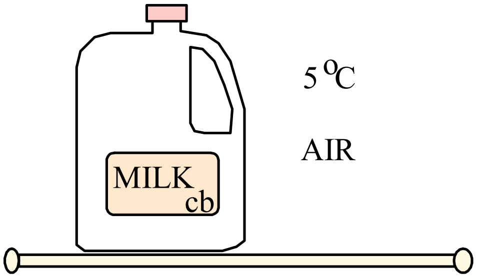
${}_1Q_2 = 3.998\ (20.98 - 104.87) = -3.988 \times 83.89 = \mathbf{-\ 334.55\ kJ}$

C.V. Refrigeration cycle $T_L = 5$ °C ; $T_H = 45$ °C, assume Carnot

Ideal : $\beta = Q_L / W = Q_L / (Q_H - Q_L) = T_L / (T_H - T_L)$

$= 278.15 / 40 = \mathbf{6.954}$

$W = Q_L / \beta = 334.55 / 6.954 = \mathbf{48.1\ kJ}$



Remark: If you calculate the work term ${}_1W_2$ you will find that it is very small, the volume does not change (liquid). The heat transfer could then have been done as $m(u_2 - u_1)$ without any change in the numbers.

7.103

An air-conditioner with a power input of 1.2 kW is working as a refrigerator ($\beta = 3$) or as a heat pump ($\beta' = 4$). It maintains an office at 20°C year round which exchanges 0.5 kW per degree temperature difference with the atmosphere. Find the maximum and minimum outside temperature for which this unit is sufficient.

Solution:

Analyze the unit in heat pump mode

Replacement heat transfer equals the loss: $\dot{Q} = 0.5\ (T_H - T_{amb})$

$$\dot{W} = \frac{\dot{Q}_H}{\beta'} = 0.5\ \frac{T_H - T_{amb}}{4}$$

$$T_H - T_{amb} = 4\ \frac{\dot{W}}{0.5} = 9.6 \text{ K}$$

Heat pump mode: Minumum $T_{amb} = 20 - 9.6 =$ **10.4 °C**

The unit as a refrigerator must cool with rate: $\dot{Q} = 0.5\ (T_{amb} - T_{house})$

$$\dot{W} = \frac{\dot{Q}_L}{\beta} = 0.5\ (T_{amb} - T_{house}) / 3$$

$$T_{amb} - T_{house} = 3\ \frac{\dot{W}}{0.5} = 7.2 \text{ K}$$

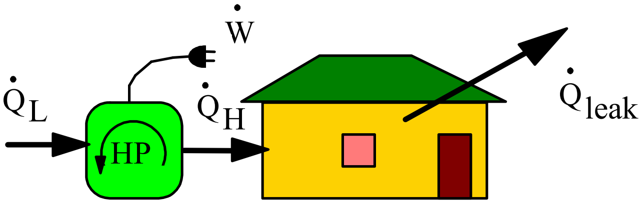
Refrigerator mode: Maximum $T_{amb} = 20 + 7.2 =$ **27.2 °C**

7.104

Make some assumption about the heat transfer rates to solve problem 7.62 when the outdoor temperature is -20^{o}C. Hint: look at the heat transfer given by Eq.7.14.

Solution:

Minimum power if we assume a Carnot cycle



We assume the heat transfer coefficient stays the same

$$\dot{Q}_H = \dot{Q}_{leak} = 25 \text{ kW} = CA\ \Delta T = CA\ [20 - (-10)] \quad \Rightarrow \quad CA = \frac{5}{6} \text{ kW/K}$$

$$\dot{Q}_{leak\ new} = CA\ \Delta T = \frac{5}{6}\ [20 - (-20)] = 33.33 \text{ kW}$$

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.15}{40} = 7.32875 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{33.333}{7.32875} = \mathbf{4.55\ kW}$$

Comment. Leak heat transfer increases and COP is lower when T outside drops.

7.105

Air in a rigid 1 m^3 box is at 300 K, 200 kPa. It is heated to 600 K by heat transfer from a reversible heat pump that receives energy from the ambient at 300 K besides the work input. Use constant specific heat at 300 K. Since the coefficient of performance changes write $dQ = m_{air} C_v dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

COP: $$\beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1V_1 / RT_1 = 200 \times 1 / 0.287 \times 300 = 2.322 \text{ kg}$$

$$dQ_H = m_{air}\, C_v\, dT_H = \beta'\, dW \cong \frac{T_H}{T_H - T_L}\, dW$$

$$=> \qquad dW = m_{air}\, C_v \left[\frac{T_H}{T_H - T_L}\right] dT_H$$

$${}_1W_2 = \int m_{air}\, C_v \left(1 - \frac{T_L}{T}\right) dT = m_{air}\, C_v \int \left(1 - \frac{T_L}{T}\right) dT$$

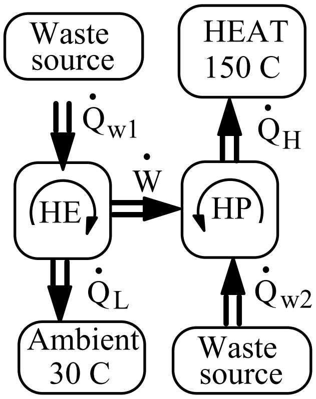
$$= m_{air}\, C_v \left[T_2 - T_1 - T_L \ln \frac{T_2}{T_1}\right]$$

$$= 2.322 \times 0.717 \left[600 - 300 - 300 \ln \frac{600}{300}\right] = \mathbf{153.1\ kJ}$$

7.106

A combination of a heat engine driving a heat pump (see Fig. P7.106) takes waste energy at 50°C as a source Q_{w1} to the heat engine rejecting heat at 30°C. The remainder Q_{w2} goes into the heat pump that delivers a Q_H at 150°C. If the total waste energy is 5 MW find the rate of energy delivered at the high temperature.

Solution:



Waste supply: $\dot{Q}_{w1} + \dot{Q}_{w2} = 5$ MW

Heat Engine:

$$\dot{W} = \eta\, \dot{Q}_{w1} = (1 - T_{L1} / T_{H1})\, \dot{Q}_{w1}$$

Heat pump:

$$\dot{W} = \dot{Q}_H / \beta_{HP} = \dot{Q}_{W2} / \beta'$$

$$= \dot{Q}_{w2} / [T_{H1} / (T_H - T_{H1})]$$

Equate the two work terms:

$$(1 - T_{L1} / T_{H1})\, \dot{Q}_{w1} = \dot{Q}_{w2} \times (T_H - T_{H1}) / T_{H1}$$

Substitute $\dot{Q}_{w1} = 5 \text{ MW} - \dot{Q}_{w2}$

$$(1 - 303.15/323.15)(5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times (150 - 50) / 323.15$$

$$20 (5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times 100 \quad \Rightarrow \quad \dot{Q}_{w2} = 0.8333 \text{ MW}$$

$$\dot{Q}_{w1} = 5 - 0.8333 = 4.1667 \text{ MW}$$

$$\dot{W} = \eta\, \dot{Q}_{w1} = 0.06189 \times 4.1667 = 0.258 \text{ MW}$$

$$\dot{Q}_H = \dot{Q}_{w2} + \dot{W} = \mathbf{1.09\ MW}$$

(For the heat pump $\beta' = 423.15 / 100 = 4.23$)

7.107

A heat pump heats a house in the winter and then reverses to cool it in the summer. The interior temperature should be 20°C in the winter and 25°C in the summer. Heat transfer through the walls and ceilings is estimated to be 2400 kJ per hour per degree temperature difference between the inside and outside.

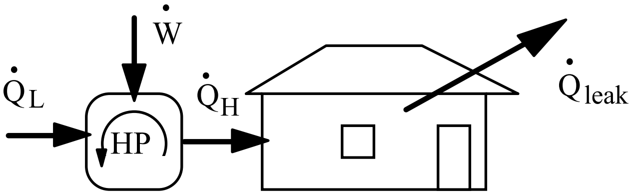
a. If the winter outside temperature is 0°C, what is the minimum power required to drive the heat pump?

b.For the same power as in part (a), what is the maximum outside summer temperature for which the house can be maintained at 25°C?

Solution:

a) Winter:

House is T_H and ambient is at T_L

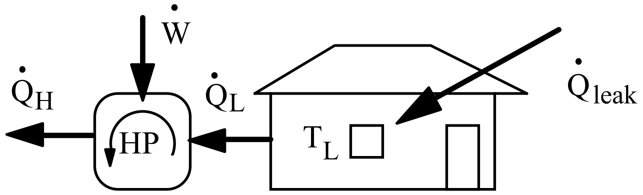


$$T_H = 20°C = 293.2 \text{ K}, \quad T_L = 0°C = 273.2 \text{ K} \quad \text{and} \quad \dot Q_H = 2400(20 - 0) \text{ kJ/h}$$

$$\beta' = \dot Q_H/\dot W_{IN} = 2400\ (20 - 0)/\ \dot W_{IN} = \frac{T_H}{T_H - T_L} = \frac{293.2}{20}$$

$$\Rightarrow \dot W_{IN} = 3275 \text{ kJ/h} = \mathbf{0.91\ kW} \quad \text{(For Carnot cycle)}$$

b)



Summer:

$T_L = T_{house}$

$T_H = T_{amb}$

$$T_L = 25°C = 298.2 \text{ K}, \quad \dot W_{IN} = 3275 \text{ kJ/h} \quad \text{and} \quad \dot Q_L = 2400(T_H - 298.2) \text{ kJ/h}$$

$$\beta = \frac{\dot Q_L}{\dot W_{IN}} = \frac{2400(T_H - 298.2)}{3275} = \frac{T_L}{T_H - T_L} = \frac{298.2}{T_H - 298.2}$$

$$\text{or,} \quad (T_H - 298.2)^2 = \frac{298.2 \times 3275}{2400} = 406.92$$

$$T_H = 318.4 \text{ K} = \mathbf{45.2°C}$$

7.108

A remote location without electricity operates a refrigerator with a bottle of propane feeding a burner to create hot gases. Sketch the setup in terms of cyclic devices and give a relation for the ratio of $\dot{Q}_L$ in the refrigerator to $\dot{Q}_{fuel}$ in the burner in terms of the various reservoir temperatures.

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta_{HE}\dot{Q}_{fuel}$

C.V.: Refrigerator: $\dot{Q}_{L2} = \beta\ \dot{W}_{HE} = \beta\ \eta_{HE}\ \dot{Q}_{fuel}$

The ratio becomes

$$\dot{Q}_{L2} / \dot{Q}_{fuel} = \beta\ \eta_{HE}$$

$$= \frac{T_{L2}}{T_{H2} - T_{L2}}\left(1 - \frac{T_{L1}}{T_{H1}}\right) \qquad \text{If Carnot devices}$$

7.109

A furnace, shown in Fig. P7.109, can deliver heat, Q_{H1} at T_{H1} and it is proposed to use this to drive a heat engine with a rejection at T_{atm} instead of direct room heating. The heat engine drives a heat pump that delivers Q_{H2} at T_{room} using the atmosphere as the cold reservoir. Find the ratio Q_{H2}/Q_{H1} as a function of the temperatures. Is this a better set-up than direct room heating from the furnace?

Solution:

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta\dot{Q}_{H1}$ where $\eta = 1 - T_{atm}/T_{H1}$

C.V.: Heat Pump: $\dot{W}_{HP} = \dot{Q}_{H2}/\beta'$ where $\beta' = T_{rm}/(T_{rm} - T_{atm})$

Work from heat engine goes into heat pump so we have

$$\dot{Q}_{H2} = \beta' \, \dot{W}_{HP} = \beta' \, \eta \, \dot{Q}_{H1}$$

and we may substitute T's for β', η. If furnace is used directly $\dot{Q}_{H2} = \dot{Q}_{H1}$, so if $\beta'\eta > 1$ this proposed setup is better. Is it? For $T_{H1} > T_{atm}$ formula shows that it is good for Carnot cycles. In actual devices it depends wether $\beta'\eta > 1$ is obtained.

7.110

Consider the rock bed thermal storage in Problem 7.65. Use the specific heat so you can write dQ_H in terms of dT_{rock} and find the expression for dW out of the heat engine. Integrate this expression over temperature and find the total heat engine work output.

Solution:

The rock provides the heat Q_H

$$dQ_H = -dU_{rock} = -mC\ dT_{rock}$$

$$dW = \eta dQ_H = -(1 - T_o / T_{rock})\ mC\ dT_{rock}$$

$$m = \rho V = 2750 \times 2 = 5500 \text{ kg}$$

$$_1W_2 = \int -(1 - T_o / T_{rock})\ mC\ dT_{rock} = -\ mC\ [T_2 - T_1 - T_o \ln \frac{T_2}{T_1}]$$

$$= -\ 5500 \times 0.89\ [\ 290 - 400 - 290 \ln \frac{290}{400}\ ]\ = \mathbf{81\ 945\ kJ}$$

7.111

On a cold (-10^{o}C) winter day a heat pump provides 20 kW to heat a house maintained at 20^{o}C and it has a COP$_{HP}$ of 4 using the maximum power available. The next day a winter storm brings the outside to -15^{o}C, assuming the same COP and the house heat loss is to the outside air. How cold is the house then?

If we look at the heat loss for the house we have

$$\dot{Q}_{loss} = 20 \text{ kW} = CA\ \Delta T \quad \Rightarrow \quad CA = \frac{20 \text{ kW}}{20 - (-10) \text{ K}} = 0.667 \text{ kW/K}$$

$$\dot{Q}_{loss} = \dot{Q}_H = COP\ \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{loss}/COP = \frac{20 \text{ kW}}{4} = 5 \text{ kW}$$

With the same COP and the same power input we can get the same $\dot{Q}_H = 20$ kW.

$$\dot{Q}_{loss} = CA\ \Delta T = 0.667 \text{ kW/K} \times [T - (-15)] \text{ K} = 20 \text{ kW}$$

$$T = 20 \text{ kW} / 0.667 \text{ (kW/K)} - 15\ ^oC = (30 - 15)\ ^oC = \mathbf{15^oC}$$

Remark: Since $\dot{Q}_H = \dot{Q}_{loss} = CA\ \Delta T$ is the same the ΔT becomes the same so the house can be kept at 30^{o}C above the ambient. In the real system the COP drops as the outdoor T drops unless the outside heat exchanger is buried under ground with a constant temperature independent upon the weather.

7.112

A Carnot heat engine operating between a high T_H and low T_L energy reservoirs has an efficiency given by the temperatures. Compare this to two combined heat engines one operating between T_H and an intermediate temperature T_M giving out work W_A and the other operating between T_M and T_L giving out W_B. The combination must have the same efficiency as the single heat engine so the heat transfer ratio $Q_H/Q_L = \psi(T_H,T_L) = [Q_H/Q_M]\ [Q_M/Q_L]$. The last two heat transfer ratios can be expressed by the same function $\psi()$ involving also the temperature T_M. Use this to show a condition the function $\psi()$ must satisfy.

The overall heat engine is a Carnot heat engine so

$$\dot{Q}_H / \dot{Q}_L = \frac{T_H}{T_L} = \psi(T_H,T_L)$$

The individual heat engines

$$\dot{Q}_H / \dot{Q}_M = \psi(T_H,T_M) \qquad \text{and} \qquad \dot{Q}_M / \dot{Q}_L = \psi(T_M,T_L)$$

Since an identity is

$$\dot{Q}_H / \dot{Q}_L = [\dot{Q}_H / \dot{Q}_M]\ \ [\dot{Q}_M / \dot{Q}_L] = \psi(T_H,T_L)$$

it follows that we have

$$\psi(T_H,T_L) = \psi(T_H,T_M) \times\ \psi(T_M,T_L)$$

Notice here that the product of the two functions must cancel the intermediate temperature T_M, this shows a condition the function $\psi()$ must satisfy. The Kelvin and Rankine temperature scales are determined by the choice of the function

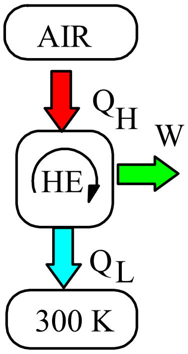
$$\psi(T_H,T_L) =\ T_H / T_L = \dot{Q}_H / \dot{Q}_L$$

satisfying the above restriction.

7.113

A 10-m^3 tank of air at 500 kPa, 600 K acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 300 K. A temperature difference of 25°C between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 400 K and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:



$$T_H = T_{air} - 25°C, \quad T_L = 300 \text{ K}$$

$$m_{air} = \frac{P_1V}{RT_1} = \frac{500 \times 10}{0.287 \times 600} = 29.04 \text{ kg}$$

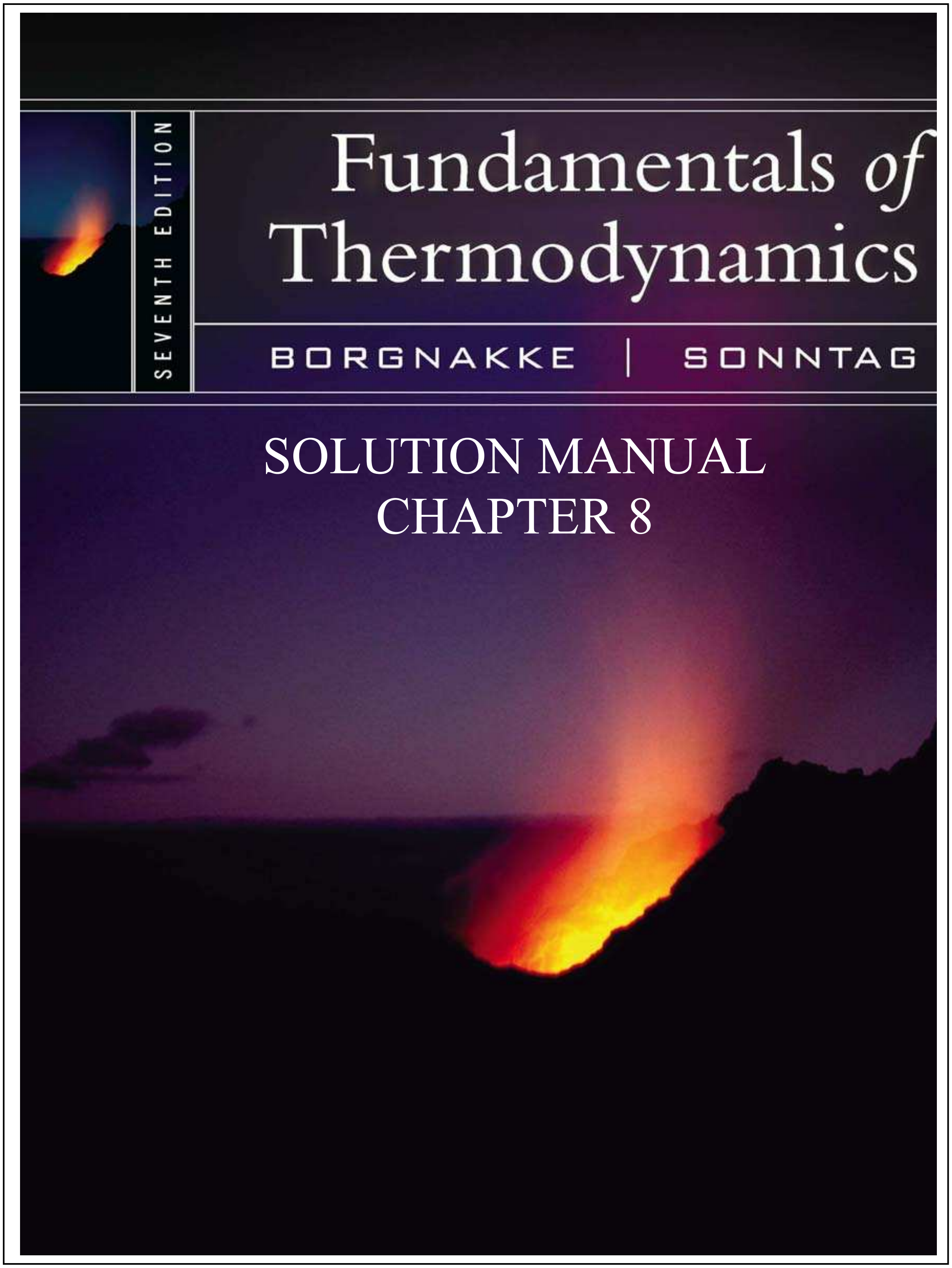
$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 25}\right) dQ_H$$

$$dQ_H = -m_{air}du = -m_{air}C_v dT_{air}$$

$$W = \int dW = -m_{air}C_v \int \left[1 - \frac{T_L}{T_a - 25}\right] dT_a$$

$$= -m_{air}C_v \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 25}{T_{a1} - 25}\right]$$

$$= -29.04 \times 0.717 \times \left[400 - 600 - 300 \ln \frac{375}{575}\right] = \mathbf{1494.3 \ kJ}$$



Fundamentals of Thermodynamics

SEVENTH EDITION

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 8

CONTENT

In-Text Concept Questions

8.a

Does Clausius say anything about the sign for $\oint dQ$?

No.

The total (net) heat transfer can be coming in like in a heat engine ($W_{out} = Q_H - Q_L$) in which case it is positive. It can also be net going out like in a refrigerator or heat pump ($W_{in} = Q_H - Q_L$) in which case the sign is negative. Finally if you look at a transmission gearbox there could be no heat transfer (first approximation) in which case the integral is zero.

8.b

Does the statement of Clausius require a constant T for the heat transfer as in a Carnot cycle?

No.

The statement for a cycle involves an integral of dQ/T so T can vary, which it does during most processes in actual devices. This just means that you cannot that easily get a closed expression for the integral.

8.c

How can you change s of a substance going through a reversible process?

From the definition of entropy

$$ds = \frac{dq}{T}$$

for a reversible process. Thus only heat transfer gives a change in s, expansion/compression involving work does not give such a contribution.

8.d

A reversible process adds heat to a substance. If T is varying does that influence the change in s?

Yes.

$$\text{Reversible:} \quad ds = \frac{dq}{T}$$

So if T goes up it means that s changes less per unit of dq, and the opposite if T decreases then s changes more per unit of dq.

8.e

Water at 100 kPa, 150^{o}C receives 75 kJ/kg in a reversible process by heat transfer. Which process changes s the most: constant T, constant v or constant P?

$$ds = \frac{dq}{T}$$

Look at the constant property lines in a T-s diagram, Fig. 8.5. The constant v line has a higher slope than the constant P line also at positive slope. Thus both the constant P and v processes have an increase in T. As T goes up the change in s is smaller.

The constant T (isothermal) process therefore changes s the most.

8.f

A liquid is compressed in a reversible adiabatic process. What is the change in T?

If the process is reversible then s is constant, $ds = \frac{dq}{T} = 0$

Change in s for a liquid (an incompressible substance) is

Eq. 8.10: $$ds = \frac{C}{T}\, dT$$

From this it follows that if ds = 0 then **T is constant**.

8.g

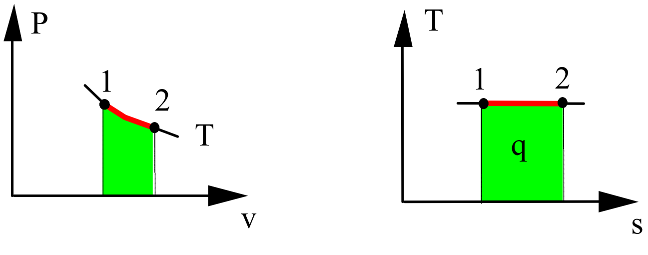
An ideal gas goes through a constant T reversible heat addition process. How do the properties (v, u, h, s, P) change (up, down or constant)?

Ideal gas: u(T), h(T) so they are both constant

Eq. 8.11 gives: $ds = dq/T + ds_{gen} = dq/T + 0 > 0$ so s goes up by q/T

Eq. 8.21 gives: $ds = (R/v)\, dv$ so v increases

Eq. 8.23 gives: $ds = -(R/P)\, dP$ so P decreases

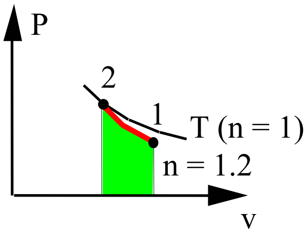


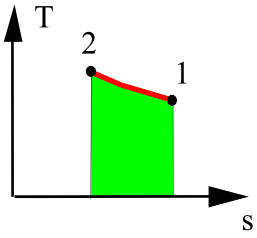
8.h

Carbon dioxide is compressed to a smaller volume in a polytropic process with n = 1.2. How do the properties (u, h, s, P, T) change (up, down or constant)?

For carbon dioxide Table A.5 k = 1.289 so we have n < k and the process curve can be recognized in Figure 8.18. From this we see a smaller volume means moving to the left in the P-v diagram and thus also up.

From P-v diagram: P up, T up
From T-s diagram Since T is up then s down.
As T is up so is h and u.





8.i

A substance has heat transfer out. Can you say anything about changes in s if the process is reversible? If it is irreversible?

Reversible: $ds = \frac{dq}{T} < 0$ since $dq < 0$

Irreversible: $ds = \frac{dq}{T} + ds_{gen} = ?$ $dq < 0$ but $ds_{gen} > 0$

You cannot say, ds depends on the magnitude of dq/T versus ds_{gen}

8.j

A substance is compressed adiabatically so P and T go up. Does that change s?

If the process is reversible then s is constant, $ds = \frac{dq}{T} = 0$

If the process is irreversible then s goes up, $ds = \frac{dq}{T} + ds_{gen} = ds_{gen} > 0$

Concept Problems

8.1

When a substance has completed a cycle, v, u, h, and s are unchanged. Did anything happen? Explain.

Yes.

During various parts of the cycle work and heat transfer may be transferred. That happens at different P and T. The net work out equals the net heat transfer in (energy conservation) so dependent upon the sign it is a heat engine or a heat pump (refrigerator). The net effect is thus a conversion of energy from one storage location to another and it may also change nature (some Q was changed to W or the opposite)

8.2

Assume a heat engine with a given Q_H. Can you say anything about Q_L if the engine is reversible? If it is irreversible?

For a reversible heat engine it must be that:

$$\oint \frac{dQ}{T} = 0 = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \qquad \text{or integrals if T not constant}$$

So as T_L is lower than T_H then Q_L must be correspondingly lower than Q_H to obtain the net zero integral.

For an irreversible heat engine we have

$$\oint \frac{dQ}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} < 0$$

This means that Q_L is larger than before (given Q_H and the T's). The irreversible heat engine rejects more energy and thus gives less out as work.

8.3

CV A is the mass inside a piston/cylinder, CV B is that plus part of the wall out to a source of ${}_1Q_2$ at T_s. Write the entropy equation for the two control volumes assuming no change of state of the piston mass or walls.

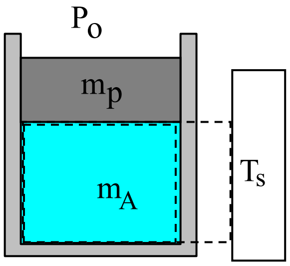


Fig. P8.3

The general entropy equation for a control mass is Eq.8.37

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} \;+\; {}_1S_{2\ \text{gen}}$$

The left hand side is storage so that depends of what is inside the C.V. and the integral is summing the dQ/T that crosses the control volume surface while the process proceeds from 1 to 2.

C.V. A: $\quad m_A\,(s_2 - s_1) = \int_1^2 \frac{dQ}{T_A} + {}_1S_{2\ \text{gen CV A}}$

C.V. B: $\quad m_A\,(s_2 - s_1) = \int_1^2 \frac{dQ}{T_s} + {}_1S_{2\ \text{gen CV B}}$

In the first equation the temperature is that of mass m_A which possibly changes from 1 to 2 whereas in the second equation it is the reservoir temperature T_s. The two entropy generation terms are also different the second one includes the first one plus any s generated in the walls that separate the mass m_A from the reservoir and there is a Q over a finite temperature difference. When the storage effect in the walls are neglected the left hand sides of the two equations are equal.

8.4

Consider the previous set-up with the mass m_A and the piston cylinder of mass m_p starting out at two different temperatures. After a while the temperature becomes uniform without any external heat transfer. Write the entropy equation storage term $(S_2 - S_1)$ for the total mass.

In this case the storage effect must be summed up over all the mass inside the control volume.

$$
\begin{aligned}
S_2 - S_1 &= m_A\,(s_2 - s_1)_A + m_P\,(s_2 - s_1)_P \\
&\approx m_A\,(s_2 - s_1)_A + m_P\,C_P \ln\left(\frac{T_2}{T_{1\,P}}\right)
\end{aligned}
$$

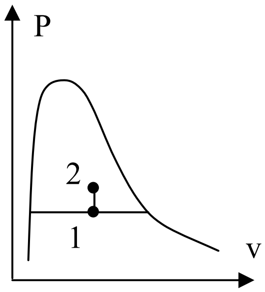
The last equation assumed a constant specific heat for the solid material of the piston, a common assumption. There is only a single temperature T_2, but there are two different temperatures for state 1 $T_{1\,A}$ and $T_{1\,P}$. The temperature T_2 would be found from the energy equation.

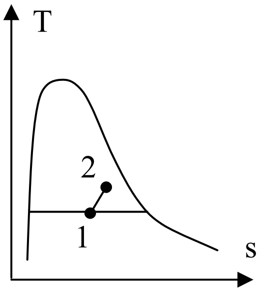
8.5

Water at 100^{o}C, quality 50% in a rigid box is heated to 110^{o}C. How do the properties (P, v, x, u and s) change? (increase, stay about the same, or decrease)

A fixed mass in a rigid box give a constant v process. So

P goes up (in the two-phase region $P = P_{sat}$ at given T)
v stays constant.
x goes up (we get closer to the saturated vapor state see P-v diagram)
u goes up (Q in and no work)
s goes up (Q in)





8.6

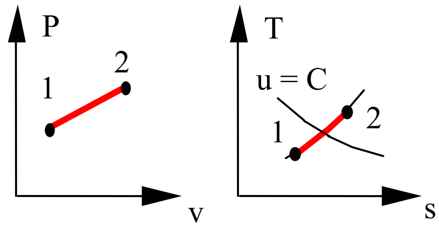
Liquid water at 20°C, 100 kPa is compressed in a piston/cylinder without any heat transfer to a pressure of 200 kPa. How do the properties (T, v, u, and s) change? (increase, stay about the same, or decrease)

Adiabatic $dq = 0$: $dq = T\ ds = 0$;
Incompressible $dv = 0$: $dw = P\ dv = 0$

(T, v, u, and s) they are all constant.
Only the pressure and enthalpy goes up.

8.7

A reversible process in a piston/cylinder is shown in Fig. P8.7. Indicate the storage change $u_2 - u_1$ and transfers ${}_1w_2$ and ${}_1q_2$ as positive, zero, or negative

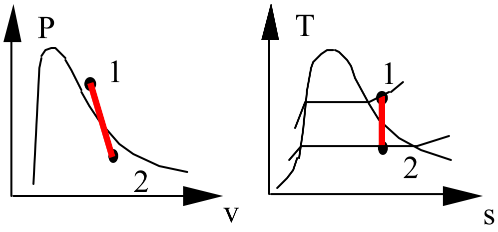


$${}_1w_2 = \int P\,dv > 0 ; \qquad {}_1q_2 = \int T\,ds > 0$$

$u_2 - u_1 > 0$ from general shape of the constant u curves. Further out in the ideal gas region the constant u curve become horizontal (u = fct(T) only).

8.8

A reversible process in a piston/cylinder is shown in Fig. P8.8. Indicate the storage change $u_2 - u_1$ and transfers ${}_1w_2$ and ${}_1q_2$ as positive, zero, or negative



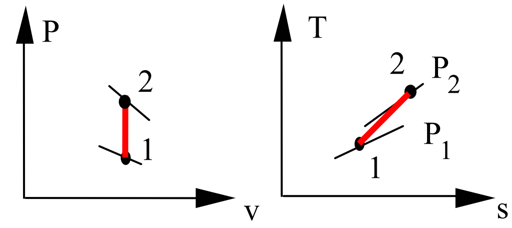
$${}_1w_2 = \int P\,dv > 0 \;; \qquad {}_1q_2 = \int T\,ds = 0$$

$$u_2 - u_1 = {}_1q_2 - {}_1w_2 < 0$$

8.9

Air at 290 K, 100 kPa in a rigid box is heated to 325 K. How do the properties (P, v, u and s) change? (increase, stay about the same, or decrease)

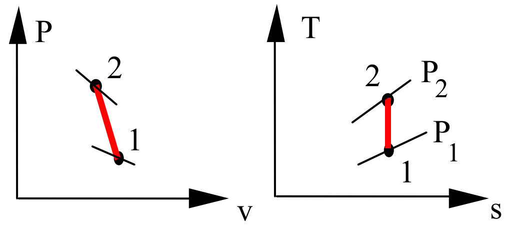
Rigid box: v = constant, (P, u, and s) all increases.



8.10

Air at 20^{O}C, 100 kPa is compressed in a piston/cylinder without any heat transfer to a pressure of 200 kPa. How do the properties (T, v, u and s) change? (increase, about the same or decrease)

T goes up,
v goes down
u goes up (work in, q = 0)
s = constant



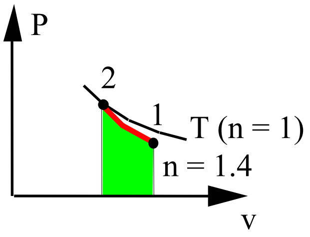
8.11

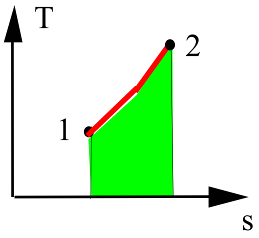
Carbon dioxide is compressed to a smaller volume in a polytropic process with n = 1.4. How do the properties (u, h, s, P, T) change (up, down or constant)?

For carbon dioxide Table A.5 k = 1.289 so we have n > k and the process curve can be recognized in Figure 8.18. From this we see a smaller volume means moving to the left in the P-v diagram and thus also up.

P up, T up. As T is up so is h and u.

From the T-s diagram as n > k then we move to larger T means s is up.





8.12

Process A: Air at 300 K, 100 kPa is heated to 310 K at constant pressure.
Process B: Heat air at 1300 K to 1310 K at constant 100 kPa.
Use the table below to compare the property changes.

	Property	$\Delta_A > \Delta_B$	$\Delta_A \approx \Delta_B$	$\Delta_A < \Delta_B$
a	$\Delta = v_2 - v_1$		√	
b	$\Delta = h_2 - h_1$			√
c	$\Delta = s_2 - s_1$	√		

a. Ideal gas $Pv = RT$ so v goes with absolute T
$\Delta v = (R/P)\ \Delta T$ thus the same

b. Since $dh = C_P\ dT$ and C_P increases with T

c. At constant P: $ds = (C_P/T)\ dT$
C_P is only 15% higher at 1300 K compared to 300 K (see Fig. 5.11)

8.13

Why do we write ΔS or $S_2 - S_1$ whereas we write $\int dQ/T$ and ${}_1S_{2\ gen}$?

This is similar to the terms in the continuity equation $m_2 - m_1$ versus m_i, m_e and the energy equation $E_2 - E_1$ versus ${}_1Q_2$, ${}_1W_2$. The first part is the **change in the storage** $S_2 - S_1$ of entropy in the mass and the second part is the transfer or generation during the process from 1 to 2. The storage terms correspond to the left hand side of the balance equation for entropy. The integral $\int dQ/T$ represents a **transfer** of entropy **across the control volume surface** during the process from 1 to 2 and the ${}_1S_{2\ gen}$ expresses the total amount of entropy being generated **inside** the control volume and both are on the right hand side of the balance equation.

8.14

A reversible heat pump has a flux of s entering as $\dot{Q}_L/T_L$. What can you say about the exit flux of s at T_H?

For the entropy equation 8.3 and 8.42, the rate of storage is zero and the generation term is zero. Thus the entropy equation becomes

$$0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + 0$$

So: $$\frac{\dot{Q}_L}{T_L} = \frac{\dot{Q}_H}{T_H} = \text{flux of s}$$

We have the same flux of s in as out matching the result in chapter 7.

8.15

An electric baseboard heater receives 1500 W of electrical power that heats the room air which loses the same amount out through the walls and windows. Specify exactly where entropy is generated in that process.

a Electrical heating wire (electrical work turned into internal energy, leaving as heat transfer).
b Heat transfer from hot wire to cooler room air, i.e. in the wire coverings
c Room air to walls
d Inside walls and windows, heat transfer over a finite ΔT
e from outside wall surface to ambient T

8.16

A 500 W electric space heater with a small fan inside heats air by blowing it over a hot electrical wire. For each control volume: a) wire at T_{wire} only b) all the room air at T_{room} and c) total room plus the heater, specify the storage, entropy transfer terms and entropy generation as rates (neglect any $\dot{Q}$ through the room walls or windows).

	Storage	$\dot{Q}$ /T	$\dot{S}_{gen}$
C.V. Wire only	0	$\frac{-500\text{ W}}{T_{wire}}$	$\frac{500\text{ W}}{T_{wire}}$
C.V. Room air	$\frac{500\text{ W}}{T_{room}}$	$\frac{500\text{ W}}{T_{wire}}$	$500\text{ W}\left(\frac{1}{T_{room}} - \frac{1}{T_{wire}}\right)$
C.V. Total room	$\frac{500\text{ W}}{T_{room}}$	0	$\frac{500\text{ W}}{T_{room}}$

Remark: Room only receives the electrical power input of 500 W.

Inequality of Clausius

8.17

Consider the steam power plant in Example 6.9 and assume an average T in the line between 1 and 2. Show that this cycle satisfies the inequality of Clausius.

Solution:

Show Clausius: $\int \frac{dQ}{T} \le 0$

For this problem we have three heat transfer terms:

$q_b = 2831$ kJ/kg, $q_{loss} = 21$ kJ/kg, $q_c = 2173.3$ kJ/kg

$$\int \frac{dq}{T} = \frac{q_b}{T_b} - \frac{q_{loss}}{T_{avg\ 1\text{-}2}} - \frac{q_c}{T_c}$$

$$= \frac{2831}{573} - \frac{21}{568} - \frac{2173.3}{318}$$

$$= -1.93 \text{ kJ/kg K} < 0 \quad \textbf{OK}$$

8.18

A heat engine receives 6 kW from a 250^{o}C source and rejects heat at 30^{o}C. Examine each of three cases with respect to the inequality of Clausius.

a. $\dot{W} = 6$ kW b. $\dot{W} = 0$ kW c. Carnot cycle

Solution:

$T_H = 250 + 273 = 523$ K ; $T_L = 30 + 273 = 303$ K

Case a) $\displaystyle\int \frac{d\dot{Q}}{T} = \frac{6000}{523} - \frac{0}{303} = 11.47 \text{ kW/K} > 0$ **Impossible**

b) $\displaystyle\int \frac{d\dot{Q}}{T} = \frac{6000}{523} - \frac{6000}{303} = -8.33 \text{ kW/K} < 0$ **OK**

c) $\displaystyle\int \frac{d\dot{Q}}{T} = 0 = \frac{6000}{523} - \frac{\dot{Q}_L}{303} \quad \Rightarrow$

$$\dot{Q}_L = \frac{303}{523} \times 6 \text{ kW} = 3.476 \text{ kW}$$

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = 2.529 \text{ kW}$$

8.19

Use the inequality of Clausius to show that heat transfer from a warm space towards a colder space without work is a possible process i.e. a heat engine with no work output.

Clausius: $\int \frac{dQ}{T} \le 0$ or $\int \frac{d\dot{Q}}{T} \le 0$

Take C.V. as the space separating the warm and cold space. It is the same Q that crosses each surface (from energy equation) so

$$\int \frac{dQ}{T} = \frac{Q}{T_{warm}} - \frac{Q}{T_{cold}} = Q\left(\frac{1}{T_{warm}} - \frac{1}{T_{cold}}\right) < 0 \qquad \text{OK}$$

8.20

Use the inequality of Clausius to show that heat transfer from a cold space towards a warmer space without work is an impossible process i.e. a heat pump with no work input.

Clausius: $\int \frac{dQ}{T} \leq 0$ or $\int \frac{d\dot{Q}}{T} \leq 0$

Take C.V. as the space separating the warm and cold space. It is the same Q that crosses each surface (from energy equation) so

$$\int \frac{dQ}{T} = \frac{Q}{T_{cold}} - \frac{Q}{T_{warm}} = Q\,(\frac{1}{T_{cold}} - \frac{1}{T_{warm}}) > 0 \quad \text{Impossible!}$$

8.21

Assume the heat engine in Problem 7.32 has a high temperature of 1200 K and a low temperature of 400 K. What does the inequality of Clausius say about each of the four cases?

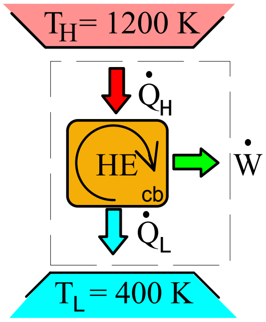
Solution:

Cases a) $\int \frac{d\dot{Q}}{T} = \frac{6}{1200} - \frac{4}{400} = -\ 0.005 \text{ kW/K} < 0$ **OK**

b) $\int \frac{d\dot{Q}}{T} = \frac{6}{1200} - \frac{0}{400} = 0.005 \text{ kW/K} > 0$ **Impossible**

c) $\int \frac{d\dot{Q}}{T} = \frac{6}{1200} - \frac{2}{400} = 0 \text{ kW/K}$ **Possible if reversible**

d) $\int \frac{d\dot{Q}}{T} = \frac{6}{1200} - \frac{6}{400} = -\ 0.001 \text{ kW/K} < 0$ **OK**



8.22

Let the steam power plant in Problem 7.35 have 700^{o}C in the boiler and 40^{o}C during the heat rejection in the condenser. Does that satisfy the inequality of Clausius? Repeat the question for the cycle operated in reverse as a refrigerator.

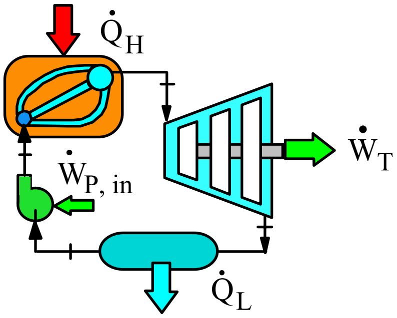
Solution:

$$\dot{Q}_H = 1 \text{ MW} \qquad\qquad \dot{Q}_L = 0.58 \text{ MW}$$

$$\int \frac{d\dot{Q}}{T} = \frac{1000}{973} - \frac{580}{313} = -0.82 \text{ kW/K} < 0 \qquad \textbf{OK}$$

Refrigerator

$$\int \frac{d\dot{Q}}{T} = \frac{580}{313} - \frac{1000}{973} = 0.82 > 0 \qquad \textbf{Cannot be possible}$$



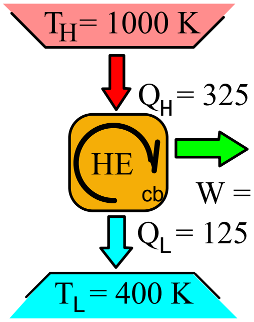
8.23

Examine the heat engine given in Problem 7.54 to see if it satisfies the inequality of Clausius.

Solution:

$Q_H = 325\text{ kJ}$ at $T_H = 1000\text{ K}$
$Q_L = 125\text{ kJ}$ at $T_L = 400\text{ K}$

$$\int \frac{dQ}{T} = \frac{325}{1000} - \frac{125}{400} = 0.0125\text{ kJ/K} > 0 \qquad \textbf{Impossible}$$



Entropy of a pure substance

8.24

Find the missing properties of T, P, s and x for water at:

a. $P = 25$ kPa, $s = 7.7$ kJ/kgK

b. $P = 10$ MPa, $u = 3400$ kJ/kg

c. $T = 150^{o}C$, $s = 7.4$ kJ/kgK

Solution:

a) Table B.1.2 $s_f < s < s_g$ so two-phase and $T = T_{sat}(P) = 64.97°C$

$$x = (s - s_f)/s_{fg} = \frac{7.70 - 0.893}{6.9383} = 0.981$$

$$h = 271.9 + 0.981 \times 2346.3 = 2573.8 \text{ kJ/kg}$$

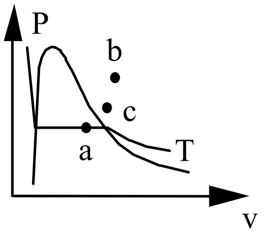
b) Table B.1.2 $u > u_g$ => Superheated vapor Table B.1.3, x = undefined

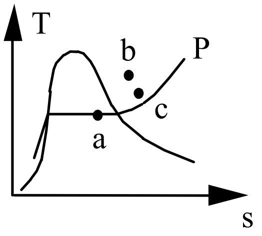
$$T \cong 682°C\ , \ s \cong 7.1223 \text{ kJ/kg K}$$

c) Table B.1.1 $s > s_g$ so superheated vapor. Table B.1.3, x = undefined

State is located between 100 and 200 kPa, so interpolate

$$P = 100 \text{ kPa} + 100 \text{ kPa} \times \frac{7.40 - 7.6133}{7.2795 - 7.6133} = 163.9 \text{ kPa}$$





8.25

Determine the missing property among P, T, s, and x for R-410a at:

a. $T = -20^{\circ}C$, $v = 0.1377\ m^3/kg$

b. $T = 20^{\circ}C$, $v = 0.01377\ m^3/kg$

c. $P = 200\ kPa$, $s = 1.409\ kJ/kgK$

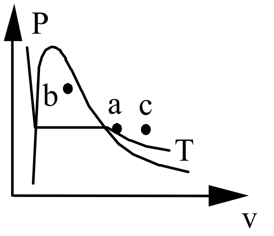
a) B.4.1: $v > v_g = 0.0648\ m^3/kg$ =>
B.4.2 superheated vapor so x is undefined
very close to 200 kPa, $s = 1.1783$ kJ/kg-K

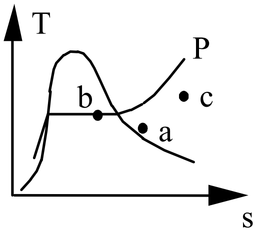
b) B.4.1: $0.000923 = v_f < v < v_g = 0.01758\ m^3/kg$ => Two-phase
$P = P_{sat} = 1444.2$ kPa

$$x = (v - v_f)/v_{fg} = \frac{0.01377 - 0.000923}{0.01666} = 0.77113$$

$$s = s_f + x\ s_{fg} = 0.3357 + 0.77113 \times 0.6627 = 0.8467 \text{ kJ/kg-K}$$

c) Table B.4.2 at 200 kPa, $s > s_g$ so superheated vapor,
x is undefined, and we find the state at $T = 60^{\circ}C$.





8.26

Find the missing properties and give the phase of the ammonia, NH_3.

a. $T = 65°C, P = 600$ kPa $s = ?\ v = ?$

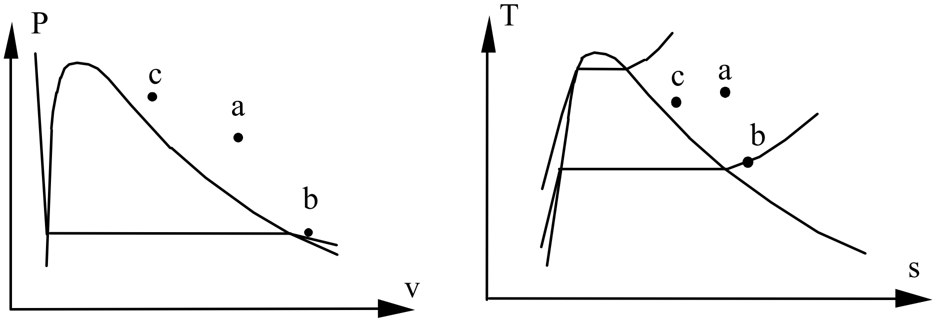
b. $T = 20°C, P = 100$ kPa $v = ?\ s = ?\ x = ?$

c. $T = 50°C, v = 0.1185\ m^3/kg$ $s = ?\ x = ?\ P = ?$

a) B.2.2 average between 60°C and 70°C
$v = (0.25981 + 0.26999)/2 = 0.26435\ m^3/kg$
$s = (5.6383 + 5.7094)/2 = 5.6739$ kJ/kgK

b) B.2.1: $P < P_{sat} = 857.5$ kPa =>
B.2.2 superheated vapor so x is undefined
$v = 1.4153\ m^3/kg$, $s = 6.2826$ kJ/kgK

c) B.2.1: $v > v_g = 0.06337\ m^3/kg$ =>
B.2.2 superheated vapor so x is undefined
very close to 1200 kPa, $s = 5.1497$ kJ/kgK



8.27

Find the entropy for the following water states and indicate each state on a T-s diagram relative to the two-phase region.

a. 250°C, v = 0.02 m^3/kg

b. 250°C, 2000 kPa

c. –2°C, 100 kPa

Solution:

a) Table B.1.1: $0.001251 = v_f < v < v_g = 0.05013\ m^3/kg$ => Two-phase

$$x = \frac{0.02 - 0.001251}{0.04887} = 0.38365$$

$s = s_f + x\, s_{fg} = 2.7927 + 0.38365 \times 3.2802 =$ **4.05 kJ/kg K**

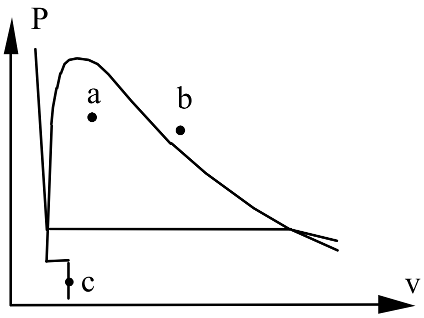
b) Table B.1.1: $P < P_{sat} = 3973$ kPa => superheated vapor B.1.3

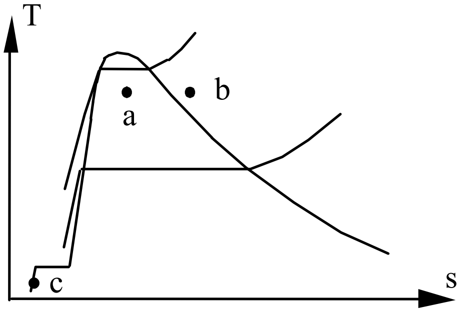
s = 6.5452 kJ/kg K

c) Table B.1.1 $T < T_{tripple} = 0.01°C$ so goto B.1.5

Table B.1.5: $P > P_{sat} = 0.5177$ kPa so compressed solid

s = –1.2369 kJ/kg K





8.28

Find the entropy for the following water states and indicate each state on a T-s diagram relative to the two-phase region.

a. $20^{o}C$, 100 kPa

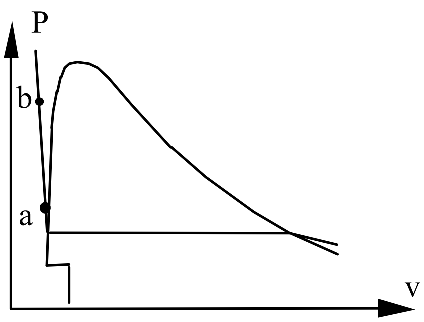
b. $20^{o}C$, 10 000 kPa

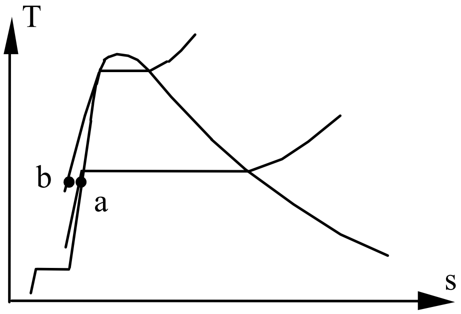
Solution:

Both states are compressed liquid states $P > P_{sat} = 2.34$ kPa from B.1.1

a) Table B.1.1: **s = 0.2966 kJ/kg K** (saturated liquid same T)

b) Table B.1.4 **s = 0.2945 kJ/kg K**





8.29

Determine the missing property among P, T, s, and x for CO_2 at:

a. $P = 1000$ kPa, $v = 0.05$ m^3/kg

b. $T = 0^oC$, $s = 1$ kJ/kgK

c. $T = 60^oC$, $s = 1.8$ kJ/kgK

a) Table B.3.1 at 1004 kPa: $v > v_g = 0.03828$ m^3/kg => Superheated vapor

B.3.2: $$T = 0 + 20\,\frac{0.05 - 0.048}{0.0524 - 0.048} = 20 \times 0.4545 = 9.09^oC$$

$$s = 1.5371 + (1.6025 - 1.5371) \times 0.4545 = 1.5514 \text{ kJ/kg-K}$$

b) Table B.3.1: $0.3344 = s_f < s < s_g = 1.1797$ kJ/kg-K => Two-phase

$$P = P_{sat} = 3485 \text{ kPa}$$

$$x = \frac{s - s_f}{s_{fg}} = \frac{1 - 0.3344}{0.8453} = 0.7874$$

c) Table B.3.1: $T > T_{critical} = 31^oC$ so superheated vapor or dense fluid.

Table B.3.2: Start at 1000 kPa, we see s = 1.72 is too low, so lower P. State is between 400 and 800 kPa at 60^oC, interpolate

$$P = 400 \text{ kPa} + 400 \text{ kPa} \times \frac{1.8 - 1.9033}{1.7660 - 1.9033} = 300.95 \text{ kPa}$$

8.30

Two kg water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process. What are the new quality and specific entropy?

Solution:

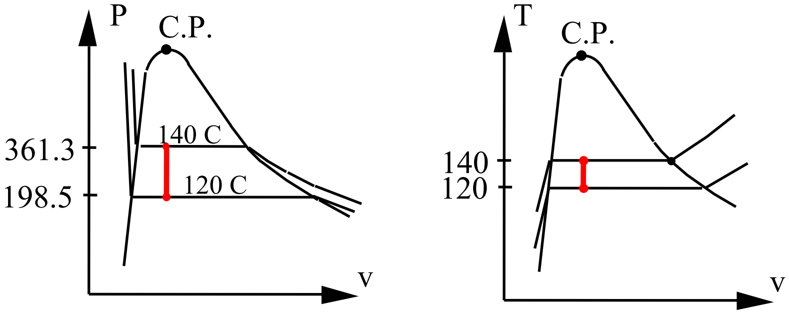
State 1 from Table B.1.1 at 120°C

$$v = v_f + x\, v_{fg} = 0.001060 + 0.25 \times 0.8908 = 0.22376 \text{ m}^3\text{/kg}$$

State 2 has same v at 140°C also from Table B.1.1

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.22376 - 0.00108}{0.50777} = \mathbf{0.4385}$$

$$s = s_f + x\, s_{fg} = 1.739 + 0.4385 \times 5.1908 = \mathbf{4.015\ kJ/kg\ K}$$



8.31

Two kg water at 200 kPa with a quality of 25% has its temperature raised 20°C in a constant pressure process. What is the change in entropy?

Solution:

State 1 from Table B.1.2 at 200 kPa

$$s = s_f + x\, s_{fg} = 1.53 + 0.25 \times 5.597 = 2.9293 \text{ kJ/kg}$$

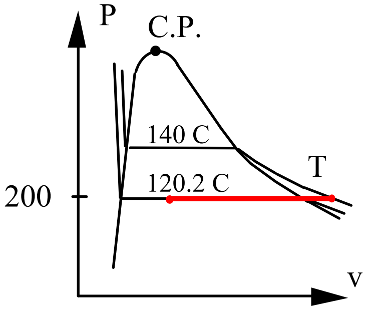
State 2 has same P from Table B.1.2 at 200 kPa

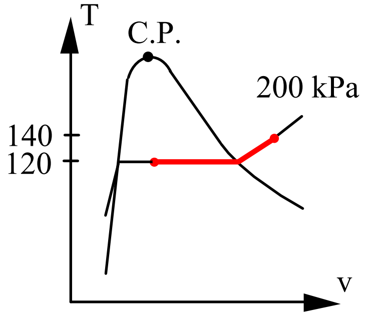
$$T_2 = T_{sat} + 20 = 120.23 + 20 = 140.23^{\circ}\text{C}$$

so state 2 is superheated vapor (x = undefined) from Table B.1.3

$$s_2 = 7.1271 + (7.2795 - 7.1271)\frac{20}{150 - 120.23} = 7.2295 \text{ kJ/kgK}$$

$$s_2 - s_1 = 7.2295 - 2.9293 = \mathbf{4.3002\ kJ/kgK}$$





8.32

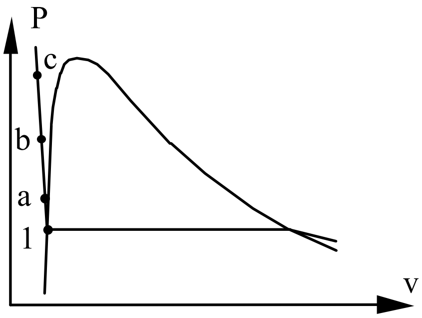
Saturated liquid water at 20^{o}C is compressed to a higher pressure with constant temperature. Find the changes in u and s when the final pressure is

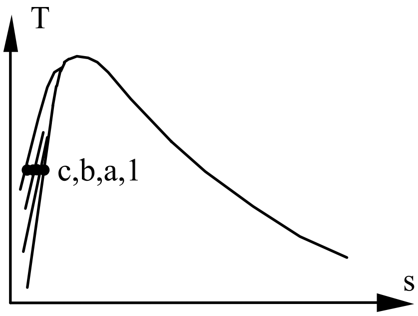
a. 500 kPa b. 2000 kPa c. 20 000 kPa

Solution:

	kJ/kg	kJ/kg K		
B.1.1:	$u_1 = 83.94$	$s_1 = 0.2966$		
B.1.4:	$u_a = 83.91$	$s_a = 0.2965$	$\Delta u = -0.03$	$\Delta s = -0.0001$
B.1.4:	$u_b = 83.82$	$s_b = 0.2962$	$\Delta u = -0.12$	$\Delta s = -0.0004$
B.1.4:	$u_c = 82.75$	$s_c = 0.2922$	$\Delta u = -1.19$	$\Delta s = -0.0044$

Nearly constant u and s, incompressible media





8.33

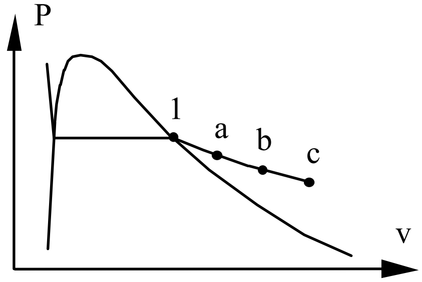
Saturated vapor water at 150^{o}C is expanded to a lower pressure with constant temperature. Find the changes in u and s when the final pressure is

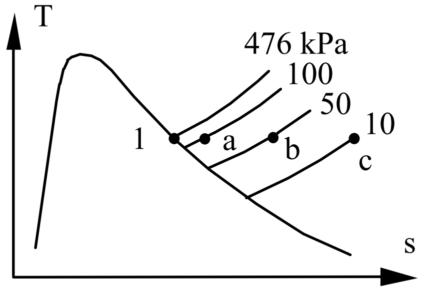
a. 100 kPa b. 50 kPa c. 10 kPa

Solution:

Table B.1.1 for the first state then B.1.3 for the a, b and c states.

kJ/kg	kJ/kg K	kJ/kg	kJ/kg K
u_1= 2559.54	s_1= 6.8378		
u_a = 2582.75	s_a = 7.6133	Δu = 23.21	Δs = 0.7755
u_b = 2585.61	s_b = 7.94	Δu = 26.07	Δs = 1.1022
u_c = 2587.86	s_c = 8.6881	Δu = 28.32	Δs = 1.8503





Remark: You approach ideal gas as P drops so u is u(T) but s is still s(T,P).

8.34

Determine the missing property among *P, T, s, x* for the following states:

a. Ammonia 25°C, $v = 0.10\ m^3/kg$
b. Ammonia 1000 kPa, s = 5.2 kJ/kg K
c. R-134a 5°C, s = 1.7 kJ/kg K
d. R-134a 50°C, s = 1.9 kJ/kg K

Solution:

	Table	**P kPa**	**T °C**	**s kJ/kg K**	**x**
a)	B2.1	1003	25	4.1601	0.7776
b)	B2.2	1000	42.53	5.2	-----
c)	B5.1	350.9	5	1.7	0.96598
d)	B5.2	232.3	50	1.9	-----

a) $x = (0.1 - 0.001658)/0.12647 = \mathbf{0.7776}$

$s = s_f + x\ s_{fg} = 1.121 + x \times 3.9083 = \mathbf{4.1601\ kJ/kg\ K}$

b) $T = 40 + 10 \times (5.2 - 5.1778)/(5.2654 - 5.1778) = \mathbf{42.53^oC}$

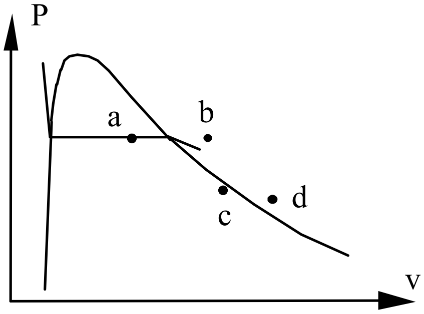
superheated vapor so x is undefined

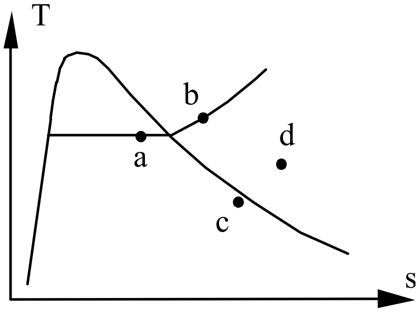
c) $x = (1.7 - 1.0243)/0.6995 = \mathbf{0.96598}$

$P = P_{sat} = \mathbf{350.9\ kPa}$

d) superheated vapor between 200 and 300 kPa

$P = 200 + 100 \times (1.9 - 1.9117)/(1.8755 - 1.9117) = \mathbf{232.3\ kPa}$





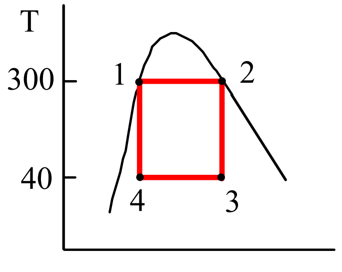
Reversible processes

8.35

Consider a Carnot-cycle heat engine with water as the working fluid. The heat transfer to the water occurs at 300°C, during which process the water changes from saturated liquid to saturated vapor. The heat is rejected from the water at 40°C. Show the cycle on a *T–s* diagram and find the quality of the water at the beginning and end of the heat rejection process. Determine the net work output per kilogram of water and the cycle thermal efficiency.

Solution:

From the definition of the Carnot cycle, two constant s and two constant T processes.



From table B.1.1

State 2 is saturated vapor so

$$s_3 = s_2 = 5.7044 \text{ kJ/kg K}$$
$$= 0.5724 + x_3(7.6845)$$

$$x_3 = \mathbf{0.6678}$$

State 1 is saturated liquid so

$$s_4 = s_1 = 3.2533 \text{ kJ/kg K} = 0.5724 + x_4(7.6845)$$

$$x_4 = \mathbf{0.3489}$$

$$\eta_{TH} = \frac{w_{NET}}{q_H} = \frac{T_H - T_L}{T_H} = \frac{260}{573.2} = \mathbf{0.4536}$$

$$q_H = T_H(s_2 - s_1) = 573.2 \text{ K } (5.7044 - 3.2533) \text{ kJ/kg K} = 1405.0 \text{ kJ/kg}$$

$$w_{NET} = \eta_{TH} \times q_H = \mathbf{637.3\ kJ/kg}$$

8.36

A piston cylinder compresses R-410a at 200 kPa, -20°C to a pressure of 1200 kPa in a reversible adiabatic process. Find the final temperature and the specific compression work?

C.V. The R-410a

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = - {}_1w_2$

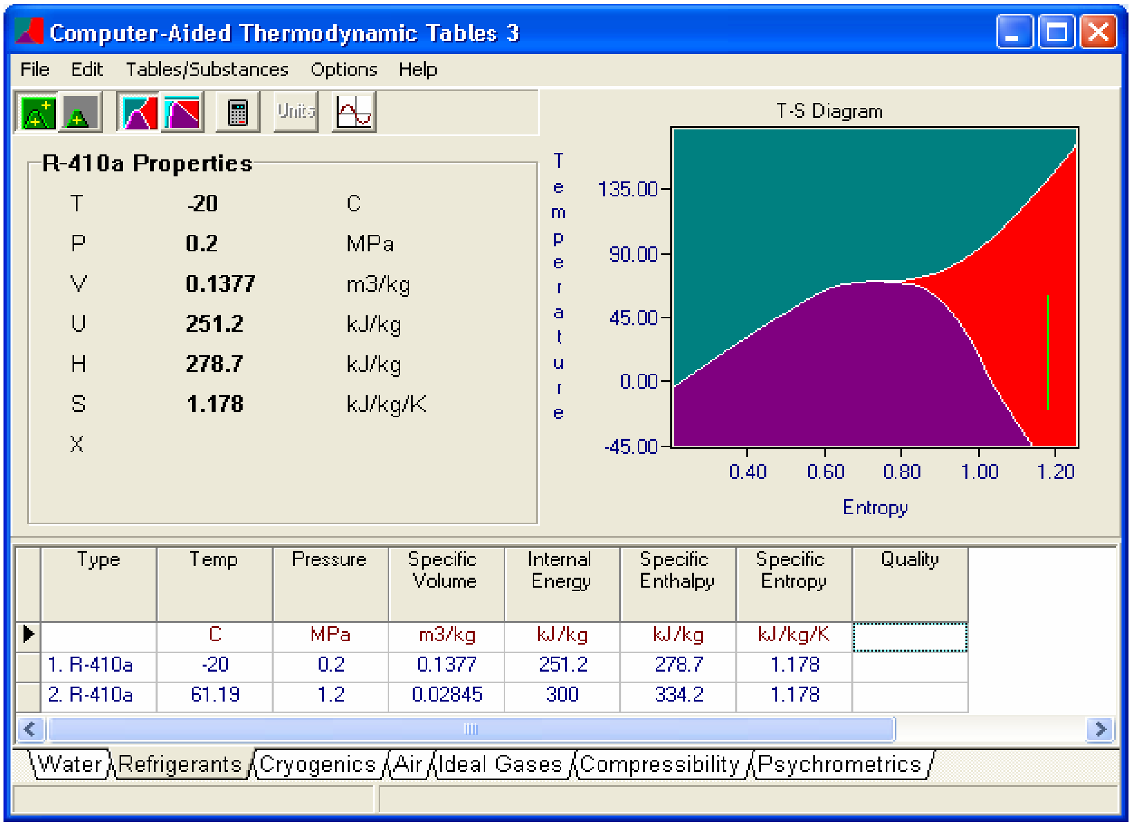
Entropy Eq.: $s_2 - s_1 = \int dq/T \; + 0 = 0 \qquad \Rightarrow \qquad s_2 = s_1$

State 1: Sup. vapor $u_1 = 251.18$ kJ/kg, $s_1 = 1.1783$ kJ/kgK

State 2: (P_2, s_2) $u_2 = 300.05$ kJ/kg (same interpolation as for T),

$$T_2 = 60 + 20 \, \frac{1.1783 - 1.1747}{1.2331 - 1.1747} = \mathbf{61.2^oC}$$

$${}_1w_2 = u_1 - u_2 = 251.18 - 300.05 = \mathbf{-48.87\ kJ/kg}$$



R-410a Properties

T	**-20**	C
P	**0.2**	MPa
V	**0.1377**	m3/kg
U	**251.2**	kJ/kg
H	**278.7**	kJ/kg
S	**1.178**	kJ/kg/K
X		

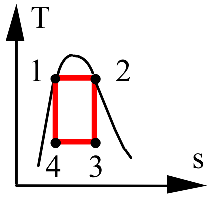
Type	Temp	Pressure	Specific Volume	Internal Energy	Specific Enthalpy	Specific Entropy	Quality
	C	MPa	m3/kg	kJ/kg	kJ/kg	kJ/kg/K	
1. R-410a	-20	0.2	0.1377	251.2	278.7	1.178	
2. R-410a	61.19	1.2	0.02845	300	334.2	1.178	

Process line shown by CATT3 program in T-s diagram

8.37

In a Carnot engine with ammonia as the working fluid, the high temperature is 60°C and as Q_H is received, the ammonia changes from saturated liquid to saturated vapor. The ammonia pressure at the low temperature is 190 kPa. Find T_L, the cycle thermal efficiency, the heat added per kilogram, and the entropy, *s*, at the beginning of the heat rejection process.

Solution:



Constant T ⇒ constant P from 1 to 2, Table B.2.1

$$q_H = \int T ds = T (s_2 - s_1) = T s_{fg}$$

$$= h_2 - h_1 = h_{fg} = \mathbf{997.0\ kJ/kg}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow \quad T_L = T_3 = T_4 = T_{sat}(P) = \mathbf{-20°C}$$

$$\eta_{cycle} = 1 - \frac{T_L}{T_H} = 1 - \frac{253.2}{333.2} = \mathbf{0.24}$$

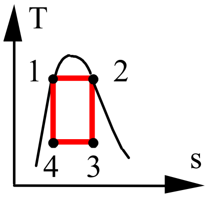
Table B.2.1: $s_3 = s_2 = s_g(60°C) = \mathbf{4.6577\ kJ/kg\ K}$

8.38

Water is used as the working fluid in a Carnot cycle heat engine, where it changes from saturated liquid to saturated vapor at 200°C as heat is added. Heat is rejected in a constant pressure process (also constant T) at 20 kPa. The heat engine powers a Carnot cycle refrigerator that operates between –15°C and +20°C. Find the heat added to the water per kg water. How much heat should be added to the water in the heat engine so the refrigerator can remove 1 kJ from the cold space?

Solution:

Carnot cycle heat engine:



Constant T $\Rightarrow$ constant P from 1 to 2, Table B.2.1

$$q_H = \int T ds = T\,(s_2 - s_1) = T\, s_{fg} = h_{fg}$$

$$= 473.15\ (4.1014) = \mathbf{1940\ kJ/kg}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow\ T_L = T_3 = T_4 = T_{sat}(P) = 60.06^oC$$

Carnot cycle refrigerator (T_L and T_H are different from above):

$$\beta_{ref} = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{273 - 15}{20 - (-15)} = \frac{258}{35} = 7.37$$

$$W = \frac{Q_L}{\beta} = \frac{1}{7.37} = 0.136\ \text{kJ}$$

The needed work comes from the heat engine

$$W = \eta_{HE}\, Q_{H\ H2O}\ ; \qquad \eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{333}{473} = 0.296$$

$$Q_{H\ H2O} = \frac{W}{\eta_{HE}} = \frac{0.136}{0.296} = \mathbf{0.46\ kJ}$$

8.39

Water at 200 kPa, $x = 1.0$ is compressed in a piston/cylinder to 1 MPa, 250°C in a reversible process. Find the sign for the work and the sign for the heat transfer.

Solution:

The process is not specified, but the beginning and end states are

State 1: Table B.1.1:

$$v_1 = 0.8857 \text{ m}^3\text{/kg}; \quad u_1 = 2529.5 \text{ kJ/kg}; \quad s_1 = 7.1271 \text{ kJ/kg K}$$

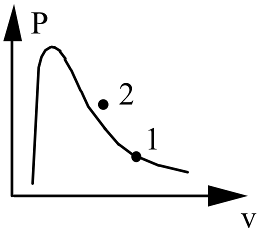
State 2: Table B.1.3:

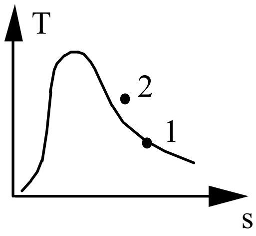
$$v_2 = 0.23268 \text{ m}^3\text{/kg}; \quad u_2 = 2709.9 \text{ kJ/kg}; \quad s_2 = 6.9246 \text{ kJ/kg K}$$

Reversible process: $dw = P\, dv$ $\qquad dq = T\, ds$

$$v_2 < v_1 \quad \Rightarrow \quad {}_1w_2 = \int P\, dv \ < 0$$

$$s_2 < s_1 \quad \Rightarrow \quad {}_1q_2 = \int T\, ds \ < 0$$





8.40

Water at 200 kPa, x = 1.0 is compressed in a piston/cylinder to 1 MPa, 350^{o}C in a reversible process. Find the sign for the work and the sign for the heat transfer.

Solution**:**

$_1w_2 = \int P\, dv$ so sign dv

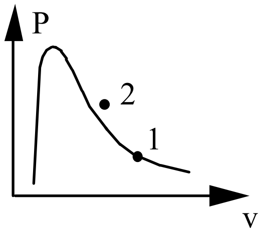
$_1q_2 = \int T\, ds$ so sign ds

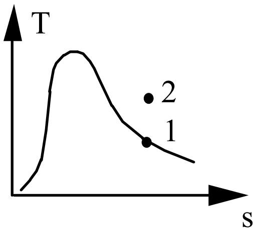
B1.2 $v_1 = 0.88573\ m^3/kg$ $s_1 = 7.1271\ kJ/kg\ K$

B1.3 $v_2 = 0.28247\ m^3/kg$ $s_2 = 7.301\ kJ/kg\ K$

$dv < 0$ => **w is negative**

$ds > 0$ => **q is positive**





8.41

R-410a at 1 MPa and 60°C is expanded in a piston cylinder to 500 kPa, 20°C in a reversible process. Find the sign for both the work and the heat transfer for this process.

Solution:

$_1w_2 = \int P\, dv$ so sign dv

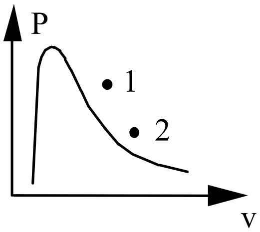
$_1q_2 = \int T\, ds$ so sign ds

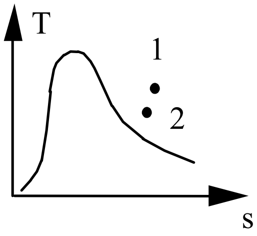
B.4.2 $v_1 = 0.03470\ m^3/kg$ $s_1 = 1.2019$ kJ/kg K

B.4.2 $v_2 = 0.06231\ m^3/kg$ $s_2 = 1.1803$ kJ/kg K

$dv > 0$ => **w is positive**

$ds < 0$ => **q is negative**





8.42

A piston cylinder maintaining constant pressure contains 0.1 kg saturated liquid water at 100°C. It is now boiled to become saturated vapor in a reversible process. Find the work term and then the heat transfer from the energy equation. Find the heat transfer from the entropy equation, is it the same?

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int \frac{dq}{T} + 0 = \frac{{}_1Q_2}{T}$

Process: $P = C \Rightarrow {}_1W_2 = m\,P(v_2 - v_1) = m\,P\,v_{fg}$

$$= 0.1\text{ kg} \times 101.3\text{ kPa} \times 1.67185\text{ m}^3\text{/kg} = \mathbf{16.936\text{ kJ}}$$

From the energy equation we get

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m\,u_{fg} + {}_1W_2$$

$$= 0.1 \times 2087.58 + 16.936 = 225.7\text{ kJ}$$

$$\text{or} \quad = m(h_2 - h_1) = m\,h_{fg} = 0.1 \times 2257.03 = \mathbf{225.7\text{ kJ}}$$

From the entropy equation we can get

$${}_1Q_2 = mT(s_2 - s_1) = m\,T\,s_{fg} = 0.1 \times 373.15 \times 6.048 = \mathbf{225.68\text{ kJ}}$$

So they are equal to within round off errors.

8.43

Consider a Carnot-cycle heat pump with R-410a as the working fluid. Heat is rejected from the R-410a at 40°C, during which process the R-410a changes from saturated vapor to saturated liquid. The heat is transferred to the R-410a at -5°C.

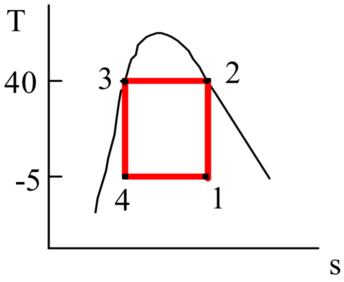
a. Show the cycle on a *T–s* diagram.

b. Find the quality of the R-410a at the beginning and end of the isothermal heat addition process at -5°C.

c. Determine the coefficient of performance for the cycle.

Solution:

a)



b) From Table B.4.1, state 3 is saturated liquid

$$s_4 = s_3 = 0.4473 \text{ kJ/kg K}$$
$$= 0.1989 + x_4(0.8477)$$
$$\Rightarrow \qquad x_4 = \mathbf{0.293}$$

State 2 is saturated vapor so from Table B.4.1

$$s_1 = s_2 = 0.9552 \text{ kJ/kg K} = 0.1989 + x_1(0.8477)$$
$$\Rightarrow \qquad x_1 = \mathbf{0.892}$$

c)

$$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{313.2}{45} = \mathbf{6.96}$$

8.44

Do Problem 8.43 using refrigerant R-134a instead of R-410a.
Consider a Carnot-cycle heat pump with R-410a as the working fluid. Heat is rejected from the R-410a at 40°C, during which process the R-410a changes from saturated vapor to saturated liquid. The heat is transferred to the R-410a at -5°C.

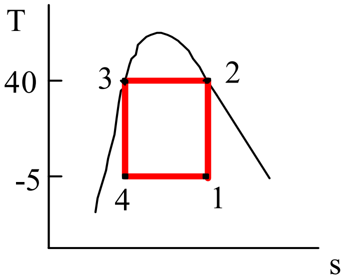
a. Show the cycle on a *T–s* diagram.

b. Find the quality of the R-410a at the beginning and end of the isothermal heat addition process at -5°C.

c. Determine the coefficient of performance for the cycle.

Solution:

a)



b) From Table B.5.1, state 3 is saturated liquid

$$s_4 = s_3 = 1.1909 \text{ kJ/kg K} = 0.9755 + x_4(0.7534)$$

$$\Rightarrow \quad x_4 = \mathbf{0.2859}$$

State 2 is saturated vapor so from Table B.5.1

$$s_1 = s_2 = 1.7123 \text{ kJ/kg K} = 0.9755 + x_1(0.7534)$$

$$\Rightarrow \quad x_1 = \mathbf{0.978}$$

c) $$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{313.2}{45} = \mathbf{6.96}$$

8.45

One kilogram of ammonia in a piston/cylinder at 50°C, 1000 kPa is expanded in a reversible isobaric process to 140°C. Find the work and heat transfer for this process.

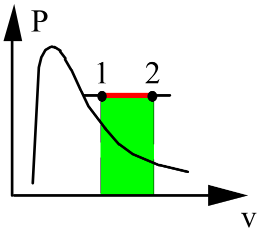
Solution:

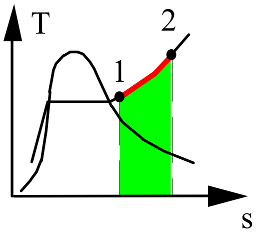
Control mass.

$m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant

$\Rightarrow\ {}_1W_2 = mP(v_2 - v_1)$





State 1: Table B.2.2 $v_1 = 0.145$ m^3/kg, $u_1 = 1391.3$ kJ/kg

State 2: Table B.2.2 $v_2 = 0.1955$ m^3/kg, $u_2 = 1566.7$ kJ/kg

${}_1W_2 = 1 \times 1000(0.1955 - 0.145) =$ **50.5 kJ**

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1 \times (1566.7 - 1391.3) + 50.5 =$ **225.9 kJ**

8.46

A piston cylinder contains 0.25 kg of R-134a at 100 kPa. It will be compressed in an adiabatic reversible process to 400 kPa and should be 70°C. What should the initial temperature be?

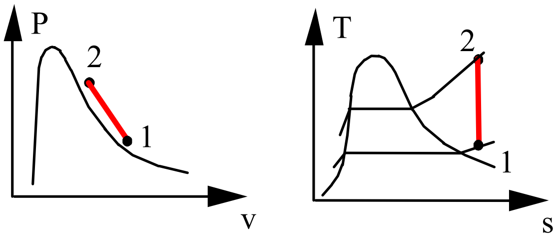
C.V. R-134a which is a control mass.

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T = 0$

State 2: $s_2 = s_1 = 1.9051$ kJ/kgK

Work backwards from state 2 to state 1

State 1: 100 kPa & s_1 => $T_1 =$ **26.4°C**



8.47

Compression and heat transfer brings carbon dioxide in a piston/cylinder from 1400 kPa, 20^{o}C to saturated vapor in an isothermal process. Find the specific heat transfer and the specific work.

Solution:

m = constant

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.3: $s_2 - s_1 = \int dq/T = {}_1q_2 /T$

Process: T = C and assume reversible $\Rightarrow$ ${}_1q_2 = T (s_2 - s_1)$

State 1: Table B.4.2:

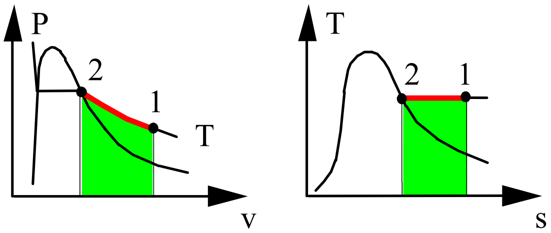
$u_1 = 259.18$ kJ/kg,

$s_1 = 1.0057$ kJ/kg K

State 2: Table B.4.1

$u_2 = 258.16$ kJ/kg,

$s_2 = 0.9984$ kJ/kg K



${}_1q_2 = (273 + 20) \times (0.9984 - 1.0057) = \mathbf{-2.14\ kJ/kg}$

${}_1w_2 = {}_1q_2 + u_1 - u_2 = -2.14 + 259.18 - 258.16$

$= \mathbf{-1.12\ kJ/kg}$

8.48

One kilogram of carbon dioxide in a piston/cylinder at 120°C, 1400 kPa, shown in Fig. P8.48, is expanded to 800 kPa in a reversible adiabatic process. Find the work and heat transfer.

Solution:

Control mass: Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

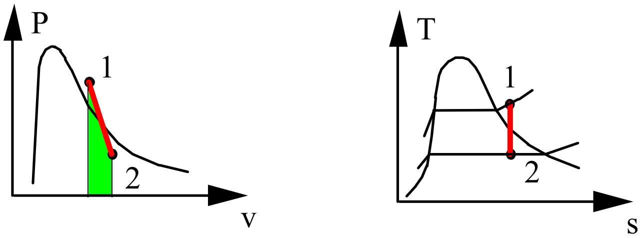
Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$ (= since reversible)

Process: $\mathbf{{}_1Q_2 = 0}$ $\Rightarrow$ $s_2 = s_1$

State 1: (P, T) Table B.3.2, $u_1 = 398.83$ kJ/kg, $s_1 = 1.8093$ kJ/kg K

State 2: P_2 , s_2 $\Rightarrow$ superheated vapor, $u_2 = 368.52$ kJ/kg

$${}_1W_2 = 1 \times (398.83 - 368.52) = \mathbf{30.3\ kJ}$$

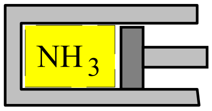


8.49

A cylinder fitted with a piston contains ammonia at 50°C, 20% quality with a volume of 1 L. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.

Solution:

C.V. Ammonia in the cylinder.



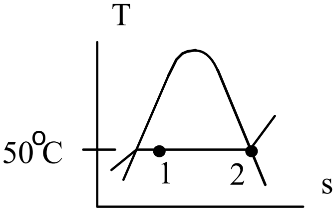


Table B.2.1: $T_1 = 50°C$, $x_1 = 0.20$, $V_1 = 1$ L

$v_1 = 0.001777 + 0.2 \times 0.06159 = 0.014095$ m^3/kg

$s_1 = 1.5121 + 0.2 \times 3.2493 = 2.1620$ kJ/kg K

$m = V_1/v_1 = 0.001/0.014095 = 0.071$ kg

$v_2 = v_g = 0.06336$ m^3/kg,

$s_2 = s_g = 4.7613$ kJ/kg K

Process: T = constant to $x_2 = 1.0$, P = constant = 2.033 MPa

From the constant pressure process

$_1W_2 = \int PdV = Pm(v_2 - v_1) = 2033 \times 0.071 \times (0.06336 - 0.014095) = \mathbf{7.11\ kJ}$

From the second law Eq.8.3 with constant T

$_1Q_2 = \int TdS = Tm(s_2 - s_1) = 323.2 \times 0.071(4.7613 - 2.1620) = \mathbf{59.65\ kJ}$

or $_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

$h_1 = 421.48 + 0.2 \times 1050.01 = 631.48$ kJ/kg, $h_2 = 1471.49$ kJ/kg

$_1Q_2 = 0.071(1471.49 - 631.48) = \mathbf{59.65\ kJ}$

8.50

Water in a piston/cylinder at 400^{o}C, 2000 kPa is expanded in a reversible adiabatic process. The specific work is measured to be 415.72 kJ/kg out. Find the final P and T and show the P-v and the T-s diagram for the process.

Solution:

C.V. Water, which is a control mass. Adiabatic so: $_1q_2 = 0$

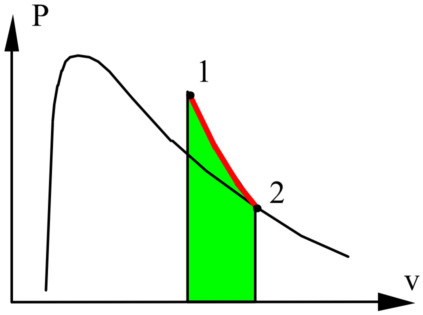
Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = -{}_1w_2$

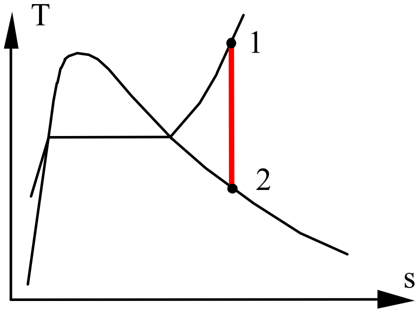
Entropy Eq.8.3: $s_2 - s_1 = \int dq/T = 0$ (= since reversible)

State 1: Table B.1.3 $u_1 = 2945.21$ kJ/kg; $s_1 = 7.127$ kJ/kg K

State 2: (s, u): $u_2 = u_1 - {}_1w_2 = 2945.21 - 415.72 = 2529.49$ kJ/kg

=> **sat. vapor 200 kPa, T = 120.23°C**





8.51

A piston cylinder has R-134a at –20°C, 100 kPa which is compressed to 500 kPa in a reversible adiabatic process. Find the final temperature and the specific work.

Solution:

C.V. R-134a, Control mass of unknown size, adiabatic $_1q_2 = 0$

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = -\,{}_1w_2$

Entropy Eq.8.3: $s_2 - s_1 = \int dq/T$

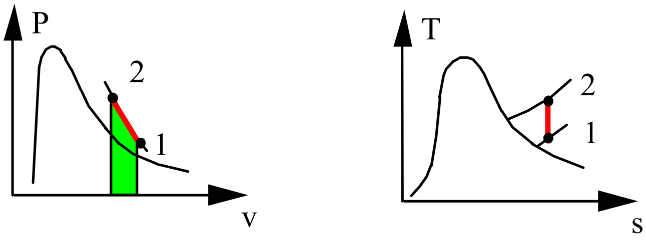
Process: Adiabatic and reversible => $s_2 = s_1$

State 1: (T, P) B.5.2 $u_1 = 367.36$ kJ/kg, $s_1 = 1.7665$ kJ/kg K

State 2: (P, s) B.5.2 $P_2 = 500$ kPa, $s_2 = s_1 = 1.7665$ kJ/kg K

very close at 30°C $u_2 = 398.99$ kJ/kg

$_1w_2 = u_2 - u_1 = 367.36 - 398.99 =$ **-31.63 kJ/kg**



8.52

A piston/cylinder has 2 kg water at 1000 kPa, 250°C which is now cooled with a constant loading on the piston. This isobaric process ends when the water has reached a state of saturated liquid. Find the work and heat transfer and sketch the process in both a P-v and a T-s diagram.

Solution:

C.V. H_2O

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

Process: $P = C \;\Rightarrow\; W = \int P\,dV = P(V_2 - V_1)$

State 1: B.1.3 $v_1 = 0.23268\ \text{m}^3/\text{kg}$, $s_1 = 6.9246$ kJ/kg K, $u_1 = 2709.91$ kJ/kg

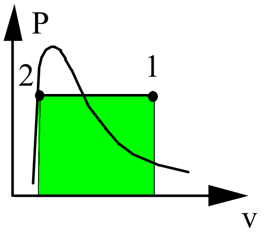
State 2: B.1.2 $v_2 = 0.001127\ \text{m}^3/\text{kg}$, $s_2 = 2.1386$ kJ/kg K, $u_2 = 761.67$ kJ/kg

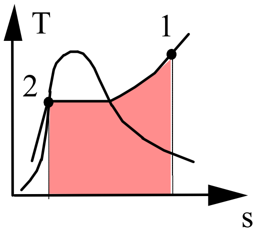
From the process equation

$${}_1W_2 = m\,P\,(v_2 - v_1) = 2 \times 1000\,(0.001127 - 0.23268) = \mathbf{-463.1\ kJ}$$

From the energy equation we get

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2\,(761.67 - 2709.91) - 463.1 = \mathbf{-4359.6\ kJ}$$





8.53

One kilogram of water at 300°C expands against a piston in a cylinder until it reaches ambient pressure, 100 kPa, at which point the water has a quality of 90.2%. It may be assumed that the expansion is reversible and adiabatic. What was the initial pressure in the cylinder and how much work is done by the water?

Solution:

C.V. Water. Process: Rev., $Q = 0$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

Process: Adiabatic $Q = 0$ and reversible => $s_2 = s_1$

State 2: $P_2 = 100$ kPa, $x_2 = 0.902$ from Table B.1.2

$$s_2 = 1.3026 + 0.902 \times 6.0568 = 6.7658 \text{ kJ/kg K}$$

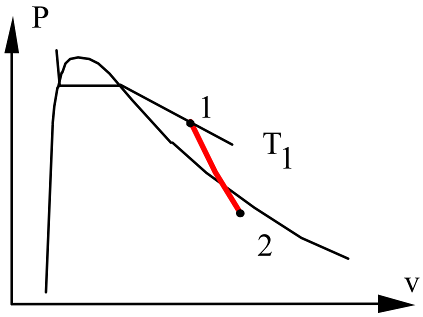
$$u_2 = 417.36 + 0.902 \times 2088.7 = 2301.4 \text{ kJ/kg}$$

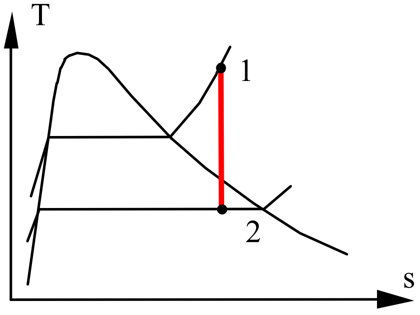
State 1 At $T_1 = 300°C$, $s_1 = 6.7658$ Find it in Table B.1.3

⇒ $P_1 =$ **2000 kPa,** $u_1 = 2772.6$ kJ/kg

From the energy equation

$${}_1W_2 = m(u_1 - u_2) = 1(2772.6 - 2301.4) = \mathbf{471.2\ kJ}$$





8.54

Water at 1000 kPa, 250°C is brought to saturated vapor in a rigid container, shown in Fig. P8.54. Find the final T and the specific heat transfer in this isometric process.

Solution:

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.3: $s_2 - s_1 = \int dq/T$

Process: $v = \text{constant} \quad \Rightarrow \quad {}_1w_2 = 0$

State 1: (T, P) Table B.1.3 $u_1 = 2709.91$ kJ/kg, $v_1 = 0.23268\ \text{m}^3/\text{kg}$

State 2: $x = 1$ and $v_2 = v_1$ so from Table B.1.1 we see $P_2 \cong 800$ kPa

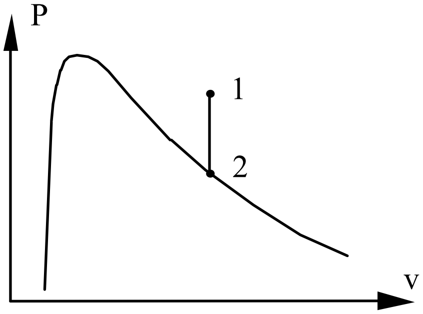
$$T_2 = 170 + 5 \times (0.23268 - 0.24283)/(0.2168 - 0.24283)$$

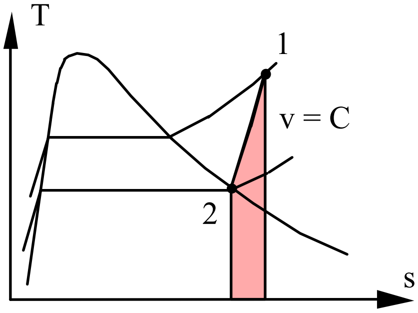
$$= 170 + 5 \times 0.38993 = \mathbf{171.95°C}$$

$$u_2 = 2576.46 + 0.38993 \times (2580.19 - 2576.46) = 2577.9 \text{ kJ/kg}$$

From the energy equation

$${}_1q_2 = u_2 - u_1 = 2577.9 - 2709.91 = \mathbf{-132\ kJ/kg}$$





Notice to get ${}_1q_2 = \int T\,ds$ we must know the function T(s) which we do not readily have for this process.

8.55

Estimate the specific heat transfer from the area in the T-s diagram and compare it to the correct value for the states and process in Problem 8.54.

Solution:

Energy Eq.5.11: $u_2 – u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.3: $s_2 – s_1 = \int dq/T$

Process: v = constant => ${}_1w_2 = 0$

State 1: (T, P) Table B.1.3 $u_1 = 2709.91$ kJ/kg, $v_1 = 0.23268$ m^3/kg,

$s_1 = 6.9246$ kJ/kg K

State 2: x = 1 and $v_2 = v_1$ so from Table B.1.1 we see $P_2 \cong 800$ kPa

$T_2 = 170 + 5 \times (0.23268 – 0.24283)/(0.2168 – 0.24283)$

$= 170 + 5 \times 0.38993 = \mathbf{171.95°C}$

$u_2 = 2576.46 + 0.38993 \times (2580.19 – 2576.46) = 2577.9$ kJ/kg

$s_2 = 6.6663 + 0.38993\ (6.6256 – 6.6663) = 6.6504$ kJ/kg K

From the energy equation

${}_1q_{2\ actual} = u_2 – u_1 = 2577.9 – 2709.91 = \mathbf{–132\ kJ/kg}$

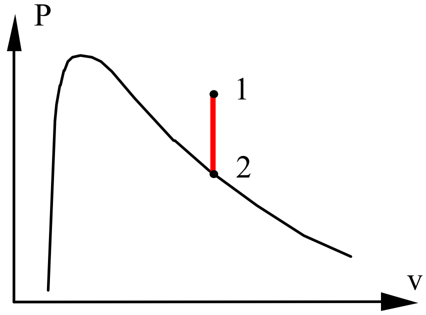
Assume a linear variation of T versus s.

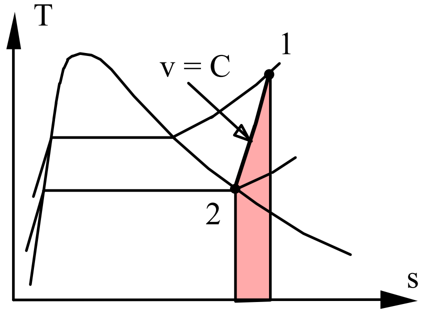
$${}_1q_2 = \int T\, ds = \text{area} \cong \frac{1}{2}(T_1 + T_2)(s_2 – s_1)$$

$$= \frac{1}{2}(171.95 + (2 \times 273.15) + 250)(6.6504 – 6.9246)$$

$= \mathbf{-132.74\ kJ/kg}$

very close i.e. the v = C curve is close to a straight line in the T-s diagram. Look at the constant v curves in Fig. E.1. In the two-phase region they curve slightly and more so in the region above the critical point.





8.56

An insulated cylinder fitted with a piston contains 0.1 kg of water at 100°C, 90% quality. The piston is moved, compressing the water until it reaches a pressure of 1.2 MPa. How much work is required in the process?

Solution:

C.V. Water in cylinder.

Energy Eq.5.11: $_1Q_2 = 0 = m(u_2 - u_1) + {_1W_2}$

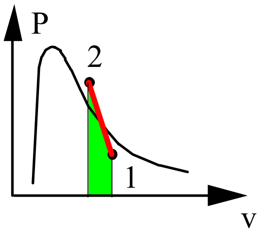
Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T = 0$ (assume reversible)

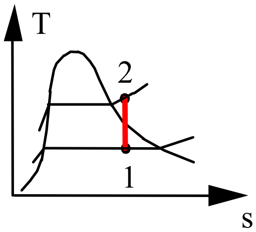
State 1: 100°C, $x_1 = 0.90$:

Table B.1.1,

$s_1 = 1.3068 + 0.90 \times 6.048$

$= 6.7500$ kJ/kg K





$u_1 = 418.91 + 0.9 \times 2087.58 = 2297.7$ kJ/kg

State 2: Given by (P, s) B.1.3 $\left.\begin{matrix} s_2 = s_1 = 6.7500 \\ P_2 = 1.2 \text{ MPa} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} T_2 = 232.3°\text{C} \\ u_2 = 2672.9 \end{matrix}\right.$

$_1W_2 = -m(u_2 - u_1) = -0.1(2672.9 - 2297.7) = \mathbf{-37.5\ kJ}$

8.57

A closed tank, *V* = 10 L, containing 5 kg of water initially at 25°C, is heated to 175°C by a heat pump that is receiving heat from the surroundings at 25°C. Assume that this process is reversible. Find the heat transfer to the water and the change in entropy.

Solution:

C.V.: Water from state 1 to state 2.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

Process: constant volume (reversible isometric) so ${}_1W_2 = 0$

State 1: $v_1 = V/m = 0.002$ from Table B.1.1

$x_1 = (0.002 - 0.001003)/43.358 = 0.000023$

$u_1 = 104.86 + 0.000023 \times 2304.9 = 104.93$ kJ/kg

$s_1 = 0.3673 + 0.000023 \times 8.1905 = 0.36759$ kJ/kg K

Continuity eq. (same mass) and V = C fixes v_2

State 2: T_2, $v_2 = v_1$ so from Table B.1.1

$x_2 = (0.002 - 0.001121)/0.21568 = 0.004075$

$u_2 = 740.16 + 0.004075 \times 1840.03 = 747.67$ kJ/kg

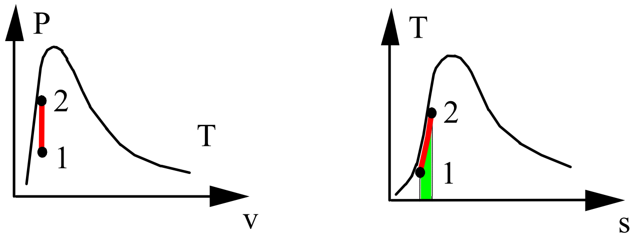
$s_2 = 2.0909 + 0.004075 \times 4.5347 = 2.1094$ kJ/kg K

Energy eq. has W = 0, thus provides heat transfer as

${}_1Q_2 = m(u_2 - u_1) = 3213.7$ kJ

The entropy change becomes

$m(s_2 - s_1) = 5(2.1094 – 0.36759) =$ **8.709 kJ/K**



Notice we do not perform the integration $\int dQ/T$ to find change in s as the equation for the dQ as a function of T is not known.

8.58

A piston/cylinder has 2 kg of R-410a at 60°C, 100 kPa which is compressed to 1000 kPa. The process happens so slowly that the temperature is constant. Find the heat transfer and work for the process assuming it to be reversible.

Solution:

CV : R-410a Control Mass

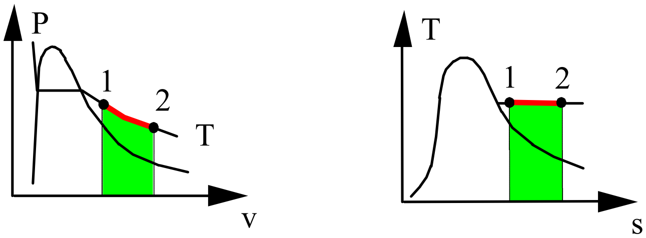
Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$;

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

Process: T = constant and assume reversible process

1: (T,P), Table B.4.2: $v_1 = 0.37833\ m^3/kg$, $u_1 = 309.4\ kJ/kg$,
$s_1 = 1.4910\ kJ/kg\ K$

2: (T,P), Table B.4.2: $v_2 = 0.03470\ m^3/kg$, $u_2 = 301.04\ kJ/kg$,
$s_2 = 1.2019\ kJ/kg\ K$



From the entropy equation (2nd law)

$${}_1Q_2 = mT(s_2 - s_1) = 2 \times 333.15\ (1.2019 - 1.4910) = \mathbf{-192.63\ kJ}$$

From the energy equation

$${}_1W_2 = {}_1Q_2 - m(u_2 - u_1) = -192.63 - 2(301.04 - 309.4) = \mathbf{-175.9\ kJ}$$

8.59

A heavily-insulated cylinder fitted with a frictionless piston, as shown in Fig. P8.59 contains ammonia at 5°C, 92.9% quality, at which point the volume is 200 L. The external force on the piston is now increased slowly, compressing the ammonia until its temperature reaches 50°C. How much work is done by the ammonia during this process?

Solution:

C.V. ammonia in cylinder, insulated so assume adiabatic $Q = 0$.

Cont.Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

State 1: $T_1 = 5^oC$, $x_1 = 0.929$, $V_1 = 200\text{ L} = 0.2\text{ m}^3$

Table B.2.1 saturated vapor, $P_1 = P_g = 515.9$ kPa

$v_1 = v_f + x_1 v_{fg} = 0.001583 + 0.929 \times 0.2414 = 0.2258\text{ m}^3/\text{kg}$,

$u_1 = u_f + x_1 u_{fg} = 202.8 + 0.929 \times 1119.2 = 1242.5$ kJ/kg

$s_1 = s_f + x_1 s_{fg} = 0.7951 + 0.929 \times 4.44715 = 4.9491$ kJ/kg K,

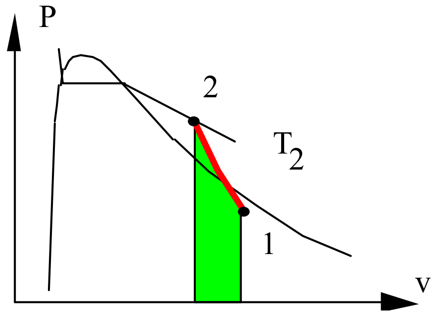
$m_1 = V_1/v_1 = 0.2 / 0.2258 = 0.886$ kg

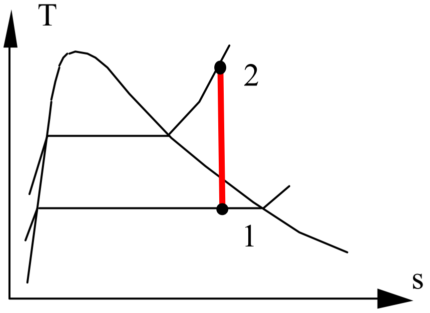
Process: $1 \rightarrow 2$ Adiabatic ${}_1Q_2 = 0$ & Reversible => $s_1 = s_2$

State 2: $T_2 = 50^oC$, $s_2 = s_1 = 4.9491$ kJ/kg K

superheated vapor, interpolate in Table B.2.2 =>

$P_2 = 1600$ kPa, $u_2 = 1364.9$ kJ/kg





Energy equation gives the work as

$${}_1W_2 = m(u_1 - u_2) = 0.886\ (1242.5 - 1364.9) = \mathbf{-108.4\ kJ}$$

8.60

A heavily insulated cylinder/piston contains ammonia at 1200 kPa, 60°C. The piston is moved, expanding the ammonia in a reversible process until the temperature is –20°C. During the process 600 kJ of work is given out by the ammonia. What was the initial volume of the cylinder?

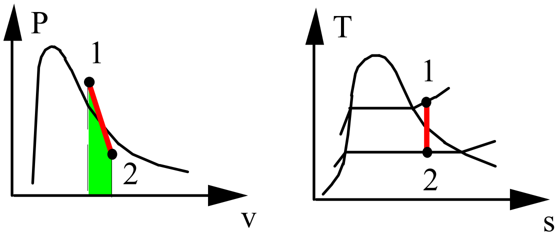
C.V. ammonia. Control mass with no heat transfer.

State 1: Table B.2.2 $v_1 = 0.1238\ m^3/kg$, $s_1 = 5.2357$ kJ/kg K

$$u_1 = h - Pv = 1553.3 - 1200 \times 0.1238 = 1404.9 \text{ kJ/kg}$$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: reversible (${}_1S_{2\ gen} = 0$) and adiabatic ($dQ = 0$) => $s_2 = s_1$



State 2: $T_2, s_2 \Rightarrow x_2 = (5.2357 - 0.3657)/5.2498 = 0.928$

$$u_2 = 88.76 + 0.928 \times 1210.7 = 1211.95 \text{ kJ/kg}$$

$${}_1Q_2 = 0 = m(u_2 - u_1) + {}_1W_2 = m(1211.95 - 1404.9) + 600$$

$$\Rightarrow \quad m = 3.110 \text{ kg}$$

$V_1 = mv_1 = 3.11 \times 0.1238 =$ **$0.385\ m^3$**

8.61

Water at 1000 kPa, 250°C is brought to saturated vapor in a piston/cylinder with an isothermal process. Find the specific work and heat transfer. Estimate the specific work from the area in the P-v diagram and compare it to the correct value.

Solution:

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.:5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

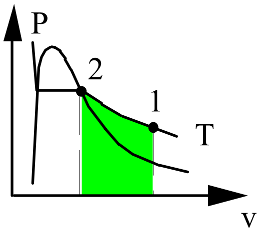
Process: T = constant, reversible

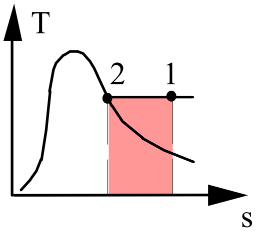
State 1: Table B.1.3:

$v_1 = 0.23268\ m^3/kg$; $u_1 = 2709.91\ kJ/kg$; $s_1 = 6.9246\ kJ/kg\ K$

State 2: (T, x) Table B.1.1 $P_2 = 3973\ kPa$

$v_2 = 0.05013\ m^3/kg$, $u_2 = 2602.37\ kJ/kg$, $s_2 = 6.0729\ kJ/kg\ K$





From the entropy equation

${}_1q_2 = \int T\,ds = T(s_2 - s_1) = (250 + 273)\ (6.0729 - 6.9246) =$ **-445.6 kJ/kg**

From the energy equation

${}_1w_2 = {}_1q_2 + u_1 - u_2 = -445.6 + 2709.91 - 2602.37 =$ **-338 kJ/kg**

Estimation of the work term from the area in the P-v diagram

$${}_1w_{2\ area} \cong \frac{1}{2}(P_1+P_2)(v_2 - v_1) = \frac{1}{2}(1000 + 3973)(0.05013 - 0.23268)$$

$$= -454\ kJ/kg$$

Not extremely accurate estimate; P-v curve not linear more like Pv = constant as curve has positive curvature the linear variation over-estimates area.

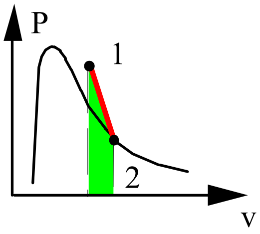
8.62

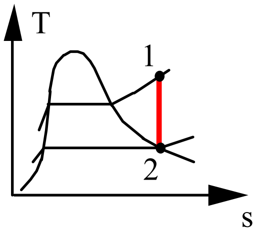
A rigid, insulated vessel contains superheated vapor steam at 3 MPa, 400°C. A valve on the vessel is opened, allowing steam to escape. The overall process is irreversible, but the steam remaining inside the vessel goes through a reversible adiabatic expansion. Determine the fraction of steam that has escaped, when the final state inside is saturated vapor.

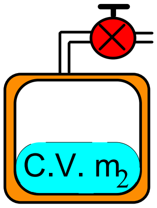
C.V.: steam remaining inside tank. Rev. & Adiabatic (inside only)

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$







Rev $({}_1S_{2\ gen} = 0)$ Adiabatic $(Q = 0)$ => $s_2 = s_1 = 6.9212 = s_G$ at T_2

$\Rightarrow$ $T_2 = 141°\text{C}$, $v_2 = v_{g\ at\ T_2} = 0.4972\ \text{m}^3/\text{kg}$

$$\frac{m_e}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{0.09936}{0.4972} = \mathbf{0.80}$$

8.63

A cylinder containing R-134a at 10°C, 150 kPa, has an initial volume of 20 L. A piston compresses the R-134a in a reversible, isothermal process until it reaches the saturated vapor state. Calculate the required work and heat transfer to accomplish this process.

Solution:

C.V. R-134a.

Cont.Eq.: $m_2 = m_1 = m$;

Energy Eq.:5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

Process: T = constant, reversible

State 1: (T, P) Table B.5.2 $u_1 = 388.36$ kJ/kg, $s_1 = 1.822$ kJ/kg K

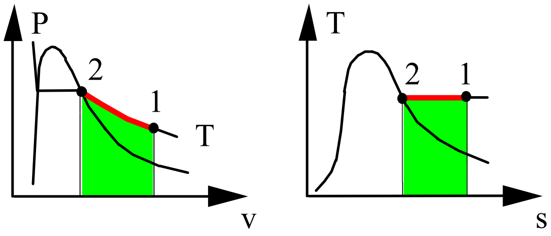
$m = V/v_1 = 0.02/0.148283 = 0.1349$ kg

State 2: (10°C, sat. vapor)

Table B.5.1

$u_2 = 383.67$ kJ/kg,

$s_2 = 1.7218$ kJ/kg K



As T is constant we can find Q by integration as

$${}_1Q_2 = \int T ds = mT(s_2 - s_1) = 0.1349 \times 283.15 \times (1.7218 - 1.822) = \mathbf{-3.83\ kJ}$$

The work is then from the energy equation

$${}_1W_2 = m(u_1 - u_2) + {}_1Q_2 = 0.1349 \times (388.36 - 383.67) - 3.83 = \mathbf{-3.197\ kJ}$$

8.64

Water at 1000 kPa, 250°C is brought to saturated vapor in a piston/cylinder with an adiabatic process. Find the final T and the specific work. Estimate the specific work from the area in the P-v diagram and compare it to the correct value.

Solution:

C.V. Water, which is a control mass with unknown size.

Energy Eq.5.11: $u_2 - u_1 = 0 - {}_1w_2$

Entropy Eq.8.3: $s_2 - s_1 = \int dq/T = 0$

Process: Adiabatic ${}_1q_2 = 0$ and reversible as used above

State 1: Table B.1.3 $v_1 = 0.23268\ \text{m}^3/\text{kg}$, $u_1 = 2709.91\ \text{kJ/kg}$,

$s_1 = 6.9246\ \text{kJ/kg K}$

State 2: Table B.1.1 $x = 1$ and $s_2 = s_1 = 6.9246\ \text{kJ/kg K}$

=> $T_2 \cong 140.56°\text{C}$, $P_2 \cong 367.34\ \text{kPa}$, $v_2 = 0.50187\ \text{m}^3/\text{kg}$,

$u_2 \cong 2550.56\ \text{kJ/kg}$

From the energy equation

$${}_1w_2 = u_1 - u_2 = 2709.91 - 2550.56 = 159.35\ \text{kJ/kg}$$

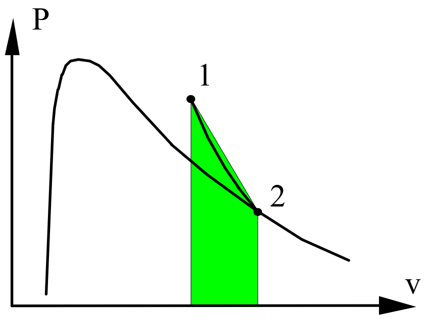
Now estimate the work term from the area in the P-v diagram

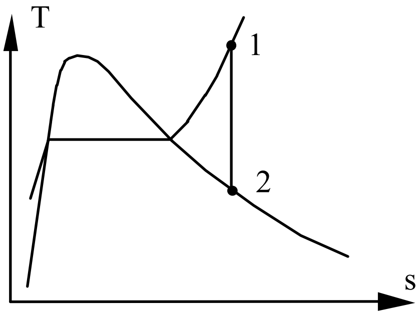
$${}_1w_2 \cong \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1}{2}(1000 + 367.34)(0.50187 - 0.23268)$$

$$= \mathbf{184\ kJ/kg}$$

The s = constant curve is <u>not</u> a straight line in the the P-v diagram, notice the straight line overestimates the area slightly.





8.65

A piston/cylinder contains 2 kg water at 200°C, 10 MPa. The piston is slowly moved to expand the water in an isothermal process to a pressure of 200 kPa. Any heat transfer takes place with an ambient at 200°C and the whole process may be assumed reversible. Sketch the process in a P-V diagram and calculate both the heat transfer and the total work.

Solution:

C.V. Water.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T = {}_1Q_2 / T$

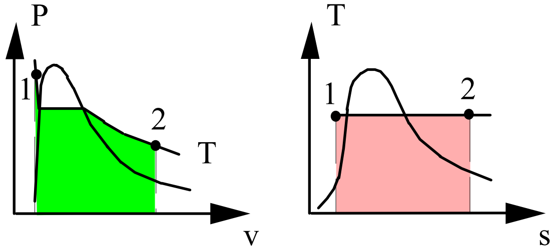
Process: $T = C$ and reversible as used in entropy equation

State 1: Table B.1.4 : $v_1 = 0.001148\ m^3/kg$, $u_1 = 844.49\ kJ/kg$,
$s_1 = 2.3178\ kJ/kg\ K$,

$V_1 = mv_1 = 0.0023\ m^3$

State 2: Table B.1.3 : $v_2 = 1.08034\ m^3/kg$, $u_2 = 2654.4\ kJ/kg$
$s_2 = 7.5066\ kJ/kg\ K$

$V_2 = mv_2 = 2.1607\ m^3$,



From the entropy equation and the process equation

$${}_1Q_2 = mT(s_2 - s_1) = 2 \times 473.15\ (7.5066 - 2.3178) = \mathbf{4910\ kJ}$$

From the energy equation

$${}_1W_2 = {}_1Q_2 - m(u_2 - u_1) = \mathbf{1290.3\ kJ}$$

8.66

Water at 1000 kPa, 250°C is brought to saturated vapor in a piston/cylinder with an isobaric process. Find the specific work and heat transfer. Estimate the specific heat transfer from the area in the T-s diagram and compare it to the correct value.

Solution:

C.V. H_2O

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.3: $s_2 - s_1 = \int dq/T$

Process: $P = C$ => $w = \int P\,dv = P(v_2 - v_1)$

1: B1.3 $v_1 = 0.23268$ m^3/kg, $s_1 = 6.9246$ kJ/kgK, $u_1 = 2709.91$ kJ/kg

2: B1.3 $v_2 = 0.19444$ m^3/kg, $s_2 = 6.5864$ kJ/kg K, $u_2 = 2583.64$ kJ/kg,
$T_2 = 179.91$°C

From the process equation

$${}_1w_2 = P\,(v_2 - v_1) = 1000\,(0.1944 - 0.23268) = \mathbf{-38.28\ kJ/kg}$$

From the energy equation

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 2583.64 - 2709.91 - 38.28 = \mathbf{-164.55\ kJ/kg}$$

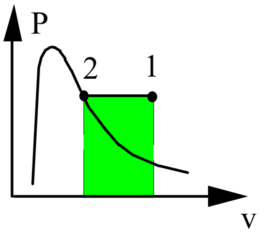
Now estimate the heat transfer from the T-s diagram.

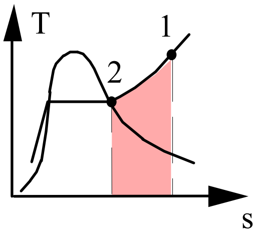
$${}_1q_2 = \int T\,ds = \text{AREA} \cong \frac{1}{2}\,(T_1 + T_2)(s_2 - s_1)$$

$$= \frac{1}{2}\,(250 + 179.91 + 2 \times 273.15)(6.5864 - 6.9246)$$

$$= 488.105 \times (-0.3382) = \mathbf{-165.1\ kJ/kg}$$

very close approximation. The P = C curve in the T-s diagram is nearly a straight line. Look at the constant P curves on Fig.E.1. Up over the critical point they curve significantly.





Entropy of a liquid or a solid

8.67

Two 5 kg blocks of steel, one at 250°C the other at 25°C, come in thermal contact. Find the final temperature and the change in entropy of the steel?

C.V. Both blocks, no external heat transfer, C from Table A.3.

Energy Eq.: $U_2 - U_1 = m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0$

$$= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})$$

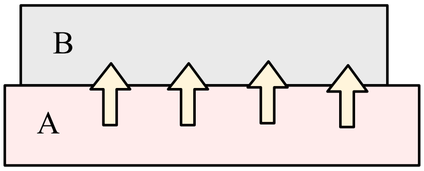
$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = \mathbf{137.5^oC}$$

Entropy Eq.8.39: $S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1S_{2\ gen}$

$$S_2 - S_1 = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}}$$

$$= 5 \times 0.46 \ln \frac{137.5 + 273.15}{250 + 273.15} + 5 \times 0.46 \ln \frac{137.5 + 273.15}{298.15}$$

$$= -0.5569 + 0.7363 = \mathbf{0.1794\ kJ/K}$$



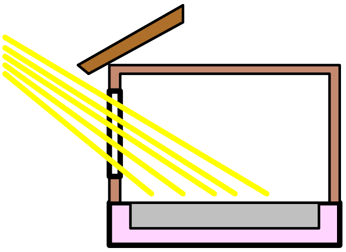
Heat transfer over a finite temperature difference is an irreversible process

8.68

A large slab of concrete, 5 × 8 × 0.3 m, is used as a thermal storage mass in a solar-heated house. If the slab cools overnight from 23°C to 18°C in an 18°C house, what is the net entropy change associated with this process?

Solution:

C.V.: Control mass concrete.



$$V = 5 \times 8 \times 0.3 = 12 \text{ m}^3$$

$$m = \rho V = 2200 \times 12 = 26\,400 \text{ kg}$$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \frac{{}_1Q_2}{T_0} + {}_1S_{2\text{ gen}}$

Process: V = constant so ${}_1W_2 = 0$

Use heat capacity (Table A.3) for change in u of the slab

$${}_1Q_2 = mC\Delta T = 26400 \text{ kg} \times 0.88 \text{ kJ/kg-K} \times (-5) \text{ K} = -116\,160 \text{ kJ}$$

We add all the storage changes as in Eq.8.39:

$$\Delta S_{\text{slab}} = m(s_2 - s_1) = m\,C \ln \frac{T_2}{T_1}$$

$$= 26400 \text{ kg} \times 0.88 \text{ kJ/kg-K} \times \ln \frac{291.2}{296.2} = -395.5 \text{ kJ/K}$$

$$\Delta S_{\text{SURR}} = \frac{-{}_1Q_2}{T_0} = \frac{+116\,160}{291.2} = +398.9 \text{ kJ/K}$$

$$\Delta S_{\text{NET}} = -395.5 + 398.9 = \mathbf{+3.4\ kJ/K}$$

$$= m(s_2 - s_1) - \frac{{}_1Q_2}{T_0} = {}_1S_{2\text{ gen}}$$

8.69

A piston cylinder has constant pressure of 2000 kPa with water at 20°C. It is now heated up to 100°C. Find the heat transfer and the entropy change using the steam tables. Repeat the calculation using constant heat capacity and incompressibility.

Solution:

C.V. Water. Constant pressure heating.

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.3: $s_2 - s_1 = {}_1q_2 / T_{SOURCE} + {}_1s_{2\ gen}$

Process: $P = P_1 \;\; => \;\; {}_1w_2 = P(v_2 - v_1)$

The energy equation then gives the heat transfer as

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1$$

Steam Tables B.1.4: $h_1 = 85.82$ kJ/kg; $s_1 = 0.2962$ kJ/kg K

$h_2 = 420.45$ kJ/kg; $s_2 = 1.3053$ kJ/kg K

$${}_1q_2 = h_2 - h_1 = -85.82 + 420.45 = \mathbf{334.63\ kJ/kg}$$

$$s_2 - s_1 = 1.3053 - 0.2962 = \mathbf{1.0091\ kJ/kg\ K}$$

Now using values from Table A.4: Liquid water $C_p = 4.18$ kJ/kg K

$$h_2 - h_1 \cong C_p(T_2 - T_1) = 4.18 \times 80 = \mathbf{334.4\ kJ/kg}$$

$$s_2 - s_1 \cong C_p \ln(T_2/T_1) = 4.18 \ln \frac{373.15}{293.15} = \mathbf{1.0086\ kJ/kg\ K}$$

Approximations are very good

8.70

A 4 L jug of milk at 25°C is placed in your refrigerator where it is cooled down to the refrigerators inside constant temperature of 5°C. Assume the milk has the property of liquid water and find the entropy change of the milk.

Solution:

C.V. Jug of milk. Control mass at constant pressure.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

State 1: Table B.1.1: $v_1 \cong v_f = 0.001003\ m^3/kg$, $s_f = 0.3673$ kJ/kg K

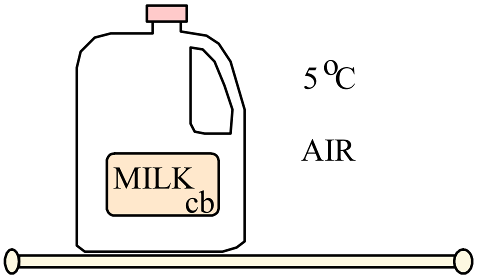
$$m = V/v = 0.004\ m^3/\ 0.001003\ (m^3/kg) = 3.988\ kg$$

State 2: Table B.1.1: $s = s_f = 0.0761$ kJ/kg K

The change of entropy becomes

$$S_2 - S_1 = m(s_2 - s_1) = 3.988\ (0.0761 - 0.3673)$$

$$= \mathbf{-\ 1.1613\ kJ/K}$$



8.71

A foundry form box with 25 kg of 200°C hot sand is dumped into a bucket with 50 L water at 15°C. Assuming no heat transfer with the surroundings and no boiling away of liquid water, calculate the net entropy change for the process.

Solution:

C.V. Sand and water, constant pressure process

$$m_{sand}(u_2 - u_1)_{sand} + m_{H_2O}(u_2 - u_1)_{H_2O} = -P(V_2 - V_1)$$

$$\Rightarrow \; m_{sand}\Delta h_{sand} + m_{H_2O}\Delta h_{H_2O} = 0$$

For this problem we could also have said that the work is nearly zero as the solid sand and the liquid water will not change volume to any measurable extent. Now we get changes in u's instead of h's. For these phases $C_V = C_P = C$ which is a consequence of the incompressibility. Now the energy equation becomes

$$m_{sand}C_{sand}\Delta T_{sand} + m_{H_2O}C_{H_2O}\Delta T_{H_2O} = 0$$

$$25 \times 0.8\times(T_2 - 200) + (50\times10^{-3}/0.001001) \times 4.184 \times (T_2 - 15) = 0$$

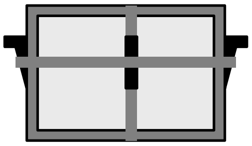
$$T_2 = 31.2°\text{C}$$

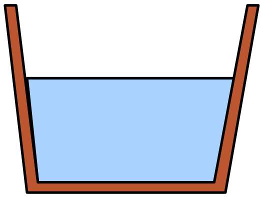
$$\Delta S = m_{sand}(s_2 - s_1) + m_{H2O}(s_2 - s_1)$$

$$= m_{sand}\, C_{sand} \ln(T_2/T_1) + m_{H2O}\, C_{H2O} \ln(T_2/T_1)$$

$$= 25 \times 0.8 \ln\left(\frac{304.3}{473.15}\right) + 49.95 \times 4.184 \ln\left(\frac{304.3}{288.15}\right) = \mathbf{2.57\ kJ/K}$$

Box holds the sand for form of the cast part





8.72

In a sink 5 liters of water at 70°C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20°C. What is the final uniform temperature and change in stored entropy neglecting any heat loss and work?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

Entropy Eq.: $S_2 - S_1 = \int dQ/T + {}_1S_{2\ gen}$

For the water: $v_f = 0.001023\ m^3/kg$, $V = 5\ L = 0.005\ m^3$; $m = V/v = 4.8876$ kg

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\ i}\,(T_2 - T_1)_i = T_2 \sum m_i C_{v\ i} - \sum m_i C_{v\ i} T_{1\ i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\ i} = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59\ \text{kJ/K}$$

Energy Eq.: $22.59\ T_2 = 4.8876 \times 4.18 \times 70 + (1\times 0.9 + 1\times 0.46 + 1\times 0.8) \times 20$

$$= 1430.11 + 43.2$$

$$T_2 = \mathbf{65.2^oC}$$

$$S_2 - S_1 = \sum m_i(s_2 - s_1)_i = \sum m_i\, C_i \ln \frac{T_2}{T_{i1}}$$

$$= 4.8876 \times 4.18 \times \ln \frac{65.22 + 273.15}{70 + 273.15}$$

$$+ 1 \times (0.9 + 0.46 + 0.8) \ln \frac{65.22 + 273.15}{20 + 273.15}$$

$$= -0.28659 + 0.30986 = \mathbf{0.02327\ kJ/K}$$

8.73

A 5-kg steel container is cured at 500°C. An amount of liquid water at 15°C, 100 kPa is added to the container so a final uniform temperature of the steel and the water becomes 75°C. Neglect any water that might evaporate during the process and any air in the container. How much water should be added and how much was the entropy changed?

CV. The steel and the water no external heat transfer nor any work.

Energy Eq.: $m_{H2O}(u_2 - u_1)_{H2O} + m_{st}(u_2 - u_1) = 0$

$m_{H2o}(313.87 - 62.98) + m_{st}C\,(T_2 - T_1) = 0$

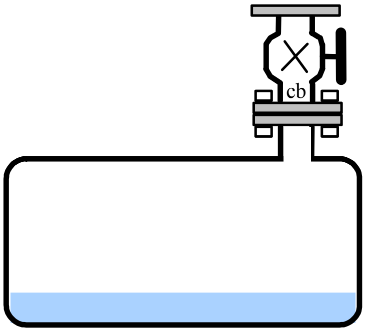
$m_{H2O} \times 250.89 + 5 \times 0.46 \times (75 - 500) = 0$

$m_{H2O} = 977.5/250.89 =$ **3.896 kg**

Entropy Eq. 8.37: $m_{H2O}\,(s_2 - s_1) + m_{st}(s_2 - s_1) = \emptyset + {}_1S_{2\text{ gen}}$

$$S_2 - S_1 = 3.896\,(1.0154 - 0.2245) + 5 \times 0.46 \ln \frac{75 + 273}{773}$$

$= 3.0813 - 1.8356 =$ **1.246 kJ/K**



8.74

A pan in an autoshop contains 5 L of engine oil at 20°C, 100 kPa. Now 2 L of hot 100°C oil is mixed into the pan. Neglect any work term and find the final temperature and the entropy change.

Solution:

Since we have no information about the oil density, we assume the same for both from Table A.4: $\rho = 885 \text{ kg/m}^3$

Energy Eq.: $m_2u_2 - m_Au_A - m_Bu_B \cong 0 - 0$

$\Delta u \cong C_v\Delta T$ so same $C_v = 1.9$ kJ/kg K for all oil states.

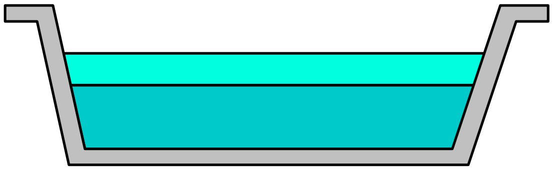
$$T_2 = \frac{m_A}{m_2} T_A + \frac{m_B}{m_2} T_B = \frac{5}{7} \times 20 + \frac{2}{7} \times 100 = 42.868^{\circ}\text{C} = \mathbf{316.02\ K}$$

$$S_2 - S_1 = m_2s_2 - m_As_A - m_Bs_B = m_A(s_2 - s_A) + m_B(s_2 - s_B)$$

$$= 0.005 \times 885 \times 1.9 \ln \frac{316.02}{293.15} + 0.002 \times 885 \times 1.9 \ln \frac{316.02}{373.15}$$

$$= 0.6316 - 0.5588 = \mathbf{+\ 0.0728\ kJ/K}$$

Entropy generation is the total change in S, recall Eq.8.39, no external Q



Oils shown before mixed to final uniform state.

8.75

A computer CPU chip consists of 50 g silicon, 20 g copper, 50 g polyvinyl chloride (plastic). It heats from 15°C to 70°C as the computer is turned on. How much did the entropy increase?

C.V. CPU chip. The process has electrical work input and no heat transfer.

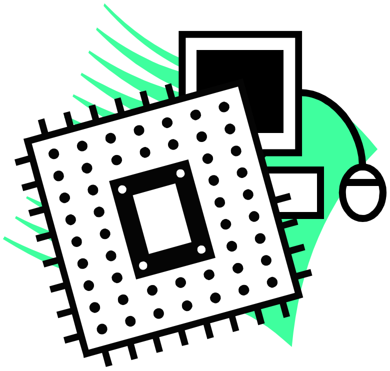
Entropy Eq.: $S_2 - S_1 = \sum m_i(s_2 - s_1)_i = \int dQ/T + {}_1S_{2\ \text{gen}} = {}_1S_{2\ \text{gen}}$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\sum m_i(s_2 - s_1)_i = \sum m_iC_i \ln(T_2 / T_1)_i = \ln(T_2/T_1) \sum m_iC_i$$

$$\sum m_iC_i = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$$S_2 - S_1 = 0.0914 \times \ln(343.15 / 288.15) = \mathbf{0.016\ kJ/K}$$



8.76

A 12 kg steel container has 0.2 kg superheated water vapor at 1000 kPa, both at 200°C. The total mass is now cooled to ambient temperature 30°C. How much heat transfer was taken out and what is the steel-water entropy change?

Solution:

C.V.: Steel and the water, control mass of constant volume.

Energy Eq.5.11: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: V = constant => ${}_1W_2 = 0$

State 1: H_20 Table B.1.3: $u_1 = 2621.9$ kJ/kg, $v_1 = 0.20596$ m^3/kg, $s_1 = 6.6939$ kJ/kg K

State 2: H_20: T_2 , $v_2 = v_1$ => from Table B.1.1

$$x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.20596 - 0.001004}{32.8922} = 0.006231$$

$$u_2 = 125.77 + x_2 \times 2290.81 = 140.04 \text{ kJ/kg}$$

$$s_2 = 0.4369 + x_2 \times 8.0164 = 0.48685 \text{ kJ/kg K}$$

$$ {}_1Q_2 = m(u_2 - u_1) = m_{steel}C_{steel} (T_2 - T_1) + m_{H2O} (u_2 - u_1)_{H2O}$$

$$= 12 \times 0.46 (30 - 200) + 0.2 (140.04 - 2621.9)$$

$$= \mathbf{-1434.8\ kJ}$$

Entropy changes from Eq.8.11 and the water tables

$$S_2 - S_1 = m_2 s_2 - m_1 s_1 = m_{steel}C_{steel} \ln \left(\frac{T_2}{T_1}\right) + m_{H2O} (s_2 - s_1)_{H2O}$$

$$=12 \times 0.46 \ln \left(\frac{303.15}{473.15}\right) + 0.2 (0.48685 - 6.6939)$$

$$= -2.4574 - 1.2414$$

$$= \mathbf{-3.699\ kJ/K}$$

8.77

Two kg of liquid lead initially at 500°C are poured into a form. It then cools at constant pressure down to room temperature of 20°C as heat is transferred to the room. The melting point of lead is 327°C and the enthalpy change between the phases, h_{if}, is 24.6 kJ/kg. The specific heats are in Tables A.3 and A.4. Calculate the net entropy change for this process.

Solution:

C.V. Lead, constant pressure process

$$m_{Pb}(u_2 - u_1)_{Pb} = {}_1Q_2 - P(V_2 - V_1)$$

We need to find changes in enthalpy (u + Pv) for each phase separately and then add the enthalpy change for the phase change.

Consider the process in several steps:

Cooling liquid to the melting temperature

Solidification of the liquid to solid

Cooling of the solid to the final temperature

$${}_1Q_2 = m_{Pb}(h_2 - h_1) = m_{Pb}(h_2 - h_{327,sol} - h_{if} + h_{327,f} - h_{500})$$

$$= 2 \times (0.138 \times (20 - 327) - 24.6 + 0.155 \times (327 - 500))$$

$$= -84.732 - 49.2 - 53.63 = -187.56 \text{ kJ}$$

$$\Delta S_{CV} = m_{Pb}[C_{p\ sol}\ln(T_2/600) - (h_{if}/600) + C_{P\ liq}\ln(600/T_1)]$$

$$= 2 \times \left[0.138 \ln \frac{293.15}{600} - \frac{24.6}{600} + 0.155 \ln \frac{600}{773.15}\right] = -0.358 \text{ kJ/K}$$

$$\Delta S_{SUR} = -{}_1Q_2/T_0 = 187.56/293.15 = 0.64 \text{ kJ/K}$$

The net entropy change from Eq.8.39 is equivalent to total entropy generation

$$\Delta S_{net} = \Delta S_{CV} + \Delta S_{SUR} = \mathbf{0.282\ kJ/K}$$

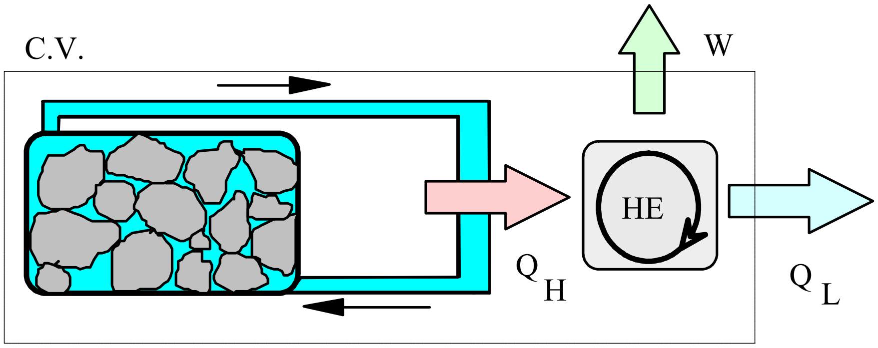
8.78

Find the total work the heat engine can give out as it receives energy from the rock bed as described in Problem 7.65 (see Fig.P 8.78). Hint: write the entropy balance equation for the control volume that is the combination of the rock bed and the heat engine.

Solution:

To get the work we must integrate over the process or do the 2nd law for a control volume around the whole setup out to T_0

C.V. Heat engine plus rock bed out to T_0. W and Q_L goes out.



Energy Eq.5.11: $(U_2 - U_1)_{rock} = - Q_L - W$

Entropy Eq.8.3: $(S_2 - S_1)_{rock} = - \frac{Q_L}{T_0} = mC \ \ln\left(\frac{T_2}{T_1}\right)$

$$= 5500 \times 0.89 \ln \frac{290}{400} = -1574.15 \text{ kJ/K}$$

$$Q_L = -T_0 \, (S_2 - S_1)_{rock} = -290 \, (-1574.15) = 456\,504 \text{ kJ}$$

The energy drop of the rock $-(U_2 - U_1)_{rock}$ equals Q_H into heat engine

$$(U_2 - U_1)_{rock} = mC \, (T_2 - T_1) = 5500 \times 0.89 \, (290 - 400) = -538\,450 \text{ kJ}$$

$$W = -(U_2 - U_1)_{rock} - Q_L = 538450 - 456504 = \mathbf{81\,946 \text{ kJ}}$$

8.79

A 5-kg aluminum radiator holds 2 kg of liquid R-134a at -10C. The setup is brought indoors and heated with 220 kJ. Find the final temperature and the change in entropy of all the mass.

Solution:

C.V. The aluminum radiator and the R-134a.

Energy Eq.5.11: $m_2u_2 – m_1u_1 = {}_1Q_2 – 0$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{al}\,(u_2 - u_1)_{al} + m_{R134a}\,(u_2 - u_1)_{R134a} = {}_1Q_2$$

Use specific heat from Table A.3 and A.4

$$m_{al}C_{al}\,(T_2 - T_1) + m_{R134a}C_{R134a} \ln (T_2 - T_1) = {}_1Q_2$$

$$T_2 - T_1 = {}_1Q_2 / [m_{al}C_{al} + m_{R134a}C_{R134a}]$$

$$= 220 / [5 \times 0.9 + 2 \times 1.43] = 29.89^{o}C$$

$$T_2 = -10 + 29.89 = 19.89^{o}C$$

Entropy change for solid (A.3) and liquid (A.4) from Eq.8.11

$$S_2 – S_1 = m_{al}\,(s_2 – s_1)_{al} + m_{R134a}\,(s_2 – s_1)_{R134a}$$

$$= m_{al}C_{al} \ln (T_2/T_1) + m_{R134a}C_{R134a} \ln (T_2/T_1)$$

$$= (5 \times 0.9 + 2 \times 1.43) \ln \frac{(19.89 + 273.15)}{-10 + 273.15}$$

$$= \mathbf{0.792\ kJ/K}$$

Entropy of ideal gases

8.80

Air inside a rigid tank is heated from 300 to 350 K. Find the entropy increase $s_2 - s_1$? What if it is from 1300 to 1350 K?

Process: V = C $\rightarrow {}_1W_2 = Ø$

Entropy change from Eq.8.17:

a) $s_2 - s_1 = C_{vo} \ln \left(\frac{T_2}{T_1}\right) = 0.717 \ln \left(\frac{350}{300}\right) =$ **0.1105 kJ/kgK**

b) $s_2 - s_1 = C_{vo} \ln \left(\frac{T_2}{T_1}\right) = 0.717 \ln \left(\frac{1350}{1300}\right) =$ **0.02706 kJ/kgK**

From A.7:

case a) $C_v \approx \Delta u / \Delta T = 36/50 = 0.72$ kJ/kg K , see A.5

case b) $C_v \approx \Delta u / \Delta T = 45.2/50 = 0.904$ kJ/kg K (25 % higher)
so result should have been 0.0341 kJ/kgK

8.81

A piston/cylinder setup contains air at 100 kPa, 400 K which is compressed to a final pressure of 1000 kPa. Consider two different processes (i) a reversible adiabatic process and (ii) a reversible isothermal process. Show both processes in P-v and a T-s diagram. Find the final temperature and the specific work for both processes.

Solution:

C.V. Air, control mass of unknown size and mass.

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.14: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen}$

Process: Reversible ${}_1s_{2\,gen} = 0$

i) $dq = 0$ so ${}_1q_2 = 0$

ii) $T = C$ so $\int dq/T = {}_1q_2/T$

i) For this process the entropy equation reduces to:

$s_2 - s_1 = 0 + 0$ so we have constant s, an isentropic process.

The relation for an ideal gas, constant s and k becomes Eq.8.32

$$T_2 = T_1(\,P_2 / P_1)^{\frac{k-1}{k}} = 400\left(\frac{1000}{100}\right)^{\frac{0.4}{1.4}} = 400 \times 10^{0.28575} = \mathbf{772\ K}$$

From the energy equation we get the work term

$${}_1w_2 = u_1 - u_2 = C_v(T_1 - T_2) = 0.717(400 - 772) = \mathbf{-266.7\ kJ/kg}$$

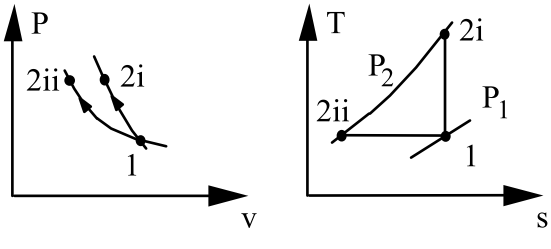
ii) For this process $T_2 = T_1$ so since ideal gas we get

$$u_2 = u_1 \text{ also } s^o_{T2} = s^o_{T1} \quad => \quad \text{Energy Eq.: } {}_1w_2 = {}_1q_2$$

Now from the entropy equation we solve for ${}_1q_2$

$${}_1w_2 = {}_1q_2 = T(s_2 - s_1) = T[s^o_{T2} - s^o_{T1} - R \ln \frac{P_2}{P_1}] = -RT \ln \frac{P_2}{P_1}$$

$$= -\,0.287 \times 400 \ln 10 = \mathbf{-264\ kJ/kg}$$



8.82

Prove that the two relations for changes in s, Eqs.8.16 and 8.17 are equivalent once we assume constant specific heat. Hint: recall the relation for specific heat in Eq. 5.27.

From Eq. 5.27: $C_{po} = C_{vo} + R$

Start with Eq.8.16: $s_2 - s_1 = C_{po} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

Now substitute Eq.5.27 to get:

$$
\begin{aligned}
s_2 - s_1 &= (C_{vo} + R) \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\
&= C_{vo} \ln \frac{T_2}{T_1} - R \left[\ln \frac{P_2}{P_1} - \ln \frac{T_2}{T_1} \right] \\
&= C_{vo} \ln \frac{T_2}{T_1} - R \ln \left(\frac{P_2}{P_1} \frac{T_1}{T_2} \right)
\end{aligned}
$$

Use the ideal gas law Pv = RT for both states to get the ratio

$$\frac{P_2 v_2}{RT_2} = \frac{P_1 v_1}{RT_1} \quad \Rightarrow \quad \frac{P_2}{P_1} \frac{T_1}{T_2} = \frac{v_1}{v_2}$$

so then we get to Eq. 8.17 as

$$
\begin{aligned}
s_2 - s_1 &= C_{vo} \ln \frac{T_2}{T_1} - R \ln \frac{v_1}{v_2} \\
&= C_{vo} \ln \frac{T_2}{T_1} + \ln \frac{v_2}{v_1}
\end{aligned}
$$

8.83

Assume an ideal gas with constant specific heats. Show the functions T(s, P = C) and T(s, v = C) mathematically and sketch them in a T-s diagram.

From Eq 8.17 when $P_2 = P_1$

$$s_2 - s_1 = C_{po} \ln \frac{T_2}{T_1}$$

so we can solve for T_2 as a function of s_2 as

$$T_2 = T_1 \exp\left[\frac{s_2 - s_1}{C_{po}} \right]$$

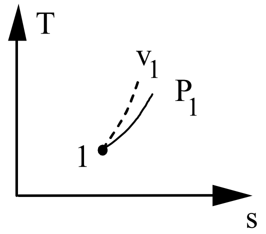
and it is an exponential function. For the other function use Eq.8.16 with $v_2 = v_1$

$$s_2 - s_1 = C_{vo} \ln \frac{T_2}{T_1}$$

so we can solve for T_2 as a function of s_2 as

$$T_2 = T_1 \exp\left[\frac{s_2 - s_1}{C_{vo}} \right]$$

which is also an exponential function. Since C_{po} is larger than C_{vo} the slope of the first function is lower than the slope of the second function.



8.84

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific entropy, using a) the steam tables, b) the ideal gas water Table A.8, and c) the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3 Superheated vapor $s_1 = 6.9299$ kJ/kgK

State 2: Table B.1.3 $s_2 = 9.7059$ kJ/kgK

$$s_2 - s_1 = 9.7059 - 6.9299 = \mathbf{2.776\ kJ/kgK}$$

b)

Table A.8 at 423.15 K: $s^o_{T1} = 11.13891$ kJ/kgK

Table A.8 at 1473.15 K: $s^o_{T2} = 13.86383$ kJ/kgK

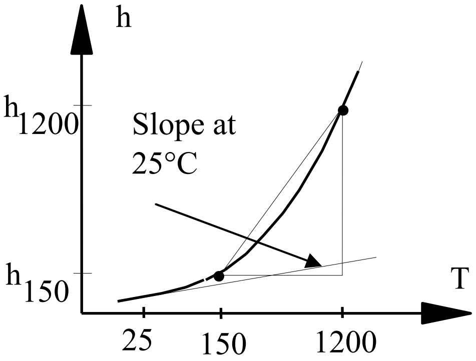
$$s_2 - s_1 = s^o_{T2} - s^o_{T1} - R \ln \frac{P_2}{P_1} = s^o_{T2} - s^o_{T1}$$

$$s^o_{T2} - s^o_{T1} = 13.86383 - 11.13891 = \mathbf{2.72492\ kJ/kgK}$$

c) Table A.5: $C_{po} = 1.872$ kJ/kgK

$$s_2 - s_1 \approx C_{po} \ln \left(\frac{T_2}{T_1}\right) = 1.872 \ln \left(\frac{1473.15}{423.15}\right) = \mathbf{2.3352\ kJ/kgK}$$

Notice how the average slope from 150°C to 1200°C is higher than the one at 25°C (= C_{po})



8.85

R-410a at 400 kPa is brought from 20°C to 120°C in a constant pressure process. Evaluate the change in specific entropy using Table B.4 and using ideal gas with $C_p = 0.81$ kJ/kgK.

Table B.4.2 $s_1 = 1.2108$ kJ/kg-K, $s_2 = 1.4788$ kJ/kg-K

$s_2 - s_1 = 1.4788 - 1.2108 =$ **0.268 kJ/kg-K**

Eq. 8.17: $s_2 - s_1 \approx C_{po} \ln \frac{T_2}{T_1} = 0.81 \ln \frac{393.15}{293.15} =$ **0.238 kJ/kg-K**

Two explanations for the difference are as the average temperature is higher than 25°C we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate $Z = Pv/RT = 0.94$).

8.86

R-410a at 300 kPa, 20^{o}C is brought to 500 kPa, 200^{o}C in a constant volume process. Evaluate the change in specific entropy using Table B.4 and using ideal gas with C_v = 0.695 kJ/kgK.

Table B.4.2 s_1 = 1.2485 kJ/kg-K, v_1 = 0.10720 m^3/kg

s_2 = 1.6413 kJ/kg-K, v_2 = 0.10714 m^3/kg (very close to v_1)

$s_2 - s_1$ = 1.6413 – 1.2485 = **0.3928 kJ/kg-K**

Eq. 8.16: $$s_2 - s_1 \approx C_{vo} \ln \frac{T_2}{T_1} = 0.695 \ln \frac{473.15}{293.15} = \mathbf{0.333\ kJ/kg\text{-}K}$$

Two explanations for the difference are as the average temperature is higher than 25^{o}C we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate Z = Pv/RT = 0.96).

8.87

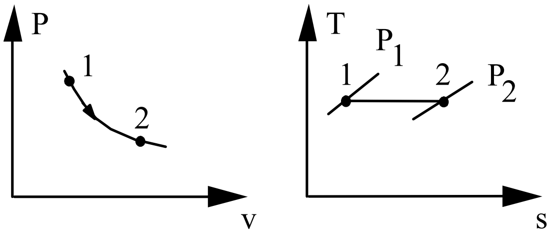
A mass of 1 kg of air contained in a cylinder at 1.5 MPa, 1000 K, expands in a reversible isothermal process to a volume 10 times larger. Calculate the heat transfer during the process and the change of entropy of the air.

Solution:

C.V. Air, control mass.

Energy Eq. 5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0$

Process: T = constant so with ideal gas => $u_2 = u_1$



From the process equation and ideal gas law

$$PV = mRT = \text{constant}$$

we can calculate the work term as in Eq.4.5

$${}_1Q_2 = {}_1W_2 = \int PdV = P_1V_1 \ln (V_2/V_1) = mRT_1 \ln (V_2/V_1)$$

$$= 1 \times 0.287 \times 1000 \ln (10) = \mathbf{660.84\ kJ}$$

The change of entropy from Eq.8.3 is

$$\Delta S_{air} = m(s_2 - s_1) = {}_1Q_2/T = 660.84/1000 = \mathbf{0.661\ kJ/K}$$

If instead we use Eq.8.17 we would get

$$\Delta S_{air} = m(s_2 - s_1) = m(C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1})$$

$$= 1 [0 + 0.287 \ln(10)] = 0.661 \text{ kJ/K}$$

consistent with the above result.

8.88

Consider a small air pistol with a cylinder volume of 1 cm^3 at 250 kPa, 27°C. The bullet acts as a piston initially held by a trigger. The bullet is released so the air expands in an adiabatic process. If the pressure should be 100 kPa as the bullet leaves the cylinder find the final volume and the work done by the air.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.5.11: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = 0$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So we realize that one piece of information is needed to get state 2.

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\,gen} = 0$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process giving $s_2 = s_1$. From Eq.8.23

$$T_2 = T_1 (P_2 / P_1)^{\frac{k-1}{k}} = 300 \left(\frac{100}{250}\right)^{\frac{0.4}{1.4}} = 300 \times 0.4^{0.28575} = 230.9 \text{ K}$$

The ideal gas law PV = mRT at both states leads to

$$V_2 = V_1\, P_1\, T_2/P_2\, T_1 = 1 \times 250 \times 230.9/100 \times 300 = \mathbf{1.92\ cm^3}$$

The work term is from Eq.8.29 or Eq.4.4 with polytropic exponent n = k

$${}_1W_2 = \frac{1}{1-k}(P_2V_2 - P_1V_1) = \frac{1}{1-1.4}(100 \times 1.92 - 250 \times 1) \times 10^{-6}$$

$$= \mathbf{0.145\ J}$$

8.89

Oxygen gas in a piston cylinder at 300 K, 100 kPa with volume 0.1 m^3 is compressed in a reversible adiabatic process to a final temperature of 700 K. Find the final pressure and volume using Table A.5.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.5.11: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\text{ gen}} = 0$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\text{ gen}} = 0$

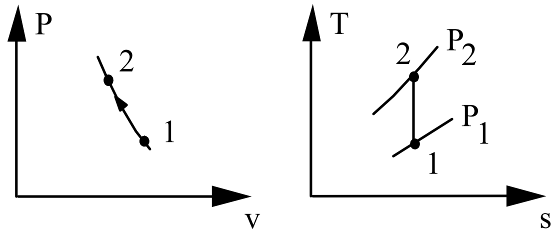
Properties: Table A.5: $k = 1.393$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.8.23

$$P_2 = P_1(T_2 / T_1)^{\frac{k}{k-1}} = \mathbf{2015\ kPa}$$

Using the ideal gas law to eliminate P from this equation leads to Eq.8.24

$$V_2 = V_1(T_2 / T_1)^{\frac{1}{1-k}} = 0.1 \times \left(\frac{700}{300}\right)^{\frac{1}{1-1.393}} = \mathbf{0.0116\ m^3}$$



8.90

Oxygen gas in a piston cylinder at 300 K, 100 kPa with volume 0.1 m^3 is compressed in a reversible adiabatic process to a final temperature of 700 K. Find the final pressure and volume using constant heat capacity from Table A.8.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.5.11: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = 0$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\,gen} = 0$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.8.19

$$s^o_{T2} - s^o_{T1} = R \ln \frac{P_2}{P_1}$$

Properties: Table A.8: $s^o_{T1} = 6.4168$, $s^o_{T2} = 7.2336$ kJ/kg K

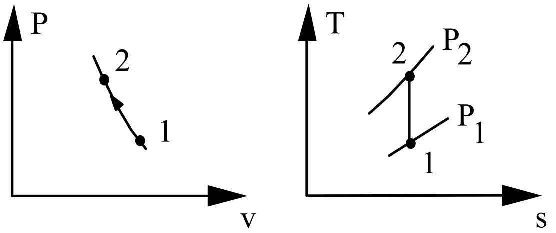
$$\frac{P_2}{P_1} = \exp\,[(s^o_{T2} - s^o_{T1})/R] = \exp\left(\frac{7.2336 - 6.4168}{0.2598}\right) = 23.1955$$

$$P_2 = 100 \times 23.1955 = \mathbf{2320\ kPa}$$

Ideal gas law: $P_1V_1 = mRT_1$ and $P_2V_2 = mRT_2$

Take the ratio of these so mR drops out to give

$$V_2 = V_1 \times (T_2 / T_1) \times (P_1 / P_2) = 0.1 \times \left(\frac{700}{300}\right) \times \left(\frac{100}{2320}\right) = \mathbf{0.01\ m^3}$$



8.91

A rigid tank contains 1 kg methane at 500 K, 1500 kPa. It is now cooled down to 300 K. Find the heat transfer and the change in entropy using ideal gas.

Ideal gas, constant volume so there is no work.

Energy Eq. 5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Use specific heat from Table A.5

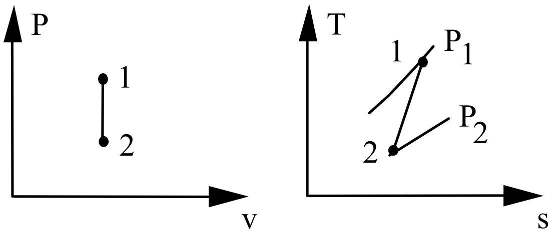
$$u_2 - u_1 = C_v (T_2 - T_1) = 1.736\ (300 - 500) = -347.2 \text{ kJ/kg}$$

$${}_1Q_2 = m(u_2 - u_1) = 1\ (-347.2) = \mathbf{-347.2\ kJ}$$

The change in s for an ideal gas, Eqs.8.16-17 and $v_2 = v_1$ gives

$$m(s_2 - s_1) = m\ [\ C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}\ ] = m\ C_{vo} \ln \frac{T_2}{T_1}$$

$$= 1 \times 1.736 \ln \frac{300}{500} = \mathbf{-\ 0.8868\ kJ/K}$$



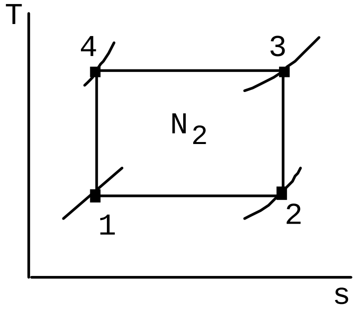
8.92

Consider a Carnot-cycle heat pump having 1 kg of nitrogen gas in a cylinder/piston arrangement. This heat pump operates between reservoirs at 300 K and 400 K. At the beginning of the low-temperature heat addition, the pressure is 1 MPa. During this process the volume triples. Analyze each of the four processes in the cycle and determine

a. The pressure, volume, and temperature at each point

b. The work and heat transfer for each process

Solution:



$T_1 = T_2 = \mathbf{300\ K}$, $T_3 = T_4 = \mathbf{400\ K}$,

$P_1 = \mathbf{1\ MPa}$, $V_2 = 3 \times V_1$

a) $P_2V_2 = P_1V_1 \Rightarrow P_2 = P_1/3 = \mathbf{0.3333\ MPa}$

$$V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.2968 \times 300}{1000} = \mathbf{0.08904\ m^3}$$

$V_2 = \mathbf{0.26712\ m^3}$

$$P_3 = P_2(T_3/T_2)^{\frac{k}{k-1}} = 0.3333\left(\frac{400}{300}\right)^{3.5} = \mathbf{0.9123\ MPa}$$

$$V_3 = V_2 \times \frac{P_2}{P_3} \times \frac{T_3}{T_2} = 0.26712 \times \frac{0.3333}{0.9123} \times \frac{400}{300} = \mathbf{0.1302\ m^3}$$

$$P_4 = P_1(T_3/T_1)^{\frac{k}{k-1}} = 1\left(\frac{400}{300}\right)^{3.5} = \mathbf{2.73707\ MPa}$$

$$V_4 = V_1 \times \frac{P_1}{P_4} \times \frac{T_4}{T_1} = 0.08904 \times \frac{1}{2.737} \times \frac{400}{300} = \mathbf{0.04337\ m^3}$$

b) $\;_1W_2 = {}_1Q_2 = mRT_1 \ln(P_1/P_2)$

$= 1 \times 0.2968 \times 300 \ln(1/0.333) = \mathbf{97.82\ kJ}$

$_3W_4 = {}_3Q_4 = mRT_3 \ln(P_3/P_4)$

$= 1 \times 0.2968 \times 400 \ln(0.9123/2.737) = \mathbf{-130.43\ kJ}$

$_2W_3 = -mC_{v0}(T_3 - T_2) = -1 \times 0.7448(400 - 300) = \mathbf{-74.48\ kJ}$

$_4W_1 = -mC_{v0}(T_1 - T_4) = -1 \times 0.7448(300 - 400) = \mathbf{+74.48\ kJ}$

$_2Q_3 = \mathbf{0}$, $_4Q_1 = \mathbf{0}$

8.93

A hydrogen gas in a piston/cylinder assembly at 280 K, 100 kPa with a volume of 0.1 m^3 is now compressed to a volume of 0.01 m^3 in a reversible adiabatic process. What is the new temperature and how much work is required?

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen} = 0 + 0 = 0$

For an isentropic process and constant heat capacity we have Eq.8.24

From Eq.8.24: $T_2 = T_1(v_2 / v_1)^{1-k} = 280\ (0.01/0.1)^{1-1.409} = \mathbf{718\ K}$

From Eq.8.39: ${}_1w_2 = \frac{R}{1-k}(T - T) = \frac{4.1243}{1-1.409}(718 - 280) = \mathbf{-4417\ kJ/kg}$

8.94

A handheld pump for a bicycle has a volume of 25 cm^3 when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so that an air pressure of 300 kPa is obtained. The outside atmosphere is at P_0, T_0. Consider two cases: (1) it is done quickly (~1 s), and (2) it is done very slowly (~1 h).

a. State assumptions about the process for each case.

b. Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

State 1: P_0, T_0 State 2: 300 kPa, ?

One piece of information must resolve the ? for a state 2 property.

Case I) Quickly means no time for heat transfer

Q = 0, so a reversible adiabatic compression.

$$u_2 - u_1 = -{}_1w_2 \;;\quad s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = 0$$

With constant s and constant heat capacity we use Eq.8.23

$$T_2 = T_1(\,P_2 / P_1)^{\frac{k-1}{k}} = 298\left(\frac{300}{101.325}\right)^{\frac{0.4}{1.4}} = \mathbf{405.3\ K}$$

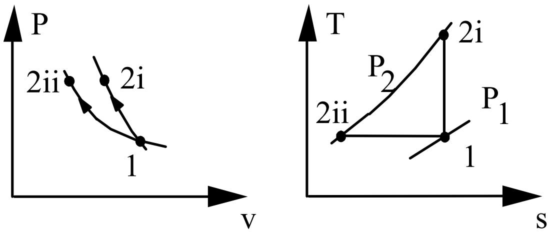
Use ideal gas law PV = mRT at both states so ratio gives

$$\Rightarrow \quad V_2 = P_1V_1T_2/T_1P_2 = \mathbf{11.48\ cm^3}$$

Case II) Slowly, time for heat transfer so T = constant = T_0.

The process is then a reversible isothermal compression.

$$T_2 = T_0 = \mathbf{298\ K} \qquad \Rightarrow \qquad V_2 = V_1P_1/P_2 = \mathbf{8.44\ cm^3}$$



8.95

An insulated piston/cylinder setup contains carbon dioxide gas at 400 kPa, 300 K which is then compressed to 3 MPa in a reversible adiabatic process. Calculate the final temperature and the specific work using a) ideal gas tables A.8 and b) using constant specific heats A.5.

Solution:

C.V. CO_2, a control mass undergoing a reversible, adiabatic process.

Energy Eq.5.11: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = 0$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\,gen} = 0$

State 1: (300 K, 400 kPa) State 2: (3000 kPa, ?)

With two terms zero in the entropy equation we have a zero for the entropy change. So this is a constant s (isentropic) expansion process, $s_2 = s_1$.

a) Table A.8 for CO_2 and Eq.8.19

$$s_2 - s_1 = 0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1)$$

$$s^o_{T2} = s^o_{T1} + R \ln \frac{P_2}{P_1} = 4.8631 + 0.1889 \ln \frac{3000}{400} = 5.2437 \text{ kJ/kg-K}$$

Now interpolate in A.8 to find T_2

$$T_2 = 450 + 50\,(5.2437 - 5.2325)/(5.3375 - 5.2325) = \mathbf{455.3\ K}$$

$${}_1w_2 = -(u_2 - u_1) = -(271 - 157.7) = \mathbf{-113.3\ kJ/kg}$$

b) Table A.5: $k = 1.289$, $C_{Vo} = 0.653$ kJ/kg K and now Eq.8.23

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \left(\frac{3000}{400}\right)^{0.224} = \mathbf{471.1\ K}$$

$${}_1w_2 = -C_{Vo}(T_2 - T_1) = -0.653\,(471.1 - 300) = \mathbf{-111.7\ kJ/kg}$$

8.96

Extend the previous problem to also solve using a constant specific heat at an average temperature from A.6 and resolve using Table B.3.

C.V. CO_2, a control mass undergoing a reversible, adiabatic process.

Energy Eq.5.11: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen} = \emptyset$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\ gen} = 0$

State 1: (300 K, 400 kPa) State 2: (3000 kPa, ?)

With two terms zero in the entropy equation we have a zero for the entropy change. So this is a constant s (isentropic) expansion process, $s_2 = s_1$.

Find a heat capacity at an average temperature from Table A.6.

Estimate $T_2 \sim 500$ K giving $T_{AVE} \sim 400$ K => $\theta = 0.4$

$$C_{Po} = 0.45 + 1.67 \times 0.4 - 1.27 \times 0.4^2 + 0.39 \times 0.4^3 = 0.9398 \text{ kJ/kg K}$$

$$C_{Vo} = C_{Po} - R = 0.9398 - 0.1889 = 0.7509, \quad k = C_{Po}/C_{Vo} = 1.2516$$

Eq.8.32:
$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \left(\frac{3000}{400}\right)^{0.201} = \mathbf{449.8\ K}$$

$$ {}_1w_2 = -C_{Vo}(T_2 - T_1) = -0.7509\ (449.8 - 300) = \mathbf{-112.5\ kJ/kg}$$

From Table B.3.2: $s_1 = 1.8102$ kJ/kg-K, $u_1 = 336.2$ kJ/kg

$T_2 = 186.7^{\circ}C =$ **459.8 K**, $u_2 = 446.9$ kJ/kg

$$ {}_1w_2 = -(u_2 - u_1) = -(446.9 - 336.2) = \mathbf{-110.7\ kJ/kg}$$

8.97

A piston/cylinder, shown in Fig. P8.97, contains air at 1380 K, 15 MPa, with $V_1 = 10\ \text{cm}^3$, $A_{cyl} = 5\ \text{cm}^2$. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, Q = 0.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -\,{}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = 0 + {}_1S_{2\ gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?, assume reversible process.

$${}_1S_{2\ gen} = 0 \quad => \quad s_2 - s_1 = 0$$

State 1: Table A.7: $u_1 = 1095.2$ kJ/kg, $s^o_{T1} = 8.5115$ kJ/kg K

$$m = P_1V_1/RT_1 = \frac{15000 \times 10{\times}10^{-6}}{0.287 \times 1380} = 0.000379 \text{ kg}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$ so from Eq.8.19

$$s^o_{T2} = s^o_{T1} + R \ln \frac{P_2}{P_1} = 8.5115 + 0.287 \ln(\frac{200}{15000}) = 7.2724 \text{ kJ/kg K}$$

Now interpolate in Table A.7 to get T_2

$$T_2 = 440 + 20\ (7.2724 - 7.25607)/(7.30142 - 7.25607) = 447.2 \text{ K}$$

$$u_2 = 315.64 + (330.31 - 315.64)\ 0.36 = 320.92 \text{ kJ/kg}$$

$$V_2 = V_1 \frac{T_2\,P_1}{T_1P_2} = \frac{10 \times 447.2 \times 15000}{1380 \times 200} = 243 \text{ cm}^3$$

$\Rightarrow$ $L_2 = V_2 / A_{cyl} = 243/5 =$ **48.6 cm**

${}_1w_2 = u_1 - u_2 = 774.3$ kJ/kg, ${}_1W_2 = m_1w_2 =$ **0.2935 kJ**

8.98

Argon in a light bulb is at 90 kPa and 20°C when it is turned on and electric input now heats it to 60°C. Find the entropy increase of the argon gas.

Solution:

C.V. Argon gas. Neglect any heat transfer.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1W_{2\text{ electrical in}}$

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\text{ gen}} = {}_1s_{2\text{ gen}}$

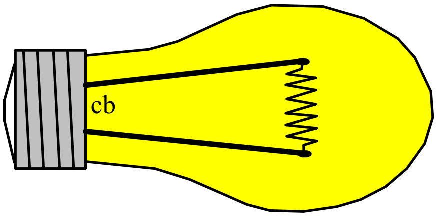
Process: v = constant and ideal gas => $P_2/P_1 = T_2/T_1$

Evaluate changes in s from Eq.8.16 or 8.17

$$s_2 - s_1 = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) \qquad \text{Eq.8.16}$$

$$= C_p \ln(T_2/T_1) - R \ln(T_2/T_1) = C_v \ln(T_2/T_1) \qquad \text{Eq.8.17}$$

$$= 0.312 \text{ kJ/kg-K} \times \ln\left[\frac{60 + 273}{20 + 273}\right] = \mathbf{0.04\ kJ/kg\ K}$$



Since there was no heat transfer but work input all the change in s is generated by the process (irreversible conversion of W to internal energy)

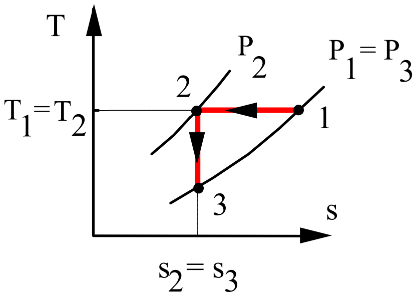
8.99

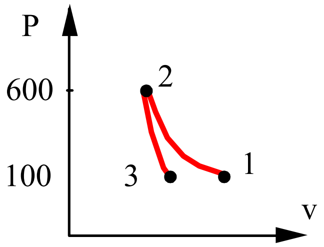
We wish to obtain a supply of cold helium gas by applying the following technique. Helium contained in a cylinder at ambient conditions, 100 kPa, 20°C, is compressed in a reversible isothermal process to 600 kPa, after which the gas is expanded back to 100 kPa in a reversible adiabatic process.

a. Show the process on a *T–s* diagram.
b. Calculate the final temperature and the net work per kilogram of helium.

Solution:

a)





b) The adiabatic reversible expansion gives constant s from the entropy equation Eq.8.37. With ideal gas and constant specific heat this gives relation in Eq.8.23

$$T_3 = T_2(P_3/P_2)^{\frac{k-1}{k}} = 293.15\ (100/600)^{0.4} = \mathbf{143.15\ K}$$

The net work is summed up over the two processes. The isothermal process has work as Eq.8.31

$$_1w_2 = -RT_1 \ln(P_2/P_1) = -2.0771 \times 293.15 \times \ln(600/100) = -1091.0\ \text{kJ/kg}$$

The adiabatic process has a work term from energy equation with no q

$$_2w_3 = C_{Vo}(T_2 - T_3) = 3.116\ (293.15 - 143.15) = +467.4\ \text{kJ/kg}$$

The net work is the sum

$$w_{NET} = -1091.0 + 467.4 = \mathbf{-623.6\ kJ/kg}$$

8.100

A 1-m^3 insulated, rigid tank contains air at 800 kPa, 25°C. A valve on the tank is opened, and the pressure inside quickly drops to 150 kPa, at which point the valve is closed. Assuming that the air remaining inside has undergone a reversible adiabatic expansion, calculate the mass withdrawn during the process.

Solution:

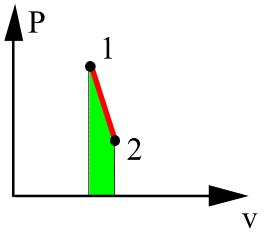
C.V.: Air remaining inside tank, m_2.

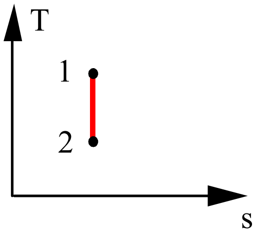
Cont.Eq.: $m_2 = m$;

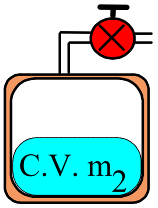
Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: adiabatic ${}_1Q_2 = 0$ and reversible ${}_1S_{2\ gen} = 0$







Entropy eq. then gives $s_2 = s_1$ and ideal gas gives the relation in Eq.8.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 298.2(150/800)^{0.286} = \mathbf{184.8\ K}$$

$m_1 = P_1V/RT_1 = (800\ \text{kPa} \times 1\ \text{m}^3)/(0.287\ \text{kJ/kgK} \times 298.2\ \text{K}) = 9.35\ \text{kg}$

$m_2 = P_2V/RT_2 = (150\ \text{kPa} \times 1\ \text{m}^3)/(0.287\ \text{kJ/kgK} \times 184.8\ \text{K}) = 2.83\ \text{kg}$

$m_e = m_1 - m_2 = \mathbf{6.52\ kg}$

8.101

Two rigid tanks, shown in Fig. P8.101, each contain 10 kg N_2 gas at 1000 K, 500 kPa. They are now thermally connected to a reversible heat pump, which heats one and cools the other with no heat transfer to the surroundings. When one tank is heated to 1500 K the process stops. Find the final (*P*, *T*) in both tanks and the work input to the heat pump, assuming constant heat capacities.

Solution:

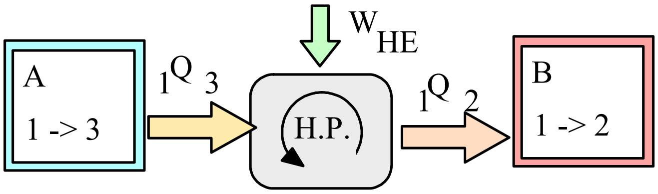
Control volume of hot tank B,

Process = constant volume & mass so no work

Energy equation Eq.5.11 and specific heat in Eq.5.20 gives

$$U_2 - U_1 \cong mC_v(T_2 - T_1) = {}_1Q_2 = 10 \times 0.7448 \times 500 = 3724 \text{ kJ}$$

$$P_2 = P_1T_2/T_1 = 1.5(P_1) = \mathbf{750\ kPa}$$



State: 1 = initial,
2 = final hot
3 = final cold

To fix temperature in cold tank, C.V.: total

For this CV only W_{HP} cross the control surface no heat transfer. The entropy equation Eq.8.37 for a reversible process becomes

$$(S_2 - S_1)_{tot} = 0 = m_{hot}\,(s_2 - s_1) + m_{cold}\,(s_3 - s_1)$$

Use specific heats to evaluate the changes in s from Eq.8.25 and division by m

$$C_{p,hot} \ln(T_2 / T_1) - R \ln(P_2 / P_1) + C_{p,cold} \ln(T_3 / T_1) - R \ln(P_3 / P_1) = 0$$

$$P_3 = P_1T_3/T_1 \quad \text{and} \quad P_2 = P_1T_2/T_1$$

Now everything is in terms of T and $C_p = C_v + R$, so

$$C_{v,hot} \ln(T_2/T_1) + C_{v,cold} \ln(T_3/T_1) = 0$$

same C_v: $T_3 = T_1(T_1/T_2) = \mathbf{667\ K}$, $P_3 = \mathbf{333\ kPa}$

$Q_{cold} = -\,{}_1Q_3 = m\,C_v\,(T_3 - T_1) = -2480$ kJ,

$W_{HP} = {}_1Q_2 + Q_{cold} = {}_1Q_2 - {}_1Q_3 = \mathbf{1244\ kJ}$

8.102

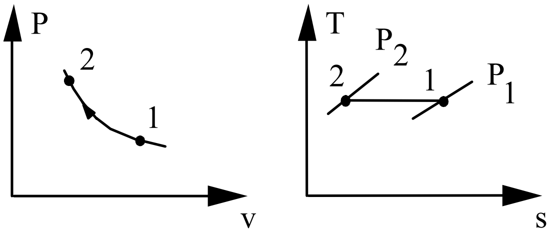
A hydrogen gas in a piston/cylinder assembly at 300 K, 100 kPa with a volume of $0.1\ m^3$ is now slowly compressed to a volume of $0.01\ m^3$ while cooling it in a reversible isothermal process. What is the final pressure, the heat transfer and the change in entropy?

Solution:

C.V. Hydrogen, control mass.

Energy Eq. 5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: T = constant so with ideal gas => $u_2 = u_1$



From the process equation ($T_2 = T_1$) and ideal gas law we get

$$P_2 = P_1\ (V_1/V_2) = 10\ P_1 = \mathbf{1000\ kPa}$$

we can calculate the work term as in Eq.4.5

$${}_1Q_2 = {}_1W_2 = \int PdV = P_1V_1 \ln (V_2/V_1)$$

$$= 100 \times 0.1 \times \ln (1/10) = \mathbf{-23.0\ kJ}$$

The change of entropy from the entropy equation Eq.8.3 is

$$m(s_2 - s_1) = {}_1Q_2/T_1 = \frac{-23}{300} = \mathbf{-0.07667\ kJ/K}$$

If instead we use Eq.8.17 we would get

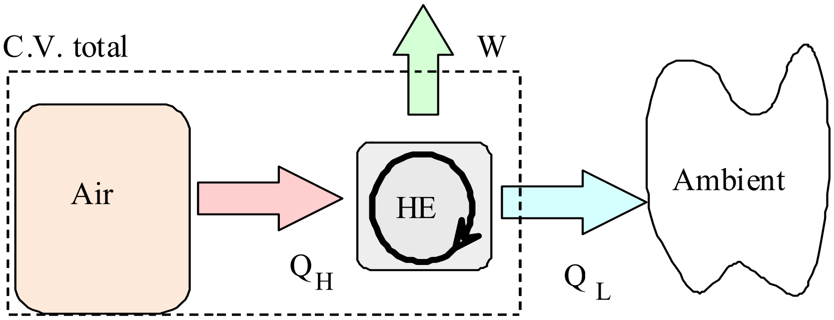
$$\Delta S = m(s_2 - s_1) = m(C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}) = m\ R \ln \frac{v_2}{v_1}$$

$$= (P_1V_1/T_1) \ln (\frac{v_2}{v_1}) = {}_1Q_2/T_1$$

consistent with the above result.

8.103

A rigid tank contains 4 kg air at 200°C, 4 MPa which acts as the hot energy reservoir for a heat engine with its cold side at 20°C shown in Fig. P.8.103. Heat transfer to the heat engine cools the air down in a reversible process to a final 20° C and then stops. Find the final air pressure and the work output of the heat engine.



Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$ (T not constant)

Process: $v = \text{constant} \Rightarrow {}_1W_2 = 0$

State 2: T_2 and $v_2 = v_1$,

$$P_2 = P_1T_2/T_1 = 4000 \times 293.15/473.15 = \mathbf{2478.3\ kPa}$$

From the energy equation

$$Q_H = -{}_1Q_2 = -m(u_2 - u_1) = -m\, C_{vo}\, (T_2 - T_1)$$
$$= -4 \times 0.717\, (293.15 - 473.15) = \mathbf{516.2\ kJ}$$

Take now CV total as the air plus heat engine out to ambient

Entropy Eq.8.3: $m(s_2 - s_1) = -\dfrac{Q_L}{T_{amb}} \Rightarrow$

$$Q_L = -mT_{amb}\,(s_2 - s_1) = -mT_{amb}\, C_{vo} \ln\frac{T_2}{T_1}$$
$$= -\,4 \times 293.15 \times 0.717 \ln(293.15/473.15) = 402.5\ \text{kJ}$$

Now the CV heat engine can give the engine work from the energy equation

Energy H.E.: $W_{HE} = Q_H - Q_L = 516.2 - 402.5 = \mathbf{113.7\ kJ}$

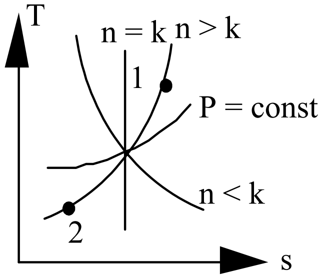
Notice to get ${}_1q_2 = \int T\, ds$ we must know the function T(s) which we do not readily have for this process.

Polytropic processes

8.104

An ideal gas having a constant specific heat undergoes a reversible polytropic expansion with exponent, *n* = 1.4. If the gas is carbon dioxide will the heat transfer for this process be positive, negative, or zero?

Solution:



CO_2: Table A.5 $k = 1.289 < n$

Since $n > k$ and $P_2 < P_1$

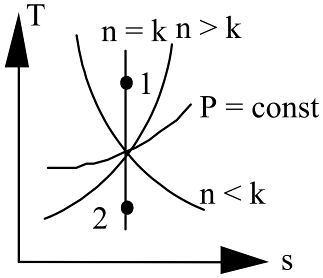
it follows that $s_2 < s_1$ and thus Q flows out.

$\mathbf{{}_1Q_2 < 0}$

8.105

Repeat the previous problem for the gas carbon monoxide, CO.

Solution:



CO: Table A.5 $k = 1.399 = n$

Since $n \approx k$ and $P_2 < P_1$

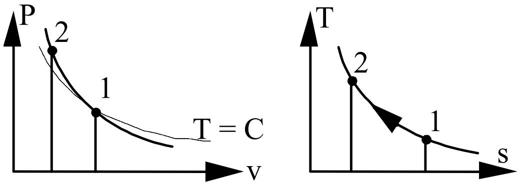
it follows that $s_2 \approx s_1$ and thus adiabatic.

$\mathbf{{}_1Q_2 \approx 0}$

8.106

Neon at 400 kPa, 20°C is brought to 100°C in a polytropic process with $n = 1.4$. Give the sign for the heat transfer and work terms and explain.

Solution:



Neon Table A.5

$k = \gamma = 1.667$ so $n < k$

$C_v = 0.618$, $R = 0.412$

From definition Eq.8.2 $ds = dq/T$ so $dq = T\,ds$

From work term $dw = P\,dv$

From figures: v goes down so work in ($\mathbf{W < 0}$);

s goes down so Q out ($\mathbf{Q < 0}$)

We can also calculate the actual specific work from Eq.8.29 and heat transfer from the energy equation as:

$${}_1w_2 = [R/(1-n)](T_2 - T_1) = -82.39 \text{ kJ/kg}$$

$$u_2 - u_1 = C_v(T_2 - T_1) = 49.432, \quad {}_1q_2 = \Delta u + {}_1w_2 = -32.958$$

$\mathbf{{}_1W_2}$ **Negative** and $\mathbf{{}_1Q_2}$ **Negative**

8.107

A piston/cylinder contains air at 300 K, 100 kPa. It is now compressed in a reversible adiabatic process to a volume 7 times as small. Use constant heat capacity and find the final pressure and temperature, the specific work and specific heat transfer for the process.

Solution:

Expansion ratio: $v_2/ v_1 = 1/7$

Process eq.: Rev. adiabatic and ideal gas gives $Pv^n = C$, with $n = k$

$$P_2 /P_1 = (v_2/v_1)^{-k} = 7^{1.4} = 15.245$$

$$P_2 = P_1 \,(7^{1.4}) = 100 \times 15.245 = \mathbf{1524.5\ kPa}$$

$$T_2 = T_1 \,(v_1/v_2)^{k-1} = 300 \times 7^{0.4} = \mathbf{653.4\ K}$$

$${}_1q_2 = \mathbf{0\ kJ/kg}$$

Polytropic process work term from Eq.8.29

$${}_1w_2 = \frac{R}{1 - k}\,(T_2 - T_1) = \frac{0.287}{-0.4}\,(653.4 - 300) = \mathbf{-253.6\ kJ/kg}$$

Notice: $C_v = R/(k-1)$ so the work term is also the change in u consistent with the energy equation.

8.108

A cylinder/piston contains 1 kg methane gas at 100 kPa, 20°C. The gas is compressed reversibly to a pressure of 800 kPa. Calculate the work required if the process is adiabatic.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3 (37): $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = \int dQ/T$

Process: ${}_1Q_2 = 0$ => $s_2 = s_1$

thus isentropic process s = const and ideal gas gives relation in Eq.8.23

$$T_2 = T_1\,(P_2/P_1)^{\frac{k-1}{k}} = 293.15\left(\frac{800}{100}\right)^{0.230} = 472.9\text{ K}$$

$${}_1W_2 = -mC_{V0}(T_2 - T_1) = -1 \times 1.736\,(472.9 - 293.15) = \mathbf{-312.0\ kJ}$$

8.109

Do the previous problem but assume the process is isothermal.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = \int dQ/T$

Process: T = constant. For ideal gas then $u_2 = u_1$ and $s^{o}_{T2} = s^{o}_{T1}$

Energy eq. gives ${}_1W_2 = {}_1Q_2$ and $\int dQ/T = {}_1Q_2/T$

with the entropy change found from Eq.8.28

$$=> \quad {}_1W_2 = {}_1Q_2 = mT(s_2 - s_1) = -mRT \ln(P_2/P_1)$$

$$= -0.5183 \times 293.2 \ln\left(\frac{800}{100}\right) = \mathbf{-316.0\ kJ}$$

8.110

Do Problem 8.108 and assume the process is polytropic with n = 1.15

Process: Pv^n = constant with n = 1.15 ;

The T-P relation is given in Eq.8.37

$$T_2 = T_1\,(P_2/P_1)^{\frac{n-1}{n}} = 293.2\left(\frac{800}{100}\right)^{0.130} = 384.2\text{ K}$$

and the work term is given by Eq.8.38

$${}_1W_2 = \int mP\,dv = m(P_2v_2 - P_1v_1)/(1 - n) = mR\,(T_2 - T_1)/(1 - n)$$

$$= 1\text{ kg} \times 0.5183\,\frac{\text{kJ}}{\text{kg-K}} \times \frac{384.2 - 293.2}{1 - 1.15}\text{ K} = \mathbf{-314.5\ kJ}$$

8.111

Hot combustion air at 1500 K expands in a polytropic process to a volume 6 times as large with n = 1.5. Find the specific boundary work and the specific heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Reversible work Eq. 8.38: ${}_1w_2 = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$

Process Eq: $Pv^n = C$; $T_2 = T_1\,(v_1/v_2)^{n-1} = 1500\left(\frac{1}{6}\right)^{0.5} = 612.4\text{ K}$

Properties from Table A.7.1: $u_1 = 1205.25$ kJ/kg, $u_2 = 444.6$ kJ/kg

$${}_1w_2 = \frac{0.287}{1 - 1.5}(612.4 - 1500) = \mathbf{509.5\ kJ/kg}$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 444.6 - 1205.25 + 509.5 = \mathbf{-251\ kJ/kg}$$

8.112

A mass of 1 kg of air contained in a cylinder at 1.5 MPa, 1000 K, expands in a reversible adiabatic process to 100 kPa. Calculate the final temperature and the work done during the process, using

a. Constant specific heat, value from Table A.5

b. The ideal gas tables, Table A.7

Solution:

C.V. Air.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: ${}_1Q_2 = 0$, ${}_1S_{2\ gen} = 0$ => $s_2 = s_1$

a) Using constant Cp from Table A.5 gives the power relation Eq.8.23.

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 1000\left(\frac{0.1}{1.5}\right)^{0.286} = \mathbf{460.9\ K}$$

$${}_1W_2 = -(U_2 - U_1) = mC_{Vo}(T_1 - T_2)$$

$$= 1\text{ kg} \times 0.717\text{ kJ/kg-K} \times (1000 - 460.9)\text{ K} = \mathbf{386.5\ kJ}$$

b) Use the standard entropy function that includes variable heat capacity from A.7.1 and Eq.8.19

$$s_2 - s_1 = s^o_{T2} - s^o_{T1} - R\ln\frac{P_2}{P_1} = 0 \quad \Rightarrow \quad s^o_{T2} = s^o_{T1} + R\ln\frac{P_2}{P_1}$$

$$s^o_{T2} = 8.13493 + 0.287\ln(100/1500) = 7.35772\text{ kJ/kg-K}$$

Interpolation gives $T_2 =$ **486 K** and $u_2 = 349.5$ kJ/kg

$${}_1W_2 = m(u_1 - u_2) = 1(759.2 - 349.5) = \mathbf{409.7\ kJ}$$

8.113

Helium in a piston/cylinder at 20°C, 100 kPa is brought to 400 K in a reversible polytropic process with exponent n = 1.25. You may assume helium is an ideal gas with constant specific heat. Find the final pressure and both the specific heat transfer and specific work.

Solution:

C.V. Helium

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = C$ & $Pv = RT$ => $Tv^{n-1} = C$

Table A.5: $C_v =$ 3.116 kJ/kg K, R = 2.0771 kJ/kg K

From the process equation and $T_1 = 293.15$, $T_2 = 400$ K

$$T_1\, v^{n-1} = T_2\, v^{n-1} \quad => \quad v_2 / v_1 = (T_1 / T_2)^{1/n-1} = 0.2885$$

$$P_2 / P_1 = (v_1 / v_2)^n = 4.73 \quad => P_2 = \mathbf{473\ kPa}$$

The work is from Eq.8.29 per unit mass

$${}_1w_2 = \int P\, dv = \int C\, v^{-n}\, dv = [\, C / (1\text{-}n)\,] \times (\, v_2{}^{1-n} - v_1{}^{1-n}\,)$$

$$= \frac{1}{1\text{-}n} (P_2\, v_2 - P_1\, v_1) = \frac{R}{1\text{-}n} (T_2 - T_1) = \mathbf{-887.7\ kJ/kg}$$

The heat transfer follows from the energy equation

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v\, (T_2 - T_1\,) + (-\,887.7) = \mathbf{-554.8\ kJ/kg}$$

8.114

The power stroke in an internal combustion engine can be approximated with a polytropic expansion. Consider air in a cylinder volume of 0.2 L at 7 MPa, 1800 K, shown in Fig. P8.114. It now expands in a reversible polytropic process with exponent, *n* = 1.5, through a volume ratio of 8:1. Show this process on *P–v* and *T–s* diagrams, and calculate the work and heat transfer for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3 (37): $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = \int dQ/T$

Process: $PV^{1.50} = \text{constant}$, $V_2/V_1 = 8$

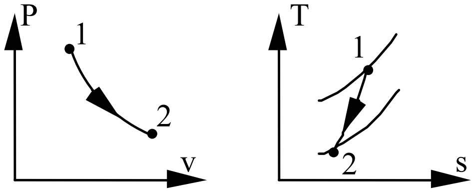
State 1: $P_1 = 7$ MPa, $T_1 = 1800$ K, $V_1 = 0.2$ L

$$m_1 = \frac{P_1V_1}{RT_1} = \frac{7000 \times 0.2 \times 10^{-3}}{0.287 \times 1800} = 2.71{\times}10^{-3} \text{ kg}$$

State 2: ($v = V_2/m$, ?) Must be on process curve so Eq.8.24 gives

$$T_2 = T_1\,(V_1/V_2)^{n-1} = 1800\,(1/8)^{0.5} = 636.4 \text{ K}$$

Table A.7: $u_1 = 1486.331$ kJ/kg and interpolate $u_2 = 463.05$ kJ/kg



Notice:
n = 1.5, k = 1.4

n > k

Work from the process expressed in Eq.8.29

$${}_1W_2 = \int PdV = mR(T_2 - T_1)/(1 - n)$$

$$= \frac{2.71{\times}10^{-3} \times 0.287(636.4 - 1800)}{1 - 1.5} = \mathbf{1.81\ kJ}$$

Heat transfer from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 2.71{\times}10^{-3} \times (463.05 - 1486.331) + 1.81 = \mathbf{-0.963\ kJ}$$

8.115

A cylinder/piston contains saturated vapor R-410a at 10°C; the volume is 10 L. The R-410a is compressed to 2 MPa, 60°C in a reversible (internally) polytropic process. Find the polytropic exponent n and calculate the work and heat transfer.

Solution:

C.V. R-410a of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{amb} + {}_1S_{2\,gen}$

Process: $P_1v_1^n = P_2v_2^n$ Eq.8.36

State 1: (T_1, x_1) Table B.4.1 $P_1 = 1085.7$ kPa, $v_1 = 0.02383$ m^3/kg

$m = V_1/v_1 = 0.01/0.02383 = 0.4196$ kg

State 2: (T_2, P_2) Table B.4.2 $v_2 = 0.01536$ m^3/kg

From process eq. $P_2/P_1 = \dfrac{2000}{1085.7} = \left(\dfrac{0.02383}{0.01536}\right)^n$ => $\mathbf{n = 1.39106}$

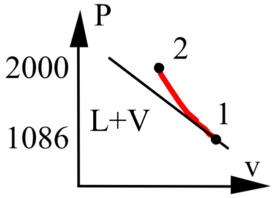
The work is from Eq.8.29

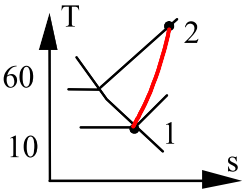
$${}_1W_2 = \int PdV = m\,\frac{P_2v_2 - P_1v_1}{1-n} = 0.4196\,\frac{2000 \times 0.01536 - 1085.7 \times 0.02383}{1 - 1.39106}$$

$= \mathbf{-5.20\ kJ}$

Heat transfer from energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.4196\,(289.9 - 255.9) - 5.20 = \mathbf{9.07\ kJ}$$





Notice:
n = 1.39, k = 1.17

n > k

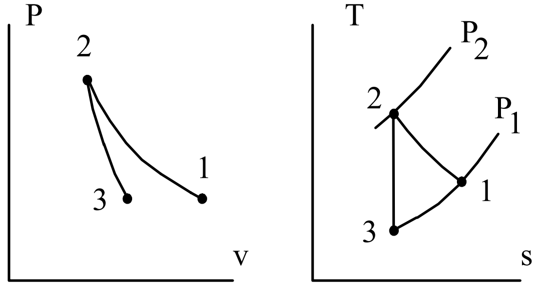
8.116

A cylinder/piston contains air at ambient conditions, 100 kPa and 20°C with a volume of 0.3 m^3. The air is compressed to 800 kPa in a reversible polytropic process with exponent, $n = 1.2$, after which it is expanded back to 100 kPa in a reversible adiabatic process.

a. Show the two processes in *P–v* and *T–s* diagrams.
b. Determine the final temperature and the net work.

Solution:

a)



$$m = P_1V_1/RT_1 = \frac{100 \times 0.3}{0.287 \times 293.2} = 0.3565 \text{ kg}$$

b) The process equation is expressed in Eq.8.37

$$T_2 = T_1(P_2/P_1)^{\frac{n-1}{n}} = 293.2 \left(\frac{800}{100}\right)^{0.167} = 414.9 \text{ K}$$

The work is from Eq.8.38

$$_1w_2 = \int_1^2 Pdv = \frac{P_2v_2-P_1v_1}{1-n} = \frac{R(T_2-T_1)}{1-n} = \frac{0.287(414.9-293.2)}{1-1.20} = -174.6 \text{ kJ/kg}$$

Isentropic relation is from Eq.8.32

$$T_3 = T_2\,(P_3/P_2)^{\frac{k-1}{k}} = 414.9 \left(\frac{100}{800}\right)^{0.286} = \mathbf{228.9\ K}$$

With zero heat transfer the energy equation gives the work

$$_2w_3 = C_{V0}(T_2 - T_3) = 0.717(414.9 - 228.9) = +133.3 \text{ kJ/kg}$$

$$w_{NET} = 0.3565(-174.6 + 133.3) = \mathbf{-14.7\ kJ}$$

Entropy generation

8.117

One kg water at 500°C and 1 kg saturated water vapor both at 200 kPa are mixed in a constant pressure and adiabatic process. Find the final temperature and the entropy generation for the process.

Solution:

Continuity Eq.: $m_2 - m_A - m_B = 0$

Energy Eq.5.11: $m_2u_2 - m_Au_A - m_Bu_B = -{}_1W_2$

Entropy Eq.8.37: $m_2s_2 - m_As_A - m_Bs_B = \int dQ/T + {}_1S_{2\text{ gen}}$

Process: $P = \text{Constant} \;\Rightarrow\; {}_1W_2 = \int PdV = P(V_2 - V_1)$

$Q = 0$

Substitute the work term into the energy equation and rearrange to get

$$m_2u_2 + P_2V_2 = m_2h_2 = m_Au_A + m_Bu_B + PV_1 = m_Ah_A + m_Bh_B$$

where the last rewrite used $PV_1 = PV_A + PV_B$.

State A1: Table B.1.3 $h_A = 3487.03$ kJ/kg, $s_A = 8.5132$ kJ/kg K

State B1: Table B.1.2 $h_B = 2706.63$ kJ/kg, $s_B = 7.1271$ kJ/kg K

Energy equation gives:

$$h_2 = \frac{m_A}{m_2} h_A + \frac{m_B}{m_2} h_B = \frac{1}{2} 3487.03 + \frac{1}{2} 2706.63 = 3096.83 \text{ kJ/kg}$$

State 2: $P_2, h_2 = 3096.83$ kJ/kg $\Rightarrow$ $s_2 = 7.9328$ kJ/kg K; $T_2 = \mathbf{312.2°C}$

With the zero heat transfer we have

$${}_1S_{2\text{ gen}} = m_2s_2 - m_As_A - m_Bs_B$$

$$= 2 \times 7.9328 - 1 \times 8.5132 - 1 \times 7.1271 = \mathbf{0.225\ kJ/K}$$

8.118

A computer chip dissipates 2 kJ of electric work over time and rejects that as heat transfer from its 50°C surface to 25°C air. How much entropy is generated in the chip? How much if any is generated outside the chip?

C.V.1 Chip with surface at 50°C, we assume chip state is constant.

Energy: $U_2 - U_1 = 0 = {}_1Q_2 - {}_1W_2 = W_{\text{electrical in}} - Q_{\text{out 1}}$

Entropy: $S_2 - S_1 = 0 = -\frac{Q_{\text{out 1}}}{T_{\text{surf}}} + {}_1S_{2\text{ gen1}}$

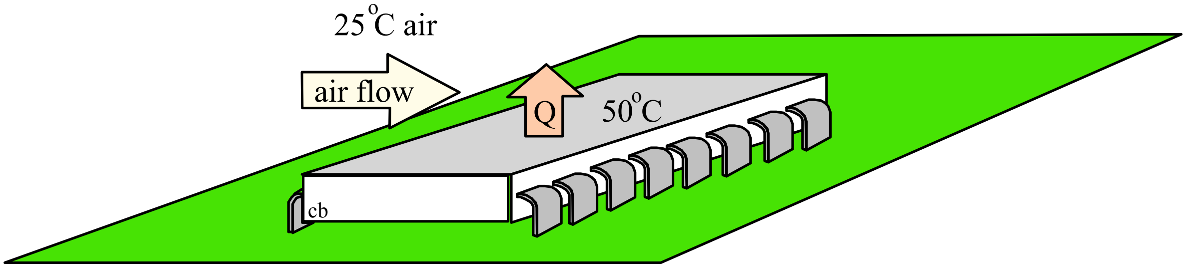
$${}_1S_{2\text{ gen1}} = \frac{Q_{\text{out 1}}}{T_{\text{surf}}} = \frac{W_{\text{electrical in}}}{T_{\text{surf}}} = \frac{2\text{ kJ}}{323.15\text{ K}} = \mathbf{6.19\ J/K}$$

C.V.2 From chip surface at 50°C to air at 25°C, assume constant state.

Energy: $U_2 - U_1 = 0 = {}_1Q_2 - {}_1W_2 = Q_{\text{out 1}} - Q_{\text{out 2}}$

Entropy: $S_2 - S_1 = 0 = \frac{Q_{\text{out1}}}{T_{\text{surf}}} - \frac{Q_{\text{out2}}}{T_{\text{air}}} + {}_1S_{2\text{ gen2}}$

$${}_1S_{2\text{ gen2}} = \frac{Q_{\text{out2}}}{T_{\text{air}}} - \frac{Q_{\text{out1}}}{T_{\text{surf}}} = \frac{2\text{ kJ}}{298.15\text{ K}} - \frac{2\text{ kJ}}{323.15\text{ K}} = \mathbf{0.519\ J/K}$$



8.119

The unrestrained expansion of the reactor water in Problem 5.50 has a final state in the two-phase region. Find the entropy generated in the process.

A water-filled reactor with volume of 1 m^3 is at 20 MPa, 360°C and placed inside a containment room as shown in Fig. P5.50. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 200 kPa.

Solution:

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

Entropy Eq.8.14: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

State 1: (T, P) Table B.1.4 $u_1 = 1702.8$ kJ/kg, $s_1 = 3.877$

Energy equation implies $u_2 = u_1 = 1702.8$ kJ/kg

State 2: $P_2 = 200$ kPa, $u_2 < u_g$ => Two-phase Table B.1.2

$x_2 = (u_2 - u_f)/\ u_{fg} = (1702.8 – 504.47)/2025.02 = 0.59176$

$v_2 = 0.001061 + 0.59176 \times 0.88467 = 0.52457\ m^3/kg$

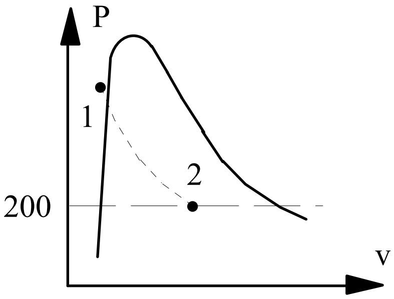
$s_2 = s_f + x_2 s_{fg} = 1.53 + 0.59176 \times 5.597 = 4.8421$ kJ/kg K

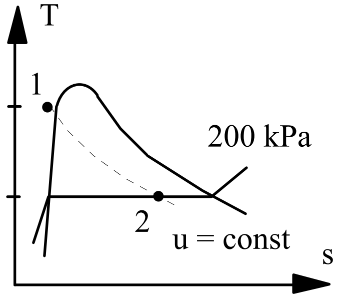
$V_2 = m_2\ v_2 = 548.5 \times 0.52457 = \mathbf{287.7\ m^3}$

From the entropy equation the generation is

${}_1S_{2\ gen} = m(s_2 - s_1) = 548.5\ (4.8421 – 3.877)$

$= \mathbf{529.4\ kJ/K}$





Entropy is generated due to the unrestrained expansion. No work was taken out as the volume goes up.

8.120

A car uses an average power of 25 hp for a one hour round trip. With a thermal efficiency of 35% how much fuel energy was used? What happened to all the energy? What change in entropy took place if we assume ambient at 20°C?

Since it is a round trip, there are no changes in storage of energy for the car after it has cooled down again. All the energy is given out to the ambient in the form of exhaust flow (hot air) and heat transfer from the radiator and underhood air flow.

$$W = \int \dot{W}\,dt = 25 \text{ hp} \times 0.7457 \text{ (kW/hp)} \times 3600 \text{ s} = 67\ 113 \text{ kJ} = \eta\, Q$$

Fuel energy used to deliver the W

$$Q = E / \eta = 67\ 113 / 0.35 = \mathbf{191\ 751\ kJ}$$

$$\Delta S = Q / T = 191\ 751 / 293.15 = \mathbf{654.1\ kJ/K}$$

All the energy (Q) ends up in the ambient at the ambient temperature.

8.121

Ammonia is contained in a rigid sealed tank unknown quality at 0^{o}C. When heated in boiling water to 100^{o}C its pressure reaches 1200 kPa. Find the initial quality, the heat transfer to the ammonia and the total entropy generation.

Solution:

C.V. Ammonia, which is a control mass of constant volume.

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Entropy Eq.8.14: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen}$

State 2: 1200 kPa, 100^{o}C => Table B.2.2

$s_2 = 5.5325$ kJ/kg K, $v_2 = 0.14347\ m^3/kg$, $u_2 = 1485.8$ kJ/kg

State 1: $v_1 = v_2$ => Table B.2.1

$x_1 = (0.14347 - 0.001566)/0.28763 = 0.49336$

$u_1 = 741.28$ kJ/kg, $s_1 = 0.7114 + x_1 \times 4.6195 = 2.9905$ kJ/kg K

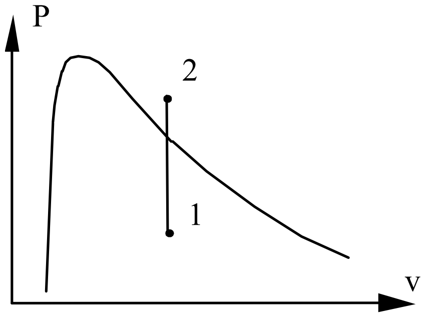
Process: V = constant => ${}_1w_2 = 0$

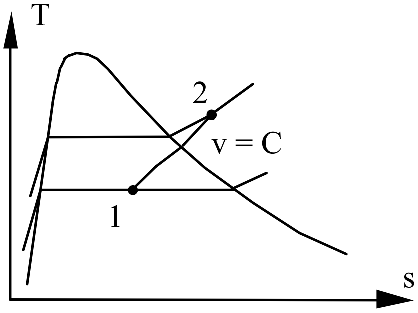
${}_1q_2 = (u_2 - u_1) = 1485.8 - 741.28 = 744.52$ kJ/kg

To get the total entropy generation take the C.V out to the water at 100^{o}C.

$${}_1s_{2\ gen} = s_2 - s_1 - {}_1q_2/T = 5.5325 - 2.9905 - 744.52/373.15$$

$$= \mathbf{0.547\ kJ/kg\ K}$$





8.122

An insulated cylinder/piston contains R-134a at 1 MPa, 50°C, with a volume of 100 L. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 100 kPa. It is claimed that the R-134a does 190 kJ of work against the piston during the process. Is that possible?

Solution:

C.V. R-134a in cylinder. Insulated so assume Q = 0.

State 1: Table B.5.2, $v_1 = 0.02185 \text{ m}^3\text{/kg}$, $u_1 = 409.39 \text{ kJ/kg}$,

$s_1 = 1.7494 \text{ kJ/kg K}$, $m = V_1/v_1 = 0.1/0.02185 = 4.577 \text{ kg}$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 190 \Rightarrow$

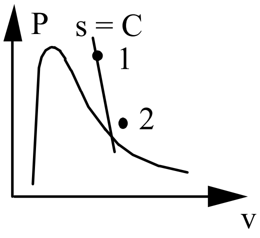
$u_2 = u_1 - {}_1W_2/m = 367.89 \text{ kJ/kg}$

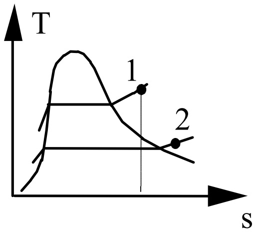
State 2: P_2 , u_2 $\Rightarrow$ Table B.5.2: $T_2 = -19.25°\text{C}$; $s_2 = 1.7689 \text{ kJ/kg K}$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2,gen} = {}_1S_{2,gen}$

${}_1S_{2,gen} = m(s_2 - s_1) = 0.0893 \text{ kJ/K}$

This is possible since ${}_1S_{2,gen} > 0$





8.123

A piece of hot metal should be cooled rapidly (quenched) to 25°C, which requires removal of 1000 kJ from the metal. The cold space that absorbs the energy could be one of three possibilities: (1) Submerge the metal into a bath of liquid water and ice, thus melting the ice. (2) Let saturated liquid R-410a at –20°C absorb the energy so that it becomes saturated vapor. (3) Absorb the energy by vaporizing liquid nitrogen at 101.3 kPa pressure.

a. Calculate the change of entropy of the cooling media for each of the three cases.

b. Discuss the significance of the results.

Solution:

a) Melting or boiling at const P & T

$$ {}_1Q_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1) $$

1) Ice melting at 0°C , Table B.1.5: $m = {}_1Q_2 / h_{ig} = \frac{1000}{333.41} = 2.9993$ kg

$$ \Delta S_{H_2O} = ms_{ig} = 2.9993(1.221) = \mathbf{3.662\ kJ/K} $$

2) R-410a boiling at -20°C, Table B.4.1: $m = {}_1Q_2 / h_{fg} = \frac{1000}{243.65} = 4.104$ kg

$$ \Delta S_{R\text{-}410a} = ms_{fg} = 4.104\ (0.9625) = \mathbf{3.950\ kJ/K} $$

3) N_2 boiling at 101.3 kPa, Table B.6.1: $m = {}_1Q_2 / h_{fg} = \frac{1000}{198.842} = 5.029$ kg

$$ \Delta S_{N_2} = ms_{fg} = 5.029(2.5708) = \mathbf{12.929\ kJ/K} $$

b) The larger the $\Delta(1/T)$ through which the Q is transferred, the larger the ΔS.
For all cases we could also just have done it as ${}_1Q_2/T$.

8.124

A cylinder fitted with a movable piston contains water at 3 MPa, 50% quality, at which point the volume is 20 L. The water now expands to 1.2 MPa as a result of receiving 600 kJ of heat from a large source at 300°C. It is claimed that the water does 124 kJ of work during this process. Is this possible?

Solution:

C.V.: H_2O in Cylinder

State 1: 3 MPa, $x_1 = 0.5$, Table B.1.2: $T_1 = 233.9^oC$

$$v_1 = v_f + x_1 v_{fg} = 0.001216 + 0.5\times 0.06546 = 0.033948 \text{ m}^3\text{/kg}$$

$$u_1 = u_f + x_1 u_{fg} = 1804.5 \text{ kJ/kg}, \quad s_1 = s_f + x_1 s_{fg} = 4.4162 \text{ kJ/kg-K}$$

$$m_1 = V_1/v_1 = 0.02 / 0.033948 = 0.589 \text{ kg}$$

1^{st} Law: $1\rightarrow 2$, $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$;

$${}_1Q_2 = 600 \text{ kJ}, \;\; {}_1W_2 = 124 \text{ kJ ?}$$

Now solve for u_2

$$u_2 = 1804.5 + (600 - 124)/0.589 = 2612.6 \text{ kJ/kg}$$

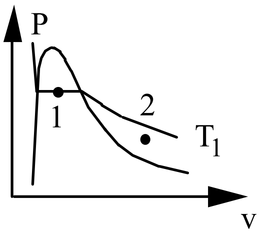
State 2: $P_2 = 1.2$ MPa : $u_2 = 2612.6$ kJ/kg Table B.1.3

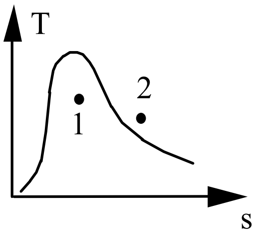
$$T_2 \cong 200^oC, \quad s_2 = 6.5898 \text{ kJ/kgK}$$

2^{nd} Law Eq.8.18: $\quad \Delta S_{net} = m(s_2 - s_1) - \frac{Q_{cv}}{T_H}$; $\quad T_H = 300^oC, \;\; Q_{CV} = {}_1Q_2$

$$\Delta S_{net} = 0.589\,(6.5898 - 4.4162) - \frac{600}{300 + 273} = 0.2335 \text{ kJ/K} \geq 0;$$

Process is possible





8.125

A mass and atmosphere loaded piston/cylinder contains 2 kg of water at 5 MPa, 100°C. Heat is added from a reservoir at 700°C to the water until it reaches 700°C. Find the work, heat transfer, and total entropy production for the system and surroundings.

Solution:

C.V. Water out to surroundings at 700°C. This is a control mass.

Energy Eq.5.11: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

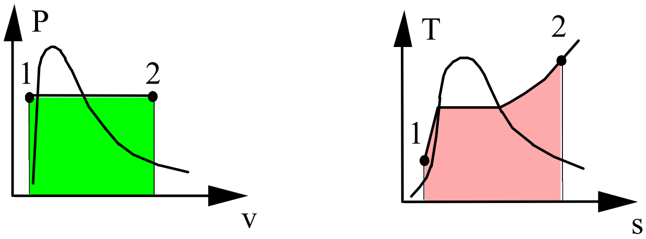
Process: $P = \text{constant}$ so ${}_1W_2 = P(V_2 - V_1) = mP(v_2 - v_1)$

State 1: Table B.1.4: $h_1 = 422.72$ kJ/kg, $u_1 = 417.52$ kJ/kg,

$s_1 = 1.303$ kJ/kg K, $v_1 = 0.00104\ m^3/kg$

State 2: Table B.1.3: $h_2 = 3900.1$ kJ/kg, $u_2 = 3457.6$ kJ/kg,

$s_2 = 7.5122$ kJ/kg K, $v_2 = 0.08849\ m^3/kg$



Work is found from the process (area in P-V diagram)

${}_1W_2 = mP(v_2 - v_1) = 2 \times 5000(0.08849 - 0.00104) =$ **874.6 kJ**

The heat transfer from the energy equation is

${}_1Q_2 = U_2 - U_1 + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1)$

${}_1Q_2 = 2(3900.1 - 422.72) =$ **6954.76 kJ**

Entropy generation from entropy equation (or Eq.8.39)

${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 2(7.5122 - 1.303) - 6954/973 =$ **5.27 kJ/K**

8.126

A piston/cylinder contains 1 kg water at 150 kPa, 20°C. The piston is loaded so pressure is linear in volume. Heat is added from a 600°C source until the water is at 1 MPa, 500°C. Find the heat transfer and the total change in entropy.

Solution:

CV H_2O out to the source, both ${}_1Q_2$ and ${}_1W_2$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = {}_1Q_2 / T_{SOURCE} + {}_1S_{2\ gen}$

Process: $P = A + BV$ => ${}_1W_2 = \int P\, dV = ½ (P_1 + P_2)(V_2 - V_1)$

State 1: B.1.1 Compressed liquid use saturated liquid at same T:

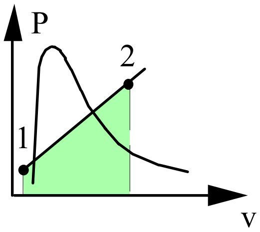
$v_1 = 0.001002\ m^3/kg$; $u_1 = 83.94\ kJ/kg$; $s_1 = 0.2966\ kJ/kg\ K$

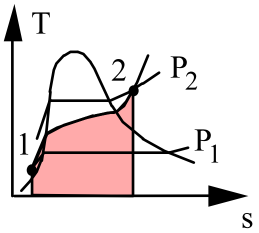
State 2: Table B.1.3 sup. vap.

$v_2 = 0.35411\ m^3/kg$

$u_2 = 3124.3\ kJ/kg$;

$s_2 = 7.7621\ kJ/kg\ K$





${}_1W_2 = ½ (1000 + 150)\ 1\ (0.35411 - 0.001002) = 203\ kJ$

${}_1Q_2 = 1(3124.3 - 83.94) + 203 =$ **3243.4 kJ**

$m(s_2 - s_1) = 1(7.7621 - 0.2968) = 7.4655\ kJ/K$

${}_1Q_2 / T_{source} = 3.7146\ kJ/K$ (for source $Q = -{}_1Q_2$ recall Eq.8.18)

${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2 / T_{SOURCE} = \Delta S_{total}$

$= \Delta S_{H2O} + \Delta S_{source} = 7.4655 - 3.7146 =$ **3.751 kJ/K**

Remark: This is an external irreversible process (delta T to the source)

8.127

A cylinder/piston contains water at 200 kPa, 200°C with a volume of 20 L. The piston is moved slowly, compressing the water to a pressure of 800 kPa. The loading on the piston is such that the product *PV* is a constant. Assuming that the room temperature is 20°C, show that this process does not violate the second law.

Solution:

C.V.: Water + cylinder out to room at 20°C

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = {}_1Q_2 / T_{room} + {}_1S_{2\ gen}$

Process: $PV = \text{constant} = Pmv \Rightarrow v_2 = P_1v_1/P_2$

$${}_1w_2 = \int Pdv = P_1v_1 \ln(v_2/v_1)$$

State 1: Table B.1.3, $v_1 = 1.0803\ m^3/kg$, $u_1 = 2654.4\ kJ/kg$,

$s_1 = 7.5066\ kJ/kg\ K$

State 2: P_2, $v_2 = P_1v_1/P_2 = 200 \times 1.0803/800 = 0.2701\ m^3/kg$

Table B.1.3: $u_2 = 2655.0\ kJ/kg$, $s_2 = 6.8822\ kJ/kg\ K$

$${}_1w_2 = 200 \times 1.0803 \ln\left(\frac{0.2701}{1.0803}\right) = -299.5\ kJ/kg$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 2655.0 - 2654.4 - 299.5 = -298.9\ kJ/kg$$

$${}_1s_{2,gen} = s_2 - s_1 - \frac{{}_1q_2}{T_{room}} = 6.8822 - 7.5066 + \frac{298.9}{293.15}$$

$= 0.395\ kJ/kg\ K > 0$ **satisfy 2nd law.**

8.128

A piston/cylinder device keeping a constant pressure has 1 kg water at 20°C and 1 kg of water at 100°C both at 500 kPa separated by a thin membrane. The membrane is broken and the water comes to a uniform state with no external heat transfer. Find the final temperature and the entropy generation for the process.

Solution:

Continuity Eq.: $m_2 - m_A - m_B = 0$

Energy Eq.5.11: $m_2u_2 - m_Au_A - m_Bu_B = -{}_1W_2$

Entropy Eq.8.37: $m_2s_2 - m_As_A - m_Bs_B = \int dQ/T + {}_1S_{2\ gen}$

Process: $P = \text{Constant} \Rightarrow {}_1W_2 = \int PdV = P(V_2 - V_1)$

$Q = 0$

Substitute the work term into the energy equation and rearrange to get

$$m_2u_2 + P_2V_2 = m_2h_2 = m_Au_A + m_Bu_B + PV_1 = m_Ah_A + m_Bh_B$$

where the last rewrite used $PV_1 = PV_A + PV_B$.

State A1: Table B.1.4 $h_A = 84.41$ kJ/kg $s_A = 0.2965$ kJ/kg K

State B1: Table B.1.4 $h_B = 419.32$ kJ/kg $s_B = 1.3065$ kJ/kg K

Energy equation gives:

$$h_2 = \frac{m_A}{m_2}h_A + \frac{m_B}{m_2}h_B = \frac{1}{2}\,84.41 + \frac{1}{2}\,419.32 = 251.865 \text{ kJ/kg}$$

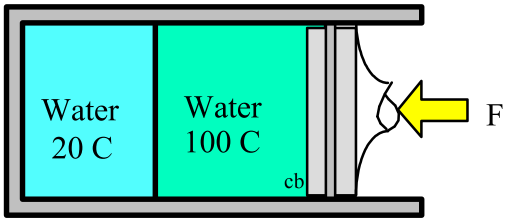
State 2: $h_2 = 251.865$ kJ/kg & $P_2 = 500$ kPa from Table B.1.4

$T_2 = 60.085°C$, $s_2 = 0.83184$ kJ/kg K

With the zero heat transfer we have

$${}_1S_{2\ gen} = m_2s_2 - m_As_A - m_Bs_B$$

$$= 2 \times 0.83184 - 1 \times 0.2965 - 1 \times 1.3065 = \mathbf{0.0607\ kJ/K}$$



8.129

A piston cylinder has 2.5 kg ammonia at 50 kPa, -20°C. Now it is heated to 50°C at constant pressure through the bottom of the cylinder from external hot gas at 200°C. Find the heat transfer to the ammonia and the total entropy generation.

Solution:

C.V. Ammonia plus space out to the hot gas.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{gas} + {}_1S_{2\ gen}$

Process: $P = C \quad \Rightarrow \quad {}_1W_2 = Pm(v_2 - v_1)$

State 1: Table B.2.2 $v_1 = 2.4463\ m^3/kg$, $h_1 = 1434.6\ kJ/kg$,

$s_1 = 6.3187\ kJ/kg\ K$

State 2: Table B.2.2 $v_2 = 3.1435\ m^3/kg$, $h_2 = 1583.5\ kJ/kg$,

$s_2 = 6.8379\ kJ/kg\ K$

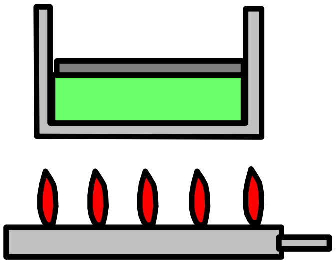
Substitute the work into the energy equation and solve for the heat transfer

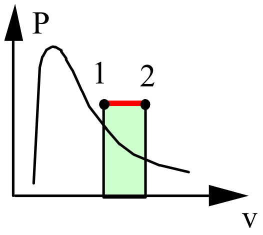
$${}_1Q_2 = m(h_2 - h_1) = 2.5\ (1583.5 – 1434.6) = \mathbf{372.25\ kJ}$$

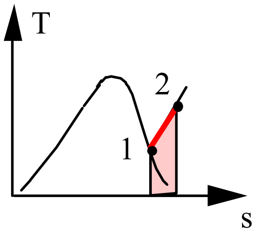
$${}_1S_{2\ gen} = m(s_2 – s_1) - {}_1Q_2/T_{gas}$$

$$= 2.5\ (6.8379 – 6.3187) – 372.25/473.15$$

$$= \mathbf{0.511\ kJ/K}$$







Remark: This is an internally reversible- externally irreversible process. The s is generated in the space between the 200°C gas and the ammonia. If there are any ΔT in the ammonia then it is also internally irreversible.

8.130

Repeat the previous Problem but include the piston/cylinder steel mass of 1 kg that we assume has the same T as the ammonia at any time.

A piston cylinder has 2.5 kg ammonia at 50 kPa, -20°C. Now it is heated to 50°C at constant pressure through the bottom of the cylinder from external hot gas at 200°C. Find the heat transfer to the ammonia and the total entropy generation.

Solution:

C.V. Ammonia plus space out to the hot gas.

Energy Eq.5.11: $m_{NH3}(u_2 - u_1) + m_{steel}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{gas} + {}_1S_{2\ gen}$

$S_2 - S_1 = m_{NH3}(s_2 - s_1) + m_{steel}(s_2 - s_1)$

Process: $P = C \quad \Rightarrow \quad {}_1W_2 = Pm_{NH3}(v_2 - v_1)_{NH3}$

State 1: (B.2.2) $v_1 = 2.4463\ m^3/kg$, $h_1 = 1434.6\ kJ/kg$, $s_1 = 6.3187\ kJ/kg\ K$

State 2: (B.2.2) $v_2 = 3.1435\ m^3/kg$, $h_2 = 1583.5\ kJ/kg$, $s_2 = 6.8379\ kJ/kg\ K$

Substitute the work into the energy equation and solve for the heat transfer

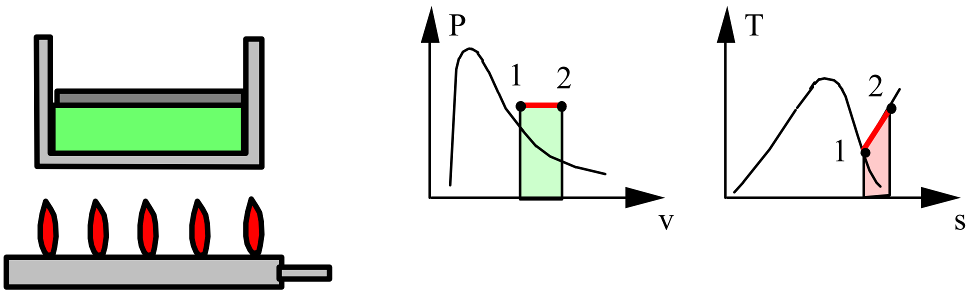
$$ {}_1Q_2 = m_{NH3}(h_2 - h_1) + m_{steel}(u_2 - u_1)$$

$$= 2.5\ (1583.5 - 1434.6) + 1 \times 0.46\ [50 - (-20)] = \mathbf{404.45\ kJ}$$

$$ {}_1S_{2\ gen} = m_{NH3}(s_2 - s_1) + m_{steel}(s_2 - s_1) - {}_1Q_2/T_{gas}$$

$$= 2.5\ (6.8379 - 6.3187) + 1 \times 0.46\ \ln(\frac{323.15}{253.15}) - \frac{404.45}{473.15}$$

$$= \mathbf{0.555\ kJ/K}$$



8.131

A piston/cylinder has ammonia at 2000 kPa, 80°C with a volume of 0.1 m^3. The piston is loaded with a linear spring and outside ambient is at 20°C, shown in Fig. P8.131. The ammonia now cools down to 20°C at which point it has a quality of 10%. Find the work, the heat transfer and the total entropy generation in the process.

CV Ammonia out to the ambient, both ${}_1Q_2$ and ${}_1W_2$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = {}_1Q_2 / T_{ambient} + {}_1S_{2\ gen}$

Process: $P = A + BV$ => ${}_1W_2 = \int P\, dV = ½\, m(P_1 + P_2)(v_2 - v_1)$

State 1: Table B.2.2

$v_1 = 0.07595\ m^3/kg,\ \ u_1 = 1421.6\ kJ/kg,\ \ s_1 = 5.0707\ kJ/kg\ K$

$m = V_1/v_1 = 0.1/0.07595 = 1.31665\ kg$

State 2: Table B.2.1

$v_2 = 0.001638 + 0.1 \times 0.14758 = 0.016396\ m^3/kg$

$u_2 = 272.89 + 0.1 \times 1059.3 = 378.82\ kJ/kg$

$s_2 = 1.0408 + 0.1 \times 4.0452 = 1.44532\ kJ/kg\ K$

$${}_1W_2 = ½\, m(P_1 + P_2)(v_2 - v_1)$$

$$= ½ \times 1.31665\ kg\ (2000 + 857.5)\ kPa\ (0.016396 - 0.07595)\ m^3/kg$$

$$= \mathbf{-\ 112\ kJ}$$

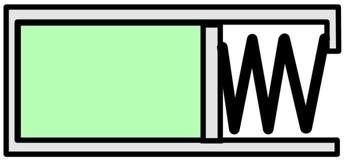
$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.31665\ (378.82 - 1421.6) - 112$$

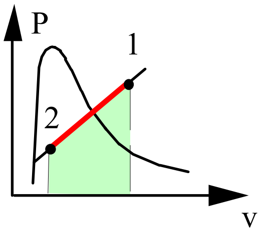
$$\mathbf{= -\ 1484.98\ kJ}$$

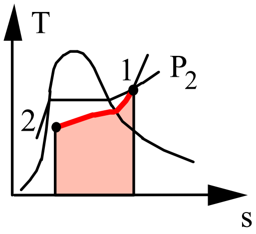
$${}_1S_{2\ gen} = m(s_2 - s_1) - ({}_1Q_2 / T_{amb})$$

$$= 1.31665\ (1.44532 - 5.0707) - \frac{-1484.98}{293.15}$$

$$= -4.77336 + 5.0656 = \mathbf{0.292\ kJ/k}$$







8.132

A 5 kg aluminum radiator holds 2 kg of liquid R-134a both at -10°C. The setup is brought indoors and heated with 220 kJ from a heat source at 100°C. Find the total entropy generation for the process assuming the R-134a remains a liquid.

Solution:

C.V. The aluminum radiator and the R-134a.

Energy Eq.5.11: $m_2u_2 - m_1u_1 = {}_1Q_2 - 0$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{al}\,(u_2 - u_1)_{al} + m_{R134a}\,(u_2 - u_1)_{R134a} = {}_1Q_2$$

Use specific heat from Table A.3 and A.4

$$m_{al}C_{al}\,(T_2 - T_1) + m_{R134a}C_{R134a}\ \ln\,(T_2 - T_1) = {}_1Q_2$$

$$T_2 - T_1 = {}_1Q_2 / [m_{al}C_{al} + m_{R134a}C_{R134a}]$$

$$= 220 / [5 \times 0.9 + 2 \times 1.43] = 29.89^{\circ}\text{C}$$

$$T_2 = -10 + 29.89 = 19.89^{\circ}\text{C}$$

Entropy generation from Eq.8.37

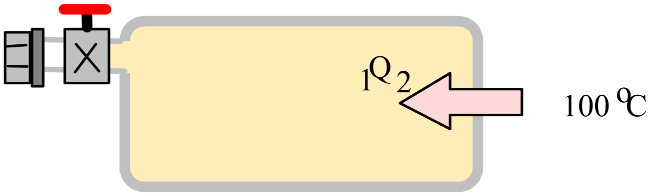
$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T$$

$$= m_{al}C_{al}\ \ln\,(T_2/T_1) + m_{R134a}C_{R134a}\ \ln\,(T_2/T_1) - \frac{{}_1Q_2}{T_{amb}}$$

$$= (5 \times 0.9 + 2 \times 1.43)\ \ln\frac{(19.89 + 273.15)}{-10 + 273.15} - \frac{220}{373.15}$$

$$= 0.7918 - 0.5896$$

$$= \mathbf{0.202\ kJ/K}$$



8.133

Two 5 kg blocks of steel, one at 250°C the other at 25°C, come in thermal contact. Find the final temperature and the total entropy generation in the process?

C.V. Both blocks, no external heat transfer, C from Table A.3.

Energy Eq.: $U_2 - U_1 = m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0$

$$= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})$$

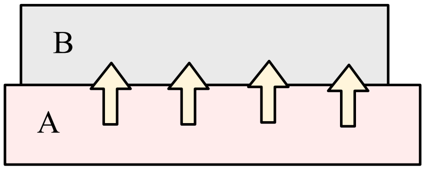
$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = \mathbf{137.5^oC}$$

Entropy Eq.: $S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1S_{2\ gen}$

$${}_1S_{2\ gen} = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}}$$

$$= 5 \times 0.46 \ln \frac{137.5 + 273.15}{250 + 273.15} + 5 \times 0.46 \ln \frac{137.5 + 273.15}{298.15}$$

$$= -0.5569 + 0.7363 = \mathbf{0.1794\ kJ/K}$$



Heat transfer over a finite temperature difference is an irreversible process

8.134

Reconsider Problem 5.60 where CO_2 is compressed from -20°C, x = 0.75 to a state of 3 MPa, 20°C in a piston/cylinder where pressure is linear in volume. Assume heat transfer is from a reservoir at 100°C and find the specific entropy generation in the process (external to the CO_2).

CV Carbon dioxide out to the source, both ${}_1Q_2$ and ${}_1W_2$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{res} + {}_1S_{2,gen}$

Process: $P = A + BV$ => ${}_1W_2 = \int P\, dV = ½\, m(P_1 + P_2)\,(v_2 - v_1)$

State 1: Table B.3.1 P = 1969.6 kPa

$v_1 = 0.000969 + 0.75 \times 0.01837 = 0.01475\ m^3/kg$,

$u_1 = 39.64 + 0.75 \times 246.25 = 224.33$ kJ/kg,

$s_1 = 0.1672 + 0.75 \times 1.1157 = 1.004$ kJ/kg-K

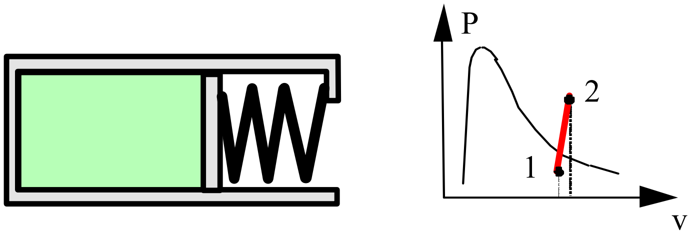
State 2: Table B.3 $v_2 = 0.01512\ m^3/kg$, $u_2 = 310.21$ kJ/kg, $s_2 = 1.3344$ kJ/kg-K

${}_1w_2 = ½\,(P_1 + P_2)(v_2 - v_1) = ½ \times (1969.6 + 3000)(0.01512 - 0.01475)$

$= 0.92$ kJ/kg

${}_1q_2 = u_2 - u_1 + {}_1w_2 = 310.21 - 224.33 + 0.92 = 86.8$ kJ/kg

${}_1s_{2,gen} = s_2 - s_1 - {}_1q_2/T_{res} = 1.3344 - 1.004 - \frac{86.8}{373.15} = \mathbf{0.098\ kJ/kg\text{-}K}$



8.135

One kilogram of ammonia (NH_3) is contained in a spring-loaded piston/cylinder, Fig. P8.135, as saturated liquid at –20°C. Heat is added from a reservoir at 100°C until a final condition of 800 kPa, 70°C is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH_3 out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$

Energy Eq.5.11: $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{res} + {}_1S_{2,gen}$

Process: $P = A + BV$ linear in V =>

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

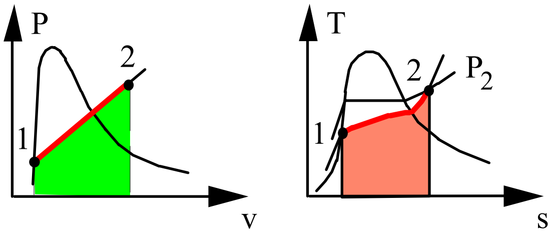
State 1: Table B.2.1

$P_1 = 190.08$ kPa,

$v_1 = 0.001504$ m^3/kg

$u_1 = 88.76$ kJ/kg,

$s_1 = 0.3657$ kJ/kg K



State 2: Table B.2.2 sup. vapor

$v_2 = 0.199$ m^3/kg, $u_2 = 1438.3$ kJ/kg, $s_2 = 5.5513$ kJ/kg K

$${}_1W_2 = \frac{1}{2}(190.08 + 800)1(0.1990 - 0.001504) = \mathbf{97.768\ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1(1438.3 - 88.76) + 97.768 = \mathbf{1447.3\ kJ}$$

$${}_1S_{2,gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 1(5.5513 - 0.3657) - \frac{1447.3}{373.15} = \mathbf{1.307\ kJ/K}$$

8.136

The water in the two tanks of Problem 5.67 receives the heat transfer from a reservoir at 300°C. Find the total entropy generation due to this process.

Two rigid tanks are filled with water. Tank A is 0.2 m^3 at 100 kPa, 150°C and tank B is 0.3 m^3 at saturated vapor 300 kPa. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 300 kPa. Give the two property values that determine the final state and heat transfer.

Take CV total A + B out to reservoir (neglect kinetic and potential energy)

Energy Eq.: $m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Entropy Eq.: $m_2 s_2 - m_A s_{A1} - m_B s_{B1} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

State A1: u = 2582.75 kJ/kg, v = 1.93636 m^3/kg, s = 7.6133 kJ/kg-K

=> $m_{A1} = V/v = 0.2/1.93636 = 0.1033$ kg

State B1: u = 2543.55 kJ/kg, v = 0.60582 m^3/kg, s = 6.9918 kJ/kg-K

=> $m_{B1} = V/v = 0.3 / 0.60582 = 0.4952$ kg

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$m_2 = m_{A1} + m_{B1} = 0.1033 + 0.4952 = 0.5985$ kg,

$V_2 = V_{A1} + V_{B1} = 0.2 + 0.3 = 0.5\ m^3$

=> $v_2 = V_2/m_2 = 0.5 /0.5985 = 0.8354\ m^3/kg$

State 2: $[P_2, v_2] = [300\ kPa, 0.8354\ m^3/kg]$

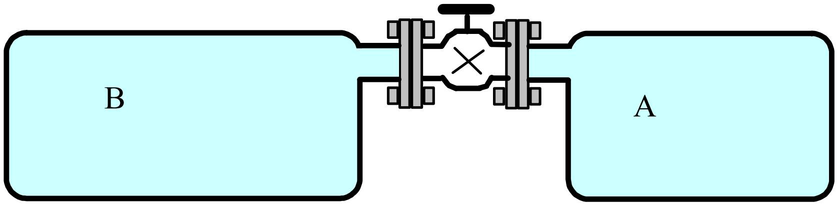
=> $T_2 = 274.76°C$ and $u_2 = 2767.32$ kJ/kg, s = 7.60835 kJ/kgK

From energy eq.

$${}_1Q_2 = 0.5985 \times 2767.32 - 0.1033 \times 2582.75 - 0.4952 \times 2543.55 = 129.9 \text{ kJ}$$

From entropy equation

$${}_1S_{2\ gen} = m_2 s_2 - m_A s_{A1} - m_B s_{B1} - {}_1Q_2/T_{res}$$
$$= 0.5985 \times 7.60835 - 0.1033 \times 7.6133 - 0.4952 \times 6.9918 - 129.9 / (273.15 + 300) = \mathbf{0.0782\ kJ/K}$$



8.137

A piston cylinder loaded so it gives constant pressure has 0.75 kg saturated vapor water at 200 kPa. It is now cooled so the volume becomes half the initial volume by heat transfer to the ambient at 20°C. Find the work, the heat transfer and the total entropy generation.

Solution:

Continuity Eq.: $m_2 - m_1 = 0$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: $P = C \quad => \quad {}_1W_2 = \int PdV = mP(v_2 - v_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

State 1: $v_1 = 0.88573\ m^3/kg$, $h_1 = 2706.63\ kJ/kg$, $s_1 = 7.1271\ kJ/kg\ K$

State 2: P_2, $v_2 = v_1/2 = 0.444286\ m3/kg \quad => \quad$ Table B.1.2

$x_2 = (0.444286 - 0.001061)/0.88467 = 0.501$

$h_2 = 504.68 + x_2 \times 2201.96 = 1607.86\ kJ/kg$

$s_2 = 1.53 + x_2 \times 5.5970 = 4.3341\ kJ/kg\ K$

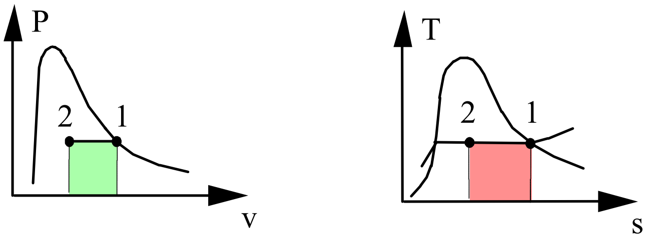
${}_1W_2 = 0.75 \times 200(0.444286 - 0.88573) =$ **-66.22 kJ**

${}_1Q_2 = 0.75(1607.86 - 2706.63) =$ **-824.1 kJ**

${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T = 0.75(4.3341 - 7.1271) - (-824.1/293.15)$

$= -2.09475 + 2.81119 =$ **0.716 kJ/K**

Notice: The process is externally irreversible (T receiving Q is not T_1)



8.138

A piston/cylinder of total 1 kg steel contains 0.5 kg ammonia at 1600 kPa both masses at 120°C. Some stops are placed so a minimum volume is 0.02 m^3, shown in Fig. P8.138. Now the whole system is cooled down to 30°C by heat transfer to the ambient at 20°C, and during the process the steel keeps same temperature as the ammonia. Find the work, the heat transfer and the total entropy generation in the process.

$$1: \quad v_1 = 0.11265 \text{ m}^3/\text{kg}, \quad u_1 = 1516.6 \text{ kJ/kg}, \quad s_1 = 5.5018 \text{ kJ/kg K}$$

$$V_1 = mv_1 = 0.05634 \text{ m}^3$$

$$\text{Stop 1a: } v_{stop} = V/m = 0.02/0.5 = 0.04 \text{ m}^3/\text{kg}$$

$$P_{stop} = P_1 \quad \Rightarrow \quad T \sim 42^{\circ}\text{C (saturated)}$$

$$2: \quad 30^{\circ}\text{C} < T_{stop} \text{ so } \quad v_2 = v_{stop} = 0.04 \text{ m}^3/\text{kg}$$

$$x_2 = \left(\frac{v_2 - v_f}{v_{fg}}\right) = \frac{0.04 - 0.00168}{0.10881} = 0.35217$$

$$u_2 = 320.46 + x_2 \times 1016.9 = 678.58 \text{ kJ/kg}$$

$$s_2 = 1.2005 + x_2 \times 3.7734 = 2.5294 \text{ kJ/kg K}$$

$${}_1W_2 = \int P\, dV = P_1 m\, (v_2 - v_1) = 1600 \times 0.5\, (0.004 - 0.11268) = -\,58.14 \text{ kJ}$$

$${}_1Q_2 = m\,(u_2 - u_1) + m_{st}(u_2 - u_1) + {}_1W_2$$

$$= 0.5(\,678.58 - 1516.6\,) + 1 \times 0.46(30 - 120) - 58.14$$

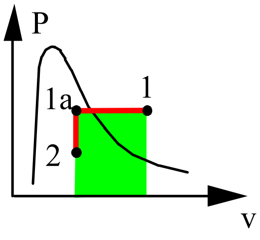
$$= -419.01 - 41.4 - 58.14 = \mathbf{-518.55 \text{ kJ}}$$

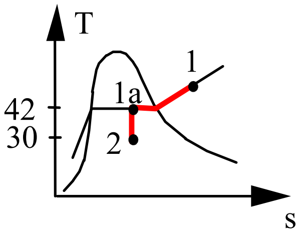
$${}_1S_{2\ gen} = m(s_2 - s_1) + m_{st}\,(s_2 - s_1) - {}_1Q_2/T_{amb}$$

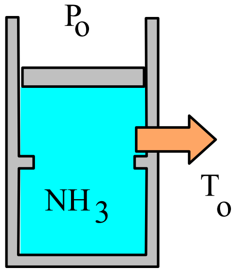
$$= 0.5\,(2.5294 - 5.5018) + 1 \times 0.46 \ln \frac{273+30}{273+120} - \frac{-518.5}{293.15}$$

$$= -\,1.4862 - 0.1196 + 1.6277$$

$$= \mathbf{0.02186 \text{ kJ/K}}$$







8.139

A hollow steel sphere with a 0.5-m inside diameter and a 2-mm thick wall contains water at 2 MPa, 250°C. The system (steel plus water) cools to the ambient temperature, 30°C. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water. This is a control mass.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = m_{H_2O}(u_2 - u_1) + m_{steel}(u_2 - u_1)$

Process: $V = \text{constant} \quad \Rightarrow \quad {}_1W_2 = 0$

$m_{steel} = (\rho V)_{steel} = 8050 \times (\pi/6)\left[(0.504)^3 - (0.5)^3\right] = 12.746 \text{ kg}$

$V_{H2O} = (\pi/6)(0.5)^3, \quad m_{H_2O} = V/v = 6.545{\times}10^{-2}/0.11144 = 0.587 \text{ kg}$

$v_2 = v_1 = 0.11144 = 0.001004 + x_2 \times 32.889 \;\Rightarrow\; x_2 = 3.358{\times}10^{-3}$

$u_2 = 125.78 + 3.358{\times}10^{-3} \times 2290.8 = 133.5 \text{ kJ/kg}$

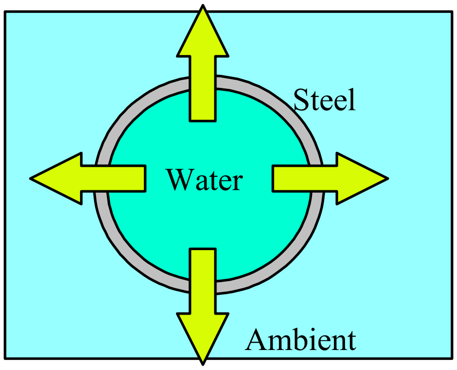
$s_2 = 0.4639 + 3.358{\times}10^{-3} \times 8.0164 = 0.4638 \text{ kJ/kg K}$

$${}_1Q_2 = m_{H_2O}(u_2 - u_1) + m_{steel}(u_2 - u_1)$$
$$= 0.587(133.5 - 2679.6) + 12.746 \times 0.48(30 - 250)$$
$$= -1494.6 + (-1346) = -2840.6 \text{ kJ}$$

$$\Delta S_{TOT} = \Delta S_{STEEL} + \Delta S_{H_2O} = 12.746 \times 0.48 \ln(303.15 / 523.15)$$
$$+ 0.587(0.4638 - 6.545) = \mathbf{-6.908 \text{ kJ/K}}$$

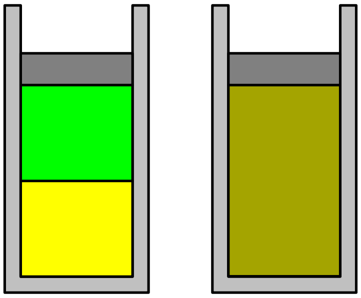
$\Delta S_{SURR} = -{}_1Q_2/T_0 = +2840.6/303.2 = \mathbf{+9.370 \text{ kJ/K}}$

$\Delta S_{NET} = -6.908 + 9.370 = \mathbf{+2.462 \text{ kJ/K}}$



8.140

One kg of air at 300 K is mixed with one kg air at 400 K in a process at a constant 100 kPa and Q = 0. Find the final T and the entropy generation in the process.



C.V. All the air.

Energy Eq.: $U_2 - U_1 = 0 - W$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\ gen}$

Process Eq.: $P = C; \quad W = P(V_2 - V_1)$

Substitute W into energy Eq.

$$U_2 - U_1 + W = U_2 - U_1 + P(V_2 - V_1) = H_2 - H_1 = 0$$

Due to the low T let us use constant specific heat

$$H_2 - H_1 = m_A(h_2 - h_1)_A + m_B(h_2 - h_1)_B$$
$$= m_AC_p(T_2 - T_{A1}) + m_BC_p(T_2 - T_{B1}) = 0$$

$$T_2 = \frac{m_AT_{A1} + m_BT_{B1}}{m_A + m_B} = \frac{1}{2}T_{A1} + \frac{1}{2}T_{B1} \mathbf{= 350\ K}$$

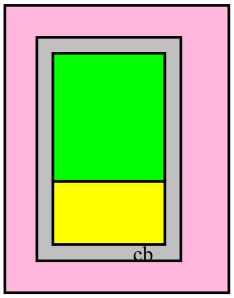
Entropy change is from Eq. 8.16 with no change in P

$${}_1S_{2\ gen} = S_2 - S_1 = m_AC_p \ln\frac{T_2}{T_{A1}} + m_BC_p \ln\frac{T_2}{T_{B1}}$$
$$= 1 \times 1.004 \ln\frac{350}{300} + 1 \times 1.004 \ln\frac{350}{400}$$
$$= 0.15477 - 0.13407 = \mathbf{0.0207\ kJ/K}$$

Remark: If you check, the volume does not change and there is no work.

8.141

One kg of air at 100 kPa is mixed with one kg air at 200 kPa, both at 300 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.



C.V. All the air.

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\ gen}$

Process Eqs.: $V = C;\quad W = 0,\ Q = 0$

States A1, B1: $u_{A1} = u_{B1}$

$V_A = m_ART_1/P_{A1};\quad V_B = m_BRT_1/P_{B1}$

$$U_2 - U_1 = m_2u_2 - m_Au_{A1} - m_Bu_{B1} = 0 \;\Rightarrow\; u_2 = (u_{A1} + u_{B1})/2 = u_{A1}$$

State 2: $\mathbf{T_2 = T_1 = 300\ K}$ (from u_2); $m_2 = m_A + m_B = 2$ kg;

$$V_2 = m_2RT_1/P_2 = V_A + V_B = m_ART_1/P_{A1} + m_BRT_1/P_{B1}$$

Divide with m_ART_1 and get

$$2/P_2 = 1/P_{A1} + 1/P_{B1} = \frac{1}{100} + \frac{1}{200} = 0.015\ \text{kPa}^{-1} \quad\Rightarrow\; P_2 = \mathbf{133.3\ kPa}$$

Entropy change from Eq. 8.25 with the same T, so only P changes

$$ {}_1S_{2\ gen} = S_2 - S_1 = -m_AR\ \ln\frac{P_2}{P_{A1}} - m_BR\ \ln\frac{P_2}{P_{B1}}$$

$$= -\,1 \times 0.287\ [\ \ln\frac{133.3}{100} + \ln\frac{133.3}{200}\ ]$$

$$= -0.287\ (0.2874 - 0.4057) = \mathbf{0.034\ kJ/K}$$

8.142

A spring loaded piston cylinder contains 1.5 kg air at 27°C and 160 kPa. It is now heated in a process where pressure is linear in volume, P = A + BV, to twice the initial volume where it reaches 900 K. Find the work, the heat transfer and the total entropy generation assuming a source at 900 K.

Solution:

C.V. Air out to the 900 K source. Since air T is lower than the source temperature we know that this is an irreversible process.

Continuity Eq.: $m_2 = m_1 = m,$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{SOURCE} + {}_1S_{2\ gen}$

Process: $P = A + BV$

State 1: (T_1, P_1) Table A.7 $u_1 = 214.36$ kJ/kg

$$V_1 = mRT_1/ P_1 = (1.5 \times 0.287 \times 300) / 160 = 0.8072 \text{ m}^3$$

State 2: $(T_2, v_2 = 2\ v_1)$ Table A.7 $u_2 = 674.824$ kJ/kg

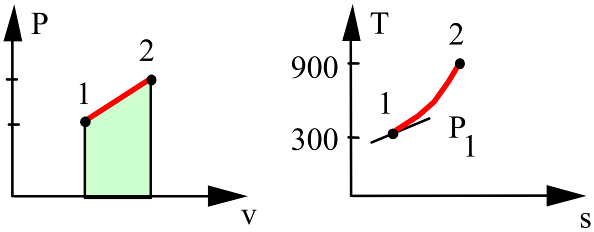
$$P_2 = RT_2/ v_2 = RT_2/2v_1 = T_2\ P_1/ 2T_1 = P_1\ T_2/2\ T_1$$
$$= 160 \times 900 / (2 \times 300) = 240 \text{ kPa}$$

From the process equation we can express the work as

$${}_1W_2 = \int PdV = 0.5 \times (P_1 + P_2)(V_2 - V_1) = 0.5 \times (P_1 + P_2) V_1$$
$$= 0.5 \times (160 + 240)\ 0.8072 = \mathbf{161.4\ kJ}$$
$${}_1Q_2 = 1.5 \times (674.824 - 214.36) + 161.4 = \mathbf{852.1\ kJ}$$

Change in s from Eq.8.19 and Table A.7 values

$${}_1S_{2\ gen} = m(s^o_{T2} - s^o_{T1} - R \ln \frac{P_2}{P_1}) - {}_1Q_2/T_{SOURCE}$$
$$= 1.5 \times [8.0158 - 6.8693 - 0.287 \ln (\frac{240}{160})] - (\frac{852.1}{900})$$
$$= 1.545 - 0.947 = \mathbf{0.598\ kJ/K}$$



8.143

Air in a rigid tank is at 900 K, 500 kPa and it now cools to the ambient temperature of 300 K by heat loss to the ambient. Find the entropy generation.

C.V. Air out to ambient. No size given so do it per unit mass.

Energy Eq.5.11: $u_2 - u_1 = C_v (T_2 - T_1) = {}_1q_2 - {}_1w_2$

Entropy Eq.8.14: $s_2 - s_1 = {}_1q_2/T_{amb} + {}_1s_{2\text{ gen tot}}$

Process: V = constant => $v_2 = v_1$ also ${}_1W_2 = 0$

Ideal gas: $P_2 = P_1 T_2 / T_1 = 500 \times 300/900 = 166.67$ kPa

From Table A.7: $u_1 = 674.82$ kJ/kg; $s_{T1} = 8.01581$ kJ/kg-K;

$u_2 = 214.36$ kJ/kg; $s_{T2} = 6.86926$ kJ/kg-K

$${}_1q_2 = u_2 - u_1 = 214.36 - 674.82 = -460.46 \text{ kJ/kg}$$

$${}_1s_{2\text{ gen tot}} = s_2 - s_1 - \frac{{}_1q_2}{T_{amb}} = s_{T2} - s_{T1} - R \ln(\frac{P_2}{P_1}) - \frac{{}_1q_2}{T_{amb}}$$

$$= 6.86926 - 8.01581 - 0.287 \ln(\frac{166.67}{500}) - \frac{-460.46}{300}$$

$$= \mathbf{0.661\ kJ/kg\text{-}K}$$

We could also have used constant specific heat being slightly less accurate.

8.144

A rigid storage tank of 1.5 m^3 contains 1 kg argon at 30°C. Heat is then transferred to the argon from a furnace operating at 1300°C until the specific entropy of the argon has increased by 0.343 kJ/kg K. Find the total heat transfer and the entropy generated in the process.

Solution:

C.V. Argon out to 1300°C. Control mass. , m = 1 kg

Argon is an ideal gas with constant heat capacity.

Energy Eq.5.11: $m (u_2 - u_1) = m C_v (T_2 - T_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = {}_1Q_2/T_{res} + {}_1S_{2\text{ gen tot}}$

Process: V = constant => $v_2 = v_1$ also ${}_1W_2 = 0$

Properties: Table A.5 R = 0.20813, C_v = 0.312 kJ/kg K

State 1: $(T_1, v_1 = V/m)$ $P_1 = mRT_1/V = 42.063$ kPa

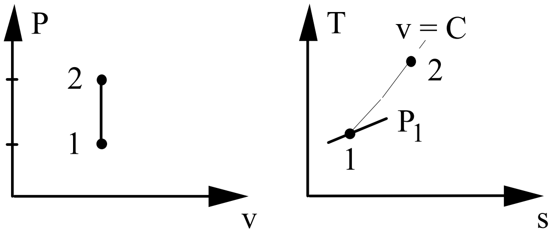
State 2: $s_2 = s_1 + 0.343$, and change in s from Eq.8.28 or Eq.8.26

$$s_2 - s_1 = C_p \ln (T_2 / T_1) - R \ln (T_2 / T_1) = C_v \ln (T_2 / T_1)$$

$$T_2 / T_1 = \exp[\frac{s_2 - s_1}{C_v}] = \exp[\frac{0.343}{0.312}] = \exp(1.09936) = 3.0$$

Pv = RT => $(P_2 / P_1)(v_2 / v_1) = T_2 / T_1 = P_2 / P_1$

$$T_2 = 3.0 \times T_1 = 909.45 \text{ K}, \quad P_2 = 3.0 \times P_1 = 126.189 \text{ kPa}$$



Heat transfer from energy equation

${}_1Q_2 = 1 \times 0.312 (909.45 - 303.15) =$ **189.2 kJ**

Entropy generation from entropy equation (2nd law)

${}_1S_{2\text{ gen tot}} = m(s_2 - s_1) - {}_1Q_2/T_{res}$

$= 1 \times 0.343 - 189.2 / (1300 + 273) =$ **0.223 kJ/K**

8.145

Argon in a light bulb is at 110 kPa, 70°C. The light is turned off so the argon cools to the ambient 20°C. Disregard the glass and any other mass and find the specific entropy generation.

Solution:

C.V. Argon gas. Neglect any heat transfer.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2$

Entropy Eq.8.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\,gen} = {}_1q_2/T_{room} + {}_1s_{2\,gen}$

Process: v = constant and ideal gas => $P_2/P_1 = T_2/T_1$

$${}_1q_2 = u_2 - u_1 = C_v\,(T_2 - T_1) = 0.312\,(20 - 70) = -15.6 \text{ kJ/kg}$$

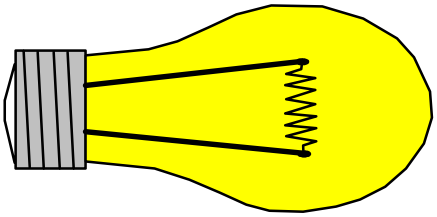
Evaluate changes in s from Eq.8.16 or 8.17

$$s_2 - s_1 = C_p \ln (T_2/T_1) - R \ln (P_2/P_1) \qquad \text{Eq.8.16}$$

$$= C_p \ln (T_2/T_1) - R \ln (T_2/T_1) = C_v \ln(T_2/T_1) \qquad \text{Eq.8.17}$$

$$= 0.312 \ln [\,(20 + 273)/(70 + 273)\,] = -0.0491 \text{ kJ/kg K}$$

$${}_1s_{2\,gen} = s_2 - s_1 - {}_1q_2/T_{room} = -0.0491 + 15.6 / 293.15 = \mathbf{0.0041 \text{ kJ/kgK}}$$



8.146

A rigid container with volume 200 L is divided into two equal volumes by a partition, shown in Fig. P8.146. Both sides contain nitrogen, one side is at 2 MPa, 200°C, and the other at 200 kPa, 100°C. The partition ruptures, and the nitrogen comes to a uniform state at 70°C. Assume the temperature of the surroundings is 20°C, determine the work done and the net entropy change for the process.

Solution:

C.V. : A + B no change in volume. $\mathbf{{}_1W_2 = 0}$

$m_{A1} = P_{A1}V_{A1}/RT_{A1} = (2000 \times 0.1)/(0.2968 \times 473.2) = 1.424 \text{ kg}$

$m_{B1} = P_{B1}V_{B1}/RT_{B1} = (200 \times 0.1)/(0.2968 \times 373.2) = 0.1806 \text{ kg}$

$P_2 = m_{TOT}RT_2/V_{TOT} = (1.6046 \times 0.2968 \times 343.2)/0.2 = 817 \text{ kPa}$

From Eq.8.25

$$\Delta S_{SYST} = 1.424\Big[1.042 \ln\frac{343.2}{473.2} - 0.2968 \ln\frac{817}{2000}\Big]$$

$$+ 0.1806\Big[1.042 \ln\frac{343.2}{373.2} - 0.2968 \ln\frac{817}{200}\Big] = -0.1894 \text{ kJ/K}$$

$${}_1Q_2 = U_2 - U_1 = 1.424 \times 0.745(70 - 200) + 0.1806 \times 0.745(70 - 100)$$

$$= -141.95 \text{ kJ}$$

From Eq.8.18

$\Delta S_{SURR} = - {}_1Q_2/T_0 = 141.95/293.2 = +0.4841 \text{ kJ/K}$

$\Delta S_{NET} = -0.1894 + 0.4841 = \mathbf{+0.2947 \text{ kJ/K}}$

8.147

Nitrogen at 200°C, 300 kPa is in a piston cylinder, volume 5 L, with the piston locked with a pin. The forces on the piston require a pressure inside of 200 kPa to balance it without the pin. The pin is removed and the piston quickly comes to its equilibrium position without any heat transfer. Find the final P, T and V and the entropy generation due to this partly unrestrained expansion.

Solution:

C.V. Nitrogen gas.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = - \int P_{eq} dV = -P_2 (V_2 - V_1)$

Entropy Eq.8.14: $m(s_2 - s_1) = 0 + {}_1S_{2\ gen}$

Process: ${}_1Q_2 = 0$ (already used), $P = P_{eq}$ after pin is out.

State 1: 200 °C, 300 kPa State 2: $P_2 = P_{eq} =$ **200 kPa**

$$m = P_1V_1/RT_1 = 300 \times 0.005 / 0.2968 \times 473.15 = 0.01068 \text{ kg}$$

The energy equation becomes

$$mu_2 + P_2V_2 = mu_1 + P_2V_1 = mh_2 \quad =>$$

$$h_2 = u_1 + P_2V_1/m = u_1 + P_2V_1\ RT_1 /P_1V_1 = u_1 + (P_2/P_1)\ RT_1$$

Solve using constant C_p, C_v

$$C_p\ T_2 = C_v\ T_1 + (P_2/P_1)\ RT_1$$

$$T_2 = T_1\ [C_v + (P_2/P_1)\ R] / C_p$$

$$= 473.15\ [0.745 + (200 / 300) \times 0.2368] / 1.042$$

$$= \mathbf{428.13\ K}$$

$$V_2 = V_1(\ T_2 / T_1) \times (\ P_1/P_2\ ) = 0.005 \times \frac{428.13}{473.15} \times \frac{300}{200}$$

$$= 0.00679 \text{ m}^3$$

$${}_1S_{2\ gen} = m(s_2 - s_1) \cong m[C_p \ln (T_2/T_1) - R \ln (P_2/ P_1)]$$

$$= P_1V_1 /RT_1\ [C_p \ln (T_2/T_1) - R \ln (P_2/ P_1)]$$

$$= 0.01068\ [1.042 \times \ln (428.13/473.15) - 0.2968 \times \ln (200 / 300)]$$

$$= \mathbf{0.000173\ kJ/K}$$

8.148

A rigid tank contains 2 kg of air at 200 kPa and ambient temperature, 20°C. An electric current now passes through a resistor inside the tank. After a total of 100 kJ of electrical work has crossed the boundary, the air temperature inside is 80°C. Is this possible?

Solution:

C.V.: Air in tank out to ambient;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$, ${}_1W_2 = -100$ kJ

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\text{ gen}} = {}_1Q_2/T_{\text{amb}} + {}_1S_{2\text{ gen}}$

Process: Constant volume and mass so $v_2 = v_1$

State 1: $T_1 = 20^{\text{o}}\text{C}$, $P_1 = 200$ kPa, $m_1 = 2$ kg

State 2: $T_2 = 80^{\text{o}}\text{C}$, $v_2 = v_1$

Ideal gas, Table A.5: $R = 0.287$ kJ/kg-K, $C_v = 0.717$ kJ/kg-K

Assume constant specific heat then energy equation gives

$${}_1Q_2 = mC_v(T_2 - T_1) + {}_1W_2 = 2 \times 0.717(80 - 20) - 100 = -14.0 \text{ kJ}$$

Change in s from Eq.8.17 (since second term drops out)

$$s_2 - s_1 = C_v \ln (T_2/T_1) + R\ln \frac{v_2}{v_1} \;; \quad v_2 = v_1, \quad \ln \frac{v_2}{v_1} = 0$$

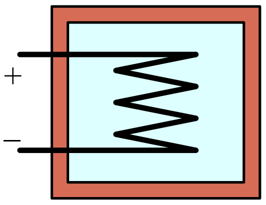
$$s_2 - s_1 = C_v\ln (T_2/T_1) = 0.1336 \text{ kJ/kg-K}$$

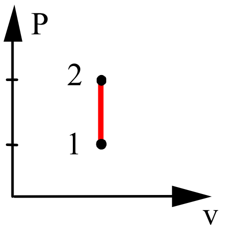
Now Eq.8.37

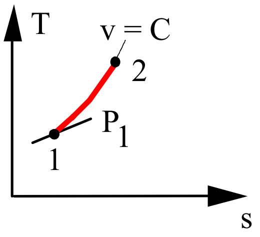
$${}_1S_{2\text{ gen}} = m(s_2 - s_1) - {}_1Q_2/T_{\text{amb}} = 2 \times 0.1336 + \frac{14}{293} = 0.315 \text{ kJ/K} \;\geq 0,$$

Process is Possible

Note: $P_2 = P_1 \frac{T_2}{T_1}$ in Eq.8.16 $s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$, results in the same answer as Eq.8.17.







8.149

The air in the tank of Problem 5.117 receives the heat transfer from a reservoir at 450 K. Find the entropy generation due to the process from 1 to 3.

C.V. Air out to reservoir.

Energy eq.: $m(u_3 - u_1) = {}_1Q_3 - {}_1W_3$

Entropy eq.: $m(s_3 - s_1) = {}_1Q_3 / T_{res} + {}_1S_{3\ gen}$

State 1: $m = P_1V_1 / RT_1 = \frac{100 \times 0.75}{0.287 \times 300} \frac{\text{kPa m}^3}{\text{kJ/kg}} = 0.871 \text{ kg}$

Process 1 to 2: Constant volume heating, $dV = 0 \Rightarrow {}_1W_2 = 0$

$P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = 133.3 \text{ kPa}$

Process 2 to 3: Isothermal expansion, $dT = 0 \Rightarrow u_3 = u_2$ and

$P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = 66.67 \text{ kPa}$

$${}_2W_3 = \int_2^3 P\,dV = P_2V_2 \ln\left(\frac{V_3}{V_2}\right) = 133.3 \times 0.75 \ln(2) = 69.3 \text{ kJ}$$

The overall process:

$${}_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = 69.3 \text{ kJ}$$

From the energy equation

$${}_1Q_3 = m(u_3 - u_1) + {}_1W_3 = m\,C_v\,(T_3 - T_1) + {}_1W_3$$

$$= 0.871 \times 0.717\,(400 - 300) + 69.3 = 131.8 \text{ kJ}$$

$${}_1S_{3\ gen} = m(s_3 - s_1) - {}_1Q_3 / T_{res}$$

$$= m\left(C_P \ln\frac{T_3}{T_1} - R \ln\frac{P_3}{P_1}\right) - {}_1Q_3 / T_{res}$$

$$= 0.871 \left[\, 1.004 \ln\frac{400}{300} - 0.287 \ln\frac{66.67}{100} \,\right] - \frac{131.8}{450}$$

$$= \mathbf{0.060\ kJ/K}$$

8.150

Nitrogen at 600 kPa, 127°C is in a 0.5 m^3 insulated tank connected to a pipe with a valve to a second insulated initially empty tank of volume 0.5 m^3, shown in Fig. P8.150. The valve is opened and the nitrogen fills both tanks at a uniform state. Find the final pressure and temperature and the entropy generation this process causes. Why is the process irreversible?

Solution:

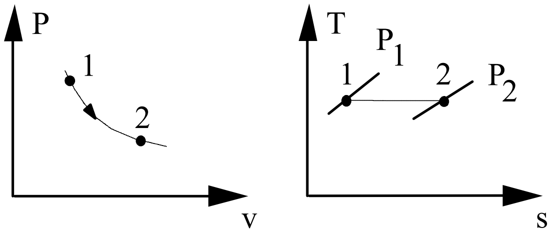
CV Both tanks + pipe + valve Insulated : Q = 0 Rigid: W = 0

Energy Eq.5.11: $m(u_2 - u_1) = 0 - 0 \quad => \quad u_2 = u_1 = u_{a1}$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1S_{2\ gen} \quad (dQ = 0)$

1: P_1 , T_1 , V_a => $m = PV/RT = (600 \times 0.5)/ (0.2968 \times 400) = 2.527$ kg

2: $V_2 = V_a + V_b$; uniform state $v_2 = V_2 / m$; $u_2 = u_{a1}$



Ideal gas u (T) => $u_2 = u_{a1}$ => $T_2 = T_{a1}$ = **400 K**

$P_2 = mR\ T_2 / V_2 = (V_1 / V_2)\ P_1 = ½ \times 600 =$ **300 kPa**

From entropy equation and Eq.8.19 for entropy change

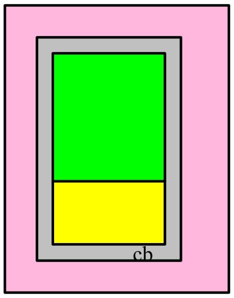
$S_{gen} = m(s_2 - s_1) = m[s_{T2} - s_{T1} - R \ln(P_2 / P_1)]$

$= m [0 - R \ln (P_2 / P_1)] = -2.527 \times 0.2968 \ln ½ =$ **0.52 kJ/K**

Irreversible due to unrestrained expansion in valve P ↓ but no work out.

8.151

One kg of carbon dioxide at 100 kPa, 500 K is mixed with two kg carbon dioxide at 200 kPa, 2000 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process using constant heat capacity from Table A.5.



C.V. All the carbon dioxide.

Continuity: $m_2 = m_A + m_B = 3$ kg

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\ gen}$

Process Eqs.: $V = C$; $W = 0$, $Q = 0$

$V_A = m_ART_1/P_{A1}$; $V_B = m_BRT_1/P_{B1}$

$$U_2 - U_1 = m_2u_2 - m_Au_{A1} - m_Bu_{B1} = 0$$

$$= m_2C_vT_2 - m_AC_vT_{A1} - m_BC_vT_{B1}$$

$$\Rightarrow T_2 = (m_AT_{A1} + m_BT_{B1})/ m_2 = \frac{1}{3} \times 500 + \frac{2}{3} \times 2000 = \mathbf{1500\ K}$$

State 2: $V_2 = m_2RT_2/P_2 = V_A + V_B = m_ART_{A1}/P_{A1} + m_BRT_{B1}/P_{B1}$

$$= 1 \times R \times 500/100 + 2 \times R \times 2000/200 = 25\ R$$

Substitute m_2, T_2 and solve for P_2

$$P_2 = 3\ R\ T_2/25\ R = 3 \times 1500 / 25 = \mathbf{180\ kPa}$$

Entropy change from Eq. 8.16

$$(s_2 - s_1)_A = C_p \ln\frac{T_2}{T_{A1}} - R \ln\frac{P_2}{P_{A1}} = 0.842 \ln\frac{1500}{500} - 0.1889 \ln\frac{180}{100}$$

$$= 0.814 \text{ kJ/kgK}$$

$$(s_2 - s_1)_B = C_p \ln\frac{T_2}{T_{B1}} - R \ln\frac{P_2}{P_{B1}} = 0.842 \ln\frac{1500}{2000} - 0.1889 \ln\frac{180}{200}$$

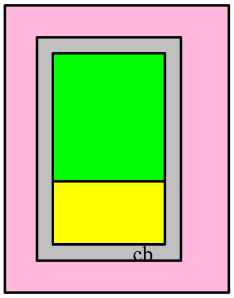
$$= -0.2223 \text{ kJ/kgK}$$

$${}_1S_{2\ gen} = S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B$$

$$= 1 \times 0.814 + 2\ (-0.2223) = \mathbf{0.369\ kJ/K}$$

8.152

One kg of carbon dioxide at 100 kPa, 500 K is mixed with two kg carbon dioxide at 200 kPa, 2000 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process using table A.8.



C.V. All the carbon dioxide.

Continuity: $m_2 = m_A + m_B = 3$ kg

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\text{ gen}}$

Process Eqs.: $V = C$; $W = 0$, $Q = 0$

$V_A = m_ART_1/P_{A1}$; $V_B = m_BRT_1/P_{B1}$

$$U_2 - U_1 = m_2u_2 - m_Au_{A1} - m_Bu_{B1} = 0$$

$$u_2 = (m_Au_{A1} + m_Bu_{B1})/\ m_2 = \frac{1}{3} \times 307.06 + \frac{2}{3} \times 1912.67 = 1377.47 \text{ kJ/kg}$$

Interpolate in Table A.8: $T_2 = 1540.2$ K, $s^o_{T2} = 6.6740$ kJ/kgK

State 2: $V_2 = m_2RT_2/P_2 = V_A + V_B = m_ART_{A1}/P_{A1} + m_BRT_{B1}/P_{B1}$

$$= 1 \times R \times 500/100 + 2 \times R \times 2000/200 = 25\ R$$

Substitute m_2, T_2 and solve for P_2

$$P_2 = 3\ R\ T_2/25\ R = 3 \times 1540.2 / 25 = \mathbf{184.82\ kPa}$$

Entropy change from Eq. 8.19

$$(s_2 - s_1)_A = s^o_{T2} - s^o_{T1} - R \ln \frac{P_2}{P_{A1}} = 6.674 - 5.3375 - 0.1889 \ln \frac{184.82}{100}$$

$$= 1.2205 \text{ kJ/kgK}$$

$$(s_2 - s_1)_B = s^o_{T2} - s^o_{T1} - R \ln \frac{P_2}{P_{B1}} = 6.674 - 7.0278 - 0.1889 \ln \frac{184.82}{200}$$

$$= -0.33889 \text{ kJ/kgK}$$

$${}_1S_{2\text{ gen}} = S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B$$

$$= 1 \times 1.2205 + 2\ (-0.33889) = \mathbf{0.543\ kJ/K}$$

8.153

A cylinder/piston contains carbon dioxide at 1 MPa, 300°C with a volume of 200 L. The total external force acting on the piston is proportional to V^3. This system is allowed to cool to room temperature, 20°C. What is the total entropy generation for the process?

Solution:

C.V. Carbon dioxide gas of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37,18: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{amb} + {}_1S_{2\ gen}$

Process: $P = CV^3$ or PV^{-3} = constant, which is polytropic with $n = -3$

State 1: (T, P) => $m = P_1V_1/RT_1 = \frac{1000 \times 0.2}{0.18892 \times 573.2} = 1.847$ kg

State 2: (T, ?) state must be on process curve and ideal gas leads to Eq.8.23

$$\Rightarrow \quad P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 1000(293.2/573.2)^{3/4} = 604.8 \text{ kPa}$$

$$V_2 = V_1(T_1/T_2)^{\frac{1}{n-1}} = 0.16914 \text{ m}^3$$

$${}_1W_2 = \int PdV = (P_2V_2 - P_1V_1)/(1-n)$$

$$= [604.8 \times 0.16914 - 1000 \times 0.2] / [1 - (-3)] = -24.4 \text{ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 1.847 \times 0.653\ (20 - 300) - 24.4 = -362.1 \text{ kJ}$$

From Eq.8.16

$$m(s_2 - s_1) = 1.847\left[0.842 \ln\frac{293.2}{573.2} - 0.18892 \ln\frac{604.8}{1000}\right]$$

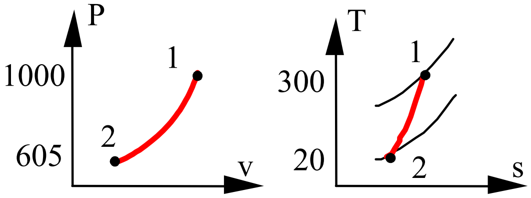
$$= 1.847[-0.4694] = -0.87 \text{ kJ/K}$$

$$\Delta S_{SURR} = -\ {}_1Q_2/T_{amb} = +362.1 / 293.2 = +1.235 \text{ kJ/K}$$

From Eq.8.37 or 8.39

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb} = \Delta S_{NET} = \Delta S_{CO2} + \Delta S_{SURR}$$

$$= -0.87 + 1.235 = \mathbf{+0.365\ kJ/K}$$



Notice:
n = -3, k = 1.3

n < k

8.154

A mass of 2 kg ethane gas at 500 kPa, 100°C, undergoes a reversible polytropic expansion with exponent, *n* = 1.3, to a final temperature of the ambient, 20°C. Calculate the total entropy generation for the process if the heat is exchanged with the ambient.

Solution:

C.V. Ethane gas of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_{amb} + {}_1S_{2\,gen}$

Process: $Pv_1^{\,n} = P_2v_2^{\,n}$

State 1: (T_1, P_1) State 2: $(T_2, ?)$ on process curve

$$P_2 = P_1\,(T_2/T_1)^{\frac{n}{n-1}} = 500\left(\frac{293.2}{373.2}\right)^{4.333} = 175.8 \text{ kPa}$$

Work is integrated in Eq.8.29

$${}_1w_2 = \int_1^2 Pdv = \frac{P_2v_2 - P_1v_1}{1-n} = \frac{R(T_2-T_1)}{1-n} = \frac{0.2765(293.2-373.2)}{1-1.30} = \mathbf{+73.7\ kJ/kg}$$

Heat transfer is from the energy equation

$${}_1q_2 = C_{V0}(T_2 - T_1) + {}_1w_2 = 1.49(293.2 - 373.2) + 73.7 = -45.5 \text{ kJ/kg}$$

Entropy change from Eq.8.16

$$s_2 - s_1 = C_{P0}\ln(T_2/T_1) - R\ln(P_2/P_1)$$

$$= 1.766\ln\frac{293.2}{373.2} - 0.2765\ln\frac{175.8}{500} = -0.1371 \text{ kJ/kg K}$$

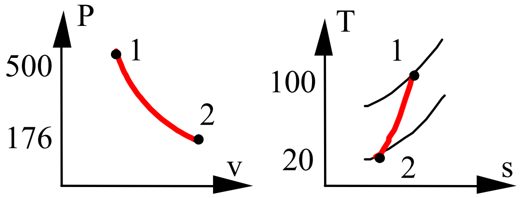
$$m(s_2 - s_1) = \Delta S_{SYST} = 2(-0.1371) = -0.2742 \text{ kJ/K}$$

$$\Delta S_{SURR} = -{}_1Q_2/T_0 = +\,2 \times 45.5/293.2 = +0.3104 \text{ kJ/K}$$

Generation from entropy equation or Eq.8.37 or 39

$${}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb} = \Delta S_{NET} = \Delta S_{SYST} + \Delta S_{SURR}$$

$$= -0.2742 + 0.3104 = \mathbf{+0.0362\ kJ/K}$$



Notice:
n = 1.3, k = 1.186

n > k

8.155

The air in the engine cylinder of Problem 5.128 looses the heat to the engine coolant at 100°C. Find the entropy generation (external to the air) using constant specific heat.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.5.11 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{amb} + {}_1S_{2\ gen}$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $$P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5 \text{ kPa}$$

$$T_2 = T_1 \ (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

From process eq.: $${}_1W_2 = \int P\ dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$$

$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = 70.64 \text{ kJ}$$

From energy eq.: $${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$$

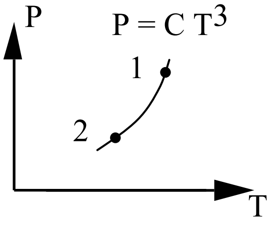
$$= 0.1 \times 0.717(569.3 - 1800) + 70.64 = -17.6 \text{ kJ}$$

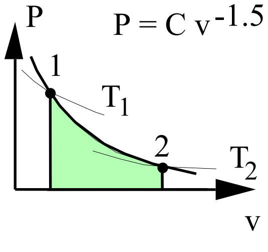
$$m(s_2 - s_1) = m\ [C_P \ln(T_2/T_1) - R \ln(P_2/P_1)]$$

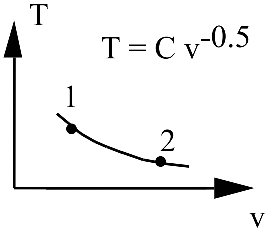
$$= 0.1\ [\ 1.004 \ln \frac{569.3}{1800} - 0.287 \ln \frac{126.5}{4000}\ ] = -0.01645 \text{ kJ/K}$$

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{amb}$$

$$= -0.011645 + 17.6/373.15 = \mathbf{0.0307\ kJ/K}$$







8.156

A cylinder/piston contains 100 L of air at 110 kPa, 25°C. The air is compressed in a reversible polytropic process to a final state of 800 kPa, 200°C. Assume the heat transfer is with the ambient at 25°C and determine the polytropic exponent *n* and the final volume of the air. Find the work done by the air, the heat transfer and the total entropy generation for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$ out to ambient.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14,18: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\,gen} = {}_1Q_2/T_0 + {}_1S_{2\,gen}$

Process: $Pv_1^n = P_2v_2^n$ Eq.8.36

State 1: (T_1, P_1) State 2: (T_2, P_2)

Thus the unknown is the exponent n.

$$m = P_1V_1/(RT_1) = 110 \times 0.1/(0.287 \times 298.15) = 0.1286 \text{ kg}$$

The relation from the process and ideal gas is in Eq.8.37

$$T_2/T_1 = (P_2/P_1)^{\frac{n-1}{n}} \quad => \quad \frac{473.15}{298.15} = \left(\frac{800}{110}\right)^{\frac{n-1}{n}} \quad \Rightarrow \quad \frac{n-1}{n} = 0.2328$$

$$\mathbf{n = 1.3034}, \quad V_2 = V_1(P_1/P_2)^{\frac{1}{n}} = 0.1\left(\frac{110}{800}\right)^{0.7672} = \mathbf{0.02182\ m^3}$$

The work is from Eq.8.38

$${}_1W_2 = \int PdV = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{800 \times 0.02182 - 110 \times 0.1}{1 - 1.3034} = \mathbf{-21.28\ kJ}$$

Heat transfer from the energy equation

$${}_1Q_2 = mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.1286 \times 0.717 \times (200 - 25) - 21.28 = \mathbf{-5.144\ kJ}$$

Entropy change from Eq.8.25

$$s_2 - s_1 = C_{P0}\ln(T_2/T_1) - R\ln(P_2/P_1)$$

$$= 1.004\ln\left(\frac{473.15}{298.15}\right) - 0.287\ln\left(\frac{800}{110}\right) = -0.106\ \frac{\text{kJ}}{\text{kg K}}$$

From the entropy equation (also Eq.8.18)

$${}_1S_{2,gen} = m(s_2 - s_1) - {}_1Q_2/T_0$$

$$= 0.1286 \times (-0.106) + (5.144/298.15) = \mathbf{0.00362\ kJ/K}$$

8.157

A piston/cylinder contains air at 300 K, 100 kPa. A reversible polytropic process with n = 1.3 brings the air to 500 K. Any heat transfer if it comes in is from a 325°C reservoir and if it goes out it is to the ambient at 300 K. Sketch the process in a P-v and a T-s diagram. Find the specific work and specific heat transfer in the process. Find the specific entropy generation (external to the air) in the process.

Solution:

Process : $Pv^n = C$

$$_1w_2 = \int P\,dv = \left(\frac{P_2v_2 - P_1v_1}{1-n}\right) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.287}{1-1.3}(500 - 300) = \mathbf{-191.3\ kJ/kg}$$

Energy equation

$$_1q_2 = u_2 - u_1 + {_1w_2} = C_v(T_2 - T_1) + {_1w_2}$$

$$= 0.717\,(500 - 300) - 191.3 = \mathbf{-47.93\ kJ/kg}$$

The $_1q_2$ is negative and thus goes out. Entropy is generated between the air and ambient.

$$s_2 - s_1 = {_1q_2}/T_{amb} + {_1s_{2\,gen}}$$

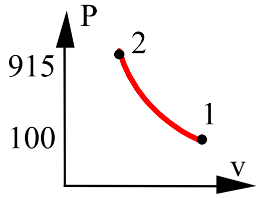
$$_1s_{2\,gen} = s_2 - s_1 - {_1q_2}/T_{amb} = C_p \ln(T_2/T_1) - R\ln(P_2/P_1) - {_1q_2}/T_{amb}$$

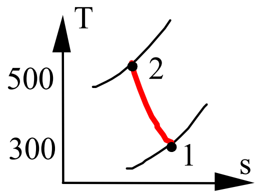
$$P_2/P_1 = (T_2/T_1)^{n/(n-1)} = (500/300)^{1.3/0.3} = 9.148$$

$$_1s_{2\,gen} = 1.004 \ln\left(\frac{500}{300}\right) - 0.287 \ln 9.148 - \left(\frac{-47.93}{300}\right)$$

$$= 0.51287 - 0.635285 + 0.15977$$

$$\mathbf{= 0.03736\ kJ/kg\ K}$$





Notice:
n = 1.3, k = 1.4

n < k

Rates or fluxes of entropy

8.158

A mass of 3 kg nitrogen gas at 2000 K, V = C, cools with 500 W. What is dS/dt?

Assume that we do not generate any s in the nitrogen then

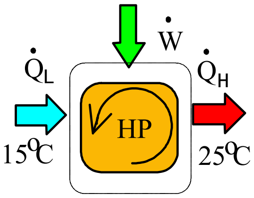
Entropy Eq. 8.42: $\dot{S}_{CV} = \frac{\dot{Q}}{T} = -\frac{500}{2000} = \mathbf{-0.25\ W/K}$

8.159

A reversible heat pump uses 1 kW of power input to heat a 25°C room, drawing energy from the outside at 15°C. Assuming every process is reversible, what are the total rates of entropy into the heat pump from the outside and from the heat pump to the room?

Solution:

C.V.TOT.



Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

Entropy Eq.: $\frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} = 0 \quad \Rightarrow \quad \dot{Q}_L = \dot{Q}_H \frac{T_L}{T_H}$

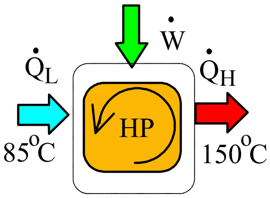
$$\dot{Q}_H \frac{T_L}{T_H} + \dot{W} = \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_H = \frac{T_H}{T_H - T_L} \dot{W}$$

$$\frac{\dot{Q}_H}{T_H} = \frac{1}{T_H - T_L} \dot{W} = \frac{1}{25 - 15} (1) = \mathbf{0.1\ kW/K}$$

$$\frac{\dot{Q}_L}{T_L} = \frac{\dot{Q}_H}{T_H} = \mathbf{0.1\ kW/K}$$

8.160

A heat pump, see problem 7.52, should upgrade 5 MW of heat at 85°C to heat delivered at 150°C. For a reversible heat pump what are the fluxes of entropy in and out of the heat pump?



C.V.TOT. Assume reversible Carnot cycle.

Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

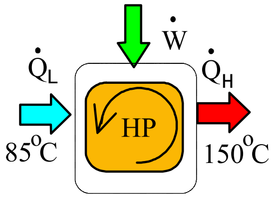
Entropy Eq.: $0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} \quad \Rightarrow \quad \frac{\dot{Q}_L}{T_L} = \frac{\dot{Q}_H}{T_H}$

The fluxes of entropy become the same as

$$\frac{\dot{Q}_H}{T_H} = \frac{\dot{Q}_L}{T_L} = \frac{5}{273.15 + 85} \frac{\text{MW}}{\text{K}} = \mathbf{0.01396\ MW/K}$$

8.161

Reconsider the heat pump in the previous problem and assume it has a COP of 2.5. What are the fluxes of entropy in and out of the heat pump and the rate of entropy generation inside it?



C.V. TOT.

Energy Eq.: $\dot{Q}_L + \dot{W} = \dot{Q}_H$

Entropy Eq.: $0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen\ tot}$

Definition of COP: $\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = 2.5; \quad \beta_{REF} = \beta_{HP} - 1 = \frac{\dot{Q}_L}{\dot{W}_{in}} = 1.50$

$$\dot{W}_{in} = \dot{Q}_L/\beta_{REF} = 5/1.50 = 3.333 \text{ MW}$$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 5 \text{ MW} + 3.333 \text{ MW} = 8.333 \text{ MW}$$

$$\frac{\dot{Q}_L}{T_L} = \frac{5}{273.15 + 85} \frac{\text{MW}}{\text{K}} = \mathbf{0.01396\ MW/K}$$

$$\frac{\dot{Q}_H}{T_H} = \frac{8.333}{273.15 + 85} \frac{\text{MW}}{\text{K}} = \mathbf{0.01969\ MW/K}$$

From the entropy equation

$$\dot{S}_{gen\ tot} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = (0.01969 - 0.01396) \frac{\text{MW}}{\text{K}} = \mathbf{5.73\ kW/K}$$

8.162

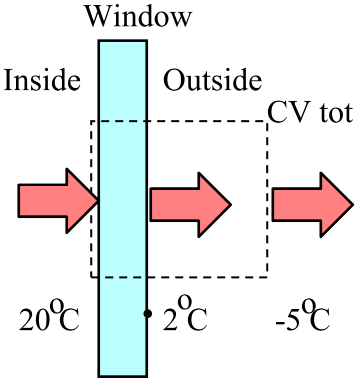
A window receives 200 W of heat transfer at the inside surface of 20°C and transmits the 200 W from its outside surface at 2°C continuing to ambient air at –5°C. Find the flux of entropy at all three surfaces and the window's rate of entropy generation.

Flux of entropy: $\dot{S} = \frac{\dot{Q}}{T}$

$$\dot{S}_{inside} = \frac{200}{293.15}\ \frac{W}{K} = 0.682\ W/K$$

$$\dot{S}_{win} = \frac{200}{275.15}\ \frac{W}{K} = 0.727\ W/K$$

$$\dot{S}_{amb} = \frac{200}{268.15}\ \frac{W}{K} = 0.746\ W/K$$



Window only: $\dot{S}_{gen\ win} = \dot{S}_{win} - \dot{S}_{inside} = 0.727 - 0.682 = \mathbf{0.045\ W/K}$

If you want to include the generation in the outside air boundary layer where T changes from 2°C to the ambient –5°C then chose the control volume as CV tot and it becomes

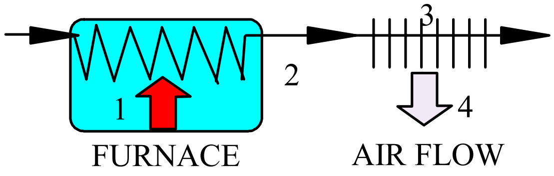
$$\dot{S}_{gen\ tot} = \dot{S}_{amb} - \dot{S}_{inside} = 0.746 - 0.682 = 0.064\ W/K$$

8.163

An amount of power, say 1000 kW, comes from a furnace at 800°C going into water vapor at 400°C. From the water the power goes to a solid metal at 200°C and then into some air at 70°C. For each location calculate the flux of *s* through a surface as ($\dot{Q}/T$). What makes the flux larger and larger?

Solution:

T_1	=>	T_2	=>	T_3	=>	T_4
furnace		vapor		metal		air



Flux of s: $F_s = \dot{Q}/T$ with T as absolute temperature.

F_{s1} = 1000/1073.15 = 0.932 kW/K, F_{s2} = 1000/673.15 = 1.486 kW/K

F_{s3} = 1000/473.15 = 2.11 kW/K, F_{s4} = 1000/343.15 = 2.91 kW/K

T	800	400	200	70	(°C)
T_{amb}	1073	673	476	646	K
Q/T	0.932	1.486	2.114	2.915	kW/K

${}_1S_{2\ gen}$ for every change in T

Q over ΔT is an irreversible process

8.164

Room air at 23°C is heated by a 2000 W space heater with a surface filament temperature of 700 K, shown in Fig. P8.164. The room at steady state looses the power to the outside which is at 7°C. Find the rate(s) of entropy generation and specify where it is made.

Solution:

For any C.V at steady state the entropy equation as a rate form is Eq.8.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

C.V. Heater Element

$$\dot{S}_{gen} = -\int d\dot{Q}/T = -(-2000/700) = \mathbf{2.857\ W/K}$$

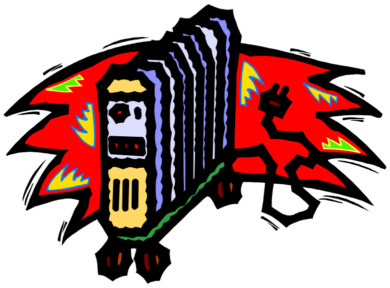
C.V. Space between heater 700 K and room 23°C

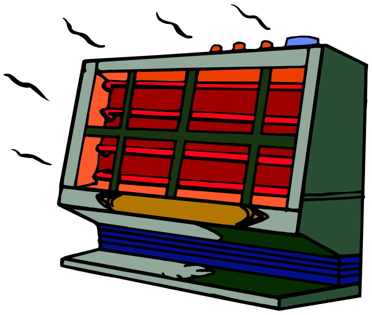
$$\dot{S}_{gen} = -\int d\dot{Q}/T = (-2000\ /\ 700) - [-2000\ /\ (23+273)] = \mathbf{3.9\ W/K}$$

C.V. Wall between 23°C inside and 7°C outside

$$\dot{S}_{gen} = -\int d\dot{Q}/T = [-2000\ /\ (23+273)] - [2000\ /\ (7+273)] = \mathbf{0.389\ W/K}$$

Notice biggest $\dot{S}_{gen}$ is for the largest change in 1/T.





8.165

A car engine block receives 2 kW at its surface of 450 K from hot combustion gases at 1500 K. Near the cooling channel the engine block transmits 2 kW out at its 400 K surface to the coolant flowing at 370 K. Finally, in the radiator the coolant at 350 K delivers the 2 kW to air which is at 25 C. Find the rate of entropy generation inside the engine block, inside the coolant and in the radiator/air combination.

For a C.V at steady state we have the entropy equation as a rate form as Eq.8.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

CV1. Engine block receives 2 kW at 450 K and it leaves at 400 K

$$\dot{S}_{gen1} = -\int d\dot{Q}/T = \frac{-2000}{450} - [\frac{-2000}{400}] = \mathbf{0.555\ W/K}$$

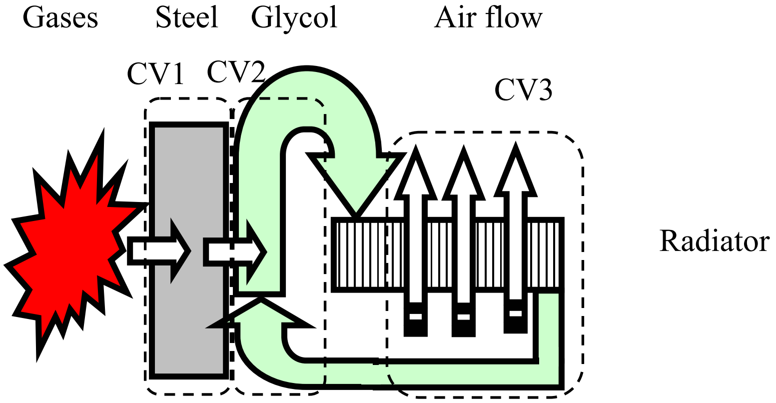
CV2. The coolant receives 2 kW at 370 K andf gives it out at 350 K

$$\dot{S}_{gen2} = -\int d\dot{Q}/T = [\frac{-2000}{370}] - [\frac{-2000}{350}] = \mathbf{0.309\ W/K}$$

CV3 Radiator to air heat transfer.

$$\dot{S}_{gen3} = -\int d\dot{Q}/T = [\frac{-2000}{350}] - [\frac{-2000}{298.15}] = \mathbf{0.994\ W/K}$$

Notice the biggest $\dot{S}_{gen}$ is for the largest change $\Delta[1/T]$



Remark: The flux of S is $\dot{Q}/T$ flowing across a surface. Notice how this flux increases as the heat transfer flows towards lower and lower T.

T [K]	1500	450	370	298.15
$\dot{Q}/T$ [W/K]	1.33	4.44	5.40	6.71

8.166

Consider an electric heater operating in steady state with 1 kW electric power input and a surface temperature of 600 K that gives heat transfer to the room air at 22^{o}C. What is the rate of entropy generation in the heating element? What is it outside?

For any C.V at steady state the entropy equation as a rate form is Eq.8.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

C.V. Heater Element. The energy equation has 1 kW in as work and 1 kW out as heat transfer

$$\dot{S}_{gen\ element} = -\int d\dot{Q}/T = -\frac{-1000}{600} = \mathbf{1.667\ W/K}$$

C.V. Space between heater 600 K and room at 22°C. This control volume has 1 kW in at 600 K and 1 kW out at 22^{o}C both are heat transfers.

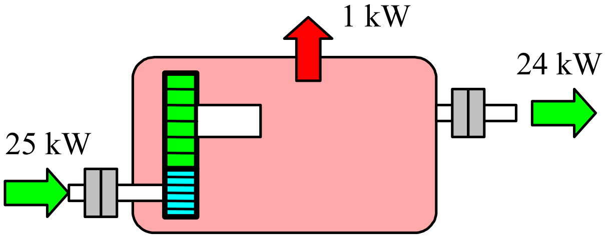
$$\dot{S}_{gen\ outside\ element} = -\int d\dot{Q}/T = -\frac{1000}{600} - [\frac{-1000}{22 + 273}] = \mathbf{1.723\ W/K}$$

8.167

The automatic transmission in a car receives 25 kW shaft work and gives out 24 kW to the drive shaft. The balance is dissipated in the hydraulic fluid and metal casing, all at 45^{o}C, which in turn transmits it to the outer atmosphere at 20^{o}C. What is the rate of entropy generation inside the transmission unit? What is it outside the unit?

Solution:

C.V. Total unit. Steady state and surface at 45^{o}C



Energy Eq:

$$0 = \dot{W}_{in} - \dot{W}_{out} - \dot{Q}_{out}$$

Entropy Eq.: $0 = - \frac{\dot{Q}_{out}}{T_{oil}} + \dot{S}_{gen}$

From energy Eq.: $\dot{Q}_{out} = \dot{W}_{in} - \dot{W}_{out} = 25 - 24 = 1 \text{ kW}$

From entropy Eq.: $\dot{S}_{gen} = \frac{\dot{Q}_{out}}{T_{oil}} = \frac{1}{273.15 + 45} \frac{\text{kW}}{\text{K}} = \mathbf{3.1\ W/K}$

C.V. From surface at 45^{o}C to atm. at 20^{o}C.

Entropy Eq.: $0 = \frac{\dot{Q}_{out}}{T_{oil}} - \frac{\dot{Q}_{out}}{T_{amb}} + \dot{S}_{gen\ outside}$

$$\dot{S}_{gen\ outside} = \dot{Q}_{out} \left[\frac{1}{T_{amb}} - \frac{1}{T_{oil}}\right] = 1 \text{ kW} \left[\frac{1}{293} - \frac{1}{318}\right] = \mathbf{0.268\ W/K}$$

8.168

A farmer runs a heat pump using 2 kW of power input. It keeps a chicken hatchery at a constant 30°C while the room loses 10 kW to the colder outside ambient at 10°C. What is the rate of entropy generated in the heat pump? What is the rate of entropy generated in the heat loss process?

Solution:

C.V. Hatchery, steady state.

To have steady state at 30°C for the hatchery

Energy Eq.: $0 = \dot{Q}_H - \dot{Q}_{Loss} \quad \Rightarrow \quad \dot{Q}_H = \dot{Q}_{Loss} = 10 \text{ kW}$

C.V. Heat pump, steady state

Energy eq.: $0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 8 \text{ kW}$

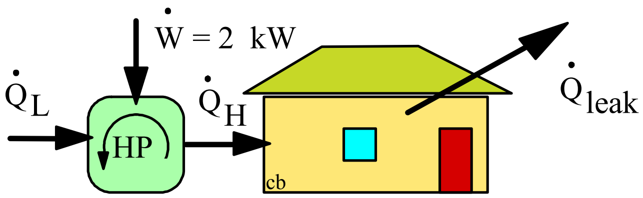
Entropy Eq.: $0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen\ HP}$

$$\dot{S}_{gen\ HP} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = \frac{10}{273 + 30} - \frac{8}{273 + 10} = \mathbf{0.00473\ kW/K}$$

C.V. From hatchery at 30°C to the ambient 10°C. This is typically the walls and the outer thin boundary layer of air. Through this goes $\dot{Q}_{Loss}$.

Entropy Eq.: $0 = \frac{\dot{Q}_{Loss}}{T_H} - \frac{\dot{Q}_{Loss}}{T_{amb}} + \dot{S}_{gen\ walls}$

$$\dot{S}_{gen\ walls} = \frac{\dot{Q}_{Loss}}{T_{amb}} - \frac{\dot{Q}_{Loss}}{T_H} = \frac{10}{283} - \frac{10}{303} = \mathbf{0.00233\ kW/K}$$



Review problems

8.169

A device brings 2 kg of ammonia from 150 kPa, -20°C to 400 kPa, 80°C in a polytropic process. Find the polytropic exponent, n, the work and the heat transfer. Find the total entropy generated assuming a source at 100°C.

Solution:

C.V. Ammonia of constant mass $m_2 = m_1 = m$ out to source.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14, 8.18: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T + {}_1S_{2\ gen}$

Process: $P_1v_1^n = P_2v_2^n$ Eq. (8.36)

State 1: Table B.2.2

$v_1 = 0.79774\ m^3/kg$, $s_1 = 5.7465\ kJ/kg\ K$, $u_1 = 1303.3\ kJ/kg$

State 2: Table B.2.2

$v_2 = 0.4216\ m^3/kg$, $s_2 = 5.9907\ kJ/kg\ K$, $u_2 = 1468.0\ kJ/kg$

$$\ln (P_2/P_1) = \ln (v_1/v_2)^n = n \times \ln (v_1/v_2)$$

$$\ln \left(\frac{400}{150}\right) = n \times \ln \left(\frac{0.79774}{0.4216}\right) = 0.98083 = n \times 0.63773$$

$\Rightarrow$ $n = \mathbf{1.538}$

The work term is integration of PdV as done in text leading to Eq.8.38

$$ {}_1W_2 = \frac{m}{1 - n}(P_2v_2 - P_1v_1)$$

$$= \frac{2}{1 - 1.538} \times (400 \times 0.4216 - 150 \times 0.79774) = \mathbf{-182.08\ kJ}$$

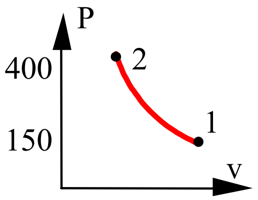
Notice we did not use Pv = RT as we used the ammonia tables.

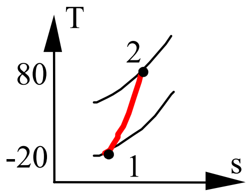
$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2\ (1468 - 1303.3) - 182.08 = \mathbf{147.3\ kJ}$$

From Eq.8.18

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T = 2\ (5.9907 - 5.7465) - \frac{147.3}{373.15}$$

$$= \mathbf{0.0936\ kJ/K}$$





Notice:
n = 1.54, k = 1.3

n > k

8.170

An insulated cylinder/piston has an initial volume of 0.15 m^3 and contains steam at 400 kPa, 200°C. The steam is expanded adiabaticly, and the work output is measured very carefully to be 30 kJ. It is claimed that the final state of the water is in the two-phase (liquid and vapor) region. What is your evaluation of the claim?

Solution:

C.V. Water.

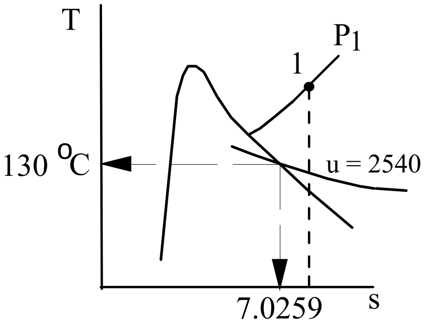
Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$

Process: ${}_1Q_2 = 0$ and reversible

State 1: (T, P) Table B.1.3

$$v_1 = 0.5342, \quad u_1 = 2646.8, \quad s_1 = 7.1706 \text{ kJ/kg K}$$



$$m = \frac{V_1}{v_1} = \frac{0.15}{0.5342} = 0.2808 \text{ kg}$$

With the assumed reversible process we have from entropy equation

$$s_2 = s_1 = 7.1706 \text{ kJ/kg K}$$

and from the energy equation

$$u_2 = u_1 - {}_1W_2/m = 2646.8 - \frac{30}{0.2808} = 2540.0 \text{ kJ/kg}$$

State 2 given by (u, s) check Table B.1.1: s_G (at $u_G = 2540$) $= 7.0259 < s_1$

⇒ **State 2 must be in superheated vapor region.**

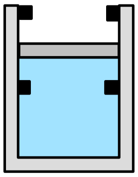
8.171

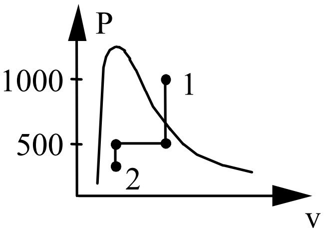
Water in a piston/cylinder is at 1 MPa, 500°C. There are two stops, a lower one at which $V_{\min} = 1\ \text{m}^3$ and an upper one at $V_{\max} = 3\ \text{m}^3$. The piston is loaded with a mass and outside atmosphere such that it floats when the pressure is 500 kPa. This setup is now cooled to 100°C by rejecting heat to the surroundings at 20°C. Find the total entropy generated in the process.

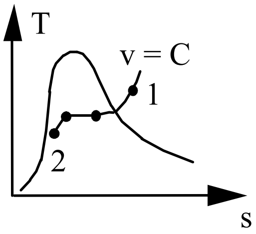
C.V. Water.

Initial state: Table B.1.3: $v_1 = 0.35411\ \text{m}^3/\text{kg}$, $u_1 = 3124.3$, $s_1 = 7.7621$

$m = V/v_1 = 3/0.35411 = 8.472\ \text{kg}$







Final state: 100°C and on line in P-V diagram.

Notice the following: $v_g(500\ \text{kPa}) = 0.3749 > v_1$, $v_1 = v_g(154°\text{C})$

$T_{sat}(500\ \text{kPa}) = 152°\text{C} > T_2$, so now piston hits bottom stops.

State 2: $v_2 = v_{bot} = V_{bot}/m = 0.118\ \text{m}^3/\text{kg}$,

$x_2 = (0.118 - 0.001044)/1.67185 = 0.0699$,

$u_2 = 418.91 + 0.0699 \times 2087.58 = 564.98\ \text{kJ/kg}$,

$s_2 = 1.3068 + 0.0699 \times 6.048 = 1.73\ \text{kJ/kg K}$

Now we can do the work and then the heat transfer from the energy equation

$_1W_2 = \int P dV = 500(V_2 - V_1) = -1000\ \text{kJ}$ $\quad (_1w_2 = -118\ \text{kJ/kg})$

$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = -22683.4\ \text{kJ}$ $\quad (_1q_2 = -2677.5\ \text{kJ/kg})$

Take C.V. total out to where we have 20°C:

$m(s_2 - s_1) = {}_1Q_2/T_0 + S_{gen} \Rightarrow$

$S_{gen} = m(s_2 - s_1) - {}_1Q_2/T_0 = 8.472\ (1.73 - 7.7621) + 22683 / 293.15$

$\mathbf{= 26.27\ kJ/K}$ $\quad (= \Delta S_{water} + \Delta S_{sur})$

8.172

Assume the heat transfer in problem 5.63 came from a 200°C reservoir. What is the total entropy generation in the process?

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0 \quad \Rightarrow m_2 = m_{A1} + m_{B1} = 0.5 + 0.5 = 1 \text{ kg}$

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m_2 s_2 - m_A s_{A1} - m_B s_{B1} = {}_1Q_2/T_{res} + {}_1S_{2\,gen}$

State A1: $v_{A1} = 0.001067 + x_{A1} \times 0.71765 = 0.072832$; $V_{A1} = mv = 0.036416 \text{ m}^3$

$u_{A1} = 535.08 + 0.1 \times 2002.14 = 735.22 \text{ kJ/kg}$;

$s_{A1} = 1.6072 + 0.1 \times 5.4455 = 2.15175 \text{ kJ/kgK}$

State B1: $v_{B1} = 1.5493 \text{ m}^3\text{/kg}$; $u_{B1} = 2966.69 \text{ kJ/kg}$; $s_{B1} = 8.2217 \text{ kJ/kg-K}$

$V_{B1} = (mv)_{B1} = 0.77465 \text{ m}^3$

State 2: If $V_2 > V_{A1}$ then $P_2 = 200$ kPa that is the piston floats.

For $(T_2, P_2) = (150°C, 200 \text{ kPa})$ => superheated vapor

$u_2 = 2576.87 \text{ kJ/kg}$; $v_2 = 0.95964 \text{ m}^3\text{/kg}$, $s_2 = 7.2795 \text{ kJ/kgK}$

$V_2 = m_2 v_2 = 0.95964 \text{ m}^3 > V_{A1}$ checks OK.

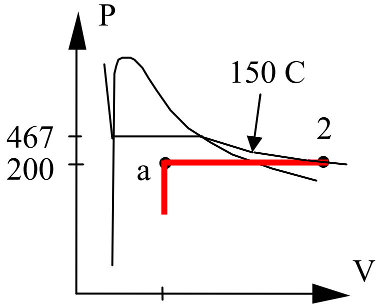
Process: ${}_1W_2 = P_2 (V_2 - V_1) = 200 (0.95964 - 0.77465 - 0.036416) = 29.715 \text{ kJ}$

From the energy and entropy equations:

$${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$$
$$= 1 \times 2576.87 - 0.5 \times 735.222 - 0.5 \times 2966.69 + 29.715 = 755.63 \text{ kJ}$$

$${}_1S_{2\,gen} = m_2 s_2 - m_A s_{A1} - m_B s_{B1} - {}_1Q_2/T_{res}$$
$$= 1 \times 7.2795 - 0.5 \times 2.15175 - 0.5 \times 8.2217 - 755.63/473.15$$
$$= \mathbf{0.496 \text{ kJ/K}}$$

The possible state 2 (P,V) combinations are shown. State a is 200 kPa, $v_a = V_{A1}/m_2 = 0.036$ and thus two-phase $T_a = 120°C$ less than T_2



8.173

A closed tank, *V* = 10 L, containing 5 kg of water initially at 25°C, is heated to 175°C by a heat pump that is receiving heat from the surroundings at 25°C. Assume that this process is reversible. Find the heat transfer to the water and the work input to the heat pump.

C.V.: Water from state 1 to state 2.

Process: constant volume (reversible isometric)

$1\text{: } v_1 = V/m = 0.002 \Rightarrow\ x_1 = (0.002 - 0.001003)/43.358 = 0.000023$

$u_1 = 104.86 + 0.000023 \times 2304.9 = 104.93 \text{ kJ/kg}$

$s_1 = 0.3673 + 0.000023 \times 8.1905 = 0.36759 \text{ kJ/kg K}$

Continuity eq. (same mass) and V = C fixes v_2

$2\text{: } T_2, v_2 = v_1 \Rightarrow$

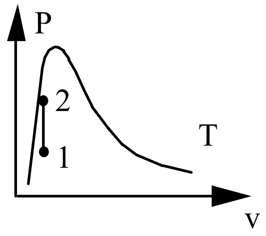
$x_2 = (0.002 - 0.001121)/0.21568 = 0.004075$

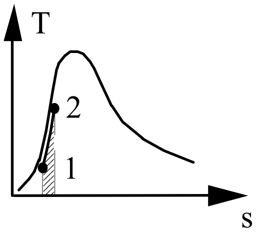
$u_2 = 740.16 + 0.004075 \times 1840.03 = 747.67 \text{ kJ/kg}$

$s_2 = 2.0909 + 0.004075 \times 4.5347 = 2.1094 \text{ kJ/kg K}$

Energy eq. has W = 0, thus provides heat transfer as

$${}_1Q_2 = m(u_2 - u_1) = \mathbf{3213.7\ kJ}$$



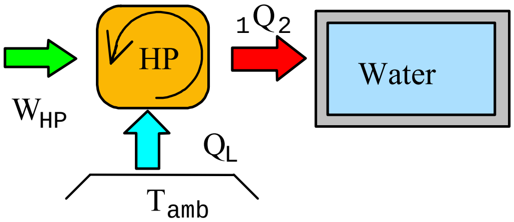


Entropy equation for the total (tank plus heat pump) control volume gives for a reversible process:

$$m(s_2 - s_1) = Q_L/T_0 \qquad \Rightarrow\ Q_L = mT_0(s_2 - s_1) = 2596.6 \text{ kJ}$$

and then the energy equation for the heat pump gives

$$W_{HP} = {}_1Q_2 - Q_L = \mathbf{617.1\ kJ}$$



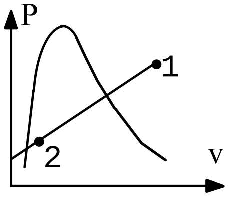
8.174

A cylinder/piston contains 3 kg of water at 500 kPa, 600°C. The piston has a cross-sectional area of 0.1 m^2 and is restrained by a linear spring with spring constant 10 kN/m. The setup is allowed to cool down to room temperature due to heat transfer to the room at 20°C. Calculate the total (water and surroundings) change in entropy for the process.

State 1: Table B.1.3,

$$v_1 = 0.8041 \text{ m}^3\text{/kg}, \quad u_1 = 3299.6 \text{ kJ/kg}, \quad s_1 = 7.3522 \text{ kJ/kg-K}$$

State 2: T_2 & on line in P-V diagram.



$$P = P_1 + (k_s/A^2_{cyl})(V - V_1)$$

Assume state 2 is two-phase, $P_2 = P_{sat}(T_2) = 2.339$ kPa

$$v_2 = v_1 + (P_2 - P_1)A^2_{cyl}/mk_s$$

$$v_2 = 0.8041 + (2.339 - 500)0.01/(3 \times 10) \;= 0.6382 = v_f + x_2v_{fg}$$

$$x_2 = (0.6382 - 0.001002)/57.7887 = 0.011,$$

$$u_2 = 109.46 \text{ kJ/kg}, \quad s_2 = 0.3887 \text{ kJ/kg-K}$$

$$_1W_2 = \frac{1}{2}(P_1 + P_2)m \times (v_2 - v_1)$$

$$= \frac{1}{2}(500 + 2.339) \times 3 \times (0.6382 - 0.8041) = -125 \text{ kJ}$$

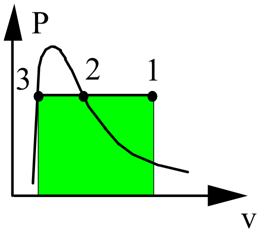
$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 3(109.46 - 3299.6) - 125 = -9695.4 \text{ kJ}$$

$$\Delta S_{tot} = S_{gen,tot} = m(s_2 - s_1) - {}_1Q_2/T_{room}$$

$$= 3(0.3887 - 7.3522) + 9695.4/293.15 = \mathbf{12.18 \ kJ/K}$$

8.175

A cylinder fitted with a frictionless piston contains water. A constant hydraulic pressure on the back face of the piston maintains a cylinder pressure of 10 MPa. Initially, the water is at 700°C, and the volume is 100 L. The water is now cooled and condensed to saturated liquid. The heat released during this process is the Q supply to a cyclic heat engine that in turn rejects heat to the ambient at 30°C. If the overall process is reversible, what is the net work output of the heat engine?



C.V.: H_2O, $1 \rightarrow 3$, this is a control mass:

Continuity Eq.: $m_1 = m_3 = m$

Energy Eq.: $m(u_3 - u_1) = {}_1Q_3 - {}_1W_3$;

Process: $P = C \Rightarrow {}_1W_3 = \int P\, dV = Pm(v_3 - v_1)$

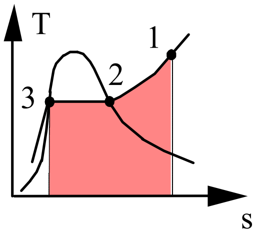
State 1: 700°C, 10 MPa, $V_1 = 100$ L Table B.1.4

$v_1 = 0.04358\ m^3/kg \Rightarrow m = m_1 = V_1/v_1 = 2.295$ kg

$h_1 = 3870.5$ kJ/kg, $s_1 = 7.1687$ kJ/kg K

State 3: $P_3 = P_1 = 10$ MPa, $x_3 = 0$ Table B.1.2

$h_3 = h_f = 1407.5$ kJ/Kg, $s_3 = s_f = 3.3595$ kJ/Kg K



$${}_1Q_3 = m(u_3 - u_1) + Pm(v_3 - v_1) = m(h_3 - h_1)$$
$$= -5652.6 \text{ kJ}$$

Heat transfer to the heat engine:

$Q_H = -{}_1Q_3 = 5652.6$ kJ

Take control volume as total water and heat engine.

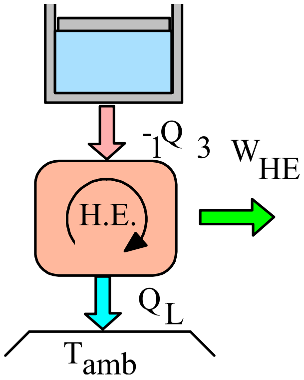
Process: Rev., $\Delta S_{net} = 0$; $T_L = 30^oC$

2nd Law: $\Delta S_{net} = m(s_3 - s_1) - Q_{cv}/T_L$;

$Q_{cv} = T_o\, m(s_3 - s_1) = -2650.6$ kJ

$\Rightarrow$ $Q_L = -Q_{cv} = 2650.6$ kJ

$W_{net} = W_{HE} = Q_H - Q_L =$ **3002 kJ**



8.176

A resistor in a heating element is a total of 0.5 kg with specific heat of 0.8 kJ/kgK. It is now receiving 500 W of electric power so it heats from 20^{o}C to 150^{o} C. Neglect external heat loss and find the time the process took and the entropy generation.

C.V. Heating element.

Energy Eq.: $m(u_2 - u_1) = {}_1W_{2\text{ in}} = \dot{W}_{\text{electrical in}}\,\Delta t$

Entropy Eq.: $m(s_2 - s_1) = 0 + {}_1S_{2\text{ gen}}$ (no heat transfer)

$$\Delta t = m(u_2 - u_1) / \dot{W}_{\text{electrical in}} = m\,C\,(T_2 - T_1) / \dot{W}_{\text{electrical in}}$$

$$= 0.5\text{ kg} \times 800\text{ J/kg-K} \times (150 - 20)\text{ K} / 500\text{ (J/s)} = \mathbf{104\ s}$$

$${}_1S_{2\text{ gen}} = m(s_2 - s_1) = m\,C\,\ln\frac{T_2}{T_1} = 0.5\text{ kg} \times 0.8\text{ kJ/kg-K}\ \ln\left(\frac{150 + 273}{20 + 273}\right)$$

$$= \mathbf{0.15\ kJ/K}$$

8.177

Two tanks contain steam, and they are both connected to a piston/cylinder as shown in Fig. P8.177. Initially the piston is at the bottom and the mass of the piston is such that a pressure of 1.4 MPa below it will be able to lift it. Steam in A is 4 kg at 7 MPa, 700°C and B has 2 kg at 3 MPa, 350°C. The two valves are opened, and the water comes to a uniform state. Find the final temperature and the total entropy generation, assuming no heat transfer.

Solution:

Control mass: All water $m_A + m_B$.

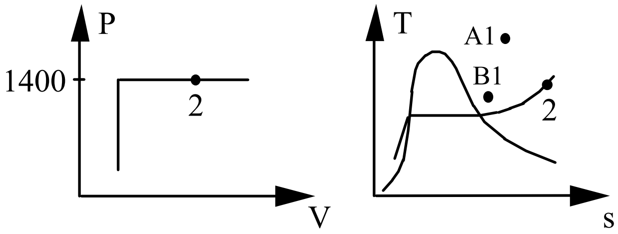
Continuity Eq.: $m_2 = m_A + m_B = 6$ kg

Energy Eq.5.11: $m_2u_2 - m_Au_{A1} - m_Bu_{B1} = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.8.14: $m_2s_2 - m_As_{A1} - m_Bs_{B1} = {}_1S_{2\text{ gen}}$

B.1.3: $v_{A1} = 0.06283$, $u_{A1} = 3448.5$, $s_{A1} = 7.3476$, $V_A = 0.2513$ m^3

B.1.3: $v_{B1} = 0.09053$, $u_{B1} = 2843.7$, $s_{B1} = 6.7428$, $V_B = 0.1811$ m^3



The only possible P, V combinations for state 2 are on the two lines.

Assume $V_2 > V_A + V_B \Rightarrow P_2 = P_{lift}$, ${}_1W_2 = P_2(V_2 - V_A - V_B)$

Substitute into energy equation:

$m_2h_2 = m_Au_{A1} + m_Bu_{B1} + P_2(V_A + V_B)$

$= 4 \times 3448.5 + 2 \times 2843.7 + 1400 \times 0.4324$

State 2: $h_2 = 3347.8$ kJ/kg, $P_2 = 1400$ kPa, $v_2 = 0.2323$, $s_2 = 7.433$

$T_2 = 441.9$ °C,

Check assumption: $V_2 = m_2v_2 = 1.394$ m^3 $> V_A + V_B$ **OK.**

${}_1S_{2\text{ gen}} = 6 \times 7.433 - 4 \times 7.3476 - 2 \times 6.7428 =$ **1.722 kJ/K**

8.178

A cylinder fitted with a piston contains 0.5 kg of R-134a at 60°C, with a quality of 50 percent. The R-134a now expands in an internally reversible polytropic process to ambient temperature, 20°C at which point the quality is 100 percent. Any heat transfer is with a constant-temperature source, which is at 60°C. Find the polytropic exponent n and show that this process satisfies the second law of thermodynamics.

Solution:

C.V.: R-134a, Internally Reversible, Polytropic Expansion: $PV^n = \text{Const.}$

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

State 1: $T_1 = 60^oC$, $x_1 = 0.5$, Table B.5.1: $P_1 = P_g = 1681.8$ kPa,

$v_1 = v_f + x_1 v_{fg} = 0.000951 + 0.5 \times 0.010511 = 0.006207\ m^3/kg$

$s_1 = s_f + x_1 s_{fg} = 1.2857 + 0.5 \times 0.4182 = 1.4948$ kJ/kg K,

$u_1 = u_f + x_1 u_{fg} = 286.19 + 0.5 \times 121.66 = 347.1$ kJ/kg

State 2: $T_2 = 20^oC$, $x_2 = 1.0$, $P_2 = P_g = 572.8$ kPa, Table B.5.1

$v_2 = v_g = 0.03606\ m^3/kg$, $s_2 = s_g = 1.7183$ kJ/kg-K

$u_2 = u_g = 389.19$ kJ/kg

Process: $PV^n = \text{Const.} \Rightarrow \frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^n \Rightarrow n = \ln\frac{P_1}{P_2} / \ln\frac{v_2}{v_1} = \mathbf{0.6122}$

$${}_1W_2 = \int PdV = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= 0.5(572.8 \times 0.03606 - 1681.8 \times 0.006207)/(1 - 0.6122) = 13.2 \text{ kJ}$$

2^{nd} Law for C.V.: R-134a plus wall out to source:

$\Delta S_{net} = m(s_2 - s_1) - \frac{Q_H}{T_H}$; Check $\Delta S_{net} \geq 0$

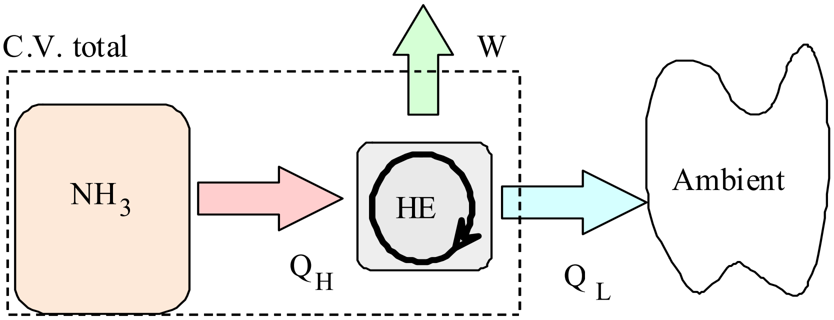
$Q_H = {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 34.2$ kJ

$\Delta S_{net} = 0.5(1.7183 - 1.4948) - 34.2/333.15 = 0.0092$ kJ/K,

$\mathbf{\Delta S_{net} > 0}$ **Process Satisfies 2^{nd} Law**

8.179

A rigid tank with 0.5 kg ammonia at 1600 kPa, 160°C is cooled in a reversible process by giving heat to a reversible heat engine that has its cold side at ambient 20°C, shown in Fig. P8.179. The ammonia eventually reaches 20°C and the process stops. Find the heat transfer from the ammonia to the heat engine and the work output of the heat engine.



C.V. Ammonia

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.3: $m(s_2 - s_1) = \int dQ/T$ (T not constant)

Process: v = constant $\Rightarrow$ ${}_1W_2 = 0$

State 1: (T, P) Table B.2.2: $u_1 = 1596.1$ kJ/kg, $v_1 = 0.12662$ m^3/kg,

$s_1 = 5.7485$ kJ/kgK

State 2: T_2 and $v_2 = v_1$ Table B.2.1 as $v_2 < v_g$

$\Rightarrow$ 2-phase, $P_2 = P_{sat} = 857.5$ kPa

$$x_2 = (v_2 - v_f)/v_{fg} = (0.12662 - 0.001638)/0.14758 = 0.846876$$

$$u_2 = u_f + x_2\ u_{fg} = 272.89 + 0.846876 \times 1059.3 = 1170 \text{ kJ/kg}$$

$$s_2 = s_f + x_2\ s_{fg} = 1.0408 + 0.846876 \times 4.0452 = 4.4666 \text{ kJ/kgK}$$

From the energy equation

$$Q_H = -{}_1Q_2 = -m(u_2 - u_1) = -0.5(1170 - 1596.1) = \mathbf{213.05\ kJ}$$

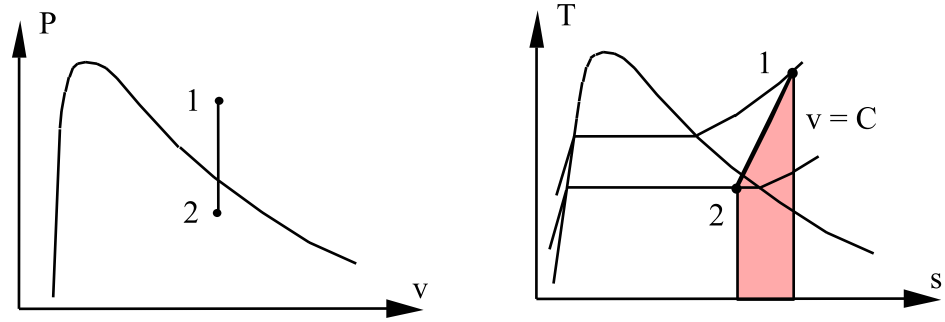
Take now CV total ammonia plus heat engine out to ambient

Entropy Eq.8.3: $m(s_2 - s_1) = -\dfrac{Q_L}{T_{amb}}$ $\Rightarrow$

$$Q_L = -mT_{amb}\,(s_2 - s_1) = -\,0.5 \times 293.15\ (4.4666 - 5.7485)$$
$$= 187.89 \text{ kJ}$$

Now the CV heat engine can give the engine work from the energy equation

Energy H.E.: $W_{HE} = Q_H - Q_L = 213.05 - 187.89 =$ **25.2 kJ**



Notice to get $_1q_2 = \int T\,ds$ we must know the function T(s) which we do not readily have for this process.

8.180

A piston/cylinder with constant loading of piston contains 1 L water at 400 kPa, quality 15%. It has some stops mounted so the maximum possible volume is 11 L. A reversible heat pump extracting heat from the ambient at 300 K, 100 kPa heats the water to 300°C. Find the total work and heat transfer for the water and the work input to the heat pump.

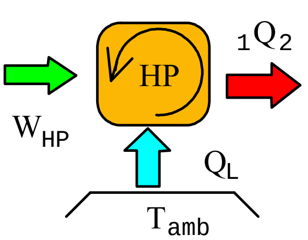
Solution: Take CV around the water and check possible P-V combinations.

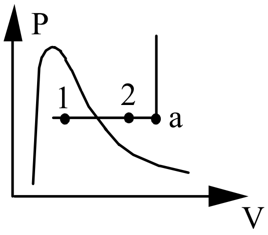
State 1: $v_1 = 0.001084 + 0.15 \times 0.46138 = 0.07029\ \text{m}^3/\text{kg}$

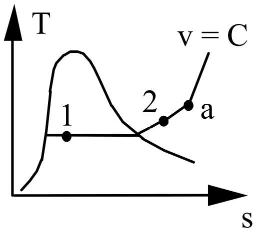
$u_1 = 604.29 + 0.15 \times 1949.26 = 896.68\ \text{kJ/kg}$

$s_1 = 1.7766 + 0.15 \times 5.1193 = 2.5445\ \text{kJ/kg K}$

$m_1 = V_1/v_1 = 0.001/0.07029 = 0.0142\ \text{kg}$







State a: $v = 11\ v_1 = 0.77319\ \text{m}^3/\text{kg}$,

400 kPa

=> Sup. vapor $T_a = 400^\circ\text{C} > T_2$

State 2: Since $T_2 < T_a$ then piston is not at stops but floating so $P_2 = 400$ kPa.

$(T, P) \Rightarrow v_2 = 0.65484\ \text{m}^3/\text{kg} \Rightarrow V_2 = (v_2/v_1) \times V_1 = 9.316\ \text{L}$

$_1W_2 = \int P\, dV = P(V_2 - V_1) = 400\ (9.316 - 1) \times 0.001 = \mathbf{3.33\ kJ}$

$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 0.0142\ (2804.8 - 896.68) + 3.33 = \mathbf{30.43\ kJ}$

Take CV as water plus the heat pump out to the ambient.

$m(s_2 - s_1) = Q_L/T_o \Rightarrow$

$Q_L = mT_o\ (s_2 - s_1) = 300 \times 0.0142\ (7.5661 - 2.5445) = 21.39\ \text{kJ}$

$W_{HP} = {_1Q_2} - Q_L = \mathbf{9.04\ kJ}$

8.181

A cylinder with a linear spring-loaded piston contains carbon dioxide gas at 2 MPa with a volume of 50 L. The device is of aluminum and has a mass of 4 kg. Everything (Al and gas) is initially at 200°C. By heat transfer the whole system cools to the ambient temperature of 25°C, at which point the gas pressure is 1.5 MPa. Find the total entropy generation for the process.

CO_2: $m = P_1V_1/RT_1 = 2000 \times 0.05/(0.18892 \times 473.2) = 1.1186 \text{ kg}$

$$V_2 = V_1(P_1/P_2)(T_2/T_1) = 0.05(2/1.5)(298.2/473.2) = 0.042 \text{ m}^3$$

$${}_1W_{2\,CO_2} = \int PdV = \frac{P_1+P_2}{2}(V_2 - V_1) = \frac{2000+1500}{2}(0.042 - 0.050) = -14.0 \text{ kJ}$$

$${}_1Q_{2\,CO_2} = mC_{V0}(T_2-T_1) + {}_1W_2 = 1.1186\times0.6529(25\text{-}200)\text{-}14.0 = -141.81 \text{ kJ}$$

$${}_1Q_{2\,Al} = mC(T_2 - T_1) = 4 \times 0.90(25 - 200) = -630 \text{ kJ}$$

System: CO_2 + Al

Entropy Eq.: $m_{CO_2}(s_2 - s_1)_{CO_2} + m_{AL}(s_2 - s_1)_{AL} = {}_1Q_2/T_0 + {}_1S_{2\,gen}$

$${}_1Q_2 = -141.81 - 630 = -771.81 \text{ kJ}$$

$$\Delta S_{SYST} = m_{CO_2}(s_2 - s_1)_{CO_2} + m_{AL}(s_2 - s_1)_{AL}$$

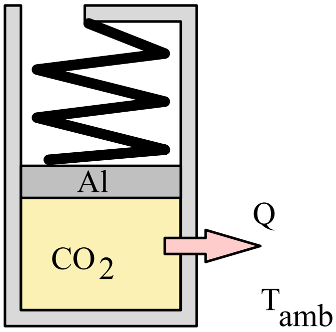
$$= 1.1186\left[0.8418 \ln\frac{298.2}{473.2} - 0.18892 \ln\frac{1.5}{2.0}\right] + 4 \times 0.9 \ln(298.2/473.2)$$

$$= -0.37407 - 1.6623 = -2.0364 \text{ kJ/K}$$

$$\Delta S_{SURR} = -({}_1Q_2/T_0) = +(771.81/298.15) = +2.5887 \text{ kJ/K}$$

$${}_1S_{2\,gen} = m_{CO_2}(s_2 - s_1)_{CO_2} + m_{AL}(s_2 - s_1)_{AL} - ({}_1Q_2/T_0)$$

$$= -2.0364 + 2.5887 = \mathbf{+0.552 \text{ kJ/K}}$$



8.182

An un-insulated cylinder fitted with a piston contains air at 500 kPa, 200°C, at which point the volume is 10 L. The external force on the piston is now varied in such a manner that the air expands to 150 kPa, 25 L volume. It is claimed that in this process the air produces 70% of the work that would have resulted from a reversible, adiabatic expansion from the same initial pressure and temperature to the same final pressure. Room temperature is 20°C.

a) What is the amount of work claimed?
b) Is this claim possible?

Solution:

C.V.: Air; $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg K, $C_v = 0.717$ kJ/kg K

State 1: $T_1 = 200^{o}C$, $P_1 = 500$ kPa, $V_1 = 10$ L $= 0.01$ m^3;

$$m_1 = V_1/v_1 = P_1V_1/RT_1 = 0.0368 \text{ kg}$$

State 2: $P_2 = 150$ kPa, $V_2 = 25$ L $= 0.025$ m^3

$\eta_s = 70\%$; Actual Work is 70% of Isentropic Work

a) Assume Reversible and Adiabatic Process; $s_1 = s_{2s}$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 473.15\ (150 / 500) = 335.4 \text{ K}$$

1st Law: ${}_1Q_{2s} = m(u_{2s} - u_1) + {}_1W_{2s}$; ${}_1Q_{2s} = 0$

Assume constant specific heat

${}_1W_{2\,s} = mC_v(T_1 - T_{2s}) = 3.63$ kJ

${}_1W_{2\,ac} = 0.7 \times {}_1W_{2\,s} =$ **2.54 kJ**

b) Use Ideal Gas Law; $T_{2\,ac} = T_1P_2V_2 / P_1V_1 = 354.9$ K

1st Law: ${}_1Q_{2\,ac} = mC_v(T_{2\,ac} - T_1) + {}_1W_{2\,ac} = -0.58$ kJ

2nd Law: $\Delta S_{net} = m(s_2 - s_1) - \frac{Q_{cv}}{T_o}$; $Q_{CV} = {}_1Q_{2\,ac}$, $T_o = 20^{o}C$

$$s_2 - s_1 = Cp \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1} = 0.0569 \text{ kJ/kg-K}$$

$\Delta S_{net} = 0.00406$ kJ/K ≥ 0 ; **Process is Possible**

8.183

A piston/cylinder assembly contains 2 kg of liquid water at 20°C, 100 kPa and it is now heated to 300°C by a source at 500°C. A pressure of 1000 kPa will lift the piston off the lower stops. Find the final volume, work, heat transfer and total entropy generation.

Solution:

C.V. Water out to source at 500°C. This is a control mass.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m(s_2 - s_1) = {}_1Q_2 / T_{SOURCE} + {}_1S_{2\ gen}$

Process: $V = V_1$ if $P < P_{LIFT}$ or $P = P_{LIFT}$ if $V > V_1$

Any state of this system must be on the two lines shown in the P-v diagram.

Initial state: Table B.1.1: $v_1 = 0.001002$, $u_1 = 83.94$, $= 0.2966$

$$V_1 = mv_1 = 2 \times 0.001002 = 0.002\ m^3$$

Final state: 300°C and on line in P-V diagram. Now check at state 1a.

State 1a: $v_{1a} = v_1$, $P = 1000$ kPa => compressed liquid $T_{1a} < 180°C$

As final state is at 300°C higher than T_{1a} we must be further out so

State 2: 1000 kPa, 300°C => Superheated vapor in Table B.1.3

$$v_2 = 0.25794,\quad u_2 = 2793.2,\quad s_2 = 7.1228$$

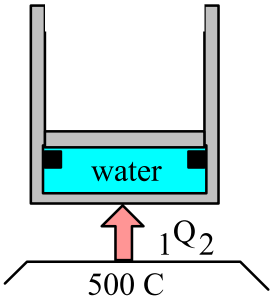
$$V_2 = mv_2 = 2 \times 0.25794 = 0.51588\ m^3$$

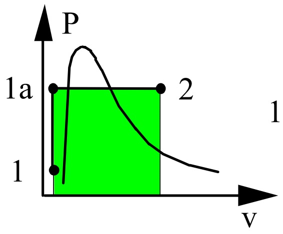
$${}_1W_2 = \int PdV = P_2(V_2 - V_1) = 1000\ (0.51588 - 0.002) = \mathbf{513.9\ kJ}$$

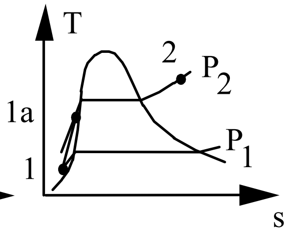
$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2(2793.2 - 83.94) + 513.9 = \mathbf{5932\ kJ}$$

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{SOURCE} = 2\ (7.1228 - 0.2966) - \frac{5932}{773.15}$$

$$= 13.652 - 7.673 = \mathbf{5.98\ kJ/K} \quad (= \Delta S_{water} + \Delta S_{sur})$$







8.184

A gas in a rigid vessel is at ambient temperature and at a pressure, P_1, slightly higher than ambient pressure, P_0. A valve on the vessel is opened, so gas escapes and the pressure drops quickly to ambient pressure. The valve is closed and after a long time the remaining gas returns to ambient temperature at which point the pressure is P_2. Develop an expression that allows a determination of the ratio of specific heats, *k*, in terms of the pressures.

C.V.: air remaining in tank,

First part of the process is an isentropic expansion s = constant.

$$P_1,\ T_0 \rightarrow P_0,\ T_x \qquad T_x/T_0 = (P_0/P_1)^{\frac{k-1}{k}}$$

Second part of the process is a constant vol. heat transfer. $P_0,\ T_x \rightarrow P_2,\ T_0$

$$\frac{P_0}{P_2} = \frac{T_x}{T_0} \;\Rightarrow\; \frac{P_0}{P_2} = \left(\frac{P_0}{P_1}\right)^{\frac{k-1}{k}} \;\rightarrow\; \mathbf{k} = \frac{\ln (P_1/P_0)}{\ln (P_1/P_2)}$$

8.185

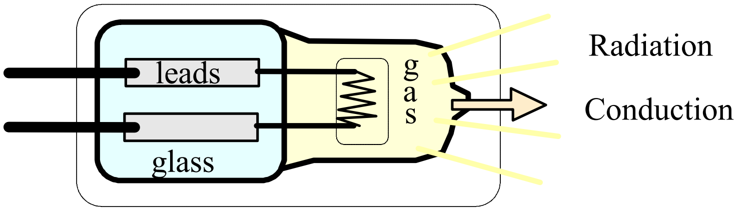
A small halogen light bulb receives an electrical power of 50 W. The small filament is at 1000 K and gives out 20% of the power as light and the rest as heat transfer to the gas, which is at 500 K; the glass is at 400 K. All the power is absorbed by the room walls at 25^{o}C. Find the rate of generation of entropy in the filament, in the total bulb including glass and the total room including bulb.

Solution:

$\dot{W}_{el} = 50$ W

$\dot{Q}_{RAD} = 10$ W

$\dot{Q}_{COND} = 40$ W



We will assume steady state and no storage in the bulb, air or room walls.

C.V. Filament steady-state

Energy Eq.5.31: $dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Entropy Eq.8.43: $dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{RAD}}{T_{FILA}} - \dfrac{\dot{Q}_{COND}}{T_{FILA}} + \dot{S}_{gen}$

$$\dot{S}_{gen} = (\dot{Q}_{RAD} + \dot{Q}_{COND})/T_{FILA} = \dot{W}_{el}/T_{FILA} = \frac{50}{1000} = \mathbf{0.05\ W/K}$$

C.V. Bulb including glass

$\dot{Q}_{RAD}$ leaves at 1000 K $\qquad$ $\dot{Q}_{COND}$ leaves at 400 K

$$\dot{S}_{gen} = \int d\dot{Q}/T = -(-10/1000) - (-40/400) = \mathbf{0.11\ W/K}$$

C.V. Total room. All energy leaves at 25°C

Eq.5.31: $dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Eq.8.43: $dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{TOT}}{T_{WALL}} + \dot{S}_{gen}$

$$\dot{S}_{gen} = \frac{\dot{Q}_{TOT}}{T_{WALL}} = 50/(25+273) = \mathbf{0.168\ W/K}$$

Solutions using the Pr and vr functions in Table A.7.2

If you would like to see more of these please let me know (claus@umich.edu) and I can prepare more of the problem solutions using these functions.

8.97

A piston/cylinder, shown in Fig. P8.97, contains air at 1380 K, 15 MPa, with $V_1 = 10\ \text{cm}^3$, $A_{cyl} = 5\ \text{cm}^2$. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, Q = 0.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = 0 + {}_1S_{2\ gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?, assume reversible process.

$${}_1S_{2\ gen} = 0 \quad \Rightarrow \quad s_2 - s_1 = 0$$

State 1: Table A.7.1: $u_1 = 1095.2$ kJ/kg,

Table A.7.2: $P_{r1} = 340.53$, $v_{r1} = 2.7024$

$$m = P_1V_1/RT_1 = \frac{15000 \times 10{\times}10^{-6}}{0.287 \times 1380} = 0.000379\ \text{kg}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$

$\Rightarrow P_{r2} = P_{r1}P_2/P_1 = 340.53{\times}200/15000 = 4.5404$

Interpolate in A.7.2 to match the P_{r2} value

$T_2 = 447$ K, $u_2 = 320.85$ kJ/kg, $v_{r2} = 65.67$

$\Rightarrow V_2 = V_1v_{r2}/v_{r1} = 10 \times 65.67 / 2.7024 =$ **$243\ \text{cm}^3$**

$\Rightarrow L_2 = V_2 /A_{cyl} = 243/5 =$ **48.6 cm**

$\Rightarrow {}_1w_2 = u_1 - u_2 = 774.4$ kJ/kg, ${}_1W_2 = m_1w_2 =$ **0.2935 kJ**

We could also have done $V_2 = V_1\ (T_2P_1/T_1P_2)$ from ideal gas law and thus did not need the vr function for this problem

8.107

A piston/cylinder contains air at 300 K, 100 kPa. It is now compressed in a reversible adiabatic process to a volume 7 times as small. Use constant heat capacity and find the final pressure and temperature, the specific work and specific heat transfer for the process.

Solution: Here we use the v_r function from Table A.7.2

Expansion ratio: $v_2/ v_1 = 1/7$

Process eq.: Rev. adiabatic and ideal gas gives $Pv^n = C$, with $n = k$

Since we know the v ratio and s is constant we use the v_r function

$v_{r1} = 179.49 \quad \Rightarrow \quad v_{r2} = v_{r1}\, v_2/ v_1 = 179.49/7 = 25.641$

Table A.7.2: Interpolate $T_2 =$ **640.7 K**

$P_2 = P_1 \times (T_2 / T_1) \times (v_1/v_2) = 100 \times (640.7/300) \times 7 =$ **1495 kPa**

Adiabatic: ${}_1q_2 =$ **0 kJ/kg**

Polytropic process work term from Eq.8.38

${}_1w_2 = -(u_2 - u_1) = -(466.37 - 214.36) =$ **-252.0 kJ/kg**

8.112

A mass of 1 kg of air contained in a cylinder at 1.5 MPa, 1000 K, expands in a reversible adiabatic process to 100 kPa. Calculate the final temperature and the work done during the process, using

a. Constant specific heat, value from Table A.5

b. The ideal gas tables, Table A.7

Solution:

C.V. Air.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: ${}_1Q_2 = 0$, ${}_1S_{2\ gen} = 0$ => $s_2 = s_1$

a) Using constant Cp from Table A.5 gives the power relation Eq.8.32.

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 1000\left(\frac{0.1}{1.5}\right)^{0.286} = \mathbf{460.9\ K}$$

$$ {}_1W_2 = -(U_2 - U_1) = mC_{Vo}(T_1 - T_2)$$

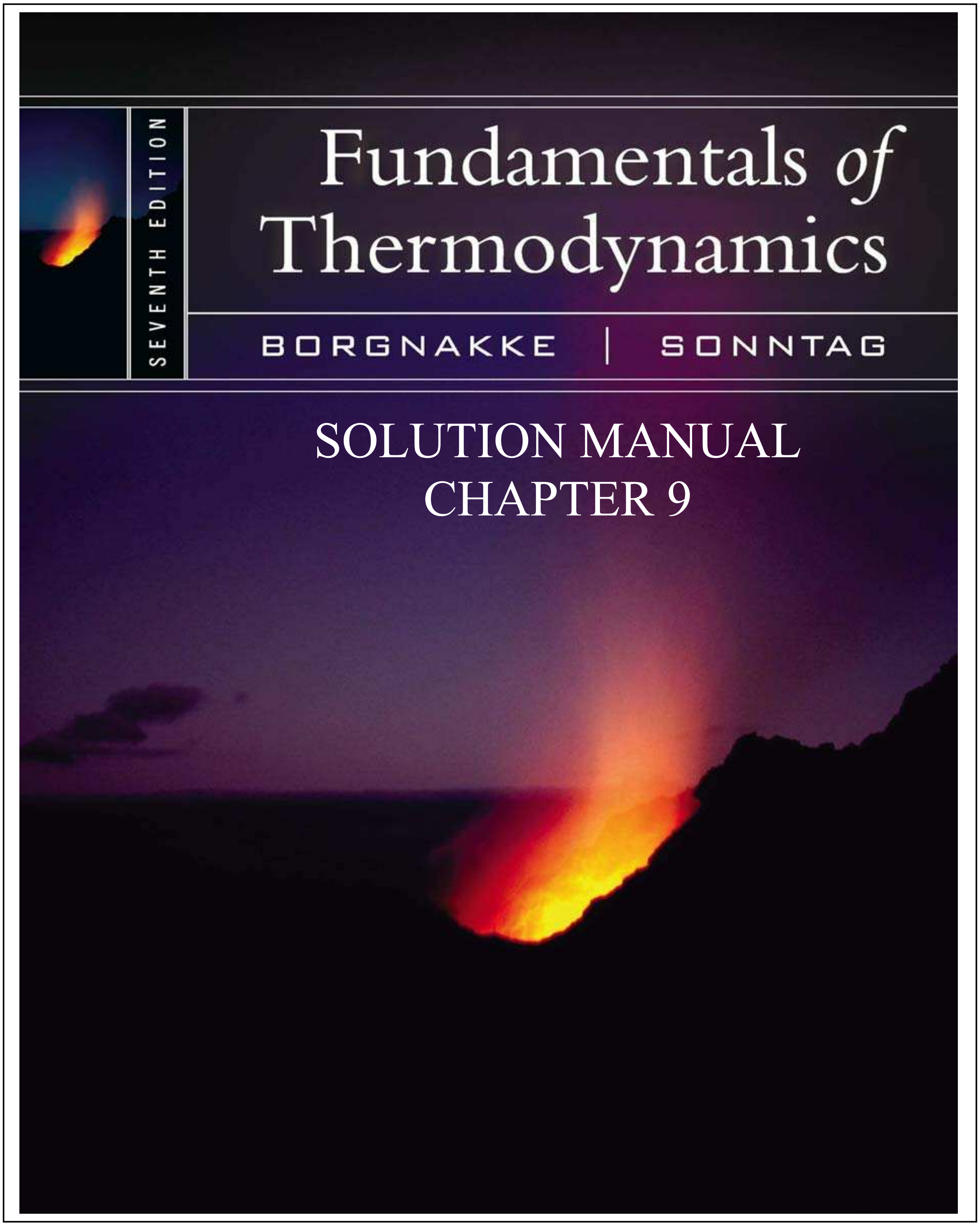
$$= 1 \times 0.717(1000 - 460.9) = \mathbf{386.5\ kJ}$$

b) Use the tabulated **reduced pressure function** that includes variable heat capacity from A.7.2

$$P_{r2} = P_{r1} \times P_2/P_1 = 91.65 \times \frac{0.1}{1.5} = 6.11$$

Interpolation gives $T_2 = \mathbf{486\ K}$ and $u_2 = 349.4$ kJ/kg

$${}_1W_2 = m(u_1 - u_2) = 1(759.2 - 349.4) = \mathbf{409.8\ kJ}$$



SEVENTH EDITION

Fundamentals of Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 9

CONTENT

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Problems re-solved with Pr and vr from Table A.7.2:
26, 30, 32, 50, 72, 170

In-Text Concept Questions

9.a

A reversible adiabatic flow of liquid water in a pump has increasing P. How about T?

Solution:

Steady state single flow: $s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen} = s_i + 0 + 0$

Adiabatic (dq = 0) means integral vanishes and reversible means $s_{gen} = 0$, so s is constant. Properties for liquid (incompressible) gives Eq.8.19

$$ds = \frac{C}{T}\, dT$$

then constant s gives constant T.

9.b

A reversible adiabatic flow of air in a compressor has increasing P. How about T?

Solution:

Steady state single flow: $$s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen} = s_i + 0 + 0$$

so s is constant. Properties for an ideal gas gives Eq.8.23 and for constant specific heat we get Eq.8.29. A higher P means a higher T, which is also the case for a variable specific heat, recall Eq.8.28 for the standard entropy.

9.c

A compressor receives R-134a at -10^{o}C, 200 kPa with an exit of 1200 kPa, 50^{o}C. What can you say about the process?

Solution:

Properties for R-134a are found in Table B.5

Inlet state: $s_i = 1.7328$ kJ/kg K

Exit state: $s_e = 1.7237$ kJ/kg K

Steady state single flow: $$s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen}$$

Since s decreases slightly and the generation term can only be positive, it must be that the heat transfer is negative (out) so the integral gives a contribution that is smaller than $-s_{gen}$.

9.d

A flow of water at some velocity out of a nozzle is used to wash a car. The water then falls to the ground. What happens to the water state in terms of **V**, T and s?

let us follow the water flow. It starts out with kinetic and potential energy of some magnitude at a compressed liquid state P, T. As the water splashes onto the car it looses its kinetic energy (it turns in to internal energy so T goes up by a very small amount). As it drops to the ground it then looses all the potential energy which goes into internal energy. Both of theses processes are irreversible so s goes up.

If the water has a temperature different from the ambient then there will also be some heat transfer to or from the water which will affect both T and s.

9.e

In a steady state single flow *s* is either constant or it increases. Is that true?
Solution:

No.

Steady state single flow: $s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen}$

Entropy can only go up or stay constant due to s_{gen}, but it can go up or down due to the heat transfer which can be positive or negative. So if the heat transfer is large enough it can overpower any entropy generation and drive s up or down.

9.f

If a flow device has the same inlet and exit pressure, can shaft work be done?

The reversible work is given by Eq.9.14

$$w = -\int v\,dP + (\mathbf{V}_i^2 - \mathbf{V}_e^2) + g\,(Z_i - Z_e)$$

For a constant pressure the first term drops out but the other two remains. Kinetic energy changes can give work out (windmill) and potential energy changes can give work out (a dam).

9.g

A polytropic flow process with $n = 0$ might be which device?

As the polytropic process is $Pv^n = C$, then $n = 0$ is a constant pressure process. This can be a pipe flow, a heat exchanger flow (heater or cooler) or a boiler.

Concept Problems

9.1

If we follow a mass element going through a reversible adiabatic flow process what can we say about the change of state?

Following a mass (this is a control mass)

$du = dq - dw = 0 - Pdv = - Pdv$; compression/expansion changes u

$ds = dq/T + ds_{gen} = 0 + 0 \quad \Rightarrow$ s = constant, isentropic process.

9.2

Which process will make the statement in concept question e) on page 267 true?

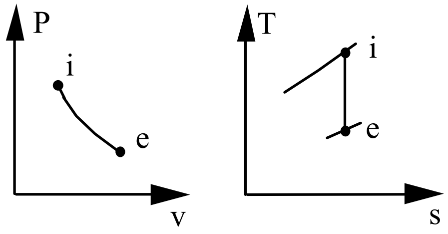
Solution:

If the process is said to be adiabatic then:

Steady state adiabatic single flow: $s_e = s_i + s_{gen} \geq s_i$

9.3

A reversible process in a steady flow with negligible kinetic and potential energy changes is shown in the diagrams. Indicate the change $h_e - h_i$ and transfers w and q as positive, zero or negative



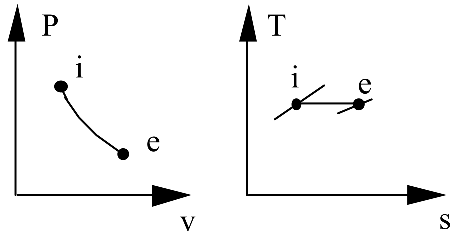
$dw = - v\ dP > 0$ P drops so work is positive out.

$dq = T\ ds = 0$ s is constant, and process reversible so adiabatic.

$h_e - h_i = q - w = 0 - w < 0$ so enthalpy drops

9.4

A reversible process in a steady flow of air with negligible kinetic and potential energy changes is shown in the diagrams. Indicate the change $h_e - h_i$ and transfers w and q as positive, zero or negative



$dw = -v\,dP > 0$ P drops so work is positive out.

$dq = T\,ds > 0$ s is increasing and process reversible so q is positive.

$h_e - h_i = 0$ as they are functions of T and thus the same.

9.5

A reversible steady isobaric flow has 1 kW of heat added with negligible changes in KE and PE, what is the work transfer?

P = C: Shaft work Eq. 9.14:

$$dw = -\,v\,dP + \Delta KE + \Delta PE - T\,ds_{gen} = 0 + 0 + 0 - 0 = \mathbf{0}$$

9.6

An air compressor has a significant heat transfer out. See example 9.4 for how high T becomes if there is no heat transfer. Is that good, or should it be insulated?

That depends on the use of the compressed air. If there is no need for the high T, say it is used for compressed air tools, then the heat transfer will lower T and result in lower specific volume reducing the work. For those applications the compressor may have fins mounted on its surface to promote the heat transfer. In very high pressure compression it is done in stages between which is a heat exchanger called an intercooler.

This is a small compressor driven by an electric motor. Used to charge air into car tires.

9.7

Friction in a pipe flow causes a slight pressure decrease and a slight temperature increase. How does that affect entropy?

Solution:

The friction converts flow work (P drops) into internal energy (T up if single phase). This is an irreversible process and s increases.

If liquid: Eq. 8.19: $ds = \frac{C}{T}\,dT$ so s follows T

If ideal gas Eq. 8.23: $ds = C_p\frac{dT}{T} - R\,\frac{dP}{P}$ (both terms increase)

9.8

To increase the work out of a turbine for a given inlet and exit pressure how should the inlet state be changed?

$$w = -\int v\, dP + \ldots.. \qquad \text{Eq.9.13}$$

For a given change in pressure boosting v will result in larger work term. So for **larger inlet T** we get a larger v and thus larger work. That is why we increase T by combustion in a gasturbine before the turbine section.

9.9

An irreversible adiabatic flow of liquid water in a pump has higher P. How about T?

Solution:

Steady state single flow: $s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen} = s_i + 0 + s_{gen}$

so s is increasing. Properties for liquid (incompressible) gives Eq.8.19 where an increase in s gives an increasse in T.

9.10

The shaft work in a pump to increase the pressure is small compared to the shaft work in an air compressor for the same pressure increase. Why?

The reversible work is given by Eq. 9.13 or 9.14 if reversible and no kinetic or potential energy changes

$$w = -\int v\, dP$$

The liquid has a very small value for v compared to a large value for a gas.

9.11

Liquid water is sprayed into the hot gases before they enter the turbine section of a large gasturbine power plant. It is claimed that the larger mass flow rate produces more work. Is that the reason?

No. More mass through the turbine does give more work, but the added mass is only a few percent. As the liquid vaporizes the specific volume increases dramatically which gives a much larger volume flow throught the turbine and that gives more work output.

$$\dot{W} = \dot{m}w = -\dot{m}\int v\, dP = -\int \dot{m}v\, dP = -\int \dot{V}\, dP$$

This should be seen relative to the small work required to bring the liquid water up to the higher turbine inlet pressure from the source of water (presumably atmospheric pressure).

9.12

A tank contains air at 400 kPa, 300 K and a valve opens up for flow out to the outside which is at 100 kPa, 300 K. What happens to the air temperature inside?

As mass flows out of the tank the pressure will drop, the air that remains basically goes through a simple (adiabatic if process is fast enough) expansion process so the temperature also drops. If the flow rate out is very small and the process thus extremely slow, enough heat transfer may take place to keep the temperature constant.

Steady state reversible processes single flow

9.13

An evaporator has R-410a at -20°C and quality 20% flowing in with the exit flow being saturated vapor at -20°C. Consider the heating to be a reversible process and find the specific heat transfer from the entropy equation.

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + q/T + 0$

$q = T\,(s_e - s_i) = T\,(s_g - s_i)$

Inlet: $s_i = 0.1154 + x_i\ 0.9625 = 0.3079$ kJ/kg-K

Exit: $s_g = 1.0779$ kJ/kg-K

$q = (273.15 - 20)\,(1.0779 - 0.3079) =$ **194.926 kJ/kg**

Remark: It fits with $h_e - h_i = (1 - x_i)\,h_{fg} = 0.8 \times 243.65 = 194.92$ kJ/kg

9.14

A reversible isothermal expander (a turbine with heat transfer) has an inlet flow of carbon dioxide at 3 MPa, 40°C and an exit flow at 1 MPa, 40°C. Find the specific heat transfer from the entropy equation and the specific work from the energy equation assuming ideal gas.

Energy Eq.6.13: $0 = h_i - h_e + q - w$

Entropy Eq.9.8: $0 = s_i - s_e + \int dq/T + s_{gen} = s_i - s_e + q/T + 0$

$$q = T\,(s_e - s_i) = T(C_{Po} \ln \frac{T_e}{T_i} - R \ln \frac{P_e}{P_i}) = - RT \ln \frac{P_e}{P_i}$$

$$= -0.1889 \text{ kJ/kg-K} \times 313.15 \text{ K} \times \ln \frac{1}{3} = \mathbf{64.99\ kJ/kg}$$

$$w = h_i - h_e + q = q = \mathbf{64.99\ kJ/kg}$$

9.15

Solve the previous Problem using Table B.3

Energy Eq.6.13: $0 = h_i - h_e + q - w$

Entropy Eq.9.8: $0 = s_i - s_e + \int dq/T + s_{gen} = s_i - s_e + q/T + 0$

Inlet state: $h_i = 378.55$ kJ/kg, $s_i = 1.4104$ kJ/kg-K

Exit state: $h_e = 398.05$ kJ/kg, $s_e = 1.6633$ kJ/kg-K

$$q = T\,(s_e - s_i) = 313.15\,(1.6633 - 1.4104) = \mathbf{79.2\ kJ/kg}$$

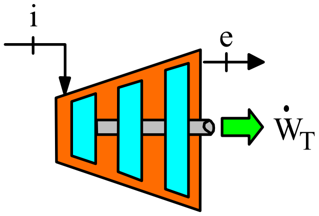
$$w = h_i - h_e + q = 378.55 - 398.05 + 79.2 = \mathbf{59.7\ kJ/kg}$$

Remark: When it is not an ideal gas h is a fct. of both T and P.

9.16

A first stage in a turbine receives steam at 10 MPa, 800°C with an exit pressure of 800 kPa. Assume the stage is adiabatic and negelect kinetic energies. Find the exit temperature and the specific work.

Solution:



C.V. Stage 1 of turbine.

The stage is adiabatic so q = 0 and we will assume reversible so $s_{gen} = 0$

Energy Eq.6.13: $w_T = h_i - h_e$

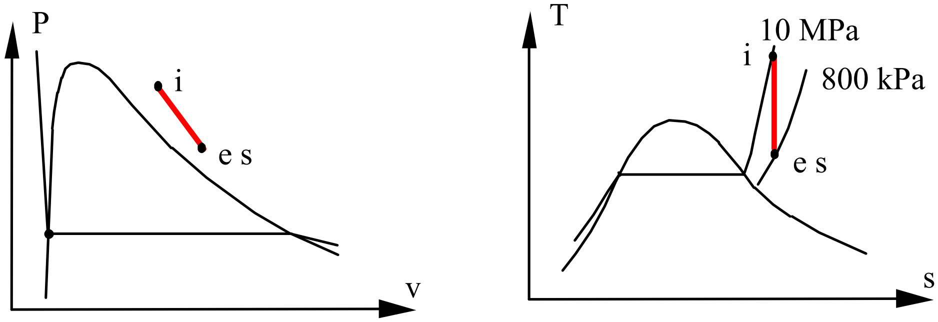
Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Inlet state: B.1.3: $h_i = 4114.9$ kJ/kg, $s_i = 7.4077$ kJ/kg K

Exit state: 800 kPa, $s = s_i$

Table B.1.3 ⇒ $T \cong$ **349.7°C,** $h_e = 3161$ kJ/kg

$w_T = 4114.9 – 3161 =$ **953.9 kJ/kg**



9.17

Steam enters a turbine at 3 MPa, 450°C, expands in a reversible adiabatic process and exhausts at 10 kPa. Changes in kinetic and potential energies between the inlet and the exit of the turbine are small. The power output of the turbine is 800 kW. What is the mass flow rate of steam through the turbine?

Solution:

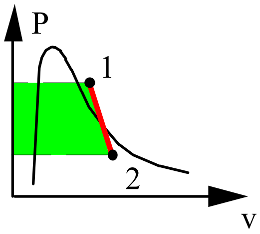
C.V. Turbine, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

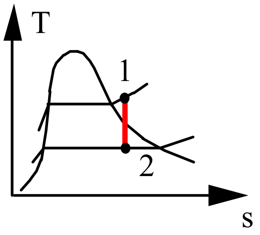
Continuity Eq.6.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.6.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_T$,

Entropy Eq.9.8: $\dot{m}s_i + \emptyset = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

Explanation for the work term is in Sect. 9.3, Eq.9.14





Inlet state: Table B.1.3 $h_i = 3344$ kJ/kg, $s_i = 7.0833$ kJ/kg K

Exit state: P_e , $s_e = s_i \Rightarrow$ Table B.1.2 saturated as $s_e < s_g$

$x_e = (7.0833 - 0.6492)/7.501 = 0.8578$,

$h_e = 191.81 + 0.8578 \times 2392.82 = 2244.4$ kJ/kg

$$\dot{m} = \dot{W}_T/w_T = \dot{W}_T/(h_i - h_e) = \frac{800}{3344 - 2244.4}\ \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{0.728\ kg/s}$$

9.18

A reversible adiabatic compressor receives 0.05 kg/s saturated vapor R-410a at 200 kPa and has an exit presure of 800 kPa. Neglect kinetic energies and find the exit temperature and the minimum power needed to drive the unit.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.6.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.6.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$,

Entropy Eq.9.8: $\dot{m}s_i + 0 = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

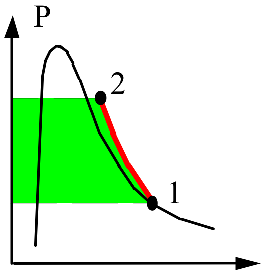
Inlet state: B 4.2.: $h_i = 264.27$ kJ/kg, $s_i = 1.1192$ kJ/kg K

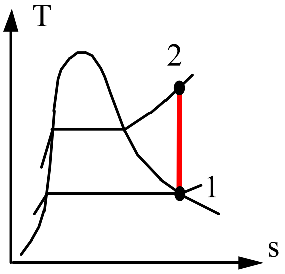
Exit state: P_e , $s_e = s_i$ $\Rightarrow$ Table B.4.2 $h_e = 302.65$ kJ/kg, $\mathbf{T_e \cong 22.7°C}$

$$-w_c = h_e - h_i = 302.65 - 264.27 = 38.38 \text{ kJ/kg}$$

$$-\dot{W}_c = \text{Power In} = -w_c\dot{m} = 38.38 \times 0.05 = \mathbf{1.92\ kW}$$

Explanation for the work term is in Sect. 9.3, Eq.9.18





9.19

In a heat pump that uses R-134a as the working fluid, the R-134a enters the compressor at 150 kPa, −10°C at a rate of 0.1 kg/s. In the compressor the R-134a is compressed in an adiabatic process to 1 MPa. Calculate the power input required to the compressor, assuming the process to be reversible.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.6.11: $\dot{m}_1 = \dot{m}_2 = \dot{m}$,

Energy Eq.6.12: $\dot{m}h_1 = \dot{m}h_2 + \dot{W}_C$,

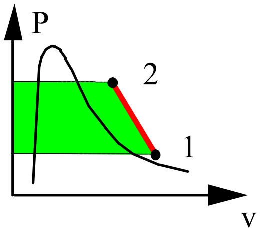
Entropy Eq.9.8: $\dot{m}s_1 + 0 = \dot{m}s_2$ (Reversible $\dot{S}_{gen} = 0$)

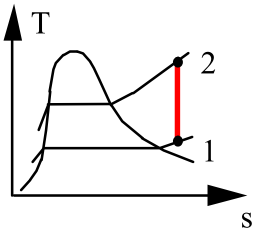
Inlet state: Table B.5.2 $h_1 = 393.84$ kJ/kg, $s_1 = 1.7606$ kJ/kg K

Exit state: $P_2 = 1$ MPa & $s_2 = s_1$ $\Rightarrow$ $h_2 = 434.9$ kJ/kg

$$\dot{W}_c = \dot{m}w_c = \dot{m}(h_1 - h_2) = 0.1 \times (393.84 - 434.9) = \mathbf{-4.1\ kW}$$

Explanation for the work term is in Sect. 9.3 Eq.9.14





9.20

A compressor in a commercial refrigerator receives R-410a at -25°C and x = 1. The exit is at 2000 kPa and the process assumed reversible and adiabatic. Neglect kinetic energies and find the exit temperature and the specific work.

CV Compressor. $q = 0$.

Energy Eq.6.13: $w_C = h_i - h_e$

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Inlet state: B.4.1: $h_i = 269.77$ kJ/kg, $s_i = 1.0893$ kJ/kg K

Exit state: 1000 kPa, $s = s_i$

Table B.4.2 ⇒ $T \cong$ **60.4°C,** $h_e = 321.13$ kJ/kg

$w_C = 269.77 – 321.13 =$ **–51.4 kJ/kg**

9.21

A boiler section boils 3 kg/s saturated liquid water at 2000 kPa to saturated vapor in a reversible constant pressure process. Assume you do not know that there is no work. Prove that there is no shaftwork using the first and second laws of thermodynamics.

Solution:

C.V. Boiler. Steady, single inlet and single exit flows.

Energy Eq.6.13: $h_i + q = w + h_e$;

Entropy Eq.9.8: $s_i + q/T = s_e$

States: Table B.1.2, $T = T_{sat} = 212.42°C = 485.57$ K

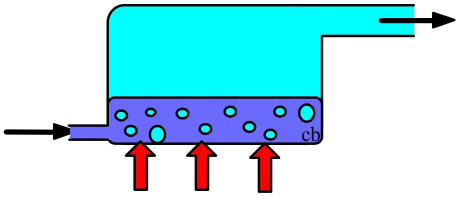
$h_i = h_f = 908.77$ kJ/kg, $s_i = 2.4473$ kJ/kg K

$h_e = h_g = 2799.51$ kJ/kg, $s_e = 6.3408$ kJ/kg K

$q = T(s_e - s_i) = 485.57(6.3408 - 2.4473) =$ **1890.6 kJ/kg**

$w = h_i + q - h_e = 908.77 + 1890.6 - 2799.51 =$ **-0.1 kJ/kg**

It should be zero (non-zero due to round off in values of s, h and T_{sat}).



Often it is a long pipe and not a chamber

9.22

Atmospheric air at -45°C, 60 kPa enters the front diffuser of a jet engine with a velocity of 900 km/h and frontal area of 1 m^2. After the adiabatic diffuser the velocity is 20 m/s. Find the diffuser exit temperature and the maximum pressure possible.

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2,$ and $h_e - h_i = C_p(T_e - T_i)$

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

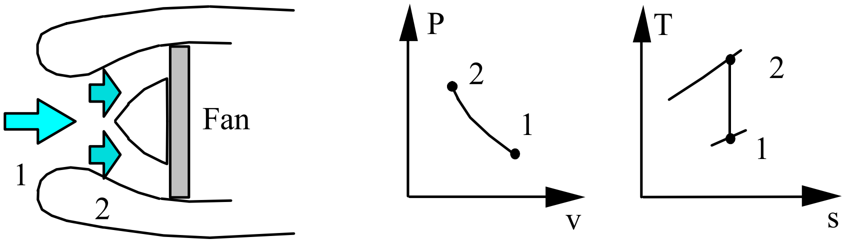
Heat capacity and ratio of specific heats from Table A.5: $C_{Po} = 1.004 \frac{\text{kJ}}{\text{kg K}}$, k = 1.4, the energy equation then gives:

$$1.004[\ T_e - (-45)] = 0.5[(900\times1000/3600)^2 - 20^2\ ]/1000 = 31.05 \text{ kJ/kg}$$

$$\Rightarrow \quad T_e = -14.05\ °C = \mathbf{259.1\ K}$$

Constant s for an ideal gas is expressed in Eq.8.23:

$$P_e = P_i\ (T_e/T_i)^{\frac{k}{k-1}} = 60\ (259.1/228.1)^{3.5} = \mathbf{93.6\ kPa}$$



9.23

A compressor is surrounded by cold R-134a so it works as an isothermal compressor. The inlet state is 0°C, 100 kPa and the exit state is saturated vapor. Find the specific heat transfer and specific work.

Solution:

C.V. Compressor. Steady, single inlet and single exit flows.

Energy Eq.6.13: $h_i + q = w + h_e$;

Entropy Eq.9.8: $s_i + q/T = s_e$

Inlet state: Table B.5.2, $h_i = 403.4$ kJ/kg, $s_i = 1.8281$ kJ/kg K

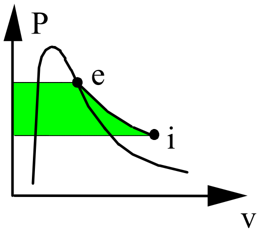
Exit state: Table B.5.1, $h_e = 398.36$ kJ/kg, $s_e = 1.7262$ kJ/kg K

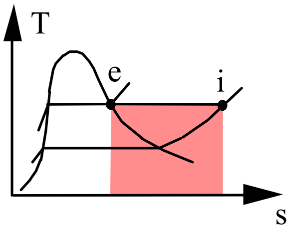
$q = T(s_e - s_i) = 273.15(1.7262 - 1.8281) = \mathbf{-\ 27.83\ kJ/kg}$

$w = 403.4 + (-27.83) - 398.36 = \mathbf{-22.8\ kJ/kg}$

Explanation for the work term is in Sect. 9.3

Eqs. 9.13 and 9.14





9.24

Consider the design of a nozzle in which nitrogen gas flowing in a pipe at 500 kPa, 200°C, and at a velocity of 10 m/s, is to be expanded to produce a velocity of 300 m/s. Determine the exit pressure and cross-sectional area of the nozzle if the mass flow rate is 0.15 kg/s, and the expansion is reversible and adiabatic.

Solution:

C.V. Nozzle. Steady flow, no work out and no heat transfer.

Energy Eq.6.13: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2$

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$

Properties Ideal gas Table A.5:

$$C_{Po} = 1.042 \frac{\text{kJ}}{\text{kg K}},\ \ R = 0.2968 \frac{\text{kJ}}{\text{kg K}},\ \ k = 1.40$$

$$h_e - h_i = C_{Po}(T_e - T_i) = 1.042(T_e - 473.2) = (10^2 - 300^2)/(2\times1000)$$

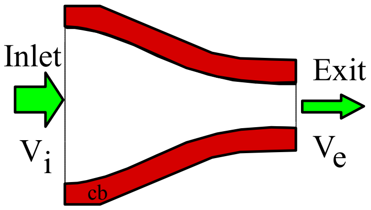
Solving for exit T: $T_e = 430$ K,

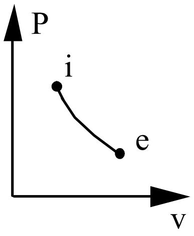
Process: $s_i = s_e$ => For ideal gas expressed in Eq.8.23

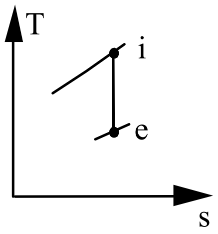
$$P_e = P_i(T_e/T_i)^{\frac{k}{k-1}} = 500\left(\frac{430}{473.2}\right)^{3.5} = \mathbf{357.6\ kPa}$$

$$v_e = RT_e/P_e = (0.2968 \times 430)/357.6 = 0.35689\ \text{m}^3/\text{kg}$$

$$A_e = \dot{m}v_e/\mathbf{V}_e = \frac{0.15 \times 0.35689}{300} = \mathbf{1.78 \times 10^{-4}\ m^2}$$







9.25

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with neglible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e + \mathbf{V}_e^2/2 \quad (Z_i = Z_e)$

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Use constant specific heat from Table A.5, $C_{Po} = 1.004 \frac{kJ}{kg\ K}$, $k = 1.4$

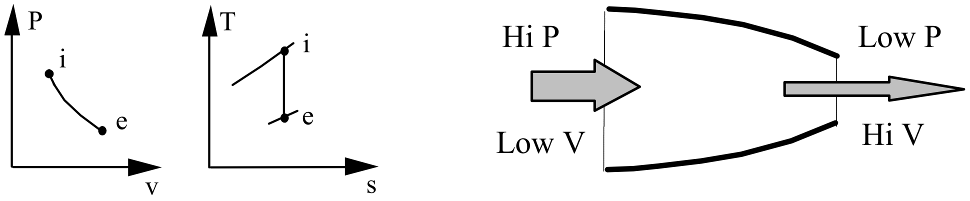
The isentropic process ($s_e = s_i$) gives Eq.8.23

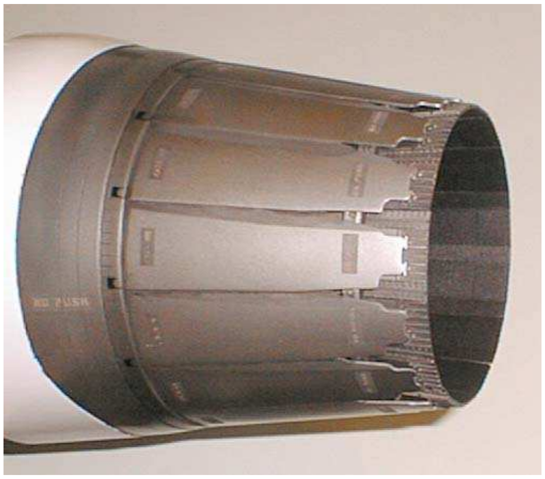
$$=> \quad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 1200\ (80/150)^{\ 0.2857} = 1002.7\text{ K}$$

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e \cong C_P(T_i - T_e)$$

$$\mathbf{V}_e = \sqrt{2\ C_P(T_i - T_e)} = \sqrt{2\times 1.004(1200\text{-}1002.7) \times 1000} = \mathbf{629.4\ m/s}$$





9.26

Do the previous problem using the air tables in A.7

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with neglible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e + \mathbf{V}_e^2/2 \quad (Z_i = Z_e)$

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Process: $q = 0, \quad s_{gen} = 0$ as used above leads to $s_e = s_i$

Inlet state: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.3460$ kJ/kg K

The constant s is rewritten from Eq.8.19 as

$$s^o_{Te} = s^o_{Ti} + R \ln(P_e / P_i) = 8.3460 + 0.287 \ln (80/150) = 8.1656$$

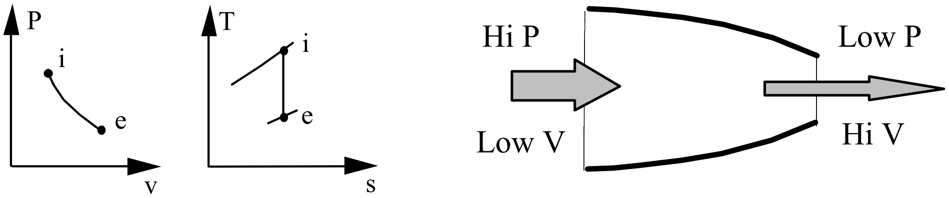
Interpolate in A.7 =>

$$T_e = 1000 + 50 \frac{8.1656 - 8.1349}{8.1908 - 8.1349} = 1027.46 \text{ K}$$

$$h_e = 1046.2 + (1103.5 - 1046.3) \times \frac{8.1656 - 8.1349}{8.1908 - 8.1349} = 1077.7$$

From the energy equation we have $\mathbf{V}_e^2/2 = h_i - h_e$, so then

$$\mathbf{V}_e = \sqrt{2\,(h_i - h_e)} = \sqrt{2(1277.8 - 1077.7) \times 1000} = \mathbf{632.6\ m/s}$$

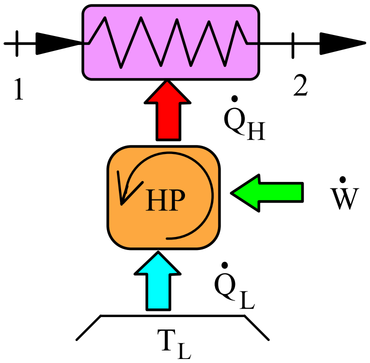


9.27

A flow of 2 kg/s saturated vapor R-410a at 500 kPa is heated at constant pressure to 60°C. The heat is supplied by a heat pump that receives heat from the ambient at 300 K and work input, shown in Fig. P9.27. Assume everything is reversible and find the rate of work input.

Solution:

C.V. Heat exchanger



Continuity Eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.: $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.4.2:

$h_1 = 274.33$ kJ/kg, $s_1 = 1.0647$ kJ/kg K

$h_2 = 342.32$ kJ/kg, $s_2 = 1.2959$ kJ/kg K

Notice we can find $\dot{Q}_H$ but the temperature T_H is not constant making it difficult to evaluate the COP of the heat pump.

C.V. Total setup and assume everything is reversible and steady state.

Energy Eq.: $\dot{m}_1 h_1 + \dot{Q}_L + \dot{W}_{in} = \dot{m}_1 h_2$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{Q}_L / T_L + 0 = \dot{m}_1 s_2$ (T_L is constant, $s_{gen} = 0$)

$$\dot{Q}_L = \dot{m}_1 T_L [s_2 - s_1] = 2 \times 300 [1.2959 - 1.0647] = 138.72 \text{ kW}$$

$$\dot{W}_{in} = \dot{m}_1 [h_2 - h_1] - \dot{Q}_L = 2 (342.32 - 274.33) - 138.72 = \mathbf{-2.74 \text{ kW}}$$

9.28

A compressor brings a hydrogen gas flow at 280 K, 100 kPa up to a pressure of 1000 kPa in a reversible process. How hot is the exit flow and what is the specific work input?

CV Compressor. Assume q = 0.

Energy Eq.6.13: $w_C = h_i - h_e \approx C_p (T_i - T_e)$

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

$T_e = T_i (P_e/P_i)^{(k-1)/k} = 280 (1000/100)^{(1.409-1)/1.409} = \mathbf{546.3\ K}$

$w_C = 14.209$ kJ/kg-K × (280 – 546.3) K = **-3783.9 kJ/kg**

Small hydrogen compressor.

9.29

A diffuser is a steady-state device in which a fluid flowing at high velocity is decelerated such that the pressure increases in the process. Air at 120 kPa, 30°C enters a diffuser with velocity 200 m/s and exits with a velocity of 20 m/s. Assuming the process is reversible and adiabatic what are the exit pressure and temperature of the air?

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2, \quad \Rightarrow \quad h_e - h_i = C_{Po}(T_e - T_i)$

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

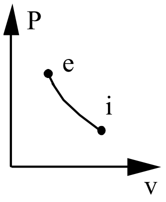
Use constant specific heat from Table A.5, $C_{Po} = 1.004 \frac{\text{kJ}}{\text{kg K}}$, $k = 1.4$

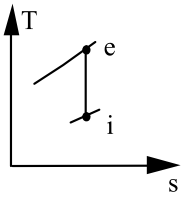
Energy equation then gives:

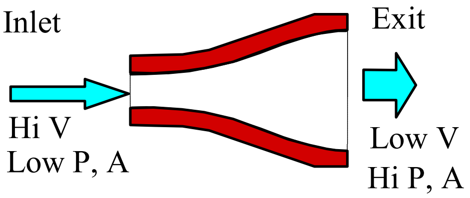
$C_{Po}(T_e - T_i) = 1.004(T_e - 303.2) = (200^2 - 20^2)/(2\times1000) \quad \Rightarrow \quad T_e = \mathbf{322.9\ K}$

The isentropic process ($s_e = s_i$) gives Eq.8.32

$$P_e = P_i(T_e/T_i)^{\frac{k}{k-1}} = 120(322.9/303.2)^{3.5} = \mathbf{149.6\ kPa}$$







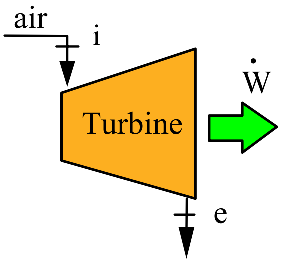
9.30

Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

a. The ideal gas tables, Table A.7

b. Constant specific heat, value at 300 K from table A.5

Solution:



C.V. Air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.6.13: $w_T = h_i - h_e$,

Entropy Eq.9.8: $s_e = s_i$

a) Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.8.28 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln(\frac{P_e}{P_i}) = 8.34596 + 0.287 \ln\left(\frac{100}{800}\right) = 7.7492 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow$ $T_e = \mathbf{706\ K}$, $h_e = 719.9$ kJ/kg

$$w = h_i - h_e = \mathbf{557.9\ kJ/kg}$$

b) Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.8.32

$$T_e = T_i\,(P_e/P_i)^{\frac{k-1}{k}} = 1200\left(\frac{100}{800}\right)^{0.286} = \mathbf{662.1\ K}$$

$$w = C_{Po}(T_i - T_e) = 1.004(1200 - 662.1) = \mathbf{539.8\ kJ/kg}$$

9.31

A highly cooled compressor brings a hydrogen gas flow at 300 K, 100 kPa up to a pressure of 1000 kPa in an isothermal process. Find the specific work assuming a reversible process.

CV Compressor. Isothermal $T_i = T_e$ so that ideal gas gives $h_i = h_e$.

Energy Eq.6.13: $w_C = h_i + q - h_e = q$

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + q/T + 0$

$q = T(s_e - s_i) = T\ [-R\ln(P_e/P_i)\ ]$

$w = q = -\ 4.1243 \times 300\ \ln(10) = $ **-2849 kJ/kg**

9.32

A compressor receives air at 290 K, 100 kPa and a shaft work of 5.5 kW from a gasoline engine. It should deliver a mass flow rate of 0.01 kg/s air to a pipeline. Find the maximum possible exit pressure of the compressor.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.6.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.6.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$,

Entropy Eq.9.8: $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

$$\dot{W}_c = \dot{m}w_c \;\Rightarrow\; -w_c = -\dot{W}/\dot{m} = 5.5/0.01 = 550 \text{ kJ/kg}$$

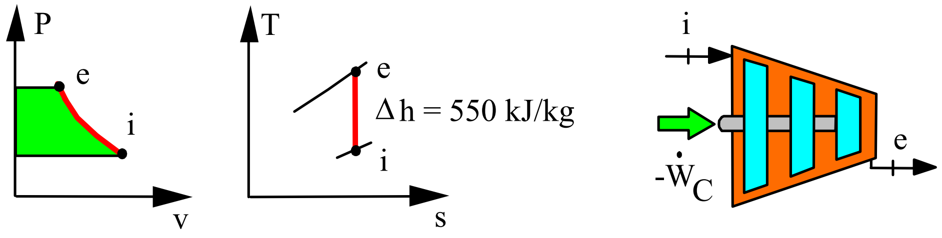
Use constant specific heat from Table A.5, $C_{Po} = 1.004$, $k = 1.4$

$$h_e = h_i + 550 \;\Rightarrow\; T_e = T_i + 550/1.004$$

$$T_e = 290 + 550/1.004 = 837.81 \text{ K}$$

$$s_i = s_e \;\Rightarrow\; P_e = P_i\,(T_e/T_i)^{\frac{k}{k-1}} \qquad \text{Eq.8.32}$$

$$P_e = 100 \times (837.81/290)^{3.5} = \mathbf{4098\ kPa}$$



9.33

An expander receives 0.5 kg/s air at 2000 kPa, 300 K with an exit state of 400 kPa, 300 K. Assume the process is reversible and isothermal. Find the rates of heat transfer and work neglecting kinetic and potential energy changes.

Solution:

C.V. Expander, single steady flow.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

Entropy Eq.: $\dot{m}s_i + \dot{Q}/T + \dot{m}s_{gen} = \dot{m}s_e$

Process: T is constant and $s_{gen} = 0$

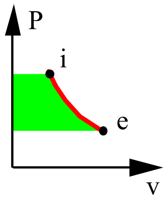
Ideal gas and isothermal gives a change in entropy by Eq. 8.24, so we can solve for the heat transfer

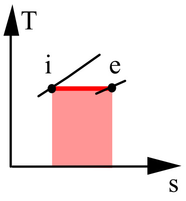
$$\dot{Q} = T\dot{m}(s_e - s_i) = -\dot{m}RT \ln \frac{P_e}{P_i}$$

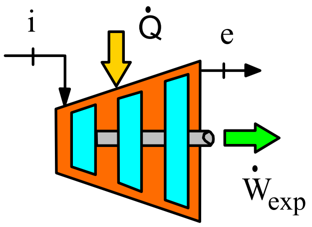
$$= -\ 0.5 \times 300 \times 0.287 \times \ln \frac{400}{2000} = \mathbf{69.3\ kW}$$

From the energy equation we get

$$\dot{W} = \dot{m}(h_i - h_e) + \dot{Q} = \dot{Q} = \mathbf{69.3\ kW}$$







9.34

A reversible steady state device receives a flow of 1 kg/s air at 400 K, 450 kPa and the air leaves at 600 K, 100 kPa. Heat transfer of 800 kW is added from a 1000 K reservoir, 100 kW rejected at 350 K and some heat transfer takes place at 500 K. Find the heat transferred at 500 K and the rate of work produced.

Solution:

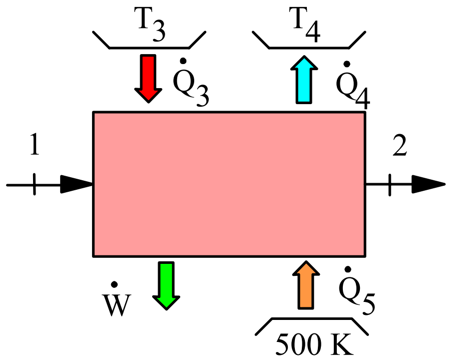
C.V. Device, single inlet and exit flows.

Energy equation, Eq.6.12:

$$\dot{m}h_1 + \dot{Q}_3 - \dot{Q}_4 + \dot{Q}_5 = \dot{m}h_2 + \dot{W}$$

Entropy equation with zero generation, Eq.9.8:

$$\dot{m}s_1 + \dot{Q}_3/T_3 - \dot{Q}_4/T_4 + \dot{Q}_5/T_5 = \dot{m}s_2$$



Solve for the unknown heat transfer using Table A.7.1 and Eq. 8.19 for change in s

$$\dot{Q}_5 = T_5\ [s_2 - s_1]\ \dot{m} + \frac{T_5}{T_4}\dot{Q}_4 - \frac{T_5}{T_3}\dot{Q}_3$$

$$= 500 \times 1\ (7.5764 - 7.1593 - 0.287 \ln\frac{100}{450}) + \frac{500}{350} \times 100 - \frac{500}{1000} \times 800$$

$$= 424.4 + 142.8 - 400 = 167.2 \text{ kW}$$

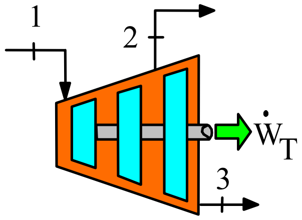
Now the work from the energy equation is

$$\dot{W} = 1 \times (401.3 - 607.3) + 800 - 100 + 167.2 = \mathbf{661.2\ kW}$$

9.35

A steam turbine in a powerplant receives 5 kg/s steam at 3000 kPa, 500^{o}C. 20% of the flow is extracted at 1000 kPa to a feedwater heater and the remainder flows out at 200 kPa. Find the two exit temperatures and the turbine power output.

C.V. Turbine. Steady flow and adiabatic q = 0.



Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.6.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

State 1: $h_1 = 3456$ kJ/kg, $s_1 = 7.234$ kJ/kgK

We also assume turbine is reversible $\dot{S}_{gen} = 0$ => $s_1 = s_2 = s_3$

State 2: (P,s) $\mathbf{T_2 = 330.6^oC}$, $h_2 = 3116$ kJ/kg

State 3: (P,s) $\mathbf{T_3 = 140.7^oC}$, $h_3 = 2750$ kJ/kg

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 5 \times 3456 - 1 \times 3116 - 4 \times 2750 = \mathbf{3164\ kW}$$

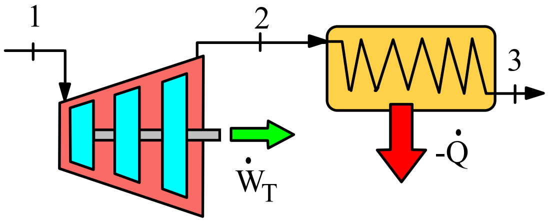
9.36

A small turbine delivers 150 kW and is supplied with steam at 700°C, 2 MPa. The exhaust passes through a heat exchanger where the pressure is 10 kPa and exits as saturated liquid. The turbine is reversible and adiabatic. Find the specific turbine work, and the heat transfer in the heat exchanger.

Solution:

Continuity Eq.6.11: Steady

$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$



Turbine: Energy Eq.6.13: $w_T = h_1 - h_2$

Entropy Eq.9.8: $s_2 = s_1 + s_{T\,gen}$

Inlet state: Table B.1.3 $h_1 = 3917.45$ kJ/kg, $s_1 = 7.9487$ kJ/kg K

Ideal turbine $s_{T\,gen} = 0$, $s_2 = s_1 = 7.9487 = s_{f2} + x\, s_{fg2}$

State 3: P = 10 kPa, $s_2 < s_g$ => saturated 2-phase in Table B.1.2

$\Rightarrow\ x_{2,s} = (s_1 - s_{f2})/s_{fg2} = (7.9487 - 0.6492)/7.501 = 0.9731$

$\Rightarrow\ h_{2,s} = h_{f2} + x\, h_{fg2} = 191.8 + 0.9731\times 2392.8 = 2520.35$ kJ/kg

$w_{T,s} = h_1 - h_{2,s} =$ **1397.05 kJ/kg**

$\dot{m} = \dot{W} / w_{T,s} = 150 / 1397 = 0.1074$ kg/s

Heat exchanger:

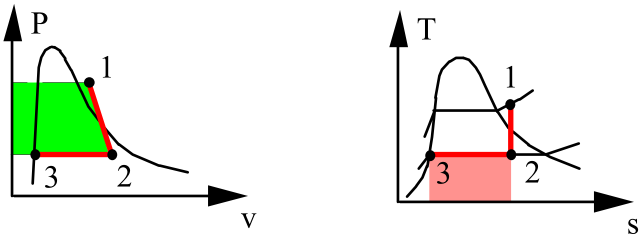
Energy Eq.6.13: $q = h_3 - h_2$,

Entropy Eq.9.8: $s_3 = s_2 + \int dq/T + s_{He\,gen}$

$q = h_3 - h_{2,s} = 191.83 - 2520.35 =$ **-2328.5 kJ/kg**

$\dot{Q} = \dot{m}\, q = 0.1074 \times (-2328.5) =$ **- 250 kW**

Explanation for the work term is in Sect. 9.3, Eq.9.14



9.37

One technique for operating a steam turbine in part-load power output is to throttle the steam to a lower pressure before it enters the turbine, as shown in Fig. P9.39. The steamline conditions are 2 MPa, 400°C, and the turbine exhaust pressure is fixed at 10 kPa. Assuming the expansion inside the turbine to be reversible and adiabatic, determine

a. The full-load specific work output of the turbine
b. The pressure the steam must be throttled to for 80% of full-load output
c. Show both processes in a *T–s* diagram.

a) C.V Turbine. Full load reversible and adiabatic

Entropy Eq.9.8 reduces to constant s so from Table B.1.3 and B.1.2

$$s_3 = s_1 = 7.1271 = 0.6493 + x_{3a} \times 7.5009$$

$$=> \quad x_{3a} = 0.8636$$

$$h_{3a} = 191.83 + 0.8636 \times 2392.8 = 2258.3 \text{ kJ/kg}$$

Energy Eq.6.13 for turbine

$${}_1w_{3a} = h_1 - h_{3a} = 3247.6 - 2258.3 = \mathbf{989.3\ kJ/kg}$$

b) The energy equation for the part load operation and notice that we have constant h in the throttle process.

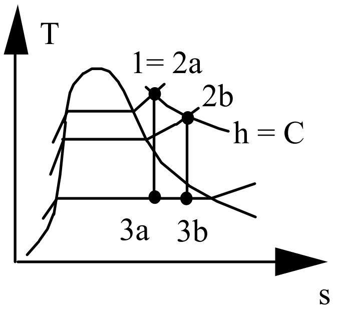
$$w_T = 0.80 \times 989.3 = 791.4 = 3247.6 - h_{3b}$$

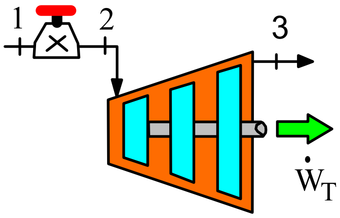
$$h_{3b} = 2456.2 = 191.83 + x_{3b} \times 2392.8 \quad => \quad x_{3b} = 0.9463$$

$$s_{3b} = 0.6492 + 0.9463 \times 7.501 = 7.7474 \text{ kJ/kg}$$

$$\left.\begin{array}{l} s_{2b} = s_{3b} = 7.7474 \\ h_{2b} = h_1 = 3247.6 \end{array}\right\} \rightarrow \quad \begin{array}{l} P_{2b} = \mathbf{510\ kPa} \\ \& \ T_{2b} = 388.4°C \end{array}$$

c)





9.38

An adiabatic air turbine receives 1 kg/s air at 1500 K, 1.6 MPa and 2 kg/s air at 400 kPa, T_2 in a setup similar to Fig. P6.76 with an exit flow at 100 kPa. What should the temperature T_2 be so the whole process can be reversible?

The process is reversible if we do not generate any entropy. Physically in this problem it means that state 2 must match the state inside the turbine so we do not mix fluid at two different temperatures (we assume the pressure inside is exactly 400 kPa).

For this reason let us select the front end as C.V. and consider the flow from state 1 to the 400 kPa. This is a single flow

Entropy Eq.9.8: $s_1 + 0/T + 0 = s_2$;

$$s_2 - s_1 = 0 = s^o_{T2} - s^o_{T1} - R \ln(P_2 / P_1)$$

$$s^o_{T2} = s^o_{T1} + R \ln(P_2 / P_1) = 8.61208 + 0.287 \ln \frac{400}{1600} = 8.2142 \text{ kJ/kg-K}$$

From A.7.1: $T_2 =$ **1071.8 K**

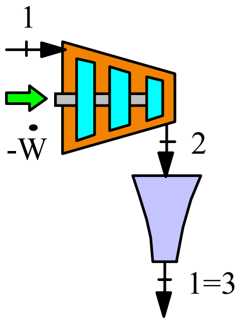
If we solve with constant specific heats we get from Eq.8.23 and k = 1.4

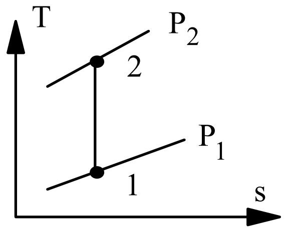
$$T_2 = T_1 (P_2 / P_1)^{(k-1)/k} = 1500 (400/1600)^{0.2857} = 1009.4 \text{ K}$$

9.39

A reversible adiabatic compression of an air flow from 20°C, 100 kPa to 200 kPa is followed by an expansion down to 100 kPa in an ideal nozzle. What are the two processes? How hot does the air get? What is the exit velocity?

Solution:





Separate control volumes around compressor and nozzle. For ideal compressor we have
inlet : 1 and exit : 2

Adiabatic : q = 0.
Reversible: $s_{gen} = 0$

Energy Eq.6.13: $h_1 + 0 = w_C + h_2$; $h_2 = h_3 + \frac{1}{2}\mathbf{V}^2$

Entropy Eq.9.8: $s_1 + 0/T + 0 = s_2$; $s_2 + 0/T = s_3$

So both processes are **isentropic**.

$$-w_C = h_2 - h_1 , \quad s_2 = s_1$$

Properties Table A.5 air: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$

Process gives constant s (isentropic) which with constant C_{Po} gives Eq.8.23

$$=> \quad T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 293.15\ (200/100)^{0.2857} = \mathbf{357.4\ K}$$

$$\Rightarrow \quad -w_C = C_{Po}(T_2 - T_1) = 1.004\ (357.4 - 293.2) = 64.457 \text{ kJ/kg}$$

The ideal nozzle then expands back down to P_1 (constant s) so state 3 equals state 1. The energy equation has no work but kinetic energy and gives:

$$\frac{1}{2}\mathbf{V}^2 = h_2 - h_1 = -w_C = 64\,457 \text{ J/kg} \quad \text{(remember conversion to J)}$$

$$\Rightarrow \quad \mathbf{V}_3 = \sqrt{2 \times 64\,457} = \mathbf{359\ m/s}$$

9.40

A turbo charger boosts the inlet air pressure to an automobile engine. It consists of an exhaust gas driven turbine directly connected to an air compressor, as shown in Fig. P9.34. For a certain engine load the conditions are given in the figure. Assume that both the turbine and the compressor are reversible and adiabatic having also the same mass flow rate. Calculate the turbine exit temperature and power output. Find also the compressor exit pressure and temperature.

Solution:

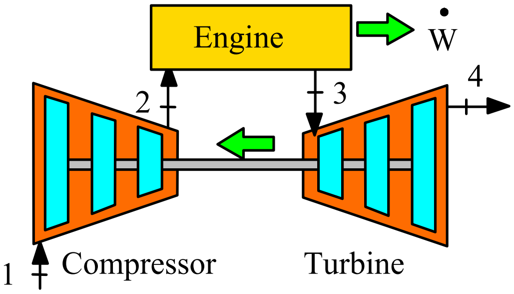
CV: Turbine, Steady single inlet and exit flows,

Process: adiabatic: $q = 0$,

reversible: $s_{gen} = 0$

EnergyEq.6.13: $w_T = h_3 - h_4$,

Entropy Eq.9.8: $s_4 = s_3$



The property relation for ideal gas gives Eq.8.23, k from Table A.5

$$s_4 = s_3 \rightarrow T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 923.2 \left(\frac{100}{170}\right)^{0.286} = \mathbf{793.2\ K}$$

The energy equation is evaluated with specific heat from Table A.5

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(923.2 - 793.2) = 130.5 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = \mathbf{13.05\ kW}$$

C.V. Compressor, steady 1 inlet and 1 exit, same flow rate as turbine.

Energy Eq.6.13: $-w_C = h_2 - h_1$,

Entropy Eq.9.8: $s_2 = s_1$

Express the energy equation for the shaft and compressor having the turbine power as input with the same mass flow rate so we get

$$-w_C = w_T = 130.5 \text{ kJ/kg} = C_{P0}(T_2 - T_1) = 1.004(T_2 - 303.2)$$

$$T_2 = \mathbf{433.2\ K}$$

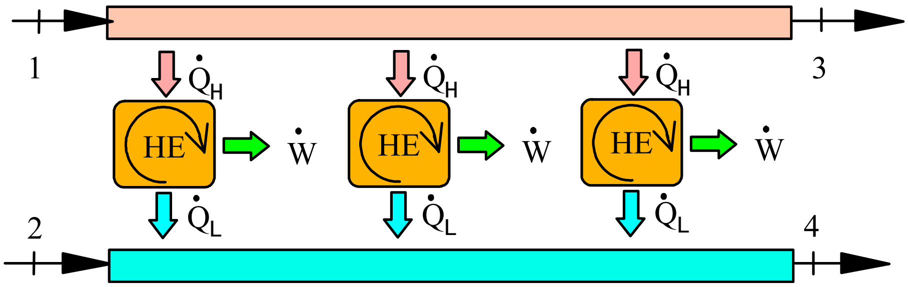
The property relation for $s_2 = s_1$ is Eq.8.23 and inverted as

$$P_2 = P_1(T_2/T_1)^{\frac{k}{k-1}} = 100\left(\frac{433.2}{303.2}\right)^{3.5} = \mathbf{348.7\ kPa}$$

9.41

Two flows of air both at 200 kPa, one has 1 kg/s at 400 K and the other has 2 kg/s at 290 K. The two lines exchange energy through a number of ideal heat engines taking energy from the hot line and rejecting it to the colder line. The two flows then leave at the same temperature. Assume the whole setup is reversible and find the exit temperature and the total power out of the heat engines.

Solution:



C.V. Total setup

Energy Eq.6.10: $\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_1h_3 + \dot{m}_2h_4 + \dot{W}_{TOT}$

Entropy Eq.9.7: $\dot{m}_1s_1 + \dot{m}_2s_2 + \dot{S}_{gen} + \int d\dot{Q}/T = \dot{m}_1s_3 + \dot{m}_2s_4$

Process: Reversible $\dot{S}_{gen} = 0$ Adiabatic $\dot{Q} = 0$

Assume the exit flow has the same pressure as the inlet flow then the pressure part of the entropy cancels out and we have

Exit same T, P => $h_3 = h_4 = h_e$; $s_3 = s_4 = s_e$

$$\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_{TOT}h_e + \dot{W}_{TOT}$$

$$\dot{m}_1s_1 + \dot{m}_2s_2 = \dot{m}_{TOT}s_e$$

$$s^o_{Te} = \frac{\dot{m}_1}{\dot{m}_{TOT}} s^o_{T1} + \frac{\dot{m}_2}{\dot{m}_{TOT}} s^o_{T2} = \frac{1}{3} \times 7.1593 + \frac{2}{3} \times 6.8352 = 6.9432 \text{ kJ/kgK}$$

Table A.7: => $T_e \cong 323$ K; $h_e = 323.6$ kJ/kg

$$\dot{W}_{TOT} = \dot{m}_1(h_1 - h_e) + \dot{m}_2(h_2 - h_e)$$

$$= 1(401.3 - 323.6) + 2(290.43 - 323.6) = \mathbf{11.36\ kW}$$

Note: The solution using constant heat capacity writes the entropy equation using Eq.8.16, the pressure terms cancel out so we get

$$\frac{1}{3} C_p \ln(T_e/T_1) + \frac{2}{3} C_p \ln(T_e/T_2) = 0 \quad \Rightarrow \quad \ln T_e = (\ln T_1 + 2 \ln T_2)/3$$

9.42

A flow of 5 kg/s water at 100 kPa, 20°C should be delivered as steam at 1000 kPa, 350°C to some application. We have a heat source at constant 500°C. If the process should be reversible how much heat transfer should we have?

CV Around unknown device out to the source surface.

Energy Eq.: $\dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$

Entropy Eq.: $\dot{m}s_i + \dot{Q}/T_S + 0 = \dot{m}s_e$ (T_S is constant, $s_{gen} = 0$)

Inlet state: $s_i = 0.2966$ kJ/kgK, Table B.1.1
Exit state: $s_e = 7.301$ kJ/kgK, Table B.1.3

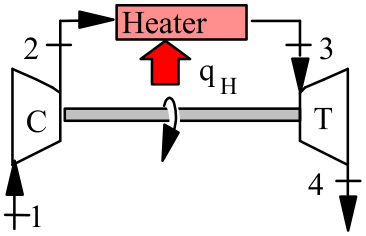
$$\dot{Q} = \dot{m}\, T_S\, (s_e - s) = 5\ 773.15\ (7.301 - 0.2966) = \mathbf{27.1\ MW}$$

The theory does not say exactly how to do it. As the pressure goes up we must have a pump or compressor and since the substance temperature is lower than the source temperature a reversible heat transfer must happen through some kind of heat engine receiving a Q from the source and delivering it to the flow extracting some work in the process.

9.43

A heat-powered portable air compressor consists of three components: (a) an adiabatic compressor; (b) a constant pressure heater (heat supplied from an outside source); and (c) an adiabatic turbine. Ambient air enters the compressor at 100 kPa, 300 K, and is compressed to 600 kPa. All of the power from the turbine goes into the compressor, and the turbine exhaust is the supply of compressed air. If this pressure is required to be 200 kPa, what must the temperature be at the exit of the heater?

Solution:



$P_2 = 600$ kPa, $P_4 = 200$ kPa

Adiabatic and reversible compressor:

Process: $q = 0$ and $s_{gen} = 0$

Energy Eq.6.13: $h - w_c = h_2$

Entropy Eq.9.8: $s_2 = s_1$

For constant specific heat the isentropic relation becomes Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.2857} = 500.8 \text{ K}$$

$$-w_c = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.5 \text{ kJ/kg}$$

Adiabatic and reversible turbine: $q = 0$ and $s_{gen} = 0$

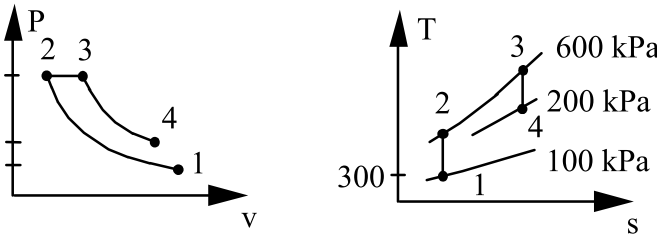
Energy Eq.6.13: $h_3 = w_T + h_4$; Entropy Eq.9.8: $s_4 = s_3$

For constant specific heat the isentropic relation becomes Eq.8.32

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = T_3(200/600)^{0.2857} = 0.7304\ T_3$$

Energy Eq. for shaft: $-w_c = w_T = C_{P0}(T_3 - T_4)$

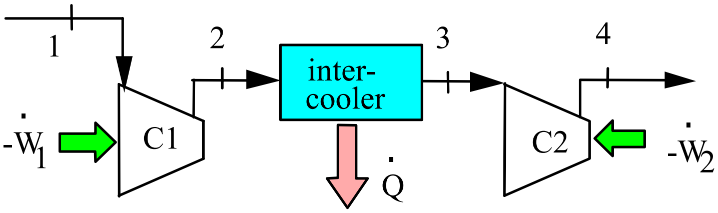
$$201.5 = 1.004\ T_3(1 - 0.7304) \quad => \quad T_3 = \mathbf{744.4\ K}$$



9.44

A two-stage compressor having an interstage cooler takes in air, 300 K, 100 kPa, and compresses it to 2 MPa, as shown in Fig. P9.44. The cooler then cools the air to 340 K, after which it enters the second stage, which has an exit pressure of 15.74 MPa. Both stages are adiabatic, and reversible. Find q in the cooler, total specific work, and compare this to the work required with no intercooler.

Solution:



C.V.: Stage 1 air, Steady flow

Process: adibatic: q = 0, reversible: $s_{gen} = 0$

Energy Eq.6.13: $-w_{C1} = h_2 - h_1$, Entropy Eq.9.8: $s_2 = s_1$

Assume constant $C_{P0} = 1.004$ from A.5 and isentropic leads to Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(2000/100)^{0.286} = 706.7 \text{ K}$$

$$w_{C1} = h_1 - h_2 = C_{P0}(T_1 - T_2) = 1.004(300 - 706.7) = \mathbf{-408.3\ kJ/kg}$$

C.V. Intercooler, no work and no changes in kinetic or potential energy.

$$q_{23} = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004(340 - 706.7) = \mathbf{-368.2\ kJ/kg}$$

C.V. Stage 2. Analysis the same as stage 1. So from Eq.8.32

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 340(15.74/2)^{0.286} = 613.4 \text{ K}$$

$$w_{C2} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(340 - 613.4) = \mathbf{-274.5\ kJ/kg}$$

Same flow rate through both stages so the total work is the sum of the two

$$w_{comp} = w_{C1} + w_{C2} = -408.3 - 274.5 = \mathbf{-682.8\ kJ/kg}$$

For no intercooler (P_2 = 15.74 MPa) same analysis as stage 1. So Eq.8.32

$$T_2 = 300(15740/100)^{0.286} = 1274.9 \text{ K}$$

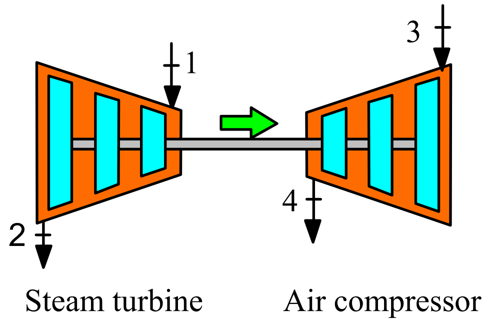
$$w_{comp} = 1.004(300 - 1274.9) = \mathbf{-978.8\ kJ/kg}$$

9.45

A certain industrial process requires a steady supply of saturated vapor steam at 200 kPa, at a rate of 0.5 kg/s. Also required is a steady supply of compressed air at 500 kPa, at a rate of 0.1 kg/s. Both are to be supplied by the process shown in Fig. P9.41. Steam is expanded in a turbine to supply the power needed to drive the air compressor, and the exhaust steam exits the turbine at the desired state. Air into the compressor is at the ambient conditions, 100 kPa, 20°C. Give the required steam inlet pressure and temperature, assuming that both the turbine and the compressor are reversible and adiabatic.

Solution:

C.V. Each device. Steady flow. Both adiabatic ($q = 0$) and reversible ($s_{gen} = 0$).



Compressor: $s_4 = s_3$ => $T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 293.2\left(\frac{500}{100}\right)^{0.286} = 464.6 \text{ K}$

$\dot{W}_C = \dot{m}_3(h_3 - h_4) = 0.1 \times 1.004(293.2 - 464.6) = -17.2 \text{ kW}$

Turbine: Energy: $\dot{W}_T = +17.2 \text{ kW} = \dot{m}_1(h_1 - h_2)$; Entropy: $s_2 = s_1$

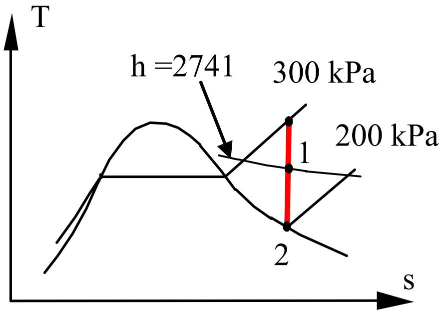


Table B.1.2: $P_2 = 200$ kPa, $x_2 = 1$ =>

$h_2 = 2706.6$ kJ/kg, $s_2 = 7.1271$ kJ/kgK

$h_1 = 2706.6 + 17.2/0.5 = 2741.0$ kJ/kg

$s_1 = s_2 = 7.1271$ kJ/kg K,

300 kPa: $s = s_2$ => $h = 2783.0$ kJ/kg

Interpolate between the 200 and 300 kPa

$$P = 200 + (300 - 200)\ \frac{2741 - 2706.63}{2783.0 - 2706.63} = \mathbf{245\ kPa}$$

$$T = 120.23 + (160.55 - 120.23)\frac{2741 - 2706.63}{2783.0 - 2706.63} = \mathbf{138.4°C}$$

If you use the software you get: At h_1, $s_1 \rightarrow P_1 =$ **242 kPa**,$T_1 =$ **138.3°C**

9.46

A certain industrial process requires a steady 0.5 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler. Local ambient conditions are 100 kPa, 20°C. Using an reversible compressor, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Solution:

Air Table A.5: $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg K, $k = 1.4$

State 1: $T_1 = T_o = 20^oC$, $P_1 = P_o = 100$ kPa, $\dot{m} = 0.5$ kg/s

State 2: $P_2 = P_3 = 500$ kPa

State 3: $T_3 = 30^oC$, $P_3 = 500$ kPa

Compressor: Assume Isentropic (adiabatic $q = 0$ and reversible $s_{gen} = 0$) From entropy equation Eq.9.8 this gives constant s which is expressed for an ideal gas in Eq.8.32

$$T_2 = T_1 \, (P_2/P_1)^{\frac{k-1}{k}} = 293.15 \, (500/100)^{0.2857} = 464.6 \text{ K}$$

Energy Eq.6.13: $q_c + h_1 = h_2 + w_c; \qquad q_c = 0,$

assume constant specific heat from Table A.5

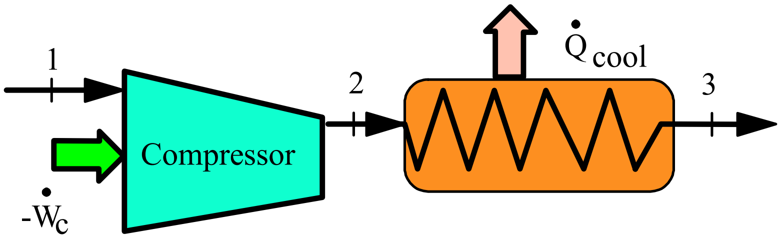
$$w_c = C_p(T_1 - T_2) = 1.004 \, (293.15 - 464.6) = -172.0 \text{ kJ/kg}$$

$$\dot{W}_C = \dot{m}w_C = \mathbf{-86 \ kW}$$

Aftercooler Energy Eq.6.13: $q + h_2 = h_3 + w; \quad w = 0,$

assume constant specific heat

$$\dot{Q} = \dot{m}q = \dot{m}C_p(T_3 - T_2) = 0.5 \times 1.004(303.15 - 464.6) = \mathbf{-81 \ kW}$$



Compressor section Aftercooler section

9.47

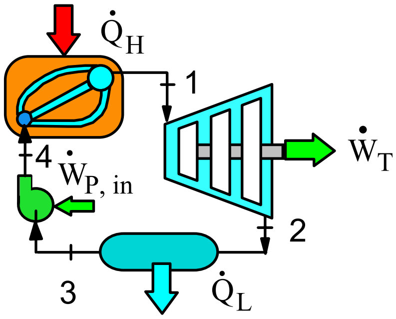
Consider a steam turbine power plant operating near critical pressure, as shown in Fig. P9.33. As a first approximation, it may be assumed that the turbine and the pump processes are reversible and adiabatic. Neglecting any changes in kinetic and potential energies, calculate

a. The specific turbine work output and the turbine exit state

b. The pump work input and enthalpy at the pump exit state

c. The thermal efficiency of the cycle

Solution:



$P_1 = P_4 = 20$ MPa

$T_1 = 700$ °C

$P_2 = P_3 = 20$ kPa

$T_3 = 40$ °C

a) State 1: (P, T) Table B.1.3 $h_1 = 3809.1$ kJ/kg, $s_1 = 6.7993$ kJ/kg K

C.V. Turbine.

Entropy Eq.9.8: $s_2 = s_1 = 6.7993$ kJ/kg K

Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766$ => $x_2 = 0.8433$

$h_2 = 251.4 + 0.8433\times 2358.33 = \mathbf{2240.1}$

Energy Eq.6.13: $w_T = h_1 - h_2 = \mathbf{1569\ kJ/kg}$

b) State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

$h_3 = 167.5$ kJ/kg, $v_3 = 0.001008$ m^3/kg

Property relation v = constant gives work from Eq.9.15 as

$w_P = - v_3(P_4 - P_3) = -0.001008(20000 – 20) = \mathbf{-20.1\ kJ/kg}$

$h_4 = h_3 - w_P = 167.5 + 20.1 = \mathbf{187.6\ kJ/kg}$

c) The heat transfer in the boiler is from energy Eq.6.13

$q_{boiler} = h_1 - h_4 = 3809.1 – 187.6 = 3621.5$ kJ/kg

$w_{net} = 1569 – 20.1 = \mathbf{1548.9\ kJ/kg}$

$$\eta_{TH} = w_{net}/q_{boiler} = \frac{1548.9}{3621.5} = \mathbf{0.428}$$

Transient processes

9.48

Air in a tank is at 300 kPa, 400 K with a volume of 2 m^3. A valve on the tank is opened to let some air escape to the ambient to a final pressure inside of 200 kPa. Find the final temperature and mass assuming a reversible adiabatic process for the air remaining inside the tank.

Solution:

C.V. Total tank.

Continuity Eq.6.15: $m_2 - m_1 = -m_{ex}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = -m_{ex}h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = -m_{ex}s_{ex} + \int dQ/T + {}_1S_{2\text{ gen}}$

Process: Adiabatic ${}_1Q_2 = 0$; rigid tank ${}_1W_2 = 0$

This has too many unknowns (we do not know state 2).

C.V. m_2 the mass that remains in the tank. This is a control mass.

Energy Eq.5.11: $m_2(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.14: $m_2(s_2 - s_1) = \int dQ/T + {}_1S_{2\text{ gen}}$

Process: Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\text{ gen}} = 0$

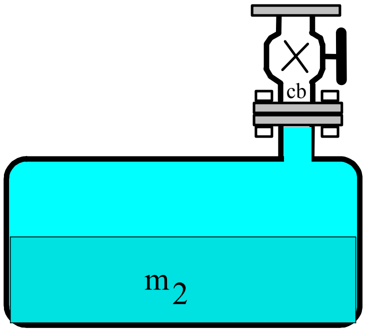
$\Rightarrow \quad s_2 = s_1$

Ideal gas and process Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 400(200/300)^{0.2857} = \mathbf{356.25\ K}$$

$$m_2 = \frac{P_2V}{RT_2} = \frac{200 \times 2}{0.287 \times 356.25} = \mathbf{3.912\ kg}$$

Notice that the work term is not zero for mass m_2. The work goes into pushing the mass m_{ex} out.



9.49

A tank contains 1 kg of carbon dioxide at 6 MPa, 60°C and it is connected to a turbine with an exhaust at 1000 kPa. The carbon dioxide flows out of the tank and through the turbine to a final state in the tank of saturated vapor is reached. If the process is adiabatic and reversible find the final mass in the tank and the turbine work output.

C.V. The tank and turbine. This is a transient problem.

Continuity Eq.6.15: $m_2 - m_1 = -m_{ex}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = -m_{ex}h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = -m_{ex}s_{ex} + \int dQ/T + {}_1S_{2\ gen}$

Process: Adiabatic ${}_1Q_2 = 0$; reversible ${}_1S_{2\ gen} = 0$

State 1: $v_1 = 0.00801\ m^3/kg$, $u_1 = 322.51\ kJ/kg$, $s_1 = 1.2789\ kJ/kg\text{-}K$

State 2: Sat. vapor, 1 property missing

C.V. m_2 the mass that remains in the tank. This is a control mass.

Process: Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\ gen} = 0$

Entropy Eq.8.14: $m_2(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = 0 + 0$

$\Rightarrow$ $s_2 = s_1$ this is the missing property.

State 2: $T_2 = -19.19°C$, $v_2 = 0.018851\ m^3/kg$, $u_2 = 285.87\ kJ/kg$

State exit: $s_{ex} = s_2 = s_1$ follows from entropy Eq. for first C.V. with the use of the continuity equation. Use 1004.5 kPa for -40°C.

$x_{ex} = (1.2789 - 0)/1.3829 = 0.924796$ => $h_{ex} = 298.17\ kJ/kg$

Tank volume constant so $V = m_1v_1 = m_2v_2$

$$m_2 = m_1\ v_1 / v_2 = 1 \times 0.00801 / 0.018851 = \mathbf{0.4249\ kg}$$

From energy eq.

$${}_1W_2 = m_1u_1 - m_2u_2 - m_{ex}h_{ex}$$
$$= 1 \times 322.51 - 0.4249 \times 285.87 - 0.5751 \times 298.17\ [kg\ kJ/kg]$$
$$= \mathbf{29.57\ kJ}$$

9.50

An underground salt mine, 100 000 m^3 in volume, contains air at 290 K, 100 kPa. The mine is used for energy storage so the local power plant pumps it up to 2.1 MPa using outside air at 290 K, 100 kPa. Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work.

Solution:

C.V. The mine volume and the pump

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = {}_1Q_2 - {}_1W_2 + m_{in}h_{in}$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = \int dQ/T + {}_1S_{2\,gen} + m_{in}s_{in}$

Process: Adiabatic ${}_1Q_2 = 0$, Process ideal ${}_1S_{2\,gen} = 0$, $s_1 = s_{in}$

$$\Rightarrow m_2s_2 = m_1s_1 + m_{in}s_{in} = (m_1 + m_{in})s_1 = m_2s_1 \Rightarrow s_2 = s_1$$

Constant s $\Rightarrow$ Eq.8.28 $s^o_{T2} = s^o_{Ti} + R \ln(P_2 / P_{in})$

$$s^o_{T2} = 6.83521 + 0.287 \ln(21) = 7.7090 \text{ kJ/kg K}$$

A.7 $\Rightarrow T_2 = \mathbf{680\ K}$, $u_2 = 496.94$ kJ/kg

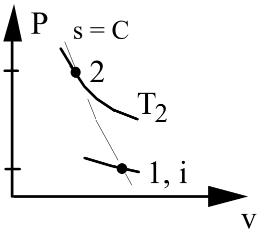
$$m_1 = P_1V_1/RT_1 = 100\times10^5/(0.287 \times 290) = 1.20149 \times 10^5 \text{ kg}$$

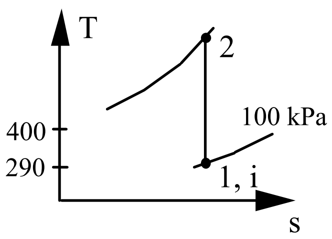
$$m_2 = P_2V_2/RT_2 = 100 \times 21\times10^5/(0.287 \times 680) = \mathbf{10.760 \times 10^5\ kg}$$

$$\Rightarrow m_{in} = 9.5585\times10^5 \text{ kg}$$

$${}_1W_2 = m_{in}h_{in} + m_1u_1 - m_2u_2$$

$$= m_{in}(290.43) + m_1(207.19) - m_2(496.94) = \mathbf{-2.322 \times 10^8\ kJ}$$





9.51

Air in a tank is at 300 kPa, 400 K with a volume of 2 m^3. A valve on the tank is opened to let some air escape to the ambient to a final pressure inside of 200 kPa. At the same time the tank is heated so the air remaining has a constant temperature. What is the mass average value of the s leaving assuming this is an internally reversible process?

Solution:

C.V. Tank, emptying process with heat transfer.

Continuity Eq.6.15: $m_2 - m_1 = -m_e$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = -m_eh_e + {}_1Q_2$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = -m_es_e + {}_1Q_2/T + 0$

Process: $T_2 = T_1$ => ${}_1Q_2$ in at 400 K

Reversible ${}_1S_{2\ gen} = 0$

State 1: Ideal gas $m_1 = P_1V/RT_1 = 300 \times 2/0.287 \times 400 = 5.2265$ kg

State 2: 200 kPa, 400 K

$m_2 = P_2V/RT_2 = 200 \times 2/0.287 \times 400 = 3.4843$ kg

=> $m_e = 1.7422$ kg

From the energy equation:

$${}_1Q_2 = m_2u_2 - m_1u_1 + m_eh_e$$

$$= 3.4843 \times 286.49 - 5.2265 \times 286.49 + 1.7422 \times 401.3$$

$$= 1.7422(401.3 - 286.49) = 200 \text{ kJ}$$

$$m_es_e = m_1s_1 - m_2s_2 + {}_1Q_2/T$$

$$= 5.2265[7.15926 - 0.287 \ln (300/100)] - 3.4843[7.15926 - 0.287 \ln (200/100)] + (200/400)$$

$$m_es_e = 35.770 - 24.252 + 0.5 = 12.018 \text{ kJ/K}$$

$s_e = 12.018/1.7422 = 6.89817 =$ **6.8982 kJ/kg K**

Note that the exit state e in this process is for the air before it is throttled across the discharge valve. The throttling process from the tank pressure to ambient pressure is a highly irreversible process.

9.52

An insulated 2 m^3 tank is to be charged with R-134a from a line flowing the refrigerant at 3 MPa. The tank is initially evacuated, and the valve is closed when the pressure inside the tank reaches 3 MPa. The line is supplied by an insulated compressor that takes in R-134a at 5°C, quality of 96.5 %, and compresses it to 3 MPa in a reversible process. Calculate the total work input to the compressor to charge the tank.

Solution:

C.V.: Compressor, R-134a. Steady 1 inlet and 1 exit flow, no heat transfer.

1st Law Eq.6.13: $q_c + h_1 = h_1 = h_2 + w_c$

Entropy Eq.9.8: $s_1 + \int dq/T + s_{gen} = s_1 + 0 = s_2$

inlet: $T_1 = 5^oC$, $x_1 = 0.965$ use Table B.5.1

$s_1 = s_f + x_1 s_{fg} = 1.0243 + 0.965 \times 0.6995 = 1.6993$ kJ/kg K,

$h_1 = h_f + x_1 h_{fg} = 206.8 + 0.965 \times 194.6 = 394.6$ kJ/kg

exit: $P_2 = 3$ MPa

From the entropy eq.: $s_2 = s_1 = 1.6993$ kJ/kg K;

$T_2 = 90^oC$, $h_2 = 436.2$ kJ/kg

$w_c = h_1 - h_2 = -41.6$ kJ/kg

C.V.: Tank; $V_T = 2\ m^3$, $P_T = 3$ MPa

1st Law Eq.6.16: $Q + m_i h_i = m_2 u_2 - m_1 u_1 + m_e h_e + W$;

Process and states have: $Q = 0$, $W = 0$, $m_e = 0$, $m_1 = 0$, $m_2 = m_i$

$u_2 = h_i = 436.2$ kJ/kg

Final state: $P_T = 3$ MPa, $u_2 = 436.2$ kJ/kg

$\rightarrow T_T = 101.9^oC$, $v_T = 0.006783\ m^3/kg$

$m_T = V_T/v_T = 294.84$ kg;

The work term is from the specific compressor work and the total mass

$-W_c = m_T(-w_c) =$ **12 295 kJ**

9.53

R-410a at 120°C, 4 MPa is in an insulated tank and flow is now allowed out to a turbine with a backup pressure of 800 kPa. The flow continue to a final tank pressure of 800 kPa and the process stops. If the initial mass was 1 kg how much mass is left in the tank and what is the turbine work assuming a reversible process?

Solution:

C.V. Total tank and turbine.

Continuity Eq.6.15: $m_2 - m_1 = -m_{ex}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = -m_{ex}h_{ex} + {}_1Q_2 - {}_1W_2$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = -m_{ex}s_{ex} + \int dQ/T + {}_1S_{2\ gen}$

Process: Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\ gen} = 0$

This has too many unknowns (we do not know state 2 only P_2).

C.V. m_2 the mass that remains in the tank. This is a control mass.

Entropy Eq.8.14: $m_2(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: Adiabatic ${}_1Q_2 = 0$; Reversible ${}_1S_{2\ gen} = 0$

$\Rightarrow \quad s_2 = s_1$

State 1: $v_1 = 0.00897\ m^3/kg$, $u_1 = 331.39\ kJ/kg$, $s_1 = 1.1529\ kJ/kg\text{-}K$

State 2 (P,s): $T_2 = 33.23°C$, $v_2 = 0.37182\ m^3/kg$, $u_2 = 281.29\ kJ/kg$

State exit: $s_{ex} = s_2 = s_1$ follows from entropy Eq. for first C.V. using the continuity eq., this is identical to state 2, $h_{ex} = 312.85\ kJ/kg$

Tank volume constant so $V = m_1v_1 = m_2v_2$

$m_2 = m_1\ v_1 / v_2 = 1 \times 0.00897 / 0.37182 = \mathbf{0.0241\ kg}$

From energy eq.

$${}_1W_2 = m_1u_1 - m_2u_2 - m_{ex}h_{ex}$$
$$= 1 \times 331.39 - 0.0241 \times 281.29 - 0.9759 \times 312.85\ [kg\ kJ/kg]$$
$$= \mathbf{19.3\ kJ}$$

Reversible shaft work, Bernoulli equation

9.54

A pump has a 2 kW motor. How much liquid water at 15^{o}C can I pump to 250 kPa from 100 kPa?

Incompressible flow (liquid water) and we assume reversible. Then the shaftwork is from Eq.9.15

$$w = -\int v\, dP = -v\, \Delta P = -0.001\ \text{m}^3/\text{kg}\ (250 - 100)\ \text{kPa}$$
$$= -\ 0.15\ \text{kJ/kg}$$

$$\dot{m} = \frac{\dot{W}}{-w} = \frac{2}{0.15} = \mathbf{13.3\ kg/s}$$

9.55

A large storage tank contains saturated liquid nitrogen at ambient pressure, 100 kPa; it is to be pumped to 500 kPa and fed to a pipeline at the rate of 0.5 kg/s. How much power input is required for the pump, assuming it to be reversible?

Solution:

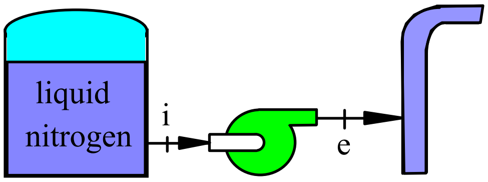
C.V. Pump, liquid is assumed to be incompressible.

Table B.6.1 at $P_i = 101.3$ kPa , $v_{Fi} = 0.00124\ m^3/kg$

Eq.9.15

$w_{PUMP} = - w_{cv} = \int v dP \approx v_{Fi}(P_e - P_i)$

$= 0.00124(500 - 101) = 0.494$ kJ/kg



$\dot{W}_{PUMP} = \dot{m} w_{PUMP} = 0.5$ kg/s (0.494 kJ/kg) = **0.247 kW**

9.56

A garden water hose has liquid water at 200 kPa, 15°C. How high a velocity can be generated in a small ideal nozzle? If you direct the water spray straight up how high will it go?

Solution:

Liquid water is incompressible and we will assume process is reversible.

Bernoulli's Eq. across the nozzle Eq.9.16: $\quad v\Delta P = \Delta(\frac{1}{2}\mathbf{V}^2)$

$$\mathbf{V} = \sqrt{2v\Delta P} = \sqrt{2\times 0.001001 \times (200\text{-}101) \times 1000} = \mathbf{14.08\ m/s}$$

Bernoulli's Eq.9.16 for the column: $\quad \Delta(\frac{1}{2}\mathbf{V}^2) = \Delta gZ$

$$\Delta Z = \Delta(\frac{1}{2}\mathbf{V}^2)/g = v\Delta P/g = 0.001001 \times (200 - 101) \times 1000/9.807 = \mathbf{10.1\ m}$$

9.57

A small pump takes in water at 20°C, 100 kPa and pumps it to 2.5 MPa at a flow rate of 100 kg/min. Find the required pump power input.

Solution:

C.V. Pump. Assume reversible pump and incompressible flow.

With single steady state flow it leads to the work in Eq.9.15

$$w_p = -\int v dP = -v_i(P_e - P_i) = -0.001002(2500 - 100) = -2.4 \text{ kJ/kg}$$

$$\dot{W}_p = \dot{m} w_p = \frac{100}{60} \frac{\text{kg/min}}{\text{sec/min}} (-2.4 \text{ kJ/kg}) = \mathbf{-4.0\ kW}$$

9.58

An irrigation pump takes water from a river at 10°C, 100 kPa and pumps it up to an open canal at a 100 m higher elevation. The pipe diameter in and out of the pump is 0.1 m and the motor driving the pump is 5 hp. Neglect kinetic energies and friction, find the maximum possible mass flow rate.

CV the pump. The flow is incompressible and steady flow. The pump work is the difference between the flow work in and out and from Bernoulli's eq. for the pipe that is equal to the potential energy increase sincle pump inlet pressure and pipe outlet pressure are the same.

$$w_p = v\,\Delta P = g\,\Delta Z = 9.81 \times 100 \text{ J/kg} = 0.981 \text{ kJ/kg}$$

The horsepower is converted from Table A.1

$$\dot{W}_{\text{motor}} = 5 \text{ hp} = 5 \times 0.746 = 3.73 \text{ kW}$$

$$\dot{m} = \dot{W}_{\text{motor}} / w_p = 3.73 / 0.981 = \mathbf{3.8\ kg/s}$$

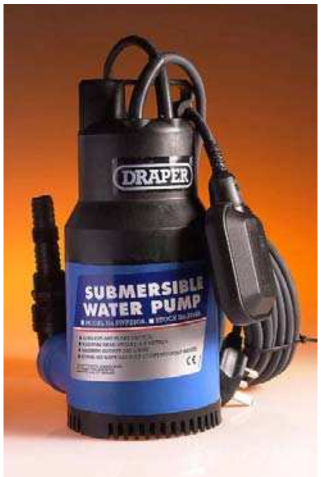
Comment:

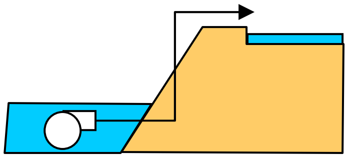
$$\dot{m} = AV/v \qquad \Rightarrow \qquad V = \frac{\dot{m}v}{A} = \frac{4\dot{m}}{\rho\,\pi\,D^2} = \frac{4 \times 3.8}{997 \times \pi \times 0.1^2} = 0.485 \text{ m/s}$$

The power to generated the kinetic energy is

$$\text{Power} = \dot{m}\,0.5\,V^2 = 3.8 \times 0.5 \times 0.485^2 = 0.447 \text{ W}$$

This is insignificant relative to the power needed for the potential energy increase.





Pump inlet and the pipe exit both have close to atmospheric pressure.

9.59

Saturated R-134a at -10°C is pumped/compressed to a pressure of 1.0 MPa at the rate of 0.5 kg/s in a reversible adiabatic process. Calculate the power required and the exit temperature for the two cases of inlet state of the R-134a:

a) quality of 100 %.

b) quality of 0 %.

Solution:

C.V.: Pump/Compressor, $\dot{m} = 0.5$ kg/s, R-134a

a) State 1: Table B.5.1, $T_1 = -10^oC$, $x_1 = 1.0$ Saturated vapor

$P_1 = P_g = 202$ kPa, $h_1 = h_g = 392.3$ kJ/kg, $s_1 = s_g = 1.7319$ kJ/kg K

Assume Compressor is isentropic, $s_2 = s_1 = 1.7319$ kJ/kg-K

$h_2 = 425.7$ kJ/kg, $\mathbf{T_2 = 45^oC}$

Energy Eq.6.13: $q_c + h_1 = h_2 + w_c$; $q_c = 0$

$w_{cs} = h_1 - h_2 = -33.4$ kJ/kg; => $\dot{W}_C = \dot{m}w_C = \mathbf{-16.7\ kW}$

b) State 1: $T_1 = -10^oC$, $x_1 = 0$ Saturated liquid. This is a pump.

$P_1 = 202$ kPa, $h_1 = h_f = 186.72$ kJ/kg, $v_1 = v_f = 0.000755$ m^3/kg

Energy Eq.6.13: $q_p + h_1 = h_2 + w_p$; $q_p = 0$

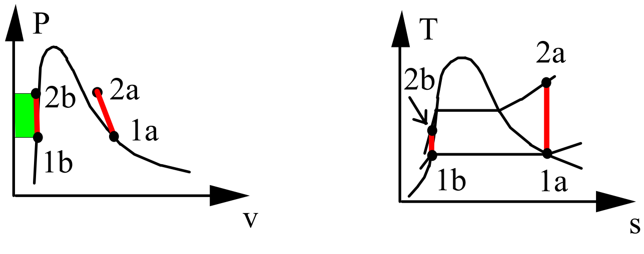
Assume Pump is isentropic and the liquid is incompressible, Eq.9.15:

$w_{ps} = - \int v\, dP = -v_1(P_2 - P_1) = -0.6$ kJ/kg

$h_2 = h_1 - w_p = 186.72 - (- 0.6) = 187.3$ kJ/kg, $P_2 = 1$ MPa

Assume State 2 is approximately a saturated liquid => $\mathbf{T_2 \cong -9.6^oC}$

$\dot{W}_P = \dot{m}w_P = \mathbf{-0.3\ kW}$



9.60

Liquid water at ambient conditions, 100 kPa, 25°C, enters a pump at the rate of 0.5 kg/s. Power input to the pump is 3 kW. Assuming the pump process to be reversible, determine the pump exit pressure and temperature.

Solution:

C.V. Pump. Steady single inlet and exit flow with no heat transfer.

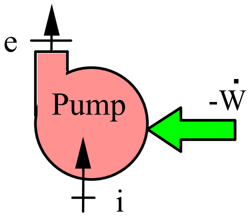
Energy Eq.6.13: $w = h_i - h_e = \dot{W}/\dot{m} = -3/0.5 = -6.0$ kJ/kg

Using also incompressible media we can use Eq.9.15

$$w = -\int vdP \approx -v_i(P_e - P_i) = -0.001003(P_e - 100)$$

from which we can solve for the exit pressure

$$P_e = 100 + 6.0/0.001003 = 6082 \text{ kPa} = \mathbf{6.082\ MPa}$$



$-\dot{W} = 3$ kW, $P_i = 100$ kPa

$T_i = 25°C$, $\dot{m} = 0.5$ kg/s

Energy Eq.: $h_e = h_i - w = 104.87 - (-6) = 110.87$ kJ/kg

Use Table B.1.4 at 5 MPa => $\mathbf{T_e = 25.3°C}$

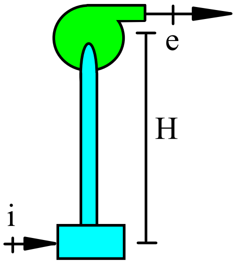
Remark:

If we use the software we get: $\left.\begin{matrix} s_i = 0.36736 = s_e \\ \text{At } s_e \ \& \ P_e \end{matrix}\right\} \rightarrow T_e = \mathbf{25.1°C}$

9.61

A small water pump on ground level has an inlet pipe down into a well at a depth H with the water at 100 kPa, 15°C. The pump delivers water at 400 kPa to a building. The absolute pressure of the water must be at least twice the saturation pressure to avoid cavitation. What is the maximum depth this setup will allow?

Solution:



C.V. Pipe in well, no work, no heat transfer

From Table B.1.1

$$P_{\text{inlet pump}} \geq 2\ P_{\text{sat, 15C}} = 2\times1.705 = 3.41 \text{ kPa}$$

Process:

Assume Δ KE $\approx \varnothing$, $v \approx$ constant. =>

Bernoulli Eq.9.16:

$$v\ \Delta P + g\ H = 0 \ \Rightarrow$$

$$1000 \times 0.001001\ (3.41 - 100) + 9.80665 \times H = 0$$

$$\Rightarrow \quad \mathbf{H = 9.86\ m}$$

Since flow has some kinetic energy and there are losses in the pipe the height is overestimated. Also the start transient would generate a very low inlet pressure (it moves flow by suction)

This pump can bring water up about 7 m by suction.

9.62

A small dam has a pipe carrying liquid water at 150 kPa, 20°C with a flow rate of 2000 kg/s in a 0.5 m diameter pipe. The pipe runs to the bottom of the dam 15 m lower into a turbine with pipe diameter 0.35 m. Assume no friction or heat transfer in the pipe and find the pressure of the turbine inlet. If the turbine exhausts to 100 kPa with negligible kinetic energy what is the rate of work?

Solution:

C.V. Pipe. Steady flow no work, no heat transfer.

States: compressed liquid B.1.1 $v_2 \approx v_1 \approx v_f = 0.001002 \text{ m}^3\text{/kg}$

Continuity Eq.6.3: $\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v$

$$\mathbf{V}_1 = \dot{m}v_1 / A_1 = 2000 \times 0.001002 / (\tfrac{\pi}{4} 0.5^2) = 10.2 \text{ m s}^{-1}$$

$$\mathbf{V}_2 = \dot{m}v_2 / A_2 = 2000 \times 0.001002 / (\tfrac{\pi}{4} 0.35^2) = 20.83 \text{ m s}^{-1}$$

From Bernoulli Eq.9.16 for the pipe (incompressible substance):

$$v(P_2 - P_1) + \tfrac{1}{2}(\mathbf{V}_2^2 - \mathbf{V}_1^2) + g(Z_2 - Z_1) = 0 \Rightarrow$$

$$P_2 = P_1 + \left[\tfrac{1}{2}(\mathbf{V}_1^2 - \mathbf{V}_2^2) + g(Z_1 - Z_2)\right]/v$$

$$= 150 + [\tfrac{1}{2}\times 10.2^2 - \tfrac{1}{2}\times 20.83^2 + 9.80665 \times 15]/(1000 \times 0.001002)$$

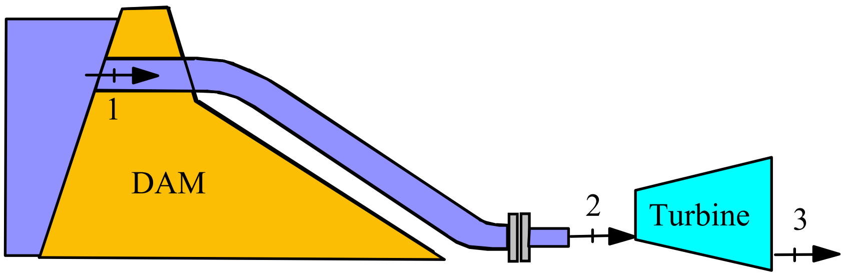
$$= 150 - 17.8 = \mathbf{132.2 \text{ kPa}}$$

Note that the pressure at the bottom should be higher due to the elevation difference but lower due to the acceleration. Now apply the energy equation Eq.9.13 for the total control volume

$$w = -\int v\, dP + \tfrac{1}{2}(\mathbf{V}_1^2 - \mathbf{V}_3^2) + g(Z_1 - Z_3)$$

$$= -0.001002\,(100 - 150) + [\tfrac{1}{2}\times 10.2^2 + 9.80665 \times 15]/1000 = 0.25 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 2000 \times 0.25 = \mathbf{500 \text{ kW}}$$



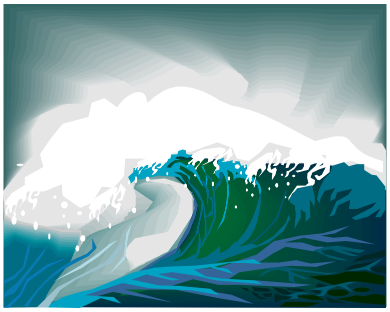
9.63

A wave comes rolling in to the beach at 2 m/s horizontal velocity. Neglect friction and find how high up (elevation) on the beach the wave will reach.

We will assume a steady reversible single flow at constant pressure and temperature for the incompressible liquid water. The water will flow in and up the sloped beach until it stops ($\mathbf{V} = 0$) so Bernoulli Eq.9.16 leads to

$$gz_{in} + \frac{1}{2}\mathbf{V}^2_{in} = gz_{ex} + 0$$

$$(z_{ex} - z_{in}) = \frac{1}{2g}\mathbf{V}^2_{in} = \frac{1}{2 \times 9.807 \text{ m/s}^2} 2^2 \text{ (m/s)}^2 = \mathbf{0.204 \text{ m}}$$



9.64

A firefighter on a ladder 25 m above ground should be able to spray water an additional 10 m up with the hose nozzle of exit diameter 2.5 cm. Assume a water pump on the ground and a reversible flow (hose, nozzle included) and find the minimum required power.

Solution:

C.V.: pump + hose + water column, total height difference 35 m. Here **V** is velocity, not volume.

Continuity Eq.6.3, 6.11: $\dot{m}_{in} = \dot{m}_{ex} = (\rho A \mathbf{V})_{nozzle}$

Energy Eq.6.12: $\dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{in} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{ex}$

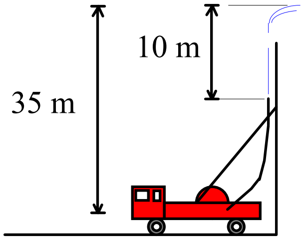
Process: $h_{in} \cong h_{ex}$, $\mathbf{V}_{in} \cong \mathbf{V}_{ex} = 0$, $z_{ex} - z_{in} = 35$ m, $\rho = 1/v \cong 1/v_f$

$$-w_p = g(z_{ex} - z_{in}) = 9.81 \times (35 - 0) = 343.2 \text{ J/kg}$$

The velocity in the exit nozzle is such that it can rise 10 m. Make that column a C.V. for which Bernoulli Eq.9.16 is:

$$gz_{noz} + \frac{1}{2}\mathbf{V}^2_{noz} = gz_{ex} + 0$$

$$\mathbf{V}_{noz} = \sqrt{2g(z_{ex} - z_{noz})} = \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$$



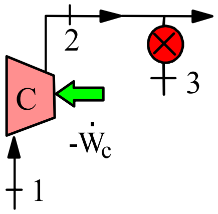
$$\dot{m} = \frac{\pi}{v_f}\left(\frac{D}{2}\right)^2 \mathbf{V}_{noz} = (\pi/4)\, 0.025^2 \times 14 / 0.001 = 6.873 \text{ kg/s}$$

$$-\dot{W}_p = -\dot{m}w_p = 6.873 \text{ kg/s} \times 343.2 \text{ J/kg} = \mathbf{2.36\ kW}$$

9.65

A pump/compressor pumps a substance from 100 kPa, 10°C to 1 MPa in a reversible adiabatic process. The exit pipe has a small crack, so that a small amount leaks to the atmosphere at 100 kPa. If the substance is (a) water, (b) R-134a, find the temperature after compression and the temperature of the leak flow as it enters the atmosphere neglecting kinetic energies.

Solution:



C.V.: Compressor, reversible adiabatic

Eq.6.13: $h_1 - w_c = h_2$; Eq.9.8: $s_1 = s_2$

State 2: P_2, $s_2 = s_1$

C.V.: Crack (Steady throttling process)

Eq.6.13: $h_3 = h_2$; Eq.9.8: $s_3 = s_2 + s_{gen}$

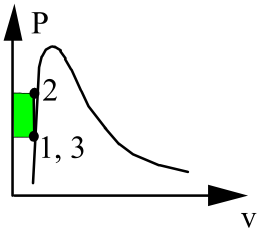
State 3: P_3, $h_3 = h_2$

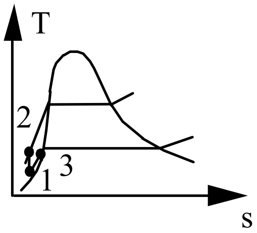
a) Water 1: compressed liquid, Table B.1.1

$$-w_c = + \int v dP = v_{f1}(P_2 - P_1) = 0.001 \times (1000 - 100) = 0.9 \text{ kJ/kg}$$

$h_2 = h_1 - w_c = 41.99 + 0.9 = 42.89$ kJ/kg => $\mathbf{T_2 = 10.2°C}$

P_3 , $h_3 = h_2$ $\Rightarrow$ compressed liquid at **~10.2°C**





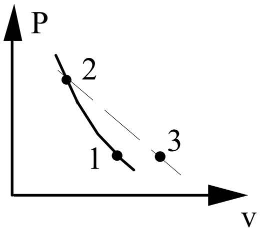
States 1 and 3 are at the same 100 kPa, and same v. You cannot separate them in the P-v fig.

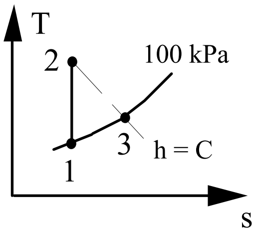
b) R-134a 1: superheated vapor, Table B.5.2, $s_1 = 1.8578$ kJ/kg K

$s_2 = s_1$ & P_2 $\Rightarrow$ $\mathbf{T_2 = 85.3°C}$, $h_2 = 468.19$ kJ/kg

$-w_c = h_2 - h_1 = 468.19 - 411.67 = 56.52$ kJ/kg

P_3 , $h_3 = h_2$ $\Rightarrow$ $\mathbf{T_3 = 74.4°C}$





9.66

A small pump is driven by a 2 kW motor with liquid water at 150 kPa, 10°C entering. Find the maximum water flow rate you can get with an exit pressure of 1 MPa and negligible kinetic energies. The exit flow goes through a small hole in a spray nozzle out to the atmosphere at 100 kPa. Find the spray velocity.

Solution:

C.V. Pump. Liquid water is incompressible so work from Eq.9.15

$$\dot{W} = \dot{m}w = -\dot{m}v(P_e - P_i) \Rightarrow$$

$$\dot{m} = \dot{W} / [-v(P_e - P_i)] = -2/[-0.001003\ (1000 - 150)] = 2.35 \text{ kg/s}$$

C.V Nozzle. No work, no heat transfer, v ≈ constant => Bernoulli Eq.9.16

$$\frac{1}{2}\mathbf{V}_{ex}^2 = v\Delta P = 0.001\ (1000 - 100) = 0.9 \text{ kJ/kg} = 900 \text{ J/kg}$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 900 \text{ J/kg}} = \mathbf{42.4\ m/s}$$

9.67

The underwater bulb nose of a container ship has a velocity relative to the ocean water as 10 m/s. What is the pressure at the front stagnation point that is 2 m down from the water surface.

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.9.16: $$v(P_e-P_i) + \frac{1}{2}(\mathbf{V}_e^2-\mathbf{V}_i^2) + g(Z_e - Z_i) = 0$$

$$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{10^2}{0.001001 \times 2000} = 49.95 \text{ kPa}$$

$$P_i = P_o + gH/v = 101 + 9.81 \times 2/ (0.001001\times 1000) = 120.6 \text{ kPa}$$

$$P_e = P_i + \Delta P = 120.6 + 49.95 = \mathbf{170.6\ kPa}$$

This container-ship is under construction and not loaded. The red line is the water line under normal load.

9.68

A speed boat has a small hole in the front of the drive with the propeller that sticks down into the water at a water depth of 0.25 m. Assume we have a stagnation point at that hole when the boat is sailing with 60 km/h, what is the total pressure there?

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.9.16: $$v(P_e - P_i) + \frac{1}{2}(\mathbf{V}_e^2 - \mathbf{V}_i^2) + g(Z_e - Z_i) = 0$$

$$\mathbf{V}_i = 60\,\frac{\text{km}}{\text{h}} \times 1000\,\frac{\text{m}}{\text{km}} \times \frac{1}{3600}\,\frac{\text{h}}{\text{s}} = 16.667 \text{ m/s}$$

$$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{16.667^2}{0.001001 \times 2000} = 138.8 \text{ kPa}$$

$$P_i = P_o + gH/v = 101 + 9.81 \times 0.25/\,(0.001001 \times 1000) = 103.45 \text{ kPa}$$

$$P_e = P_i + \Delta P = 103.45 + 138.8 = \mathbf{242.3\ kPa}$$

Remark: This is very fast for a boat

9.69

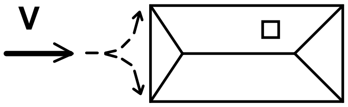
Atmospheric air at 100 kPa, 17°C blows at 60 km/h towards the side of a building. Assume the air is nearly incompressible find the pressure and the temperature at the stagnation point (zero velocity) on the wall.

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.9.16:

$$v(P_e - P_i) + \frac{1}{2}(\mathbf{V}_e^2 - \mathbf{V}_i^2) + g(Z_e - Z_i) = 0$$



$$\mathbf{V}_i = 60 \frac{\text{km}}{\text{h}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{1}{3600} \frac{\text{h}}{\text{s}} = 16.667 \text{ m/s}$$

$$v = \frac{RT_i}{P_i} = \frac{0.287 \times 290.15}{100} = 0.8323 \frac{\text{m}^3}{\text{kg}}$$

$$\Delta P = \frac{1}{2v}\mathbf{V}_i^2 = \frac{16.667^2}{0.8323 \times 2000} = 0.17 \text{ kPa}$$

$$P_e = P_i + \Delta P = 100.17 \text{ kPa}$$

Then Eq.8.23 for an isentropic process:

$$T_e = T_i\,(P_e/P_i)^{0.286} = 290.15 \times 1.0005 = \mathbf{290.3\ K}$$

Very small effect due to low velocity and air is light (large specific volume)

9.70

You drive on the highway with 120 km/h on a day with 17°C, 100 kPa atmosphere. When you put your hand out of the window flat against the wind you feel the force from the air stagnating, i.e. it comes to relative zero velocity on your skin. Assume the air is nearly incompressible and find the air temperature and pressure right on your hand.

Solution:

Energy Eq.6.13: $\frac{1}{2}\mathbf{V}^2 + h_o = h_{st}$

$$T_{st} = T_o + \frac{1}{2}\mathbf{V}^2/C_p = 17 + \frac{1}{2}[(120\times1000)/3600]^2 \times (1/1004)$$

$$= 17 + 555.5/1004 = \mathbf{17.6°C}$$

$$v = RT_o/P_o = 0.287 \times 290/100 = 0.8323 \text{ m}^3\text{/kg}$$

From Bernoulli Eq.9.16:

$$v\Delta P = \frac{1}{2}\mathbf{V}^2$$

$$P_{st} = P_o + \frac{1}{2}\mathbf{V}^2/v = 100 + 555.5/(0.8323 \times 1000) = \mathbf{100.67\ kPa}$$

9.71

An air flow at 100 kPa, 290 K, 200 m/s is directed towards a wall. At the wall the flow stagnates (comes to zero velocity) without any heat transfer. Find the stagnation pressure a) assuming incompressible flow b) assume an adiabatic compression. Hint: T comes from the energy equation.

Solution:

Ideal gas: $v = RT_o/P_o = 0.287 \times 290/100 = 0.8323 \text{ m}^3\text{/kg}$

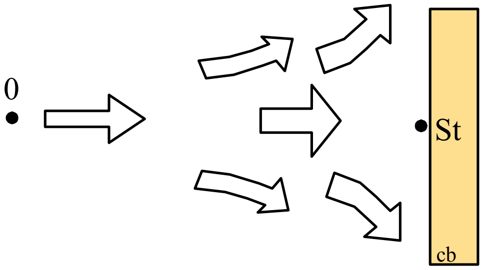
Kinetic energy: $\frac{1}{2}\mathbf{V}^2 = \frac{1}{2}(90^2/1000) = 4.05 \text{ kJ/kg}$

a) Reversible and incompressible gives Bernoulli Eq.9.16:

$$\Delta P = \frac{1}{2}\mathbf{V}^2/v = 4.05/0.8323$$

$$= 4.866 \text{ kPa}$$

$$P_{st} = P_o + \Delta P = 104.9 \text{ kPa}$$



b) adiabatic compression

Energy Eq.6.13: $\frac{1}{2}\mathbf{V}^2 + h_o = h_{st}$

$$h_{st} - h_o = \frac{1}{2}\mathbf{V}^2 = C_p\Delta T$$

$$\Delta T = \frac{1}{2}\mathbf{V}^2/C_p = 4.05/1.004 = 4.03°\text{C}$$

$$=> \; T_{st} = 290 + 4.03 = 294 \text{ K}$$

Entropy Eq.9.8 assume also reversible process:

$$s_o + s_{gen} + \int(1/T)\, dq = s_{st}$$

as $dq = 0$ and $s_{gen} = 0$ then it follows that $s = \text{constant}$

This relation gives Eq.8.23:

$$P_{st} = P_o\left(\frac{T_{st}}{T_o}\right)^{\frac{k}{k-1}} = 100 \times (294/290)^{3.5} = \mathbf{105\ kPa}$$

9.72

Calculate the air temperature and pressure at the stagnation point right in front of a meteorite entering the atmosphere (-50 °C, 50 kPa) with a velocity of 2000 m/s. Do this assuming air is incompressible at the given state and repeat for air being a compressible substance going through an adiabatic compression.

Solution:

Kinetic energy: $\frac{1}{2}\mathbf{V}^2 = \frac{1}{2}(2000)^2/1000 = 2000$ kJ/kg

Ideal gas: $v_{atm} = RT/P = 0.287 \times 223/50 = 1.28\ m^3/kg$

a) incompressible

Energy Eq.6.13: $\Delta h = \frac{1}{2}\mathbf{V}^2 = 2000$ kJ/kg

If A.5 $\Delta T = \Delta h/C_p = 1992$ K unreasonable, too high for that C_p

Use A.7: $h_{st} = h_o + \frac{1}{2}\mathbf{V}^2 = 223.22 + 2000 = 2223.3$ kJ/kg

$T_{st} = 1977$ K

Bernoulli (incompressible) Eq.9.17:

$$\Delta P = P_{st} - P_o = \frac{1}{2}\mathbf{V}^2/v = 2000/1.28 = 1562.5 \text{ kPa}$$

$$P_{st} = 1562.5 + 50 = 1612.5 \text{ kPa}$$

b) compressible

$T_{st} = 1977$ K the same energy equation.

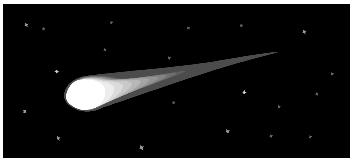
From A.7.1: $s^o_{T\,st} = 8.9517$ kJ/kg K; $s^o_{T\,o} = 6.5712$ kJ/kg K

Eq.8.28:

$$P_{st} = P_o \times e^{(s^o_{T\,st} - s^o_{T\,o})/R}$$

$$= 50 \times \exp\left[\frac{8.9517 - 6.5712}{0.287}\right]$$

$$= \mathbf{200\ 075\ kPa}$$

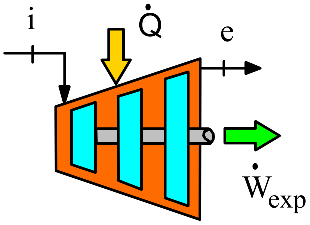


Notice that this is highly compressible, v is not constant.

9.73

Helium gas enters a steady-flow expander at 800 kPa, 300°C, and exits at 120 kPa. The mass flow rate is 0.2 kg/s, and the expansion process can be considered as a reversible polytropic process with exponent, n = 1.3. Calculate the power output of the expander.

Solution:



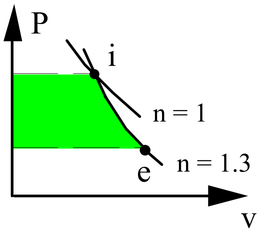
CV: expander, reversible polytropic process.
From Eq.8.37:

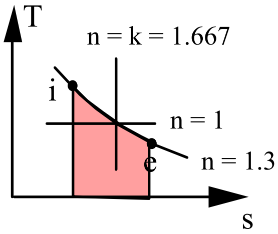
$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{n-1}{n}} = 573.2 \left(\frac{120}{800}\right)^{\frac{0.3}{1.3}} = 370\text{ K}$$

Work evaluated from Eq.9.17

$$w = -\int v dP = -\frac{nR}{n-1}(T_e - T_i) = \frac{-1.3 \times 2.07703}{0.3}(370 - 573.2)$$
$$= 1828.9 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 0.2 \times 1828.9 = \mathbf{365.8\ kW}$$





9.74

A flow of air at 100 kPa, 300 K enters a device and goes through a polytropic process with n = 1.3 before it exits at 1000 K. Find the exit pressure, the specific work and heat transfer using constant specific heats.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13: $h_1 + q = h_2 + w$ Neglect kinetic, potential energies

Entropy Eq.9.8: $s_1 + \int dq/T + s_{gen} = s_2$

$T_e = 1000$ K; $T_i = 300$ K; $P_i = 100$ kPa

Process Eq.8.37: $P_e = P_i\,(T_e/T_i)^{\frac{n}{n-1}} = 100\,(1000/300)^{\frac{1.3}{0.3}} = \mathbf{18\ 442\ kPa}$

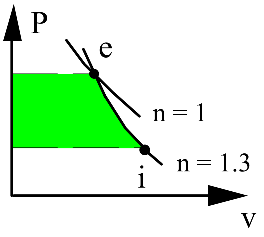
and the process leads to Eq.9.17 for the work term

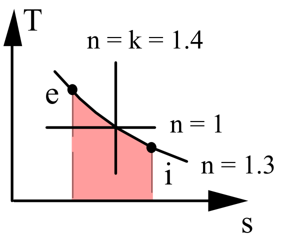
$$w = \frac{n}{n-1} R\,(T_e - T_i) = (1.3/{-0.3}) \times 0.287 \text{ kJ/kg-K} \times (1000 - 300) \text{ K}$$

$= \mathbf{-\ 849.3\ kJ/kg}$

$q = h_e - h_i + w = C_P\,(T_e - T_i) + w = 1.004(1000 - 300) - 849.3$

$= \mathbf{-146.5\ kJ/kg}$





Notice:
dP > 0
so dw <0

ds < 0
so dq < 0

9.75

Solver the previous problem but use the air tables A.7

Air at 100 kPa, 300 K, flows through a device at steady state with the exit at 1000 K during which it went through a polytropic process with n = 1.3. Find the exit pressure, the specific work and heat transfer.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13: $h_1 + q = h_2 + w$ Neglect kinetic, potential energies

Entropy Eq.9.8: $s_1 + \int dq/T + s_{gen} = s_2$

$T_e = 1000$ K; $T_i = 300$ K; $P_i = 100$ kPa

Process Eq.8.37: $P_e = P_i\,(T_e/\,T_i)^{\frac{n}{n-1}} = 100\,(1000/300)^{\frac{1.3}{0.3}} = \mathbf{18\ 442\ kPa}$

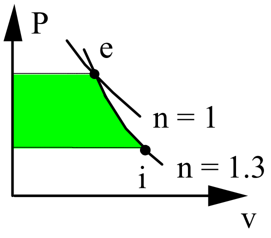
and the process leads to Eq.9.19 for the work term

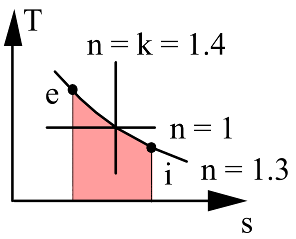
$$w = \frac{n}{n-1}\,R\,(T_e - T_i) = (1.3/\text{-}0.3) \times 0.287\ \text{kJ/kg-K} \times (1000 - 300)\ \text{K}$$

$$= \mathbf{-\ 849.3\ kJ/kg}$$

$$q = h_e - h_i + w = 1046.2 - 300.5 - 849.3$$

$$= \mathbf{-103.6\ kJ/kg}$$





Notice:
dP > 0
so dw <0

ds < 0
so dq < 0

9.76

A flow of 4 kg/s ammonia goes through a device in a polytropic process with an inlet state of 150 kPa, -20°C and an exit state of 400 kPa, 80°C. Find the polytropic exponent n, the specific work and heat transfer.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13: $h_1 + q = h_2 + w$ Neglect kinetic, potential energies

Entropy Eq.9.8: $s_1 + \int dq/T + s_{gen} = s_2$

Process Eq.8.37: $P_1 v_1^n = P_2 v_2^n$:

State 1: Table B.2.2 $v_1 = 0.79774$, $s_1 = 5.7465$ kJ/kg K, $h_1 = 1422.9$ kJ/kg

State 2: Table B.2.2 $v_2 = 0.4216$, $s_2 = 5.9907$ kJ/kg K, $h_2 = 1636.7$ kJ/kg

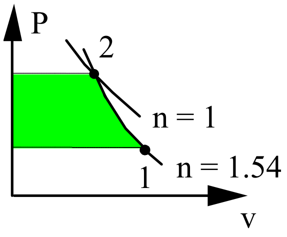
$$\ln (P_2/P_1) = n \ln (v_1/v_2) \quad => \quad 0.98083 = n \times 0.63772$$

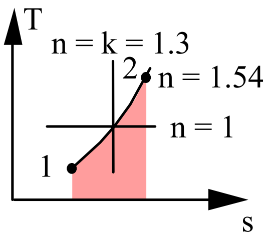
$$n = \ln (P_2/P_1) / \ln (v_1/v_2) = \mathbf{1.538}$$

From the process and the integration of v dP gives Eq.9.19.

$$w_{shaft} = -\frac{n}{n-1} (P_2 v_2 - P_1 v_1) = -2.8587 (168.64 - 119.66) = \mathbf{-140.0\ kJ/kg}$$

$$q = h_2 + w - h_1 = 1636.7 - 1422.9 - 140 = \mathbf{73.8\ kJ/kg}$$





Notice:
dP > 0
so dw <0

ds > 0
so dq > 0

9.77

An expansion in a gas turbine can be approximated with a polytropic process with exponent n = 1.25. The inlet air is at 1200 K, 800 kPa and the exit pressure is 125 kPa with a mass flow rate of 0.75 kg/s. Find the turbine heat transfer and power output.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13: $h_i + q = h_e + w$ Neglect kinetic, potential energies

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_e$

Process Eq.8.37:

$$T_e = T_i\,(P_e/P_i)^{\frac{n-1}{n}} = 1200\,(125/800)^{\frac{0.25}{1.25}} = 827.84\text{ K}$$

so the exit enthalpy is from Table A.7.1

$$h_e = 822.2 + \frac{27.84}{50}(877.4 - 822.2) = 852.94\text{ kJ/kg}$$

The process leads to Eq.9.17 for the work term

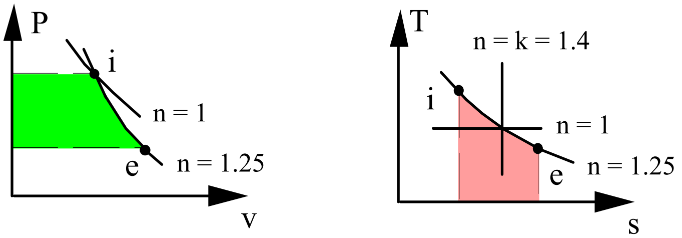
$$\dot{W} = \dot{m}w = -\dot{m}\frac{nR}{n-1}(T_e - T_i) = -0.75\,\frac{1.25 \times 0.287}{0.25} \times (827.84 - 1200)$$

$= \mathbf{400.5\ kW}$

Energy equation gives

$$\dot{Q} = \dot{m}q = \dot{m}(h_e - h_i) + \dot{W} = 0.75(852.94 - 1277.81) + 400.5$$

$= -318.65 + 400.5 = \mathbf{81.9\ kW}$



Notice:
dP < 0
so dw > 0

ds > 0
so dq > 0

Notice this process has some heat transfer in during expansion which is unusual. The typical process would have n = 1.5 with a heat loss.

Steady state irreversible processes

9.78

Consider the steam turbine in Example 6.6. Is this a reversible process?

Solution:

At the given states

Table B.1.3: $s_i = 6.9552$ kJ/kg K; $s_e = 7.3593$ kJ/kg K

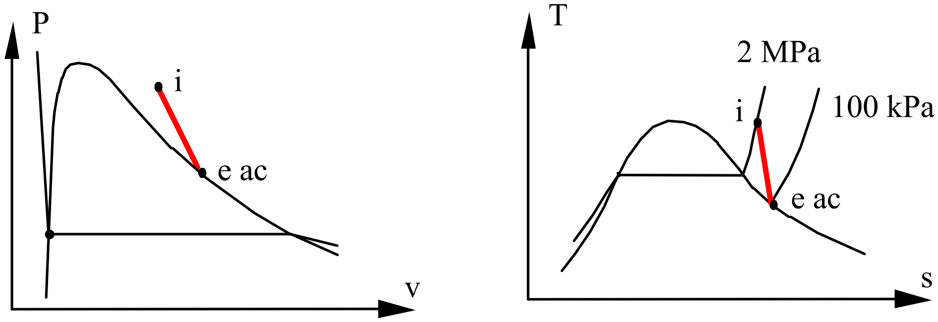
Do the second law for the turbine, Eq.9.8

$$\dot{m}_e s_e = \dot{m}_i s_i + \int d\dot{Q}/T + \dot{S}_{gen}$$

$$s_e = s_i + \int dq/T + s_{gen}$$

$$s_{gen} = s_e - s_i - \int dq/T = 7.3593 - 6.9552 - (\text{negative}) > 0$$

Entropy goes up even if q goes out. **This is an irreversible process**.



9.79

A large condenser in a steam power plant dumps 15 MW by condensing saturated water vapor at 45°C to saturated liquid. What is the water flow rate and the entropy generation rate with an ambient at 25°C?

Solution:

This process transfers heat over a finite temperature difference between the water inside the condenser and the outside ambient (cooling water from the sea, lake or river or atmospheric air)

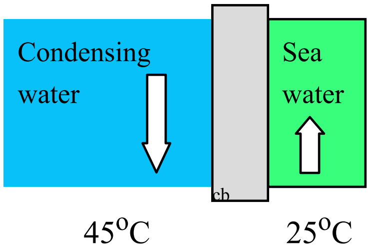
C.V. The Condensing water flow

Energy Eq.: $0 = \dot{m}\,(h_g - h_f) - \dot{Q}_{out}$

$$\dot{m} = \dot{Q}_{out} / h_{fg} = \frac{15\ 000}{2394.77}\ \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{6.264\ kg/s}$$

C.V. The wall that separates the inside 45°C water from the ambient at 25°C.

Entropy Eq. 9.1 for steady state operation:



$$\frac{dS}{dt} = 0 = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen} = \frac{\dot{Q}}{T_{45}} - \frac{\dot{Q}}{T_{25}} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \frac{15}{25 + 273}\ \frac{\text{MW}}{\text{K}} - \frac{15}{45 + 273}\ \frac{\text{MW}}{\text{K}} = \mathbf{3.17\ \frac{kW}{K}}$$

9.80

The throttle process described in Example 6.5 is an irreversible process. Find the entropy generation per kg ammonia in the throttling process.

Solution:

The process is adiabatic and irreversible. The consideration with the energy given in the example resulted in a constant h and two-phase exit flow.

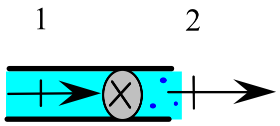
Table B.2.1: $s_i = 1.2792$ kJ/kg K

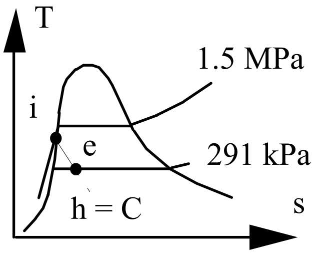
Table B.2.1: $s_e = s_f + x_e\, s_{fg} = 0.5408 + 0.1638 \times 4.9265$

$= 1.34776$ kJ/kg K

We assumed no heat transfer so the entropy equation Eq.9.8 gives

$s_{gen} = s_e - s_i - \int dq/T = 1.34776 – 1.2792 – 0 =$ **0.0686 kJ/kg K**





9.81

R-134a at 30^{o}C, 800 kPa is throttled in a steady flow to a lower pressure so it comes out at –10^{o}C. What is the specific entropy generation?

Solution:

The process is adiabatic and irreversible. The consideration of the energy given in example 6.5 resulted in a constant h and two-phase exit flow.

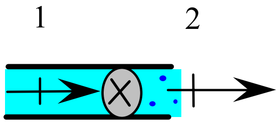
Table B.4.1: $h_i = 241.79$ kJ/kg, $s_i = 1.143$ kJ/kg K (compressed liquid)

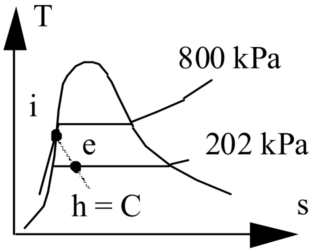
State 2: -10^{o}C, $h_e = h_i < h_g$ so two-phase $x_e = (h_e – h_f)/h_{fg} = 0.267$

Table B.4.1: $s_e = s_f + x_e\, s_{fg} = 0.9507 + 0.267 \times 0.7812 = 1.16$ kJ/kg K

We assumed no heat transfer so the entropy equation Eq.9.8 gives

$$s_{gen} = s_e - s_i - \int dq/T = 1.16 – 1.143 – 0 = \mathbf{0.017\ kJ/kg\ K}$$

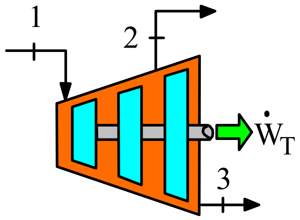




9.82

Analyze the steam turbine described in Problem 6.64. Is it possible?

Solution:



C.V. Turbine. Steady flow and adiabatic.

Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.6.10: $\dot{m}_1h_1 = \dot{m}_2h_2 + \dot{m}_3h_3 + \dot{W}$

Entropy Eq.9.7: $\dot{m}_1s_1 + \dot{S}_{gen} = \dot{m}_2s_2 + \dot{m}_3s_3$

States from Table B.1.3: $s_1 = 6.6775$, $s_2 = 6.9562$, $s_3 = 7.14413$ kJ/kg K

$\dot{S}_{gen} = 20×6.9562 + 80×7.14413 - 100×6.6775 =$ **42.9 kW/K** **> 0**

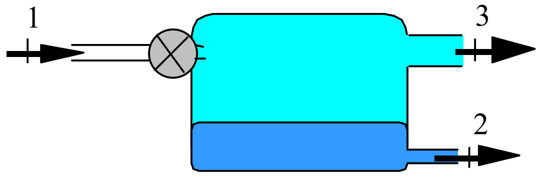
Since it is positive => **possible.**

Notice the entropy is increasing through turbine: $s_1 < s_2 < s_3$

9.83

A geothermal supply of hot water at 500 kPa, 150°C is fed to an insulated flash evaporator at the rate of 1.5 kg/s. A stream of saturated liquid at 200 kPa is drained from the bottom of the chamber and a stream of saturated vapor at 200 kPa is drawn from the top and fed to a turbine. Find the rate of entropy generation in the flash evaporator.

Solution:



Two-phase out of the valve. The liquid drops to the bottom.

Continuity Eq.6.9: $\dot m_1 = \dot m_2 + \dot m_3$

Energy Eq.6.10: $\dot m_1 h_1 = \dot m_2 h_2 + \dot m_3 h_3$

Entropy Eq.9.7: $\dot m_1 s_1 + \dot S_{gen} + \int d\dot Q/T \;\; = \dot m_2 s_2 + \dot m_3 s_3$

Process: $\dot Q = 0$, irreversible (throttle)

B.1.1 $h_1 = 632.18$ kJ/kg, $s_1 = 1.8417$ kJ/kg K

B.1.2 $h_3 = 2706.63$ kJ/kg, $s_3 = 7.1271$ kJ/kg K,

$h_2 = 504.68$ kJ/kg, $s_2 = 1.53$ kJ/kg K

From the energy equation we solve for the flow rate

$$\dot m_3 = \dot m_1(h_1 - h_2)/(h_3 - h_2) = 1.5 \times 0.0579 = 0.08685 \text{ kg/s}$$

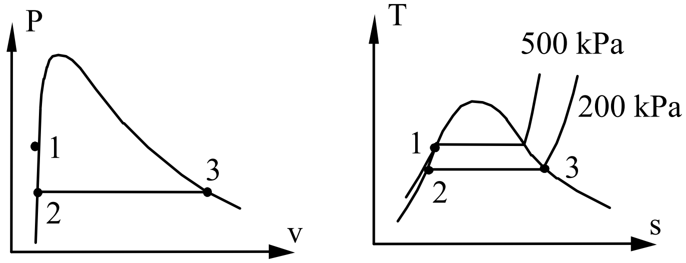
Continuity equation gives: $\dot m_2 = \dot m_1 - \dot m_2 = 1.41315$ kg/s

Entropy equation now leads to

$$\dot S_{gen} = \dot m_2 s_2 + \dot m_3 s_3 - \dot m_1 s_1$$

$$= 1.41315 \times 1.53 + 0.08685 \times 7.127 - 1.5 \times 1.8417$$

$= \mathbf{0.01855\ kW/K}$

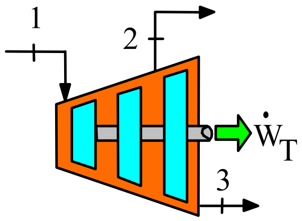


9.84

A steam turbine has an inlet of 2 kg/s water at 1000 kPa and 350°C with velocity of 15 m/s. The exit is at 100 kPa, 150°C and very low velocity. Find the power produced and the rate of entropy generation.

Solution:

C.V. Turbine. Steady flow and adiabatic.



Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.6.10: $\dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2) = \dot{m}_2 h_2 + \dot{W}$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$

States from Table B.1.3: $h_1 = 3158$ kJ/kg, $s_1 = 7.301$ kJ/kgK,

$h_2 = 2776$ kJ/kg, $s_2 = 7.613$ kJ/kgK

$$\dot{W} = \dot{m}_1(h_1 + \tfrac{1}{2}\mathbf{V}^2 - h_2) = 2\,(3158 + \tfrac{1}{2}\,\frac{15^2}{1000} - 2776) = \mathbf{764\ kW}$$

$$\dot{S}_{gen} = \dot{m}_1(s_2 - s_1) = 2\,(7.613 - 7.301) = \mathbf{0.624\ kW/K}$$

9.85

A large supply line has a steady flow of R-410a at 1000 kPa, 60°C. It is used in three different adiabatic devices shown in Fig. P9.85, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 300 kPa. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

Inlet state: B.4.2: $h_i = 335.75$ kJ/kg , $s_i = 1.2019$ kJ/kg-K

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e$ ($Z_i = Z_e$ and $\mathbf{V}$'s are small)

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Exit state: $h_e = h_i = 335.75$ kJ/kg, $T_e =$ **47.9°C,** $s_e = 1.332$ kJ/kg-K

$s_{gen} = s_e - s_i = 1.332 - 1.2019 =$ **0.2 kJ/kg K**

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The isentropic process ($s_e = s_i$) gives from B.4.2

$\mathbf{T_e = 4.6^oC}$, $\mathbf{s_{gen} = 0}$, $h_e = 296.775$ kJ/kg

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 335.75 - 296.775 = 38.975 \text{ kJ/kg}$$

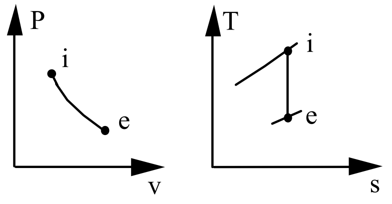
$$\mathbf{V}_e = \sqrt{2 \times 38.975 \times 1000} = \mathbf{279.2\ m/s}$$

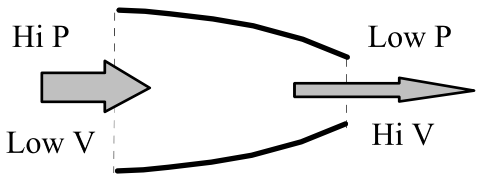
Turbine:

Process: Reversible and adiabatic => same as for nozzle except w, $\mathbf{V}_e = 0$

Energy Eq.6.13: $h_i = h_e + w$ ($Z_i = Z_e$)

$\mathbf{T_e = 4.6^oC}$, $\mathbf{s_{gen} = 0}$, $h_e = 296.775$ kJ/kg





9.86

Two flowstreams of water, one at 0.6 MPa, saturated vapor, and the other at 0.6 MPa, 600°C, mix adiabatically in a steady flow process to produce a single flow out at 0.6 MPa, 400°C. Find the total entropy generation for this process.

Solution:

1: B.1.2 $h_1 = 2756.8$ kJ/kg, $s_1 = 6.760$ kJ/kg K
2: B.1.3 $h_2 = 3700.9$ kJ/kg, $s_2 = 8.2674$ kJ/kg K
3: B.1.3 $h_3 = 3270.3$ kJ/kg, $s_3 = 7.7078$ kJ/kg K

Continuity Eq.6.9: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2,$

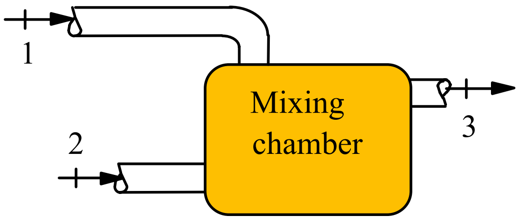
Energy Eq.6.10: $\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$

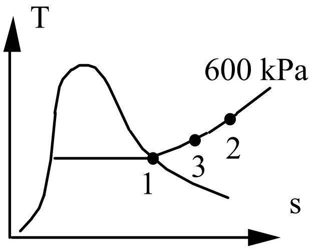
$$\Rightarrow \quad \dot{m}_1/\dot{m}_3 = (h_3 - h_2) / (h_1 - h_2) = 0.456$$

Entropy Eq.9.7: $\dot{m}_3 s_3 = \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} \quad \Rightarrow$

$$\dot{S}_{gen}/\dot{m}_3 = s_3 - (\dot{m}_1/\dot{m}_3)\, s_1 - (\dot{m}_2/\dot{m}_3)\, s_2$$

$$= 7.7078 - 0.456 \times 6.760 - 0.544 \times 8.2674 = \mathbf{0.128\ kJ/kg\ K}$$





The mixing process generates entropy. The two inlet flows could have exchanged energy (they have different T) through some heat engines and produced work, the process failed to do that, thus irreversible.

9.87

A mixing chamber receives 5 kg/min ammonia as saturated liquid at −20°C from one line and ammonia at 40°C, 250 kPa from another line through a valve. The chamber also receives 325 kJ/min energy as heat transferred from a 40°C reservoir. This should produce saturated ammonia vapor at −20°C in the exit line. What is the mass flow rate in the second line and what is the total entropy generation in the process?

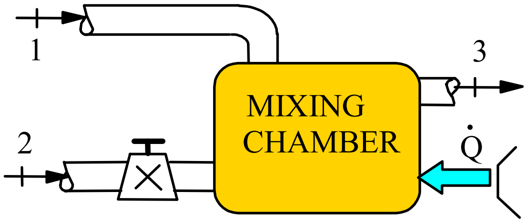
Solution:

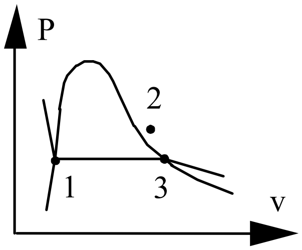
CV: Mixing chamber out to reservoir

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{Q}/T_{res} + \dot{S}_{gen} = \dot{m}_3 s_3$





From the energy equation:

$$\dot{m}_2 = \left[(\dot{m}_1(h_1 - h_3) + \dot{Q}\right]/(h_3 - h_2)$$

$$= \left[5 \times (89.05 - 1418.05) + 325\right] / (1418.05 - 1551.7)$$

$$= \mathbf{47.288\ kg/min} \quad \Rightarrow \quad \dot{m}_3 = 52.288 \text{ kg/min}$$

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{Q}/T_{res}$$

$$= 52.288 \times 5.6158 - 5 \times 0.3657 - 47.288 \times 5.9599 - 325/313.15$$

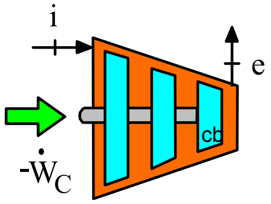
$$= \mathbf{8.94\ kJ/K\ min}$$

9.88

A compressor in a commercial refrigerator receives R-410a at -25°C and x = 1. The exit is at 1800 kPa and 60°C. Neglect kinetic energies and find the specific entropy generation.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer q = 0 and $Z_1 = Z_2$



Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

From Table B.4.1 : $s_i = 1.0893$ kJ/kgK

From Table B.4.2 : $s_e = 1.1076$ kJ/kgK

Entropy generation becomes

$$s_{gen} = s_e - s_i = 1.1076 - 1.0893 = \mathbf{0.0183\ kJ/kgK}$$

9.89

A condenser in a power plant receives 5 kg/s steam at 15 kPa, quality 90% and rejects the heat to cooling water with an average temperature of 17°C. Find the power given to the cooling water in this constant pressure process and the total rate of enropy generation when condenser exit is saturated liquid.

Solution:

C.V. Condenser. Steady state with no shaft work term.

Energy Eq.6.12: $\dot{m}\, h_i + \dot{Q} = \dot{m} h_e$

Entropy Eq.9.8: $\dot{m}\, s_i + \dot{Q}/T + \dot{S}_{gen} = \dot{m}\, s_e$

Properties are from Table B.1.2

$h_i = 225.91 + 0.9 \times 2373.14 = 2361.74$ kJ/kg , $h_e = 225.91$ kJ/kg

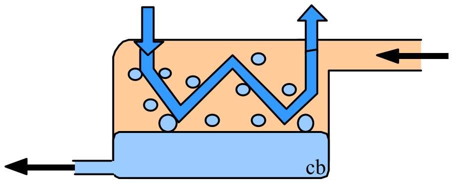
$s_i = 0.7548 + 0.9 \times 7.2536 = 7.283$ kJ/kg K, $s_e = 0.7548$ kJ/kg K

$\dot{Q}_{out} = -\dot{Q} = \dot{m}\,(h_i - h_e) = 5(2361.74 - 225.91) =$ **10679 kW**

$\dot{S}_{gen} = \dot{m}\,(s_e - s_i) + \dot{Q}_{out}/T$

$= 5(0.7548 - 7.283) + 10679/(273 + 17)$

$= -32.641 + 36.824 =$ **4.183 kW/K**



Often the cooling media flows inside a long pipe carrying the energy away.

9.90

Carbon dioxide at 300 K, 200 kPa is brought through a steady device where it is heated to 500 K by a 600 K reservoir in a constant pressure process. Find the specific work, specific heat transfer and specific entropy generation.

Solution:

C.V. Heater and walls out to the source. Steady single inlet and exit flows.

Since the pressure is constant and there are no changes in kinetic or potential energy between the inlet and exit flows the work is zero. **w = 0**

Continuity Eq.6.11: $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.6.13: $h_i + q = h_e$

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_e = s_i + q/T_{source} + s_{gen}$

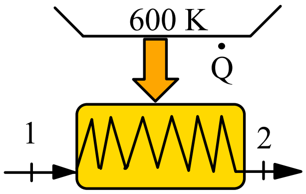
Properties are from Table A.8 so the energy equation gives

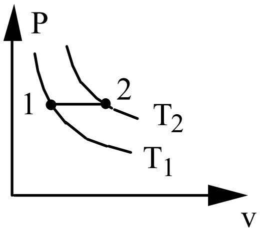
$q = h_e - h_i = 401.52 – 214.38 = \mathbf{187.1\ kJ/kg}$

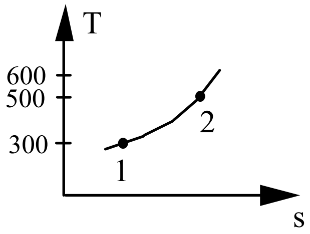
From the entropy equation

$s_{gen} = s_e - s_i - q/T_{source} = (5.3375 – 4.8631) - 187.1/600$

$= 0.4744 - 0.3118 = \mathbf{0.1626\ kJ/kg\ K}$







9.91

A heat exchanger that follows a compressor receives 0.1 kg/s air at 1000 kPa, 500 K and cools it in a constant pressure process to 320 K. The heat is absorbed by ambient ait at 300 K. Find the total rate of entropy generation.

Solution:

C.V. Heat exchanger to ambient, steady constant pressure so no work.

Energy Eq.6.12: $\dot{m}h_i = \dot{m}h_e + \dot{Q}_{out}$

Entropy Eq.9.8: $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e + \dot{Q}_{out}/T$

Using Table A.5 and Eq.8.25 for change in s

$$\dot{Q}_{out} = \dot{m}(h_i - h_e) = \dot{m}C_{Po}(T_i - T_e) = 0.1 \times 1.004(500 - 320) = 18.07 \text{ kW}$$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) + \dot{Q}_{out}/T = \dot{m}C_{Po} \ln(T_e/T_i) + \dot{Q}_{out}/T$$

$$= 0.1 \times 1.004 \ln(320/500) + 18.07/300$$

$$= \mathbf{0.0154\ kW/K}$$

Using Table A.7.1 and Eq. 8.19 for change in entropy

$h_{500} = 503.36$ kJ/kg, $h_{320} = 320.58$ kJ/kg;

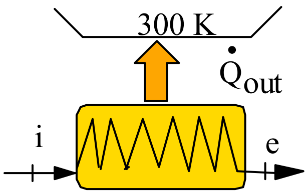
$s_{T500} = 7.38692$ kJ/kg K, $s_{T320} = 6.93413$ kJ/kg K

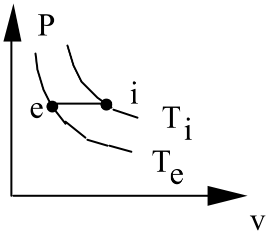
$$\dot{Q}_{out} = \dot{m}(h_i - h_e) = 0.1\ (503.36 - 320.58) = 18.19 \text{ kW}$$

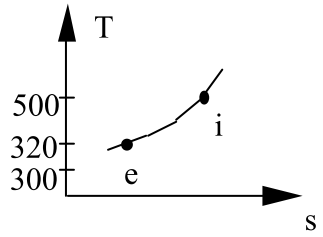
$$\dot{S}_{gen} = \dot{m}(s_e - s_i) + \dot{Q}_{out}/T$$

$$= 0.1(6.93413 - 7.38692) + 18.19/300$$

$$= \mathbf{0.0156\ kW/K}$$







9.92

Air at 1000 kPa, 300 K is throttled to 500 kPa. What is the specific entropy generation?

Solution:

C.V. Throttle, single flow, steady state. We neglect kinetic and potential energies and there are no heat transfer and shaft work terms.

Energy Eq. 6.13: $h_i = h_e \quad \Rightarrow \quad T_i = T_e$ (ideal gas)

Entropy Eq. 9.9: $s_e = s_i + \int_i^e \frac{dq}{T} + s_{gen} = s_i + s_{gen}$

Change in s Eq.8.24: $s_e - s_i = \int_i^e C_p \frac{dT}{T} - R \ln \frac{P_e}{P_i} = - R \ln \frac{P_e}{P_i}$

$$s_{gen} = s_e - s_i = - 0.287 \ln \left(\frac{500}{1000}\right) = \mathbf{0.2} \ \mathbf{\frac{kJ}{kg\ K}}$$

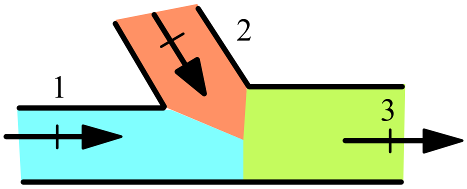
9.93

Two flows of air both at 200 kPa; one has 1 kg/s at 400 K and the other has 2 kg/s at 290 K. The two flows are mixed together in an insulated box to produce a single exit flow at 200 kPa. Find the exit temperature and the total rate of entropy generation.

Solution:

Continuity Eq.6.9:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 1 + 2 = 3 \text{ kg/s}$$



Energy Eq.6.10:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

Using constant specific heats from A.5 and Eq.8.16 for s change.

Divide the energy equation with $\dot{m}_3 C_{Po}$

$$T_3 = (\dot{m}_1/\dot{m}_3)T_1 + (\dot{m}_2/\dot{m}_3)T_2 = \frac{1}{3} \times 400 + \frac{2}{3} \times 290 = \mathbf{326.67\ K}$$

$$\dot{S}_{gen} = \dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2)$$

$$= 1 \times 1.004 \ln (326.67/400) + 2 \times 1.004 \ln (326.67/290)$$

$$= \mathbf{0.0358\ kW/K}$$

Using A.7.1 and Eq.8.19 for change in s.

$$h_3 = (\dot{m}_1/\dot{m}_3)h_1 + (\dot{m}_2/\dot{m}_3)h_2 = \frac{1}{3} \times 401.3 + \frac{2}{3} \times 290.43 = 327.39 \text{ kJ/kg}$$

From A.7.1: $T_3 = \mathbf{326.77\ K}$ $\qquad s_{T_3} = 6.9548$ kJ/kg K

$$\dot{S}_{gen} = 1(6.9548 - 7.15926) + 2(6.9548 - 6.83521)$$

$$= \mathbf{0.0347\ kW/K}$$

The pressure correction part of the entropy terms cancel out as all three states have the same pressure.

9.94

Methane at 1 MPa, 300 K is throttled through a valve to 100 kPa. Assume no change in the kinetic energy and ideal gas behavior. What is the specific entropy generation?

Continuity Eq.6.11: $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy Eq.6.13: $h_i + 0 = h_e$

Entropy Eq.9.8, 9.9: $s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

Properties are from Table B.7.2 so the energy equation gives

$$h_e = h_i = 618.76 \text{ kJ/kg} \quad \Rightarrow \quad T_e = 296 \text{ K}, \; s_e = 11.5979 \text{ kJ/kg-K}$$

$$s_{gen} = s_e - s_i = 11.5979 - 10.4138 = \mathbf{1.184\ kJ/kgK}$$

9.95

Air at 327°C, 400 kPa with a volume flow 1 m^3/s runs through an adiabatic turbine with exhaust pressure of 100 kPa. Neglect kinetic energies and use constant specific heats. Find the lowest and highest possible exit temperature. For each case find also the rate of work and the rate of entropy generation.

Solution:

C.V Turbine. Steady single inlet and exit flows, q = 0.

Inlet state: (T, P) $\quad v_i = RT_i/P_i = 0.287 \times 600/400 = 0.4305 \ m^3/kg$

$$\dot{m} = \dot{V}/v_i = 1/0.4305 = 2.323 \text{ kg/s}$$

The lowest exit T is for maximum work out i.e. reversible case

Process: Reversible and adiabatic => constant s from Eq.9.8

Eq.8.32: $\quad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 600 \times (100/400)^{0.2857} = \mathbf{403.8\ K}$

$$\Rightarrow \quad w = h_i - h_e = C_{Po}(T_i - T_e) = 1.004 \times (600 - 403.8) = 197 \text{ kJ/kg}$$

$\dot{W}_T = \dot{m}w = 2.323 \times 197 = \mathbf{457.6\ kW}$ and $\dot{\mathbf{S}}_{\mathbf{gen}} = \mathbf{0}$

Highest exit T occurs when there is no work out, throttling

$q = \varnothing; \quad \mathbf{w = \varnothing} \quad \Rightarrow \quad h_i - h_e = 0 \quad \Rightarrow \quad T_e = T_i = \mathbf{600\ K}$

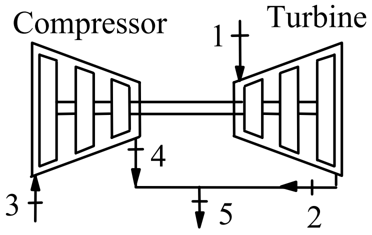
$$\dot{S}_{gen} = \dot{m}(s_e - s_i) = -\dot{m}R \ln\frac{P_e}{P_i} = -2.323 \times 0.287 \ln\frac{100}{400} = \mathbf{0.924\ kW/K}$$

9.96

In a heat-driven refrigerator with ammonia as the working fluid, a turbine with inlet conditions of 2.0 MPa, 70°C is used to drive a compressor with inlet saturated vapor at –20°C. The exhausts, both at 1.2 MPa, are then mixed together. The ratio of the mass flow rate to the turbine to the total exit flow was measured to be 0.62. Can this be true?

Solution:

Assume the compressor and the turbine are both adiabatic.



C.V. Total:

Continuity Eq.6.11: $\dot{m}_5 = \dot{m}_1 + \dot{m}_3$

Energy Eq.6.10: $\dot{m}_5 h_5 = \dot{m}_1 h_1 + \dot{m}_3 h_3$

Entropy: $\dot{m}_5 s_5 = \dot{m}_1 s_1 + \dot{m}_3 s_3 + \dot{S}_{C.V.,gen}$

$$s_5 = y s_1 + (1-y) s_3 + \dot{S}_{C.V.,gen}/\dot{m}_5$$

Assume $y = \dot{m}_1/\dot{m}_5 = 0.62$

State 1: Table B.2.2 $h_1 = 1542.7$ kJ/kg, $s_1 = 4.982$ kJ/kg K,

State 3: Table B.2.1 $h_3 = 1418.1$ kJ/kg, $s_3 = 5.616$ kJ/kg K

Solve for exit state 5 in the energy equation

$$h_5 = y h_1 + (1-y) h_3 = 0.62 \times 1542.7 + (1 - 0.62)1418.1 = 1495.4 \text{ kJ/kg}$$

State 5: $h_5 = 1495.4$ kJ/kg, $P_5 = 1200$ kPa $\Rightarrow$ $s_5 = 5.056$ kJ/kg K

Now check the 2nd law, entropy generation

$$\Rightarrow \dot{S}_{C.V.,gen}/\dot{m}_5 = s_5 - y s_1 - (1-y) s_3 = \mathbf{-0.1669} \textbf{ Impossible}$$

The problem could also have been solved assuming a reversible process and then find the needed flow rate ratio y. Then y would have been found larger than 0.62 so the stated process can not be true.

9.97

A large supply line has a steady air flow at 500 K, 200 kPa. It is used in three different adiabatic devices shown in Fig. P9.85, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 100 kPa. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e$ ($Z_i = Z_e$ and $\mathbf{V}$'s are small)

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Since it is air we have h = h(T) so same h means same $T_e = T_i =$ **500 K**

$$s_{gen} = s_e - s_i = s^o_{Te} - s^o_{Ti} - R \ln(P_e / P_i) = 0 - 0.287 \ln(1/2) = \mathbf{0.2\ kJ/kg\ K}$$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

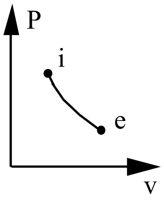
Use constant specific heat from Table A.5, $C_{Po} = 1.004 \frac{\text{kJ}}{\text{kg K}}$, k = 1.4

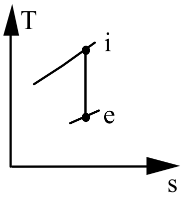
The isentropic process ($s_e = s_i$) gives Eq.8.32

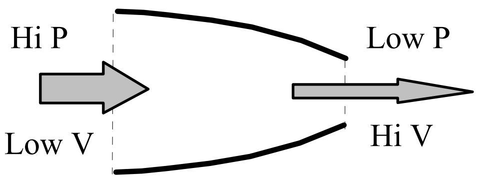
$$\Rightarrow \quad T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 500\ (100/200)^{0.2857} = \mathbf{410\ K}$$

The energy equation becomes: $\mathbf{V}_e^2/2 = h_i - h_e \cong C_P(T_i - T_e)$

$$\mathbf{V}_e = \sqrt{2\ C_P(T_i - T_e)} = \sqrt{2 \times 1.004(500\text{-}410) \times 1000} = \mathbf{424\ m/s}$$







Turbine:

Process: Reversible and adiabatic => constant s from Eq.9.8

Eq.8.32: $T_e = T_i(P_e/P_i)^{\frac{k-1}{k}} = 500 \times (100/200)^{0.2857} = \mathbf{410\ K}$

$$\Rightarrow \quad w = h_i - h_e = C_{Po}(T_i - T_e) = 1.004 \times (500 - 410) = 90\ \text{kJ/kg}$$

9.98

Repeat the previous problem for the throttle and the nozzle when the inlet air temperature is 2500 K and use the air tables.

C.V. Throttle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e$ ($Z_i = Z_e$ and $\mathbf{V}$'s are small)

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + s_{gen}$

Since it is air we have h = h(T) so same h means same $T_e = T_i =$ **2500 K**

$$s_{gen} = s_e - s_i = s^o_{Te} - s^o_{Ti} - R \ln(P_e / P_i) = 0 - 0.287 \ln(1/2) = \mathbf{0.2\ kJ/kg\ K}$$

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e + \mathbf{V}_e^2/2$ ($Z_i = Z_e$)

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

The isentropic process ($s_e = s_i$) gives Eq.8.32

$$0 = s_e - s_i = s^o_{Te} - s^o_{Ti} - R \ln(P_e / P_i)$$

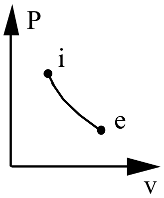
=> $s^o_{Te} = s^o_{Ti} + R \ln(P_e / P_i) = 9.24781 + 0.287 \ln(100/200) = 9.04888$

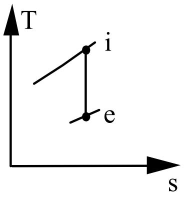
$T =$ **2136.6 K,** $h_e = 2422.86$ kJ/kg

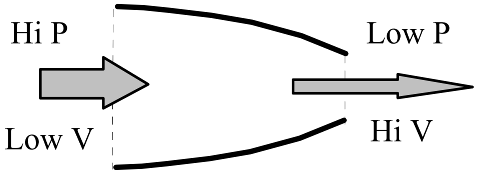
The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 2883.06 - 2422.86 = 460.2 \text{ kJ/kg}$$

$$\mathbf{V}_e = \sqrt{2 \times 1000 \times 460.2} = \mathbf{959\ m/s}$$





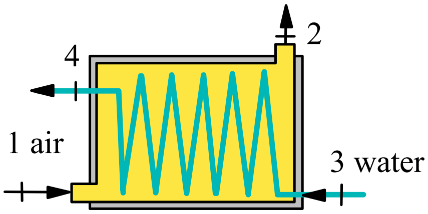


9.99

A counter flowing heat exchanger has one line with 2 kg/s at 125 kPa, 1000 K entering and the air is leaving at 100 kPa, 400 K. The other line has 0.5 kg/s water coming in at 200 kPa, 20°C and leaving at 200 kPa. What is the exit temperature of the water and the total rate of entropy generation?

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.



Energy Eq.6.10: $\dot{m}_{AIR}\Delta h_{AIR} = \dot{m}_{H2O}\Delta h_{H2O}$

From A.7: $h_1 - h_2 = 1046.22 - 401.3 = 644.92$ kJ/kg

From B.1.2 $h_3 = 83.94$ kJ/kg; $s_3 = 0.2966$ kJ/kg K

$h_4 - h_3 = (\dot{m}_{AIR}/\dot{m}_{H2O})(h_1 - h_2) = (2/0.5)644.92 = 2579.68$ kJ/kg

$h_4 = h_3 + 2579.68 = 2663.62$ kJ/kg $< h_g$ at 200 kPa

$T_4 = T_{sat} = 120.23°C$,

$x_4 = (2663.62 - 504.68)/2201.96 = 0.9805$,

$s_4 = 1.53 + x_4\ 5.597 = 7.01786$ kJ/kg K

From entropy Eq.9.7

$$\dot{S}_{gen} = \dot{m}_{H2O}(s_4 - s_3) + \dot{m}_{AIR}(s_2 - s_1)$$

$$= 0.5(7.01786 - 0.2966) + 2(7.1593 - 8.1349 - 0.287 \ln (100/125))$$

$$= 3.3606 - 1.823 = \mathbf{1.54\ kW/K}$$

9.100

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.005 kg/s and exits as saturated vapor. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Assume the heat transfer comes from a 300 K source and find the rates of entropy generation in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

Table B.6.1: $h_1 = -81.53$ kJ/kg, $s_1 = 3.294$ kJ/kgK,

$h_2 = 86.85$ kJ/kg, $s_2 = 5.041$ kJ/kgK

Table B.6.2: $h_3 = 289.05$ kJ/kg, $s_3 = 6.238$ kJ/kgK

Energy Eq.6.13: $q_{boiler} = h_2 - h_1 = 86.85 - (-81.53) = 168.38$ kJ/kg

$$\dot{Q}_{boiler} = \dot{m}_1 q_{boiler} = 0.005 \times 168.38 = 0.842 \text{ kW}$$

Entropy Eq.: $\dot{S}_{gen} = \dot{m}_1(s_2 - s_1) - \dot{Q}_{boiler}/T_{source}$

$= 0.005(5.041\text{-}3.294) - 0.842/300 = \mathbf{0.0059\ kW/K}$

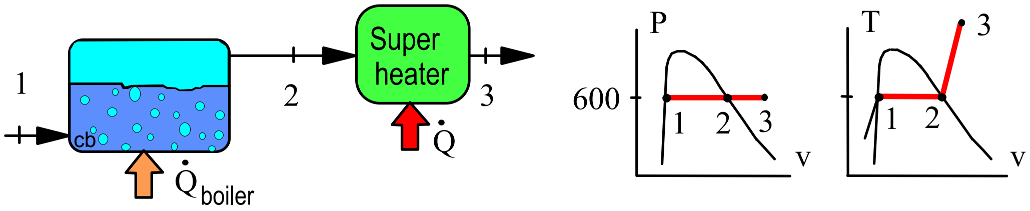
C.V. Superheater (same approximations as for boiler)

Energy Eq.6.13: $q_{sup\ heater} = h_3 - h_2 = 289.05 - 86.85 = 202.2$ kJ/kg

$$\dot{Q}_{sup\ heater} = \dot{m}_2 q_{sup\ heater} = 0.005 \times 202.2 = 1.01 \text{ kW}$$

Entropy Eq.: $\dot{S}_{gen} = \dot{m}_1(s_3 - s_2) - \dot{Q}_{sup\ heater}/T_{source}$

$= 0.005(6.238\text{-}5.041) - 1.01/300 = \mathbf{0.00262\ kW/K}$



9.101

One type of feedwater heater for preheating the water before entering a boiler operates on the principle of mixing the water with steam that has been bled from the turbine. For the states as shown in Fig. P9.59, calculate the rate of net entropy increase for the process, assuming the process to be steady flow and adiabatic.

Solution:

CV: Feedwater heater, Steady flow, no external heat transfer.

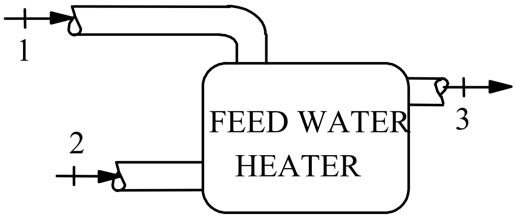
Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

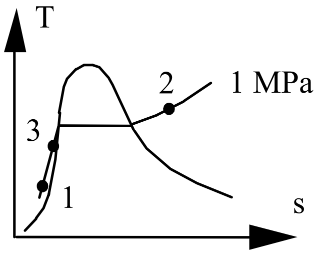
Energy Eq.6.10: $\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1)h_2 = \dot{m}_3 h_3$

Properties: All states are given by (P,T) table B.1.1 and B.1.3

$h_1 = 168.42$, $h_2 = 2828$, $h_3 = 675.8$ all kJ/kg

$s_1 = 0.572$, $s_2 = 6.694$, $s_3 = 1.9422$ all kJ/kg K





Solve for the flow rate from the energy equation

$$\dot{m}_1 = \frac{\dot{m}_3(h_3 - h_2)}{(h_1 - h_2)} = \frac{4(675.8 - 2828)}{(168.42 - 2828)} = 3.237 \text{ kg/s}$$

$$\Rightarrow \quad \dot{m}_2 = 4 - 3.237 = 0.763 \text{ kg/s}$$

The second law for steady flow, $\dot{S}_{CV} = 0$, and no heat transfer, Eq.9.7:

$$\dot{S}_{C.V.,gen} = \dot{S}_{SURR} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

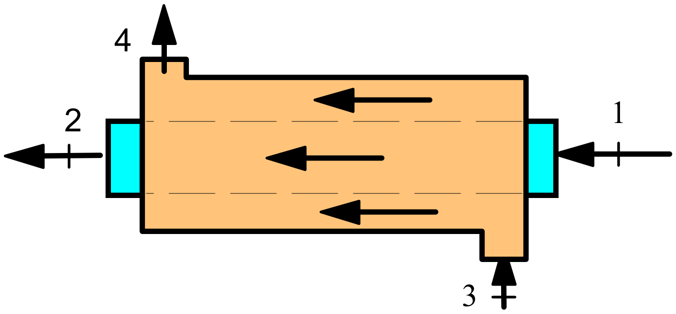
$$= 4(1.9422) - 3.237(0.572) - 0.763(6.694) = \mathbf{0.8097\ kJ/K\ s}$$

9.102

A coflowing (same direction) heat exchanger has one line with 0.25 kg/s oxygen at 17°C, 200 kPa entering and the other line has 0.6 kg/s nitrogen at 150 kPa, 500 K entering. The heat exchanger is very long so the two flows exit at the same temperature. Use constant heat capacities and find the exit temperature and the total rate of entropy generation.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.



Energy Eq.6.10: $\dot{m}_{O2}h_1 + \dot{m}_{N2}h_3 = \dot{m}_{O2}h_2 + \dot{m}_{N2}h_4$

Same exit temperature so $T_4 = T_2$ with values from Table A.5

$$\dot{m}_{O2}C_{P\ O2}T_1 + \dot{m}_{N2}C_{P\ N2}T_3 = (\dot{m}_{O2}C_{P\ O2} + \dot{m}_{N2}C_{P\ N2})T_2$$

$$T_2 = \frac{0.25 \times 0.922\times 290 + 0.6 \times 1.042 \times 500}{0.25 \times 0.922 + 0.6 \times 1.042} = \frac{379.45}{0.8557}$$

$= \mathbf{443.4\ K}$

Entropy Eq.9.7 gives for the generation

$$\dot{S}_{gen} = \dot{m}_{O2}(s_2 - s_1) \quad + \dot{m}_{N2}(s_4 - s_3)$$

$$= \dot{m}_{O2}C_P \ln (T_2/T_1) + \dot{m}_{N2}C_P \ln (T_4/T_3)$$

$$= 0.25 \times 0.922 \ln (443.4/290) + 0.6 \times 1.042 \ln (443.4/500)$$

$$= 0.0979 – 0.0751 = \mathbf{0.0228\ kW/K}$$

9.103

A steam turbine in a power plant receives steam at 3000 kPa, 500°C. The turbine has two exit flows, one is 20% of the flow at 1000 kPa, 350°C to a feedwater heater and the remainder flows out at 200 kPa, 200°C. Find the specific turbine work and the specific entropy generation both per kg flow in.

C.V. Steam turbine (x = 0.2 = extraction fraction)

Energy Eq.6.13: $w = h_1 - xh_2 - (1 - x)h_3$

Entropy Eq.9.8: $s_2 = s_1 + s_{gen\ HP}$ (full flow rate)

Entropy Eq.9.8: $s_3 = s_2 + s_{gen\ LP}$ [flow rate is fraction (1-x)]

Overall entropy gen: $s_{gen\ HP} = s_{gen\ HP} + (1 - x)\ s_{gen\ LP}$

Inlet state: Table B.1.3 $h_1 = 3456.48$ kJ/kg; $s_1 = 7.2337$ kJ/kg K

Extraction state: $h_2 = 3157.65$ kJ/kg, $s_2 = 7.3010$ kJ/kg K

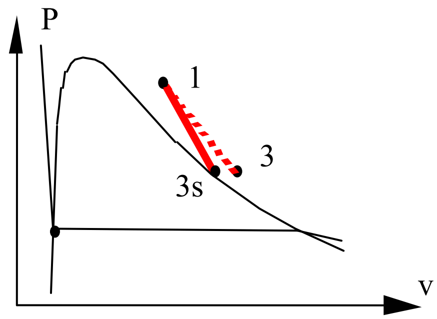
Exit (actual) state: Table B.1.3 $h_3 = 2870.46$; $s_3 = 7.5066$ kJ/kg K

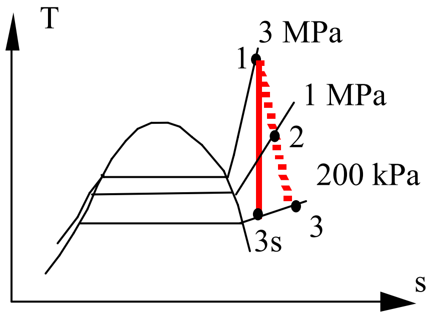
Actual turbine energy equation

$$w = 3456.48 - 0.2 \times 3157.65 - 0.8 \times 2870.46 = \mathbf{528.58\ kJ/kg}$$

$$s_{gen\ tot} = 7.301 - 7.2337 + 0.8\ (7.5066 - 7.301)$$

$$= \mathbf{0.232\ kJ/kgK}$$





9.104

Carbon dioxide used as a natural refrigerant flows through a cooler at 10 MPa, which is supercritical so no condensation occurs. The inlet is at 200^{o}C and the exit is at 40^{o}C. Assume the heat transfer is to the ambient at 20^{o}C and find the specific entropy generation.

C.V. Heat exchanger to ambient, steady constant pressure so no work.

Energy Eq.6.12: $h_i = h_e + q_{out}$

Entropy Eq.9.8: $s_i + s_{gen} = s_e + q_{out}/T$

Using Table B.3

$$q_{out} = (h_i - h_e) = 519.49 - 200.14 = 319.35 \text{ kJ/kg}$$

$$s_{gen} = s_e - s_i + q_{out}/T = 0.6906 - 1.5705 + \frac{319.35}{293.15 \text{ K}} = \mathbf{0.2095 \ kW/K}$$

9.105

A supply of 5 kg/s ammonia at 500 kPa, 20°C is needed. Two sources are available one is saturated liquid at 20°C and the other is at 500 kPa and 140°C. Flows from the two sources are fed through valves to an insulated mixing chamber, which then produces the desired output state. Find the two source mass flow rates and the total rate of entropy generation by this setup.

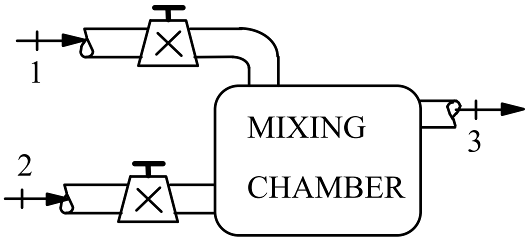
Solution:

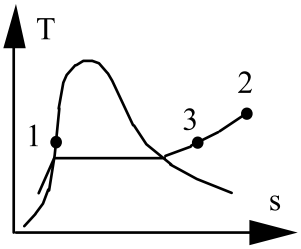
C.V. mixing chamber + valve. Steady, no heat transfer, no work.

Continuity Eq.6.9: $\dot m_1 + \dot m_2 = \dot m_3;$

Energy Eq.6.10: $\dot m_1 h_1 + \dot m_2 h_2 = \dot m_3 h_3$

Entropy Eq.9.7: $\dot m_1 s_1 + \dot m_2 s_2 + \dot S_{gen} = \dot m_3 s_3$





State 1: Table B.2.1 $h_1 = 273.4$ kJ/kg, $s_1 = 1.0408$ kJ/kg K

State 2: Table B.2.2 $h_2 = 1773.8$ kJ/kg, $s_2 = 6.2422$ kJ/kg K

State 3: Table B.2.2 $h_3 = 1488.3$ kJ/kg, $s_3 = 5.4244$ kJ/kg K

As all states are known the energy equation establishes the ratio of mass flow rates and the entropy equation provides the entropy generation.

$$\dot m_1 h_1 + (\dot m_3 - \dot m_2) h_2 = \dot m_3 h_3 \quad \Rightarrow \quad \dot m_1 = \dot m_3 \frac{h_3 - h_2}{h_1 - h_2} = 0.952 \text{ kg/s}$$

$$\dot m_2 = \dot m_3 - \dot m_1 = 4.05 \text{ kg/s}$$

$$\dot S_{gen} = 5 \times 5.4244 - 0.95 \times 1.0408 - 4.05 \times 6.2422 = \mathbf{0.852\ kW/K}$$

Transient processes

9.106

Calculate the specific entropy generated in the filling process given in Example 6.11.

Solution:

C.V. Cannister filling process where: ${}_1Q_2 = 0$; ${}_1W_2 = 0$; $m_1 = 0$

Continuity Eq.6.15: $m_2 - 0 = m_{in}$;

Energy Eq.6.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0 \quad \Rightarrow \quad u_2 = h_{line}$

Entropy Eq.9.11: $m_2s_2 - 0 = m_{in}s_{line} + 0 + {}_1S_{2\ gen}$

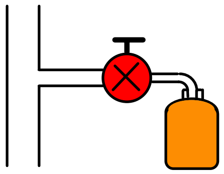
Inlet state : 1.4 MPa, 300°C, $h_i = 3040.4$ kJ/kg, $s_i = 6.9533$ kJ/kg K

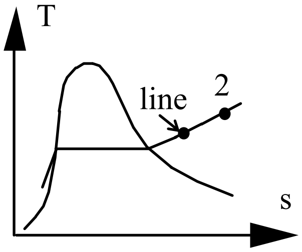
final state: 1.4 MPa, $u_2 = h_i = 3040.4$ kJ/kg

=> $T_2 = 452°C$, $s_2 = 7.45896$ kJ/kg K

${}_1S_{2\ gen} = m_2(s_2 - s_i)$

${}_1s_{2\ gen} = s_2 - s_i = 7.45896 – 6.9533 =$ **0.506 kJ/kg K**





9.107

An initially empty 0.1 m^3 cannister is filled with R-410a from a line flowing saturated liquid at −5°C. This is done quickly such that the process is adiabatic. Find the final mass, liquid and vapor volumes, if any, in the cannister. Is the process reversible?

Solution:

C.V. Cannister filling process where: $_1Q_2 = 0$; $_1W_2 = 0$; $m_1 = 0$

Continuity Eq.6.15: $m_2 - 0 = m_{in}$;

Energy Eq.6.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0 \quad \Rightarrow \quad u_2 = h_{line}$

1: Table B.4.1 $\quad u_f = 49.65,\ u_{fg} = 137.16,\ h_f = 50.22$ all kJ/kg

2: $P_2 = P_{line}$; $u_2 = h_{line} \Rightarrow$ 2 phase $u_2 > u_f$; $u_2 = u_f + x_2u_{fg}$

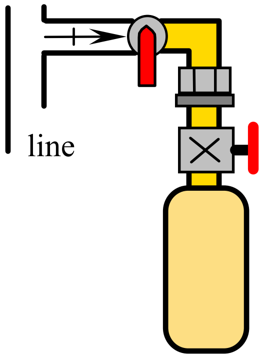
$x_2 = (50.22 - 49.65)/137.16 = 0.004156$

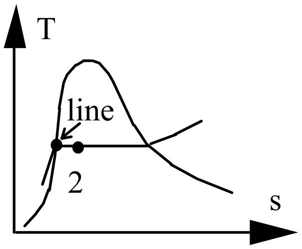
$\Rightarrow v_2 = v_f + x_2v_{fg} = 0.000841 + 0.004156\times0.03764 = 0.0009974\ m^3/kg$

$\Rightarrow m_2 = V/v_2 =$ **100.26 kg** ; $m_f = 99.843$ kg; $m_g = 0.417$ kg

$V_f = m_fv_f =$ **0.084** m^3; $V_g = m_gv_g =$ **0.016 m^3**

Process is irreversible (throttling) $s_2 > s_f$





9.108

Calculate the total entropy generated in the filling process given in Example 6.12.

Solution:

Since the solution to the problem is done in the example we will just add the second law analysis to that.

Initial state: Table B.1.2: $s_1 = 6.9404$ kJ/kg K

Final state: Table B.1.3: $s_2 = 6.9533 + \frac{42}{50} \times (7.1359 - 6.9533) = 7.1067 \frac{\text{kJ}}{\text{kg K}}$

Inlet state: Table B.1.3: $s_i = 6.9533$ kJ/kg K

Entropy Eq.9.11: $m_2s_2 - m_1s_1 = m_is_i + {}_1S_{2\text{ gen}}$

Now solve for the generation

$$
\begin{aligned}
{}_1S_{2\text{ gen}} &= m_2s_2 - m_1s_1 - m_is_i \\
&= 2.026 \times 7.1067 - 0.763 \times 6.9404 - 1.263 \times 6.9533 \\
&= \mathbf{0.32\ kJ/K} \quad > 0
\end{aligned}
$$

9.109

A 1-m^3 rigid tank contains 100 kg R-410a at ambient temperature, 15°C. A valve on top of the tank is opened, and saturated vapor is throttled to ambient pressure, 100 kPa, and flows to a collector system. During the process the temperature inside the tank remains at 15°C. The valve is closed when no more liquid remains inside. Calculate the heat transfer to the tank and total entropy generation in the process.

Solution:

C.V. Tank out to surroundings. Rigid tank so no work term.

Continuity Eq.6.15: $m_2 - m_1 = -m_e$;

Energy Eq.6.16: $m_2u_2 - m_1u_1 = Q_{CV} - m_eh_e$

Entropy Eq.9.11: $m_2s_2 - m_1s_1 = Q_{CV}/T_{SUR} - m_es_e + S_{gen}$

State 1: Table B.4.1, $v_1 = V_1/m_1 = 1/100 = 0.000904 + x_1\ 0.01955$

$x_1 = 0.46527$, $u_1 = 80.02 + 0.46527 \times 177.1 = 162.42$ kJ/kg

$s_1 = 0.3083 + 0.46527 \times 0.6998 = 0.6339$; $h_e = h_g = 282.79$ kJ/kg

State 2: $v_2 = v_g = 0.02045$, $u_2 = u_g = 257.12$, $s_2 = 1.0081$ kJ/kg K

Exit state: $h_e = 282.79$, $P_e = 100$ kPa $\rightarrow T_e = -18.65°C$, $s_e = 1.2917$ kJ/kgK

$m_2 = 1/0.02045 = 48.9$ kg; $m_e = 100 - 48.9 = 51.1$ kg

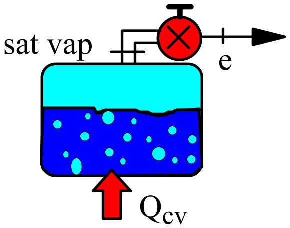
$Q_{CV} = m_2u_2 - m_1u_1 + m_eh_e$

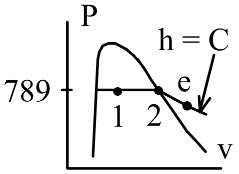
$= 48.9 \times 257.12 - 100 \times 162.42 + 51.1 \times 282.79 =$ **10 782 kJ**

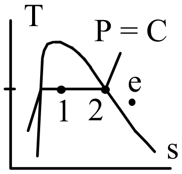
$S_{gen} = m_2s_2 - m_1s_1 + m_es_e - Q_{CV}/T_{SUR}$

$= 48.9 \times 1.0081 - 100 \times 0.6339 + 51.1 \times 1.2917 - 10\ 782 / 288.15$

= **14.5 kJ/K**







9.110

A 1 L can of R-134a is at room temperature 20^{o}C with a quality of 50%. A leak in the top valve allows vapor to escape and heat transfer from the room takes place so we reach final state of 5^{o}C with a quality of 100%. Find the mass that escaped, the heat transfer and the entropy generation not including that made in the valve.

CV The can of R-134a not including the nozzle/valve out to ambient 20^{o}C

Continuity Eq.: $m_2 - m_1 = - m_e$

Energy Eq.: $m_2u_2 - m_1u_1 = - m_eh_e + {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m_2s_2 - m_1s_1 = - m_es_e + \int dQ/T + {}_1S_{2\ gen}$

Process Eq.: V = constant => ${}_1W_2 = \int PdV = 0$

State 1: (T,x) $v_1 = v_f + x_1v_{fg} = 0.000817 + 0.5\ 0.03524 = 0.018437\ m^3/kg$

$u_1 = u_f + x_1u_{fg} = 227.03 + 0.5\ 162.16 = 308.11$ kJ/kg

$s_1 = s_f + x_1s_{fg} = 1.0963 + 0.5\ 0.622 = 1.4073$ kJ/kg

$m_1 = V / v_1 = 0.001 / 0.018437 = 0.05424$ kg

State 2: (T,x)

$v_2 = v_g = 0.05833\ m^3/kg$, $u_2 = u_g = 380.85$ kJ/kg, $s_2 = s_g = 1.7239$ kJ/kg K

$m_2 = V/v_2 = 0.001 / 0.05833 = 0.017144$ kg

Exit state e: Saturated vapor starting at 20^{o}C ending at 5^{o}C so we take an average

$h_e = 0.5(h_{e1} + h_{e2}) = 0.5\ (409.84 + 401.32) = 405.58$ kJ/kg

$s_e = 0.5(s_{e1} + s_{e2}) = 0.5\ (1.7183 + 1.7239) = 1.7211$ kJ/kg K

$m_e = m_1 - m_2 = \mathbf{0.0371\ kg}$

The heat transfer from the energy equation becomes

${}_1Q_2 = m_2u_2 - m_1u_1 + m_eh_e = 6.5293 - 16.7119 + 15.047 = \mathbf{4.864\ kJ}$

${}_1S_{2\ gen} = m_2s_2 - m_1s_1 + m_es_e - {}_1Q_2/T_{amb}$

$= 0.029555 - 0.076332 + 0.063853 - 0.016592 = \mathbf{0.000484\ kJ/K}$

9.111

An empty cannister of 0.002 m^3 is filled with R-134a from a line flowing saturated liquid R-134a at 0°C. The filling is done quickly so it is adiabatic. Find the final mass in the cannister and the total entropy generation.

Solution:

C.V. Cannister filling process where: ${}_1Q_2 = 0$; ${}_1W_2 = 0$; $m_1 = 0$

Continuity Eq.6.15: $m_2 - 0 = m_{in}$;

Energy Eq.6.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0 \quad \Rightarrow \quad u_2 = h_{line}$

Entropy Eq.9.11: $m_2s_2 - 0 = m_{in}s_{line} + 0 + {}_1S_{2\ gen}$

Inlet state: Table B.5.1 $\quad h_{line} = 200$ kJ/kg, $\quad s_{line} = 1.0$ kJ/kg K

State 2: $\quad P_2 = P_{line}$ and $u_2 = h_{line} = 200$ kJ/kg $> u_f$

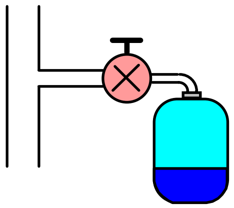
$x_2 = (200 - 199.77) / 178.24 = 0.00129$

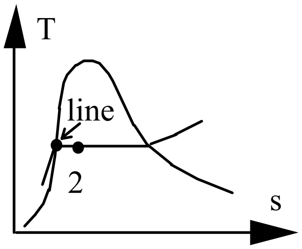
$v_2 = 0.000773 + x_2\, 0.06842 = 0.000861$ m^3/kg

$s_2 = 1.0 + x_2\, 0.7262 = 1.000937$ kJ/kg K

$m_2 = V / v_2 = 0.002/0.000861 =$ **2.323 kg**

${}_1S_{2\ gen} = m_2(s_2 - s_{line}) = 2.323\ (1.00094 - 1) =$ **0.00218 kJ/K**





9.112

An 0.2 m^3 initially empty container is filled with water from a line at 500 kPa, 200°C until there is no more flow. Assume the process is adiabatic and find the final mass, final temperature and the total entropy generation.

Solution:

C.V. The container volume and any valve out to line.

Continuity Eq.6.15: $m_2 - m_1 = m_2 = m_i$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = m_2u_2 = {}_1Q_2 - {}_1W_2 + m_ih_i = m_ih_i$

Entropy Eq.9.11: $m_2s_2 - m_1s_1 = m_2s_2 = \int dQ/T + {}_1S_{2\ gen} + m_is_i$

Process: Adiabatic ${}_1Q_2 = 0$, Rigid ${}_1W_2 = 0$ Flow stops $P_2 = P_{line}$

State i: $h_i = 2855.37$ kJ/kg; $s_i = 7.0592$ kJ/kg K

State 2: 500 kPa, $u_2 = h_i = 2855.37$ kJ/kg => Table B.1.3

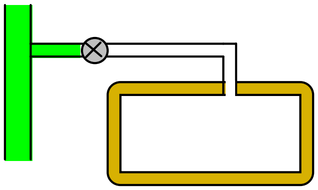
$T_2 \cong$ **332.9°C** , $s_2 = 7.5737$ kJ/kg, $v_2 = 0.55387\ m^3/kg$

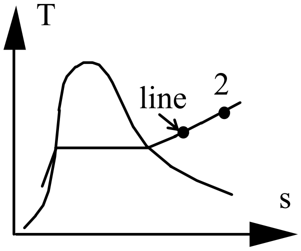
$m_2 = V/v_2 = 0.2/0.55387 =$ **0.361 kg**

From the entropy equation

$${}_1S_{2\ gen} = m_2s_2 - m_2s_i$$

$$= 0.361(7.5737 - 7.0592) = \mathbf{0.186\ kJ/K}$$





9.113

A 10 m tall 0.1 m diameter pipe is filled with liquid water at 20°C. It is open at the top to the atmosphere, 100 kPa, and a small nozzle is mounted in the bottom. The water is now let out through the nozzle splashing out to the ground until the pipe is empty. Find the water initial exit velocity, the average kinetic energy in the exit flow and the total entropy generation for the process.

Total mass: $m = \rho A H = \rho \frac{\pi}{4} D^2 H = 998 \frac{\pi}{4} 0.1^2 \times 10 = 78.383 \text{ kg}$

Bernoulli: $\frac{1}{2}\mathbf{V}^2 = gH \quad \Rightarrow \quad \mathbf{V}_1 = \sqrt{2gH_1} = \sqrt{2 \times 9.807 \times 10} = \mathbf{14\ m/s}$

$$\frac{1}{2}\mathbf{V}_{avg}^2 = gH_{avg} = g\frac{1}{2}H_1 = 9.807 \times 5 = \mathbf{49\ m^2/s^2} \quad \text{(J/kg)}$$

All the energy (average kinetic energy) is dispersed in the ambient at 20°C so

$$S_{gen} = \frac{Q}{T} = \frac{m}{2T}\mathbf{V}_{avg}^2 = \frac{78.383 \text{ kg} \times 49 \text{ J/kg}}{293.15 \text{ K}} = \mathbf{13.1\ J/K}$$

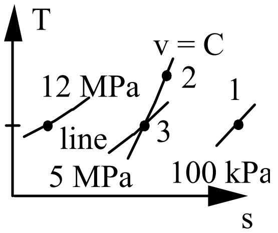
9.114

Air from a line at 12 MPa, 15°C, flows into a 500-L rigid tank that initially contained air at ambient conditions, 100 kPa, 15°C. The process occurs rapidly and is essentially adiabatic. The valve is closed when the pressure inside reaches some value, P_2. The tank eventually cools to room temperature, at which time the pressure inside is 5 MPa. What is the pressure P_2? What is the net entropy change for the overall process?

Solution:

CV: Tank. Mass flows in, so this is transient. Find the mass first

$$m_1 = P_1V/RT_1 = \frac{100 \times 0.5}{0.287 \times 288.2} = 0.604 \text{ kg}$$



Fill to P_2, then cool to $T_3 = 15°C$, $P_3 = 5$ MPa

$$m_3 = m_2 = P_3V/RT_3$$

$$= \frac{5000 \times 0.5}{0.287 \times 288.2} = 30.225 \text{ kg}$$

Mass: $m_i = m_2 - m_1 = 30.225 - 0.604 = 29.621$ kg

In the process 1-2 heat transfer = 0

1st law Eq.6.16: $m_ih_i = m_2u_2 - m_1u_1$; $m_iC_{P0}T_i = m_2C_{V0}T_2 - m_1C_{V0}T_1$

$$T_2 = \frac{(29.621 \times 1.004 + 0.604 \times 0.717) \times 288.2}{30.225 \times 0.717} = 401.2 \text{ K}$$

$$P_2 = m_2RT_2/V = (30.225 \times 0.287 \times 401.2)/0.5 = \mathbf{6.960\ MPa}$$

Consider now the total process from the start to the finish at state 3.

Energy Eq.6.16: $Q_{CV} + m_ih_i = m_2u_3 - m_1u_1 = m_2h_3 - m_1h_1 - (P_3 - P_1)V$

But, since $T_i = T_3 = T_1$, $m_ih_i = m_2h_3 - m_1h_1$

$\Rightarrow Q_{CV} = -(P_3 - P_1)V = -(5000 - 100)0.5 = -2450$ kJ

From Eqs.9.24-9.26

$$\Delta S_{NET} = m_3s_3 - m_1s_1 - m_is_i - Q_{CV}/T_0 = m_3(s_3 - s_i) - m_1(s_1 - s_i) - Q_{CV}/T_0$$

$$= 30.225\left[0\text{-}0.287 \ln\frac{5}{12}\right] - 0.604\left[0\text{-}0.287 \ln\frac{0.1}{12}\right] + (2450 / 288.2)$$

$$= \mathbf{15.265\ kJ/K}$$

9.115

An initially empty canister of volume 0.2 m^3 is filled with carbon dioxide from a line at 1000 kPa, 500 K. Assume the process is adiabatic and the flow continues until it stops by itself. Use constant heat capacity to solve for the final mass and temperature of the carbon dioxide in the canister and the total entropy generated by the process.

Solution:

C.V. Cannister + valve out to line. No boundary/shaft work, $m_1 = 0$; $Q = 0$.

Continuity Eq.6.15: $m_2 - 0 = m_i$

Energy Eq.6.16: $m_2\, u_2 - 0 = m_i\, h_i$

Entropy Eq.9.11: $m_2 s_2 - 0 = m_i s_i + {}_1S_{2\ gen}$

State 2: $P_2 = P_i$ and $u_2 = h_i = h_{line} = h_2 - RT_2$ (ideal gas)

To reduce or eliminate guess use: $h_2 - h_{line} = C_{Po}(T_2 - T_{line})$

Energy Eq. becomes: $C_{Po}(T_2 - T_{line}) - RT_2 = 0$

$$T_2 = T_{line}\, C_{Po}/(C_{Po} - R) = T_{line}\, C_{Po}/C_{Vo} = k\, T_{line}$$

Use A.5: $C_P = 0.842 \frac{kJ}{kg\ K}$, $k = 1.289$ => $T_2 = 1.289 \times 500 =$ **644.5 K**

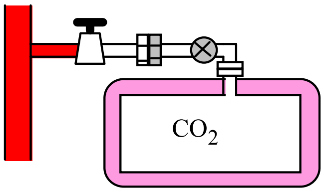
$m_2 = P_2V/RT_2 = 1000 \times 0.2/(0.1889 \times 644.5) =$ **1.643 kg**

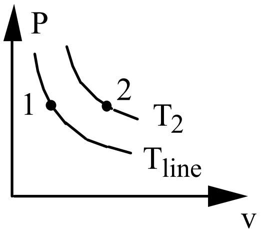
$${}_1S_{2\ gen} = m_2\,(s_2 - s_i) = m_2[\, C_P \ln(T_2 / T_{line}) - R \ln(P_2 / P_{line})]$$

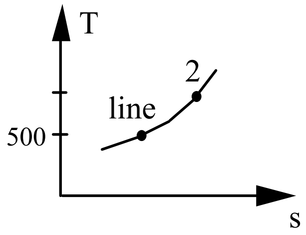
$= 1.644[0.842 \times \ln(1.289) - 0] =$ **0.351 kJ/K**

If we use A.8 at 550 K: $C_P = 1.045 \frac{kJ}{kg\ K}$, $k = 1.22$

=> $T_2 = 610$ K, $m_2 = 1.735$ kg







9.116

A can of volume 0.2 m^3 is empty and filled with carbon dioxide from a line at 3000 kPa, 60^oC. The process is adiabatic and stops when the can is full. Use Table B.3 to find the final temperature and the entropy generation.

Solution:

C.V. Cannister filling process where: ${}_1Q_2 = \emptyset$; ${}_1W_2 = \emptyset$; $m_1 = \emptyset$

Continuity Eq.6.15: $m_2 - \emptyset = m_{in}$;

Energy Eq.6.16: $m_2u_2 - \emptyset = m_{in}h_{line} + \emptyset + \emptyset \quad \Rightarrow \quad u_2 = h_{line}$

Entropy Eq.9.11: $m_2s_2 - 0 = m_{in}s_{line} + {}_1S_{2\ gen}$

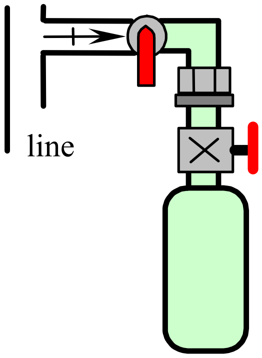
1: Table B.3.2 $h_{line} = 400.19$ kJ/kg, $s_{line} = 1.4773$ kJ/kg-K

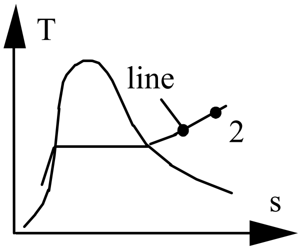
2: $P_2 = P_{line}$; $u_2 = h_{line} \Rightarrow$ $T_2 = \mathbf{129.6^oC}$, $s_2 = 1.6735$ kJ/kg-K

$v_2 = 0.024012\ m^3/kg \quad \Rightarrow \quad m_2 = V/v_2 = 8.329$ kg

${}_1S_{2\ gen} = m_2s_2 - m_{in}s_{line} = m_2(s_2 - s_{line})$

$= 8.329\ (1.6735 - 1.4773) = \mathbf{1.634\ kJ/K}$





9.117

A cook filled a pressure cooker with 3 kg water at 20°C and a small amount of air and forgot about it. The pressure cooker has a vent valve so if P > 200 kPa steam escapes to maintain a pressure of 200 kPa. How much entropy was generated in the throttling of the steam through the vent to 100 kPa when half the original mass has escaped?

Solution:

The pressure cooker goes through a transient process as it heats water up to the boiling temperature at 200 kPa then heats more as saturated vapor at 200 kPa escapes. The throttling process is steady state as it flows from saturated vapor at 200 kPa to 100 kPa which we assume is a constant h process.

C.V. Pressure cooker, no work.

Continuity Eq.6.15: $m_2 - m_1 = -m_e$

Energy Eq.6.16: $m_2 u_2 - m_1 u_1 = -m_e h_e + {}_1Q_2$

Entropy Eq.9.11: $m_2 s_2 - m_1 s_1 = -m_e s_e + \int dQ/T + {}_1S_{2\ gen}$

State 1: $v_1 = v_f = 0.001002\ m^3/kg \quad V = m_1 v_1 = 0.003006\ m^3$

State 2: $m_2 = m_1/2 = 1.5$ kg, $v_2 = V/m_2 = 2v_1$, $P_2 = 200$ kPa

Exit: $h_e = h_g = 2706.63$ kJ/kg, $s_e = s_g = 7.1271$ kJ/kg K

So we can find the needed heat transfer and entropy generation if we know the C.V. surface temperature T. If we assume T for water then ${}_1S_{2\ gen} = 0$, which is an internally reversible externally irreversible process, there is a ΔT between the water and the source.

C.V. Valve, steady flow from state e (200 kPa) to state 3 (at 100 kPa).

Energy Eq.: $h_3 = h_e$

Entropy Eq.: $s_3 = s_e + {}_es_{3\ gen}$ generation in valve (throttle)

State 3: 100 kPa, $h_3 = 2706.63$ kJ/kg Table B.1.3 ⇒

$$T_3 = 99.62 + (150\text{-}99.62)\frac{2706.63 - 2675.46}{2776.38 - 2675.46} = 115.2°C$$

$$s_3 = 7.3593 + (7.6133 - 7.3593)\ 0.30886 = 7.4378 \text{ kJ/kg K}$$

$${}_eS_{3\ gen} = m_e(s_3 - s_e) = 1.5\ (7.4378 - 7.1271) = \mathbf{0.466\ kJ/K}$$

9.118

A balloon is filled with air from a line at 200 kPa, 300 K to a final state of 110 kPa, 300 K with a mass of 0.1 kg air. Assume the pressure is proportional to the balloon volume as: P = 100 kPa + CV. Find the heat transfer to/from the ambient at 300 K and the total entropy generation.

C.V. Balloon out to the ambient. Assume $m_1 = 0$

Continuity Eq.6.15: $m_2 - 0 = m_{in}$;

Energy Eq.6.16: $m_2u_2 - 0 = m_{in}h_{in} + {}_1Q_2 - {}_1W_2$

Entropy Eq.9.11: $m_2s_2 - 0 = m_{in}s_{in} + \int \frac{dQ}{T} + {}_1S_{2\ gen} = m_{in}s_{in} + \frac{{}_1Q_2}{T} + {}_1S_{2\ gen}$

Process Eq.: $P = A + C\ V$, $A = 100$ kPa

State 2 (P, T):

$$V_2 = m_2RT_2 / P_2 = \frac{0.1 \times 0.287 \times 300}{110} = 0.078273 \text{ m}^3$$

$$P_2 = A + CV_2 \quad \Rightarrow \quad C = (P_2 - 100) / V_2 = 127.758 \text{ kPa/m}^3$$

Inlet state: $h_{in} = h_2 = u_2 + P_2v_2$, $s_{in} = s_2 - R \ln(\frac{P_{in}}{P_2})$

$${}_1W_2 = \int P\ dV = \int A + CV\ dV = A\ (V_2 - 0) + \frac{1}{2}C\ (V_2^2 - 0)$$
$$= 100 \times 0.078273 + \frac{1}{2}\ 127.758 \times 0.078273^2$$
$$= 8.219 \text{ kJ} \quad [= \frac{1}{2}(P_o + P_2)V_2 = \text{area in P-V diagram}]$$

$${}_1Q_2 = m_2(u_2 - h_{line}) + {}_1W_2 = - P_2V_2 + {}_1W_2$$
$$= - 110 \times 0.078273 + 8.219 = \mathbf{-0.391\ kJ}$$

$${}_1S_{2\ gen} = m_2(s_2 - s_{in}) - \frac{{}_1Q_2}{T} = m_2\ R \ln (\frac{P_{in}}{P_2}) - \frac{{}_1Q_2}{T}$$
$$= 0.1 \times 0.287 \ln(\frac{200}{110}) + \frac{0.391}{300} = \mathbf{0.0185\ kJ/K}$$

Device efficiency

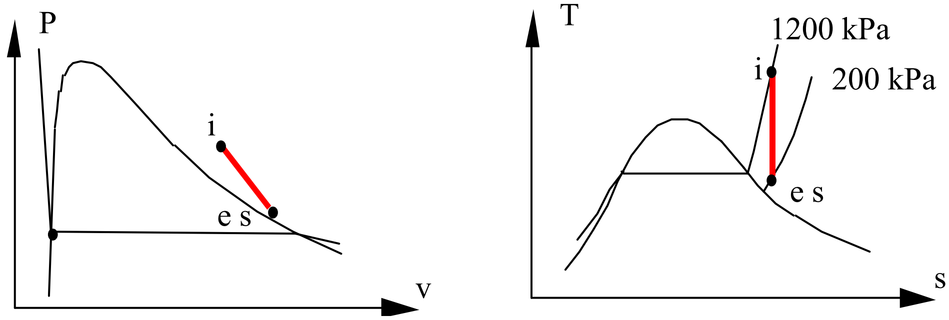
9.119

A steam turbine inlet is at 1200 kPa, 500°C. The exit is at 200 kPa. What is the lowest possible exit temperature? Which efficiency does that correspond to?

We would expect the lowest possible exit temperature when the maximum amount of work is taken out. This happens in a reversible process so if we assume it is adiabatic this becomes an isentropic process.

Exit: 200 kPa, $s = s_{in} = 7.6758$ kJ/kg K ⇒ **$T = 241.9^oC$**

The efficiency from Eq.9.26 measures the turbine relative to an isentropic turbine, so the **efficiency** will be **100%**.



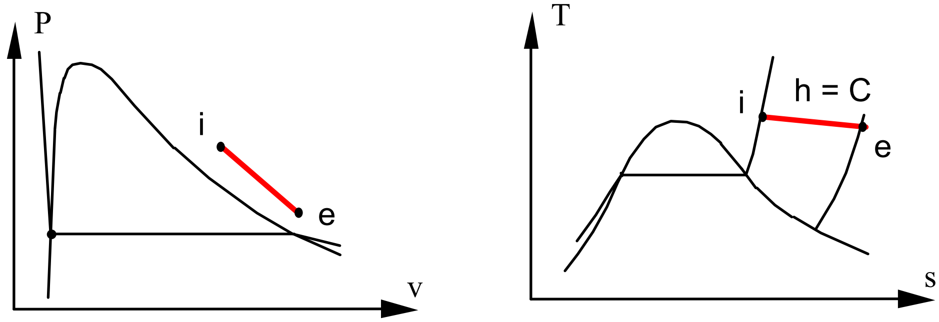
9.120

A steam turbine inlet is at 1200 kPa, 500^{o}C. The exit is at 200 kPa. What is the highest possible exit temperature? Which efficiency does that correspond to?

The highest possible exit temperature would be if we did not get any work out, i.e. the turbine broke down. Now we have a throttle process with constant h assuming we do not have a significant exit velocity.

Exit: 200 kPa, $h = h_{in} = 3476.28$ kJ/kg $\Rightarrow$ $\mathbf{T = 495^oC}$

Efficiency: $\eta = \frac{w}{w_s} = \mathbf{0}$



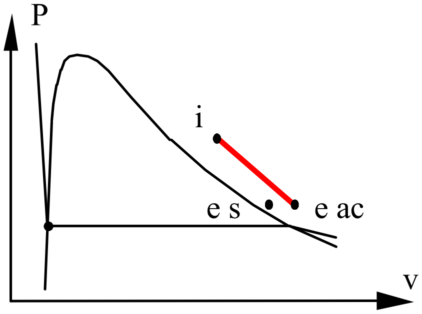
Remark: Since process is irreversible there is no area under curve in T-s diagram that correspond to a q, nor is there any area in the P-v diagram corresponding to a shaft work.

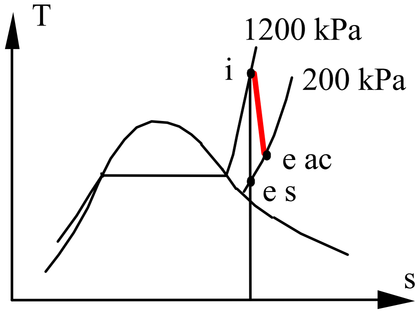
9.121

A steam turbine inlet is at 1200 kPa, 500°C. The exit is at 200 kPa, 275°C. What is the isentropic efficiency?

Inlet: $h_{in} = 3476.28$ kJ/kg, $s_{in} = 7.6758$ kJ/kg K
Exit: $h_{ex} = 3021.4$ kJ/kg, $s_{ex} = 7.8006$ kJ/kg K
Ideal Exit: 200 kPa, $s = s_{in} = 7.6758$ kJ/kg K $\Rightarrow$ $h_s = 2954.7$ kJ/kg

$$w_{ac} = h_{in} - h_{ex} = 3476.28 - 3021.4 = 454.9 \text{ kJ/kg}$$
$$w_s = h_{in} - h_s = 3476.28 - 2954.7 = 521.6 \text{ kJ/kg}$$
$$\eta = \frac{w_{ac}}{w_s} = \frac{454.9}{521.6} = \mathbf{0.872}$$



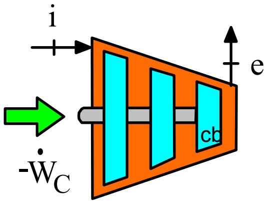


9.122

A compressor in a commercial refrigerator receives R-22 at -25°C and x = 1. The exit is at 1000 kPa and 60°C. Neglect kinetic energies and find the isentropic compressor efficiency.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer q = 0 and $Z_1 = Z_2$



Energy Eq.6.13: $h_i + 0 = w_C + h_e$;

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_e = s_i + 0 + s_{gen}$

From Table B.4.1 : $h_i = 239.92$ kJ/kg, $s_i = 0.9685$ kJ/kgK

From Table B.4.2 : $h_e = 286.97$ kJ/kg, $s_e = 0.9893$ kJ/kgK

Energy equation gives

$w_{C\ ac} = h_i - h_e = 239.92 - 286.97 = -47.05$ kJ/kgK

The ideal compressor has an exit state e,s: 1000 kPa, 0.9685 kJ/kgK

Table B.4.2 ⇒ $T_{e\ s} \cong 51.4°C$, $h_{e\ s} = 280.1$ kJ/kg

$w_{C\ s} = 239.9 - 280.1 = -40.2$ kJ/kg

The isentropic efficiency measures the actual compressor to the ideal one

$\eta = w_{C\ s} / w_{C\ ac} = -40.2 / -47.05 = \mathbf{0.854}$

9.123

The exit velocity of a nozzle is 500 m/s. If $\eta_{\text{nozzle}} = 0.88$ what is the ideal exit velocity?

The nozzle efficiency is given by Eq. 9.29 and since we have the actual exit velocity we get

$$\mathbf{V}^2_{e\,s} = \mathbf{V}^2_{ac}/\eta_{\text{nozzle}} \quad \Rightarrow$$

$$\mathbf{V}_{e\,s} = \mathbf{V}_{ac}/\sqrt{\eta_{\text{nozzle}}} = 500 \,/\, \sqrt{0.88} = \mathbf{533\ m/s}$$

9.124

Find the isentropic efficiency of the R-134a compressor in Example 6.10

Solution:

State 1: Table B.5.2 $h_1 = 387.2$ kJ/kg; $s_1 = 1.7665$ kJ/kg K

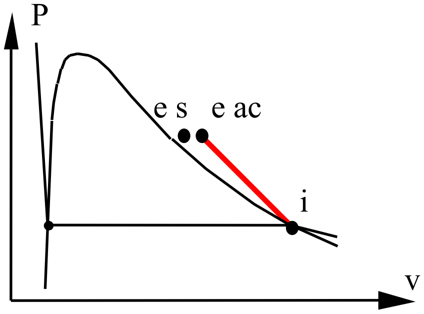
State 2ac: $h_2 = 435.1$ kJ/kg

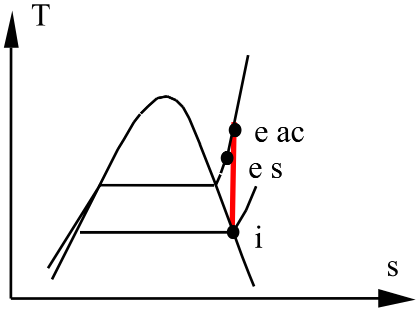
State 2s: s = 1.7665 kJ/kg K, 800 kPa => h = 431.8 kJ/kg; T = 46.8°C

$-w_{c\,s} = h_{2s} - h_{\,1} = 431.8 - 387.2 = 44.6$ kJ/kg

$-w_{ac} = 5/0.1 = 50$ kJ/kg

$\eta = w_{c\,s}/\,w_{ac} = 44.6/50 = \mathbf{0.89}$





9.125

Steam enters a turbine at 300°C, 600 kPa and exhausts as saturated vapor at 20 kPa. What is the isentropic efficiency?

Solution:

C.V. Turbine. Steady single inlet and exit flow.

To get the efficiency we must compare the actual turbine to the ideal one (the reference).

Energy Eq.6.13: $w_T = h_1 - h_2$;

Entropy Eq.9.8: $s_{2s} = s_1 + s_{gen} = s_1$

Process: Ideal $s_{gen} = 0$

State 1: Table B.1.3 $h_1 = 3061.63$ kJ/kg, $s_1 = 7.3723$ kJ/kg K

State 2s: 20 kPa, $s_{2s} = s_1 = 7.3723$ kJ/kg K $< s_g$ so two-phase

$$x_{2s} = \frac{s - s_f}{s_{fg}} = \frac{7.3723 - 0.8319}{7.0766} = 0.92423$$

$$h_{2s} = h_f + x_{2s}\, h_{fg} = 251.38 + x_{2s} \times 2358.33 = 2431.0 \text{ kJ/kg}$$

$$w_{Ts} = h_1 - h_{2s} = 3061.63 – 2431.0 = 630.61 \text{ kJ/kg}$$

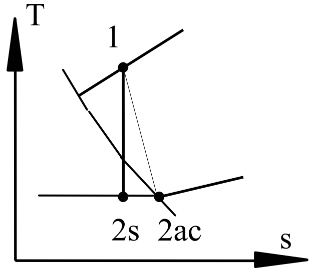
State 2ac: Table B.1.2 $h_{2ac} = 2609.7$ kJ/kg, $s_{2ac} = 7.9085$ kJ/kg K

Now we can consider the actual turbine from energy Eq.6.13:

$$w^T_{ac} = h_1 - h_{2ac} = 3061.63 – 2609.7 = 451.93$$

Then the efficiency from Eq. 9.26

$$\eta_T = w^T_{ac} / w_{Ts} = 451.93/630.61 = \mathbf{0.717}$$



9.126

An emergency drain pump should be able to pump 0.1 m^3/s liquid water at 15°C, 10 m vertically up delivering it with a velocity of 20 m/s. It is estimated that the pump, pipe and nozzle have a combined isentropic efficiency expressed for the pump as 60%. How much power is needed to drive the pump?

Solution:

C.V. Pump, pipe and nozzle together. Steady flow, no heat transfer.

Consider the ideal case first (it is the reference for the efficiency).

Energy Eq.6.12: $\dot{m}_i(h_i + \mathbf{V}^2{}_i/2 + gZ_i) + \dot{W}_{in} = \dot{m}_e(h_e + \mathbf{V}^2{}_e/2 + gZ_e)$

Solve for work and use reversible process Eq.9.15

$$\dot{W}_{ins} = \dot{m}\,[h_e - h_i + (\mathbf{V}^2{}_e - \mathbf{V}^2{}_i)/2 + g(Z_e - Z_i)]$$

$$= \dot{m}[(\,P_e - P_i)v + \mathbf{V}^2{}_e/2 + g\Delta Z]$$

$$\dot{m} = \dot{V}/v = 0.1/0.001001 = 99.9 \text{ kg/s}$$

$$\dot{W}_{ins} = 99.9[0 + (20^2/2) \times (1/1000) + 9.807 \times (10/1000)]$$

$$= 99.9(0.2 + 0.09807) = 29.8 \text{ kW}$$

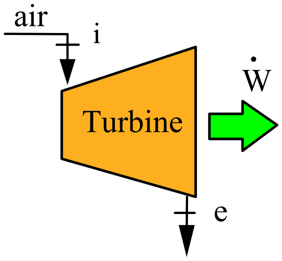
With the estimated efficiency the actual work, Eq.9.27 is

$$\dot{W}_{in\ actual} = \dot{W}_{in\ s}/\eta = 29.8/0.6 = 49.7 \text{ kW} = \mathbf{50\ kW}$$

9.127

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa and an isentropic efficiency of 87%. Find the exit temperature.

Solution:



C.V. Ideal air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.6.13: $w_T = h_i - h_e$,

Entropy Eq.9.8: $s_e = s_i$

Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.8.28 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln(\frac{P_e}{P_i}) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow$ $T_{e\,s} = 761.9$ K, $h_{e\,s} = 780.52$ kJ/kg

$$w_{T\,s} = h_i - h_{e\,s} = 1277.81 - 780.52 = 497.3 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T\,ac} = \eta_T\, w_{T\,s} = 0.87 \times 497.3 = 432.65 \text{ kJ/kg} = h_i - h_{e\,ac}$$

$$h_{e\,ac} = h_i - w_{T\,ac} = 1277.81 - 432.65 = 845.16 \text{ kJ/kg}$$

Interpolate in A.7.1 $\Rightarrow$ $T_{e\,ac} =$ **820.8 K**

If constant specific heats are used we get

Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.8.32

$$T_{e\,s} = T_i\,(P_e/P_i)^{\frac{k-1}{k}} = 1200\left(\frac{200}{1200}\right)^{0.286} = 719.2 \text{ K}$$

$$w_{T\,s} = C_{Po}(T_i - T_{e\,s}) = 1.004(1200 - 719.2) = 482.72 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T\,ac} = \eta_T\, w_{T\,s} = 0.87 \times 482.72 = 419.97 \text{ kJ/kg} = C_{Po}(T_i - T_{e\,ac})$$

$$T_{e\,ac} = T_i - w_{T\,ac}/\,C_{Po} = 1200 - 419.97/1.004 = \mathbf{781.7\ K}$$

9.128

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa. Find the lowest possible exit temperature. Which efficiency does that correspond to?

Solution:

Look at the T-s diagram for the possible processes. We notice that the lowest exit T is for the isentropic process (the ideal turbine)

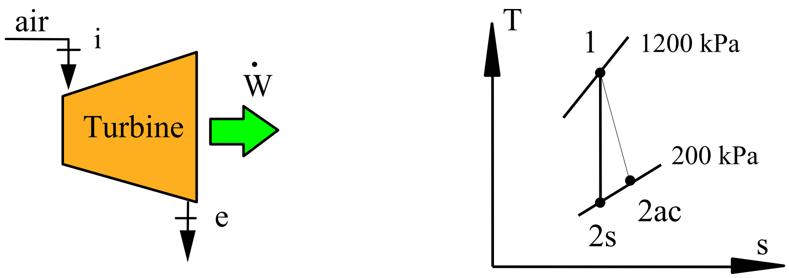


Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.8.28 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln(\frac{P_e}{P_i}) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow$ $T_{e\,s} = \mathbf{761.9\ K}$

This is an efficiency of **100%**

9.129

Liquid water enters a pump at 15°C, 100 kPa, and exits at a pressure of 5 MPa. If the isentropic efficiency of the pump is 75%, determine the enthalpy (steam table reference) of the water at the pump exit.

Solution:

CV: pump $\dot{Q}_{CV} \approx 0, \quad \Delta KE \approx 0, \quad \Delta PE \approx 0$

2nd law, reversible (ideal) process: $s_{es} = s_i \Rightarrow$

Eq.9.18 for work term.

$$w_s = -\int_i^{es} v dP \approx -v_i(P_e - P_i) = -0.001001 \text{ m}^3\text{/kg } (5000 - 100) \text{ kPa}$$

$$= -4.905 \text{ kJ/kg}$$

Real process Eq.9.28: $w = w_s/\eta_s = -4.905/0.75 = -6.54$ kJ/kg

Energy Eq.6.13: $h_e = h_i - w = 62.99 + 6.54 =$ **69.53 kJ/kg**

9.130

Ammonia is brought from saturated vapor at 300 kPa to 1400 kPa, 140^{o}C in a steady flow adiabatic compressor. Find the compressor specific work, entropy generation and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w_C = h_i - h_e$

Entropy Eq.9.9: $s_e = s_i + s_{gen}$

States: 1: B.2.2 $h_i = 1431.7$ kJ/kg, $s_i = 5.4565$ kJ/kg-K

2: B.2.2 $h_e = 1752.8$ kJ/kg, $s_e = 5.7023$ kJ/kg-K

$$-w_C = h_e - h_i = 1752.8 - 1431.7 = 321.1 \text{ kJ/kg}$$

Ideal compressor. We find the exit state from (P,s).

State 2s: P_e, $s_{e\,s} = s_i = 5.4565$ kJ/kg-K ⇨ $h_{e\,s} = 1656.08$ kJ/kg

$$-w_{C\,s} = h_{2s} - h_i = 1656.08 - 1431.7 = 224.38 \text{ kJ/kg}$$

$$\eta_C = -w_{C\,s} / -w_C = \frac{224.38}{321.1} = \mathbf{0.699}$$

9.131

A centrifugal compressor takes in ambient air at 100 kPa, 15°C, and discharges it at 450 kPa. The compressor has an isentropic efficiency of 80%. What is your best estimate for the discharge temperature?

Solution:

C.V. Compressor. Assume adiabatic, no kinetic energy is important.

Energy Eq.6.13: $w = h_1 - h_2$

Entropy Eq.9.8: $s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

We will solve using constant specific heat.

State 2 for the ideal, $s_{gen} = 0$ so $s_2 = s_1$ and it becomes:

Eq.8.23: $T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 288.15\ (450 / 100)^{0.2857} = 442.83\ K$

$w_s = h_1 - h_{2s} = C_p\ (T_1 - T_{2s}) = 1.004\ (288.15 - 442.83) = -155.3\ kJ/kg$

The actual work from definition Eq.9.27 and then energy equation:

$w_{ac} = w_s/\eta = -155.3 / 0.8 = -194.12\ kJ/kg = h_1 - h_2 = C_p(T_1 - T_2)$

$\Rightarrow\ T_2 = T_1 - w_{ac} / C_p$

$= 288.15 + 194.12/1.004 =$ **481.5 K**

Solving using Table A.7.1 instead will give

State 1: Table A.7.1: $s^o_{T1} = 6.82869\ kJ/kg\ K$

Now constant s for the ideal is done with Eq.8.19

$s^o_{T2s} = s^o_{T1} + R\ \ln(\frac{P_2}{P_1}) = 6.82869 + 0.287\ \ln(\frac{450}{100}) = 7.26036\ kJ/kg\ K$

From A.7.1: $T_{2s} = 442.1\ K$ and $h_{2s} = 443.86\ kJ/kg$

$w_s = h_1 - h_{2s} = 288.57 - 443.86 = -155.29\ kJ/kg$

The actual work from definition Eq.9.27 and then energy equation:

$w_{ac} = w_s/\eta = -155.29 / 0.8 = -194.11\ kJ/kg$

$\Rightarrow\ h_2 = 194.11 + 288.57 = 482.68$, Table A.7.1: $T_2 =$ **480 K**

The answer is very close to the previous one due to the modest T's.

9.132

A compressor is used to bring saturated water vapor at 1 MPa up to 17.5 MPa, where the actual exit temperature is 650°C. Find the isentropic compressor efficiency and the entropy generation.

Solution:

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w = h_1 - h_2$

Entropy Eq.9.9: $s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

States: 1: B.1.2 $h_1 = 2778.1$ kJ/kg, $s_1 = 6.5865$ kJ/kg K

2ac: B.1.3 $h_{2,AC} = 3693.9$ kJ/kg, $s_{2,AC} = 6.7357$ kJ/kg K

2s: B.1.3 $(P, s = s_1)$ $h_{2,s} = 3560.1$ kJ/kg

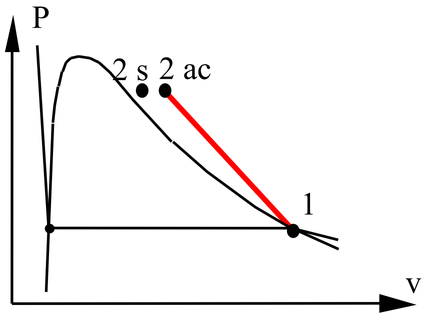
IDEAL:
$-w_{c,s} = h_{2,s} - h_1 = 782$ kJ/kg

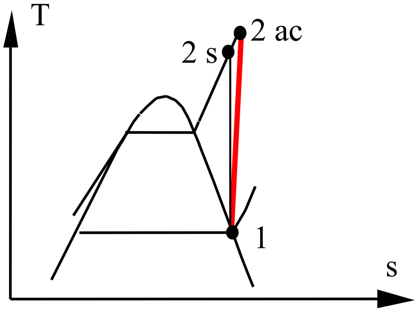
ACTUAL:
$-w_{C,AC} = h_{2,AC} - h_1 = 915.8$ kJ/kg

Definition Eq.9.28: $\eta_c = w_{c,s}/w_{c,AC} = \mathbf{0.8539 \sim 85\%}$

Entropy Eq.9.8:

$s_{gen} = s_{2\,ac} - s_1 = 6.7357 - 6.5865 = \mathbf{0.1492\ kJ/kg\ K}$





9.133

A refrigerator uses carbon dioxide that is brought from 1 MPa, -20^{o}C to 6 MPa using 2 kW power input to the compressor with a flow rate of 0.02 kg/s. Find the compressor exit temperature and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $-w_C = h_2 - h_1 = \frac{\dot{W}}{\dot{m}} = \frac{2 \text{ kW}}{0.02 \text{ kg/s}} = 100 \text{ kJ/kg}$

Entropy Eq.9.9: $s_2 = s_1 + s_{gen}$

States: 1: B.3.2 $h_1 = 342.31$ kJ/kg, $s_1 = 1.4655$ kJ/kg-K

2: B.3.2 $h_2 = h_1 - w_C = 442.31$ kJ/kg ⇨ $T_2 =$ **117.7^{o}C**

Ideal compressor. We find the exit state from (P,s).

State 2s: P_2, $s_{2s} = s_1 = 1.4655$ kJ/kg-K ⇨ $h_{2s} = 437.55$ kJ/kg

$$-w_{C\,s} = h_{2s} - h_1 = 437.55 - 342.31 = 95.24 \text{ kJ/kg}$$

$$\eta_C = -w_{C\,s} / -w_C = \frac{95.24}{100} = \mathbf{0.952}$$

9.134

Find the isentropic efficiency for the compressor in Problem 6.57.

A compressor in an air-conditioner receives saturated vapor R-410a at 400 kPa and brings it to 1.8 MPa, $60^{o}C$ in an adiabatic compression. Find the flow rate for a compressor work of 2 kW?

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w = h_1 - h_2$

Entropy Eq.9.9: $s_2 = s_1 + s_{gen}$

States: 1: B.4.2 $h_1 = 271.9$ kJ/kg, $s_1 = 1.0779$ kJ/kg-K

2: B.4.2 $h_2 = 323.92$ kJ/kg

$$-w_C = h_2 - h_1 = 323.92 - 271.9 = 52.02 \text{ kJ/kg}$$

Ideal compressor. We find the exit state from (P,s).

State 2s: P_2, $s_{2s} = s_1 = 1.0779$ kJ/kg-K ⇨ $h_{2s} = 314.33$ kJ/kg

$$-w_{C\,s} = h_{2s} - h_1 = 314.33 - 271.9 = 42.43 \text{ kJ/kg}$$

$$\eta_C = -w_{C\,s} / -w_C = \frac{42.43}{52.02} = \mathbf{0.816}$$

9.135

A pump receives water at 100 kPa, 15°C and a power input of 1.5 kW. The pump has an isentropic efficiency of 75% and it should flow 1.2 kg/s delivered at 30 m/s exit velocity. How high an exit pressure can the pump produce?

Solution:

CV Pump. We will assume the ideal and actual pumps have same exit pressure, then we can analyse the ideal pump.

Specific work: $w_{ac} = 1.5/1.2 = 1.25$ kJ/kg

Ideal work Eq.9.27: $w_s = \eta\ w_{ac} = 0.75 \times 1.25 = 0.9375$ kJ/kg

As the water is incompressible (liquid) we get

Energy Eq.9.14:

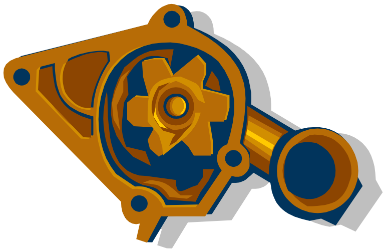
$$w_s = (P_e - P_i)v + \mathbf{V}^2_e/2 = (P_e - P_i)0.001001 + (30^2/2)/1000$$

$$= (P_e - P_i)0.001001 + 0.45$$

Solve for the pressure difference

$$P_e - P_i = (w_s - 0.45)/0.001001 = 487 \text{ kPa}$$

$$\mathbf{P_e = 587\ kPa}$$



Water pump from a car

9.136

A turbine receives air at 1500 K, 1000 kPa and expands it to 100 kPa. The turbine has an isentropic efficiency of 85%. Find the actual turbine exit air temperature and the specific entropy increase in the actual turbine.

Solution:

C.V. Turbine. steady single inlet and exit flow.

To analyze the actual turbine we must first do the ideal one (the reference).

Energy Eq.6.13: $w_T = h_1 - h_2$;

Entropy Eq.9.8: $s_2 = s_1 + s_{gen} = s_1$

Entropy change in Eq.8.19 and Table A.7.1:

$$s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 8.61208 + 0.287 \ln(100/1000) = 7.95124$$

Interpolate in A.7 => $T_{2s} = 849.2$, $h_{2s} = 876.56$ =>

$$w_T = 1635.8 - 876.56 = 759.24 \text{ kJ/kg}$$

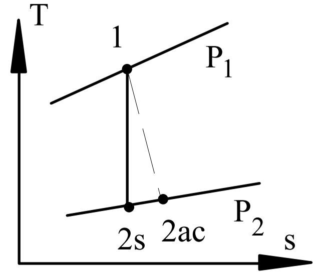
Now we can consider the actual turbine from Eq.9.26 and Eq.6.13:

$$w^T_{ac} = \eta_T\, w_T = 0.85 \times 759.24 = 645.35 = h_1 - h_{2ac}$$

$$\Rightarrow \quad h_{2ac} = h_1 - w^T_{ac} = 990.45 \quad \Rightarrow \quad \mathbf{T_{2ac} = 951\ K}$$

The entropy balance equation is solved for the generation term

$$s_{gen} = s_{2ac} - s_1 = 8.078 - 8.6121 - 0.287 \ln(100/1000) = \mathbf{0.1268\ kJ/kg\ K}$$



9.137

Carbon dioxide, CO_2, enters an adiabatic compressor at 100 kPa, 300 K, and exits at 1000 kPa, 520 K. Find the compressor efficiency and the entropy generation for the process.

Solution:

C.V. Ideal compressor. We will assume constant heat capacity.

Energy Eq.6.13: $w_c = h_1 - h_2$,

Entropy Eq.9.8: $s_2 = s_1 : \; T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300\left(\frac{1000}{100}\right)^{0.2242} = 502.7 \text{ K}$

$w_{cs} = C_p(T_1 - T_{2s}) = 0.842(300\text{-}502.7) = -170.67$ kJ/kg

C.V. Actual compressor

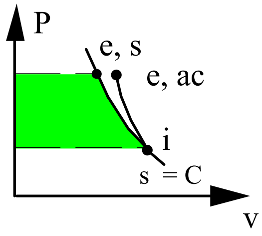
$w_{cac} = C_p(T_1 - T_{2ac}) = 0.842(300 - 520) = -185.2$ kJ/kg

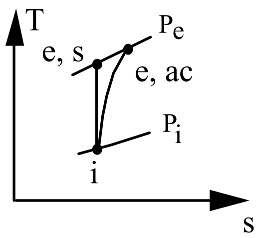
$\eta_c = w_{cs}/w_{cac} = -170.67/(-185.2) =$ **0.92**

Use Eq.8.16 for the change in entropy

$s_{gen} = s_{2ac} - s_1 = C_p \ln (T_{2ac}/T_1) - R \ln (P_2/P_1)$

$= 0.842 \ln(520 / 300) - 0.1889 \ln(1000 / 100) =$ **0.028 kJ/kg K**





Constant heat capacity is not the best approximation. It would be more accurate to use Table A.8. Entropy change in Eq.8.19 and Table A.8:

$s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 4.8631 + 0.1889 \ln(1000/100) = 5.29806$

Interpolate in A.8 => $T_{2s} = 481$ K, $h_{2s} = 382.807$ kJ/kg =>

$-w_{cs} = 382.807 - 214.38 = 168.43$ kJ/kg; $-w_{cac} = 422.12 - 214.38 = 207.74$ kJ/kg

$\eta_c = w_{cs}/w_{cac} = -168.43/(-207.74) =$ **0.81**

$s_{gen} = s_{2ac} - s_1 = 5.3767 - 4.8631 - 0.1889 \ln(10) = 0.0786$ kJ/kgK

9.138

A small air turbine with an isentropic efficiency of 80% should produce 270 kJ/kg of work. The inlet temperature is 1000 K and it exhausts to the atmosphere. Find the required inlet pressure and the exhaust temperature.

Solution:

C.V. Turbine actual energy Eq.6.13:

$$w = h_i - h_{e,ac} = 270 \text{ kJ/kg}$$

Table A.7: $h_i = 1046.22 \Rightarrow h_{e,ac} = 776.22$ kJ/kg, $\mathbf{T_e = 757.9\ K}$

C.V. Ideal turbine, Eq.9.27 and energy Eq.6.13:

$$w_s = w/\eta_s = 270/0.8 = 337.5 = h_i - h_{e,s} \Rightarrow h_{e,s} = 708.72 \text{ kJ/kg}$$

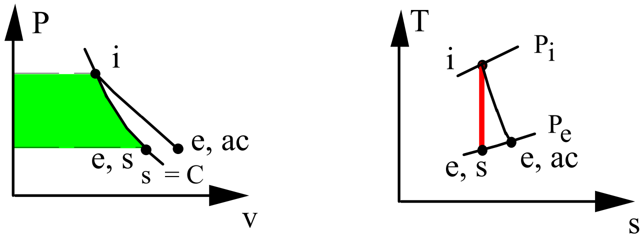
From Table A.7: $T_{e,s} = 695.5$ K

Entropy Eq.9.8: $s_i = s_{e,s}$ adiabatic and reversible

To relate the entropy to the pressure use Eq.8.19 inverted and standard entropy from Table A.7.1:

$$P_e/P_i = \exp[\,(s^o_{Te} - s^o_{Ti}\,)\,/\,R\,] = \exp[(7.733 - 8.13493)/0.287] = 0.2465$$

$$P_i = P_e\,/\,0.2465 = 101.3/0.2465 = \mathbf{411\ kPa}$$



If constant heat capacity were used

$$T_e = T_i - w/C_p = 1000 - 270/1.004 = 731 \text{ K}$$

C.V. Ideal turbine, Eq.9.26 and energy Eq.6.13:

$$w_s = w/\eta_s = 270/0.8 = 337.5 \text{ kJ/kg} = h_i - h_{e,s} = C_p(T_i - T_{e,s})$$

$$T_{e,s} = T_i - w_s/C_p = 1000 - 337.5/1.004 = 663.8 \text{ K}$$

Eq.9.8 (adibatic and reversible) gives constant s and relation is Eq.8.23

$$P_e/P_i = (T_e/T_i)^{k/(k-1)} \Rightarrow P_i = 101.3\ (1000/663.8)^{3.5} = 425 \text{ kPa}$$

9.139

The small turbine in Problem 9.31 was ideal. Assume instead the isentropic turbine efficiency is 88%. Find the actual specific turbine work and the entropy generated in the turbine.

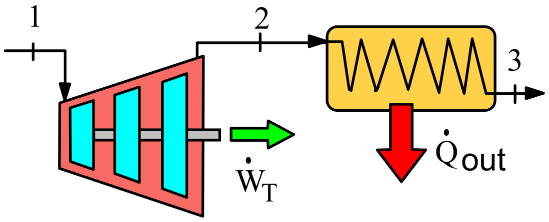
Solution:

Continuity Eq.6.11: (Steady)

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$$

Turbine: Energy Eq.6.13:

$$w_T = h_1 - h_2$$



Entropy Eq.9.8: $s_2 = s_1 + s_{T\ gen}$

Inlet state: Table B.1.3 $h_1 = 3917.45$ kJ/kg, $s_1 = 7.9487$ kJ/kg K

Ideal turbine $s_{T\ gen} = 0$, $s_2 = s_1 = 7.9487 = s_{f2} + x\ s_{fg2}$

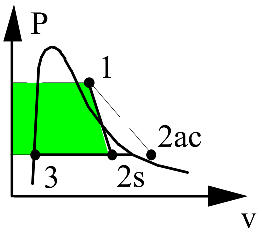
State 2: P = 10 kPa, $s_2 < s_g$ => saturated 2-phase in Table B.1.2

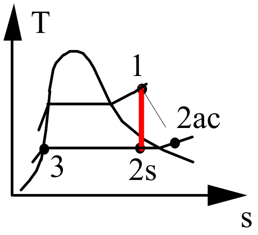
$\Rightarrow\ x_{2,s} = (s_1 - s_{f2})/s_{fg2} = (7.9487 - 0.6492)/7.501 = 0.9731$

$\Rightarrow\ h_{2,s} = h_{f2} + x \times h_{fg2} = 191.8 + 0.9731 \times 2392.8 = 2520.35$ kJ/kg

$w_{T,s} = h_1 - h_{2,s} = 1397.05$ kJ/kg

Explanation for the reversible work term is in sect. 9.3 Eq.9.14





$w_{T,AC} = \eta \times w_{T,s} =$ **1229.9 kJ/kg**

$= h_1 - h_{2,AC}\ \Rightarrow\ h_{2,AC} = h_1 - w_{T,AC} = 2687.5$ kJ/kg

$\Rightarrow\ T_{2,AC} = 100°C,\ s_{2,AC} = 8.4479$ kJ/kg-K

$s_{T\ gen} = s_{2,AC} - s_1 =$ **0.4992 kJ/kg K**

9.140

A compressor in an industrial air-conditioner compresses ammonia from a state of saturated vapor at 150 kPa to a pressure 800 kPa. At the exit, the temperature is measured to be 100°C and the mass flow rate is 0.5 kg/s. What is the required motor size for this compressor and what is its isentropic efficiency?

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w = h_1 - h_2$

Entropy Eq.9.8: $s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

States: 1: B.2.2: $h_1 = 1410.9$ kJ/kg, $v_1 = 0.7787$ m^3/kg, $s_1 = 5.6983$ kJ/kg K

2ac: B.2.3 $h_{2,AC} = 1670.6$ kJ/kg, $v_{2,AC} = 0.21949$ m^3/kg

2s: B.2.3 $(P, s = s_1)$ $h_{2,s} = 1649.8$ kJ/kg, $T_{2,s} = 91.4^{o}$C

ACTUAL:

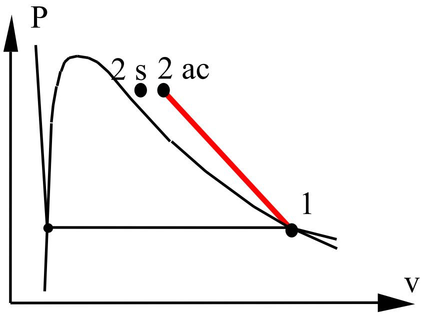
$$-w_{C,AC} = h_{2,AC} - h_1 = 1670.6 - 1410.9 = 259.7 \text{ kJ/kg}$$

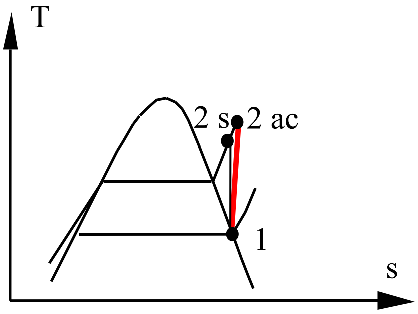
$$\dot{W}_{in} = \dot{m}\,(-w_{C,AC}) = 0.5 \text{ kg/s} \times 259.7 \text{ kJ/kg} = \mathbf{130\ kW}$$

IDEAL:

$$-w_{c,s} = h_{2,s} - h_1 = 1649.8 - 1410.9 = 238.9 \text{ kJ/kg}$$

Definition Eq.9.27: $\eta_c = w_{c,s}/w_{c,AC} = \mathbf{0.92}$





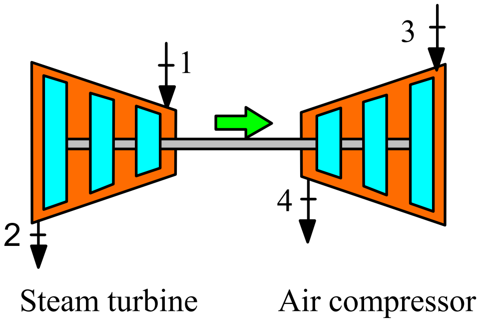
9.141

Repeat Problem 9.45 assuming the steam turbine and the air compressor each have an isentropic efficiency of 80%.

A certain industrial process requires a steady supply of saturated vapor steam at 200 kPa, at a rate of 0.5 kg/s. Also required is a steady supply of compressed air at 500 kPa, at a rate of 0.1 kg/s. Both are to be supplied by the process shown in Fig. P9.41. Steam is expanded in a turbine to supply the power needed to drive the air compressor, and the exhaust steam exits the turbine at the desired state. Air into the compressor is at the ambient conditions, 100 kPa, 20°C. Give the required steam inlet pressure and temperature, assuming that both the turbine and the compressor are reversible and adiabatic.

Solution:

C.V. Each device. Steady flow. Both adiabatic ($q = 0$) and actual devices ($s_{gen} > 0$) given by η_{sT} and η_{sc}.



$$\text{Air Eq.8.32,}\quad T_{4s} = T_3(P_4/P_3)^{\frac{k-1}{k}} = 293.2\left(\frac{500}{100}\right)^{0.286} = 464.6\text{ K}$$

$$\dot{W}_{Cs} = \dot{m}_3(h_3 - h_{4s}) = 0.1 \times 1.004(293.2 - 464.6) = -17.21\text{ kW}$$

$$\dot{W}_{Cs} = \dot{m}_3(h_3 - h_4) = \dot{W}_{Cs}/\eta_{sc} = -17.2/0.80 = -21.5\text{ kW}$$

Now the actual turbine must supply the actual compressor work. The actual state 2 is given so we must work backwards to state 1.

$$\dot{W}_T = +21.5\text{ kW} = \dot{m}_1(h_1 - h_2) = 0.5(h_1 - 2706.6)$$

$$\Rightarrow\quad h_1 = 2749.6\text{ kJ/kg}$$

Also, $\eta_{sT} = 0.80 = (h_1 - h_2)/(h_1 - h_{2s}) = 43/(2749.6 - h_{2s})$

$\Rightarrow h_{2s} = 2695.8$ kJ/kg

$2695.8 = 504.7 + x_{2s}(2706.6 - 504.7) \quad => \quad x_{2s} = 0.9951$

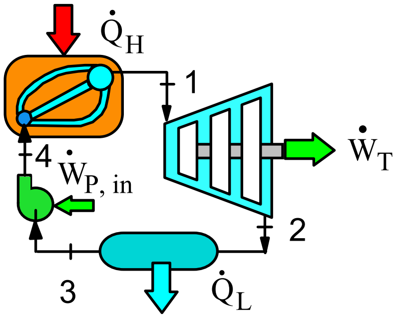
$s_{2s} = 1.5301 + 0.9951(7.1271 - 1.5301) = 7.0996$ kJ/kg K

$(s_1 = s_{2s}, h_1) \rightarrow P_1 =$ **269 kPa**, $T_1 =$ **143.5°C**

9.142

Repeat Problem 9.47 assuming the turbine and the pump each have an isentropic efficiency of 85%.

Solution:



$P_1 = P_4 = 20$ MPa

$T_1 = 700$ °C

$P_2 = P_3 = 20$ kPa

$T_3 = 40$ °C

$\eta_P = \eta_T = 85\%$

a) State 1: (P, T) Table B.1.3 $h_1 = 3809.1$ kJ/kg, $s_1 = 6.7993$ kJ/kg K

C.V. Turbine. First we do the ideal, then the actual.

Entropy Eq.9.8: $s_2 = s_1 = 6.7993$ kJ/kg K

Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766$ => $x_2 = 0.8433$

$h_{2\ s} = 251.4 + 0.8433 \times 2358.33 = 2240.1$ kJ/kg

Energy Eq.6.13: $w_{T\ s} = h_1 - h_{2\ s} = 1569$ kJ/kg

$w_{T\ AC} = \eta_T w_{T\ s} = \mathbf{1333.65} = h_1 - h_{2\ AC}$

$h_{2\ AC} = h_1 - w_{T\ AC} = 2475.45$ kJ/kg;

$\mathbf{x_{2,AC}} = (2475.45 - 251.4)/2358.3 = \mathbf{0.943}$ **,** $\mathbf{T_{2,AC} = 60.06°C}$

b)

State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

$h_3 = 167.54$ kJ/kg, $v_3 = 0.001008$ m^3/kg

$w_{P\ s} = - v_3(P_4 - P_3) = -0.001008(20000 – 20) = -20.1$ kJ/kg

$-w_{P,AC} = -w_{P,s}/\eta_\rho = 20.1/0.85 = \mathbf{23.7} = h_{4,AC} - h_3$

$h_{4,AC} = \mathbf{191.2}$ $T_{4,AC} \cong 45.7°C$

c) The heat transfer in the boiler is from energy Eq.6.13

$q_{boiler} = h_1 - h_4 = 3809.1 – 191.2 = 3617.9$ kJ/kg

$w_{net} = 1333.65 – 23.7 = \mathbf{1310\ kJ/kg}$

$$\eta_{TH} = w_{net}/q_{boiler} = \frac{1310}{3617.9} = \mathbf{0.362}$$

9.143

Assume an actual compressor has the same exit pressure and specific heat transfer as the ideal isothermal compressor in Problem 9.23 with an isothermal efficiency of 80%. Find the specific work and exit temperature for the actual compressor.

Solution:

C.V. Compressor. Steady, single inlet and single exit flows.

Energy Eq.6.13: $h_i + q = w + h_e$;

Entropy Eq.9.8: $s_i + q/T = s_e$

Inlet state: Table B.5.2, $h_i = 403.4$ kJ/kg, $s_i = 1.8281$ kJ/kg K

Exit state: Table B.5.1, $h_e = 398.36$ kJ/kg, $s_e = 1.7262$ kJ/kg K

$$q = T(s_e - s_i) = 273.15(1.7262 - 1.8281) = -27.83 \text{ kJ/kg}$$

$$w = 403.4 + (-27.83) - 398.36 = -22.8 \text{ kJ/kg}$$

From Eq.9.28 for a cooled compressor

$$w_{ac} = w_T / \eta = -22.8/0.8 = \mathbf{28.5\ kJ/kg}$$

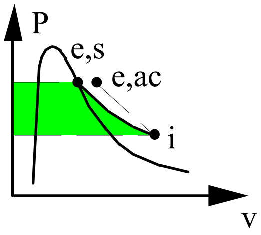
Now the energy equation gives

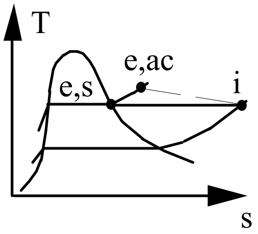
$$h_e = h_i + q - w_{ac} = 403.4 + (-27.83) + 28.5 = 404.07$$

$T_{e\ ac} \approx$ **6°C** $\qquad P_e = 294$ kPa

Explanation for the reversible work term is in Sect. 9.3

Eqs. 9.13 and 9.14





9.144

Air enters an insulated turbine at 50°C, and exits the turbine at - 30°C, 100 kPa. The isentropic turbine efficiency is 70% and the inlet volumetric flow rate is 20 L/s. What is the turbine inlet pressure and the turbine power output?

Solution:

C.V.: Turbine, $\eta_s = 0.7$, Insulated

Air table A.5: $C_p = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$

Inlet: $T_i = 50^oC$, $\dot{V}_i = 20 \text{ L/s} = 0.02 \text{ m}^3\text{/s}$;

$$\dot{m} = P\dot{V}/RT = 100 \times 0.02/(0.287 \times 323.15) = 0.099 \text{ kg/s}$$

Exit (actual): $T_e = -30^oC$, $P_e = 100$ kPa

1st Law Steady state Eq.6.13: $q_T + h_i = h_e + w_T$; $q_T = 0$

Assume Constant Specific Heat

$w_T = h_i - h_e = C_p(T_i - T_e) = 80.3$ kJ/kg

$w_{Ts} = w/\eta = 114.7$ kJ/kg, $w_{Ts} = C_p(T_i - T_{es})$

Solve for $T_{es} = 208.9$ K

Isentropic Process Eq.8.32: $P_e = P_i\,(T_e / T_i)^{\frac{k}{k-1}}$ => $\mathbf{P_i = 461}$ **kPa**

$\dot{W}_T = \dot{m}w_T = 0.099 \times 80.3 =$ **7.98 kW**

9.145

Find the isentropic efficiency of the nozzle in example 6.4.

Solution:

C.V. adiabatic nozzle with known inlet state and velocity.

Inlet state: B.1.3 $h_i = 2850.1$ kJ/kg; $s_i = 6.9665$ kJ/kg K

Process ideal: adiabatic and reversible Eq.9.8 gives constant s

ideal exit, (150 kPa, s) ; $x_{es} = (6.9665 - 1.4335)/5.7897 = 0.9557$

$h_{es} = h_f + x_{es}\, h_{fg} = 2594.9$ kJ/kg

$\mathbf{V}_{es}^2/2 = h_i - h_{es} + \mathbf{V}_i^2/2 = 2850.1 - 2594.9 + (50^2)/2000 = 256.45$ kJ/kg

$\mathbf{V}_{es} = 716.2$ m/s

From Eq.9.29,

$$\eta_{noz.} = (\mathbf{V}_e^2/2)/(\mathbf{V}_{es}^2/2) = 180/256.45 = \mathbf{0.70}$$

9.146

Air enters an insulated compressor at ambient conditions, 100 kPa, 20°C, at the rate of 0.1 kg/s and exits at 200°C. The isentropic efficiency of the compressor is 70%. What is the exit pressure? How much power is required to drive the compressor? Assume the ideal and actual compressor has the same exit pressure.

Solution:

C.V. Compressor: P_1, T_1, T_e(real), $\eta_{s\ COMP}$ known, assume constant C_{P0}

Energy Eq.6.13 for real: $-w = C_{P0}(T_e - T_i) = 1.004(200 - 20) = 180.72$

Ideal $-w_s = -w \times \eta_s = 180.72 \times 0.70 = 126.5$

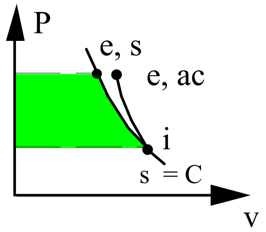
Energy Eq.6.13 for ideal:

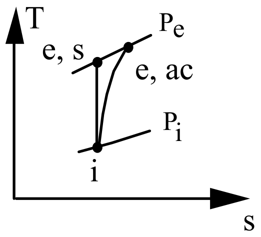
$126.5 = C_{P0}(T_{es} - T_i) = 1.004(T_{es} - 293.2), \quad T_{es} = 419.2\ \text{K}$

Constant entropy for ideal as in Eq.8.23:

$$P_e = P_i(T_{es}/T_i)^{\frac{k}{k-1}} = 100(419.2/293.20)^{3.5} = \mathbf{349\ kPa}$$

$$-\dot{W}_{REAL} = \dot{m}(-w) = 0.1 \times 180.72 = \mathbf{18.07\ kW}$$





9.147

A nozzle in a high pressure liquid water sprayer has an area of 0.5 cm^2. It receives water at 250 kPa, 20°C and the exit pressure is 100 kPa. Neglect the inlet kinetic energy and assume a nozzle isentropic efficiency of 85%. Find the ideal nozzle exit velocity and the actual nozzle mass flow rate.

Solution:

C.V. Nozzle. Liquid water is incompressible v ≈ constant, no work, no heat transfer => Bernoulli Eq.9.16

$$\frac{1}{2}\mathbf{V}_{ex}^2 - 0 = v(P_i - P_e) = 0.001002\ (250 - 100) = 0.1503\ \text{kJ/kg}$$

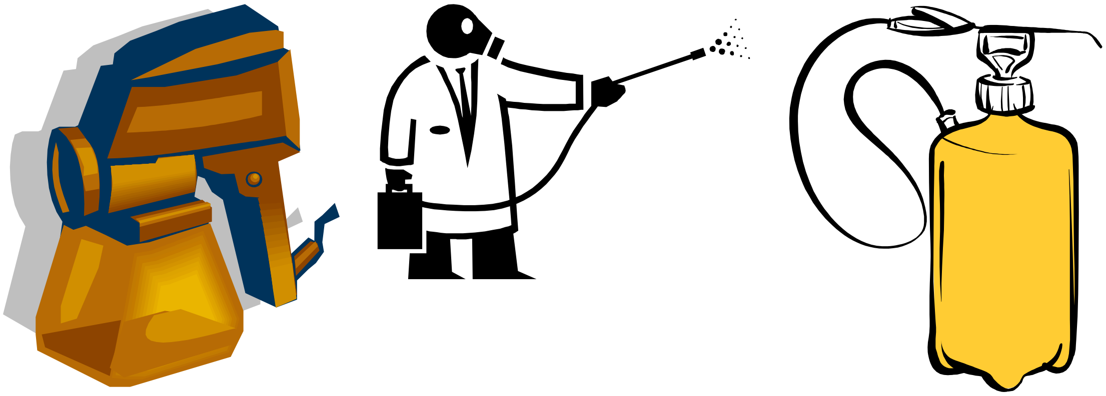
$$\mathbf{V}_{ex} = \sqrt{2 \times 0.1503 \times 1000\ \text{J/kg}} = \mathbf{17.34\ m\ s^{-1}}$$

This was the ideal nozzle now we can do the actual nozzle, Eq. 9.29

$$\frac{1}{2}\mathbf{V}_{ex\ ac}^2 = \eta\ \frac{1}{2}\mathbf{V}_{ex}^2 = 0.85 \times 0.1503 = 0.12776\ \text{kJ/kg}$$

$$\mathbf{V}_{ex\ ac} = \sqrt{2 \times 0.12776 \times 1000\ \text{J/kg}} = 15.99\ \text{m s}^{-1}$$

$$\dot{m} = \rho A\mathbf{V}_{ex\ ac} = A\mathbf{V}_{ex\ ac}/v = 0.5 \times 10^{-4} \times 15.99 / 0.001002 = \mathbf{0.798\ kg/s}$$



These are examples of relatively low pressure spray systems.

9.148

Redo Problem 9.64 if the water pump has an isentropic efficiency of 85% (hose, nozzle included).

Solution:

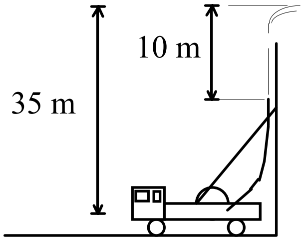
C.V.: pump + hose + water column, height difference 35 m. **V** is velocity.

Continuity Eq.6.11: $\dot{m}_{in} = \dot{m}_{ex} = (\rho A\mathbf{V})_{nozzle}$;

Energy Eq.6.12: $\dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{in} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{ex}$

Process: $h_{in} \cong h_{ex}$, $\mathbf{V}_{in} \cong \mathbf{V}_{ex} = 0$,
$z_{ex} - z_{in} = 35$ m , $\rho = 1/v \cong 1/v_f$

$-w_p = g(z_{ex} - z_{in}) = 9.80665(35 - 0) = 343.2$ J/kg



The velocity in nozzle is such that it can rise 10 m, so make that column C.V.

$$gz_{noz} + \frac{1}{2}\mathbf{V}^2_{noz} = gz_{ex} + 0$$

$$\Rightarrow \mathbf{V}_{noz} = \sqrt{2g(z_{ex} - z_{noz})} = \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$$

$$\dot{m} = (\pi/v_f)\,(D^2/4)\,\mathbf{V}_{noz} = (\pi/4)\ 0.025^2 \times 14 / 0.001 = 6.873 \text{ kg/s ;}$$

$$-\dot{W}_p = \dot{m}(-w_p)/\eta = 6.872 \times 0.343/0.85 = \mathbf{2.77\ kW}$$

9.149

Air flows into an insulated nozzle at 1 MPa, 1200 K with 15 m/s and mass flow rate of 2 kg/s. It expands to 650 kPa and exit temperature is 1100 K. Find the exit velocity, and the nozzle efficiency.

Solution:

C.V. Nozzle. Steady 1 inlet and 1 exit flows, no heat transfer, no work.

Energy Eq.6.13: $h_i + (1/2)\mathbf{V}_i^2 = h_e + (1/2)\mathbf{V}_e^2$

Entropy Eq.9.8: $s_i + s_{gen} = s_e$

Ideal nozzle $s_{gen} = 0$ and assume same exit pressure as actual nozzle. Instead of using the standard entropy from Table A.7 and Eq.8.19 let us use a constant heat capacity at the average T and Eq.8.23. First from A.7.1

$$C_{p\ 1150} = \frac{1277.81 - 1161.18}{1200 - 1100} = 1.166 \text{ kJ/kg K};$$

$$C_v = C_{p\ 1150} - R = 1.166 - 0.287 = 0.8793, \quad k = C_{p\ 1150}/C_v = 1.326$$

Notice how they differ from Table A.5 values.

$$T_{e\ s} = T_i\ (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{650}{1000}\right)^{0.24585} = 1079.4 \text{ K}$$

$$\frac{1}{2}\mathbf{V}_{e\ s}^2 = \frac{1}{2}\mathbf{V}_i^2 + C(T_i - T_{e\ s}) = \frac{1}{2}\times 15^2 + 1.166(1200 - 1079.4) \times 1000$$

$$= 112.5 + 140619.6 = 140732 \text{ J/kg} \quad \Rightarrow \quad \mathbf{V}_{e\ s} = 530.5 \text{ m/s}$$

Actual nozzle with given exit temperature

$$\frac{1}{2}\mathbf{V}_{e\ ac}^2 = \frac{1}{2}\mathbf{V}_i^2 + h_i - h_{e\ ac} = 112.5 + 1.166(1200 - 1100) \times 1000$$

$$= 116712.5 \text{ J/kg}$$

$$\Rightarrow \quad \mathbf{V}_{e\ ac} = \mathbf{483\ m/s}$$

$$\eta_{noz} = (\tfrac{1}{2}\mathbf{V}_{e\ ac}^2 - \tfrac{1}{2}\mathbf{V}_i^2)/(\tfrac{1}{2}\mathbf{V}_{e\ s}^2 - \tfrac{1}{2}\mathbf{V}_i^2) =$$

$$= (h_i - h_{e,\ AC})/(h_i - h_{e,\ s}) = \frac{116600}{140619.6} = \mathbf{0.829}$$

9.150

A nozzle is required to produce a flow of air at 200 m/s at 20°C, 100 kPa. It is estimated that the nozzle has an isentropic efficiency of 92%. What nozzle inlet pressure and temperature is required assuming the inlet kinetic energy is negligible?

Solution:

C.V. Air nozzle: P_e, T_e(real), V_e(real), η_s(real)

For the real process: $h_i = h_e + \mathbf{V}_e^2/2$ or

$T_i = T_e + \mathbf{V}_e^2/2C_{P0} = 293.2 + 200^2/2 \times 1000 \times 1.004 = \mathbf{313.1\ K}$

For the ideal process, from Eq.9.29:

$\mathbf{V}_{es}^2/2 = \mathbf{V}_e^2/2\eta_s = 200^2/2 \times 1000 \times 0.92 \quad = 21.74$ kJ/kg

and $\quad h_i = h_{es} + (\mathbf{V}_{es}^2/2)$

$T_{es} = T_i - \mathbf{V}_{es}^2/(2C_{P0}) = 313.1 - 21.74/1.004 = 291.4$ K

The constant s relation in Eq.8.23 gives

$$\Rightarrow \quad P_i = P_e\,(T_i/T_{es})^{\frac{k}{k-1}} = 100\left(\frac{313.1}{291.4}\right)^{3.50} = \mathbf{128.6\ kPa}$$

9.151

A water-cooled air compressor takes air in at 20°C, 90 kPa and compresses it to 500 kPa. The isothermal efficiency is 80% and the actual compressor has the same heat transfer as the ideal one. Find the specific compressor work and the exit temperature.

Solution:

Ideal isothermal compressor exit 500 kPa, 20°C

Reversible process: $dq = T\,ds \;\; => \;\; q = T(s_e - s_i)$

$$q = T(s_e - s_i) = T[s^o_{Te} - s^o_{T1} - R \ln(P_e / P_i)]$$

$$= - RT \ln (P_e / P_i) = - 0.287 \times 293.15 \ln (500/90) \; = - 144.3 \text{ kJ/kg}$$

As same temperature for the ideal compressor $h_e = h_i \;\Rightarrow$

$$w = q = -144.3 \text{ kJ/kg} \;\; => \;\; w_{ac} = w / \eta = \mathbf{- 180.3\ kJ/kg}, \quad q_{ac} = q$$

Now for the actual compressor energy equation becomes

$$q_{ac} + h_i = h_{e\,ac} + w_{ac} \Rightarrow$$

$$h_{e\,ac} - h_i = q_{ac} - w_{ac} = - 144.3 - (-180.3) = 36 \text{ kJ/kg} \;\approx C_p\,(T_{e\,ac} - T_i)$$

$$T_{e\,ac} = T_i + 36/1.004 = \mathbf{55.9°C}$$

Review Problems

9.152

A flow of saturated liquid R-410a at 200 kPa in an evaporator is brought to a state of superheated vapor at 200 kPa, 40^{o}C. Assume the process is reversible find the specific heat transfer and specific work.

C.V. Evaporator. From the device we know that potential and kinetic energies are not important (see chapter 6).

Since the pressure is constant and the process is reversible from Eq.9.14

$$w = -\int v dP + 0 + 0 - 0 = \mathbf{0}$$

From energy equation

$$h_i + q = w + h_e = h_e; \qquad q = h_e - h_i$$

State i: $h_i = 4.18$ kJ/kg, State e: $h_e = 328.68$ kJ/kg

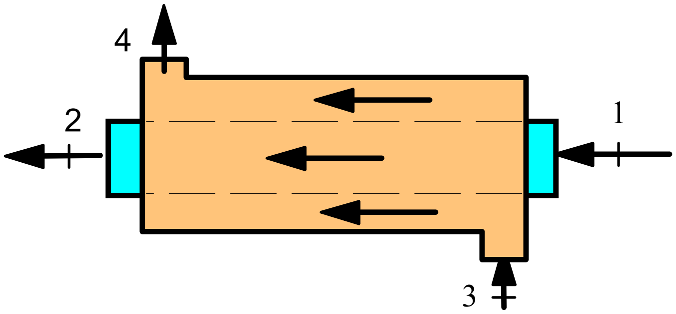
$$q = h_e - h_i = 328.68 - 4.18 = \mathbf{324.5\ kJ/kg}$$

9.153

A coflowing heat exchanger has one line with 2 kg/s saturated water vapor at 100 kPa entering. The other line is 1 kg/s air at 200 kPa, 1200 K. The heat exchanger is very long so the two flows exit at the same temperature. Find the exit temperature by trial and error. Calculate the rate of entropy generation.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.
No W, no external Q



Flows: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{H_2O}$; $\dot{m}_3 = \dot{m}_4 = \dot{m}_{air}$

Energy: $\dot{m}_{H_2O} (h_2 - h_1) = \dot{m}_{air} (h_3 - h_4)$

State 1: Table B.1.2 $h_1 = 2675.5$ kJ/kg State 2: 100 kPa, T_2

State 3: Table A.7 $h_3 = 1277.8$ kJ/kg, State 4: 200 kPa, T_2

Only one unknown T_2 and one equation the energy equation:

$2(h_2 - 2675.5) = 1(1277.8 - h_4) \quad => \quad 2h_2 + h_4 = 6628.8$ kW

At 500 K: $h_2 = 2902.0$, $h_4 = 503.36$ => LHS = 6307 too small

At 700 K: $h_2 = 3334.8$, $h_4 = 713.56$ => LHS = 7383 too large

Linear interpolation $T_2 = 560$ K, $h_2 = 3048.3$, $h_4 = 565.47$ => LHS = 6662

Final states are with $\mathbf{T_2 = 554.4\ K = 281\ °C}$

H2O: Table B.1.3, $h_2 = 3036.8$ kJ/kg, $s_2 = 8.1473$, $s_1 = 7.3593$ kJ/kg K

AIR: Table A.7, $h_4 = 559.65$ kJ/kg, $s_{T4} = 7.4936$, $s_{T3} = 8.3460$ kJ/kg K

The entropy balance equation, Eq.9.7, is solved for the generation term:

$$\dot{S}_{gen} = \dot{m}_{H_2O} (s_2 - s_1) + \dot{m}_{air} (s_4 - s_3)$$

$$= 2(8.1473 - 7.3593) + 1 (7.4936 - 8.3460) = \mathbf{0.724\ kW/K}$$

No pressure correction is needed as the air pressure for 4 and 3 is the same.

9.154

A flow of R-410a at 2000 kPa, 40^{o}C in an isothermal expander is brought to a state of 1000 kPa in a reversible process. Find the specific heat transfer and work.

C.V. Expander. Steady reversible, single inlet and exit flow. Some q and w

Energy Eq.6.13: $h_i + q = w + h_e$;

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_e$

Process: T = constant so $\int dq/T = q/T$ and reversible $s_{gen} = 0$

State i: $h_i = 295.49$ kJ/kg, $s_i = 1.0099$ kJ/kg-K

State e: $h_e = 316.05$ kJ/kg, $s_e = 1.1409$ kJ/kg-K

From entropy equation

$q = T (s_e - s_i) = 313.15 \text{ K } (1.1409 – 1.0099) \text{ kJ/kg-K} = $ **41.023 kJ/kg**

From the energy equation

$w = h_i – h_e + q = 295.49 – 316.05 + 41.023 = $ **330.46 kJ/kg**

9.155

A vortex tube has an air inlet flow at 20°C, 200 kPa and two exit flows of 100 kPa, one at 0°C and the other at 40°C. The tube has no external heat transfer and no work and all the flows are steady and have negligible kinetic energy. Find the fraction of the inlet flow that comes out at 0°C. Is this setup possible?

Solution:

C.V. The vortex tube. Steady, single inlet and two exit flows. No q or w.

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$; Energy: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

States all given by temperature and pressure. Use constant heat capacity to evaluate changes in h and s. Solve for $x = \dot{m}_2/\dot{m}_1$ from the energy equation

$\dot{m}_3/\dot{m}_1 = 1 - x$; $h_1 = x\, h_2 + (1-x)\, h_3$

$\Rightarrow\ x = (h_1 - h_3)/(h_2 - h_3) = (T_1 - T_3)/(T_2 - T_3) = (20–40)/(0–40) = 0.5$

Evaluate the entropy generation

$$\dot{S}_{gen}/\dot{m}_1 = x\, s_2 + (1-x)s_3 - s_1 = 0.5(s_2 - s_1) + 0.5(s_3 - s_1)$$

$$= 0.5\,[C_p \ln(T_2 / T_1) – R \ln(P_2 / P_1)] + 0.5[C_p \ln(T_3 / T_1) – R \ln(P_3/ P_1)]$$

$$= 0.5\left[1.004 \ln\left(\frac{273.15}{293.15}\right) - 0.287 \ln\left(\frac{100}{200}\right)\right]$$

$$+ 0.5\left[1.004 \ln\left(\frac{313.15}{293.15}\right) - 0.287 \ln\left(\frac{100}{200}\right)\right]$$

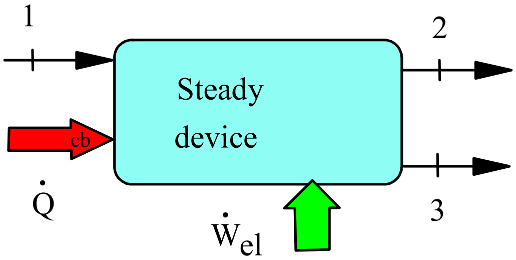
$= \mathbf{0.1966\ kJ/kg\ K\ \ > 0}$ **So this is possible**.

9.156

A stream of ammonia enters a steady flow device at 100 kPa, 50°C, at the rate of 1 kg/s. Two streams exit the device at equal mass flow rates; one is at 200 kPa, 50°C, and the other as saturated liquid at 10°C. It is claimed that the device operates in a room at 25°C on an electrical power input of 250 kW. Is this possible?

Solution:

Control volume: Steady device out to ambient 25°C.



Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{Q} + \dot{W}_{el} = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{Q}/T_{room} + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

State 1: Table B.2.2, $h_1 = 1581.2$ kJ/kg, $s_1 = 6.4943$ kJ/kg K

State 2: Table B.2.2 $h_2 = 1576.6$ kJ/kg, $s_2 = 6.1453$ kJ/kg K

State 3: Table B.2.1 $h_3 = 226.97$ kJ/kg, $s_3 = 0.8779$ kJ/kg K

From the energy equation

$$\dot{Q} = 0.5 \times 1576.6 + 0.5 \times 226.97 - 1 \times 1581.2 - 250 = -929.4 \text{ kW}$$

From the entropy equation

$$\dot{S}_{gen} = 0.5\times 6.1453 + 0.5 \times 0.8779 - 1 \times 6.4943 - (-929.4)/298.15$$
$$= 0.1345 \text{ kW/K} > 0$$

since $\dot{S}_{gen} > 0$ this is possible

9.157

In a heat-powered refrigerator, a turbine is used to drive the compressor using the same working fluid. Consider the combination shown in Fig. P9.157 where the turbine produces just enough power to drive the compressor and the two exit flows are mixed together. List any assumptions made and find the ratio of mass flow rates $\dot{m}_3/\dot{m}_1$ and T_5 (x_5 if in two-phase region) if the turbine and the compressor are reversible and adiabatic

Solution:

CV: compressor

$s_{2S} = s_1 = 1.0779$ kJ/kg K $\rightarrow$ $h_{2S} = 317.43$ kJ/kg

$w_{SC} = h_1 - h_{2S} = 271.89 - 317.43 = -45.54$ kJ/kg

CV: turbine

$s_{4S} = s_3 = 1.0850$ kJ/kgK and P_{4S} ⇨ $h_{4S} = 319.72$ kJ/kg

$w_{ST} = h_3 - h_{4S} = 341.29 - 319.72 = 21.57$ kJ/kg

$$\text{As } \dot{w}_{TURB} = -\dot{w}_{COMP}\,, \quad \dot{m}_3/\dot{m}_1 = -\frac{w_{SC}}{w_{ST}} = \frac{45.54}{21.57} = \mathbf{2.111}$$

CV: mixing portion

$\dot{m}_1 h_{2S} + \dot{m}_3 h_{4S} = (\dot{m}_1 + \dot{m}_3) h_5$

$1 \times 317.43 + 2.111 \times 319.72 = 3.111\ h_5$

$\Rightarrow h_5 = 318.984$ kJ/kg => $T_5 = \mathbf{58.7°C}$

9.158

Carbon dioxide flows through a device entering at 300 K, 200 kPa and leaving at 500 K. The process is steady state polytropic with n = 3.8 and heat transfer comes from a 600 K source. Find the specific work, specific heat transfer and the specific entropy generation due to this process.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13: $h_i + q = h_e + w$ Neglect kinetic, potential energies

Entropy Eq.9.8: $s_i + \int dq/T + s_{gen} = s_e$

Process Eq.8.28:

$$P_e = P_i \, (T_e / T_i)^{\frac{n}{n-1}} = 200(500/300)^{\frac{3.8}{2.8}} = 400 \text{ kPa}$$

and the process leads to Eq.9.17 for the work term

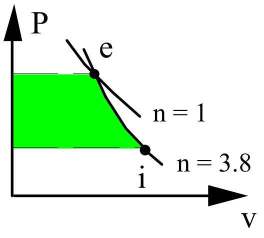
$$w = -\frac{n}{n-1} R \, (T_e - T_i) = -\frac{3.8}{2.8} \times 0.1889 \times (500 - 300) = \mathbf{-51.3 \ kJ/kg}$$

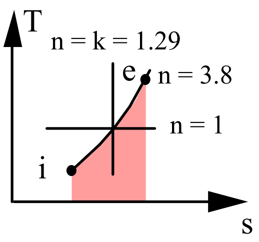
Energy equation gives

$$q = h_e - h_i + w = 401.52 - 214.38 - 51.3 = \mathbf{135.8 \ kJ/kg}$$

Entropy equation gives (CV out to source)

$$s_{gen} = s_e - s_i - q/T_{source} = s^o_{Te} - s^o_{Ti} - R \ln(P_e / P_i) - q/T_{source}$$
$$= 5.3375 - 4.8631 - 0.1889 \ln (400/200) - (135.8/600)$$
$$= \mathbf{0.117 \ kJ/kg \ K}$$





Notice:
dP > 0
so dw <0

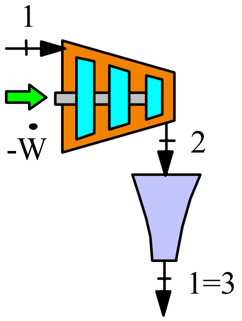
ds > 0
so dq > 0

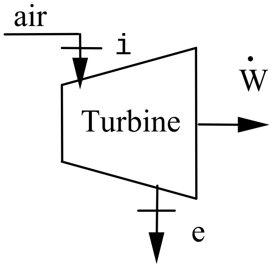
Notice process is externally irreversible, ΔT between source and CO_2

9.159

Air at 100 kPa, 17°C is compressed to 400 kPa after which it is expanded through a nozzle back to the atmosphere. The compressor and the nozzle are both reversible and adiabatic and kinetic energy in and out of the compressor can be neglected. Find the compressor work and its exit temperature and find the nozzle exit velocity.

Solution:





Separate control volumes around compressor and nozzle. For ideal compressor we have inlet : 1 and exit : 2

Adiabatic : $q = 0$.
Reversible: $s_{gen} = 0$

Energy Eq.6.13: $h_1 + 0 = w_C + h_2$;

Entropy Eq.9.8: $s_1 + 0/T + 0 = s_2$

$$- w_C = h_2 - h_1 , \quad s_2 = s_1$$

Properties Table A.5 air: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$

Process gives constant s (isentropic) which with constant C_{Po} gives Eq.8.32

$$=> \quad T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290\ (400/100)^{0.2857} = \mathbf{430.9\ K}$$

$$\Rightarrow \quad -w_C = C_{Po}(T_2 - T_1) = 1.004\ (430.9 - 290) = \mathbf{141.46\ kJ/kg}$$

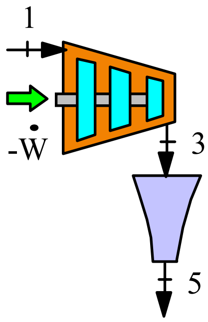
The ideal nozzle then expands back down to P_1 (constant s) so state 3 equals state 1. The energy equation has no work but kinetic energy and gives:

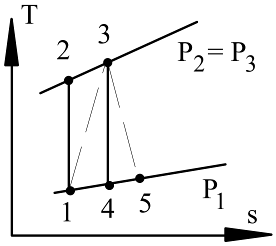
$$\tfrac{1}{2}\mathbf{V}^2 = h_2 - h_1 = -w_C = 141\ 460 \text{ J/kg} \quad \text{(remember conversion to J)}$$

$$\Rightarrow \quad \mathbf{V}_3 = \sqrt{2\times141460} = \mathbf{531.9\ m/s}$$

9.160

Assume both the compressor and the nozzle in Problem 9.37 have an isentropic efficiency of 90% the rest being unchanged. Find the actual compressor work and its exit temperature and find the actual nozzle exit velocity.





C.V. Ideal compressor, inlet: 1 exit: 2

Adiabatic : $q = 0$.
Reversible: $s_{gen} = 0$

Energy Eq.6.13: $h_1 + 0 = w_C + h_2$;

Entropy Eq.9.8: $s_1 + 0/T + 0 = s_2$

$$- w_{Cs} = h_2 - h_1 , \quad s_2 = s_1$$

Properties use air Table A.5: $C_{Po} = 1.004 \frac{kJ}{kg\ K}$, $R = 0.287 \frac{kJ}{kg\ K}$, $k = 1.4$,

Process gives constant s (isentropic) which with constant C_{Po} gives Eq.8.32

$$=> \quad T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290\ (400/100)^{\ 0.2857} = \mathbf{430.9\ K}$$

$$\Rightarrow \quad -w_{Cs} = C_{Po}(T_2 - T_1) = 1.004\ (430.9 - 290) = \mathbf{141.46\ kJ/kg}$$

The ideal nozzle then expands back down to state 1 (constant s). The actual compressor discharges at state 3 however, so we have:

$$w_C = w_{Cs}/\eta_C = \mathbf{-157.18} \quad \Rightarrow \quad T_3 = T_1 - w_C/C_p = \mathbf{446.6\ K}$$

Nozzle receives air at 3 and exhausts at 5. We must do the ideal (exit at 4) first.

$$s_4 = s_3 \ \Rightarrow \ \text{Eq.8.32:} \qquad T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 300.5\ K$$

$$\frac{1}{2}\mathbf{V}_s^2 = C_p(T_3 - T_4) = 146.68 \ \Rightarrow \ \frac{1}{2}\mathbf{V}_{ac}^2 = 132\ kJ/kg \ \Rightarrow V_{ac} = \mathbf{513.8\ m/s}$$

If we need it, the actual nozzle exit (5) can be found:

$$T_5 = T_3 - \ \mathbf{V}_{ac}^2/2C_p = \ 315\ K$$

9.161

An insulated piston/cylinder contains R-410a at 20°C, 85% quality, at a cylinder volume of 50 L. A valve at the closed end of the cylinder is connected to a line flowing R-410a at 2 MPa, 60°C. The valve is now opened, allowing R-410a to flow in, and at the same time the external force on the piston is decreased, and the piston moves. When the valve is closed, the cylinder contents are at 800 kPa, 20°C, and a positive work of 50 kJ has been done against the external force. What is the final volume of the cylinder? Does this process violate the second law of thermodynamics?

Solution:

C.V. Cylinder volume. A transient problem.

Continuity Eq.: $m_2 - m_1 = m_i$

Energy Eq.: $m_2u_2 - m_1u_1 = {}_1Q_2 + m_ih_i - {}_1W_2$

Entropy Eq.: $m_2s_2 - m_1s_1 = {}_1Q_2/T + m_is_i + {}_1S_{2\text{ gen}}$

Process: ${}_1Q_2 = 0$, ${}_1W_2 = 50$ kJ

State 1: $T_1 = 20^oC$, $x_1 = 0.85$, $V_1 = 50 \text{ L} = 0.05 \text{ m}^3$

$P_1 = P_g = 1444.2$ kPa, $u_1 = u_f + x_1u_{fg} = 232.62$ kJ/kg

$v_1 = v_f + x_1v_{fg} = 0.000923 + 0.85\times0.01666 = 0.015084 \text{ m}^3\text{/kg}$,

$s_1 = s_f + x_1s_{fg} = 0.3357 + 0.85\times0.6627 = 0.8990$ kJ/kg K

$m_1 = V_1/v_1 = 0.050 / 0.015084 = 3.3148$ kg

State 2: $T_2 = 20^oC$, $P_2 = 800$ kPa, superheated, $v_2 = 0.03693 \text{ m}^3\text{/kg}$,

$u_2 = 270.47$ kJ/kg, $s_2 = 1.1105$ kJ/kg K

Inlet: $T_i = 60^oC$, $P_i = 2$ MPa, $h_i = 320.62$ kJ/kg, $s_i = 1.0878$ kJ/kg K

Solve for the mass m_2 from the energy equation (the only unknown)

$$m_2 = [m_1u_1 - {}_1W_2 - m_1h_i] / [u_2 - h_i]$$

$$= \frac{3.3148 \times 232.62 - 50 - 3.3148 \times 320.62}{270.47 - 320.62} = 6.8136 \text{ kg}$$

$$\mathbf{V_2 = m_2v_2 = 0.2516\ m^3}$$

Now check the second law

$${}_1S_{2\text{ gen}} = m_2s_2 - m_1s_1 - {}_1Q_2/T - m_is_i$$

$$= 6.8136 \times 1.1105 - 3.3148 \times 0.8990 - 0 - (6.8136 - 3.3148)1.0878$$

$= \mathbf{0.7805\ kJ/K \geq 0}$, **Satisfies 2nd Law**

9.162

A certain industrial process requires a steady 0.5 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler, see Fig. P9.46. Local ambient conditions are 100 kPa, 20°C. Using an isentropic compressor efficiency of 80%, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Air table A.5: $R = 0.287$ kJ/kg-K, $C_p = 1.004$ kJ/kg-K, $k = 1.4$

State 1: $T_1 = T_o = 20^oC$, $P_1 = P_o = 100$ kPa, $\dot{m} = 0.5$ kg/s

State 2: $P_2 = P_3 = 500$ kPa

State 3: $T_3 = 30^oC$, $P_3 = 500$ kPa

We have $\eta_s = 80\ \% = w_{Cs}/w_{C\ ac}$

Compressor: First do the ideal (Isentropic)

$$T_{2s} = T_1\ (P_2/P_1)^{\frac{k-1}{k}} = 293.15\ \ (500/100)^{0.2857} = 464.6 \text{ K}$$

Energy Eq.: $q_c + h_1 = h_2 + w_c$; $q_c = 0$, assume constant specific heat

$$w_{cs} = C_p(T_1 - T_{2s}) = 1.004\ (293.15 - 464.6) = -172.0 \text{ kJ/kg}$$

$\eta_s = w_{Cs}/w_{C\ ac}$, $w_{C\ ac} = w_{Cs}/\eta_s = -215$, $\dot{W}_C = \dot{m}w_C =$ **-107.5 kW**

$w_{C\ ac} = C_p\ (T_1 - T_2)$, solve for $T_2 = 507.5$ K

Aftercooler:

Energy Eq.: $q + h_2 = h_3 + w$; $w = 0$, assume constant specific heat

$$q = C_p\ (T_3 - T_2) = 1.004(303.15 - 507.5) = -205 \text{ kJ/kg,}$$

$\dot{Q} = \dot{m}q =$ **-102.5 kW**

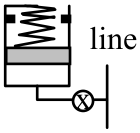
9.163

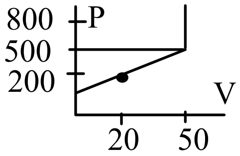
A frictionless piston/cylinder is loaded with a linear spring, spring constant 100 kN/m and the piston cross-sectional area is 0.1 m^2. The cylinder initial volume of 20 L contains air at 200 kPa and ambient temperature, 10°C. The cylinder has a set of stops that prevent its volume from exceeding 50 L. A valve connects to a line flowing air at 800 kPa, 50°C. The valve is now opened, allowing air to flow in until the cylinder pressure reaches 800 kPa, at which point the temperature inside the cylinder is 80°C. The valve is then closed and the process ends.

a) Is the piston at the stops at the final state?

b) Taking the inside of the cylinder as a control volume, calculate the heat transfer during the process.

c) Calculate the net entropy change for this process.





$T_o = 10^oC = 283.15$ K

$A_p = 0.1\ m^2,\ V_{stop} = 50$ L

Air from Table A.5: $R = 0.287,\ C_p = 1.004,\ C_v = 0.717$ kJ/kg-K

State 1: $T_1 = 10^oC,\ P_1 = 200$ kPa, $V_1 = 20\ L = 0.02\ m^3$,

$m_1 = P_1V_1/RT_1 = 200 \times 0.02/(0.287 \times 283.15) = 0.0492$ kg

State 2: $T_2 = 80^oC,\ \ P_2 = 800$ kPa, Inlet: $T_i = 50^oC,\ \ P_i = 800$ kPa

a) $P_{stop} = P_1 + \frac{k_s}{A_p^2}(V_{stop} - V_1) = 500$ kPa, $\mathbf{P_2 > P_{stop}}$ **→ Piston hits stops**

$V_2 = V_{stop} = 50$ L, $m_2 = PV/RT = 0.3946$ kg

b) 1^{st} Law: ${}_1Q_2 + m_ih_i = m_2u_2 - m_1u_1 + m_eh_e + {}_1W_2;\ m_e = 0,\ m_i = m_2 - m_1$

${}_1W_2 = \int P\ dV = (P_1 + P_{stop})(V_{stop} - V_1)/2 = 10.5$ kJ

Assume constant specific heat

${}_1Q_2 = m_2C_vT_2 - m_1C_vT_1 - (m_2 - m_1)\ C_pT_i + {}_1W_2$ **= -11.6 kJ**

c) 2^{nd} Law:

$$\Delta S_{net} = m_2s_2 - m_1s_1 - m_is_i - \frac{Q_{cv}}{T_o} = m_2(s_2 - s_i) - m_1(s_1 - s_i) - \frac{Q_{cv}}{T_o}$$

$s_2 - s_i = C_p \ln(T_2 / T_i) - R \ln(P_2 / P_i) = 0.08907$ kJ/kg-K $(P_2 = P_i)$

$s_1 - s_i = C_p \ln(T_1 / T_i) - R \ln(P_1 / P_i) = 0.26529$ kJ/kg-K

$\mathbf{\Delta S_{net} = 0.063}$ **kJ/K**

9.164

Air enters an insulated turbine at 50°C, and exits the turbine at - 30°C, 100 kPa. The isentropic turbine efficiency is 70% and the inlet volumetric flow rate is 20 L/s. What is the turbine inlet pressure and the turbine power output?

C.V.: Turbine, $\eta_s = 0.7$, Insulated

Air: $C_p = 1.004$ kJ/kg-K, $R = 0.287$ kJ/kg-K, $k = 1.4$

Inlet: $T_i = 50^oC$, $\dot{V}_i = 20$ L/s $= 0.02$ m^3/s

Exit: $T_e = -30^oC$, $P_e = 100$ kPa

a) 1^{st} Law steady flow: $q + h_i = h_e + w_T$; $q = 0$

Assume Constant Specific Heat

$w_T = h_i - h_e = C_p(T_i - T_e) = 80.3$ kJ/kg

$w_{Ts} = w/\eta = 114.7$ kJ/kg, $w_{Ts} = C_p(T_i - T_{es})$

Solve for $T_{es} = 208.9$ K

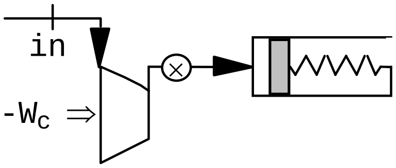
Isentropic Process: $P_e = P_i\,(T_e / T_i)^{\frac{k}{k-1}}$ => $\mathbf{P_i = 461\ kPa}$

b) $\dot{W}_T = \dot{m}w_T$; $\dot{m} = P\dot{V}/RT = 0.099$ kg/s => $\dot{W}_T =$ **7.98 kW**

9.165

An initially empty spring-loaded piston/cylinder requires 100 kPa to float the piston. A compressor with a line and valve now charges the cylinder with water to a final pressure of 1.4 MPa at which point the volume is 0.6 m^3, state 2. The inlet condition to the reversible adiabatic compressor is saturated vapor at 100 kPa. After charging the valve is closed and the water eventually cools to room temperature, 20°C, state 3. Find the final mass of water, the piston work from 1 to 2, the required compressor work, and the final pressure, P_3.

Solution:



Process 1→2: transient, adiabatic.
for C.V. compressor + cylinder
Assume process is reversible

Continuity: $m_2 - 0 = m_{in}$, Energy: $m_2u_2 - \emptyset = (m_{in}h_{in}) - W_c - {}_1W_2$

Entropy Eq.: $m_2s_2 - \emptyset = m_{in}s_{in} + 0 \Rightarrow s_2 = s_{in}$

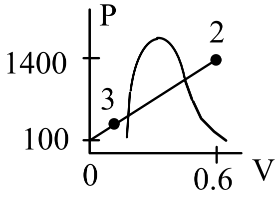
Inlet state: Table B.1.2, $h_{in} = 2675.5$ kJ/kg, $s_{in} = 7.3594$ kJ/kg K

$${}_1W_2 = \int PdV = \frac{1}{2}(P_{float} + P_2)(V_2 - \emptyset) = \frac{1}{2}(100+1400)0.6 = \mathbf{450\ kJ}$$

State 2: P_2 , $s_2 = s_{in}$ Table B.1.3 $\Rightarrow$ $v_2 = 0.2243$, $u_2 = 2984.4$ kJ/kg

$m_2 = V_2/v_2 = 0.6/0.2243 =$ **2.675 kg**

$W_c = m_{in}h_{in} - m_2u_2 - {}_1W_2 = 2.675 \times (2675.5 - 2984.4) - 450 =$ **-1276.3 kJ**



State 3 must be on line & 20°C

Assume 2-phase ⇒ $P_3 = P_{sat}(20°C) = 2.339$ kPa

less than P_{float} so compressed liquid

Table B.1.1: $v_3 \cong v_f(20°C) = 0.001002 \Rightarrow V_3 = m_3v_3 = 0.00268\ m^3$

On line: $P_3 = 100 + (1400 - 100) \times 0.00268/0.6 =$ **105.8 kPa**

9.166

Consider the scheme shown in Fig. P9.166 for producing fresh water from salt water. The conditions are as shown in the figure. Assume that the properties of salt water are the same as for pure water, and that the pump is reversible and adiabatic.

a. Determine the ratio $(\dot{m}_7/\dot{m}_1)$, the fraction of salt water purified.

b. Determine the input quantities, w_P and q_H.

c. Make a second law analysis of the overall system.

C.V. Flash evaporator: Steady flow, no external q, no work.

Energy Eq.: $\dot{m}_1 h_4 = (\dot{m}_1 - \dot{m}_7)h_5 + \dot{m}_7 h_6$

Table B.1.1 or $632.4 = \big(1 - (\dot{m}_7/\dot{m}_1)\big)\ 417.46 + (\dot{m}_7/\dot{m}_1)\ 2675.5$

$\Rightarrow \dot{m}_7/\dot{m}_1 = \mathbf{0.0952}$

C.V. Pump steady flow, incompressible liq.:

$w_P = -\int v dP \approx -v_1(P_2 - P_1) = -\ 0.001001(700 - 100) = \mathbf{-0.6\ kJ/kg}$

$h_2 = h_1 - w_P = 62.99 + 0.6 = 63.6$ kJ/kg

C.V. Heat exchanger: $h_2 + (\dot{m}_7/\dot{m}_1)h_6 = h_3 + (\dot{m}_7/\dot{m}_1)h_7$

$63.6 + 0.0952 \times 2675.5 = h_3 + 0.0952 \times 146.68 \Rightarrow h_3 = 304.3$ kJ/kg

C.V. Heater: $q_H = h_4 - h_3 = 632.4 - 304.3 = \mathbf{328.1\ kJ/kg}$

CV: entire unit, entropy equation per unit mass flow rate at state 1

$S_{C.V.,gen} = -\ q_H/T_H + \big(1 - (\dot{m}_7/\dot{m}_1)\big)s_5 + (\dot{m}_7/\dot{m}_1)s_7 - s_1$

$= (-328.1/473.15) + 0.9048 \times 1.3026 + 0.0952 \times 0.5053 - 0.2245$

$= 0.3088$ kJ/K kg m_1

9.167

A rigid 1.0 m^3 tank contains water initially at 120°C, with 50 % liquid and 50% vapor, by volume. A pressure-relief valve on the top of the tank is set to 1.0 MPa (the tank pressure cannot exceed 1.0 MPa - water will be discharged instead). Heat is now transferred to the tank from a 200°C heat source until the tank contains saturated vapor at 1.0 MPa. Calculate the heat transfer to the tank and show that this process does not violate the second law.

Solution:

C.V. Tank and walls out to the source. Neglect storage in walls. There is flow out and no boundary or shaft work.

Continuity Eq.6.15: $m_2 - m_1 = - m_e$

Energy Eq.6.16: $m_2 u_2 - m_1 u_1 = - m_e h_e + {}_1Q_2$

Entropy Eq.9.12: $m_2 s_2 - m_1 s_1 = - m_e s_e + \int dQ/T + {}_1S_{2\ gen}$

State 1: $T_1 = 120^oC$, Table B.1.1

$v_f = 0.00106\ m^3/kg$, $m_{liq} = 0.5V_1/v_f = 471.7$ kg

$v_g = 0.8919\ m^3/kg$, $m_g = 0.5V_1/v_g = 0.56$ kg,

$m_1 = 472.26$ kg, $x_1 = m_g/m_1 = 0.001186$

$u_1 = u_f + x_1 u_{fg} = 503.5 + 0.001186 \times 2025.8 = 505.88$ kJ/kg,

$s_1 = s_f + x_1 s_{fg} = 1.5275 + 0.001186 \times 5.602 = 1.5341$ kJ/kg-K

State 2: $P_2 = 1.0$ MPa, sat. vap. $x_2 = 1.0$, $V_2 = 1 m^3$

$v_2 = v_g = 0.19444\ m^3/kg$, $m_2 = V_2/v_2 = 5.14$ kg

$u_2 = u_g = 2583.6$ kJ/kg, $s_2 = s_g = 6.5864$ kJ/kg-K

Exit: $P_e = 1.0$ MPa, sat. vap. $x_e = 1.0$, $h_e = h_g = 2778.1$ kJ/kg,

$s_e = s_g = 6.5864$ kJ/kg, $m_e = m_1 - m_2 = 467.12$ kg

From the energy equation we get

$${}_1Q_2 = m_2 u_2 - m_1 u_1 + m_e h_e = \mathbf{1\ 072\ 080\ kJ}$$

From the entropy Eq.9.24 (with 9.25 and 9.26) we get

$${}_1S_{2\ gen} = m_2 s_2 - m_1 s_1 + m_e s_e - \frac{{}_1Q_2}{T_H}\ ; \qquad T_H = 200^oC = 473\ K$$

$${}_1S_{2\ gen} = \Delta S_{net} = \mathbf{120.4\ kJ \geq 0} \quad \textbf{Process Satisfies 2}^{\textbf{nd}} \textbf{ Law}$$

9.168

A jet-ejector pump, shown schematically in Fig. P9.168, is a device in which a low-pressure (secondary) fluid is compressed by entrainment in a high-velocity (primary) fluid stream. The compression results from the deceleration in a diffuser. For purposes of analysis this can be considered as equivalent to the turbine-compressor unit shown in Fig. P9.157 with the states 1, 3, and 5 corresponding to those in Fig. P9.168. Consider a steam jet-pump with state 1 as saturated vapor at 35 kPa; state 3 is 300 kPa, 150°C; and the discharge pressure, P_5, is 100 kPa.

a. Calculate the ideal mass flow ratio, $\dot{m}_1/\dot{m}_3$.

b. The efficiency of a jet pump is defined as $\eta = (\dot{m}_1/\dot{m}_3)_{actual} / (\dot{m}_1/\dot{m}_3)_{ideal}$ for the same inlet conditions and discharge pressure. Determine the discharge temperature of the jet pump if its efficiency is 10%.

a) ideal processes (isen. comp. & exp.)

$$\left.\begin{matrix}\text{expands 3-4s}\\ \text{comp 1-2s}\end{matrix}\right\} \text{then mix at const. P}$$

$$s_{4s} = s_3 = 7.0778 = 1.3026 + x_{4s} \times 6.0568 \quad \Rightarrow \quad x_{4s} = 0.9535$$

$$h_{4s} = 417.46 + 0.9535 \times 2258.0 = 2570.5 \text{ kJ/kg}$$

$$s_{2s} = s_1 = 7.7193 \rightarrow T_{2s} = 174°\text{C} \;\; \& \;\; h_{2s} = 2823.8 \text{ kJ/kg}$$

$$\dot{m}_1(h_{2s} - h_1) = \dot{m}_3(h_3 - h_{4s})$$

$$\Rightarrow (\dot{m}_1/\dot{m}_3)_{IDEAL} = \frac{2761.0 - 2570.5}{2823.8 - 2631.1} = \mathbf{0.9886}$$

b) real processes with jet pump eff. = 0.10

$$\Rightarrow (\dot{m}_1/\dot{m}_3)_{ACTUAL} = 0.10 \times 0.9886 = 0.09886$$

$$\text{1st law} \quad \dot{m}_1 h_1 + \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_3) h_5$$

$$0.09886 \times 2631.1 + 1 \times 2761.0 = 1.09896\, h_5$$

State 5: $h_5 = 2749.3$ kJ/kg, $P_5 = 100$ kPa => $T_5 = \mathbf{136.5\ ^oC}$

9.169

A horizontal, insulated cylinder has a frictionless piston held against stops by an external force of 500 kN. The piston cross-sectional area is 0.5 m^2, and the initial volume is 0.25 m^3. Argon gas in the cylinder is at 200 kPa, 100°C. A valve is now opened to a line flowing argon at 1.2 MPa, 200°C, and gas flows in until the cylinder pressure just balances the external force, at which point the valve is closed. Use constant heat capacity to verify that the final temperature is 645 K and find the total entropy generation.

Solution:

The process has inlet flow, no work (volume constant) and no heat transfer.

Continuity Eq.6.15: $m_2 - m_1 = m_i$

Energy Eq.6.16: $m_2 u_2 - m_1 u_1 = m_i h_i$

$$m_1 = P_1V_1/RT_1 = 200 \times 0.25/(0.2081 \times 373.15) = 0.644 \text{ kg}$$

Force balance: $P_2A = F \quad \Rightarrow \quad P_2 = \frac{500}{0.5} = 1000 \text{ kPa}$

For argon use constant heat capacities so the energy equation is:

$$m_2\ C_{Vo}\, T_2 - m_1\, C_{Vo}\, T_1\ = (m_2\ - m_1\ )\, C_{Po}\, T_{\text{in}}$$

We know P_2 so only 1 unknown for state 2.

Use ideal gas law to write $m_2T_2 = P_2V_1/R$ and $m_1\, T_1\ = P_1V_1/R$

and divide the energy equation with C_{Vo} to solve for the change in mass

$$(P_2\, V_1\ - P_1V_1)/R\ = (m_2\ - m_1\ )\, (C_{Po}/C_{Vo}\ )\, T_{\text{in}}$$

$$(m_2\ - m_1\ ) = (P_2 - P_1)V_1/(R\ k\ T_{\text{in}}\ )$$

$$= (1000 - 200)\times 0.25/(0.2081\times 1.667\times 473.15) = 1.219 \text{ kg}$$

$$m_2 = 1.219 + 0.644 = 1.863 \text{ kg.}$$

$$T_2 = P_2V_1/(m_2R) = 1000\times 0.25/(1.863\times 0.2081) = 645 \text{ K} \quad \textbf{OK}$$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = m_is_i + 0 + {}_1S_{2\text{ gen}}$

$${}_1S_{2\text{ gen}}\ = m_1(s_2 - s_1) + (m_2 - m_1)(s_2 - s_i)$$

$$= m_1\Big[C_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}\Big] + (m_2 - m_1)\Big[C_p \ln\frac{T_2}{T_i} - R \ln\frac{P_2}{P_i}\Big]$$

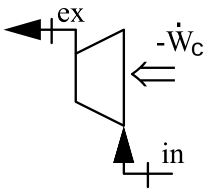
$$= 0.644[\ 0.52 \ln\frac{645}{373.15} - 0.2081 \ln\frac{1000}{200}]$$

$$+ 1.219[\ 0.52 \ln\frac{645}{473.15} - 0.2081 \ln\frac{1000}{1200}]$$

$$= - 0.03242 + 0.24265 = \mathbf{0.21\ kJ/K}$$

9.170

Supercharging of an engine is used to increase the inlet air density so that more fuel can be added, the result of which is an increased power output. Assume that ambient air, 100 kPa and 27°C, enters the supercharger at a rate of 250 L/s. The supercharger (compressor) has an isentropic efficiency of 75%, and uses 20 kW of power input. Assume that the ideal and actual compressor have the same exit pressure. Find the ideal specific work and verify that the exit pressure is 175 kPa. Find the percent increase in air density entering the engine due to the supercharger and the entropy generation.



C.V.: Air in compressor (steady flow)

Cont: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m} = \dot{V}/v_{in} = 0.29$ kg/s

Energy: $\dot{m}h_{in} - \dot{W} = \dot{m}h_{ex}$ Assume: $\dot{Q} = 0$

Entropy: $\dot{m}s_{in} + \dot{S}_{gen} = \dot{m}s_{ex}$

$$v_{in} = \frac{RT_{in}}{P_{in}} = 0.8614 \text{ m}^3\text{/kg},\ \ s^o_{Ti} = 6.86975 \text{ kJ/kg K},\ \ h_{in} = 300.62 \text{ kJ/kg}$$

$$\eta_c = w_{C\,s}/w_{C\,ac} \Rightarrow\ \ -\dot{W}_S = -\dot{W}_{AC} \times \eta_c = 15 \text{ kW}$$

$$-w_{C\,s} = -\dot{W}_S/\dot{m} = 51.724 \text{ kJ/kg},\quad -w_{C\,ac} = 68.966 \text{ kJ/kg}$$

Table A.7: $h_{ex\,s} = h_{in} - w_{C\,s} = 300.62 + 51.724 = 352.3$ kJ/kg

$$\Rightarrow T_{ex\,s} = 351.5 \text{ K},\ \ s^o_{Te} = 7.02830 \text{ kJ/kg K}$$

$$P_{ex} = P_{in} \times e^{(s^o_{T\,ex} - s^o_{T\,in})/R} = 100 \times \exp\left[\frac{7.0283 - 6.86975}{0.287}\right]$$

$$= \mathbf{173.75\ kPa}$$

The actual exit state is

$$h_{ex\,ac} = h_{in} - w_{C\,ac} = 369.6 \text{ kJ/kg} \Rightarrow\ T_{ex\,ac} = 368.6 \text{ K}$$

$$v_{ex} = RT_{ex}/P_{ex} = 0.6088 \text{ m}^3\text{/kg},\ \ s^o_{Tex\,ac} = 7.0764$$

$$\rho_{ex}/\rho_{in} = v_{in}/v_{ex} = 0.8614/0.6088 = \mathbf{1.415\ \ or\ \ 41.5\%\ increase}$$

$$s_{gen} = s_{ex} - s_{in} = 7.0764 - 6.86975 - 0.287 \ln\left(\frac{173.75}{100}\right) = \mathbf{0.0481\ kJ/kg\ K}$$

9.171

A rigid steel bottle, V = 0.25 m^3, contains air at 100 kPa, 300 K. The bottle is now charged with air from a line at 260 K, 6 MPa to a bottle pressure of 5 MPa, state 2, and the valve is closed. Assume that the process is adiabatic, and the charge always is uniform. In storage, the bottle slowly returns to room temperature at 300 K, state 3. Find the final mass, the temperature T_2, the final pressure P_3, the heat transfer ${}_1Q_3$ and the total entropy generation.

C.V. Bottle. Flow in, no work, no heat transfer.

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$;

Energy Eq.6.16: $m_2u_2 - m_1u_1 = m_{in}h_{in}$

State 1 and inlet: Table A.7, $u_1 = 214.36$ kJ/kg, $h_{in} = 260.32$ kJ/kg

$$m_1 = P_1V/RT_1 = (100 \times 0.25)/(0.287 \times 300) = 0.290 \text{ kg}$$

$$m_2 = P_2V/RT_2 = 5000 \times 0.25/(0.287 \times T_2) = 4355.4/T_2$$

Substitute into energy equation

$$u_2 + 0.00306\ T_2 = 260.32$$

Now trial and error on T_2

$T_2 = 360$ => LHS = 258.63 (low);

$T_2 = 370$ => LHS = 265.88 (high)

Interpolation $T_2 = 362.3$ K (LHS = 260.3 OK)

$m_2 = 4355.4/362.3 = 12.022$ kg ; $P_3 = m_2RT_3/V =$ **4140 kPa**

Now use the energy equation from the beginning to the final state

$${}_1Q_3 = m_2u_3 - m_1u_1 - m_{in}h_{in} = (12.022 - 0.29)\ 214.36 - 11.732 \times 260.32$$

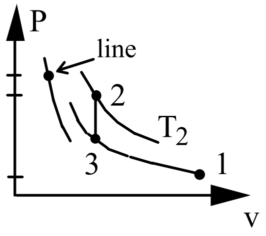
$$= \mathbf{-539.2\ kJ}$$

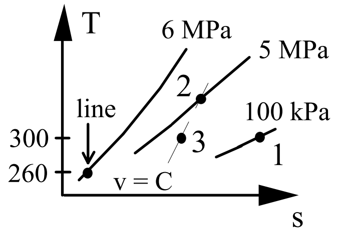
Entropy equation from state 1 to state 3 with change in s from Eq.8.28

$$S_{gen} = m_2s_3 - m_1s_1 - m_{in}s_{in} - {}_1Q_3/T = m_2(s_3 - s_{in}) - m_1(s_1 - s_{in}) - {}_1Q_3/T$$

$$= 12.022[6.8693 - 6.7256 - R \ln(4140/6000)]$$

$$- 0.29[6.8693 - 6.7256 - R \ln(100/6000)] + 539.2/300 = \mathbf{4.423\ kJ/K}$$





Problem could have been solved with constant specific heats from A.5 in which case we would get the energy explicit in T_2 (no iterations).

9.172

A certain industrial process requires a steady 0.5 kg/s of air at 200 m/s, at the condition of 150 kPa, 300 K. This air is to be the exhaust from a specially designed turbine whose inlet pressure is 400 kPa. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent n = 1.20.

a) What is the turbine inlet temperature?

b) What are the power output and heat transfer rate for the turbine?

c) Calculate the rate of net entropy increase, if the heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature.

Solution:

C.V. Turbine, this has heat transfer, PV^n = Constant, n = 1.2

Process polytropic Eq.8.37: $T_e / T_i = (P_e/P_i)^{\frac{n-1}{n}} \Rightarrow T_i = 353.3\text{ K}$

Energy Eq.6.12: $\dot{m}_i(h + \mathbf{V}^2/2)_{in} + \dot{Q} = \dot{m}_{ex}(h + \mathbf{V}^2/2)_{ex} + \dot{W}_T$

Reversible shaft work in a polytropic process, Eq.9.14 and Eq.9.19:

$$w_T = -\int v\, dP + (\mathbf{V}_i^2 - \mathbf{V}_e^2)/2 = -\frac{n}{n-1}(P_e v_e - P_i v_i) + (\mathbf{V}_i^2 - \mathbf{V}_e^2)/2$$

$$= -\frac{n}{n-1}R\,(T_e - T_i) - \mathbf{V}_e^2/2 = 71.8\text{ kJ/kg}$$

$\dot{W}_T = \dot{m}w_T =$ **35.9 kW**

Assume constant specific heat in the energy equation

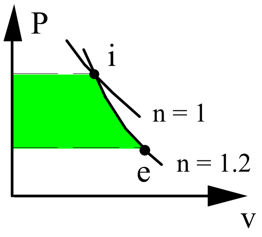
$\dot{Q} = \dot{m}[C_P\,(T_e - T_i) + \mathbf{V}_e^2/2\,] + \dot{W}_T =$ **19.2 kW**

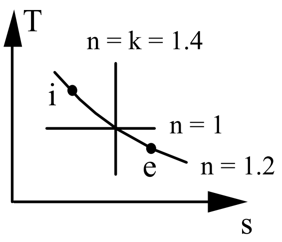
Entropy Eq.9.7 or 9.23 with change in entropy from Eq.8.25:

$dS_{net}/dt = \dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}_H/T_H, \quad T_H = T_i + 100 = 453.3\text{ K}$

$s_e - s_i = C_P \ln(T_e / T_i) - R \ln(P_e / P_i) = 0.1174\text{ kJ/kg K}$

$dS_{net}/dt = 0.5 \times 0.1174 - 19.2/453.3 = 0.0163\text{ kW/K}$





Problems solved with P_r and v_r functions

9.26

Do the previous problem using the air tables in A.7
The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with neglible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:
C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i = h_e + \mathbf{V}_e^2/2 \quad (Z_i = Z_e)$

Entropy Eq.9.8: $s_e = s_i + \int dq/T + s_{gen} = s_i + 0 + 0$

Process: $q = 0$, $s_{gen} = 0$ as used above leads to $s_e = s_i$

Inlet state: $h_i = 1277.8$ kJ/kg, $P_{r\,i} = 191.17$

The constant s is done using the P_r function from A.7.2

$$P_{r\,e} = P_{r\,i}\,(P_e / P_i) = 191.17\ (80/150) = 101.957$$

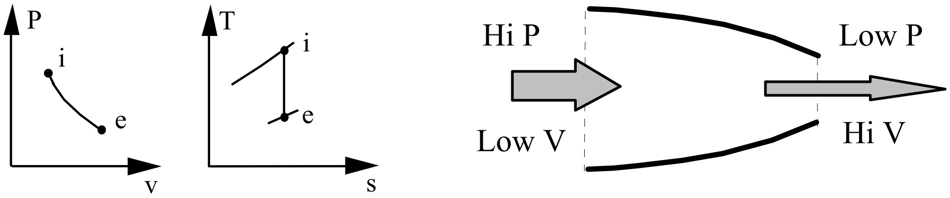
Interpolate in A.7 =>

$$T_e = 1000 + 50\,\frac{101.957 - 91.651}{111.35 - 91.651} = 1026.16 \text{ K}$$

$$h_e = 1046.2 + 0.5232 \times (1103.5 - 1046.2) = 1076.2 \text{ kJ/kg}$$

From the energy equation we have $\mathbf{V}_e^2/2 = h_i - h_e$, so then

$$\mathbf{V}_e = \sqrt{2\,(h_i - h_e)} = \sqrt{2(1277.8 - 1076.2) \text{ kJ/kg} \times 1000 \text{ J/kJ}} = \mathbf{635\ m/s}$$



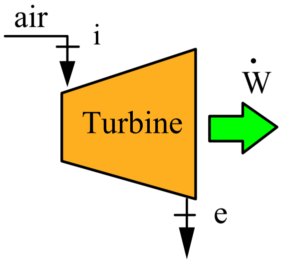
9.30

Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

a. The ideal gas tables, Table A.7

b. Constant specific heat, value at 300 K from table A.5

Solution:



C.V. Air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.6.13: $w_T = h_i - h_e$,

Entropy Eq.9.8: $s_e = s_i$

a) Table A.7: $h_i = 1277.8$ kJ/kg, $P_{r\,i} = 191.17$

The constant s process is done using the P_r function from A.7.2

$$\Rightarrow P_{r\,e} = P_{r\,i}\,(P_e / P_i) = 191.17 \left(\frac{100}{800}\right) = 23.896$$

Interpolate in A.7.1 $\Rightarrow$ $T_e =$ **705.7 K**, $h_e = 719.7$ kJ/kg

$w = h_i - h_e = 1277.8 – 719.7 =$ **558.1 kJ/kg**

b) Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.8.32

$$T_e = T_i\,(P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{100}{800}\right)^{0.286} = \mathbf{662.1\ K}$$

$w = C_{Po}(T_i - T_e) = 1.004(1200 - 662.1) =$ **539.8 kJ/kg**

9.32

A compressor receives air at 290 K, 100 kPa and a shaft work of 5.5 kW from a gasoline engine. It should deliver a mass flow rate of 0.01 kg/s air to a pipeline. Find the maximum possible exit pressure of the compressor.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq.6.11: $\dot{m}_i = \dot{m}_e = \dot{m}$,

Energy Eq.6.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$,

Entropy Eq.9.8: $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

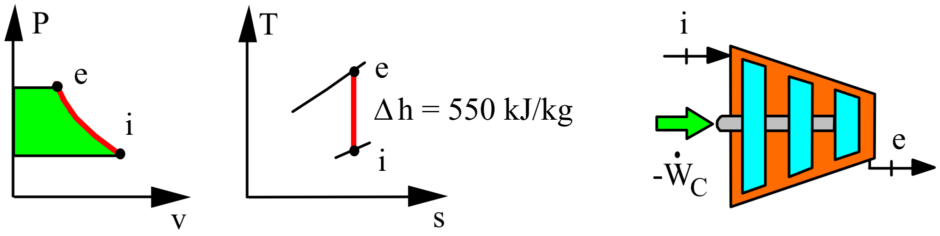
$$\dot{W}_c = \dot{m}w_c \quad => \quad -w_c = -\dot{W}/\dot{m} = 5.5/0.01 = 550 \text{ kJ/kg}$$

Use Table A.7, $h_i = 290.43$ kJ/kg, $P_{r\,i} = 0.9899$

$$h_e = h_i + (-w_c) = 290.43 + 550 = 840.43 \text{ kJ/kg}$$

A.7 => $T_e = 816.5$ K, $P_{r\,e} = 41.717$

$$P_e = P_i\,(P_{r\,e}/P_{r\,i}) = 100 \times (41.717/0.9899) = \mathbf{4214\ kPa}$$



9.50

An underground saltmine, 100 000 m^3 in volume, contains air at 290 K, 100 kPa. The mine is used for energy storage so the local power plant pumps it up to 2.1 MPa using outside air at 290 K, 100 kPa. Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work.

Solution:

C.V. The mine volume and the pump

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = {}_1Q_2 - {}_1W_2 + m_{in}h_{in}$

Entropy Eq.9.12: $m_2s_2 - m_1s_1 = \int dQ/T + {}_1S_{2\,gen} + m_{in}s_{in}$

Process: Adiabatic ${}_1Q_2 = 0$, Process ideal ${}_1S_{2\,gen} = 0$, $s_1 = s_{in}$

$\Rightarrow m_2s_2 = m_1s_1 + m_{in}s_{in} = (m_1 + m_{in})s_1 = m_2s_1 \Rightarrow s_2 = s_1$

Constant s $\Rightarrow$ $P_{r2} = P_{r\,i}\,(P_2 / P_i) = 0.9899 \left(\frac{2100}{100}\right) = 20.7879$

A.7.2 $\Rightarrow T_2 = \mathbf{680\ K}$, $u_2 = 496.94$ kJ/kg

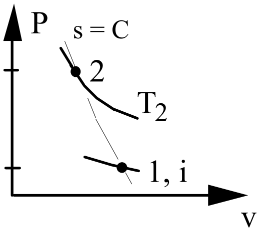
$m_1 = P_1V_1/RT_1 = 100{\times}10^5/(0.287 \times 290) = 1.20149 \times 10^5$ kg

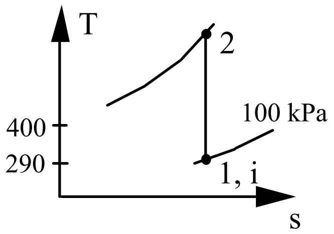
$m_2 = P_2V_2/RT_2 = 100 \times 21{\times}10^5/(0.287 \times 680) = \mathbf{10.760 \times 10^5\ kg}$

$\Rightarrow m_{in} = 9.5585{\times}10^5$ kg

${}_1W_2 = m_{in}h_{in} + m_1u_1 - m_2u_2$

$= m_{in}(290.43) + m_1(207.19) - m_2(496.94) = \mathbf{-2.322 \times 10^8\ kJ}$





9.72

Calculate the air temperature and pressure at the stagnation point right in front of a meteorite entering the atmosphere (-50 °C, 50 kPa) with a velocity of 2000 m/s. Do this assuming air is incompressible at the given state and repeat for air being a compressible substance going through an adiabatic compression.

Solution:

Kinetic energy: $\frac{1}{2}\mathbf{V}^2 = \frac{1}{2}(2000)^2/1000 = 2000$ kJ/kg

Ideal gas: $v_{atm} = RT/P = 0.287 \times 223/50 = 1.28$ m^3/kg

a) incompressible

Energy Eq.6.13: $\Delta h = \frac{1}{2}\mathbf{V}^2 = 2000$ kJ/kg

If A.5 $\Delta T = \Delta h/C_p = 1992$ K unreasonable, too high for that C_p

Use A.7: $h_{st} = h_o + \frac{1}{2}\mathbf{V}^2 = 223.22 + 2000 = 2223.3$ kJ/kg

$T_{st} = 1977$ K

Bernoulli (incompressible) Eq.9.17:

$$\Delta P = P_{st} - P_o = \frac{1}{2}\mathbf{V}^2/v = 2000/1.28 = 1562.5 \text{ kPa}$$

$$P_{st} = 1562.5 + 50 = 1612.5 \text{ kPa}$$

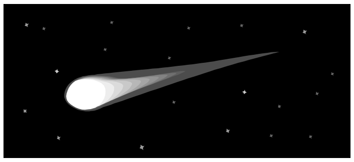
b) compressible

$T_{st} = 1977$ K the same energy equation.

From A.7.2: Stagnation point $P_{r\,st} = 1580.3$; Free $P_{r\,o} = 0.39809$

$$P_{st} = P_o \times \frac{P_{r\,st}}{P_{r\,o}} = 50 \times \frac{1580.3}{0.39809}$$

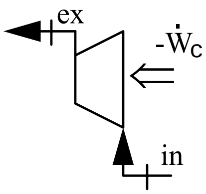
$= \mathbf{198\ 485\ kPa}$



Notice that this is highly compressible, v is not constant.

9.170

Supercharging of an engine is used to increase the inlet air density so that more fuel can be added, the result of which is an increased power output. Assume that ambient air, 100 kPa and 27°C, enters the supercharger at a rate of 250 L/s. The supercharger (compressor) has an isentropic efficiency of 75%, and uses 20 kW of power input. Assume that the ideal and actual compressor have the same exit pressure. Find the ideal specific work and verify that the exit pressure is 175 kPa. Find the percent increase in air density entering the engine due to the supercharger and the entropy generation.



C.V.: Air in compressor (steady flow)

Cont: $\dot{m}_{in} = \dot{m}_{ex} = \dot{m} = \dot{V}/v_{in} = 0.29$ kg/s

Energy: $\dot{m}h_{in} - \dot{W} = \dot{m}h_{ex}$ Assume: $\dot{Q} = 0$

Entropy: $\dot{m}s_{in} + \dot{S}_{gen} = \dot{m}s_{ex}$

Inlet state: $v_{in} = RT_{in}/P_{in} = 0.8614 \text{ m}^3/\text{kg}$, $P_{r\,in} = 1.1167$

$\eta_c = w_{C\,s}/w_{C\,ac} \Rightarrow -\dot{W}_S = -\dot{W}_{AC} \times \eta_c = 15$ kW

$-w_{C\,s} = -\dot{W}_S/\dot{m} = 51.724$ kJ/kg, $-w_{C\,ac} = 68.966$ kJ/kg

Table A.7: $h_{ex\,s} = h_{in} - w_{C\,s} = 300.62 + 51.724 = 352.3$ kJ/kg

$\Rightarrow T_{ex\,s} = 351.5$ K, $P_{r\,ex} = 1.949$

$P_{ex} = P_{in} \times P_{r\,ex}/P_{r\,in} = 100 \times 1.949 / 1.1167 =$ **174.5 kPa**

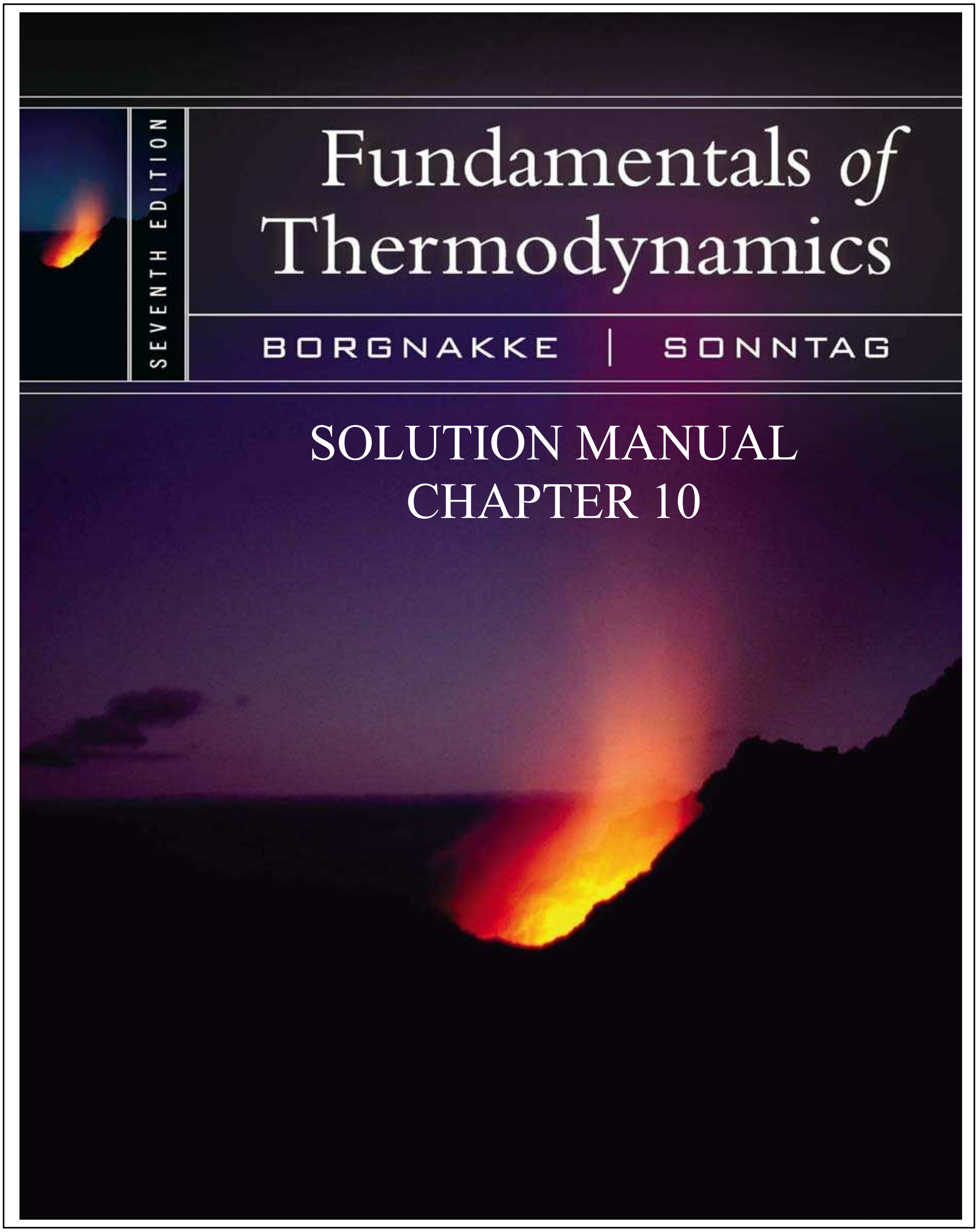
The actual exit state is

$h_{ex\,ac} = h_{in} - w_{C\,ac} = 369.6$ kJ/kg $\Rightarrow T_{ex\,ac} = 368.6$ K

$v_{ex} = RT_{ex}/P_{ex} = 0.606 \text{ m}^3/\text{kg}$

$\rho_{ex}/\rho_{in} = v_{in}/v_{ex} = 0.8614/0.606 =$ **1.42 or 42 % increase**

$s_{gen} = s_{ex} - s_{in} = 7.0767 - 6.8693 - 0.287 \ln(174/100)] =$ **0.0484 kJ/kg K**



Fundamentals of Thermodynamics

SEVENTH EDITION

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 10

CONTENT

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Problems re-solved using Pr and vr functions
from table A.7.2: 30, 65

In-Text Concept Questions

10.a

Can I have any energy transfer as heat transfer that is 100% available?

By definition the possible amount of work that can be obtained equals the exergy (availability). The maximum is limited to that out of a reversible heat engine, if constant T then that is the Carnot heat engine

$$W = (1 - \frac{T_o}{T})Q$$

So we get a maximum for an infinite high temperature T, where we approach an efficiency of one. In practice you do not have such a source (the closest would be solar radiation) and secondly no material could contain matter at very high T so a cycle process can proceed (the closest would be a plasma suspended by a magnetic field as in a tokamak).

10.b

Is electrical work 100% available?

Yes. By definition work is 100% exergy or availability.

10.c

A nozzle does not have any actual work involved, how should you then interpret the reversible work?

The purpose of the nozzle is not to make work, but the reversible work can be used to generate kinetic energy in addition to what comes out of the actual nozzle. The reversible work plus the actual kinetic energy would equal the possible total kinetic energy out of the reversible nozzle. The other point is that the reversible nozzle has a heat transfer in (if the actual nozzle is irreversible) also unusual for a nozzle.

10.d

If an actual control volume process is reversible what can you say about the work term?

The work term is the maximum possible. If the work is positive it is the maximum possible work out and if the work is negative it is the minimum work possible that must be supplied.

10.e

Can entropy change in a control volume process that is reversible?

Yes it can. A flow in or out at a state with a different entropy than the average entropy inside or a heat transfer will change the entropy. All the terms that changes the entropy are transfer terms so there is no net increase (generation) of entropy.

10.f

Energy can be stored as internal energy, potential energy or kinetic energy. Are those energy forms all 100% available?

The internal energy is only partly available, a process like an expansion can give out work or if it cools by heat transfer out it is a Q out that is only partly available as work. Potential energy like from gravitation, mgH, or a compressed spring or a charged battery are forms that are close to 100% available with only small losses present. Kinetic energy like in a fly-wheel or motion of a mass can be transferred to work out with losses depending on the mechanical system.

10.g

We cannot create nor destroy energy, can we create or destroy exergy?

Yes. Every process that is irreversible to some degree destroys exergy. This destruction is directly proportional to the entropy generation. We cannot create exergy at most it can stay constant, which is the case for a reversible process.

10.h

In a turbine what is the source of exergy?

The flow into the turbine is the source, it provides the conditions that allows the turbine to give work out. If the turbine exit flow has useful exergy we can write the source as the inlet flow exergy minus the exit flow exergy

10.i

In a pump what is the source of exergy?

The shaft work is the input that drives the pump, which in turn pushes on the flow to generate a higher pressure exit flow. The increase in the flow exergy is the desired output, that is expressing the increase in P in terms of exergy.

10.j

In a pump what gains exergy?

The higher pressure exit flow is the desired output. When we express that in terms of availability it becomes the increase in the flow exergy that is the output, that is expressing the increase in P in terms of exergy.

10.k

In a heat engine what is the source of exergy?

Generally it is the high temperature heat transfer. However, if the rejected heat transfer at the low temperature has any useful exergy we can also count the difference as the source.

10.l

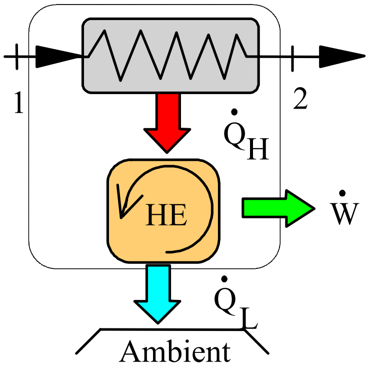
In a heat pump what is the source of exergy?

The work input is what drives the heat pump.

10.m

In Eq.10.39 for the heat engine the source of exergy was written as a heat transfer. How does the expression look like if the source is a flow of hot gas being cooled down as it gives energy to the heat engine?

Look at a heat exchanger that provides such a set-up in Fig. 10.3



C.V. Heat exchanger plus H.E.

Energy Eq.6.12:

$$0 = \dot{m}_1 h_1 - \dot{m}_1 h_2 - \dot{W} - \dot{Q}_L$$

Entropy Eq.9.2 (assume reversible):

$$0 = \dot{m}_1 s_1 - \dot{m}_1 s_2 - \dot{Q}_L/T_o + \dot{S}_{gen}$$

Solve for $\dot{Q}_L$ and sustitute into energy Eq.

$$\dot{Q}_L = T_o\, \dot{m}_1\, (s_1 - s_2) + T_o\dot{S}_{gen}$$

$$\begin{aligned}\dot{W} &= \dot{m}_1\, [\, h_1 - h_2 - T_o\, (s_1 - s_2)] - T_o\dot{S}_{gen}\\ &= \dot{m}_1\, (\psi_1 - \psi_2) - T_o\dot{S}_{gen}\\ &= \eta_{HE\ II}\ \dot{m}_1\, (\psi_1 - \psi_2)\end{aligned}$$

So the H.E. work is a fraction of the flow exergy.

Concept-Study Guide Problems

10.1

Why does the reversible C.V. counterpart to the actual C.V. have the same storage and flow terms?

If you want to compare two devices they should be comparable in as many respects as possible to be useful. Specifically when we want to find the possible work output all the aspects that can generate work are important and must be considered. The flows in and out and the storage effect are closely tied to the possible work output and thus should be the same for the two control volumes. Obviously one control volume that receives a high exergy flow or depletes its stored exergy can give out more work than a control volume that does not have such effects.

10.2

Can one of the heat transfers in Eq.10.5-6 be to or from the ambient?

Yes it can, then the reversible heat transfer we find from the ambient to balance the entropy is in addition to the one already listed. In many practical devices there is an actual heat transfer loss to the surroundings because of elevated temperatures inside the device.

10.3

All the energy in the ocean is that available?

No. Since the ocean is at the ambient T (it **is** the ambient) it is not possible to extract any work from it. You can extract wave energy (wind generated kinetic energy) or run turbines from the tide flow of water (moon generated kinetic energy). However, since the ocean temperature is not uniform there are a few locations where cold and warmer water flows close to each other like at different depths. In that case a heat engine can operate due to the temperature difference.

10.4

Does a reversible process change the availability if there is no work involved?

Yes. There can be heat transfer involved and that has an availability associated with it, which then equals the change of availability of the substance. A flow can enter and exit a device with different levels of availability. What is unchanged is the total (CV plus surroundings) availability that remains constant, but it may be redistributed.

10.5

Is the reversible work between two states the same as ideal work for the device?

No. It depends on the definition of ideal work. The ideal device does not necessarily have the same exit state as the actual device. An ideal turbine is approximated as a reversible adiabatic device so the ideal work is the isentropic work. The reversible work is between the inlet state and the actual exit state that do not necessarily have the same entropy.

10.6

When is the reversible work the same as the isentropic work?

That happens when the inlet and exit states (or beginning and end states) have the same entropy. For a reversible adiabatic process in a C.V (which then is isentropic) the two work terms become the same.

10.7

If I heat some cold liquid water to T_o, do I increase its availability?

No. You decrease its availability by bringing it closer to T_o, where it has zero availability, if we neglect pressure effects. Any substance at a T different from ambient (higher or lower) has a positive availability since you can run a heat engine using the two temperatures as the hot and cold reservoir, respectively. For a T lower than the ambient it means that the ambient is the hot side of the heat engine.

10.8

Are reversible work and availability (exergy) connected?

Yes. They are very similar. Reversible work is usually defined as the reversible work that can be obtained between two states, inlet-exit or beginning to end. Availability is a property of a given state and defined as the reversible work that can be obtained by changing the state of the substance from the given state to the dead state (ambient).

10.9

Consider availability (exergy) associated with a flow. The total exergy is based on the thermodynamic state, the kinetic and potential energies. Can they all be negative?

No. By virtue of its definition kinetic energy can only be positive. The potential energy is measured from a reference elevation (standard sea level or a local elevation) so it can be negative. The thermodynamic state can only have a positive exergy the smallest it can be is zero if it is the ambient dead state.

10.10

Verify that Eq.10.29 reduces to Eq.10.14 for a steady state process

The expression for the reversible work is

$$\dot{W}^{rev} = \dot{\Phi}_q + \sum \dot{m}_i \, \psi_i - \sum \dot{m}_e \, \psi_e - \dot{\Phi}_{CV} + P_o \dot{V} \qquad (10.29)$$

Definition of steady state is no storage effect so:

$$\dot{V} = 0; \qquad \dot{\Phi}_{CV} = 0$$

the last two terms in Eq.10.29 drop out. Eq.10.14 is also written for a single flow so the summations are each a single term.

$$\sum \dot{m}_i \, \psi_i - \sum \dot{m}_e \, \psi_e = \dot{m} \, \psi_i - \dot{m} \, \psi_e = \dot{m} \, (\psi_i - \psi_e)$$

The heat transfer term is from eq.10.21and with single flow

$$\dot{\Phi}_q = \sum (1 - \frac{T_o}{T_j}) \dot{Q}_j = \dot{m} \sum (1 - \frac{T_o}{T_j}) q_j \qquad (10.21)$$

Now we have

$$\dot{W}^{rev} = \dot{m} \sum (1 - \frac{T_o}{T_j}) q_j + \dot{m} \, (\psi_i - \psi_e)$$

divide by the mass flow rate to get

$$w^{rev} = \sum (1 - \frac{T_o}{T_j}) q_j + \psi_i - \psi_e$$

The last difference in exergies are from Eq.10.23

$$\psi_i - \psi_e = (h_{tot\ i} - T_o s_i) - (h_{tot\ e} - T_o s_e)$$

which when substituted gives Eq.10.14.

10.11

What is the second law efficiency of a Carnot Heat engine?

The Carnot Heat engine is by definition reversible and thus it delivers the maximum amount of work possible. It has a second law eficiency of 100%.

10.12

What is the second law efficiency of a reversible heat engine?

Since the reversible heat engine has no entropy generation it produces the maximum work possible and the actual work is the reversible work so it has a second law efficiency of 100%.

10.13

For a nozzle what is the output and input (source) expressed in exergies?

For the nozzle a high pressure low velocity inlet flow generates a higher velocity in the exit flow at the expense of the pressure. The desired output is the higher velocity expressed as kinetic energy, which is part of exergy. The source is the inlet high pressure expressed in flow exergy.

10.14

Is the exergy equation independent of the energy and entropy equations?

No. The exergy equation is derived from the other balance equations by defining the exergy from the state properties and the reference dead state.

10.15

Use the exergy balance equation to find the efficiency of a steady state Carnot heat engine operating between two fixed temperature reservoirs?

The exergy balance equation, Eq.10.36, for this case looks like

$$0 = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H - \left(1 - \frac{T_o}{T_L}\right) \dot{Q}_L - \dot{W} + 0 + 0 - 0 - 0$$

Steady state (LHS = 0 and dV/dt = 0, no mass flow terms, Carnot cycle so reversible and the destruction is then zero. From the energy equation we have

$$0 = \dot{Q}_H - \dot{Q}_L - \dot{W}$$

which we can subtract from the exergy balance equation to get

$$0 = -\frac{T_o}{T_H} \dot{Q}_H + \frac{T_o}{T_L} \dot{Q}_L$$

Solve for one heat transfer in terms of the other

$$\dot{Q}_L = \frac{T_L}{T_H} \dot{Q}_H$$

The work from the energy equation is

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = \dot{Q}_H \left[1 - \frac{T_L}{T_H} \right]$$

from which we can read the Carnot cycle efficiency as we found in Chapter 7.

Available Energy, Reversible work

10.16

Find the availability of 100 kW delivered at 500 K when the ambient is 300 K.

Solution:

The availability of an amount of heat transfer equals the possible work that can be extracted. This is the work out of a Carnot heat engine with heat transfer to the ambient as the other reservoir. The result is from Chapter 7 as also shown in Eq. 10.1 and Eq. 10.36

$$\dot{\Phi} = \dot{W}_{\text{rev HE}} = (1 - \frac{T_o}{T})\dot{Q} = (1 - \frac{300}{500})\ 100\ \text{kW} = \mathbf{40\ kW}$$

10.17

A control mass gives out 10 kJ of energy in the form of

a. Electrical work from a battery

b. Mechanical work from a spring

c. Heat transfer at 500°C

Find the change in availability of the control mass for each of the three cases.

Solution:

a) Work is availability $\Delta\Phi = -W_{el} = \mathbf{-10\ kJ}$

b) Work is availability $\Delta\Phi = -W_{spring} = \mathbf{-10\ kJ}$

c) Give the heat transfer to a Carnot heat engine and W is availability

$$\Delta\Phi = -\left[1 - \frac{T_0}{T_H}\right] Q_{out} = -\left(1 - \frac{298.15}{773.15}\right) 10 = \mathbf{-6.14\ kJ}$$

10.18

A refrigerator should remove 1.5 kW from the cold space at -10°C while it rejects heat to the kitchen at 25°C. Find the reversible work.

The reversible work is related to the Carnot cycle work as the the two reservoirs are at constant temperatures.

$$\dot{W} = \dot{Q}_L / \beta ; \qquad \beta = \dot{Q}_L/\dot{W}_{carnot} = \frac{T_L}{T_H - T_L} = \frac{263.15}{25 - (-10)} = 7.52$$

In general we have defined the reversible work with the standard sign definition

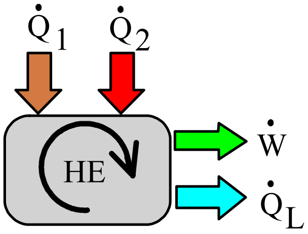
$$\dot{W}^{rev} = -\dot{Q}_L / \beta = -\frac{1.5}{7.52} \text{ kW} = \mathbf{-0.2\ kW}$$

10.19

A heat engine receives 5 kW at 800 K and 10 kW at 1000 K rejecting energy by heat transfer at 600 K. Assume it is reversible and find the power output. How much power could be produced if it could reject energy at $T_o = 298$ K?

Solution:

C.V. The heat engine, this is in steady state.



Energy Eq.: $0 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L - \dot{W}$

Entropy Eq.: $0 = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_L}{T_L} + 0$

Now solve for $\dot{Q}_L$ from the entropy equation

$$\dot{Q}_L = \frac{T_L}{T_1}\dot{Q}_1 + \frac{T_L}{T_2}\dot{Q}_2 = \frac{600}{800} \times 5 + \frac{600}{1000} \times 10 = 9.75 \text{ kW}$$

Substitute into the energy equation and solve for the work term

$$\dot{W} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L = 5 + 10 - 9.75 = \mathbf{5.25\ kW}$$

For a low temperature of 298 K we can get

$$\dot{Q}_{L2} = \frac{298}{600}\dot{Q}_L = 4.843 \text{ kW}$$

$$\dot{W} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_L = 5 + 10 - 4.843 = \mathbf{10.16\ kW}$$

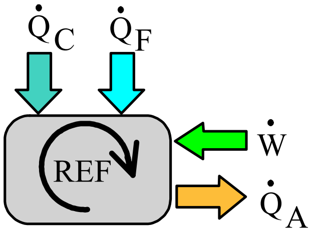
Remark: Notice the large increase in the power output.

10.20

A household refrigerator has a freezer at T_F and a cold space at T_C from which energy is removed and rejected to the ambient at T_A as shown in Fig. P10.27. Assume that the rate of heat transfer from the cold space, $\dot{Q}_C$, is the same as from the freezer, $\dot{Q}_F$, find an expression for the minimum power into the heat pump. Evaluate this power when $T_A = 20°C$, $T_C = 5°C$, $T_F = -10°C$, and $\dot{Q}_F = 3$ kW.

Solution:

C.V. Refrigerator (heat pump), Steady, no external flows except heat transfer.



Energy Eq.: $\dot{Q}_F + \dot{Q}_c + \dot{W}_{in} = \dot{Q}_A$

(amount rejected to ambient)

Reversible gives maximum work (minimum work in) as from Eq. 10.1 or 10.10 on rate form (standard sign notation for work).

$$\dot{W}^{rev} = \dot{Q}_F\left[1 - \frac{T_A}{T_F}\right] + \dot{Q}_c\left[1 - \frac{T_A}{T_C}\right] - \dot{Q}_A\left[1 - \frac{T_A}{T_A}\right]$$

$$= 3\left[1 - \frac{293.15}{263.15}\right] + 3\left[1 - \frac{293.15}{278.15}\right] - 0$$

$$= \mathbf{-0.504\ kW} \qquad \text{(negative so work goes in)}$$

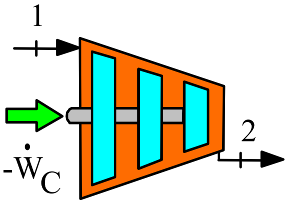
10.21

The compressor in a refrigerator takes refrigerant R-134a in at 100 kPa, –20°C and compresses it to 1 MPa, 40°C. With the room at 20°C find the minimum compressor work.

Solution:

C.V. Compressor out to ambient. Minimum work in is the reversible work.

Steady flow, 1 inlet and 2 exit



Energy Eq.: $w_c = h_1 - h_2 + q^{rev}$

Entropy Eq.: $s_2 = s_1 + \int dq/T + s_{gen} = s_1 + q^{rev}/T_o + 0$

$$\Rightarrow \quad q^{rev} = T_o(s_2 - s_1)$$

$$w_{c\ min} = h_1 - h_2 + T_o(s_2 - s_1)$$

$$= 387.22 - 420.25 + 293.15 \times (1.7148 - 1.7665)$$

$$= \mathbf{-48.19\ kJ/kg}$$

10.22

Find the specific reversible work for a R-134a compressor with inlet state of –20°C, 100 kPa and an exit state of 600 kPa, 50°C. Use a 25°C ambient temperature.

Solution:

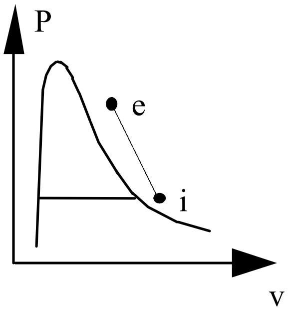
This is a steady state flow device for which the reversible work is given by Eq.10.9. The compressor is also assumed to be adiabatic so $q = 0$

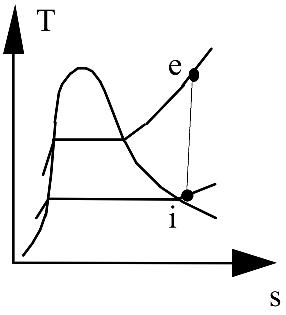
$$w^{rev} = T_o(s_e - s_i) - (h_e - h_i)$$

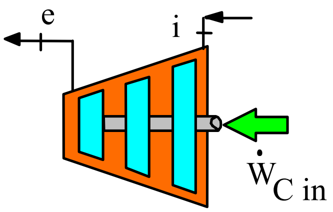
Table B.5.2: $h_i = 387.22$ kJ/kg; $s_i = 1.7665$ kJ/kg K

$h_e = 438.59$ kJ/kg; $s_e = 1.8084$ kJ/kg K

$w^{rev} = 298.15\ (1.8084 - 1.7665) - (438.59 - 387.22) =$ **-38.878 kJ/kg**





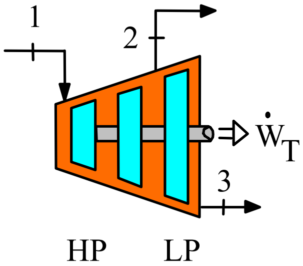


10.23

Calculate the reversible work out of the two-stage turbine shown in Problem 6.80, assuming the ambient is at 25°C. Compare this to the actual work which was found to be 18.08 MW.

C.V. Turbine. Steady flow, 1 inlet and 2 exits.

Use Eq. 10.12 for each flow stream with q = 0 for adiabatic turbine.



Supply state 1: 20 kg/s at 10 MPa, 500°C

Process steam 2: 5 kg/s, 0.5 MPa, 155°C,

Exit state 3: 20 kPa, x = 0.9

Table B.1.3: $h_1 = 3373.7$, $h_2 = 2755.9$ kJ/kg,

$s_1 = 6.5966$, $s_2 = 6.8382$ kJ/kg K

Table B.1.2: $h_3 = 251.4 + 0.9 \times 2358.3 = 2373.9$ kJ/kg,

$s_3 = 0.8319 + 0.9 \times 7.0766 = 7.2009$ kJ/kg K

$$\dot{W}^{rev} = (\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3) - T_0(\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3)$$

$$= 20 \times 3373.7 - 5 \times 2755.9 - 15 \times 2373.9$$

$$- 298.15\ (20 \times 6.5966 - 5 \times 6.8382 - 15 \times 7.2009)$$

$$= \mathbf{21.14\ MW} = \dot{W}^{ac} + \dot{Q}^{rev} = 18084 \text{ kW} + 3062.7 \text{ kW}$$

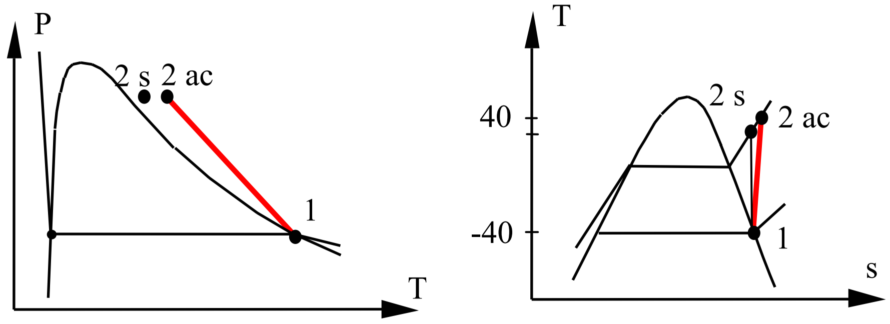
10.24

A compressor in a refrigerator receives R-410a at 150 kPa, -40^{o}C and it brings it up to 600 kPa, 40^{o}C in an adiabatic compression. Find the specific reversible work.

States: 1: B.4.2 $h_1 = 263.99$ kJ/kg, $s_1 = 1.1489$ kJ/kg K

2: B.4.2 $h_2 = 322.64$ kJ/kg, $s_2 = 1.2152$ kJ/kg K

$$w^{rev} = \psi_1 - \psi_2 = h_1 - h_2 - T_0 (s_1 - s_2)$$
$$= 263.99 - 322.64 - 298.15 (1.1489 - 1.2152)$$
$$= -58.65 + 19.767 = -38.9 \text{ kJ/kg}$$



10.25

An air compressor takes air in at the state of the surroundings 100 kPa, 300 K. The air exits at 400 kPa, 200°C at the rate of 2 kg/s. Determine the minimum compressor work input.

C.V. Compressor, Steady flow, minimum work in is reversible work.

$\psi_1 = 0$ at ambient conditions

Get the properties from the air table A.7.1 and correct standard entropy for the pressure

$$s_0 - s_2 = s^{o}_{T_0} - s^{o}_{T_2} - R\ \ln(P_0/P_2)$$

$$= 6.86926 - 7.3303 - 0.287\ \ln(100/400) = -0.06317\ \text{kJ/kg K}$$

$$\psi_2 = h_2 - h_0 + T_0(s_0 - s_2) = 475.79 - 300.473 + 300\ (-0.06317)$$

$$= 156.365\ \text{kJ/kg}$$

$$\mathbf{-\dot{W}^{REV}} = \dot{m}(\psi_2 - \psi_1) = 2\ \text{kg/s}\ (156.365 – 0)\ \text{kJ/kg} = \mathbf{312.73\ kW} = \mathbf{\dot{W}_c}$$

10.26

Find the specific reversible work for a steam turbine with inlet 4 MPa, 500°C and an actual exit state of 100 kPa, x = 1.0 with a 25°C ambient.

Solution:

Steam turbine $T_o = 25°C = 298.15$ K

Inlet state: Table B.1.3 $h_i = 3445.2$ kJ/kg; $s_i = 7.090$ kJ/kg K

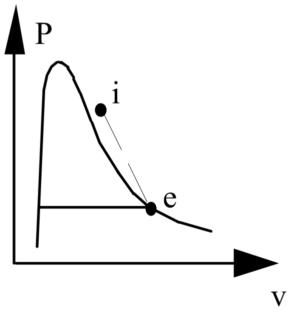
Exit state: Table B.1.2 $h_e = 2675.5$ kJ/kg; $s_e = 7.3593$ kJ/kg K

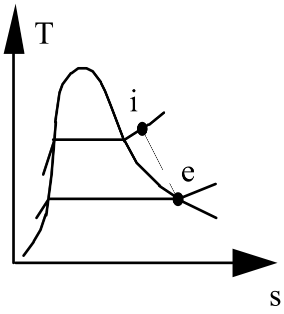
From Eq.9.39,

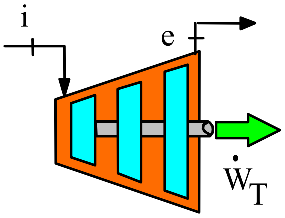
$$w^{rev} = (h_i - T_o s_i) - (h_e - T_o s_e) = (h_i - h_e) + T_o(s_e - s_i)$$

$$= (3445.2 - 2675.5) + 298.2(7.3593 - 7.0900)$$

$$= 769.7 + 80.3 = \mathbf{850.0\ kJ/kg}$$







10.27

A steam turbine receives steam at 6 MPa, 800°C. It has a heat loss of 49.7 kJ/kg and an isentropic efficiency of 90%. For an exit pressure of 15 kPa and surroundings at 20°C, find the actual work and the reversible work between the inlet and the exit.

C.V. Reversible adiabatic turbine (isentropic)

$w_T = h_i - h_{e,s}$; $s_{e,s} = s_i = 7.6566$ kJ/kg K, $h_i = 4132.7$ kJ/kg

$x_{e,s} = (7.6566 - 0.7548)/7.2536 = 0.9515$,

$h_{e,s} = 225.91 + 0.9515\times2373.14 = 2483.9$ kJ/kg

$w_{T,s} = 4132.7 - 2483.9 = 1648.79$ kJ/kg

C.V. Actual turbine

$w_{T,ac} = \eta w_{T,s} =$ **1483.91 kJ/kg**

$= h_i - h_{e,ac} - q_{loss} \Rightarrow$

$h_{e,ac} = h_i - q_{loss} - w_{T,ac} = 4132.7 - 49.7 - 1483.91 = 2599.1$ kJ/kg

Actual exit state: P, h $\Rightarrow$ saturated vapor, $s_{e,ac} = 8.0085$ kJ/kg K

C.V. Reversible process, work from Eq.10.12

$$q^R = T_0(s_{e,ac} - s_i) = 293.15 \times (8.0085 - 7.6566) = 103.16 \frac{kJ}{kg}$$

$w^R = h_i - h_{e,ac} + q^R = 4132.7 - 2599.1 + 103.16 =$ **1636.8 kJ/kg**

10.28

An air flow of 5 kg/min at 1500 K, 125 kPa goes through a constant pressure heat exchanger, giving energy to a heat engine shown in Figure P10.35. The air exits at 500 K and the ambient is at 298 K, 100 kPa. Find the rate of heat transfer delivered to the engine and the power the engine can produce.

Solution:

C.V. Heat exchanger

Continuity eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.6.12: $\dot{m}_1 h_1 = \dot{m}_1 h_2 + \dot{Q}_H$

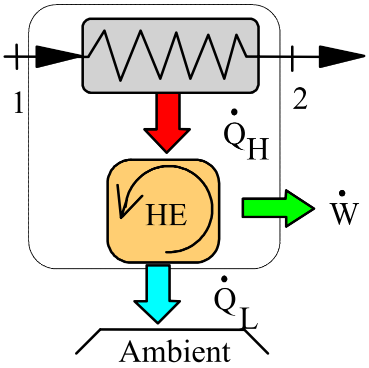


Table A.7.1: $h_1 = 1635.8$ kJ/kg,

$h_2 = 503.36$ kJ/kg, $s_1 = 8.61209$ kJ/kg K

$s_2 = 7.38692$ kJ/kg K

$$\dot{Q}_H = \dot{m}(h_1 - h_2) = \frac{5}{60}\frac{\text{kg}}{\text{s}}(1635.8 - 503.36)\frac{\text{kJ}}{\text{kg}} = \mathbf{94.37\ kW}$$

Notice T_H is not constant so we do not know the heat engine efficiency.

C.V. Total system for which we will write the second law.

Entropy Equation 9.8: $\dot{m}\, s_1 + \dot{S}_{gen} = \dot{m}\, s_2 + \dot{Q}_L/T_o$

Process: Assume reversible $\dot{S}_{gen} = 0$, and $P = C$ for air

$$\dot{Q}_L = T_o\, \dot{m}\, (s_1 - s_2) = 298\ \text{K}\ \frac{5}{60}\frac{\text{kg}}{\text{s}}(8.61209 - 7.38692)\frac{\text{kJ}}{\text{kg K}}$$

$$= 30.425\ \text{kW}$$

Energy equation for the heat engine gives the work as

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = 94.37 - 30.425 = \mathbf{63.9\ kW}$$

10.29

Water at 1000^{o}C, 15 MPa is flowing through a heat exchanger giving off energy to come out as saturated liquid water at 15 MPa in a steady flow process. Find the specific heat transfer and the specific flow-availability (exergy) the water has delivered.

Let us fix the states first from Table B.1.2 and B.1.3:

State 1 (in): $h = 4596.6$ kJ/kg, $s = 7.6347$ kJ/kg K

State 2 (ex): $h = 1610.45$ kJ/kg, $s = 3.6847$ $T = 342.24^oC = 615.4$ K

CV Heat exchanger, no work.

Energy Eq.: $q_{out} = h_{in} - h_{ex} = 4596.6 - 1610.45 =$ **2986.15 kJ/kg**

Flow availability: $\Delta\psi = \psi_{in} - \psi_{ex} = (h - T_o s)_{in} - (h - T_o s)_{ex}$

$= q_{out} - T_o(s_{in} - s_{ex})$

$= 2986.15 - 298.15(7.6347 - 3.6847)$

= **1808.5 kJ/kg**

10.30

An air compressor receives atmospheric air at T_0 = 17°C, 100 kPa, and compresses it up to 1400 kPa. The compressor has an isentropic efficiency of 88% and it loses energy by heat transfer to the atmosphere as 10% of the isentropic work. Find the actual exit temperature and the reversible work.

C.V. Compressor

Isentropic: $w_{c,in,s} = h_{e,s} - h_i$; $s_{e,s} = s_i$

From table A.7.1 and entropy equation we get

$$s^o_{Te\,s} = s^o_{Ti} + R \ln (P_e/P_i) = 6.83521 + 0.287 \ln(14) = 7.59262 \text{ kJ/kgK}$$

Back interpolate in Table A.7: $\Rightarrow$ $h_{e,s}$ = 617.23 kJ/kg

$$w_{c,in,s} = 617.23 - 290.43 = 326.8 \text{ kJ/kg}$$

Actual: $w_{c,in,ac} = w_{c,in,s}/\eta_c$ = 371.36 kJ/kg; q_{loss} = 32.68 kJ/kg

$$w_{c,in,ac} + h_i = h_{e,ac} + q_{loss}$$

$$\Rightarrow h_{e,ac} = 290.43 + 371.36 - 32.68 = 629.1 \text{ kJ/kg}$$

$$\Rightarrow T_{e,ac} = \mathbf{621\ K}$$

Reversible: $w^{rev} = h_i - h_{e,ac} + T_0(s_{e,ac} - s_i)$

$$= 290.43 - 629.1 + 290.15 \times (7.6120 - 6.8357)$$

$$= -338.67 + 225.38 = \mathbf{-113.3\ kJ/kg}$$

Since q_{loss} is also to the atmosphere it is the net q exchanged with the ambient that explains the change in s.

10.31

Air flows through a constant pressure heating device, shown in Fig. P10.32. It is heated up in a reversible process with a work input of 200 kJ/kg air flowing. The device exchanges heat with the ambient at 300 K. The air enters at 300 K, 400 kPa. Assuming constant specific heat develop an expression for the exit temperature and solve for it by iterations.

C.V. Total out to T_0

Energy Eq.: $h_1 + q_0^{rev} - w^{rev} = h_2$

Entropy Eq.: $s_1 + q_0^{rev}/T_0 = s_2 \quad \Rightarrow \quad q_0^{rev} = T_0(s_2 - s_1)$

$h_2 - h_1 = T_0(s_2 - s_1) - w^{rev}$ (same as Eq. 10.12)

Constant C_p gives: $C_p(T_2 - T_1) = T_0C_p \ln (T_2/T_1) + 200$

The energy equation becomes

$$\mathbf{T_2 - T_0 \ln(\frac{T_2}{T_1}) = T_1 + \frac{200}{C_p}}$$

$T_1 = 300$ K, $C_p = 1.004$ kJ/kg K, $T_0 = 300$ K

$$T_2 - 300 \ln\left(\frac{T_2}{300}\right) = 300 + \frac{200}{1.004} = 499.3 \text{ K}$$

Now trial and error on T_2

At 600 K LHS = 392 (too low)

At 800 K LHS = 505.75

Linear interpolation gives $\mathbf{T_2 = 790\ K}$ (LHS = 499.5 **OK**)

10.32

A rock bed consists of 6000 kg granite and is at 70°C. A small house with lumped mass of 12000 kg wood and 1000 kg iron is at 15°C. They are now brought to a uniform final temperature with no external heat transfer by connecting the house and rock bed through some heat engines. If the process is reversible, find the final temperature and the work done in the process.

Solution:

Take C.V. Total (rockbed and heat engine)

Energy Eq.: $m_{rock}(u_2 - u_1) + m_{wood}(u_2 - u_1) + m_{Fe}(u_2 - u_1) = -{}_1W_2$

Entropy Eq.: $m_{rock}(s_2 - s_1) + m_{wood}(s_2 - s_1) + m_{Fe}(s_2 - s_1) = 0$

$$(mC)_{rock}\ln\frac{T_2}{T_1} + (mC)_{wood}\ln\frac{T_2}{T_1} + (mC)_{Fe}\ln\frac{T_2}{T_1} = 0$$

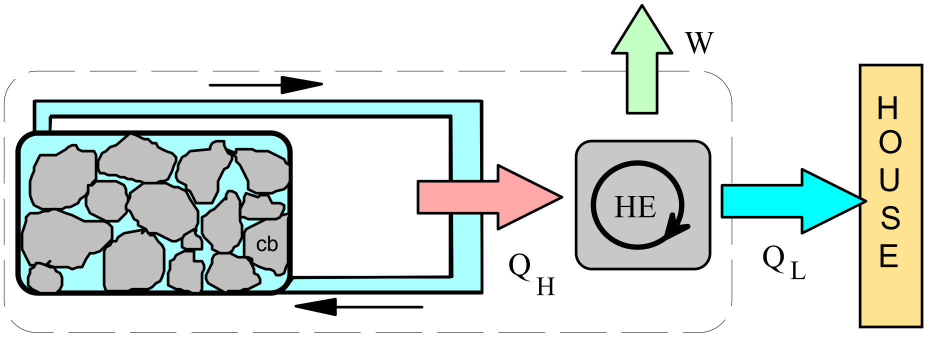
$$6000 \times 0.89 \ln (T_2/343.15) + 12000 \times 1.26 \ln (T_2/288.15)$$
$$+ 1000 \times 0.46 \ln (T_2/288.15) = 0$$

=> $\mathbf{T_2 = 301.3\ K}$

Now from the energy equation

$$-{}_1W_2 = 6000 \times 0.89(301.3 - 343.15)$$
$$+ (12000 \times 1.26 + 460)(301.3 - 288.15)$$

$\Rightarrow {}_1W_2 =$ **18 602 kJ**

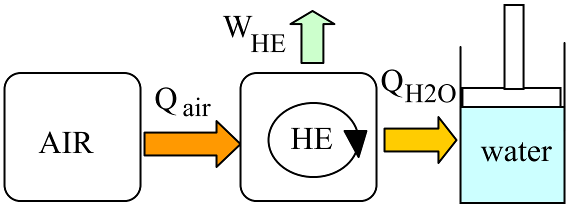


10.33

A constant pressure piston/cylinder has 1 kg of saturated liquid water at 100 kPa. A rigid tank contains air at 1000 K, 1000 kPa. They are now thermally connected by a reversible heat engine cooling the air tank and boiling the water to saturated vapor. Find the required amount of air and the work out of the heat engine.

CV. Tank, piston/cyl. and the heat engine.

The minimum amount of air is when T_2 of air equals water T = 99.62 C.



State: 1 = initial air
2 = final air
3 = initial water
4 = final water

For this CV only W_{HE} and W_{pist} cross the control surface no heat transfer. The entropy equation Eq.8.14 for a reversible process becomes

$$(S_2 - S_1)_{tot} = m_{air}(s_2 - s_1) + m_{H2O}(s_4 - s_3) = 0 + 0$$

$$m_{H2O}(s_4 - s_3) = m_{H2O}\, s_{fg} = 1 \text{ kg} \times 6.0568 \text{ kJ/kg-K} = 6.0568 \text{ kJ/K}$$

$$s_2 - s_1 = C_v \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} = C_v \ln\frac{T_2}{T_1} = 0.717 \ln\frac{372.77}{1000} = -0.70753 \frac{\text{kJ}}{\text{kg-K}}$$

Now solve for the air mass from entropy equation

$$m_{air} = \mathbf{-}m_{H2O}(s_4 - s_3) / (s_2 - s_1) = -\frac{6.0568}{-0.70753} = \mathbf{8.5605\ kg}$$

Let us find the two heat transfers

Energy Eq, water: $m_{H2O}(u_4 - u_3) = Q_{H2O} - W_{H2O}$; $W_{H2O} = Pm(v_4 - v_3)$

$$Q_{H2O} = m_{H2O}(u_4 - u_3) + W_{H2O} = m_{H2O}(h_4 - h_3) = m_{H2O}\, h_{fg} = 2258.02 \text{ kJ}$$

Energy Eq. air: $m_{air}(u_2 - u_1) = -Q_{air} - 0 = m_{air}\, C_v\, (T_2 - T_1)$

$$Q_{air} = 8.5605 \text{ kg } 0.717\ (1000 - 372.77) = 3849.86 \text{ kJ}$$

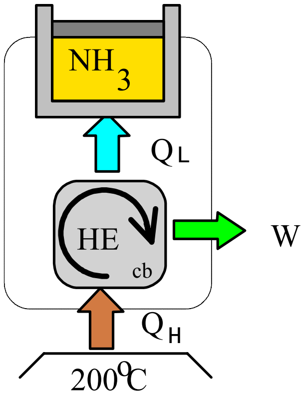
Now the work out of the heat enguine is

$$W_{HE} = Q_{air} - Q_{H2O} = 3849.86 - 2258.02 = \mathbf{1591.84\ kJ}$$

10.34

A piston/cylinder has forces on the piston so it keeps constant pressure. It contains 2 kg of ammonia at 1 MPa, 40°C and is now heated to 100°C by a reversible heat engine that receives heat from a 200°C source. Find the work out of the heat engine.

C.V. Ammonia plus heat engine



Energy: $m_{am}(u_2 - u_1) = {}_1Q_{2,200} - W_{H.E.} - {}_1W_{2,pist}$

Entropy: $m_{am}(s_2 - s_1) = {}_1Q_2/T_{res} + 0$

$\Rightarrow \quad {}_1Q_2 = m_{am}(s_2 - s_1)T_{res}$

Process: $P = \text{const.} \Rightarrow {}_1W_2 = P(v_2 - v_1)m_{am}$

Substitute the piston work term and heat transfer into the energy equation

$W_{H.E.} = m_{am}(s_2 - s_1)T_{res} - m_{am}(h_2 - h_1)$

Table B.2.2: $h_1 = 1508.5$ kJ/kg, $s_1 = 5.1778$ kJ/kg K,

$h_2 = 1664.3$ kJ/kg, $s_2 = 5.6342$ kJ/kg K

$$W_{H.E.} = 2 \times \left[(5.6342 - 5.1778)473.15 - (1664.3 - 1508.5)\right] = \mathbf{120.3\ kJ}$$

10.35

A basement is flooded with 6 m^3 of water at 15°C. It is pumped out with a small pump driven by a 0.75 kW electric motor. The hose can reach 8 m vertically up and to ensure the water can flow over the edge of a dike it should have a velocity of 15 m/s at that point generated by a nozzle, see Fig. P10.35. Find the maximum flow rate you can get and how fast the basement can be emptied.

C.V. Pump plus hose and nozzle, single steady state flow. For maximum flow rate assume reversible process:

$$w^{rev} = T_o (s_2 - s_1) - (h_{2\ tot} - h_{1\ tot}) + 0 \qquad (10.9)$$

Since we have no heat transfer and reversible s is constant and with a liquid flow T is also constant so $h_2 = h_1$. We could also have used Bernoulli equation.

$$w^{rev} = -\left(\frac{1}{2}V_2^2 + gH - 0\right) = -\frac{1}{2}\,15^2 + 9.81 \times 8 = -191 \text{ J/kg}$$

$$\dot{m} = \dot{W}/(-w^{rev}) = \frac{0.75 \text{ kW}}{0.191 \text{ kJ/kg}} = \mathbf{3.925\ kg/s}$$

$$m = V/v = 6 \text{ m}^3 / 0.001 \text{ m}^3\text{/kg} = 6000 \text{ kg}$$

$$\Delta t = m / \dot{m} = \frac{6000 \text{ kg}}{3.925 \text{ kg/s}} = 1528 \text{ sec} = \mathbf{25.5\ min.}$$

Irreversibility

10.36

A 20°C room is heated with a 1500 W electric baseboard heater. What is the rate of irreversibility?

Let us not consider any storage in the room or any heat loss but just the effect of the heating process. Electric power comes in to the heater and leaves as heat transfer to the room air.

C.V. heater

Entropy Eq.: $0 = -\frac{\dot{Q}}{T_{air}} + \dot{S}_{gen} \quad \Rightarrow \quad \dot{S}_{gen} = \frac{\dot{Q}}{T_{air}}$

Irreversibility: $\dot{I} = T_o\,\dot{S}_{gen} = 293.15\text{ K} \times \frac{1500\text{ W}}{293.15\text{ K}} = \mathbf{1500\ W}$

10.37

A refrigerator removes 1.5 kW from the cold space at -10°C using 750 W of power input while it rejects heat to the kitchen at 25°C. Find the rate of irreversibility.

C.V. Refrigerator

Energy Eq.: $0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_H = \dot{Q}_L + \dot{W}$

Entropy Eq.: $0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{gen} \quad \Rightarrow \quad \dot{S}_{gen} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L}$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 1.5 \text{ kW} + 0.75 \text{ kW} = 2.25 \text{ kW}$$

$$\dot{S}_{gen} = \frac{2250 \text{ W}}{298.15 \text{ K}} - \frac{1500 \text{ W}}{263.15 \text{ K}} = 1.8464 \text{ W/K}$$

Irreversibility: $\dot{I} = T_o \dot{S}_{gen} = 298.15 \text{ K} \times 1.8464 \text{ W/K} = \mathbf{550\ W}$

10.38

Calculate the irreversibility for the condenser in Problem 9.89 assuming an ambient temperature at 17°C.

Solution:

C.V. Condenser. Steady state with no shaft work term.

Energy Equation 6.12: $\dot{m}\, h_i + \dot{Q} = \dot{m} h_e$

Entropy Equation 9.8: $\dot{m}\, s_i + \dot{Q}/T + \dot{S}_{gen} = \dot{m}\, s_e$

Properties are from Table B.1.2

$h_i = 225.91 + 0.9 \times 2373.14 = 2361.74$ kJ/kg , $h_e = 225.91$ kJ/kg

$s_i = 0.7548 + 0.9 \times 7.2536 = 7.283$ kJ/kg K, $s_e = 0.7548$ kJ/kg K

From the energy equation

$\dot{Q}_{out} = -\dot{Q} = \dot{m}\,(h_i - h_e) = 5(2361.74 - 225.91) = 10679$ kW

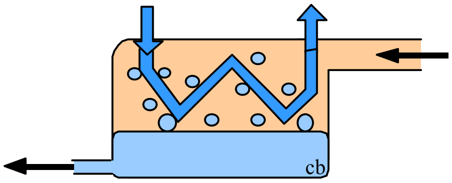
From the entropy equation

$\dot{S}_{gen} = \dot{m}\,(s_e - s_i) + \dot{Q}_{out}/T = 5(0.7548 - 7.283) + 10679/(273.15 + 17)$

$= -32.641 + 36.805 = 4.164$ kW/K

From Eq.10.11 times $\dot{m}$,

$\dot{I} = T_o\, \dot{S}_{gen} = 290.15 \times 4.164 =$ **1208 kW**



Often the cooling
media flows inside a
long pipe carrying
the energy away.

10.39

The throttle process in Example 6.5 is an irreversible process. Find the reversible work and irreversibility assuming an ambient temperature at 25°C.

Solution:

C.V. Throttle. Steady state, adiabatic $q = 0$ and no shaft work $w = 0$.

Inlet state: B.2.1 $h_i = 346.8$ kJ/kg; $s_i = 1.2792$ kJ/kg K

Energy Eq.6.13: $h_e = h_i$

Exit state: B.2.1 $P = 291$ kPa, $h_e = h_i$ which is two-phase

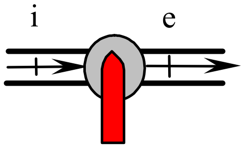
$$s_e = s_f + x s_{fg} = 0.5408 + 0.1638 \times 4.9265 = 1.3478 \text{ kJ/kg K}$$

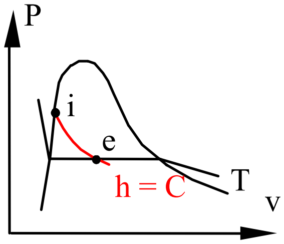
The reversible work is the difference in availability also equal to the expression in Eq.10.9 or 10.36 and 10.37

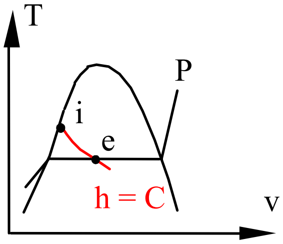
$$w^{rev} = \psi_i - \psi_e = (h_i - T_o s_i) - (h_e - T_o s_e) = (h_i - h_e) + T_o(s_e - s_i)$$

$$= 0 + 298.15\ (1.2792 - 1.3478) = \mathbf{20.45\ kJ/kg}$$

$$i = w^{rev} - w = 20.45 - 0 = \mathbf{20.45\ kJ/kg}$$







10.40

A compressor in a refrigerator receives R-410a at 150 kPa, -40°C and it brings it up to 600 kPa, 40°C in an adiabatic compression. Find the specific work, entropy generation and irreversibility.

States: 1: B.4.2 $h_1 = 263.99$ kJ/kg, $s_1 = 1.1489$ kJ/kg K

2: B.4.2 $h_2 = 322.64$ kJ/kg, $s_2 = 1.2152$ kJ/kg K

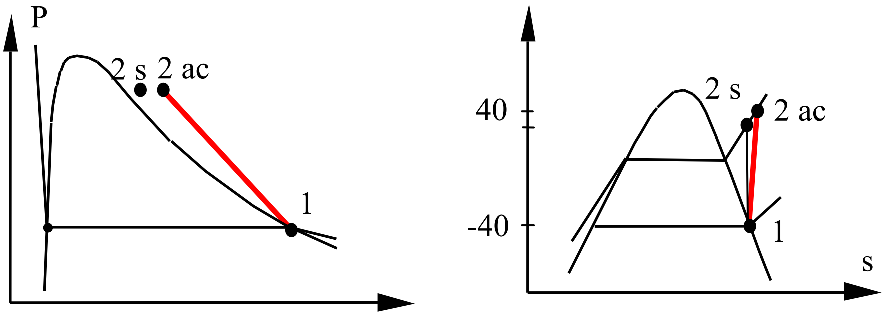
C.V. Compressor, steady state, q = 0, no KE, PE energies.

Energy Eq.: $w = h_1 - h_2 = 263.99 - 322.64 =$ **–58.65 kJ/kg**

Entropy Eq.: $0 = s_1 - s_2 + s_{gen}$ ⇨

$s_{gen} = s_2 - s_1 = 1.2152 - 1.1489 =$ **0.0663 kJ/kg-K**

Irreversibility: $i = T_o\, s_{gen} = 298.15\text{ K} \times 0.0663\text{ kJ/kg-K} =$ **19.77 kJ/kg**

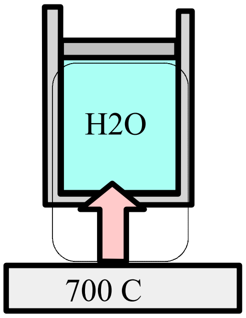


10.41

A constant pressure piston/cylinder contains 2 kg of water at 5 MPa and 100°C. Heat is added from a reservoir at 700°C to the water until it reaches 700°C. We want to find the total irreversibility in the process.

Solution:

C.V. Piston cylinder out to the reservoir (incl. the walls).



Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m(s_2 - s_1) = {}_1Q_2/T_{res} + {}_1S_{2\,gen}$

State 1: $h_1 = 422.71$ kJ/kg, $s_1 = 1.303$ kJ/kg K

State 2: $h_2 = 3900.13$ kJ/kg, $s_2 = 7.5122$ kJ/kg K

Process: $P = C \quad \Rightarrow \quad {}_1W_2 = P(V_2 - V_1)$

From the energy equation we get

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2(3900.13 - 422.71) = 6954.8 \text{ kJ}$$

From the entropy equation we get

$${}_1S_{2\,gen} = m(s_2 - s_1) - \frac{{}_1Q_2}{T_{res}} = 2(7.5122 - 1.303) - \frac{6954.8}{273 + 700} = 5.2717 \frac{\text{kJ}}{\text{K}}$$

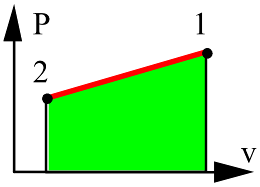
Now the irreversibility is from Eq. 10.19

$${}_1I_2 = m\ {}_1i_2 = T_o\ {}_1S_{2\,gen} = 298.15 \text{ K} \times 5.2717 \frac{\text{kJ}}{\text{K}} = \mathbf{1572\ kJ}$$

10.42

Calculate the reversible work and irreversibility for the process described in Problem 5.114, assuming that the heat transfer is with the surroundings at 20°C.

Solution:



Linear spring gives

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

Equation of state: PV = mRT

State 1: $V_1 = mRT_1/P_1 = 2 \times 0.1889 \times 673.15 /500 = 0.5087 \text{ m}^3$

State 2: $V_2 = mRT_2/P_2 = 2 \times 0.1889 \times 313.15 /300 = 0.3944 \text{ m}^3$

$${}_1W_2 = \frac{1}{2}(500 + 300)(0.3944 - 0.5087) = -45.72 \text{ kJ}$$

From Figure 5.11: $C_p(T_{avg}) = 5.25 \text{ R} = 0.99 \Rightarrow C_v = 0.803 = C_p - R$

For comparison the value from Table A.5 at 300 K is $C_v = 0.653$ kJ/kg K

$${}_1Q_2 = mC_v(T_2 - T_1) + {}_1W_2 = 2 \times 0.803(40 - 400) - 45.72 = -623.9 \text{ kJ}$$

$${}_1W_2^{rev} = T_o(S_2 - S_1) - (U_2 - U_1) + {}_1Q_2\,(1 - T_o/T_H)$$

$$= T_o m(s_2 - s_1) + {}_1W_2^{ac} - {}_1Q_2\, T_o/T_o$$

$$= T_o m[C_P \ln(T_2 / T_1) - R \ln(P_2 / P_1)] + {}_1W_2^{ac} - {}_1Q_2$$

$$= 293.15 \times 2\, [\, 0.99 \ln(313/673) - 0.1889 \ln(300/500)] - 45.72 + 623.9$$

$$= -387.8 - 45.72 + 623.9 = \mathbf{190.4\ kJ}$$

$${}_1I_2 = {}_1W_2^{rev} - {}_1W_2^{ac} = 190.4 - (-45.72) = \mathbf{236.1\ kJ}$$

10.43

A constant "flow" of steel parts 2 kg/s at 20$^{\mathrm{o}}$C goes into a furnace where they are heat treated to 900$^{\mathrm{o}}$C by a source at an average 1250 K. Find the reversible work and the irreversibility in this process?

Energy Eq.: $0 = \dot{m}(h_i - h_e) + \dot{Q}$

Entropy Eq.: $0 = \dot{m}(s_i - s_e) + \dot{Q} / T_s + \dot{S}_{gen}$

$$\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_P\,(T_e - T_i) = 2\text{ kg/s} \times 0.46\text{ kJ/kg-K} \times 880\text{ K}$$
$$= 809.6\text{ kW}$$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q} / T_s = \dot{m}C_P \ln(T_e/T_i) - \dot{Q} / T_s$$
$$= 2\text{ kg/s} \times 0.46\text{ kJ/kg-K} \times \ln(1173 / 293) - 809.6\text{ kW} / 1250\text{ K}$$
$$= 0.6285\text{ kW/K}$$

$\dot{I} = T_0\dot{S}_{gen} = 293\text{ K} \times 0.6285\text{ kW/K} =$ **184.15 kW**

The reversible work is equal to the irreversibility plus the actual work, Eq.10.12

$\dot{W}^{rev} = \dot{I} + \dot{W}_{ac} = \dot{I} =$ **184.15 kW**

10.44

Fresh water can be produced from saltwater by evaporation and subsequent condensation. An example is shown in Fig. P10.44, where 150-kg/s saltwater, state 1, comes from the condenser in a large power plant. The water is throttled to the saturated pressure in the flash evaporator and the vapor, state 2, is then condensed by cooling with sea water. As the evaporation takes place below atmospheric pressure, pumps must bring the liquid water flows back up to P_0. Assume that the saltwater has the same properties as pure water, the ambient is at 20°C and that there are no external heat transfers. With the states as shown in the table below find the irreversibility in the throttling valve and in the condenser.

State	1	2	3	4	5	6	7	8
T [°C]	30	25	25	--	23	--	17	20
h [kJ/kg]	125.77	2547.2			96.5		71.37	83.96
s [kJ/kg K]	0.4369	8.558			0.3392		0.2535	0.2966

C.V. Valve. $P_2 = P_{sat}(T_2 = T_3) = 3.169$ kPa

Continuity Eq.: $\dot{m}_1 = m_{ex} = \dot{m}_2 + \dot{m}_3$

Energy Eq.: $h_1 = h_e$; Entropy Eq.: $s_1 + s_{gen} = s_e$

$h_e = h_1 \Rightarrow x_e = (125.77 - 104.87)/2442.3 = 0.008558$

$\Rightarrow s_e = 0.3673 + 0.008558 \times 8.1905 = 0.4374$ kJ/kg K

$\dot{m}_2 = (1 - x_e)\dot{m}_1 = 148.716$ kg/s

$s_{gen} = s_e - s_1 = 0.4374 - 0.4369 = 0.000494$ kJ/kg K

$\dot{I} = \dot{m}T_0s_{gen} = 150 \times 293.15 \times 0.000494 =$ **21.72 kW**

C.V. Condenser.

Energy Eq.: $\dot{m}_2h_2 + \dot{m}_7h_7 = \dot{m}_2h_5 + \dot{m}_7h_8 \Rightarrow$

$$\dot{m}_7 = \dot{m}_2 \times (h_2 - h_5)/(h_8 - h_7) = 148.716 \times \frac{2547.2 - 96.5}{83.96 - 71.37} = 28\,948 \frac{\text{kg}}{\text{s}}$$

Entropy Eq.: $\dot{m}_2s_2 + \dot{m}_7s_7 + \dot{S}_{gen} = \dot{m}_2s_5 + \dot{m}_7s_8$

$\dot{I} = T_0\dot{S}_{gen} = T_0 \left[\dot{m}_2(s_5 - s_2) + \dot{m}_7(s_8 - s_7)\right]$

$= 293.15[148.716(0.3392 - 8.558) + 28948(0.2966 - 0.2535)]$

$= 293.15 \times 25.392 =$ **7444 kW**

10.45

Two flows of air both at 200 kPa of equal flow rates mix in an insulated mixing chamber. One flow is at 1500 K and the other is at 300 K. Find the irreversibility in the process per kilogram of air flowing out.

C.V. Mixing chamber

Continuity Eq..: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2\,\dot{m}_1$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_1 h_2 = 2\,\dot{m}_1 h_3$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{m}_1 s_2 + \dot{S}_{gen} = 2\,\dot{m}_1 s_3$

Properties from Table A.7

$h_3 = (h_1 + h_2)/2 = (300.47 + 1635.8)/2 = 968.14$ kJ/kg

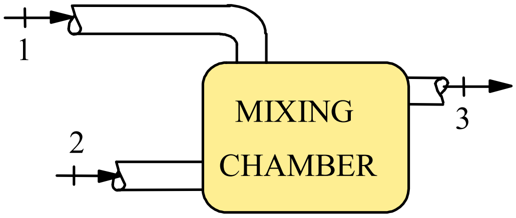
$\Rightarrow\ s^{o}_{T3} = 8.0536$ kJ/kg K (linear interpolation)

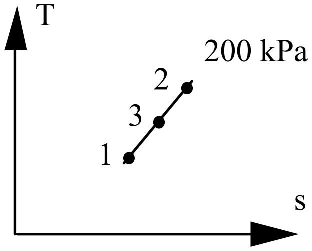
From the entropy equation (same P so no correction for that)

$\dot{S}_{gen}/2\dot{m}_1 = s_3 - (s_1 + s_2)/2 = 8.0536 - (6.86926 + 8.61208)/2$

$= 0.3129$ kJ/kg K

$i = \dot{I}/2\dot{m}_1 = T_0\,\dot{S}_{gen}/2\dot{m}_1 = 298.15 \times 0.3129 =$ **93.29 kJ/kg**





10.46

A computer CPU chip consists of 50 g silicon, 20 g copper, and 50g polyvinyl chloride (plastic). It now heats from ambient 25^{o}C to 70^{o}C in an adiabatic process as the computer is turned on. Find the amount of irreversibility.

C.V. CPU chip. The process has electrical work input and no heat transfer.

Entropy Eq.: $S_2 - S_1 = \sum m_i(s_2 - s_1)_i = \int dQ/T + {}_1S_{2\ gen} = {}_1S_{2\ gen}$

Irreversibility: ${}_1I_2 = T_0\ {}_1S_{2\ gen}$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

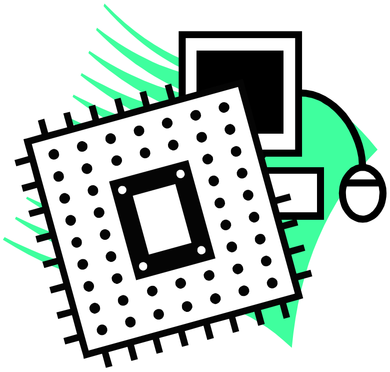
$$\sum m_i(s_2 - s_1)_i = \sum m_iC_i \ln(T_2 / T_1)_i = \ln (T_2/T_1) \sum m_iC_i$$

$$\sum m_iC_i = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$${}_1S_{2\ gen} = S_2 - S_1 = 0.0914 \times \ln (343.15 / 288.15) = 0.016 \text{ kJ/K}$$

$${}_1I_2 = T_0\ {}_1S_{2\ gen} = 298.15 \text{ K} \times 0.016 \text{ kJ/K}$$

= **4.77 kJ**



10.47

R-134a is filled into an insulated 0.2 m^3 initially empty container from a line at 500 kPa, saturated vapor until the flow stops by itself. Find the final mass and temperature in the container and the total irreversibility in the process.

Solution:

C.V. Cannister filling process where: ${}_1Q_2 = 0$; ${}_1W_2 = 0$; $m_1 = 0$

Continuity Eq.6.15: $m_2 - 0 = m_{in}$;

Energy Eq.6.16: $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0 \quad \Rightarrow \quad u_2 = h_{line}$

Entropy Eq.9.11: $m_2s_2 - 0 = m_{in}s_{line} + 0 + {}_1S_{2\ gen}$

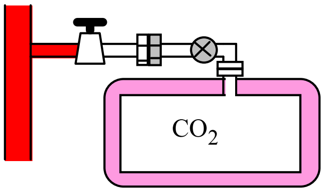
Inlet state: Table B.5.1 $h_{line} = 407.45$ kJ/kg, $s_{line} = 1.7198$ kJ/kg K

State 2: $P_2 = P_{line}$ and $u_2 = h_{line} = 407.45 \text{ kJ/kg} > u_g \Rightarrow T_2 = \mathbf{40°C}$

$v_2 = 0.04656 \text{ m}^3\text{/kg}, \quad s_2 = 1.7971 \text{ kJ/kg K}$

$m_2 = V / v_2 = 0.2/0.04656 = \mathbf{4.296\ kg}$

${}_1S_{2\ gen} = m_2(s_2 - s_{line}) = 4.296\ (1.7971 – 1.7198) = \mathbf{0.332\ kJ/K}$



10.48

Air enters the turbocharger compressor (see Fig. P10.45), of an automotive engine at 100 kPa, 30°C, and exits at 170 kPa. The air is cooled by 50°C in an intercooler before entering the engine. The isentropic efficiency of the compressor is 75%. Determine the temperature of the air entering the engine and the irreversibility of the compression-cooling process.

Solution:

a) Compressor. First ideal which is reversible adiabatic, constant s:

$$T_{2S} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 303.2\left(\frac{170}{100}\right)^{0.286} = 352.9 \text{ K}$$

$$w_S = C_{P0}(T_1 - T_{2S}) = 1.004(303.2 - 352.9) = -49.9 \text{ kJ/kg}$$

Now the actual compressor

$$w = w_S/\eta_S = -49.9/0.75 = -66.5 \text{ kJ/kg} = C_P(T_1 - T_2)$$

$$\Rightarrow T_2 = 369.5 \text{ K}$$

$$T_3(\text{to engine}) = T_2 - \Delta T_{\text{INTERCOOLER}} = 369.5 - 50$$

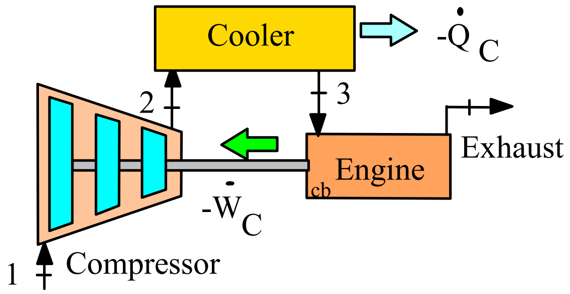
$$= 319.5 \text{ K} = \mathbf{46.3°C}$$

b) Irreversibility from Eq.10.13 with rev. work from Eq.10.12, ($q = 0$ at T_H)

$$s_3 - s_1 = 1.004 \ln\left(\frac{319.4}{303.2}\right) - 0.287 \ln\left(\frac{170}{100}\right) = -0.1001 \ \frac{\text{kJ}}{\text{kg K}}$$

$$i = T(s_3 - s_1) - (h_3 - h_1) - w = T(s_3 - s_1) - C_P(T_3 - T_1) \ - C_P(T_1 - T_2)$$

$$= 303.2(-0.1001) - 1.004(-50) = \mathbf{+19.8 \ kJ/kg}$$



10.49

A rock bed consists of 6000 kg granite and is at 70°C. A small house with lumped mass of 12000 kg wood and 1000 kg iron is at 15°C. They are now brought to a uniform final temperature by circulating water between the rock bed and the house. Find the final temperature and the irreversibility of the process, assuming an ambient at 15°C.

C.V. Total Rockbed and house. No work, no Q irreversible process.

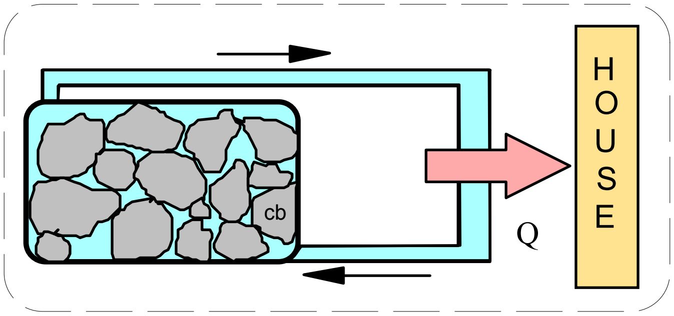
Energy Eq.: $(mC)_{rock}(T_2 - 70) + (mC_{wood} + mC_{Fe})(T_2 - 15) = 0$

$T_2 = \mathbf{29.0°C = 302.2\ K}$

Entropy Eq.: $S_2 – S_1 = \sum m_i(s_2 - s_1)_i = 0 + S_{gen}$

$$S_{gen} = \sum m_i(s_2 - s_1)_i = 5340 \ln \frac{302.2}{343.15} + 15580 \ln \frac{302.2}{288.15} = 63.13 \text{ kJ/K}$$

${}_1I_2 = (T_0)_1S_{2,gen} = 288.15 \times 63.13 = \mathbf{18\ 191\ kJ}$



10.50

A car air-conditioning unit has a 0.5-kg aluminum storage cylinder that is sealed with a valve and it contains 2 L of refrigerant R-134a at 500 kPa and both are at room temperature 20°C. It is now installed in a car sitting outside where the whole system cools down to ambient temperature at –10°C. What is the irreversibility of this process?

C.V. Aluminum and R-134a

Energy Eq.: $m_{Al}(u_2 - u_1)_{Al} + m_R(u_2 - u_1)_R = {}_1Q_2 - {}_1W_2 \quad ({}_1W_2 = 0)$

Entropy Eq.: $m_{AL}(s_2 - s_1)_{Al} + m_R(s_2 - s_1)_R = {}_1Q_2/T_0 + {}_1S_{2\ gen}$

Table A.3 for specific heat $C_{v,Al} = C_{p,Al} = 0.9$ kJ/kg K

$(u_2 - u_1)_{Al} = C_{v,Al}(T_2 - T_1) = 0.9(-10 - 20) = - 27$ kJ/kg

$(s_2 - s_1)_{Al} = C_{p,Al} \ln(T_2/T_1) = 0.9 \ln(263.15/293.15) = -0.09716$ kJ/kg K

Table B.5.2: $v_1 = 0.04226\ m^3/kg$, $u_1 = 390.5$ kJ/kg,

$s_1 = 1.7342$ kJ/kg K, $m_{R134a} = V/v_1 = 0.0473$ kg

$v_2 = v_1 = 0.04226\ \&\ T_2 \quad \Rightarrow \quad x_2 = (0.04226 - 0.000755)/0.09845 = 0.4216$

$u_2 = 186.57 + 0.4216\times185.7 = 264.9$ kJ/kg,

$s_2 = 0.9507 + 0.4216\times0.7812 = 1.2801$ kJ/kg K

${}_1Q_2 = 0.5 \times (-27) + 0.0473(264.9 - 390.5) = - 19.44$ kJ

$${}_1S_{2\ gen} = 0.5\ (-0.09716) + 0.0473(1.2801 - 1.7342) + \frac{19.44}{263.15} = 0.003815 \text{ kJ/K}$$

${}_1I_2 = T_0\ (\,{}_1S_{2\ gen}\,) = 263.15 \times 0.003815 =$ **1.0 kJ**

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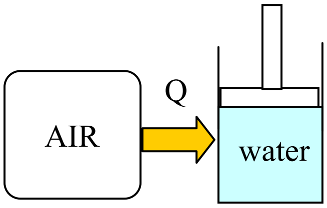
10.51

A constant pressure piston/cylinder has 1 kg of saturated liquid water at 100 kPa. A rigid tank contains air at 1000 K, 1000 kPa. They are now thermally connected by conduction through the walls cooling the air tank and bringing the water to saturated vapor. Find the required amount of air and the irreversibility of the process assuming no external heat transfer.



State: 1 = initial air
2 = final air
3 = initial water
4 = final water we assume $T_4 = T_2$

Take CV as each of the masses. This gives Q from the two energy equations.

$$Q = m_{H2O}(u_4 - u_3) + W_{H2O} = m_{H2O}(h_4 - h_3) = m_{H2O}\, h_{fg} = 2258.02 \text{ kJ}$$

$$Q = -m_{air}(u_2 - u_1) = -m_{air}\, C_v\, (T_2 - T_1)$$

$$m_{air} = m_{H2O}\, h_{fg} / C_v\, (T_1 - T_2) = \frac{2258.02}{0.717\ (1000 - 372.77)} = \mathbf{5.0209\ kg}$$

The entropy equation Eq.8.14 for a reversible process becomes

$$(S_2 - S_1)_{tot} = m_{air}(s_2 - s_1) + m_{H2O}(s_4 - s_3) = 0 + {}_1S_{2\ gen}$$

$$m_{H2O}(s_4 - s_3) = m_{H2O}\, s_{fg} = 1 \text{ kg} \times 6.0568 \text{ kJ/kg-K} = 6.0568 \text{ kJ/K}$$

$$s_2 - s_1 = C_v \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} = C_v \ln\frac{T_2}{T_1} = 0.717 \ln\frac{372.77}{1000} = -0.70753\ \frac{\text{kJ}}{\text{kg-K}}$$

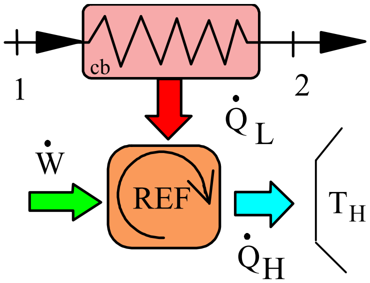
Now solve for the entropy generation

$${}_1S_{2\ gen} = m_{air}(s_2 - s_1) + m_{H2O}(s_4 - s_3)$$
$$= 5.0209\ (-0.70753) + 6.0568 = 2.5044 \text{ kJ/K}$$

$${}_1I_2 = T_0\ (\,{}_1S_{2\ gen}\,) = 298.15 \times 2.5044 = \mathbf{746.6\ kJ}$$

10.52

The water cooler in Problem 7.25 operates steady state. Find the rate of exergy destruction (irreversibility).



The mass flow rate is

$$\dot{m} = \boldsymbol{\rho}\dot{V} = \frac{25 \times 10^{-3}}{0.001002}\,\frac{1}{3600}\ \text{kg/s} = 6.93\ \text{g/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$

$$= 6.93 \times 10^{-3} \times 4.18 \times (18 - 10) = 0.2318\ \text{kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad ⇨ \quad \dot{W} = \dot{Q}_L / \beta = 0.2318 / 2.5 = 0.093\ \text{kW}$$

Energy equation for the refrigerator

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 0.2318\ \text{kW} + 0.093\ \text{kW} = 0.325\ \text{kW}$$

C.V. Total

Entropy Eq.: $0 = \dot{m}(s_1 - s_2) - \dot{Q}_H/T_H + \dot{S}_{gen}$

$$\dot{S}_{gen} = \dot{Q}_H/T_H - \dot{m}(s_1 - s_2) = \dot{Q}_H/T_H - \dot{m}C_p \ln(T_1/T_2)$$

$$= \frac{0.325\ \text{kW}}{298.15\ \text{K}} - 0.00693\ \text{kg/s} \times 4.18\ \text{kJ/kg-K} \times \ln\frac{291.15}{283.15}$$

$$= 0.000283\ \text{kW/K}$$

$$\dot{I} = T_0\, \dot{S}_{gen} = 298.15\ \text{K} \times 0.000283\ \text{kW/K} = \mathbf{0.0844\ kW}$$

Availability (exergy)

10.53

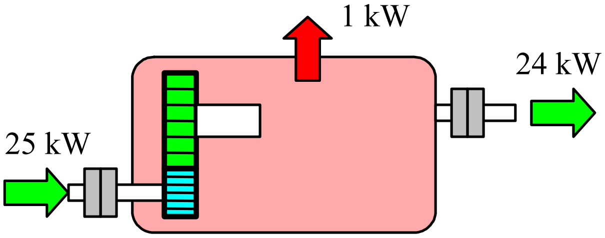
Find all exergy transfers in problem 8.167

The automatic transmission in a car receives 25 kW shaft work and gives out 24 kW to the drive shaft. The balance is dissipated in the hydraulic fluid and metal casing, all at 45°C, which in turn transmits it to the outer atmosphere at 20°C. What is the rate of entropy generation inside the transmission unit? What is it outside the unit?

Solution:

C.V. Total unit. Steady state and surface at 45°C

Exergy transfer:



$$\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right)\dot{Q}_H; \qquad \dot{\Phi}_W = \dot{W};$$

The two shaft works are 100% exergy

$\dot{\Phi}_W = \dot{W} = 25$ kW coming in and $\dot{\Phi}_W = \dot{W} = 24$ kW leaving

The remaingning balance of energy leaves as heat transfer from the 45°C surface

$$\dot{\Phi}_{\text{out with Q}} = \left(1 - \frac{T_o}{T_{\text{Surface}}}\right)\dot{Q}_{\text{out}} = \left(1 - \frac{293.15}{273.15 + 45}\right) 1 \text{ kW} = \mathbf{0.0786\ kW}$$

Here we used 293.15 for the reference T as it was the ambient. Once the heat transfer is redistributed at the ambient T the exergy transfer is zero.

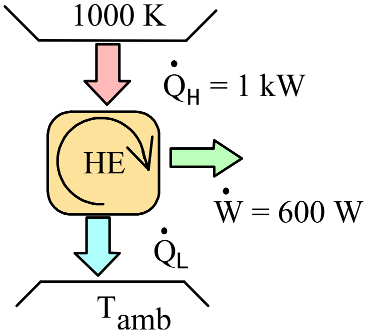
10.54

A heat engine receives 1 kW heat transfer at 1000 K and gives out 600 W as work with the rest as heat transfer to the ambient. What are the fluxes of exergy in and out?

Exergy flux in: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H = \left(1 - \frac{298.15}{1000}\right) 1\text{ kW} = \mathbf{0.702\ kW}$

Exergy flux out: $\dot{\Phi}_L = \left(1 - \frac{T_o}{T_L}\right) \dot{Q}_L = \mathbf{0}$ $\quad (T_L = T_o)$

The other exergy flux out is the power $\dot{\Phi}_{out} = \dot{W} = \mathbf{0.6\ kW}$



10.55

A heat pump has a coefficient of performance of 2 using a power input of 3 kW. Its low temperature is T_o and the high temperature is 80^oC, with an ambient at T_o. Find the fluxes of exergy associated with the energy fluxes in and out.

First let us do the energies in and out

$$COP = \beta = \frac{\dot{Q}_H}{\dot{W}} \quad => \quad \dot{Q}_H = \beta\, \dot{W} = 2 \times 3 \text{ kW} = 6 \text{ kW}$$

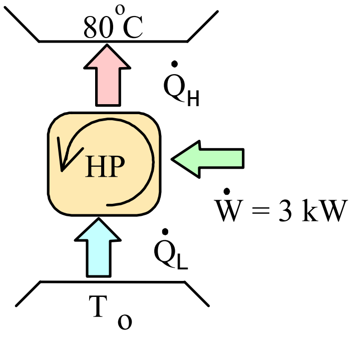
Energy Eq.: $\dot{Q}_L = \dot{Q}_H - \dot{W} = 6 - 3 = 3 \text{ kW}$

Exergy flux in: $\dot{\Phi}_L = \left(1 - \frac{T_o}{T_L}\right) \dot{Q}_L = \mathbf{0} \qquad (T_L = T_o)$

Exergy flux in: $\dot{\Phi}_W = \dot{W} = \mathbf{3\ kW}$

Exergy flux out: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H = \left(1 - \frac{298.15}{353.15}\right) 6 \text{ kW} = \mathbf{0.935\ kW}$

Remark: The process then destroys (3 – 0.935) kW of exergy.



10.56

A flow of air at 1000 kPa, 300 K is throttled to 500 kPa. What is the irreversibility? What is the drop in flow availability?

A throttle process is constant enthalpy if we neglect kinetic energies.

Process: $h_e = h_i$ so ideal gas => $T_e = T_i$

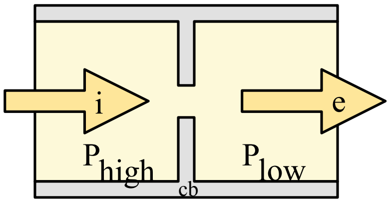
Entropy Eq.: $s_e - s_i = s_{gen} = s^o_{Te} - s^o_{Ti} - R \ln \frac{P_e}{P_i} = 0 - R \ln \frac{P_e}{P_i}$

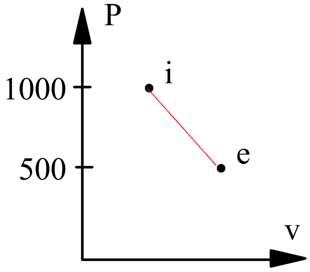
$$s_{gen} = - 0.287 \ln (500 / 1000) = 0.2 \text{ kJ/kg K}$$

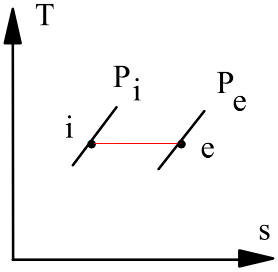
Eq.10.11: $i = T_o\ s_{gen} = 298\ 0.2 = \mathbf{59.6\ kJ/kg}$

The drop in availability is exergy destruction, which is the irreversibility

$$\Delta\psi = i = \mathbf{59.6\ kJ/kg}$$







10.57

Find the change in availability from inlet to exit of the condenser in Problem 9.42.

Solution:

Condenser of Prob. 9.42 has inlet equal to turbine exit.

State 2: $P_2 = 20$ kPa; $s_2 = s_1 = 6.7993$ kJ/kg K

$=> x_2 = (6.7993 – 0.8319)/7.0766 = 0.8433$

$h_2 = 2240.1$ kJ/kg

State 3: $P_2 = P_3$; $T_3 = 40$°C; Compressed liquid assume sat.liq. same T

Table B.1.1 $h_3 = 167.5$ kJ/kg; $s_3 = 0.5724$ kJ/kg K

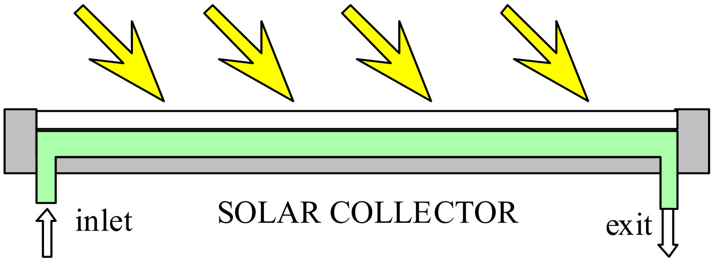
From Eq.10.24 or 10.37

$$\begin{aligned}\psi_3 - \psi_2 &= (h_3 - T_o s_3) – (h_2 - T_o s_2)\\ &= (h_3 - h_2) – T_o(s_3 - s_2)\\ &= (167.5 – 2240.1) – 298.2(0.5724 – 6.7993\ )\\ &= –2072.6 + 1856.9 = \mathbf{-215.7\ kJ/kg}\end{aligned}$$

10.58

A steady stream of R-410a at ambient temperature, 20°C and 800 kPa enters a solar collector and it exits at 80°C, 600 kPa. Calculate the change in exergy of the R410a.

Solution:



Inlet (T,P) Table B.4.2: $h_i = 300.02$ kJ/kg, $s_i = 1.1105$ kJ/kg K

Exit (T,P) Table B.4.2: $h_e = 359.67$ kJ/kg, $s_e = 1.3265$ kJ/kg K

From Eq.10.22 or 10.23

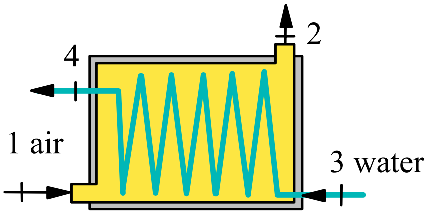
$$\Delta\psi_{ie} = \psi_e - \psi_i = (h_e - h_i) - T_0(s_e - s_i) = (359.67 - 300.02)$$
$$- 293.15(1.3265 - 1.1105) = \mathbf{-3.67\ kJ/kg}$$

Remark: it is negative due to the pressure loss.

10.59

Calculate the change in availability (kW) of the two flows in Problem 9.61.

Solution:



The two flows in the heat exchanger exchanges energy and thus also exergy (availability). Fist find state 4

Air A.7: $h_1 = 1046.22$, $h_2 = 401.3$ kJ/kg,

$s^o_{T1} = 8.1349$, $s^o_{T2} = 7.1593$ kJ/kg K

Water B.1.1: $h_3 = 83.94$ kJ/kg, $s_3 = 0.2966$ kJ/kg K

Energy Eq.6.10: $\dot{m}_{AIR}\Delta h_{AIR} = \dot{m}_{H2O}\Delta h_{H2O}$

$$h_4 - h_3 = (\dot{m}_{AIR}/\dot{m}_{H2O})(h_1 - h_2) = (2/0.5)644.92 = 2579.68 \text{ kJ/kg}$$

$$h_4 = h_3 + 2579.68 = 2663.62 \;<\; h_g \quad \text{at 200 kPa}$$

$$T_4 = T_{sat} = 120.23°\text{C},$$

$$x_4 = (2663.62 – 504.68)/2201.96 = 0.9805,$$

$$s_4 = 1.53 + x_4\, 5.597 = 7.01786 \text{ kJ/kg K}$$

We consider each flow separately and for each flow availability is Eq.10.24, include mass flow rate as in Eq.10.36, use $T_o = 20$ C

For the air flow:

$$\dot{m}_1(\psi_1 - \psi_2) = \dot{m}_1 [\, h_1 - h_2 - T_o (s_1 - s_2) \,]$$

$$= 2 [\, 1046.22 - 401.3 - 293.2(8.1349 - 7.1593 - 0.287 \ln \frac{125}{100})]$$

$$= 2 (644.92 - 267.22) = \mathbf{755.4\ kW}$$

For the water flow:

$$\dot{m}_3(\psi_4 - \psi_3) = \dot{m}_3 [\, h_4 - h_3 - T_o (s_4 - s_3) \,]$$

$$= 0.5 [\, 2663.62 - 83.94 - 293.2(7.01786 - 0.2966)]$$

$$= 0.5[\, 2579.68 - 1970.7 \,] = \mathbf{304.7\ kW}$$

10.60

Consider the springtime melting of ice in the mountains, which gives cold water running in a river at 2°C while the air temperature is 20°C. What is the availability of the water relative to the temperature of the ambient?

Solution:

$\psi = h_1 - h_0 - T_0(s_1 - s_0)$ flow availability from Eq.10.24

Approximate both states as saturated liquid from Table B.1.1

$\psi = 8.392 - 83.96 - 293.15(0.03044 - 0.2966) = \mathbf{2.457\ kJ/kg}$

Why is it positive? As the water is brought to 20°C it can be heated with q_L from a heat engine using q_H from atmosphere $T_H = T_0$ thus giving out work.

10.61

Nitrogen flows in a pipe with velocity 300 m/s at 500 kPa, 300°C. What is its availability with respect to an ambient at 100 kPa, 20°C?

Solution:

From the availability or exergy in Eq.10.24

$$\psi = h_1 - h_0 + (1/2)\mathbf{V}_1^2 - T_0(s_1 - s_0)$$

$$= C_p(T_1 - T_0) + (1/2)\mathbf{V}_1^2 - T_0\Big[C_p \ln(\frac{T_1}{T_0}) - R \ln(\frac{P_1}{P_0})\Big]$$

$$= 1.042(300 - 20) + \frac{300^2}{2000} - 293.15\left(1.042 \ln\left(\frac{573.15}{293.15}\right) - 0.2968 \ln\left(\frac{500}{100}\right)\right)$$

= **272 kJ/kg**

Notice that the high velocity does give a significant contribution.

10.62

A power plant has an overall thermal efficiency of 40% receiving 100 MW of heat transfer from hot gases at an average of 1300 K and rejects heat transfer at $50^{o}C$ from the condenser to a river at ambient temperature $20^{o}C$. Find the rate of both energy and exergy a) from the hot gases and b) from the condenser.

C.V. High T heat exchanger.

Energy flux in: $\dot{Q}_H = 100$ MW

Exergy flux in: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H = \left(1 - \frac{293.15}{1300}\right) 100 \text{ MW} = \mathbf{77.45\ MW}$

C.V. condenser.

Overall power plant: $\dot{W} = \eta \dot{Q}_H$ so from energy equation

Energy flux out $\dot{Q}_L = \dot{Q}_H - \dot{W} = (1 - \eta)\, \dot{Q}_H = 0.6 \times 100 \text{ MW} = \mathbf{60\ MW}$

Exergy flux out: $\dot{\Phi}_L = \left(1 - \frac{T_o}{T_L}\right) \dot{Q}_L = \left(1 - \frac{293.15}{323.15}\right) 60 \text{ MW} = \mathbf{5.57\ MW}$

10.63

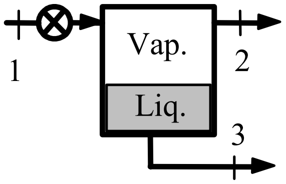
A geothermal source provides 10 kg/s of hot water at 500 kPa, 150°C flowing into a flash evaporator that separates vapor and liquid at 200 kPa. Find the three fluxes of availability (inlet and two outlets) and the irreversibility rate.

C.V. Flash evaporator chamber. Steady flow with no work or heat transfer.

Cont. Eq.: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

Entropy Eq.: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$



B.1.1: $h_o = 104.87$, $s_o = 0.3673$, $h_1 = 632.18$, $s_1 = 1.8417$

B.1.2: $h_2 = 2706.63$, $s_2 = 7.1271$, $h_3 = 504.68$, $s_3 = 1.530$

$$h_1 = xh_2 + (1 - x)\, h_3 \;\; => \;\; x = \dot{m}_2/\dot{m}_1 = \frac{h_1 - h_3}{h_2 - h_3} = 0.0579$$

$$\dot{m}_2 = x\dot{m}_1 = 0.579 \text{ kg/s} \qquad \dot{m}_3 = (1\text{-}x)\dot{m}_1 = 9.421 \text{ kg/s}$$

$$\dot{S}_{gen} = 0.579 \times 7.1271 + 9.421 \times 1.53 - 10 \times 1.8417 = 0.124 \text{ kW/K}$$

Flow availability Eq.10.22: $\psi = (h - T_o s) - (h_o - T_o s_o) = h - h_o - T_o(s - s_o)$

$\psi_1 = 632.18 - 104.87 - 298.15\,(1.8417 - 0.3673) = 87.72$ kJ/kg

$\psi_2 = 2706.63 - 104.87 - 298.15\,(7.1271 - 0.3673) = 586.33$ kJ/kg

$\psi_3 = 504.68 - 104.87 - 298.15\,(1.53 - 0.3673) = 53.15$ kJ/kg

$\dot{m}_1\,\psi_1 = \mathbf{877.2\ kW}$ $\quad\dot{m}_2\psi_2 = \mathbf{339.5\ kW}$ $\quad\dot{m}_3\psi_3 = \mathbf{500.7\ kW}$

$\dot{I} = \dot{m}_1\,\psi_1 - \dot{m}_2\,\psi_2 - \dot{m}_3\psi_3 = \mathbf{37\ kW}$ $\;(= T_o\,\dot{S}_{gen}\,)$

10.64

A steady-flow device receives R-410a at 40°C, 800 kPa and it exits at 40°C, 100 kPa. Assume a reversible isothermal process. Find the change in specific exergy.

C.V. The steady flow device assume $T_0 = 25^{o}C$

Inlet (T,P) Table B.4.2: $h_i = 319.42$ kJ/kg, $s_i = 1.1746$ kJ/kg K

Exit (T,P) Table B.4.2: $h_e = 330.12$ kJ/kg, $s_e = 1.4380$ kJ/kg K

From Eq.10.22 or 10.23

$$\Delta\psi_{ie} = \psi_e - \psi_i = (h_e - h_i) - T_0(s_e - s_i) = (330.12 - 319.42)$$
$$- 298.2(1.4380 - 1.1746) = \mathbf{-67.85\ kJ/kg}$$

The device is like an expander, P drops so work is out and s increases so heat transfer comes in.

10.65

An air compressor is used to charge an initially empty 200-L tank with air up to 5 MPa. The air inlet to the compressor is at 100 kPa, 17°C and the compressor isentropic efficiency is 80%. Find the total compressor work and the change in availability of the air.

C.V. Tank + compressor Transient process with constant inlet conditions, no heat transfer.

Continuity: $m_2 - m_1 = m_{in}$ $(m_1 = 0)$ Energy: $m_2u_2 = m_{in}h_{in} - {}_1W_2$

Entropy: $m_2s_2 = m_{in}s_{in} + {}_1S_{2\ gen}$

Reversible compressor: ${}_1S_{2\ GEN} = 0$ $\Rightarrow$ $s_2 = s_{in}$

State 1: $v_1 = RT_1/P_1 = 0.8323\ m^3/kg$,

State inlet, Table A.7.1: $h_{in} = 290.43$ kJ/kg, $s^o_{Tin} = 6.83521$ kJ/kg K

Eq.8.28: $s^o_{T2} = s^o_{Tin} + R \ln (\frac{P_2}{P_{in}}) = 6.83521 + 0.287 \ln (\frac{5000}{100}) = 7.95796$

Table A.7.1 $\Rightarrow$ $T_{2,s} = 854.6$ K, $u_{2,s} = 637.25$ kJ/kg

$\Rightarrow$ ${}_1w_{2,s} = h_{in} - u_{2,s} = 290.43 – 637.25 = -346.82$ kJ/kg

Actual compressor: ${}_1w_{2,AC} = {}_1w_{2,s}/\eta_c = -433.53$ kJ/kg

$u_{2,AC} = h_{in} - {}_1w_{2,AC} = 290.43 –(-433.53) = 723.96$ kJ/kg

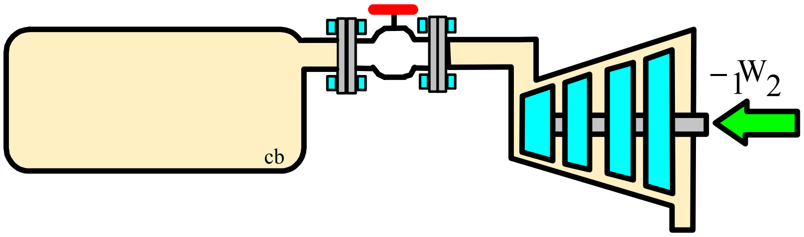
$\Rightarrow$ $T_{2,AC} = 958.5$ K, $s^o_{T2\ ac} = 8.08655$ kJ/kg K

State 2 $\boxed{u, P}$ $v_2 = RT_2/P_2 = 0.05502\ m^3/kg$ so $m_2 = V_2/v_2 = 3.635$ kg

$\Rightarrow$ ${}_1W_2 = m_2 ({}_1w_{2,AC}) =$ **-1575.9 kJ**

$m_2(\phi_2 - \phi_1) = m_2[u_2 - u_1 + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$

$= 3.635 [723.96 - 207.19 + 100(0.05502 - 0.8323) - 290[8.08655 - 6.83521 - 0.287 \ln(5000/100)] =$ **1460.4 kJ**



10.66

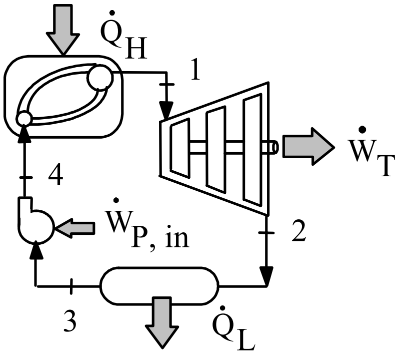
Find the availability at all 4 states in the power plant of Problem 9.42 with an ambient at 298 K.

Solution:

Flow availability from Eq.10.24 neglecting kinetic and potential energy is:

$$\psi = h - h_0 - T_0(s - s_0)$$

so we need (h,s) for all four states.



$P_1 = P_4 = 20$ MPa, $T_1 = 700$ °C
$h_1 = 3809.1$ kJ/kg,
$s_1 = 6.7993$ kJ/kg K
$P_2 = P_3 = 20$ kPa, $T_3 = 40$ °C
State 3: (P, T) Comp. liquid, take sat. liquid Table B.1.1
$h_3 = 167.5$ kJ/kg,
$v_3 = 0.001008$ m^3/kg

C.V. Turbine.

Entropy Eq.9.8: $s_2 = s_1 = 6.7993$ kJ/kg K

Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766 \quad => \quad x_2 = 0.8433$

$h_2 = 251.4 + 0.8433\times 2358.33 = 2240.1$ kJ/kg

$w_T = h_1 - h_2 = 3809.1 - 2240.1 = 1569$ kJ/kg

CV. Pump, property relation in Eq.9.13 gives work from Eq.9.18 as

$w_P = - v_3(P_4 - P_3) = -0.001008(20000 – 20) = -20.1$ kJ/kg

$h_4 = h_3 - w_P = 167.5 + 20.1 = 187.6$ kJ/kg

Flow availability from Eq.10.24 and notice that since turbine work and pump work are reversible they represent also change in avalability.

$\psi_1 = h_1 - h_0 - T_0(s_1 - s_0) = 3809.1 – 104.87 - 298 (6.7993 – 0.3673)$
$= 1787.5$ kJ/kg

$\psi_2 = h_2 - h_0 - T_0(s_2 - s_0) = \psi_1 - w_T = 1787.5 - 1569 = 218.5$ kJ/kg

$\psi_3 = h_3 - h_0 - T_0(s_3 - s_0) = 167.5 - 104.87 - 298(0.5724 - 0.3673)$
$= 1.51$ kJ/kg

$\psi_4 = h_4 - h_0 - T_0(s_4 - s_0) = \psi_3 - w_P = 1.51 + 20.1 = 21.61$ kJ/kg

10.67

Calculate the availability of the water at the initial and final states of Problem 8.126, and the irreversibility of the process.

State properties

1: $u_1 = 83.94$ kJ/kg, $s_1 = 0.2966$ kJ/kg K, , $v_1 = 0.001\ m^3/kg$

2: $u_2 = 3124.3$ kJ/kg, $s_2 = 7.7621$ kJ/kg K, $v_2 = 0.354\ m^3/kg$

0: $u_o = 104.86$ kJ/kg, $s_o = 0.3673$ kJ/kg K, $v_o = 0.001003\ m^3/kg$

Process transfers: ${}_1W_2^{ac} = 203$ kJ, ${}_1Q_2^{ac} = 3243.4$ kJ, $T_H = 873.15$ K

$$\phi = (u - T_o s) - (u_o - T_o s_o) + P_o(v - v_o)$$

$$\phi_1 = (83.94 - 298.15\times0.2966) - (104.86 - 298.15\times0.3673) + 100\ (0.001002 - 0.001003) = 0.159 \text{ kJ/kg}$$

$$\phi_2 = (3124.3 - 298.15\times7.7621) - (104.86 - 298.15\times0.3673) + 100\ (0.35411 - 0.001003) = 850 \text{ kJ/kg}$$

$$ {}_1I_2 = m(\phi_1 - \phi_2) + [1 - (T_o/T_H)]\,{}_1Q_2^{ac} - {}_1W_2^{ac} + P_o(V_2 - V_1)$$

$$= -849.84 + (1 - \frac{298.15}{873.15})\ 3243.4 - 203 + 100\ (0.3541 - 0.001)$$

$$= -849.84 + 2135.9 - 203 + 35.31 = \mathbf{1118.\ kJ}$$

[($S_{gen} = 3.75$ kJ/K; $T_oS_{gen} = 1118$ kJ so OK]

10.68

Air flows at 1500 K, 100 kPa through a constant pressure heat exchanger giving energy to a heat engine and comes out at 500 K. What is the constant temperature the same heat transfer should be delivered at to provide the same availability?

Solution:

C.V. Heat exchanger

Continuity eq.: $\dot{m}_1 = \dot{m}_2$;

Energy Eq.6.12: $\dot{m}_1 h_1 = \dot{m}_1 h_2 + \dot{Q}_H$

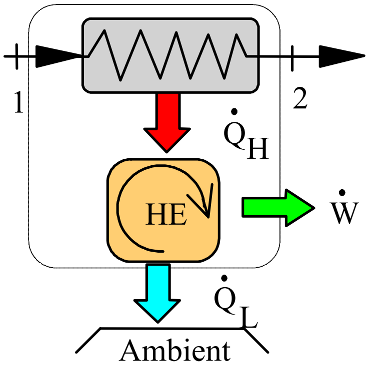


Table A.7.1: $h_1 = 1635.8$ kJ/kg,

$h_2 = 503.36$ kJ/kg, $s_1 = 8.61209$ kJ/kg K

$s_2 = 7.38692$ kJ/kg K

$q_{out} = h_1 - h_2 = 1635.8 - 503.36 = 1132.4$ kJ/kg

Availability from heat transfer at T: $\Delta\psi = (1 - \frac{T_o}{T_H})\, q_{out} = \psi_1 - \psi_2$

Eq.10.37: $\psi_1 - \psi_2 = h_1 - h_2 - T_o (s_1 - s_2)$

$= 1132.4 - 298.15 (8.6121 - 7.38692)$

$= 1132.4 - 356.3 = 767.1$ kJ/kg

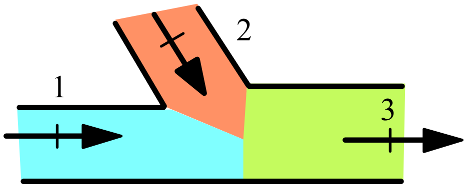
$$1 - \frac{T_o}{T_H} = (\psi_1 - \psi_2) / q_{out} = 767.1 / 1132.4 = 0.6774$$

$$\frac{T_o}{T_H} = 0.3226 \quad => \quad \mathbf{T_H = 924\ K}$$

10.69

A flow of 0.1 kg/s hot water at 80°C is mixed with a flow of 0.2 kg/s cold water at 20°C in a shower fixture. What is the rate of exergy destruction (irreversibility) for this process?

Solution:



Continuity Eq.6.9:

$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$= 0.1 + 0.2 = 0.3$ kg/s

Energy Eq.6.10:

$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

Use specific heat from A.4 and Eq.8.11 for s change or Table B.1.1 saturated liquid values. Divide the energy equation with $\dot{m}_3 C_P$

$$T_3 = (\dot{m}_1/\dot{m}_3)T_1 + (\dot{m}_2/\dot{m}_3)T_2 = \frac{1}{3} \times 80 + \frac{2}{3} \times 20 = 40^{o}C$$

$$\dot{S}_{gen} = \dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2)$$

$$= 0.1 \times 4.18 \ln (313.15/353.15) + 0.2 \times 4.18 \ln (313.15/293.15)$$

$$= 0.00493 \text{ kW/K}$$

$$\dot{\Phi}_{Destruction} = \dot{I} = T_o \dot{S}_{gen} = 298.15 \times 0.00493 = \mathbf{1.47\ kW}$$

You could also have decided to use 20°C for the ambient T in this problem.

10.70

An electric stove has one heating element at $300^{o}C$ getting 500 W of electric power. It transfers 90% of the power to 1 kg water in a kettle initially at $20^{o}C$, 100 kPa, the rest 10% leaks to the room air. The water at a uniform T is brought to the boiling point. At the start of the process what is the rate of availability transfer by: a) electrical input b) from heating element and c) into the water at T_{water}.

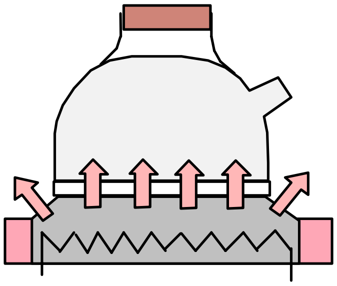
a) Work is availability $\dot{\Phi} = \dot{W} = \mathbf{500\ W}$

b) Heat transfer at $300^{o}C$ is only partly availability

$$\dot{\Phi} = \left(1 - \frac{T_o}{T_H}\right)\dot{Q} = \left(1 - \frac{293.15}{273.15 + 300}\right) 500 = \mathbf{244\ W}$$

c) Water receives heat transfer at $20^{o}C$ as 90% of 500 W

$$\dot{\Phi} = \left(1 - \frac{T_o}{T_{water}}\right)\dot{Q} = \left(1 - \frac{293.15}{273.15 + 20}\right) 450 = \mathbf{0\ W}$$



500 W at $300^{o}C$

10.71

A water kettle has 1 kg of saturated liquid water at P_o. It is on an electric stove that heats it from a hot surface at 500 K. Water vapor escapes from the kettle and when the last liquid drop disappears the stove is turned off. Find the destruction of exergy two places: a) between the hot surface and the water and b) between the electrical wire input and the hot surface.

C.V. The water, this is a control mass. We bring it from saturated liquid to saturated vapor same pressure.

Energy eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 \Rightarrow {}_1Q_2 = m\,(h_2 - h_1) = mh_{fg} = 2257 \text{ kJ}$

C.V. Kettle bottom, consider the hot surface to water heat transfer.

$$\psi_{in} = \left(1 - \frac{T_o}{T_{H\text{ surface}}}\right) {}_1Q_2 = \left(1 - \frac{273.15}{500}\right) 2257 \text{ kJ} = 1024 \text{ kJ}$$

$$\psi_{out} = \left(1 - \frac{T_o}{T_{water}}\right) {}_1Q_2 = \left(1 - \frac{273.15}{373.15}\right) 2257 \text{ kJ} = 605 \text{ kJ}$$

The destruction is the drop in exergy

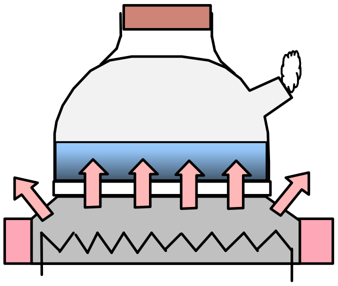
$$\psi_{destruction} = \psi_{in} - \psi_{out} = 1024 - 605 = \mathbf{419\ kJ}$$

C.V. Wire and hot plate.

$$\psi_{in} = W_{electrical} = {}_1Q_2 = 2257 \text{ kJ}$$

$$\psi_{out} = \left(1 - \frac{T_o}{T_{H\text{ surface}}}\right) {}_1Q_2 = \left(1 - \frac{273.15}{500}\right) 2257 \text{ kJ} = 1024 \text{ kJ}$$

$$\psi_{destruction} = \psi_{in} - \psi_{out} = 2257 - 1024 = \mathbf{1233\ kJ}$$



Comment: We could also have found the destruction of exergy by finding the entropy generation terms and then $\psi_{destruction} = T_o\, S_{gen}$

10.72

A 10-kg iron disk brake on a car is initially at 10°C. Suddenly the brake pad hangs up, increasing the brake temperature by friction to 110°C while the car maintains constant speed. Find the change in availability of the disk and the energy depletion of the car's gas tank due to this process alone. Assume that the engine has a thermal efficiency of 35%.

Solution:

All the friction work is turned into internal energy of the disk brake.

Energy eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 \;\Rightarrow\; {}_1Q_2 = m_{Fe}C_{Fe}(T_2 - T_1)$

$${}_1Q_2 = 10 \times 0.45 \times (110 - 10) = 450 \text{ kJ}$$

Neglect the work to the surroundings at P_0, so change in availability is from Eq.10.27

$$\Delta\phi = m(u_2 - u_1) - T_0 m(s_2 - s_1)$$

Change in s for a solid, Eq.8.20

$$m(s_2\text{-}s_1) = mC \ln(T_2/T_1) = 10 \times 0.45 \times \ln\left(\frac{383.15}{283.15}\right) = 1.361 \text{ kJ/K}$$

$$\Delta\phi = 450 - 283.15 \times 1.361 = \mathbf{64.63\ kJ}$$

$$W_{engine} = \eta_{th}Q_{gas} = {}_1Q_2 = \text{Friction work}$$

$$Q_{gas} = {}_1Q_2/\eta_{th} = 450/0.35 = \mathbf{1285.7\ kJ}$$

10.73

Water as saturated liquid at 200 kPa goes through a constant pressure heat exchanger as shown in Fig. P10.62. The heat input is supplied from a reversible heat pump extracting heat from the surroundings at 17°C. The water flow rate is 2 kg/min and the whole process is reversible, that is, there is no overall net entropy change. If the heat pump receives 40 kW of work find the water exit state and the increase in availability of the water.

C.V. Heat exchanger + heat pump.

$\dot{m}_1 = \dot{m}_2 = 2$ kg/min, $\quad \dot{m}_1 h_1 + \dot{Q}_0 + \dot{W}_{in} = \dot{m}_1 h_2, \quad \dot{m}_1 s_1 + \dot{Q}_0/T_0 = \dot{m}_1 s_2$

Substitute $\dot{Q}_0$ into energy equation and divide by $\dot{m}_1$

$$h_1 - T_0 s_1 + w_{in} = h_2 - T_0 s_2$$

LHS = $504.7 - 290.15 \times 1.5301 + 40 \times 60/2 = 1260.7$ kJ/kg

State 2: P_2 , $\; h_2 - T_0 s_2 = 1260.7$ kJ/kg

At sat. vap. $h_g - T_0 s_g = 638.8$ so state 2 is superheated vapor at 200 kPa.

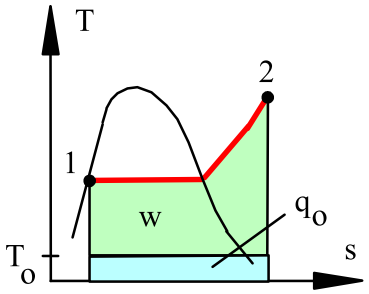
At 600°C: $\quad h_2 - T_0 s_2 = 3703.96 - 290.15 \times 8.7769 = 1157.34$ kJ/kg

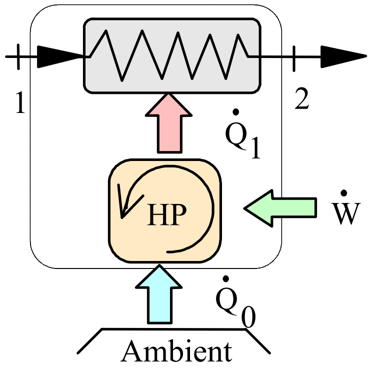
At 700°C: $\quad h_2 - T_0 s_2 = 3927.66 - 290.15 \times 9.0194 = 1310.68$ kJ/kg

Linear interpolation $\Rightarrow$ $\mathbf{T_2 = 667°C}$

$\Delta\psi = (h_2 - T_0 s_2) - (h_1 - T_0 s_1) = w_{in} =$ **1200 kJ/kg**

$= 1260.7 - 504.7 + 290.15 \times 1.5301 \approx$ **1200 kJ/kg**





10.74

Ammonia, 2 kg, at 400 kPa, 40°C is in a piston/cylinder together with an unknown mass of saturated liquid ammonia at 400 kPa. The piston is loaded so it keeps constant pressure and the two masses are allowed to mix to a final uniform state of saturated vapor without external heat transfer. Find the total exergy destruction in the process.

C.V. All the ammonia.

Continuity Eq.: $m_2 - m_A - m_B = 0$

Energy Eq.: $m_2u_2 - m_Au_A - m_Bu_B = Q - W$

Entropy Eq.: $m_2s_2 - m_As_A - m_Bs_B = \int dQ/T + {}_1S_{2\,gen}$

Process: $Q = 0,\ P = C \quad \text{so } W = P(V_2 - V_A - V_B)$

State A: $h_A = 1543.6$ kJ/kg, $s_A = 5.7111$ kJ/kg-K, $m_A = 2$ kg

State B: $h_B = 171.65$ kJ/kg, $s_B = 0.6793$ kJ/kg-K, $m_B = ?$ kg

State 2: $h_2 = 1440.2$ kJ/kg, $s_2 = 5.3559$ kJ/kg-K, $m_2 = m_A + m_B$

Insert the process equation into the energy equation

$$m_2h_2 - m_Ah_A - m_Bh_B = 0$$

Now solve the energy equation for the unknown mass m_B

$$m_B = m_A \frac{h_2 - h_A}{h_B - h_2} = 2 \text{ kg} \frac{1440.2 - 1543.6}{171.65 - 1440.2} = 0.163 \text{ kg}$$

Solve for entropy generation

$$\begin{aligned} {}_1S_{2\,gen} &= m_2s_2 - m_As_A - m_Bs_B \\ &= 2.163 \times 5.3559 - 2 \times 5.7111 - 0.163 \times 0.6793 \\ &= 0.051886 \text{ kJ/K} \end{aligned}$$

Exergy destruction

$$\psi_{destruction} = T_o\, S_{gen} = 298.15 \times 0.051886 = \mathbf{15.47\ kJ}$$

10.75

A 1 kg block of copper at 350°C is quenched in a 10 kg oil bath initially at ambient temperature of 20°C. Calculate the final uniform temperature (no heat transfer to/from ambient) and the change of availability of the system (copper and oil).

Solution:

C.V. Copper and oil. $C_{co} = 0.42$ kJ/kg K, $C_{oil} = 1.8$ kJ/kg K

$$m_2u_2 - m_1u_1 = {}_1Q_2 - {}_1W_2 = 0 = m_{co}C_{co}(T_2 - T_1)_{co} + (mC)_{oil}(T_2 - T_1)_{oil}$$

$$1 \times 0.42\ (T_2 - 350) + 10 \times 1.8\ (T_2 - 20) = 0$$

$$18.42\ T_2 = 507 \qquad => \qquad T = 27.5°\text{C} = 300.65\text{ K}$$

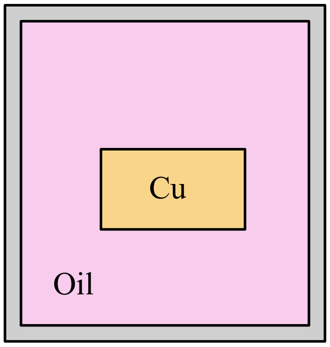
For each mass copper and oil, we neglect work term (v = C) so Eq.10.22 is

$$(\phi_2 - \phi_1) = u_2 - u_1 - T_o(s_2 - s_1) = mC\ [(T_2 - T_1) - T_o\ln\ (T_2 / T_1)\ ]$$

$$m_{cv}(\phi_2 - \phi_1)_{cv} + m_{oil}\ (\phi_2 - \phi_1)_{oil} =$$

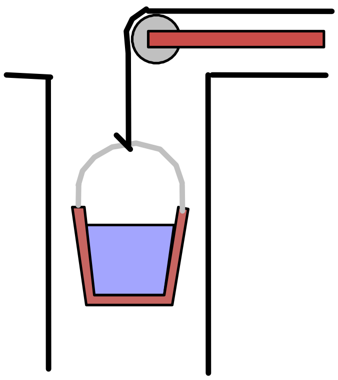
$$= 0.42 \times [(-322.5) - 293.15 \ln \frac{300.65}{623.15}\ ] + 10 \times 1.8\ [7.5 - 293.15 \ln \frac{300.65}{293.15}\ ]$$

$$= - 45.713 + 1.698 = \mathbf{- 44.0\ kJ}$$



10.76

A wooden bucket (2 kg) with 10 kg hot liquid water, both at 85°C, is lowered 400 m down into a mineshaft. What is the availability of the bucket and water with respect to the surface ambient at 20°C?



C.V. Bucket and water. Both thermal availability and potential energy terms.

$v_1 \approx v_0$ for both wood and water so work to atm. is zero.

Use constant heat capacity table A.3 for wood and table B.1.1 (sat. liq.) for water.
From Eq.10.27

$$\phi_1 - \phi_0 = m_{wood}[u_1 - u_0 - T_0(s_1 - s_0)] + m_{H_2O}[u_1 - u_0 - T_0(s_1 - s_0)] + m_{tot}g(z_1 - z_0)$$

$$= 2\text{ kg }[1.26 \frac{\text{kJ}}{\text{kg K}} (85 - 20)\text{ K} - 293.15\text{ K} \times 1.26 \frac{\text{kJ}}{\text{kg K}} \times \ln\frac{273.15 + 85}{293.15}]$$

$$+ 10\text{ kg }[\,355.82 - 83.94 - 293(1.1342 - 0.2966)]\text{ kJ/kg}$$

$$+ 12\text{ kg} \times 9.807\ (\text{m/s}^2) \times (-400)\text{ m} /1000\ (\text{J/kJ})$$

$$= 15.85 + 263.38 - 47.07 = \mathbf{232.2\ kJ}$$

Exergy Balance Equation

10.77

Apply the exergy equation to solve Problem 10.18

A refrigerator should remove 1.5 kW from the cold space at -10°C while it rejects heat to the kitchen at 25°C. Find the reversible work.

The exergy balance equation is Eq.10.38

$$\frac{d\Phi}{dt} = \sum \left(1 - \frac{T_o}{T}\right) \dot{Q}_{cv} - \dot{W}_{cv} + P_o \frac{dV}{dt} + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - T_o \dot{S}_{gen}$$

$$0 \quad = (1 - \frac{T_o}{T_L})\, \dot{Q}_L - (1 - \frac{T_o}{T_H})\, \dot{Q}_H - \dot{W}_{cv} + 0 + 0 - 0 - 0$$

The two d/dt terms are zero due to steady state, no flow and reversible give zero for the last three terms so we get the equation we can solve for the work

$$\dot{W}_{cv} = (1 - \frac{T_o}{T_L})\, \dot{Q}_L - (1 - \frac{T_o}{T_H})\, \dot{Q}_H$$

To solve we need the heat transfer $\dot{Q}_H$ as we do have $\dot{Q}_L = 1.5$ kW. We need to use the entropy equation to get

$$\dot{Q}_H = (T_H/T_L)\, \dot{Q}_L = \frac{298.15}{263.15}\, 1.5 \text{ kW} = 1.6995 \text{ kW}$$

$$\dot{W}_{cv} = (1 - \frac{T_o}{T_L})\, \dot{Q}_L - (1 - \frac{T_o}{T_H})\, \dot{Q}_H = \dot{Q}_L - \dot{Q}_H + \frac{T_o}{T_H}\dot{Q}_H - \frac{T_o}{T_L}\dot{Q}_L$$

$$= \dot{Q}_L - \dot{Q}_H = 1.5 - 1.6995 = -0.1995 \text{ kW} = \mathbf{-0.2\ kW}$$

10.78

Apply the exergy equation to solve Problem 10.36 with $T_o = 20^oC$.

A 20^oC room is heated with a 1500 W electric baseboard heater. What is the rate of irreversibility?

The exergy balance equation is Eq.10.38

$$\frac{d\Phi}{dt} = \sum \left(1 - \frac{T_o}{T}\right) \dot{Q}_{cv} - \dot{W}_{cv} + P_o \frac{dV}{dt} + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - T_o \dot{S}_{gen}$$

For the heater we have steady state (d/dt terms are zero), one $\dot{Q}_{cv}$ out at T_o, electrical work in and no flow terms leaving us with

$$0 = \left(1 - \frac{T_o}{T_o}\right) \times \dot{Q}_{cv} + \dot{W}_{electrical} - T_o \dot{S}_{gen}$$

$$\dot{I} = T_o\, \dot{S}_{gen} = \dot{W}_{electrical} = \mathbf{1500\ W}$$

10.79

Find the specific flow exergy in and out of the steam turbine in Example 9.1 assuming an ambient at 293 K. Use the exergy balance equation to find the reversible specific work. Does this calculation of specific work depend on T_o?

Solution:

The specific flow exergy is from Eq. 10.37

$$\psi_i = h_i + \frac{1}{2}\mathbf{V}_i^2 - T_o s_i - (h_o - T_o s_o)$$

Reference state: $h_o = 83.94$ kJ/kg, $s_o = 0.2966$ kJ/kg K,

$$h_o - T_o s_o = -2.9638 \text{ kJ/kg}$$

The properties are listed in Example 9.1 so the specific flow exergies are

$$\psi_i = 3051.2 + 1.25 - 293 \times 7.1228 - (-2.9638) = \mathbf{968.43\ kJ/kg}$$

$$\psi_e = 2655.0 + 20 - 293 \times 7.1228 - (-2.9638) = \mathbf{590.98\ kJ/kg}$$

The reversible work is from Eq.10.39, with $q = 0$ and $s_{gen} = 0$, so

$$w = \psi_i - \psi_e = 968.43 - 590.98 = \mathbf{377.45\ kJ/kg}$$

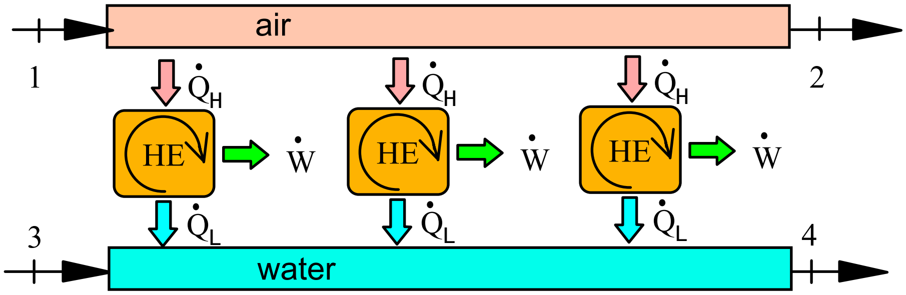
The offset T_o terms drop out as we take the difference and also ($s_i = s_e$)

$$\psi_i - \psi_e = h_i - h_e - T_o(s_i - s_e) = h_i - h_e$$

Notice since the turbine is reversible we get the same as in Example 9.1

10.80

A counterflowing heat exchanger cools air at 600 K, 400 kPa to 320 K using a supply of water at 20°C, 200 kPa. The water flow rate is 0.1 kg/s and the air flow rate is 1 kg/s. Assume this can be done in a reversible process by the use of heat engines and neglect kinetic energy changes. Find the water exit temperature and the power out of the heat engine(s).



C.V. Total

Energy eq.: $\dot{m}_a h_1 + \dot{m}_{H_2O} h_3 = \dot{m}_a h_2 + \dot{m}_{H_2O} h_4 + \dot{W}$

Entropy Eq,: $\dot{m}_a s_1 + \dot{m}_{H_2O} s_3 = \dot{m}_a s_2 + \dot{m}_{H_2O} s_4 \qquad (s_{gen} = 0)$

Table A.7: $h_1 = 607.316$ kJ/kg, $s^o_{T1} = 7.57638$ kJ/kg K

Table A.7: $h_2 = 320.576$ kJ/kg, $s^o_{T2} = 6.93413$ kJ/kg K,

Table B.1.1: $h_3 = 83.96$ kJ/kg, $s_3 = 0.2966$ kJ/kg K

From the entropy equation we first find state 4

$$s_4 = (\dot{m}_a/\dot{m}_{H_2O})(s_1 - s_2) + s_3 = (1/0.1)(7.57638 - 6.93413) + 0.2966 = 6.7191$$

4: $P_4 = P_3$, s_4 ⇒ Table B.1.2: $x_4 = (6.7191-1.530)/5.597 = 0.9271$,

$h_4 = 504.68 + 0.9271 \times 2201.96 = 2546.1$ kJ/kg, $T_4 =$ **120.20°C**

From the energy equation

$$\dot{W} = \dot{m}_a(h_1 - h_2) + \dot{m}_{H_2O}(h_3 - h_4)$$

$$= 1(607.32 - 320.58) + 0.1(83.96 - 2546.1) = \mathbf{40.53\ kW}$$

The work could be found from the exergy balance equation (requires state 4) as

$$\dot{W} = \dot{m}_a(\psi_1 - \psi_2) + \dot{m}_{H_2O}(\psi_3 - \psi_4)$$

Since the entropies (including $\dot{m}$'s) balance out it is the same as the energy Eq.

10.81

Apply the exergy equation to solve Problem 10.37.

A refrigerator removes 1.5 kW from the cold space at -10^{o}C using 750 W of power input while it rejects heat to the kitchen at 25^{o}C. Find the rate of irreversibility.

The exergy balance equation is Eq.10.38

$$\frac{d\Phi}{dt} = \sum \left(1 - \frac{T_o}{T}\right) \dot{Q}_{cv} - \dot{W}_{cv} + P_o \frac{dV}{dt} + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - T_o \dot{S}_{gen}$$

$$0 \quad = (1 - \frac{T_o}{T_L})\, \dot{Q}_L - (1 - \frac{T_o}{T_H})\, \dot{Q}_H - \dot{W}_{cv} + 0 + 0 - 0 - T_o \dot{S}_{gen}$$

The two d/dt terms are zero due to steady state and no flow. Now solve for the irreversibility, notice the $\dot{Q}_H$ term drops out since $T_H = T_o$.

$$\begin{aligned} \dot{I} = T_o\, \dot{S}_{gen} &= (1 - \frac{T_o}{T_L})\, \dot{Q}_L - \dot{W}_{cv} \\ &= (1 - \frac{298.15}{263.15})\ 1.5 \text{ kW} - (-0.75) \text{ kW} \\ &= -0.2 \text{ kW} + 0.75 \text{ kW} = \mathbf{0.55\ kW} \end{aligned}$$

10.82

Estimate some reasonable temperatures to use and find all the fluxes of exergy in the refrigerator given in Example 7.2

We will assume the following temperatures:

Ambient: $T = 20^oC$ usually it is the kitchen air.

Low T: $T = 5^oC$ (refrigerator) $T= -10^oC$ (freezer)

$$\dot{\Phi}_W = \dot{W} = \mathbf{150\ W}$$

$$\dot{\Phi}_H = (1 - \frac{T_o}{T_H})\,\dot{Q}_H = (1 - \frac{T_{amb}}{T_{amb}})\,\dot{Q}_H = \mathbf{0}$$

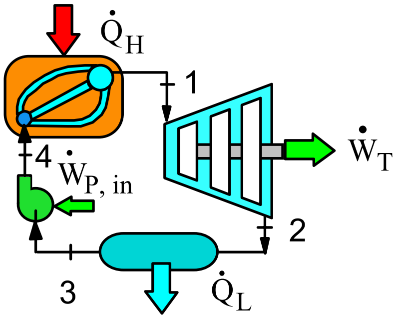
$$\dot{\Phi}_L = (1 - \frac{T_o}{T_L})\,\dot{Q}_L = (1 - \frac{293}{278})\,250 = \mathbf{-13.5\ W}$$

I.e. the flux goes into the cold space! Why? As you cool it $T < T_o$ and you increase its availability (exergy), it is further away from the ambient.

10.83

Consider the condenser in Problem 9.42. Find the specific energy and exergy that are given out, assuming an ambient at 20°C. Find also the specific exergy destruction in the process.

Solution:



Condenser from state 2 to state 3

$P_2 = P_3 = 20$ kPa

$T_3 = 40$ °C

State 1: (P, T) Table B.1.3

$h_1 = 3809.1$ kJ/kg, $s_1 = 6.7993$ kJ/kg K

C.V. Turbine.

Entropy Eq.9.8: $s_2 = s_1 = 6.7993$ kJ/kg K

Table B.1.2 $s_2 = 0.8319 + x_2 \times 7.0766 \Rightarrow x_2 = 0.8433$

$h_2 = 251.4 + 0.8433 \times 2358.33 = 2240.1$ kJ/kg

State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

$h_3 = 167.54$ kJ/kg, $s_3 = 0.5724$ kJ/kg K

C.V. Condenser

Energy Eq.: $q_L = h_2 - h_3 = 2240.1 - 167.54 =$ **2072.56 kJ/kg**

Exergy Eq.: $\Delta\psi = \psi_2 - \psi_3 = h_2 - h_3 - T_o(s_2 - s_3)$

$= 2072.56 - 293.15(6.7993 - 0.5724)$

$=$ **247.1 kJ/kg** going out

Since all the exergy that goes out ends up at the ambient where it has zero exergy, the destruction equals the outgoing exergy.

$\psi_{destr} = \Delta\psi =$ **247.1 kJ/kg**

Notice the condenser gives out a large amount of energy but little exergy.

10.84

Evaluate the steady state exergy fluxes due to a heat transfer of 250 W through a wall with 600 K on one side and 400 K on the other side. What is the exergy destruction in the wall.

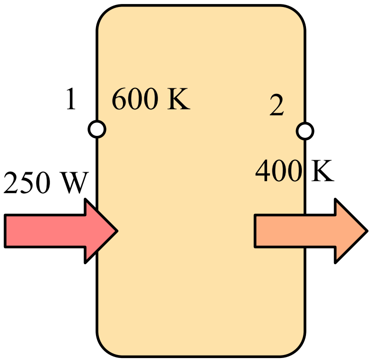
Solution:

Exergy flux due to a $\dot{Q}$ term Eq.10.36:

$$\dot{\Phi}_Q = (1 - \frac{T_o}{T})\,\dot{Q}$$

$$\dot{\Phi}_1 = (1 - \frac{T_o}{T_1})\,\dot{Q} = (1 - \frac{298}{600})\,250 = 125.8 \text{ W}$$

$$\dot{\Phi}_2 = (1 - \frac{T_o}{T_2})\,\dot{Q} = (1 - \frac{298}{400})\,250 = 63.8 \text{ W}$$



Steady state state so no storage and Eq.10.36 is

$$0 = \dot{\Phi}_1 - \dot{\Phi}_2 - \dot{\Phi}_{destr.}$$

$$\dot{\Phi}_{destr.} = \dot{\Phi}_1 - \dot{\Phi}_2 = 125.8 - 63.8 = \mathbf{62\ W}$$

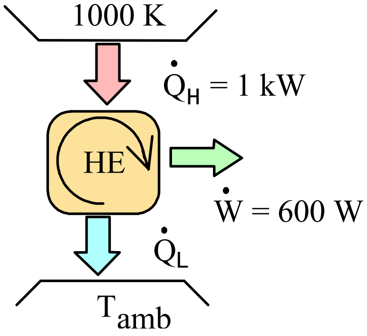
10.85

Apply the exergy equation to find the exergy destruction for Problem 10.54

Exergy flux in: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H = \left(1 - \frac{298.15}{1000}\right) 1 \text{ kW} =$ **0.702 kW**

Exergy flux out: $\dot{\Phi}_L = \left(1 - \frac{T_o}{T_L}\right) \dot{Q}_L = \mathbf{0} \qquad (T_L = T_o)$

The other exergy flux out is the power $\dot{\Phi}_{out} = \dot{W} =$ **0.6 kW**



The exergy balance equation Eq.10. for the HE becomes (steady state)

$$0 = \dot{\Phi}_H - \dot{\Phi}_L - \dot{W} - \dot{\Phi}_{destruction}$$

$$\dot{\Phi}_{destruction} = \dot{\Phi}_H - \dot{\Phi}_L - \dot{W} = 702 \text{ W} - 0 - 600 \text{ W} = \mathbf{102\ W}$$

10.86

The condenser in a power plant cools 10 kg/s water at 10 kPa, quality 90% so it comes out as saturated liquid at 10 kPa. The cooling is done by ocean-water coming in at ambient $15^{\circ}C$ and returned to the ocean at $20^{\circ}C$. Find the transfer out of the water and the transfer into the ocean-water of both energy and exergy (4 terms).

Solution:

C.V. Water line. No work but heat transfer out.

Energy Eq.: $\dot{Q}_{out} = \dot{m}\,(h_1 - h_2) = 10(2345.35 - 191.81) = \mathbf{21\ 535\ kW}$

C.V. Ocean water line. No work but heat transfer in equals water heattransfer out

Energy Eq.: $q = h_4 - h_3 = 83.94 - 62.98 = 20.96$ kJ/kg

$\dot{m}_{ocean} = \dot{Q}_{out}/q = 21\ 535 / 20.96 = 1027.4$ kg/s

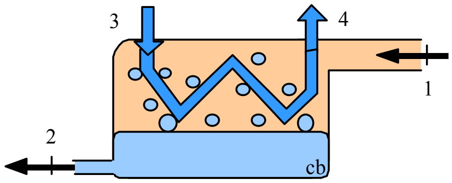
Exergy out of the water follows Eq.10.37

$$\dot{\Phi}_{out} = \dot{m}(\psi_1 - \psi_2) = \dot{m}\,[\,h_1 - h_2 - T_o\,(s_1 - s_2)\,]$$
$$= 10\,[\,2345.35 - 191.81 - 288.15(7.4001 - 0.6492)]$$
$$= \mathbf{2082.3\ kW}$$

Exergy into the ocean water

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}(\psi_4 - \psi_3) = \dot{m}_{ocean}\,[\,h_4 - h_3 - T_o(s_4 - s_3)]$$
$$= 1027.4\,[\,20.96 - 288.15(0.2966 - 0.2245)]$$
$$= \mathbf{189.4\ kW}$$

Notice there is a large amount of energy exchanged but very little exergy.



Often the cooling media flows inside a long pipe carrying the energy away.

10.87

Consider the car engine in Example 7.1 and assume the fuel energy is delivered at a constant 1500 K. The 70% of the energy that is lost is 40% exhaust flow at 900 K and the remainder 30% heat transfer to the walls at 450 K goes on to the coolant fluid at 370 K, finally ending up in atmospheric air at ambient 20°C. Find all the energy and exergy flows for this heat engine. Find also the exergy destruction and where that is done.

From the example in the text we get: $\dot{Q}_L = 0.7\,\dot{Q}_H = 233$ kW

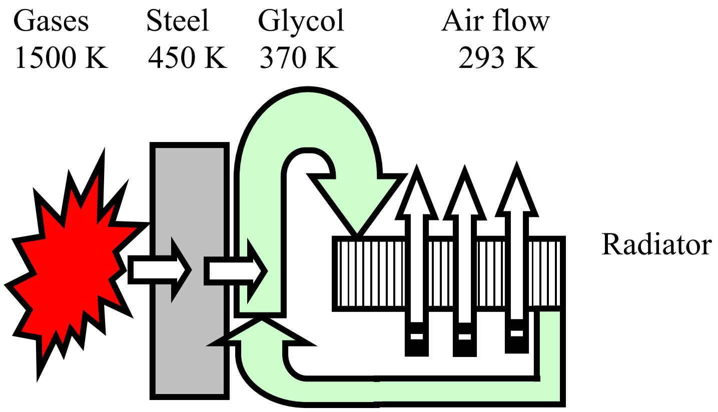
This is separated into two fluxes:

$\dot{Q}_{L1} = 0.4\,\dot{Q}_H = 133$ kW @900 K

$\dot{Q}_{L2} = 0.3\,\dot{Q}_H = 100$ kW @450 K

$= \dot{Q}_{L3} = 100$ kW @370 K

$= \dot{Q}_{L4} = 100$ kW @293 K



Assume all the fuel energy is delivered at 1500 K then that has an exergy of

$$\dot{\Phi}_{QH} = (1 - \frac{T_o}{T_H})\,\dot{Q}_H = (1 - \frac{293}{1500})\,333 = 267.9 \text{ kW}$$

Work is exergy

$$\dot{\Phi}_W = \dot{W} = 100 \text{ kW}$$

The exhaust flow exergy becomes

$$\dot{\Phi}_{QL1} = (1 - \frac{T_o}{T_{L1}})\,\dot{Q}_H = (1 - \frac{293}{900})\,133 = 89.7 \text{ kW}$$

The heat transfer through the coolant keeps dropping in temperature and are

$$\dot{\Phi}_{QL2} = (1 - \frac{T_o}{T_{L2}})\,\dot{Q}_{QL2} = (1 - \frac{293}{450})\,100 = 34.9\text{ kW}$$

$$\dot{\Phi}_{QL3} = (1 - \frac{T_o}{T_{L3}})\,\dot{Q}_{QL3} = (1 - \frac{293}{370})\,100 = 20.8\text{ kW}$$

$$\dot{\Phi}_{QL4} = (1 - \frac{T_o}{T_{L4}})\,\dot{Q}_{QL4} = (1 - \frac{293}{293})\,100 = 0\text{ kW}$$

The destruction terms are the drops in exergy fluxes. For the inside of the cylinder

$$\dot{\Phi}_{\text{Destruction inside}} = \dot{\Phi}_{QH} - \dot{W} - \dot{\Phi}_{QL1} - \dot{\Phi}_{QL2} = 43.3\text{ kW}$$

Outside the cylinder, the wall that separates the gas from the coolant gives

$$\dot{\Phi}_{\text{Destruction wall}} = \dot{\Phi}_{QL2} - \dot{\Phi}_{QL3} = 14.1\text{ kW}$$

The wall in the radiator separating the coolant from the air gives

$$\dot{\Phi}_{\text{Destruction radiator}} = \dot{\Phi}_{QL3} - \dot{\Phi}_{QL4} = 20.8\text{ kW}$$

10.88

Use the exergy balance equation to solve for the work in Problem 10.33.
A piston/cylinder has forces on the piston so it keeps constant pressure. It contains 2 kg of ammonia at 1 MPa, 40°C and is now heated to 100°C by a reversible heat engine that receives heat from a 200°C source. Find the work out of the heat engine.

Solution:

To evaluate it we need the change in exergy Eq.10.43

$$\Phi_2 - \Phi_1 = m_{am}(u_2 - u_1) + P_o(V_2 - V_1) - m_{am}T_o(s_2 - s_1)$$

The work in Eq.10.44 ($W = W_{H.E.} + {}_1W_{2,pist}$) is from the exergy Eq.10.42

$$W = P_o(V_2 - V_1) + (1 - \frac{T_o}{T_H})\,{}_1Q_2 - (\Phi_2 - \Phi_1) - 0$$

$$= (1 - \frac{T_o}{T_H})\,{}_1Q_2 - m_{am}(u_2 - u_1) + m_{am}T_o(s_2 - s_1)$$

Now we must evaluate the three terms on the RHS and the work ${}_1W_{2,pist}$.

State 1: $u_1 = 1369.8$ kJ/kg, $v_1 = 0.13868$ m^3/kg, $s_1 = 5.1778$ kJ/kg K

State 2: $u_2 = 1490.5$ kJ/kg, $v_2 = 0.17389$ m^3/kg, $s_2 = 5.6342$ kJ/kg K

$${}_1W_{2,pist} = m_{am}P(v_2 - v_1) = 2 \times 1000\ (0.17389 - 0.13868) = 70.42\text{ kJ}$$

C.V. Heat engine and ammnia (otherwise we involve another Q)

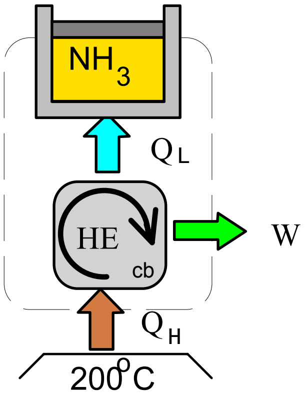
Entropy: $m_{am}(s_2 - s_1) = {}_1Q_2/T_H + 0$

$\Rightarrow\ {}_1Q_2 = T_H\, m_{am}(s_2 - s_1)$

$= 473.15 \times 2\ (5.6342 - 5.1778)$

$= 431.89$ kJ

Substitute this heat transfer into the work term



$$W = (1 - \frac{298.15}{473.15})\ 431.89 - 2(1490.5 - 1369.8) + 2\times298.15(5.6342 - 5.1778)$$

$$= 159.74 - 241.4 + 272.15 = 190.49\text{ kJ}$$

$W_{H.E.} = W - {}_1W_{2,pist} = 190.49 - 70.42 =$ **120.0 kJ**

Device Second-Law Efficiency

10.89

A heat engine receives 1 kW heat transfer at 1000 K and gives out 600 W as work with the rest as heat transfer to the ambient. Find its first and second law efficiencies.

First law efficiency is based on the energies

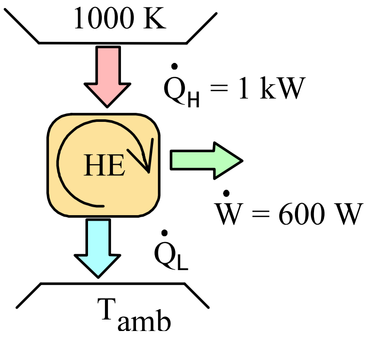
$$\eta_I = \dot{W}/\dot{Q}_H = \frac{0.6}{1} = \mathbf{0.6}$$

The second law efficiency is based on work out versus availability in

Exergy flux in: $\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H = \left(1 - \frac{298.15}{1000}\right) 1\text{ kW} = 0.702\text{ kW}$

$$\eta_{II} = \frac{\dot{W}}{\dot{\Phi}_H} = \frac{0.6}{0.702} = \mathbf{0.855}$$

Notice the exergy flux in is equal to the Carnot heat engine power output given 1 kW at 1000 K and rejecting energy to the ambient.



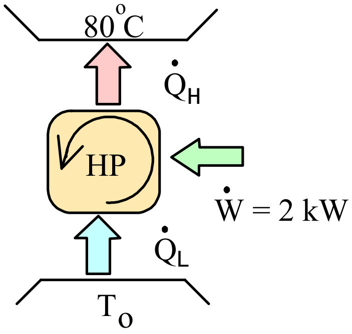
10.90

Find the second law efficiency of the heat pump in problem 10.55.

The second law efficiency is a ratio of exergies namely what we want out divided by what we have to put in. Exergy from first term on RHS Eq. 10.36

$$\dot{\Phi}_H = \left(1 - \frac{T_o}{T_H}\right) \dot{Q}_H; \qquad \dot{Q}_H = \beta\, \dot{W} = 2 \times 2 \text{ kW} = 4 \text{ kW}$$

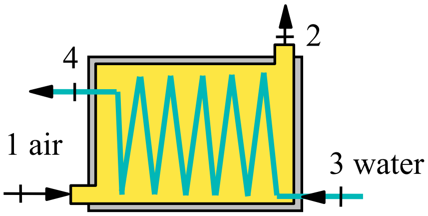
$$\eta_{II} = \frac{\dot{\Phi}_H}{\dot{W}} = \left(1 - \frac{T_o}{T_H}\right) \frac{\dot{Q}_H}{\dot{W}} = \left(1 - \frac{298.15}{353.15}\right) \frac{4}{2} = \mathbf{0.31}$$



10.91

A heat exchanger increases the availability of 3 kg/s water by 1650 kJ/kg using 10 kg/s air coming in at 1400 K and leaving with 600 kJ/kg less availability. What are the irreversibility and the second law efficiency?

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.



The irreversibility is the destruction of exergy (availability) so

$$\dot{I} = \dot{\Phi}_{\text{destruction}} = \dot{\Phi}_{\text{in}} - \dot{\Phi}_{\text{out}} = 10 \times 600 - 3 \times 1650 = \mathbf{1050\ kW}$$

The second law efficiency, Eq.10.32

$$\eta_{II} = \dot{\Phi}_{\text{out}} / \dot{\Phi}_{\text{in}} = \frac{3 \times 1650}{10 \times 600} = \mathbf{0.825}$$

10.92

A steam turbine inlet is at 1200 kPa, 500°C. The actual exit is at 300 kPa with an actual work of 407 kJ/kg. What is its second law efficiency?

The second law efficiency is the actual work out measured relative to the reversible work out, Eq. 10.29.

Steam turbine $T_o = 25°\text{C} = 298.15\text{ K}$

Inlet state: Table B.1.3 $h_i = 3476.28\text{ kJ/kg}$; $s_i = 7.6758\text{ kJ/kg K}$

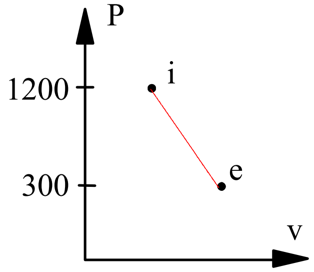
Actual turbine energy Eq.: $h_e = h_i - w_{ac} = 3476.28 – 407 = 3069.28\text{ kJ/kg}$

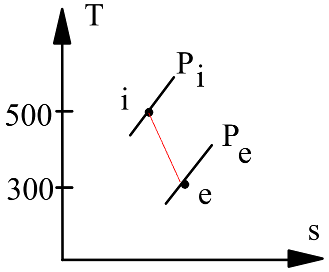
Actual exit state: Table B.1.3 $T_e = 300^o\text{C}$; $s_e = 7.7022\text{ kJ/kg K}$

From Eq.10.9,

$$w^{rev} = (h_i - T_o s_i) – (h_e - T_o s_e) = (h_i - h_e) + T_o(s_e - s_i)$$
$$= (3476.28 – 3069.28) + 298.15(7.7022 – 7.6758)$$
$$= 407 + 7.87 = 414.9\text{ kJ/kg}$$

$\eta_{II} = w_{ac}/w^{rev} = 407 / 414.9 = \mathbf{0.98}$





10.93

Find the isentropic efficiency and the second law efficiency for the compressor in Problem 10.24.

A compressor in a refrigerator receives R-410a at 150 kPa, -40°C and it brings it up to 600 kPa, 40°C in an adiabatic compression. Find the specific reversible work.

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w = h_1 - h_2$

Entropy Eq.9.8: $s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

States: 1: B.4.2 $h_1 = 263.99$ kJ/kg, $s_1 = 1.1489$ kJ/kg K

2: B.4.2 $h_2 = 322.64$ kJ/kg, $s_2 = 1.2152$ kJ/kg K

2s: B.4.2 $(P, s = s_1)$ $h_{2,s} = 302.67$ kJ/kg

IDEAL: $-w_{c,s} = h_{2,s} - h_1 = 38.68$ kJ/kg

ACTUAL: $-w_{C,AC} = h_{2,AC} - h_1 = 58.65$ kJ/kg

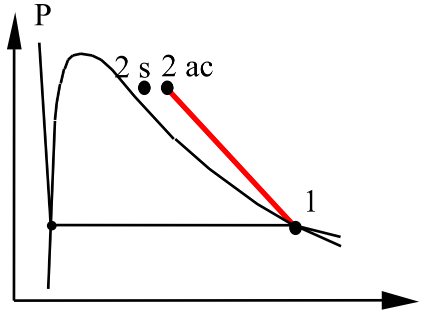
Definition Eq.9.27: $\eta_c = w_{c,s}/w_{c,AC} = \mathbf{0.659}$

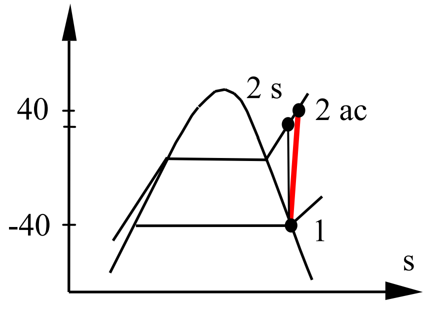
Rev. work Eq.9.39: $w_{rev} = \psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2)$

$= 263.99 - 322.64 - 298.15\ (1.1489 - 1.2152)$

$= -58.65 + 19.767 = -38.9$ kJ/kg

2nd law efficiency: $\eta_{II} = \frac{w_{rev}}{w_{ac}} = \frac{-38.9}{-58.65} = \mathbf{0.663} \quad \left(= \frac{w_{ac} + i}{w_{ac}} \right)$





10.94

A steam turbine has inlet at 4 MPa, 500°C and actual exit of 100 kPa, x = 1.0. Find its first law (isentropic) and its second law efficiencies.

Solution:

C.V. Steam turbine

Energy Eq.6.13: $w = h_i - h_e$

Entropy Eq.9.8: $s_e = s_i + s_{gen}$

Inlet state: Table B.1.3 $h_i = 3445.2$ kJ/kg; $s_i = 7.0900$ kJ/kg K

Exit (actual) state: Table B.1.2 $h_e = 2675.5$; $s_e = 7.3593$ kJ/kg K

Actual turbine energy equation

$$w = h_i - h_e = 769.7 \text{ kJ/kg}$$

Ideal turbine reversible process so $s_{gen} = 0$ giving

$$s_{es} = s_i = 70900 = 1.3025 + x_{es} \times 6.0568$$

$$x_{es} = 0.9555, \quad h_{es} = 417.4 + 0.9555 \times 2258.0 = 2575.0 \text{ kJ/kg}$$

The energy equation for the ideal gives

$$w_s = h_i - h_{es} = 870.2 \text{ kJ/kg}$$

The first law efficiency is the ratio of the two work terms

$$\eta_s = w/w_s = \mathbf{0.885}$$

The reversible work for the actual turbine states is, Eq.10.9

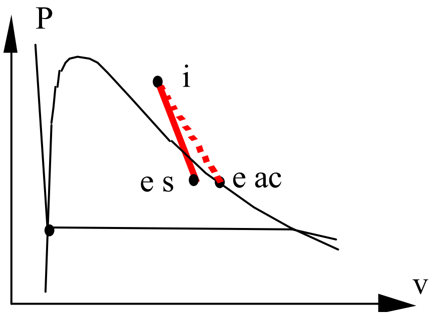
$$w^{rev} = (h_i - h_e) + T_o(s_e - s_i)$$

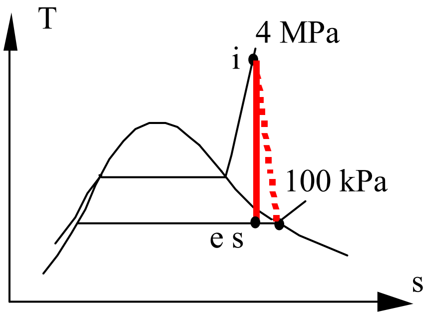
$$= 769.7 + 298.2(7.3593 – 7.0900)$$

$$= 769.7 + 80.3 = 850.0 \text{ kJ/kg}$$

Second law efficiency Eq.10.29

$$\eta_{2^{nd}\ Law} = w/w_{rev} = 769.7/850.0 = \mathbf{0.906}$$





10.95

Steam enters a turbine at 25 MPa, 550°C and exits at 5 MPa, 325°C at a flow rate of 70 kg/s. Determine the total power output of the turbine, its isentropic efficiency and the second law efficiency.

Solution:

$h_i = 3335.6$ kJ/kg, $s_i = 6.1765$ kJ/kg K,

$h_e = 2996.5$ kJ/kg, $s_e = 6.3289$ kJ/kg K

Actual turbine: $w_{T,ac} = h_i - h_e = 339.1$ kJ/kg

$$\dot{W} = \dot{m}\, w_{T,ac} = 70 \times 339.1 = \mathbf{23\ 740\ kW}$$

Isentropic turbine has a different exit state: $s_{e,s} = s_i \Rightarrow h_{e,s} = 2906.6$ kJ/kg

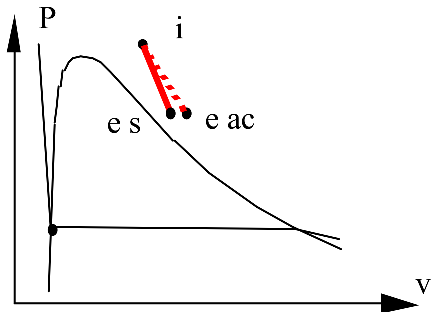
$$w_{T,s} = h_i - h_{e,s} = 429 \text{ kJ/kg}$$

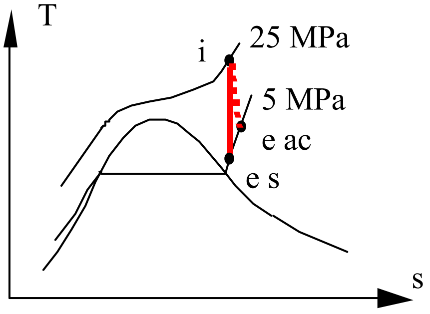
Rev. turbine has the same exit state as the actual turbine:

$$w_{rev} = w_{T,ac} + T_0(s_e - s_i) = 339.1 + 45.44 = 384.54 \text{ kJ/kg}$$

Eq.9.27: $\eta_T = w_{T,ac}/w_{T,s} = 339.1/429 = \mathbf{0.79}$

Eq.10.29: $\eta_{II} = w_{T,ac}/w_{rev} = 339.1/384.54 = \mathbf{0.88}$





10.96

Find the isentropic and the second law efficiencies for a turbine. It receives steam at 3000 kPa, 500°C and has two exit flows, one at 1000 kPa, 350°C with 20% of the flow and the remainder at 200 kPa, 200°C.

C.V. Steam turbine ($x = 0.2$ = extraction fraction)

Energy Eq.6.13: $w = h_1 - xh_2 - (1 - x)h_3$

Entropy Eq.9.8: $s_e = s_1 + s_{gen}$

Inlet state: Table B.1.3 $h_1 = 3456.48$ kJ/kg; $s_1 = 7.2337$ kJ/kg K

Extraction state: $h_2 = 3157.65$ kJ/kg, $s_2 = 7.3010$ kJ/kg K

Exit (actual) state: Table B.1.3 $h_3 = 2870.46$; $s_3 = 7.5066$ kJ/kg K

Actual turbine energy equation

$$w = 3456.48 - 0.2 \times 3157.65 - 0.8 \times 2870.46 = \mathbf{528.58\ kJ/kg}$$

Ideal turbine reversible process so $s_{gen} = 0$ giving

$$s_{2s} = s_1 = 7.2337 \Rightarrow h_{2s} = 3117.43 \text{ kJ/kg}$$

$$s_{3s} = s_1 = 7.2337 \Rightarrow h_{3s} = 2750.12 \text{ kJ/kg}$$

The energy equation for the ideal turbine gives

$$w_s = h_1 - xh_{2s} - (1 - x)h_{3s}$$

$$= 3456.48 - 0.2 \times 3117.43 - 0.8 \times 2750.12 = 632.9 \text{ kJ/kg}$$

The first law efficiency is the ratio of the two work terms

$$\eta_s = w/w_s = \mathbf{0.835}$$

The reversible work for the actual turbine states is, Eq.9.39

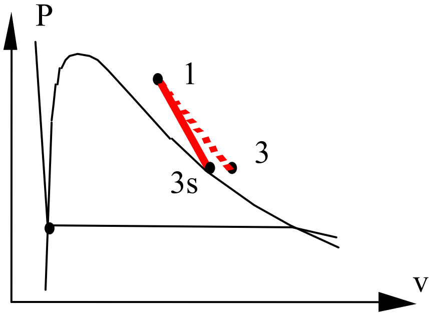
$$w^{rev} = [h_1 - xh_2 - (1 - x)h_3] - T_o[\, s_1 - xs_2 - (1 - x)s_3]$$

$$= 528.58 - 298.2(7.2337 - 0.2 \times 7.301 - 0.8 \times 7.5066)$$

$$= 528.58 + 69.1 = 597.68 \text{ kJ/kg}$$

Second law efficiency Eq.9.44

$$\eta_{II} = w/w_{rev} = 528.58/597.68 = \mathbf{0.884}$$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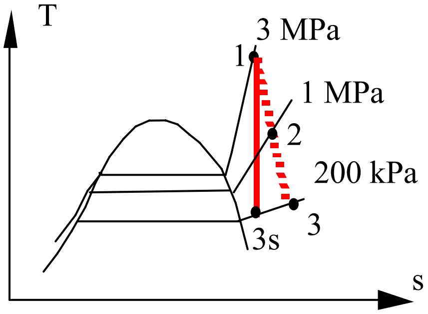

T
3 MPa
1
1 MPa
2
200 kPa
3
3s
s

10.97

A heat engine operating with an evironment at 298 K produces 5 kW of power output with a first law efficiency of 50%. It has a second law efficiency of 80% and T_L = 310 K. Find all the energy and exergy transfers in and out.

Solution:

From the definition of the first law efficiency (conversion efficiency)

$$\dot{Q}_H = \dot{W} / \eta_I = \frac{5}{0.5} = \mathbf{10\ kW}$$

Energy Eq.: $\dot{Q}_L = \dot{Q}_H - \dot{W} = 10 – 5 = \mathbf{5\ kW}$

Exergy output: $\dot{\Phi}_W = \dot{W} = \mathbf{5\ kW}$

From the definition of the second law efficiency $\eta_{II} = \dot{W}/\dot{\Phi}_H$, this requires that we assume the availability rejected at 310 K is lost and not counted, otherwise the efficiency should be $\eta_{II} = \dot{W}/(\dot{\Phi}_H - \dot{\Phi}_L)$.

Exergy from source: $\dot{\Phi}_H = (1 - \frac{T_o}{T_H})\, \dot{Q}_H = \frac{5}{0.8} = \mathbf{6.25\ kW}$

Exergy rejected: $\dot{\Phi}_L = (1 - \frac{T_o}{T_L})\, \dot{Q}_L = (1 - \frac{298}{310})\ 5 = \mathbf{0.194\ kW}$

Notice from the $\dot{\Phi}_H$ form we could find the single characteristic T_H as

$$(1 - \frac{T_o}{T_H}) = 6.25\text{ kW} / \dot{Q}_H = 0.625 \quad => \quad T_H = 795\text{ K}$$

10.98

A steam turbine inlet is at 1200 kPa, 500°C. The actual exit is at 200 kPa, 300°C. What are the isentropic efficiency and its second law efficiency?

Solution:

C.V. Turbine actual, steady state and adiabatic.

Inlet state: Table B.1.3: $h_i = 3476.28$ kJ/kg, $s_i = 7.6758$ kJ/kg K

Exit state: Table B.1.3: $h_e = 3071.79$ kJ/kg, $s_e = 7.8926$ kJ/kg K

Energy Eq.: $w_{Tac} = h_i - h_e = 3476.28 - 3071.79 = 404.49$ kJ/kg

C.V. Turbine isentropic, steady state, reversible and adiabatic.

Isentropic exit state: 200 kPa, $s = s_i$ => $h_{es} = 2954.7$ kJ/kg

Energy eq.: $w_{T\,s} = h_i - h_{es} = 3476.28 - 2954.7 = 521.58$ kJ/kg

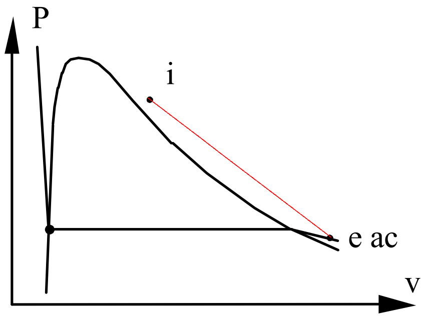
$$\eta_I = w_{Tac}/w_{T\,s} = \frac{404.49}{521.58} = \mathbf{0.776}$$

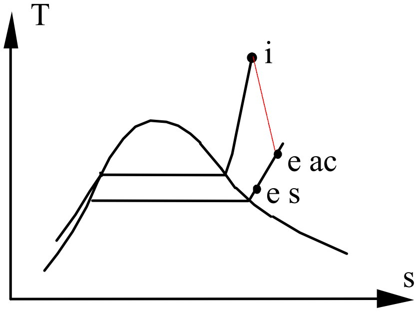
Reversible work for actual turbine is from Eq.10.9 or 10.39

$$w_T^{rev} = \psi_i - \psi_e = h_i - h_e - T_0(s_i - s_e) = w_{Tac} - T_0(s_i - s_e)$$
$$= 404.49 - 298.15(7.6758 - 7.8926) = 469.13 \text{ kJ/kg}$$

Then the second law efficiency is in Eq.10.29

$$\eta_{II} = w_{Tac}/w_T^{rev} = \frac{404.49}{469.13} = \mathbf{0.862}$$





10.99

Air flows into a heat engine at ambient conditions 100 kPa, 300 K, as shown in Fig. P10.99. Energy is supplied as 1200 kJ per kg air from a 1500 K source and in some part of the process a heat transfer loss of 300 kJ/kg air happens at 750 K. The air leaves the engine at 100 kPa, 800 K. Find the first and the second law efficiencies.

C.V. Engine out to reservoirs

$$h_i + q_{1500} = q_{750} + h_e + w$$

$$w_{ac} = h_i + q_{1500} - q_{750} - h_e$$

$$= 300.47 + 1200 - 300 - 822.20 = 378.27 \text{ kJ/kg}$$

$$\eta_{TH} = w/q_{1500} = \mathbf{0.3152}$$

For second law efficiency also a q to/from ambient

$$s_i + (q_{1500}/T_H) + (q_0/T_0) = (q_{750}/T_m) + s_e$$

$$q_0 = T_0(s_e - s_i) + (T_0/T_m)q_{750} - (T_0/T_H)q_{1500}$$

$$= 300\left(7.88514 - 6.86925 - 0.287 \ln\frac{100}{100}\right) + \frac{300}{750}\,300$$

$$-(300/1500)\ 1200 = 184.764 \text{ kJ/kg}$$

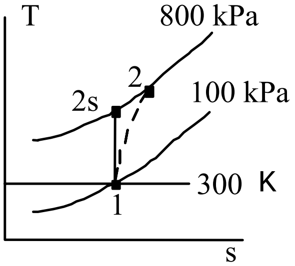
$$w^{rev} = h_i - h_e + q_{1500} - q_{750} + q_0 = w_{ac} + q_0 = 563.03 \text{ kJ/kg}$$

$$\eta_{II} = w_{ac}/w_{rev} = 378.27/563.03 = \mathbf{0.672}$$

10.100

Air enters a compressor at ambient conditions, 100 kPa, 300 K, and exits at 800 kPa. If the isentropic compressor efficiency is 85%, what is the second-law efficiency of the compressor process?

Solution:



Ideal (isentropic, Eq.8.32)

$T_{2s} = 300(8)^{0.286} = 543.8$ K

$-w_s = 1.004(543.8 - 300) = 244.6$ kJ/kg

$$-w = \frac{-w_s}{\eta_s} = \frac{244.6}{0.85} = 287.8 \text{ kJ/kg K}$$

$$T_2 = T_1 + \frac{-w}{C_{P0}} = 300 + \frac{287.8}{1.004} = 586.8 \text{ K}$$

Eq.8.25: $s_2 - s_1 = 1.004 \ln(586.8/300) - 0.287 \ln 8 = 0.07645$

Availability, Eq.10.24

$\psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) = 287.8 - 300(0.07645) = 264.9$ kJ/kg

2nd law efficiency, Eq.10.29 or 10.30 (but for a compressor):

$$\eta_{2\text{nd Law}} = \frac{\psi_2 - \psi_1}{-w} = \frac{264.9}{287.8} = \mathbf{0.92}$$

10.101

A compressor is used to bring saturated water vapor at 1 MPa up to 17.5 MPa, where the actual exit temperature is 650°C. Find the irreversibility and the second-law efficiency.

Solution:

Inlet state: Table B.1.2 $h_i = 2778.1$ kJ/kg, $s_i = 6.5864$ kJ/kg K

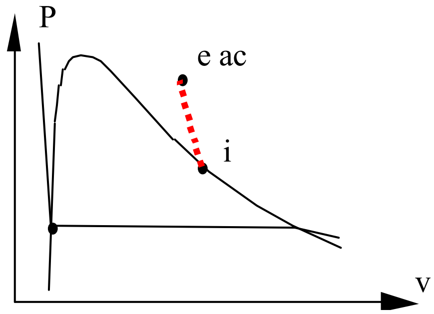
Actual compressor Table B.1.3: $h_{e,ac} = 3693.9$ kJ/kg, $s_{e,ac} = 6.7356$ kJ/kg K

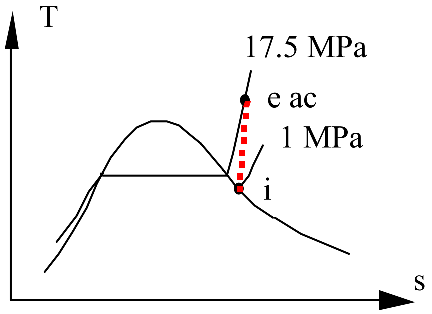
Energy Eq. Actual compressor: $-w_{c,ac} = h_{e,ac} - h_i = 915.8$ kJ/kg

From Eq.10.11: $i = T_0(s_{e,ac} - s_i) = 298.15\ (6.7356 - 6.5864) = \mathbf{44.48\ kJ/kg}$

From Eq.10.10: $w_{rev} = i + w_{c,ac} = -915.8 + 44.48 = -871.32$ kJ/kg

$\eta_{II} = -w_{rev}/w_{c,ac} = 871.32/915.8 = \mathbf{0.951}$





10.102

Use the exergy equation to analyze the compressor in Example 6.10 to find its second law efficiency assuming an ambient at 20^{o}C.

C.V. The R-134a compressor. Steady flow. We need to find the reversible work and compare that to the actual work.

Notice the heat loss goes out to ambient T and has thus 0 exergy.

Exergy eq.: 10.36: $0 = \dot{m}(\psi_1 - \psi_2) + (-\dot{W}^{rev}_{comp}) + 0$

$$-\dot{W}^{rev}_{comp} = \dot{m} [h_2 - h_1 - T_o (s_2 - s_1)]$$

$$= 0.1 \text{ kg/s} [435.1 – 387.2 \;\; – 293.15 \text{ K} \times (1.7768 – 1.7665) \frac{\text{kJ}}{\text{kg K}}]$$

$$= 4.5 \text{ kW}$$

$$\eta_{II} = -\dot{W}^{rev}_{comp} / -\dot{W}^{ac}_{comp} = \frac{4.5}{5} = \mathbf{0.90}$$

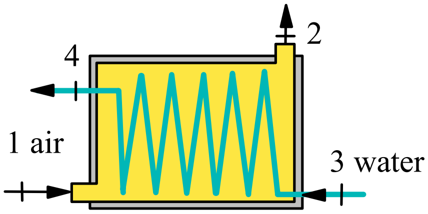
For a real device this is a little high.

10.103

Calculate the second law efficiency of the counter flowing heat exchanger in Problem 9.99 with an ambient at 20°C.

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.



Heat exchanger Prob 9.61 with $T_o = 20°C$ solve first for state 4.

Energy Eq.6.10: $\dot{m}_{AIR}\Delta h_{AIR} = \dot{m}_{H2O}\Delta h_{H2O}$

From A.7: $h_1 - h_2 = 1046.22 – 401.3 = 644.92$ kJ/kg

From B.1.2 $h_3 = 83.94$ kJ/kg; $s_3 = 0.2966$ kJ/kg K

$$h_4 - h_3 = (\dot{m}_{AIR}/\dot{m}_{H2O})(h_1 - h_2) = (2/0.5)644.92 = 2579.68 \text{ kJ/kg}$$

$$h_4 = h_3 + 2579.68 = 2663.62 < h_g \quad \text{at 200 kPa}$$

$$T_4 = T_{sat} = 120.23°C,$$

$$x_4 = (2663.62 – 504.68)/2201.96 = 0.9805,$$

$$s_4 = 1.53 + x_4\, 5.597 = 7.01786 \text{ kJ/kg K}$$

We need the change in availability for each flow from Eq.10.24

$$(\psi_1 - \psi_2) = (h_1 - h_2) + T_o(s_2 - s_1)$$

$$= (1046.2 – 401.3) + 293.2(7.1593 – 8.1349 – 0.287 \ln(100/125)$$

$$= 644.9 + 293.2(-0.91156) = 377.6 \text{ kJ/kg}$$

$$(\psi_4 - \psi_3) = (h_4 - h_3) + T_o(s_4 - s_3)$$

$$= (2663.6 – 83.9) – 293.2(7.0179 – 0.2966)$$

$$= 2579.9 – 1970.7 = 609.0$$

Efficiency from Eq.10.30

$$\eta_{2^{nd}\,Law} = [\dot{m}_w(\psi_4 - \psi_3)]/[\dot{m}_A(\psi_1 - \psi_2)]$$

$$= (0.5 \times 609.0)/(2 \times 377.6) = \mathbf{0.403}$$

10.104

An air-compressor receives air at 290 K, 100 kPa and brings it up to a higher pressure in an adiabatic process. The actual specific work is 210 kJ/kg and the isentropic efficiency is 82%. Find the exit pressure and the second law efficiency.

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $w = h_1 - h_2$

Entropy Eq.9.8: $s_2 = s_1 + s_{gen}$

We have two different cases, the ideal and the actual compressor.

$$-w_{C,AC} = h_{2,AC} - h_1 = 210 \text{ kJ/kg} = w_{c,s} / \eta_c$$

$$T_{2,AC} = T_1 - w_{C,AC} / C_P = 290 + 210 / 1.004 = 499.2 \text{ K}$$

For the isentropic compressor we get

$$-w_{c,s} = -\eta_c\, w_{C,AC} = 0.82 \times 210 = 172.2 \text{ kJ/kg} = h_{2,s} - h_1$$

$$T_{2,s} = T_1 - w_{c,s}/ C_P = 290 + 172.2/1.004 = 461.5 \text{ K}$$

$$P_2 = P_1 (T_{2,s} / T_1)^{k/k-1} = 100 (461.5/290)^{3.5} = \mathbf{508.4\ kPa}$$

$$w^{rev} = (\psi_1 - \psi_{2,AC}) = (h_1 - h_{2,AC}) + T_o(s_{2,AC} - s_1)$$

$$= w_{C,AC} + T_o[\, C_P \ln(T_{2,ac} / T_1) - R \ln (P_2 / P_1)]$$

$$= -210 + 298.15 \left[\, 1.004 \ln(\frac{290}{499.2}) - 0.287 \ln(\frac{508.4}{100}) \right]$$

$$= -210 + 23.4 = -\,186.6 \text{ kJ/kg}$$

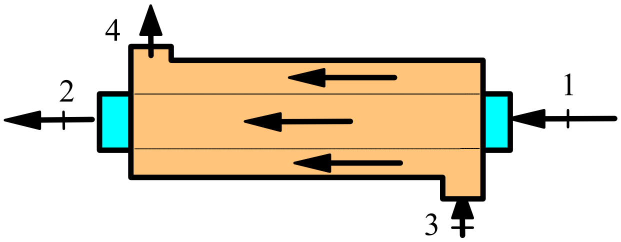
$$\eta_{II} = w^{rev} / w_{C,AC} = \frac{-186.6}{-210} = \mathbf{0.88}$$

10.105

Calculate the second law efficiency of the coflowing heat exchanger in Problem 9.62 with an ambient at 17°C.

Solution:

C.V. Heat exchanger, steady 2 flows in and two flows out.



First solve for the exit temperature in Problem 9.62

C.V. Heat exchanger, steady 2 flows in and two flows out.

Energy Eq.6.10: $\dot{m}_{O2}h_1 + \dot{m}_{N2}h_3 = \dot{m}_{O2}h_2 + \dot{m}_{N2}h_4$

Same exit tempearture so $T_4 = T_2$ with values from Table A.5

$$\dot{m}_{O2}C_{P\ O2}T_1 + \dot{m}_{N2}C_{P\ N2}T_3 = (\dot{m}_{O2}C_{P\ O2} + \dot{m}_{N2}C_{P\ N2})T_2$$

$$T_2 = \frac{0.25 \times 0.922 \times 290 + 0.6 \times 1.042 \times 500}{0.25 \times 0.922 + 0.6 \times 1.042} = \frac{379.45}{0.8557} = 443.4 \text{ K}$$

The second law efficiency for a heat exchanger is the ratio of the availability gain by one fluid divided by the availability drop in the other fluid. We thus have to find the change of availability in both flows.

For each flow availability in Eq.10.24 include mass flow rate as in Eq.10.36

For the oxygen flow:

$$\dot{m}_{O2}(\psi_2 - \psi_1) = \dot{m}_{O2} [\, h_2 - h_1 - T_o (\, s_2 - s_1 \,) \,]$$

$$= \dot{m}_{O2} [\, C_P(T_2 - T_1) - T_o [\, C_P \ln(T_2 / T_1) - R \ln(P_2 / P_1) \,]$$

$$= \dot{m}_{O2}C_P [\, T_2 - T_1 - T_o\ln(T_2 / T_1) \,]$$

$$= 0.25 \times 0.922 [\, 443.4 - 290 - 290 \ln(443.4/290) \,]$$

$$= 6.977 \text{ kW}$$

For the nitrogen flow

$$\dot{m}_{N2}(\psi_3 - \psi_4) = \dot{m}_{N2}C_P [\, T_3 - T_4 - T_o\ln(T_3 / T_4) \,]$$

$$= 0.6 \times 1.042 [\, 500 - 443.4 - 290 \ln(500/443.4) \,] = 13.6 \text{ kW}$$

From Eq.10.30

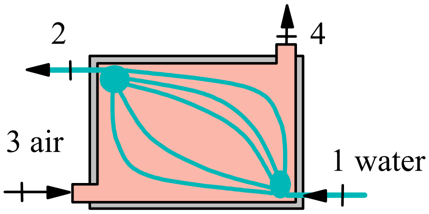
$$\eta_{2^{nd}\text{ Law}} = \frac{\dot{m}_{O2}(\psi_1 - \psi_2)}{\dot{m}_{N2}(\psi_3 - \psi_4)} = \frac{6.977}{13.6} = \mathbf{0.513}$$

10.106

A flow of 2 kg/s water at 1000 kPa, 80°C goes into a constant pressure boiler where the water is heated to 400°C. Assume the hot gases that heats the water is air coming in at 1200 K and leaving at 900 K as in a counter flowing heat exchanger. Find the total rate of irreversibility in the process and the second law efficiency of the boiler setup.

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside. We need to find the air mass flow rate.



Energy Eq.: $\dot{m}_{H2O}(h_2 - h_1) = \dot{m}_{air}(h_3 - h_4)$

$$\dot{m}_{air} = \dot{m}_{H2O} \frac{h_2 - h_1}{h_3 - h_4} = 2 \frac{3263.88 - 335.64}{1277.81 - 933.15} = 16.992 \text{ kg/s}$$

Availability increase of the water flow

$$\dot{m}_{H2O}(\psi_2 - \psi_1) = \dot{m}_{H2O}[h_2 - h_1 - T_o(s_2 - s_1)]$$
$$= 2\ [3263.88 - 335.64 - 298.15(7.465 - 1.0746)]$$
$$= 2\ [\ 2928.24 - 1905.3\ ] = 2045.9 \text{ kW}$$

Availability decrease of the air flow

$$\dot{m}_{air}(\psi_3 - \psi_4) = \dot{m}_{air}[h_3 - h_4 - T_o(s_3 - s_4)]$$
$$= 16.992\ [1277.81 - 933.15 - 298.15(8.34596 - 8.01581)]$$
$$= 4183.9 \text{ kW}$$

The irreversibility is the destruction of exergy

$$\dot{I} = \dot{\Phi}_{destruction} = \dot{\Phi}_{in} - \dot{\Phi}_{out} = 4183.9 \text{ kW} - 2045.9 \text{ kW} = \mathbf{2138\ kW}$$

$$\eta_{2^{nd}\ Law} = \frac{\dot{m}_{H2O}(\psi_2 - \psi_1)}{\dot{m}_{air}(\psi_3 - \psi_4)} = \frac{2045.9}{4183.9} = \mathbf{0.489}$$

10.107

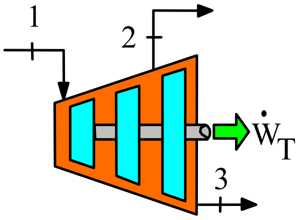
A steam turbine receives 5 kg/s steam at 400°C, 10 MPa. One flow of 0.8 kg/s is extracted at 2.5 MPa as saturated vapor and the remainder runs out at 1500 kPa with a quality of 0.975. Find the second law efficiency of the turbine.

C.V. Turbine. Steady flow and adiabatic q = 0.

Continuity Eq.6.9: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$;

Energy Eq.6.10: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$

Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3$



State 1: $h_1 = 3096.46$ kJ/kg, $s_1 = 6.2119$ kJ/kg-K

State 2: $h_2 = 2803.07$ kJ/kg, $s_2 = 6.2574$ kJ/kg-K

State 3: $h_3 = 844.87 + 0.975 \times 1947.28 = 2743.5$ kJ/kg,

$s_3 = 2.315 + 0.975 \times 4.1298 = 6.34156$ kJ/kg-K

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 5 \times 3096.46 - 0.8 \times 2803.07 - 4.2 \times 2743.5$$
$$= 1717 \text{ kW}$$

This work is now compared to the reversible work possible

$$\dot{W}^{rev} = \dot{m}_1 \psi_1 - \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3$$
$$= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 - T_o \left[\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 \right]$$
$$= 1717 \text{ kW} - 298.15 \left[5 \times 6.2119 - 0.8 \times 6.2574 - 4.2 \times 6.34156\right] \text{ kW}$$
$$= 1717 \text{ kW} + 173.2 \text{ kW} = 1890 \text{ kW}$$

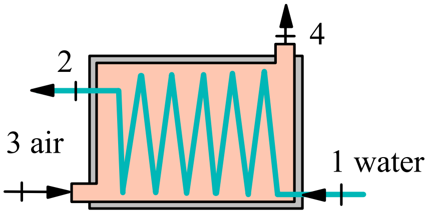
$$\eta_{2^{nd}\text{ Law}} = \dot{W} / \dot{W}^{rev} = \frac{1717}{1890} = \mathbf{0.908}$$

10.108

A heat exchanger brings 10 kg/s water from 100°C to 500°C at 2000 kPa using air coming in at 1400 K and leaving at 460 K. What is the second law efficiency?

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside. We need to find the air mass flow rate.



Energy Eq.: $\dot{m}_{H2O}(h_2 - h_1) = \dot{m}_{air}(h_3 - h_4)$

$$\dot{m}_{air} = \dot{m}_{H2O}\,\frac{h_2 - h_1}{h_3 - h_4} = 10\,\frac{3467.55 - 420.45}{1515.27 - 462.34} = 28.939 \text{ kg/s}$$

Availability increase of the water flow

$$\dot{m}_{H2O}(\psi_2 - \psi_1) = \dot{m}_{H2O}[h_2 - h_1 - T_o(s_2 - s_1)]$$
$$= 10\,[\,3467.55 – 420.45 – 298.15(7.4316 – 1.3053)]$$
$$= 10\,[\,3047.1 – 1826.56\,] = 12\,205 \text{ kW}$$

Availability decrease of the air flow

$$\dot{m}_{air}(\psi_3 - \psi_4) = \dot{m}_{air}[h_3 - h_4 – T_o(s_3 - s_4)]$$
$$= 28.939\,[1515.27 – 462.34 – 298.15(8.52891 – 7.30142)]$$
$$= 19\,880 \text{ kW}$$

$$\eta_{2^{nd}\text{ Law}} = \frac{\dot{m}_{H2O}(\psi_2 - \psi_1)}{\dot{m}_{air}(\psi_3 - \psi_4)} = \frac{12\,205}{19\,880} = \mathbf{0.614}$$

Review Problems

10.109

Calculate the irreversibility for the process described in Problem 6.133, assuming that heat transfer is with the surroundings at 17°C.

Solution:

C.V. Cylinder volume out to $T_o = 17\ ^oC$.

Continuity Eq.6.15: $m_2 - m_1 = m_{in}$

Energy Eq.6.16: $m_2u_2 - m_1u_1 = m_{in}h_{line} + {}_1Q_2 - {}_1W_2$

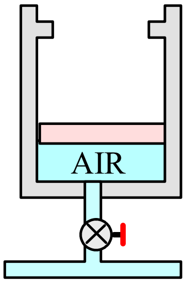
Entropy Eq.9.12: $m_2\, s_2 - m_1s_1 = m_is_i + {}_1Q_2 / T_o + {}_1S_{2\ gen}$

Process: P_1 is constant to stops, then constant V to state 2 at P_2

State 1: P_1, T_1 $\quad m_1 = \dfrac{P_1V}{RT_1} = \dfrac{300 \times 0.25}{0.287 \times 290.2} = 0.90\text{ kg}$

State 2:

Open to $P_2 = 400\text{ kPa}, T_2 = 350\text{ K}$



$$m_2 = \frac{400 \times 1}{0.287 \times 350} = 3.982\text{ kg}$$

$$m_i = 3.982 - 0.90 = 3.082\text{ kg}$$

Only work while constant P

$${}_1W_2 = P_1(V_2 - V_1) = 300(1 - 0.25) = 225\text{ kJ}$$

Energy eq.:

$${}_1Q_2 = m_2u_2 - m_1u_1 + {}_1W_2 - m_ih_i$$
$$= 3.982 \times 0.717 \times 350 - 0.90 \times 0.717 \times 290.2 + 225$$
$$- 3.082 \times 1.004 \times 600 = -819.2\text{ kJ}$$

Entropy eq. gives

$$T_o\ {}_1S_{2\ gen} = I = T_o\ [\ m_1\ (s_2 - s_1) + m_i\ (s_2 - s_i)] - {}_1Q_2$$

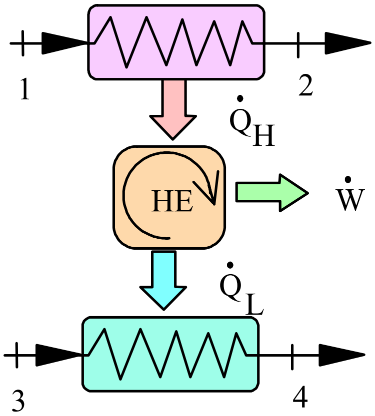
$$= 290.15[0.9(C_p \ln \frac{350}{290} - R \ln \frac{400}{300}) + 3.082(C_p\ln \frac{350}{600} - R \ln \frac{400}{500})]$$
$$- (- 819.2\text{ kJ})$$

$$= 290.15\ (0.0956 - 1.4705) + 819.2$$

$$= \mathbf{420.3\ kJ}$$

10.110

The high-temperature heat source for a cyclic heat engine is a steady flow heat exchanger where R-134a enters at 80°C, saturated vapor, and exits at 80°C, saturated liquid at a flow rate of 5 kg/s. Heat is rejected from the heat engine to a steady flow heat exchanger where air enters at 150 kPa and ambient temperature 20°C, and exits at 125 kPa, 70°C. The rate of irreversibility for the overall process is 175 kW. Calculate the mass flow rate of the air and the thermal efficiency of the heat engine.



C.V. R-134a Heat Exchanger,

$\dot{m}_{R134a} = 5$ kg/s, Table B.5.1

Inlet: $T_1 = 80^{o}C$, sat. vapor $x_1 = 1.0$,

$h_1 = h_g = 429.189$ kJ/kg,

$s_1 = s_g = 1.6862$ kJ/kg-K

Exit: $T_2 = 80^{o}C$, sat. liquid $x_2 = 0.0$

$h_2 = h_f = 322.794$ kJ/kg,

$s_2 = s_f = 1.3849$ kJ/kg-K

C.V. Air Heat Exchanger, $C_p = 1.004$ kJ/kg-K, $R = 0.287$ kJ/kg-K

Inlet: $T_3 = 20^{o}C$, $P_3 = 150$ kPa Exit: $T_4 = 70^{o}C$, $P_4 = 125$ kPa

$$s_4 - s_3 = C_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{P_4}{P_3}\right) = 0.2103 \text{ kJ/kg-K}$$

Entropy Eq. for the total system as control volume (since we know $\dot{I}$):

$$\dot{I} / T_o = \dot{S}_{net} = \dot{m}_{R134a} (s_2 - s_1) + \dot{m}_{air}(s_4 - s_3)$$

$$\dot{m}_{air} = [\dot{I}/T_o - \dot{m}_{R134a} (s_2 - s_1)]/(s_4 - s_3) = \mathbf{10.0\ kg/s}$$

Energy Eq. for each line: $\dot{Q} + \dot{m}h_{in} = \dot{m}h_{ex} + \dot{W}$; $\dot{W} = 0$

R-134a: ${}_1\dot{Q}_2 = -\dot{Q}_H = \dot{m}_{R134a}(h_2 - h_1) = -532$ kW

Air: $\dot{Q}_L = {}_3\dot{Q}_4 = \dot{m}_{air}(h_4 - h_3) = \dot{m}_{air} C_p(T_4 - T_3) = 501.8$ kW

Control volume heat engine

$$\dot{W}_{net} = \dot{Q}_H - \dot{Q}_L = 532 - 501.8 = 30.2 \text{ kW};$$

$$\eta_{th} = \dot{W}_{net} / \dot{Q}_H = \mathbf{0.057}, \quad \textbf{or 5.7\%}$$

10.111

Calculate the availability of the system (aluminum plus gas) at the initial and final states of Problem 8.137, and also the process irreversibility.

State 1: $T_1 = 200\ ^oC$, $v_1 = V_1/\ m = 0.05\ /\ 1.1186 = 0.0447\ m^3/kg$

State 2: $v_2 = v_1 \times (2\ /\ 1.5) \times (298.15\ /\ 473.15) = 0.03756\ m^3/kg$

The metal does not change volume, so the combined is using Eq.10.22 as

$$\phi_1 = m_{gas}\phi_{gas} + m_{Al}\phi_{Al}$$

$$= m_{gas}[u_1\text{-}u_o\text{-}T_o(s_1 - s_o)]_{cv} + m_{gas}P_o(v_1\text{-}v_o) + m_{Al}[u_1\text{-}u_o\ \text{-}T_o(s_1\text{-}s_o)]_{Al}$$

$$= m_{gas}C_v\ (T_1 - T_o) - \ m_{gas}T_o\ [C_p \ln\frac{T_1}{T_o} - R \ln\frac{P_1}{P_o}\ ] + m_{gas}P_o\ (v_1 - v_o)$$

$$+ m_{Al}\ [C\ (T_1 - T_o) - T_oC \ln\ (T_1/T_o)\ ]_{Al}$$

$$\phi_1 = 1.1186\ [\ 0.653(200\text{-}25) - 298.15\ (0.842 \ln\frac{473.15}{298.15} - 0.18892 \ln\frac{2000}{100}\ )$$

$$+ 100\ (0.0447 - 0.5633\ )\ ] + 4 \times 0.90\ [\ 200\ \text{-}25 - 298.15 \ln\frac{473.15}{298.15}\ ]$$

$$= 128.88 + 134.3 = 263.2\ kJ$$

$$\phi_2 = 1.1186\ [\ 0.653(25 - 25) - 298.15\ (0.842 \ln\frac{298.15}{298.15} - 0.18892 \ln\frac{1500}{100}\ )$$

$$+ 100\ (0.03756 - 0.5633\ )\ ] + 4 \times 0.9\ [\ 25\ \text{-}25 - 298.15 \ln\frac{298.15}{298.15}\ ]$$

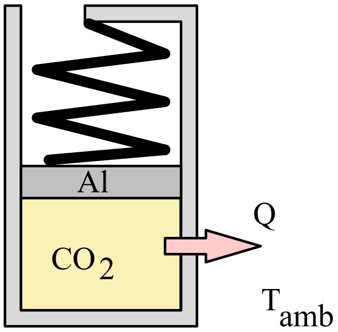
$$= 111.82 + 0 = 111.82\ kJ$$

The irreversibility is as in Eq.10.28

$${}_1I_2 = \phi_1 - \phi_2 + [1 - (T_0/T_H)]\ {}_1Q_2 - {}_1W_2^{AC} + P_om(\ V_2 - V_1)$$

$$= 263.2 - 111.82 + 0 - (-14) + 100 \times 1.1186\ (0.03756 - 0.0447) = 164.58\ kJ$$

$$[(S_{gen} = 0.552\ kJ/K; \qquad T_oS_{gen} = 164.58\ kJ \quad \text{so OK}\ ]$$



10.112

A rigid container with volume 200 L is divided into two equal volumes by a partition. Both sides contains nitrogen, one side is at 2 MPa, 300°C, and the other at 1 MPa, 50°C. The partition ruptures, and the nitrogen comes to a uniform state at 100°C. Assuming the surroundings are at 25°C find the actual heat transfer and the irreversibility in the process.

Solution:

C.V. Total container

Continuity Eq.: $m_2 - m_A - m_B = 0$

Energy Eq.: $m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = {}_1Q_2/T_{sur} + {}_1S_{s\ gen}$

Process: $V = C \quad => \quad {}_1W_2 = 0$

From the initial state we get the mass as

$$m_2 = m_A + m_B = \frac{P_{A1}V_A}{RT_{A1}} + \frac{P_{B1}V_B}{RT_{B1}}$$

$$= \frac{2000\times0.1}{0.2968\times573.15} + \frac{1000\times0.1}{0.2968\times323.15} = 1.176 + 1.043 = 2.219 \text{ kg}$$

$$P_2 = m_2RT_2/V_{tot} = 2.219 \times 0.2968 \times 373.15/0.2 = 1228.8 \text{ kPa}$$

From the energy equation we get the heat transfer as the change in U

$${}_1Q_2 = m_AC_v(T_2 - T_1)_A + m_BC_v(T_2 - T_1)_B$$

$$= 1.176 \times 0.745 \times (100 - 300) + 1.043 \times 0.745 \times (100 - 50)$$

$$= \mathbf{-136.4\ kJ}$$

The entropy changes are found from Eq.8.25

$$(s_2 - s_1)_A = 1.042 \times \ln\frac{373.15}{573.15} - 0.2968 \times \ln\frac{1228.8}{2000} = -0.09356 \text{ kJ/kg K}$$

$$(s_2 - s_1)_B = 1.042 \times \ln\frac{373.15}{323.15} - 0.2968 \times \ln\frac{1228.8}{1000} = 0.0887 \text{ kJ/kg K}$$

The entropy generation follows from the entropy equation

$${}_1S_{2,gen} = 1.176\times (-0.09356) + 1.043\times 0.0887 + 136.4/298.15 = 0.4396 \text{ kJ/K}$$

Now the irreversibility comes from Eq. 10.19

$${}_1I_2 = T_0 \times {}_1S_{2,gen} = \mathbf{131.08\ kJ}$$

10.113

Consider the heat engine in Problem 10.??79. The exit temperature was given as 800 K, but what are the theoretical limits for this temperature? Find the lowest and the highest, assuming the heat transfers are as given. For each case give the first and second law efficiency.

The **lowest exhaust temperature** will occur when the maximum amount of work is delivered which is a reversible process. Assume no other heat transfers then

2nd law: $s_i + q_H/T_H + 0 = s_e + q_m/T_m$

$$s_e - s_i = q_H/T_H - q_m/T_m = s^\circ_{Te} - s^\circ_{Ti} - R\ \ln(P_e/P_i)$$

$$s^\circ_{Te} = s^\circ_{Ti} + R\ \ln(P_e/P_i) + q_H/T_H - q_m/T_m$$

$$= 6.86926 + 0.287\ \ln(100/100) + 1200/1500 - 300/750$$

$$= 7.26926 \text{ kJ/kg K}$$

Table A.7.1 ⇒ $T_{e,min}$ = **446 K**, h_e = 447.9 kJ/kg

$$h_i + q_{1500} = q_{750} + h_e + w$$

$$w_{rev} = h_i + q_{1500} - q_{750} - h_e = 300.47 + 1200 - 300 - 447.9$$

$$= 752.57 \text{ kJ/kg}$$

$$\eta_I = \eta_{TH} = \frac{w_{rev}}{q_{1500}} = \frac{752.57}{1200} = \mathbf{0.627}$$

The second law efficiency measures the work relative to the source of availability and not q_{1500}. So

$$\eta_{II} = \frac{w_{rev}}{(1 - T_o/T_H)q_{1500}} = \frac{752.57}{(1 - 300/1500)1200} = \frac{752.57}{960} = \mathbf{0.784}$$

The **maximum exhaust temperature** occurs with no work out

$$h_i + q_H = q_m + h_e \Rightarrow h_e = 300.473 + 1200 - 300 = 1200.5 \text{ kJ/kg}$$

Table A.7.1 ⇒ $T_{e,max}$ = **1134 K**

Now : $w_{ac} = 0$ so $\eta_I = \eta_{II} =$ **0**

10.114

A small air gun has 1 cm^3 air at 250 kPa, 27°C. The piston is a bullet of mass 20 g. What is the potential highest velocity with which the bullet can leave?

Solution:

The availability of the air can give the bullet kinetic energy expressed in the exergy balance Eq.10.42 (no heat transfer and reversible),

$$\Phi_2 - \Phi_1 = m(u_2 - u_1) + P_o(V_2 - V_1) - mT_o(s_2 - s_1) = -{}_1W_2 + P_o(V_2 - V_1)$$

Ideal gas so: $m = PV/RT = \frac{250 \times 1 \times 10^{-6}}{0.287 \times 300} = 2.9 \times 10^{-6}$ kg

The second state with the lowest exergy to give maximum velocity is the dead state and we take $T_o = 20^oC$. Now solve for the exergy change

$$\Phi_2 - \Phi_1 = -m(u_2 - u_1) + mT_o(s_2 - s_1)$$

$$= mC_v(T_1 - T_2) + mT_o\, [\, C_p \ln(\frac{T_2}{T_1}) - R \ln(\frac{P_2}{P_1})\,]$$

$$= 2.9 \times 10^{-6}\, [\, 0.717(27 - 20) + 293.15\, (1.004 \ln\frac{293}{300} - 0.287 \ln\frac{100}{250})]$$

$$= 0.0002180 \text{ kJ} = 0.218 \text{ J} = \frac{1}{2}\, m_{bullet}\mathbf{V}_{ex}^2$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 0.218/0.020} = \mathbf{4.67\ m/s}$$

Comment: Notice that an isentropic expansion from 250 kPa to 100 kPa will give the final air temperature as 230.9 K but less work out. The above process is not adiabatic but Q is transferred from ambient at T_o.

10.115

Find the irreversibility in the cooling process of the glass plate in Problem 6.130.

In a glass factory a 2 m wide sheet of glass at 1500 K comes out of the final rollers that fix the thickness at 5 mm with a speed of 0.5 m/s. Cooling air in the amount of 20 kg/s comes in at 17°C from a slot 2 m wide and flows parallel with the glass. Suppose this setup is very long so the glass and air comes to nearly the same temperature (a co-flowing heat exchanger) what is the exit temperature?

Energy Eq.: $\dot{m}_{glass}h_{glass\ 1} + \dot{m}_{air}h_{air\ 2} = \dot{m}_{glass}h_{glass\ 3} + \dot{m}_{air}h_{air\ 4}$

$$\dot{m}_{glass} = \rho\dot{V} = \rho AV = 2500\times 2 \times 0.005\times 0.5 = 12.5 \text{ kg/s}$$

$$\dot{m}_{glass}C_{glass}\ (T_3 - T_1) + \dot{m}_{air}\ C_{Pa}\ (T_4 - T_2) = \varnothing$$

$$T_4 = T_3\ ,\ C_{glass} = 0.80 \text{ kJ/kg K},\quad C_{Pa} = 1.004 \text{ kJ/kg K}$$

$$T_3 = \frac{\dot{m}_{glass}C_{glass}\ T_1 + \dot{m}_{air}C_{Pa}\ T_2}{\dot{m}_{glass}C_{glass} + \dot{m}_{air}C_{Pa}} = \frac{12.5\times0.80\times1500 + 20\times1.004\times290}{12.5\times0.80 + 20\times1.004}$$

$$= 692.3 \text{ K}$$

To find the irreversibility we need the entropy generation term

Entropy Eq.: $\dot{m}_{glass}s_{glass\ 1} + \dot{m}_{air}s_{air\ 2} + \dot{S}_{gen} = \dot{m}_{glass}s_{glass\ 3} + \dot{m}_{air}s_{air\ 4}$

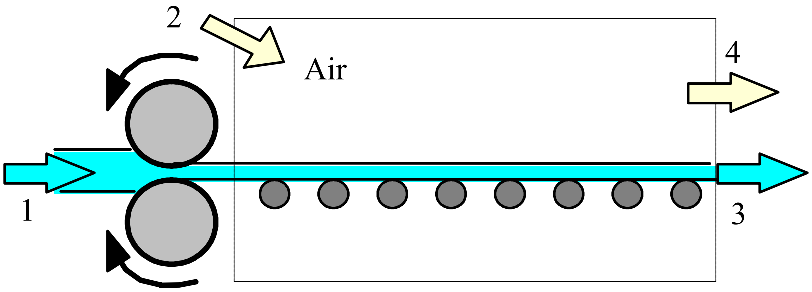
$$\dot{S}_{gen} = \dot{m}_{glass}\ (s_3 - s_1)_{glass} + \dot{m}_{air}(s_4 - s_2)_{air}$$

$$= \dot{m}_{glass}\ C_{glass}\ \ln(T_3 / T_1) + \dot{m}_{air}\ C_{Pa}\ \ln(T_4 / T_2)$$

$$= 12.5 \times 0.8 \times \ln(692.3 / 1500) + 20 \times 1.004 \times \ln(692.3/290)$$

$$= -7.732 \text{ kW/K} + 17.472 \text{ kW/K} = 9.74 \text{ kW/K}$$

$$\dot{I} = T_o\ \dot{S}_{gen} = 298.15 \text{ K} \times 9.74 \text{ kW/K} = \mathbf{2904\ kW}$$



10.116

Consider the nozzle in Problem 9.147. What is the second law efficiency for the nozzle?

A nozzle in a high pressure liquid water sprayer has an area of 0.5 cm^2. It receives water at 250 kPa, 20°C and the exit pressure is 100 kPa. Neglect the inlet kinetic energy and assume a nozzle isentropic efficiency of 85%. Find the ideal nozzle exit velocity and the actual nozzle mass flow rate.

Solution:

C.V. Nozzle. Liquid water is incompressible v ≈ constant, no work, no heat transfer => Bernoulli Eq.9.17

$$\frac{1}{2}\mathbf{V}_{ex}^2 - 0 = v(P_i - P_e) = 0.001002\ (250 - 100) = 0.1503 \text{ kJ/kg}$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 0.1503 \times 1000 \text{ J/kg}} = \mathbf{17.34\ m\ s^{-1}}$$

This was the ideal nozzle now we can do the actual nozzle, Eq. 9.30

$$\frac{1}{2}\mathbf{V}_{ex\ ac}^2 = \eta\,\frac{1}{2}\mathbf{V}_{ex}^2 = 0.85 \times 0.1503 = 0.12776 \text{ kJ/kg}$$

$$\mathbf{V}_{ex\ ac} = \sqrt{2 \times 0.12776 \times 1000 \text{ J/kg}} = 15.99 \text{ m s}^{-1}$$

The second law efficiency is the actual nozzle compare to a reversible process between the inlet and actual exit states. However here there is no work so the actual exit state then must have the reversible possible kinetic energy.

Energy actual nozzle: $h_i + 0 = h_e + \frac{1}{2}\mathbf{V}_{ex\ ac}^2$ same Z, no q and no w.

The reversible process has zero change in exergies from Eq.10.36 as

$$0 = 0 - 0 + 0 + \psi_i - \psi_e - 0$$

$$\psi_i = \psi_e = h_i + 0 - T_o\, s_i = h_e + \frac{1}{2}\mathbf{V}_{ex\ rev}^2 - T_o s_e$$

$$\frac{1}{2}\mathbf{V}_{ex\ rev}^2 = h_i - h_e + T_o\,(s_e - s_i) = \frac{1}{2}\mathbf{V}_{ex\ ac}^2 + T_o\, s_{gen}$$

We can not get properties for these states accurately enough by interpolation to carry out the calculations. With the computer program we can get:

Inlet: $h_i = 84.173$ kJ/kg, $s_i = 0.29652$ kJ/kg K

Exit,s: $h_{e\ s} = 84.023$ kJ/kg, $T_{e\ s} = 19.998$°C, $\frac{1}{2}\mathbf{V}_{ex}^2 = 0.15$ kJ/kg

Exit,ac: $\frac{1}{2}\mathbf{V}_{ex\ ac}^2 = 0.1275$ kJ/kg, $h_e = 84.173 - 0.1275 = 84.0455$ kJ/kg

(P, h) => $s_e = 0.29659$ kJ/kg K, T = 20.003°C

$$\frac{1}{2}\mathbf{V}_{ex\ rev}^2 = \frac{1}{2}\mathbf{V}_{ex\ ac}^2 + T_o\, s_{gen} = 0.1275 + 293.15(0.29659 - 0.29652)$$
$$= 0.148 \text{ kJ/kg}$$

$$\eta_{II} = 0.1275/0.148 = \mathbf{0.86}$$

10.117

Air in a piston/cylinder arrangement, shown in Fig. P10.117, is at 200 kPa, 300 K with a volume of 0.5 m^3. If the piston is at the stops, the volume is 1 m^3 and a pressure of 400 kPa is required. The air is then heated from the initial state to 1500 K by a 1900 K reservoir. Find the total irreversibility in the process assuming surroundings are at 20°C.

Solution:

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq,: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: $P = P_0 + \alpha(V-V_0)$ if $V \le V_{stop}$

Information: $P_{stop} = P_0 + \alpha(V_{stop}-V_0)$

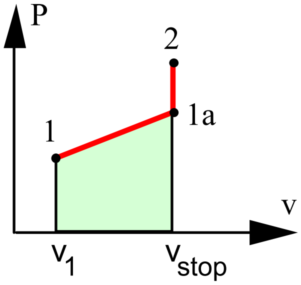
Eq. of state ⇒ $T_{stop} = T_1P_{stop}V_{stop}/P_1V_1 = 1200 < T_2$

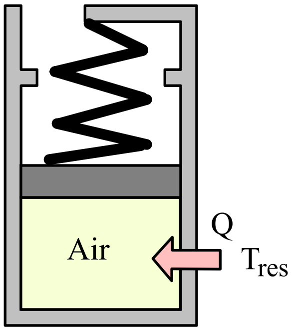
So the piston will hit the stops => $V_2 = V_{stop}$

$$P_2 = (T_2/T_{stop})\, P_{stop} = (1500/1200)\ 400 = 500 \text{ kPa} = 2.5\ P_1$$

State 1:

$$m_2 = m_1 = \frac{P_1V_1}{RT_1} = \frac{200 \times 0.5}{0.287 \times 300} = 1.161 \text{ kg}$$





$${}_1W_2 = \tfrac{1}{2}(P_1 + P_{stop})(V_{stop} - V_1) = \tfrac{1}{2}(200 + 400)(1 - 0.5) = 150 \text{ kJ}$$

$${}_1Q_2 = M(u_2 - u_1) + {}_1W_2 = 1.161(1205.25 - 214.36) + 150 = 1301 \text{ kJ}$$

$$s_2 - s_1 = s^{o}_{T2} - s^{o}_{T1} - R\ln(P_2/P_1) = 8.6121 - 6.8693 - 0.287 \ln 2.5 = 1.48 \text{ kJ/kg K}$$

Take control volume as total out to reservoir at T_{RES}

$${}_1S_{2\ gen\ tot} = m(s_2 - s_2) - {}_1Q_2/T_{RES} = 1.034 \text{ kJ/K}$$

$${}_1I_2 = T_0\left({}_1S_{2\ gen}\right) = 293.15 \times 1.034 = \mathbf{303\ kJ}$$

10.118

A 1 kg rigid steel tank contains 1.2 kg of R-134a at 20°C and 500 kPa. Now the set-up is placed in a freezer that brings it to -20°C. The freezer operates in a 20°C kitchen and has a coefficient of performance that is half that of a Carnot refrigerator. Find the heat transfer out of the R-134a, the extra work input to the refrigerator due to this process and the total irreversibility including the refrigerator.

C.V. The R-134a.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq,: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen}$

Process: $V = C$ so ${}_1W_2 = 0$

State 1: $v_1 = 0.04226\ m^3/kg$, $u_1 = 390.52$ kJ/kg, $s_1 = 1.7342$ kJ/kg-K

State 2: $v_2 = v_1$, -20°C => 2 phase state

$x_2 = (v_2 - v_f)/v_{fg} = (0.04226 - 0.000738)/0.14576 = 0.28486$

$u_2 = 173.65 + x_2 \times 192.85 = 228.59$ kJ/kg,

$s_2 = 0.9007 + x_2 \times 0.8388 = 1.13964$ kJ/kg-K

${}_1Q_2 = m(u_2 - u_1) = 1.2\ (228.59 - 390.52) = \mathbf{-194.3\ kJ}$

Refrigerator: $\beta = 0.5\ \beta_{Carnot} = 0.5 \dfrac{T_L}{T_H - T_L} = 0.5 \dfrac{253.15}{20 - (-20)} = 3.164$

$W_{ref} = Q_L/\beta = 194.3\ kJ/\ 3.164 = \mathbf{61.41\ kJ}$

C.V. Total incl. refrigerator. Storage effect in R-134a, otherwise transfer terms.

Energy Eq.: $m(u_2 - u_1) = W_{ref} - Q_H$

Entropy Eq,: $m(s_2 - s_1) = -Q_H/T_0 + {}_1S_{2\ gen}$

$Q_H = W_{ref} - m(u_2 - u_1) = 61.41 - (-194.3) = 255.71$ kJ

$${}_1S_{2\ gen} = m(s_2 - s_1) + \frac{Q_H}{T_0} = 1.2(1.13964 - 1.7342) + \frac{255.71}{293.15}$$

$$= -0.713472 + 0.872284 = 0.158812\ kJ/K$$

${}_1I_2 = T_0\ \left({}_1S_{2\ gen}\right) = 293.15 \times 0.158812 = \mathbf{46.56\ kJ}$

10.119

A piston/cylinder arrangement has a load on the piston so it maintains constant pressure. It contains 1 kg of steam at 500 kPa, 50% quality. Heat from a reservoir at 700°C brings the steam to 600°C. Find the second-law efficiency for this process. Note that no formula is given for this particular case so determine a reasonable expression for it.

Solution:

1: Table B.1.2 $P_1, x_1 \Rightarrow v_1 = 0.001093 + 0.5\times0.3738 = 0.188\ m^3/kg$,

$h_1 = 640.21 + 0.5\times2108.47 = 1694.5$ kJ/kg,

$s_1 = 1.8606 + 0.5\times4.9606 = 4.341$ kJ/kg K

2: $P_2 = P_1, T_2 \Rightarrow v_2 = 0.8041,\ \ h_2 = 3701.7$ kJ/kg, $s_2 = 8.3521$ kJ/kg K

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1)$

${}_1Q_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1) = 2007.2$ kJ

${}_1W_2 = Pm(v_2 - v_1) = 308.05$ kJ

${}_1W_{2\text{ to atm}} = P_0m(v_2 - v_1) = 61.61$ kJ

Useful work out $= {}_1W_2 - {}_1W_{2\text{ to atm}} = 246.44$ kJ

$$\Delta\phi_{\text{reservoir}} = (1 - T_0/T_{\text{res}})\,{}_1Q_2 = \left(1 - \frac{298.15}{973.15}\right) 2007.2 = 1392.2 \text{ kJ}$$

$\eta_{II} = W_{net}/\Delta\phi = \mathbf{0.177}$

10.120

Consider the nozzle in Problem 9.147. What is the second law efficiency for the nozzle?

Error this is also problem 10.116

10.121

A jet of air at 200 m/s flows at 25°C, 100 kPa towards a wall where the jet flow stagnates and leaves at very low velocity. Consider the process to be adiabatic and reversible. Use the exergy equation and the second law to find the stagnation temperature and pressure.

Solution:

C.V. From free flow to stagnation point. Reversible adiabatic steady flow.

Exergy Eq.10.36: $0 = \dot{m}\psi_i - \dot{m}\psi_e - \dot{\Phi}_{destr.}$

Entropy Eq.: $0 = \dot{m}s_i - \dot{m}s_e + \int \dot{m}dq/T + \dot{m}s_{gen} = \dot{m}s_i - \dot{m}s_e + 0 + 0$

Process: Reversible $\dot{\Phi}_{destr.} = 0$, $s_{gen} = 0$, adiabatic $q = 0$

From exergy Eq.: $\psi_e - \psi_i = 0 = h_e - T_o s_e - h_i + T_o s_i - \frac{1}{2}\mathbf{V}_i^2$

From entropy Eq.: $s_e = s_i$, so entropy terms drop out

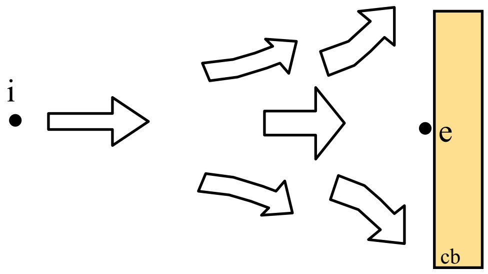
Exergy eq. now leads to: $h_e = h_i + \frac{1}{2}\mathbf{V}_i^2 \;\Rightarrow\; T_e = T_i + \frac{1}{2}\mathbf{V}_i^2 / C_p$

$$T_e = 25 + \frac{1}{2}\frac{200^2 \text{ J/kg}}{1004 \text{ J/kg K}} = \mathbf{44.92^oC}$$

Eq.8.32: $P_e = P_i\,(T_e/T_e)^{\frac{k}{k-1}} = 100\left(\frac{273 + 44.92}{273 + 25}\right)^{1.4/0.4} = \mathbf{125.4\ kPa}$

State i is the free stream state.

State e is the stagnation state.



10.122

Air in a piston/cylinder arrangement is at 110 kPa, 25°C, with a volume of 50 L. It goes through a reversible polytropic process to a final state of 700 kPa, 500 K, and exchanges heat with the ambient at 25°C through a reversible device. Find the total work (including the external device) and the heat transfer from the ambient.

C.V. Total out to ambient

$$m_a(u_2 - u_1) = {}_1Q_2 - {}_1W_{2,tot}, \quad m_a(s_2 - s_1) = {}_1Q_2/T_0$$

$$m_a = 110 \times 0.05/0.287 \times 298.15 = 0.0643 \text{ kg}$$

$${}_1Q_2 = T_0 m_a(s_2 - s_1) = 298.15 \times 0.0643\Big[7.3869 - 6.8631 - 0.287 \ln (700/110)\Big] = -0.14 \text{ kJ}$$

$${}_1W_{2,tot} = {}_1Q_2 - m_a(u_2 - u_1) = -0.14 - 0.0643 \times (359.844 - 213.037) = \mathbf{-9.58\ kJ}$$

10.123

Consider the light bulb in Problem 8.185. What are the fluxes of exergy at the various locations mentioned? What are the exergy destruction in the filament, the entire bulb including the glass and the entire room including the bulb? The light does not affect the gas or the glass in the bulb but it gets absorbed on the room walls.

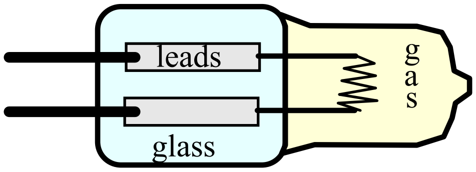
A small halogen light bulb receives an electrical power of 50 W. The small filament is at 1000 K and gives out 20% of the power as light and the rest as heat transfer to the gas, which is at 500 K; the glass is at 400 K. All the power is absorbed by the room walls at 25°C. Find the rate of generation of entropy in the filament, in the total bulb including glass and the total room including bulb.

Solution:

$\dot{W}_{el} = 50$ W

$\dot{Q}_{RAD} = 10$ W

$\dot{Q}_{COND} = 40$ W



We will assume steady state and no storage in the bulb, air or room walls.

C.V. Filament steady-state

Energy Eq.5.31: $dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Entropy Eq.8.43: $dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{RAD}}{T_{FILA}} - \dfrac{\dot{Q}_{COND}}{T_{FILA}} + \dot{S}_{gen}$

$$\dot{I} = T_o\dot{S}_{gen} = (\dot{Q}_{RAD} + \dot{Q}_{COND})\frac{T_o}{T_{FILA}} = T_o\frac{\dot{W}_{el}}{T_{FILA}} = 298\,\frac{50}{1000} = \mathbf{14.9\ W}$$

C.V. Bulb including glass

$\dot{Q}_{RAD}$ leaves at 1000 K $\qquad$ $\dot{Q}_{COND}$ leaves at 400 K

$\dot{I} = T_o\dot{S}_{gen} = T_o\int d\dot{Q}/T = 298[-(-10/1000) - (-40/400)] = \mathbf{32.8\ W}$

C.V. Total room. All energy leaves at 25°C

Eq.5.31: $dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$

Eq.8.43: $dS_{c.v.}/dt = 0 = -\dfrac{\dot{Q}_{TOT}}{T_{WALL}} + \dot{S}_{gen}$

$$\dot{I} = T_o\dot{S}_{gen} = T_o\frac{\dot{Q}_{TOT}}{T_{WALL}} = 298\,\frac{50}{25+273} = \mathbf{50\ W}$$

Since the room is at T_o then all the incoming exergy is destroyed.

10.124

Consider the irreversible process in Problem 8.128. Assume that the process could be done reversibly by adding heat engines/pumps between tanks A and B and the cylinder. The total system is insulated, so there is no heat transfer to or from the ambient. Find the final state, the work given out to the piston and the total work to or from the heat engines/pumps.

C.V. Water $m_A + m_B$ + heat engines. No Qexternal, only ${}_1W_{2,cyl} + W_{HE}$

$m_2 = m_{A1} + m_{B1} = 6$ kg, $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = -{}_1W_{2,cyl} - W_{HE}$

$m_2s_2 - m_{A1}s_{A1} - m_{B1}s_{B1} = \emptyset + \emptyset$

$v_{A1} = 0.06283$ $u_{A1} = 3448.5$ $s_{A1} = 7.3476$ $V_A = 0.2513\ m^3$

$v_{B1} = 0.09053$ $u_{B1} = 2843.7$ $s_{B1} = 6.7428$ $V_B = 0.1811\ m^3$

$m_2s_2 = 4\times7.3476 + 2\times6.7428 = 42.876 \Rightarrow$ $s_2 = 7.146$ kJ/kg K

If $P_2 < P_{lift} = 1.4$ MPa then

$V_{2'} = V_A + V_B = 0.4324\ m^3$, $v_{2'} = 0.07207\ m^3/kg$

$(P_{lift}, s_2) \Rightarrow$ $v_2 = 0.20135 \Rightarrow$ $V_2 = 1.208\ m^3 > V_{2'}$ OK

$\Rightarrow P_2 = P_{lift} =$ **1.4 MPa** $u_2 = 2874.2$ kJ/kg

${}_1W_{2,cyl} = P_{lift}(V_2 - V_A - V_B) = 1400\times(1.208 - 0.4324) =$ **1085.84 kJ**

$W_{HE} = m_{A1}u_{A1} + m_{B1}u_{B1} - m_2u_2 - {}_1W_{2,cyl}$

$= 4 \times 3447.8 + 2 \times 2843.7 - 6 \times 2874.2 - 1085.84 =$ **1147.6 kJ**

10.125

Air enters a steady-flow turbine at 1600 K and exhausts to the atmosphere at 1000 K. The second law efficiency is 85%. What is the turbine inlet pressure?

C.V.: Turbine, exits to atmosphere so assume $P_e = 100$ kPa

Inlet: $T_i = 1600$ K, Table A.7: $h_i = 1757.3$ kJ/kg, $s^o_i = 8.1349$ kJ/kg K

Exit: $T_e = 1000$ K, $h_e = 1046.2$ kJ/kg, $s^o_e = 8.6905$ kJ/kg K

1st Law: $q + h_i = h_e + w; \; q = 0 \quad \Rightarrow \quad w = (h_i - h_e) = 711.1$ kJ/kg

2nd Law: $\psi_i - \psi_e = w/\eta_{2ndLaw} = 711.1/0.85 = 836.6$ kJ/kg

$\psi_i - \psi_e = (h_i - h_e) - T_o(s_i - s_e) = 836.6$ kJ/kg

$h_i - h_e = w = 711.1$ kJ/kg, assume $T_o = 25^oC \rightarrow s_i - s_e = 0.4209$ kJ/kg-K

$s_i - s_e = s^o_e - s^o_i - R \ln(P_i/P_e) = 0.4209$ kJ/kg K $\quad \Rightarrow \quad P_e/P_i = 30.03;$

$P_i =$ **3003 kPa**

Problems Solved Using Pr and vr Functions

10.30

An air compressor receives atmospheric air at $T_0 = 17°C$, 100 kPa, and compresses it up to 1400 kPa. The compressor has an isentropic efficiency of 88% and it loses energy by heat transfer to the atmosphere as 10% of the isentropic work. Find the actual exit temperature and the reversible work.

C.V. Compressor

Isentropic: $w_{c,in,s} = h_{e,s} - h_i$; $s_{e,s} = s_i$

Table A.7: $P_{r,e,s} = P_{r,i} \times (P_e/P_i) = 0.9917 \times 14 = 13.884$

$\Rightarrow h_{e,s} = 617.51$ kJ/kg

$w_{c,in,s} = 617.51 - 290.58 = 326.93$ kJ/kg

Actual: $w_{c,in,ac} = w_{c,in,s}/\eta_c = 371.51$; $q_{loss} = 32.693$ kJ/kg

$w_{c,in,ac} + h_i = h_{e,ac} + q_{loss}$

=> $h_{e,ac} = 290.58 + 371.51 - 32.693 = 629.4$ kJ/kg

=> $T_{e,ac} = 621$ K

Reversible: $w^{rev} = h_i - h_{e,ac} + T_0(s_{e,ac} - s_i)$

$= 290.58 - 629.4 + 290.15 \times (7.6121 - 6.8357)$

$= -338.82 + 225.42 =$ **-113.4 kJ/kg**

Since q_{loss} is also to the atmosphere it is not included as it will not be reversible.

10.65

An air compressor is used to charge an initially empty 200-L tank with air up to 5 MPa. The air inlet to the compressor is at 100 kPa, 17°C and the compressor isentropic efficiency is 80%. Find the total compressor work and the change in availability of the air.

Solution:

C.V. Tank + compressor Transient process with constant inlet conditions, no heat transfer.

Continuity: $m_2 - m_1 = m_{in}$ ($m_1 = 0$) Energy: $m_2u_2 = m_{in}h_{in} - {}_1W_2$

Entropy: $m_2s_2 = m_{in}s_{in} + {}_1S_{2\ gen}$

Reversible compressor: ${}_1S_{2\ GEN} = 0 \Rightarrow s_2 = s_{in}$

State 1: $v_1 = RT_1/P_1 = 0.8323\ m^3/kg$,

State inlet, Table A.7.1: $h_{in} = 290.43$ kJ/kg, $s^o_{Tin} = 6.8352$ kJ/kg K

Table A.7.2 $P_{rin} = 0.9899$ used for constant s process

Table A.7.2 $\Rightarrow P_{r2} = P_{rin}(P_2/P_{in}) = 0.9899 \times (5000/100) = 49.495$

$\Rightarrow T_{2,s} = 855$ K, $u_{2,s} = 637.2$ kJ/kg

$\Rightarrow {}_1w_{2,s} = h_{in} - u_{2,s} = 290.43 – 637.2 = -346.77$ kJ/kg

Actual compressor: ${}_1w_{2,AC} = {}_1w_{2,s}/\eta_c = -433.46$ kJ/kg

$u_{2,AC} = h_{in} - {}_1w_{2,AC} = 290.43 – (-433.46) = 723.89$ kJ/kg

Backinterpolate in Table A.7.1 $\Rightarrow T_{2,AC} = 958$ K, $s^o_{T2,AC} = 8.0867$ kJ/kg K

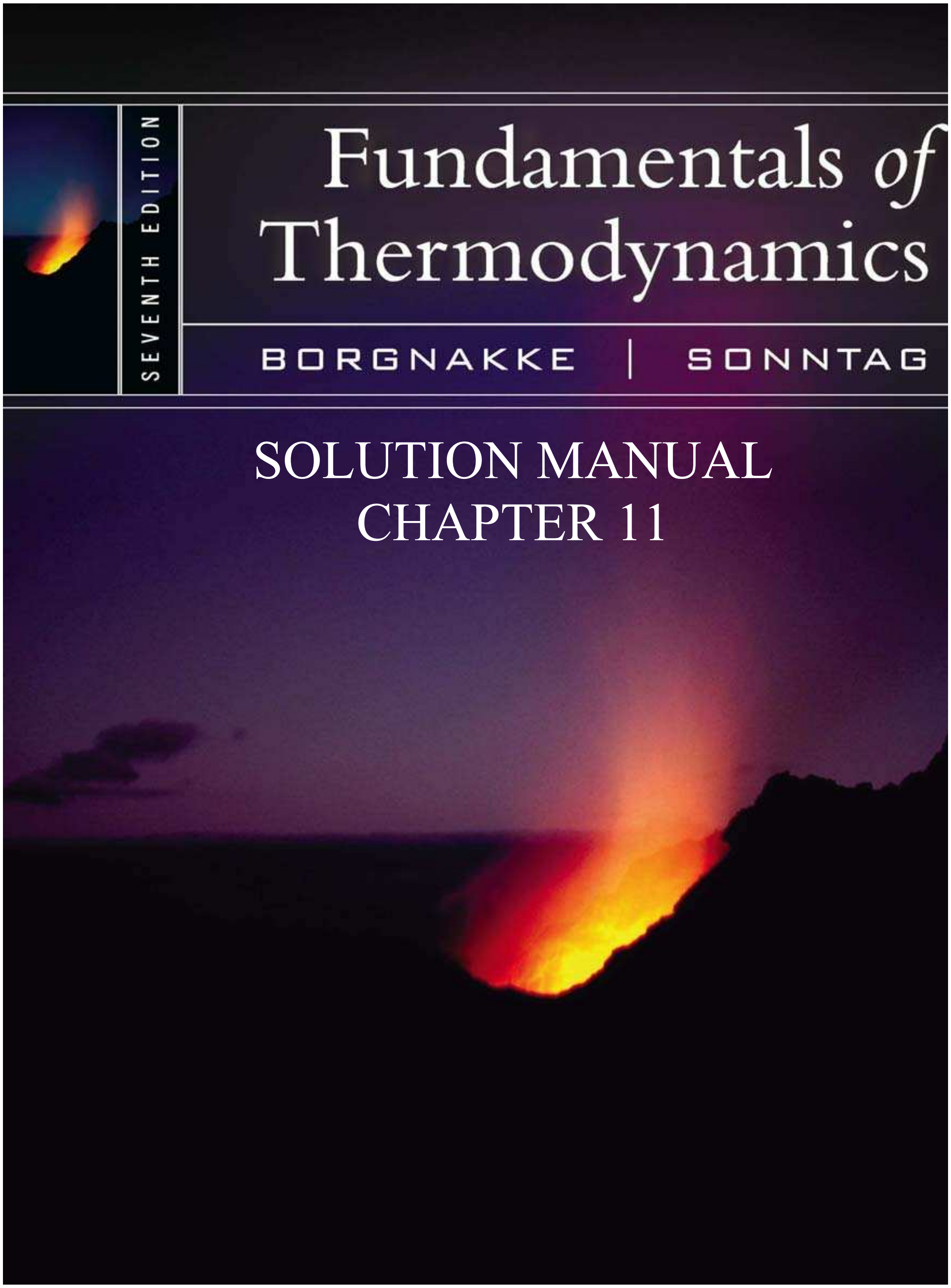
$\Rightarrow v_2 = RT_2/P_2 = 0.055\ m^3/kg$

State 2 $\boxed{u, P}$ $m_2 = V_2/v_2 = 3.636$ kg $\Rightarrow {}_1W_2 = m_2({}_1w_{2,AC}) = \mathbf{-1576\ kJ}$

$$m_2(\phi_2 - \phi_1) = m_2[u_2 - u_1 + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$$

$$= 3.636\ [723.89 - 207.19 + 100(0.055 - 0.8323) - 290[8.0867 - 6.8352 - 0.287 \ln(5000/100)] = \mathbf{1460.3\ kJ}$$

Here we used Eq.8.28 for the change in entropy.



SEVENTH EDITION

Fundamentals of Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 11

CONTENT

In-Text Concept Questions

11.a

Consider a Rankine cycle without superheat. How many single properties are needed to determine the cycle? Repeat the answer for a cycle with superheat.

a. No superheat. Two single properties.
High pressure (or temperature) and **low pressure** (or **temperature**). This assumes the condenser output is saturated liquid and the boiler output is saturated vapor. Physically the high pressure is determined by the pump and the low temperature is determined by the cooling medium.

b. Superheat. Three single properties.
High pressure and **temperature** and **low pressure** (or **temperature**). This assumes the condenser output is saturated liquid. Physically the high pressure is determined by the pump and the high temperature by the heat transfer from the hot source. The low temperature is determined by the cooling medium.

11.b

Which component determines the high pressure in a Rankine cycle? What determines the low pressure?

The high pressure in the Rankine cycle is determined by the pump. The low pressure is determined as the saturation pressure for the temperature you can cool to in the condenser.

11.c

What is the difference between an open and a closed feedwater heater?

The open feedwater heater mixes the two flows at the extraction pressure and thus requires two feedwater pumps.

The closed feedwater heater does not mix the flows but let them exchange energy (it is a two fluid heat exchanger). The flows do not have to be at the same pressure. The condensing source flow is dumped into the next lower pressure feedwater heater or the condenser or it is pumped up to line pressure by a drip pump and added to the feedwater line.

11.d

In a cogenerating power plant, what is cogenerated?

The electricity is cogenerated. The main product is a steam supply.

11.e

A refrigerator in my 20^{o}C kitchen uses R-134a and I want to make ice cubes at –5^{o}C. What is the minimum high P and the maximum low P it can use?

Since the R-134a must give heat transfer out to the kitchen air at 20^{o}C, it must at least be that hot at state 3.

From Table B.5.1: $P_3 = P_2 = P_{sat}$ = **573 kPa** is minimum high P.

Since the R-134a must absorb heat transfer at the freezers –5^{o}C, it must at least be that cold at state 4.

From Table B.5.1: $P_1 = P_4 = P_{sat}$ = **245 kPa** is maximum low P.

11.f

How many parameters are needed to completely determine a standard vapor compression refrigeration cycle?

Two parameters: The high pressure and the low pressure. This assumes the exit of the condenser is saturated liquid and the exit of the evaporator is saturated vapor.

Concept-Study Guide Problems

11.1

Is a steam power plant running in a Carnot cycle? Name the four processes.

No. It runs in a Rankine cycle.

1-2:	An isentropic compression (constant s)	Pump
2-3:	An isobaric heating (constant P)	Boiler
3-4:	An isentropic expansion (constant s)	Turbine
4-1:	An isobaric cooling, heat rejection (constant P)	Condenser

11.2

Raising the boiler pressure in a Rankine cycle for fixed superheat and condenser temperatures in what direction do these change: turbine work, pump work and turbine exit T or x.

Turbine work:	about the same P up, but v down
Turbine exit T:	same if it was two-phase, down if sup. vapor
Turbine exit x:	down
Pump work:	up

11.3

For other properties fixed in a Rankine cycle raising the condenser temperature causes changes in which work and heat transfer terms?

This results in less turbine work out.
An increase in heat rejection.
A small reduction in pump work.
A small reduction in boiler heat addition.

11.4

Mention two benefits of a reheat cycle.

The reheat raises the average temperature at which you add heat.

The reheat process brings the states at the lower pressure further out in the superheated vapor region and thus raises the quality (if two-phase) in the last turbine section.

11.5

What is the benefit of the moisture separator in the powerplant of Problem 6.106?

You avoid larger droplets in the turbine and raise the quality for the later stages.

11.6

Instead of the moisture separator in Problem 6.106 what could have been done to remove any liquid in the flow?

A reheat could be done to re-boil the liquid and even superheat it.

11.7

Can the energy removed in a power plant condenser be useful?

Yes.

In some applications it can be used for heating buildings locally or as district heating. Other uses could be to heat greenhouses or as general process steam in a food process or paper mill. These applications are all based on economics and scale. The condenser then has to operate at a higher temperature than it otherwise would.

11.8

If the district heating, see Fig.1.1, should supply hot water at 90°C what is the lowest possible condenser pressure with water as the working substance?

The condenser temperature must be higher than 90°C for which the saturation pressure is 70.14 kPa.

P > **70.14 kPa**

11.9

What is the mass flow rate through the condensate pump in Fig. 11.14?

We need to check the continuity equation around several CVs.

Do control volume around HP turbine:

Number in 1000 kg/h: $0 = + 320 - 28 - 28 - 12 -$ out to LP turbine
out to LP turbine = 252 000 kg/h

which matches with Fig.: 227 000 in condenser + 25 000 from trap

Condensate pump (main) has **252 000 kg/h**

11.10

A heat pump for a 20^{o}C house uses R-410a and the outside is at –5^{o}C. What is the minimum high P and the maximum low P it can use?

As the heat pump must be able to heat at 20^{o}C that becomes the smallest possible condensing temperature and thus $P > P_{sat}$ = **1444 kPa**.

It must absorb heat from –5^{o}C and thus must be colder in the evaporation process so $P < P_{sat}$ = **679 kPa.**

11.11

A heat pump uses carbon dioxide and it is required that it condenses at a minimum of 22^{o}C and receives energy from the outside on a winter day at -10^{o}C. What restrictions does that place on the operating pressures?

The high pressure $P > P_{sat} = 6003$ kPa, close to critical $P = 7377$ kPa

The low pressure $P < P_{sat} = 2649$ kPa

Notice for carbon dioxide that the low pressure is fairly high.

11.12

Since any heat transfer is driven by a temperature difference, how does that affect all the real cycles relative to the ideal cycles?

Heat transfers are given as $\dot{Q} = CA\ \Delta T$ so to have a reasonable rate the area and the temperature difference must be large. The working substance then must have a different temperature than the ambient it exchanges energy with. This gives a smaller temperature difference for a heat engine with a lower efficiency as a result. The refrigerator or heat pump must have the working substance with a higher temperature difference than the reservoirs and thus a lower coefficient of performance (COP).

The smaller CA is, the larger ΔT must be for a certain magnitude of the heat transfer rate. This can be a design problem, think about the front end air intake grill for a modern car which is very small compared to a car 20 years ago.

Simple Rankine cycles

11.13

A steam power plant as shown in Fig. 11.3 operating in a Rankine cycle has saturated vapor at 3.0 MPa leaving the boiler. The turbine exhausts to the condenser operating at 10 kPa. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 10) = 3.02$ kJ/kg

$\Rightarrow h_2 = h_1 + w_p = 191.81 + 3.02 = 194.83$ kJ/kg

C.V. Boiler : $q_H = h_3 - h_2 = 2804.14 - 194.83 =$ **2609.3 kJ/kg**

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

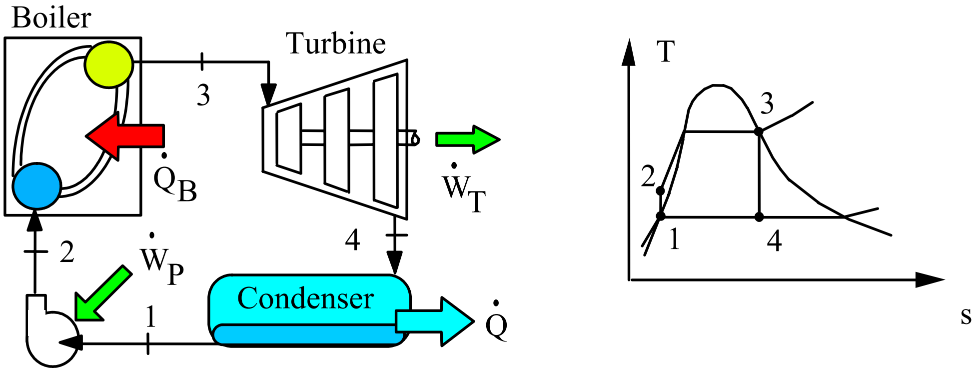
$s_4 = s_3 = 6.1869 = 0.6492 + x_4 (7.501) \Rightarrow x_4 = 0.7383$

$\Rightarrow h_4 = 191.81 + 0.7383 (2392.82) = 1958.34$ kJ/kg

$w_T = 2804.14 - 1958.34 =$ **845.8 kJ/kg**

C.V. Condenser : $q_L = h_4 - h_1 = 1958.34 - 191.81 =$ **1766.5 kJ/kg**

$\eta_{cycle} = w_{net} / q_H = (w_T + w_p) / q_H = (845.8 - 3.0) / 2609.3 =$ **0.323**



11.14

Consider a solar-energy-powered ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the solar collector at 175°C, and the condenser pressure is 10 kPa. Determine the thermal efficiency of this cycle.

Solution:

C.V. H_2O ideal Rankine cycle

State 3: $T_3 = 175°C \quad \Rightarrow \quad P_3 = P_{G\ 175°C} = 892$ kPa, $s_3 = 6.6256$ kJ/kg K

CV Turbine adiabatic and reversible so second law gives

$s_4 = s_3 = 6.6256 = 0.6493 + x_4 \times 7.5009 \quad => \quad x_4 = 0.797$

$h_4 = 191.83 + 0.797 \times 2392.8 = 2098.3$ kJ/kg

The energy equation gives

$w_T = h_3 - h_4 = 2773.6 - 2098.3 = 675.3$ kJ/kg

C.V. pump and incompressible liquid gives work into pump

$w_P = v_1(P_2 - P_1) = 0.00101(892 - 10) = 0.89$ kJ/kg

$h_2 = h_1 + w_P = 191.83 + 0.89 = 192.72$ kJ/kg

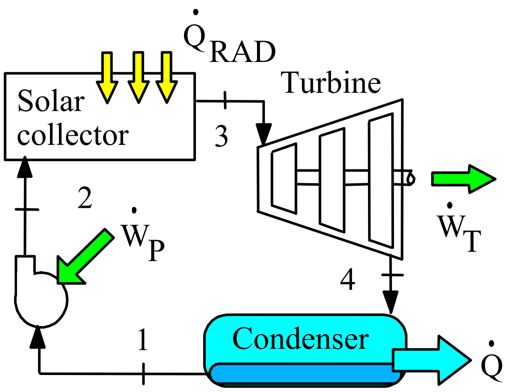
C.V. boiler gives the heat transfer from the energy equation as

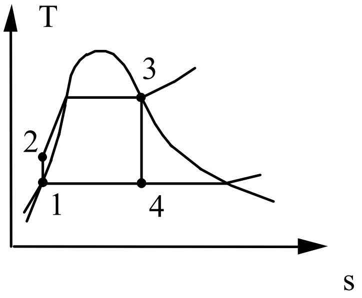
$q_H = h_3 - h_2 = 2773.6 - 192.72 = 2580.9$ kJ/kg

The cycle net work and efficiency are found as

$w_{NET} = w_T - w_P = 675.3 - 0.89 = 674.4$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 674.4/2580.9 = \mathbf{0.261}$





11.15

A power plant for a polar expedition uses ammonia which is heated to 80°C at 1000 kPa in the boiler and the condenser is maintained at -15°C. Find the cycle efficiency.

Solution:

Standard Rankine cycle with superheat. From the listed information we get from Table B.2.2

State 1: $h_1 = 111.66$ kJ/kg, $v_1 = 0.001519$ m^3/kg,

$P_1 = 236.3$ kPa, $s = 0.4538$ kJ/kgK

State 3: $h_3 = 1614.6$ kJ/kg, $s_3 = 5.4971$ kJ/kgK

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4$;

Entropy: $s_4 = s_3 = 5.4971$ kJ/kg K

$$\Rightarrow \; x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{5.4971 - 0.4538}{5.0859} = 0.9916 \; ;$$

$$h_4 = 111.66 + 0.9916 \times 1312.9 = 1413.56 \text{ kJ/kg}$$

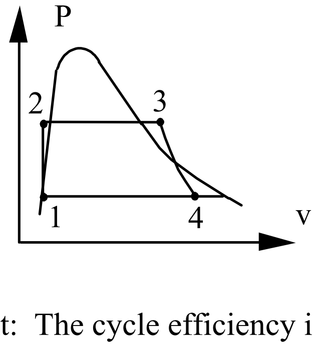
$$w_{T,s} = 1614.6 - 1413.56 = 201.04 \text{ kJ/kg}$$

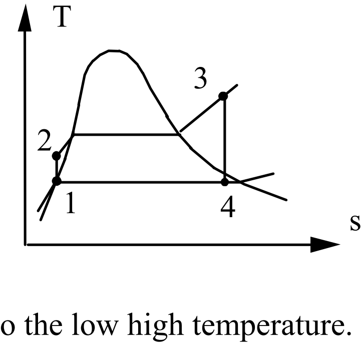
C.V. Pump: $w_P = \int v \, dP = v_1(P_2 - P_1) = 0.001519(1000 – 236.3) = 1.16$ kJ/kg

$$\Rightarrow \; h_2 = h_1 + w_P = 111.66 + 1.16 = 112.8 \text{ kJ/kg}$$

C.V. Boiler: $q_H = h_3 - h_2 = 1614.6 – 112.8 = 1501.8$ kJ/kg

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{201.04 – 1.16}{1501.8} = \mathbf{0.133}$$





Comment: The cycle efficiency is low due to the low high temperature.

11.16

A Rankine cycle with R-410a has the boiler at 3 MPa superheating to 180°C and the condenser operates at 800 kPa. Find all four energy transfers and the cycle efficiency.

State 1: $v_1 = 0.000855\ \text{m}^3/\text{kg}$, $h_1 = 57.76$ kJ/kg (at 0°C)

State 3: $h_3 = 445.09$ kJ/kg, $s_3 = 1.3661$ kJ/kg-K

State 4: (800 kPa, $s = s_3$) $h_4 = 385.97$ kJ/kg interpolated sup. vap.

C.V. Pump: $w_P = \int v\, dP = v_1(P_2 - P_1) = 0.000855\ (3000 - 800) = \mathbf{1.881\ kJ/kg}$

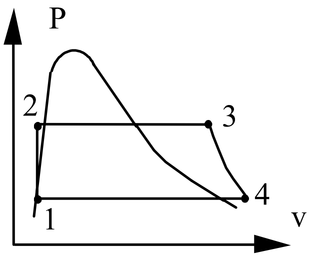
$\Rightarrow\ h_2 = h_1 + w_P = 57.76 + 1.881 = 59.64$ kJ/kg

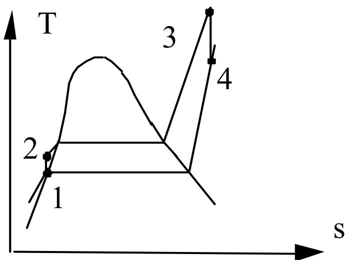
C.V. Boiler: $q_H = h_3 - h_2 = 445.09 - 59.64 = \mathbf{385.45\ kJ/kg}$

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4 = 445.09 - 385.97 = \mathbf{59.12\ kJ/kg}$

C.V. Condenser: $q_L = h_4 - h_1 = 385.97 - 57.76 = \mathbf{328.21\ kJ/kg}$

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{59.12 - 1.881}{385.45} = \mathbf{0.148}$$





11.17

A utility runs a Rankine cycle with a water boiler at 3.0 MPa and the cycle has the highest and lowest temperatures of 450°C and 45°C respectively. Find the plant efficiency and the efficiency of a Carnot cycle with the same temperatures.

Solution:

The states properties from Tables B.1.1 and B.1.3

$1\text{: } 45^{o}\text{C}, x = 0 \quad => \quad h_1 = 188.42, \;\; v_1 = 0.00101 \text{ m}^3\text{/kg}, \;\; P_{sat} = 9.6 \text{ kPa}$

$3\text{: } 3.0 \text{ MPa}, \quad 450^{o}\text{C} \quad => \quad h_3 = 3344 \text{ kJ/kg}, \quad s_3 = 7.0833 \text{ kJ/kg K}$

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101\ (3000 - 9.6) = 3.02 \text{ kJ/kg}$$

$$=> \; h_2 = h_1 + w_p = 188.42 + 3.02 = 191.44 \text{ kJ/kg}$$

C.V. Boiler : $q_H = h_3 - h_2 = 3344 - 191 = 3152.56$ kJ/kg

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 7.0833 = 0.6386 + x_4\ (7.5261) \quad => \quad x_4 = 0.8563$$

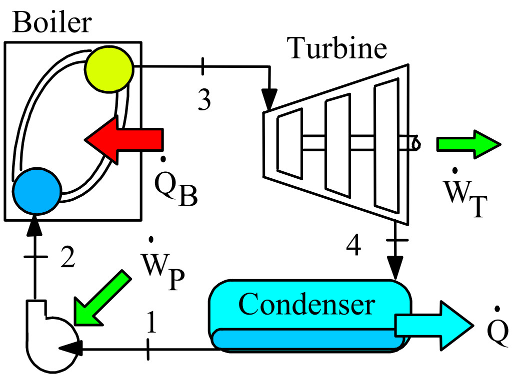
$$=> \; h_4 = 188.42 + 0.8563\ (2394.77) = 2239.06 \text{ kJ/kg}$$

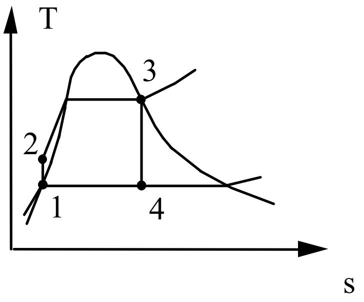
$$w_T = 3344 - 2239.06 = 1105 \text{ kJ/kg}$$

C.V. Condenser : $q_L = h_4 - h_1 = 2239.06 - 188.42 = 2050.64$ kJ/kg

$$\eta_{cycle} = w_{net} / q_H = (w_T + w_p) / q_H = (1105 - 3.02) / 3152.56 = \mathbf{0.349}$$

$$\eta_{carnot} = 1 - T_L / T_H = 1 - \frac{273.15 + 45}{273.15 + 450} = \mathbf{0.56}$$





11.18

A steam power plant has a high pressure of 3 MPa and it maintains 60°C in the condenser. A condensing turbine is used, but the quality should not be lower than 90% at any state in the turbine. Find the specific work and heat transfer in all components and the cycle efficiency.

Solution:

Based on the standard Rankine cycle and Table B.1.

State 1: Sat. liquid. $P_1 = 19.94$ kPa, $h_1 = 251.11$ kJ/kg, $v_1 = 0.001017$ m^3/kg

Consider C.V. pump

Energy: $h_2 - h_1 = w_p = v_1 (P_2 - P_1) = 0.001017 (3000 – 19.94) =$ **3.03 kJ/kg**

State 2: $P_2 = 3000$ kPa, $h_2 = h_1 + w_p = 251.11 + 3.03 = 254.1$ kJ/kg

State 4: $P_4 = P_1 = 19.94$ kPa, $x = 0.9$

$$s_4 = s_f + x_4 s_{fg} = 0.8311 + 0.9 \times 7.0784 = 7.20166 \text{ kJ/kg-K}$$

$$h_4 = h_f + x_4 h_{fg} = 251.11 + 0.9 \times 2358.48 = 2373.74 \text{ kJ/kg}$$

Consider the turbine for which $s_4 = s_3$.

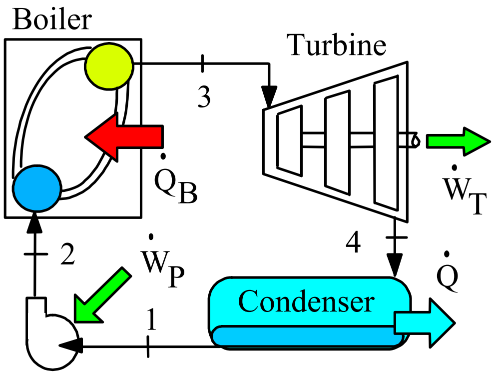
State 3: Table B.2.2 3000 kPa, $s_3 = 7.20166$ kJ/kg K => $h_3 = 3432.5$ kJ/kg

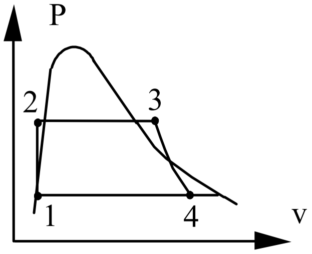
Boiler: $q_H = h_3 – h_2 = 3432.5 – 254.1 =$ **3178.4 kJ/kg**

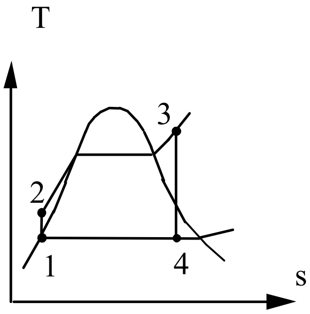
Turbine: $w_T = h_3 – h_4 = 3432.5 – 2373.74 =$ **1058.8 kJ/kg**

Condenser: $q_L = h_4 – h = 2373.74 – 251.1 =$ **2122.6 kJ/kg**

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = \dfrac{1058.8 - 3.03}{3178.4} =$ **0.332**







11.19

A low temperature power plant operates with R-410a maintaining -20°C in the condenser, a high pressure of 3 MPa with superheat. Find the temperature out of the boiler/superheater so the turbine exit temperature is 60°C and find the overall cycle efficiency.

State 1: $P_1 = 399.6$ kPa, $v_1 = 0.000803$ m^3/kg, $h_1 = 28.24$ kJ/kg

State 4: $P_4 = P_1 \approx 400$ kPa, $h_4 = 343.58$ kJ/kg, $s_4 = 1.3242$ kJ/kg-K

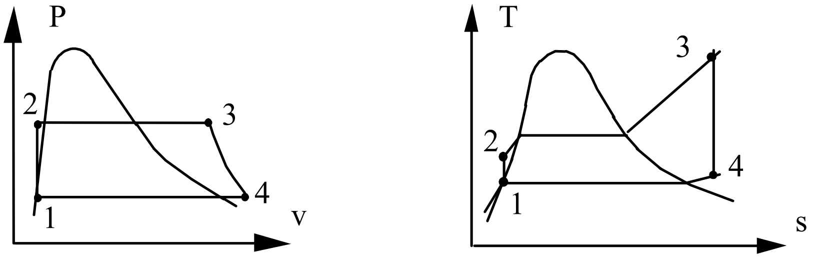
State 3: 3 MPa, $s = s_4$ ⇨ $h_3 = 426.56$ kJ/kg, $T_3 = \mathbf{143.6^oC}$

Pump: $w_P = v_1 (P_2 - P_1) = 0.000803\ (3000 – 399.6) = 2.09$ kJ/kg

Boiler: $q_H = h_3 – h_2 = 426.56 – (28.24 + 2.09) = 396.23$ kJ/kg

Turbine: $w_T = h_3 – h_4 = 426.56 – 343.58 = 82.98$ kJ/kg

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = \dfrac{82.98 - 2.09}{396.23} = \mathbf{0.204}$



11.20

A steam power plant operating in an ideal Rankine cycle has a high pressure of 5 MPa and a low pressure of 15 kPa. The turbine exhaust state should have a quality of at least 95% and the turbine power generated should be 7.5 MW. Find the necessary boiler exit temperature and the total mass flow rate.

Solution:

C.V. Turbine assume adiabatic and reversible.

Energy: $w_T = h_3 - h_4$; Entropy: $s_4 = s_3$

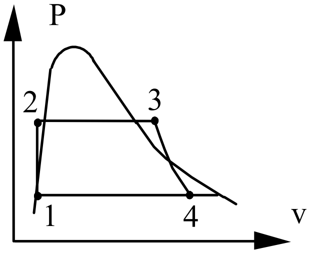
Since the exit state is given we can relate that to the inlet state from entropy.

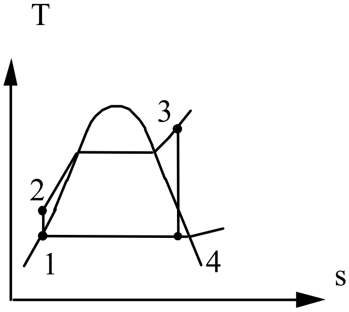
4: 15 kPa, $x_4 = 0.95$ => $s_4 = 7.6458$ kJ/kg K, $h_4 = 2480.4$ kJ/kg

3: $s_3 = s_4$, P_3 ⇒ $h_3 = 4036.7$ kJ/kg, $T_3 =$ **758°C**

$w_T = h_3 - h_4 = 4036.7 - 2480.4 = 1556.3$ kJ/kg

$\dot{m} = \dot{W}_T/w_T = 7.5 \times 1000/1556.3 =$ **4.82 kg/s**





11.21

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use R-134a Table B.5)

$$w_P = h_2 - h_1 = \int_1^2 v dP \approx v_1(P_2 - P_1)$$

$$= 0.000873(2926.2 - 1017.0) = 1.67 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 256.54 + 1.67 = 258.21 \text{ kJ/kg}$$

CV: Boiler

$$q_H = h_3 - h_2 = 428.10 - 258.21 = 169.89 \text{ kJ/kg}$$

CV: Turbine

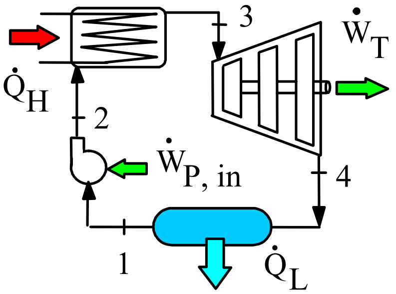
$$s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \quad => \quad x_4 = 0.9346$$

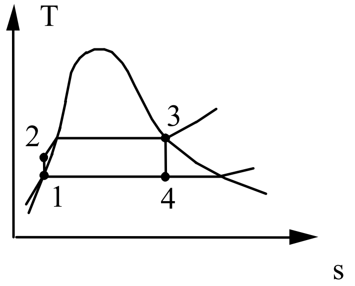
$$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96 \text{ kJ/kg}$

$$w_{NET} = w_T - w_P = 18.96 - 1.67 = 17.29 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 17.29/169.89 = \mathbf{0.102}$$





11.22

Do Problem 11.21 with R-410a as the working fluid and boiler exit at 4000 kPa, 70°C.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use R-410a Table B.4)

$$w_P = h_2 - h_1 = \int_1^2 v dP \approx v_1(P_2-P_1) = 0.001025(4000 - 2420.7) = 1.619 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 124.09 + 1.619 = 125.71 \text{ kJ/kg}$$

CV: Boiler: $q_H = h_3 - h_2 = 287.88 - 125.71 = 162.17$ kJ/kg

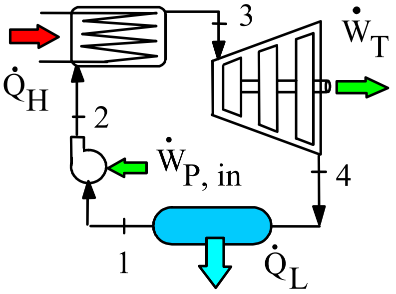
CV: Turbine

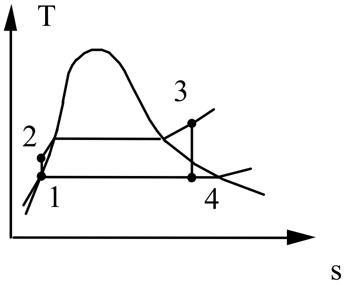
$$s_4 = s_3 = 0.93396 = 0.4473 + x_4 \times 0.5079, \Rightarrow \qquad x_4 = 0.9582$$

$$h_4 = 124.09 + 0.9582 \times 159.04 = 276.48 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 287.88 - 276.48 = 11.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (11.4 - 1.62)/162.17 = \mathbf{0.060}$$





11.23

Do Problem 11.21 with ammonia as the working fluid.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use Ammonia Table B.2)

$$w_P = h_2 - h_1 = \int_1^2 v\,dP = v_1(P_2 - P_1)$$

$$= 0.001725(4608.6 - 1554.9) = 5.27 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 371.43 + 5.27 = 376.7 \text{ kJ/kg}$$

CV: Boiler

$$q_H = h_3 - h_2 = 1447.8 - 376.7 = 1071.1 \text{ kJ/kg}$$

CV: Turbine

$$s_4 = s_3 = 4.3901 = 1.3574 + x_4 \times 3.5088 \quad => \quad x_4 = 0.8643$$

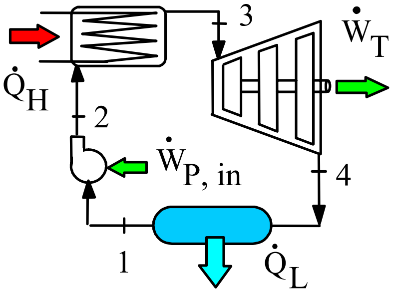
$$h_4 = 371.43 + 0.8643 \times 1098.8 = 1321.13 \text{ kJ/kg}$$

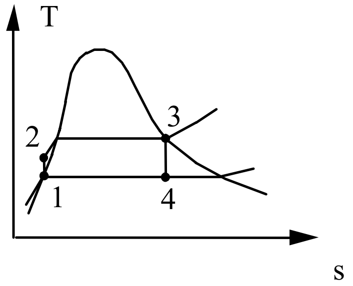
Energy Eq.:

$$w_T = h_3 - h_4 = 1447.8 - 1321.13 = 126.67 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_P = 126.67 - 5.27 = 121.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 121.4/1071.1 = \mathbf{0.113}$$

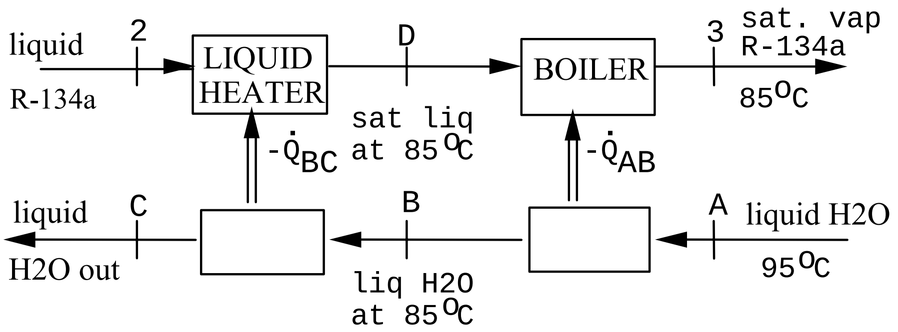




11.24

Consider the boiler in Problem 11.21 where the geothermal hot water brings the R-134a to saturated vapor. Assume a counter flowing heat exchanger arrangement. The geothermal water temperature should be equal to or greater than the R-134a temperature at any location inside the heat exchanger. The point with the smallest temperature difference between the source and the working fluid is called the pinch point. If 2 kg/s of geothermal water is available at 95°C, what is the maximum power output of this cycle for R-134a as the working fluid? (hint: split the heat exchanger C.V. into two so the pinch point with $\Delta T = 0$, T = 85°C appears).

2 kg/s of water is available at 95 ^{o}C for the boiler. The restrictive factor is the boiling temperature of 85° C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.



CV Pump: $w_P = v_1(P_2-P_1) = 0.000873(2926.2 - 1017.0) = 1.67$ kJ/kg

$$h_2 = h_1 + w_P = 256.54 + 1.67 = 258.21 \text{ kJ/kg}$$

Write the energy equation for the first section A-B and D-3:

$$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12 \text{ kW}$$

$$= \dot{m}_{R134A}(428.1 - 332.65) \Rightarrow \dot{m}_{R134A} = 0.8813 \text{ kg/s}$$

To be sure that the boiling temp. is the restrictive factor, calculate T_C from the energy equation for the remaining section:

$$-\dot{Q}_{AC} = 0.8813(332.65 - 258.21) = 65.60 \text{ kW} = 2(355.88 - h_C)$$

$$\Rightarrow h_C = 323.1 \text{ kJ/kg}, \; T_C = 77.2°C > T_2 = 40°C \; \textbf{OK}$$

CV: Turbine: $s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \;\; => \;\; x_4 = 0.9346$

$$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14 \text{ kJ/kg}$$

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96$ kJ/kg

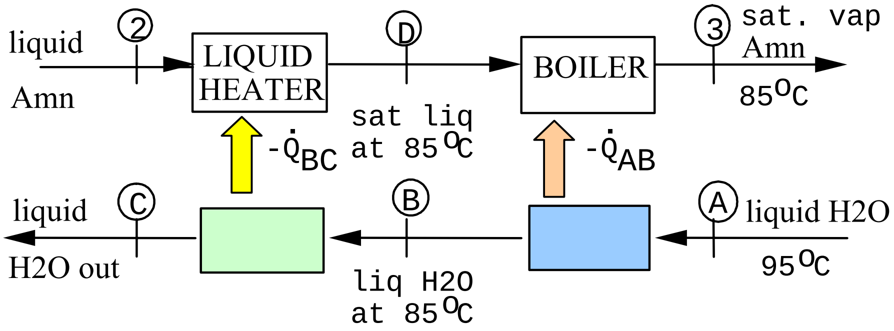
Cycle: $w_{NET} = w_T - w_P = 18.96 - 1.67 = 17.29$ kJ/kg

$$\dot{W}_{NET} = \dot{m}_{R134A}w_{NET} = 0.8813 \times 17.29 = \mathbf{15.24\ kW}$$

11.25

Do the previous problem with ammonia as the working fluid.

A flow with 2 kg/s of water is available at 95°C for the boiler. The restrictive factor is the boiling temperature of 85°C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.



State 1: $40^{\circ}C$, 1554.9 kPa, $v_1 = 0.001725\ m^3/kg$

CV Pump: $w_P = v_1(P_2 - P_1) = 0.001725(4608.6 - 1554.9) = 5.27$ kJ/kg

$h_2 = h_1 + w_P = 371.43 + 5.27 = 376.7$ kJ/kg

$$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12 \text{ kW}$$

$$= \dot{m}_{amn}(1447.8 - 609.21) \ \Rightarrow\ \dot{m}_{amn} = 0.100 \text{ kg/s}$$

To verify that $T_D = T_3$ is the restrictive factor, find T_C.

$$-\dot{Q}_{AC} = 0.100(609.21 - 376.7) = 23.25 = 2.0(355.88 - h_C)$$

$h_C = 344.25$ kJ/kg $\Rightarrow$ $T_C = 82.5^{\circ}C > T_2 = 40^{\circ}C$ **OK**

CV: Turbine

$s_4 = s_3 = 4.3901 = 1.3574 + x_4 \times 3.5088$ => $x_4 = 0.8643$

$h_4 = 371.43 + 0.8643 \times 1098.8 = 1321.13$ kJ/kg

Energy Eq.:

$w_T = h_3 - h_4 = 1447.8 - 1321.13 = 126.67$ kJ/kg

$w_{NET} = w_T - w_P = 126.67 - 5.27 = 121.4$ kJ/kg

$\dot{W}_{NET} = \dot{m}_{amn} w_{NET} = 0.1 \times 121.4 =$ **12.14 kW**

11.26

A low temperature power plant operates with carbon dioxide maintaining -10^{o}C in the condenser, a high pressure of 6 MPa and it superheats to 100^{o}C. Find the turbine exit temperature and the overall cycle efficiency.

State 1: $v_1 = 0.001017\ m^3/kg$, $h_1 = 63.62$ kJ/kg, $P_1 = 2648.7$ kPa

State 3: $h_3 = 421.69$ kJ/kg, $s_3 = 1.4241$ kJ/kg-K

State 4: (2648.7 kPa, $s = s_3$) $h_4 = 372.5$ kJ/kg interpolated sup. vap.

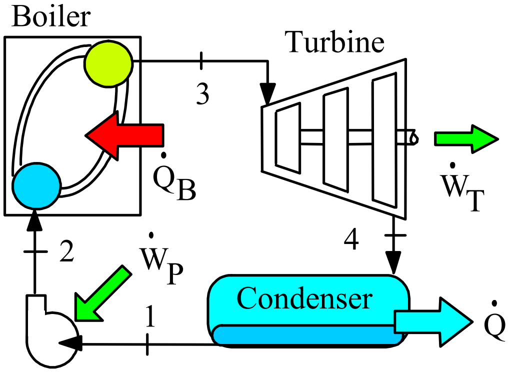
C.V. Pump: $w_P = \int v\, dP = v_1(P_2 - P_1) = 0.001017\ (6000 – 2648.7) =$ **3.408 kJ/kg**

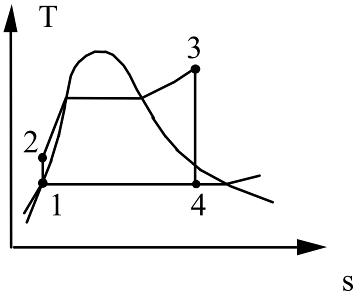
$\Rightarrow h_2 = h_1 + w_P = 63.62 + 3.408 = 67.03$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 421.69 – 67.03 =$ **354.66 kJ/kg**

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4 = 445.09 - 385.97 =$ **59.12 kJ/kg**

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{59.12 – 1.881}{385.45} = \mathbf{0.148}$$





11.27

Consider the ammonia Rankine-cycle power plant shown in Fig. P11.27. The plant was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

a. Determine the turbine power output and the pump power input for the cycle.
b. Determine the mass flow rate of water through each heat exchanger.
c. What is the thermal efficiency of this power plant?

Solution:

a) C.V. Turbine. Assume reversible and adiabatic.

$$s_2 = s_1 = 5.0863 = 0.8779 + x_2 \times 4.3269 \quad => \quad x_2 = 0.9726$$

$$h_2 = 227.08 + 0.9726 \times 1225.09 = 1418.6 \text{ kJ/kg}$$

$$w_T = h_1 - h_2 = 1460.29 - 1418.6 = 41.69 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 1000 \times 41.69 = \mathbf{41\ 690\ kW}$$

Pump: $w_P \approx v_3(P_4 - P_3) = 0.0016(857 - 615) = 0.387$ kJ/kg

$$\dot{W}_P = \dot{m}w_P = 1000 \times 0.387 = \mathbf{387\ kW}$$

b) Consider condenser heat transfer to the low T water

$$\dot{Q}_{\text{to low T H2O}} = 1000(1418.6 - 227.08) = 1.1915\times10^6 \text{ kW}$$

$$\dot{m}_{\text{low T H2O}} = \frac{1.1915\times10^6}{29.38 - 20.98} = \mathbf{141\ 850\ kg/s}$$

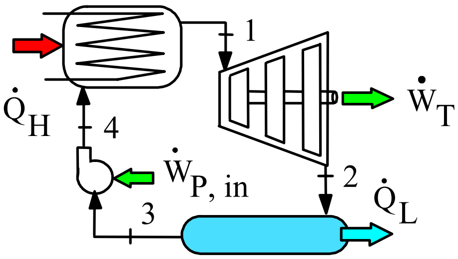
$$h_4 = h_3 + w_P = 227.08 + 0.39 = 227.47 \text{ kJ/kg}$$

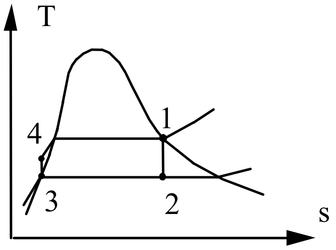
Now consider the boiler heat transfer from the high T water

$$\dot{Q}_{\text{from high T H2O}} = 1000(1460.29 - 227.47) = 1.2328\times10^6 \text{ kW}$$

$$\dot{m}_{\text{high T H2O}} = \frac{1.2328\times10^6}{104.87 - 96.50} = \mathbf{147\ 290\ kg/s}$$

c) $$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \frac{41\ 690 - 387}{1.2328\times10^6} = \mathbf{0.033}$$





11.28

Do problem 11.27 with carbon dioxide as the working fluid.

Consider the ammonia Rankine-cycle power plant shown in Fig. P11.27. The plant was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

a. Determine the turbine power output and the pump power input for the cycle.
b. Determine the mass flow rate of water through each heat exchanger.
c. What is the thermal efficiency of this power plant?

Solution:

a) C.V. Turbine. Assume reversible and adiabatic.

$$s_2 = s_1 = 1.0406 = 0.4228 + x_2 \times 0.6963 \qquad => \qquad x_2 = 0.88726$$

$$h_2 = 112.83 + 0.88726 \times 197.15 = 287.75 \text{ kJ/kg}$$

$$w_T = h_1 - h_2 = 294.96 - 287.75 = 7.206 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 1000 \times 7.206 = \mathbf{7\ 206\ kW}$$

Pump: $w_P \approx v_3(P_4 - P_3) = 0.001161(5729 - 4502) = 1.425$ kJ/kg

$$\dot{W}_P = \dot{m}w_P = 1000 \times 1.425 = \mathbf{1425\ kW}$$

b) Consider condenser heat transfer to the low T water

$$\dot{Q}_{\text{to low T H2O}} = 1000(287.75 - 112.83) = 0.2749 \times 10^6 \text{ kW}$$

$$\dot{m}_{\text{low T H2O}} = \frac{0.2749 \times 10^6}{29.38 - 20.98} = \mathbf{32\ 728\ kg/s}$$

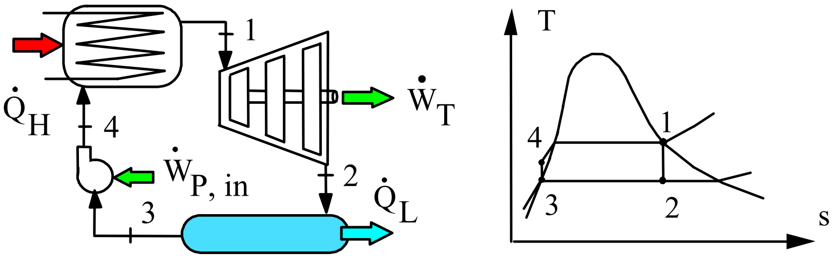
$$h_4 = h_3 + w_P = 112.83 + 1.425 = 114.26 \text{ kJ/kg}$$

Now consider the boiler heat transfer from the high T water

$$\dot{Q}_{\text{from high T H2O}} = 1000(294.96 - 114.26) = 0.1807 \times 10^6 \text{ kW}$$

$$\dot{m}_{\text{high T H2O}} = \frac{0.1807 \times 10^6}{104.87 - 96.50} = \mathbf{21\ 589\ kg/s}$$

c) $$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \frac{7\ 206 - 1425}{0.1807 \times 10^6} = \mathbf{0.032}$$


$\dot{Q}_H$
1
$\dot{W}_T$
4
$\dot{W}_{P,\ in}$
2
3
$\dot{Q}_L$
T
4
1
3
2
s

11.29

A smaller power plant produces 25 kg/s steam at 3 MPa, 600^{o}C in the boiler. It cools the condenser with ocean water coming in at 12^{o}C and returned at 15^{o}C so the condenser exit is at 45^{o}C. Find the net power output and the required mass flow rate of ocean water.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45^{o}C, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m^3/kg, $P_{sat} = 9.59$ kPa

3: 3.0 MPa, 600^{o}C: $h_3 = 3682.34$ kJ/kg, $s_3 = 7.5084$ kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 9.6) = 3.02$ kJ/kg

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$s_4 = s_3 = 7.5084 = 0.6386 + x_4 (7.5261) \quad => \quad x_4 = 0.9128$

$=> h_4 = 188.42 + 0.9128 (2394.77) = 2374.4$ kJ/kg

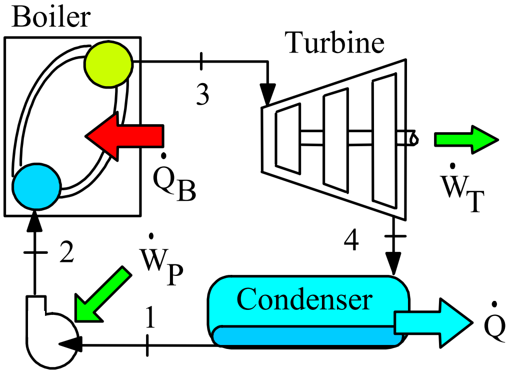
$w_T = 3682.34 – 2374.4 = 1307.94$ kJ/kg

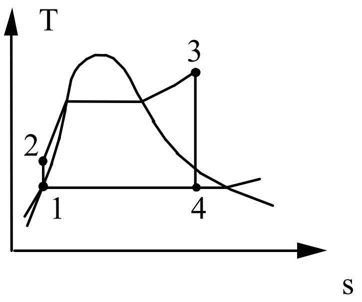
$\dot{W}_{NET} = \dot{m}(w_T – w_p) = 25 (1307.94 – 3.02) =$ **32.6 MW**

C.V. Condenser : $q_L = h_4 - h_1 = 2374.4 - 188.42 = 2186$ kJ/kg

$\dot{Q}_L = \dot{m}q_L = 25 \times 2186 = 54.65 \text{ MW} = \dot{m}_{ocean} C_p \Delta T$

$\dot{m}_{ocean} = \dot{Q}_L / C_p \Delta T = 54\,650 / (4.18 \times 3) =$ **4358 kg/s**





11.30

The power plant in Problem 11.13 is modified to have a super heater section following the boiler so the steam leaves the super heater at 3.0 MPa, 400°C. Find the specific work and heat transfer in each of the ideal components and the cycle efficiency.

Solution:

C.V. Pump: $w_P = \int v\,dP = v_1(P_2 - P_1) = 0.00101(3000 - 10) = \mathbf{3.02\ kJ/kg}$

$\Rightarrow\ h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 3230.82 – 194.83 = \mathbf{3036\ kJ/kg}$

C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4$;

Entropy: $s_4 = s_3 = 6.9211$ kJ/kg K

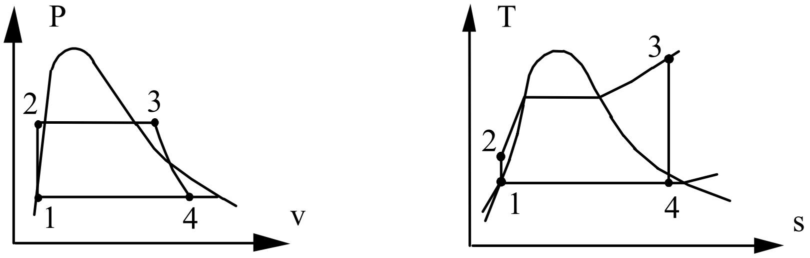
$$\Rightarrow\ x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.9211 - 0.6492}{7.501} = 0.83614\ ;$$

$h_4 = 191.81 + 0.83614 \times 2392.82 = 2192.5$ kJ/kg

$w_{T,s} = 3230.82 - 2192.5 = \mathbf{1038.3\ kJ/kg}$

C.V. Condenser: $q_C = h_4 - h_1 = 2192.5 - 191.81 = \mathbf{2000.7\ kJ/kg}$

$$\eta_{CYCLE} = w_{NET}/q_H = \frac{1038.3 – 3.02}{3036} = \mathbf{0.341}$$



11.31

Consider an ideal Rankine cycle using water with a high-pressure side of the cycle at a supercritical pressure. Such a cycle has a potential advantage of minimizing local temperature differences between the fluids in the steam generator, such as the instance in which the high-temperature energy source is the hot exhaust gas from a gas-turbine engine. Calculate the thermal efficiency of the cycle if the state entering the turbine is 30 MPa, 550°C, and the condenser pressure is 10 kPa. What is the steam quality at the turbine exit?

Solution:

For the efficiency we need the net work and steam generator heat transfer.

C.V. Pump. For this high exit pressure we use Table B.1.4

State 1: $s_1 = 0.6492$ kJ/kg K, $h_1 = 191.81$ kJ/kg

Entropy Eq.: $s_2 = s_1 \quad => \quad h_2 = 222.5$ kJ/kg

$w_p = h_2 - h_1 = 30.69$ kJ/kg

C.V. Turbine. Assume reversible and adiabatic.

Entropy Eq.: $s_4 = s_3 = 6.0342 = 0.6492 + x_4 \times 7.501$

$x_4 =$ **0.7179** Very low for a turbine exhaust

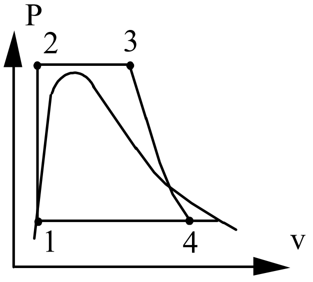
$h_4 = 191.81 + x_4 \times 2392.82 = 1909.63$, $h_3 = 3275.36$ kJ/kg

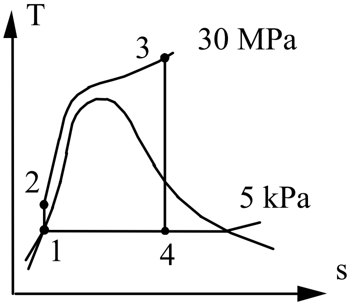
$w_T = h_3 - h_4 = 1365.7$ kJ/kg

Steam generator: $q_H = h_3 - h_2 = 3052.9$ kJ/kg

$w_{NET} = w_T - w_p = 1365.7 - 30.69 = 1335$ kJ/kg

$\eta = w_{NET}/q_H = 1335 / 3052.9 =$ **0.437**





11.32

Find the mass flow rate in Problem 11.26 so the turbine can produce 1 MW.

A low temperature power plant operates with carbon dioxide maintaining -10^{o}C in the condenser, a high pressure of 6 MPa and it superheats to 100^{o}C. Find the turbine exit temperature and the overall cycle efficiency.

State 3: $h_3 = 421.69$ kJ/kg, $s_3 = 1.4241$ kJ/kg-K

State 4: (800 kPa, $s = s_3$) $h_4 = 385.97$ kJ/kg interpolated sup. vap.

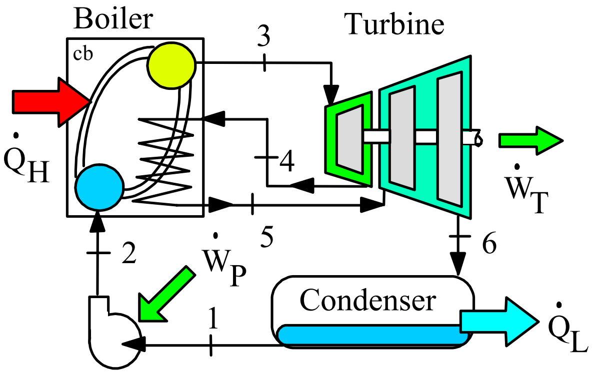
C.V. Tubine: Energy: $w_{T,s} = h_3 - h_4 = 445.09 - 385.97 = 59.12$ kJ/kg

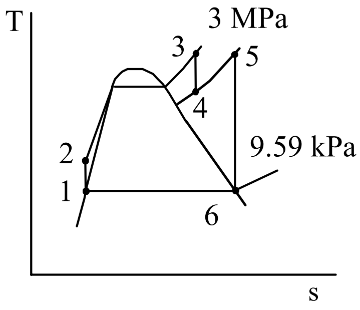
$\dot{m}$ **=** $\dot{W}_{T,s} / w_{T,s} = 1000$ kW/ 59.12 kJ/kg = **16.9 kg/s**

Reheat Cycles

11.33

A smaller power plant produces steam at 3 MPa, 600°C in the boiler. It keeps the condenser at 45°C by transfer of 10 MW out as heat transfer. The first turbine section expands to 500 kPa and then flow is reheated followed by the expansion in the low pressure turbine. Find the reheat temperature so the turbine output is saturated vapor. For this reheat find the total turbine power output and the boiler heat transfer.





The states properties from Tables B.1.1 and B.1.3

1: 45°C, x = 0: h_1 = 188.42 kJ/kg, v_1 = 0.00101 m^3/kg, P_{sat} = 9.59 kPa

3: 3.0 MPa, 600°C: h_3 = 3682.34 kJ/kg, s_3 = 7.5084 kJ/kg K

6: 45°C, x = 1: h_6 = 2583.19 kJ/kg, s_6 = 8.1647 kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3000 - 9.59) = 3.02$ kJ/kg

$h_2 = h_1 + w_p = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. HP Turbine section

Entropy Eq.: $s_4 = s_3$ => h_4 = 3093.26 kJ/kg; T_4 = 314°C

C.V. LP Turbine section

Entropy Eq.: $s_6 = s_5$ = 8.1647 kJ/kg K => state 5

State 5: 500 kPa, s_5 => h_5 = 3547.55 kJ/kg, T_5 = **529°C**

C.V. Condenser.

Energy Eq.: $q_L = h_6 - h_1 = h_{fg} = 2394.77$ kJ/kg

$$\dot{m} = \dot{Q}_L / q_L = 10\,000 / 2394.77 = 4.176 \text{ kg/s}$$

Both turbine sections

$$\dot{W}_{T,tot} = \dot{m}w_{T,tot} = \dot{m}(h_3 - h_4 + h_5 - h_6)$$

$$= 4.176\ (3682.34 - 3093.26 + 3547.55 - 2583.19) = \mathbf{6487\ kW}$$

Both boiler sections

$$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$$

$$= 4.176\ (3682.34 - 191.44 + 3547.55 - 3093.26) = \mathbf{16\,475\ kW}$$

11.34

A smaller power plant produces 25 kg/s steam at 3 MPa, 600^{o}C in the boiler. It cools the condenser with ocean water so the condenser exit is at 45^{o}C. There is a reheat done at 500 kPa up to 400^{o}C and then expansion in the low pressure turbine. Find the net power output and the total heat transfer in the boiler.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45^{o}C, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m^3/kg, $P_{sat} = 9.59$ kPa

3: 3.0 MPa, 600^{o}C: $h_3 = 3682.34$ kJ/kg, $s_3 = 7.5084$ kJ/kg K

5: 500 kPa, 400^{o}C: $h_5 = 3271.83$ kJ/kg, $s_5 = 7.7937$ kJ/kg K

C.V. Pump Reversible and adiabatic. Incompressible flow so

Energy: $w_p = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101\ (3000 - 9.6) = 3.02$ kJ/kg

C.V. LP Turbine section

Entropy Eq.: $s_6 = s_5 = 7.7937$ kJ/kg K => two-phase state

$$x_6 = (s_6 - s_f)/s_{fg} = \frac{7.7937 - 0.6386}{7.5261} = 0.9507$$

$$h_6 = 188.42 + 0.9507 \times 2394.77 = 2465.1 \text{ kJ/kg}$$

Both turbine sections

$$w_{T,tot} = h_3 - h_4 + h_5 - h_6$$

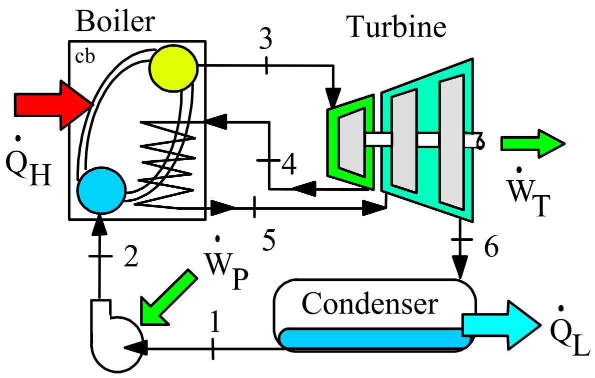
$$= 3682.34 - 3093.26 + 3271.83 - 2465.1 = 1395.81 \text{ kJ/kg}$$

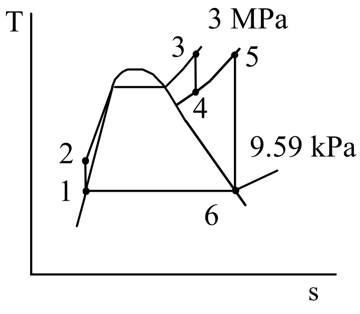
$$\dot{W}_{net} = \dot{W}_T - \dot{W}_p = \dot{m}(w_{T,tot} - w_p) = 25\ (1395.81 - 3.02) = \mathbf{34\ 820\ kW}$$

Both boiler sections

$$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$$

$$= 25\ (3682.34 - 191.44 + 3271.83 - 3093.26) = \mathbf{91\ 737\ kW}$$





11.35

Consider the supercritical cycle in problem 11.31 and assume the turbine first expands to 3 MPa then a reheat to 500°C with a further expansion in the low pressure turbine to 10 kPa. Find the combined specific turbine work and the total specific heat transfer in the boiler.

For the efficiency we need the net work and steam generator heat transfer.

C.V. Pump. For this high exit pressure we use Table B.1.4

State 1: $s_1 = 0.6492$ kJ/kg K, $h_1 = 191.81$ kJ/kg

Entropy Eq.: $s_2 = s_1 \quad => \quad h_2 = 222.5$ kJ/kg

State 3: $h_3 = 3275.36$ kJ/kg, $s_3 = 6.0342$ kJ/kg-K

C.V. Turbine section 1. Assume reversible and adiabatic.

Entropy Eq.: $s_4 = s_3 = 6.0342 = 2.6456 + x_4 \times 3.5412, \quad x_4 = 0.956907$

$h_4 = 1008.41 + x_4 \times 1795.73 = 2726.76$ kJ/kg,

State 5: $h_5 = 3456.48$ kJ/kg, $s_5 = 7.2337$ kJ/kg-K

C.V. Turbine section 2. Assume reversible and adiabatic.

Entropy Eq.: $s_6 = s_5 = 7.2337 = 0.6492 + x_6 \times 7.501, \quad x_6 = 0.87782$

$h_6 = 191.81 + x_6 \times 2392.82 = 2292.27$ kJ/kg

Steam generator: $q_H = h_3 - h_2 + h_5 - h_4$

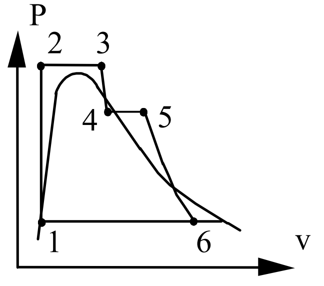
$= 3275.36 - 222.5 + 3456.48 - 2726.76$

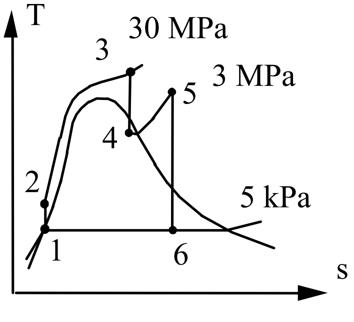
$= 3052.86 + 729.72 =$ **3782.6 kJ/kg**

Turbine: $w_T = h_3 - h_4 + h_5 - h_6$

$= 3275.36 - 2726.76 + 3456.48 - 2292.27$

$= 548.6 + 1164.21 =$ **1712.8 kJ/kg**





11.36

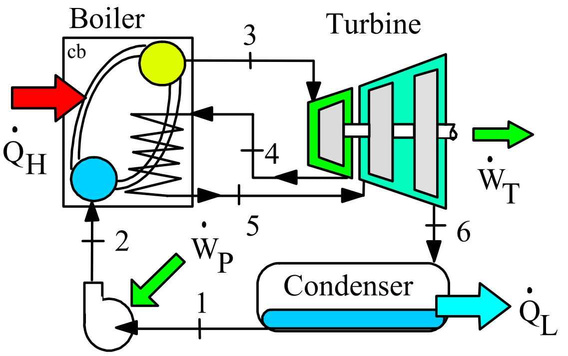
Consider an ideal steam reheat cycle where steam enters the high-pressure turbine at 3.0 MPa, 400°C, and then expands to 0.8 MPa. It is then reheated to 400°C and expands to 10 kPa in the low-pressure turbine. Calculate the cycle thermal efficiency and the moisture content of the steam leaving the low-pressure turbine.

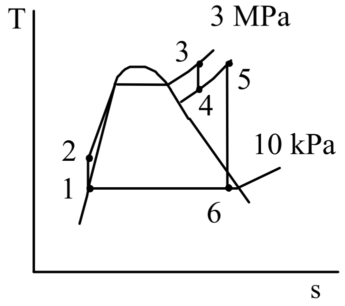
Solution:

C.V. Pump reversible, adiabatic and assume incompressible flow

$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg,

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg





C.V. HP Turbine section

$P_3 = 3$ MPa, $T_3 = 400^{\circ}C$ => $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

$s_4 = s_3$ => $h_4 = 2891.6$ kJ/kg;

C.V. LP Turbine section

State 5: $400^{\circ}C$, 0.8 MPa => $h_5 = 3267.1$ kJ/kg, $s_5 = 7.5715$ kJ/kg K

Entropy Eq.: $s_6 = s_5 = 7.5715$ kJ/kg K => two-phase state

$$x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{7.5715 - 0.6492}{7.501} = 0.92285 = \mathbf{0.923}$$

$$h_6 = 191.81 + 0.92285 \times 2392.82 = 2400 \text{ kJ/kg}$$

$w_{T,tot} = h_3 - h_4 + h_5 - h_6 = 3230.82 - 2891.6 + 3267.1 - 2400 = 1237.8$ kJ/kg

$q_{H1} = h_3 - h_2 = 3230.82 - 194.83 = 3036$ kJ/kg

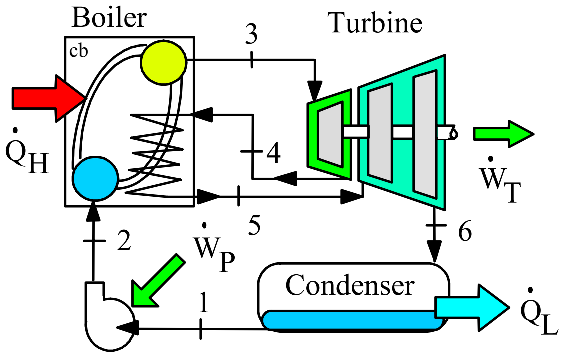
$q_H = q_{H1} + h_5 - h_4 = 3036 + 3267.1 - 2891.6 = 3411.5$ kJ/kg

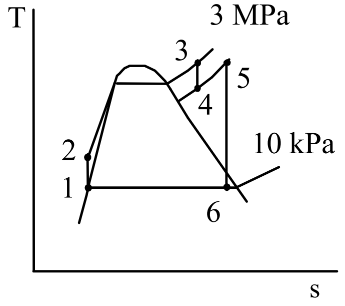
$\eta_{CYCLE} = (w_{T,tot} - w_P)/q_H = (1237.8 - 3.02)/3411.5 = \mathbf{0.362}$

11.37

The reheat pressure affects the operating variables and thus turbine performance. Repeat Problem 11.33 twice, using 0.6 and 1.0 MPa for the reheat pressure.

Solution





C.V. Pump reversible, adiabatic and assume incompressible flow

$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg,

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

State 3: 3 MPa, 400°C => $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

Low T boiler section: $q_{H1} = h_3 - h_2 = 3230.82 - 194.83 = 3035.99$ kJ/kg

State 4: P_4, $s_4 = s_3$

For $P_4 = 1$ MPa: $h_4 = 2940.85$ kJ/kg state 4 is sup. vapor

State 5: 400°C, $P_5 = P_4$ => $h_5 = 3263.9$ kJ/kg, $s_5 = 7.465$ kJ/kg K,

For $P_4 = 0.6$ MPa: $h_4 = 2793.2$ kJ/kg state 4 is sup. vapor

State 5: 400°C, $P_5 = P_4$ => $h_5 = 3270.3$ kJ/kg, $s_5 = 7.7078$ kJ/kg K,

State 6: 10 kPa, $s_6 = s_5$ => $x_6 = (s_6 - s_f)/s_{fg}$

Total turbine work: $w_{T,tot} = h_3 - h_4 + h_5 - h_6$

Total boiler H.Tr.: $q_H = q_{H1} + h_5 - h_4$

Cycle efficiency: $\eta_{CYCLE} = (w_{T,tot} - w_P)/q_H$

P_4=P_5	x_6	h_6	w_T	q_H	η_{CYCLE}
1	0.9087	2366	1187.9	3359.0	0.3527
0.6	0.9410	2443.5	1228.0	3437.7	0.3563

Notice the very small changes in efficiency.

11.38

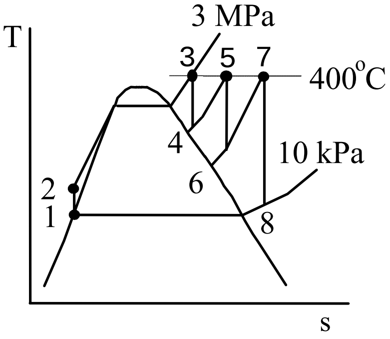
The effect of a number of reheat stages on the ideal steam reheat cycle is to be studied. Repeat Problem 11.33 using two reheat stages, one stage at 1.2 MPa and the second at 0.2 MPa, instead of the single reheat stage at 0.8 MPa.

C.V. Pump reversible, adiabatic and assume incompressible flow, work in

$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg,

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

$P_4 = P_5 = 1.2$ MPa, $P_6 = P_7 = 0.2$ MPa



3: $h_3 = 3230.82$ kJ/kg, $s_3 = 6.9211$ kJ/kg K

4: P_4, $s_4 = s_3 \Rightarrow$ sup. vap. $h_4 = 2985.3$

5: $h_5 = 3260.7$ kJ/kg, $s_5 = 7.3773$ kJ/kg K

6: P_6, $s_6 = s_5 \Rightarrow$ sup. vapor

$h_6 = 2811.2$ kJ/kg

7: $h_7 = 3276.5$ kJ/kg, $s_7 = 8.2217$ kJ/kg K

8: P_8, $s_8 = s_7 \Rightarrow$ sup. vapor $h_8 = 2607.9$ kJ/kg

Total turbine work, same flow rate through all sections

$w_T = (h_3 - h_4) + (h_5 - h_6) + (h_7 - h_8) = 245.5 + 449.5 + 668.6 = 1363.6$ kJ/kg

Total heat transfer in boiler, same flow rate through all sections

$q_H = (h_3 - h_2) + (h_5 - h_4) + (h_7 - h_6) = 3036 + 319.8 + 465.3 = 3821.1$ kJ/kg

Cycle efficiency: $\eta_{TH} = \dfrac{w_T - w_P}{q_H} = \dfrac{1363.6 - 3.02}{3821.1} = \mathbf{0.356}$

Open Feedwater Heaters

11.39

A power plant for a polar expedition uses ammonia and the boiler exit is 80°C, 1000 kPa and the condenser operates at -15°C. A single open "feed water" heater operates at 400 kPa with an exit state of saturated liquid. Find the mass fraction extracted in the turbine.

CV Feedwater heater.

States given and fixed from knowing Fig. 11.10:

5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kgK

3: $h_3 = 171.226$ kJ/kg

1: $h_1 = 111.66$ kJ/kg, $v_1 = 0.001519$ m^3/kg

Analyze the pump: $h_2 = h_1 + w_{P1} = h_1 + v_1 (P_2 - P_1)$

$= 111.66 + 0.001519 \text{ m}^3\text{/kg} \times (400 - 236.3) \text{ kPa}$

$= 111.909$ kJ/kg

Analyze the turbine: 6: 400 kPa, $s_6 = s_1 \;\; => \;\; h_6 = 1479.6$ kJ/kg

Analyze the FWH leads to Eq.11.5:

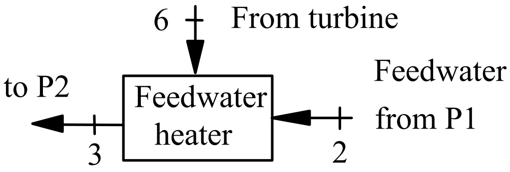
$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{171.226 - 111.909}{1479.6 - 11.909} = \mathbf{0.0434}$$

11.40

An open feedwater heater in a regenerative steam power cycle receives 20 kg/s of water at 100°C, 2 MPa. The extraction steam from the turbine enters the heater at 2 MPa, 275°C, and all the feedwater leaves as saturated liquid. What is the required mass flow rate of the extraction steam?

Solution:

The complete diagram is as in Figure 11.10 in main text.



C.V Feedwater heater

Continuity Eq.: $\dot{m}_2 + \dot{m}_6 = \dot{m}_3$

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_3 h_3 = (\dot{m}_2 + \dot{m}_6)\, h_3$

Table B.1.4: $h_2 = 420.45$ kJ/kg, Table B.1.2: $h_3 = 908.77$ kJ/kg

Table B.1.3: $h_6 = 2963$ kJ/kg, this is interpolated

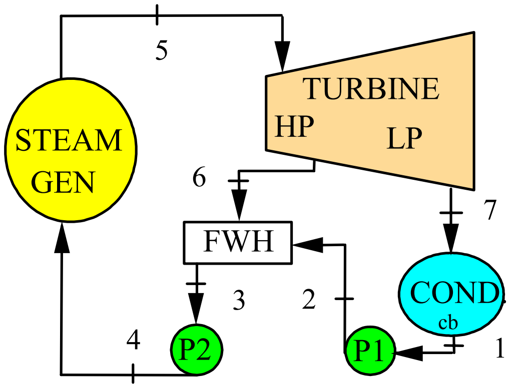
With the values substituted into the energy equation we get

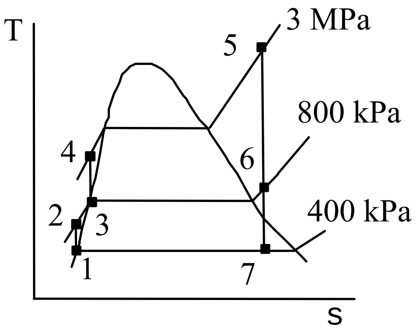
$$\dot{m}_6 = \dot{m}_2 \frac{h_3 - h_2}{h_6 - h_3} = 20 \times \frac{908.77 - 420.45}{2963 - 908.77} = \mathbf{4.754\ kg/s}$$

Remark: For lower pressures at state 2 where Table B.1.4 may not have an entry the corresponding saturated liquid at same T from Table B.1.1 is used.

11.41

A low temperature power plant operates with R-410a maintaining -20°C in the condenser, a high pressure of 3 MPa with superheat to 80°C. There is one open "feed water" heater operating at 800 kPa with an exit as saturated liquid at 0°C. Find the extraction fraction of the flow out of the turbine and the turbine work per unit mass flowing through the boiler.





State 1: $x_1 = 0$, $h_1 = 28.24$ kJ/kg, $v_1 = 0.000803$ m^3/kg

State 3: $x_3 = 0$, $h_3 = 57.76$ kJ/kg, $v_3 = 0.000855$ m^3/kg

State 5: $h_5 = 329.1$ kJ/kg, $s_5 = 1.076$ kJ/kg K

State 6: $s_6 = s_5 \;\; => \;\; T_6 = 10.2^{o}C, \quad h_6 = 290.1$ kJ/kg

State 7: $s_7 = s_5 \;\; => \;\; x_7 = (s_7 - s_f)/s_{fg} = 0.9983, \quad h_7 = 271.5$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.000803\ (800 - 400) = 0.32 \text{ kJ/kg}$$

$$=> \; h_2 = h_1 + w_{P1} = 28.24 + 0.32 = 28.56 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

Energy Eq.: $(1 - y)\, h_2 + y\, h_6 = 1\, h_3$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{57.76 - 28.56}{290.1 - 28.56} = \mathbf{0.1116}$$

C.V. Turbine

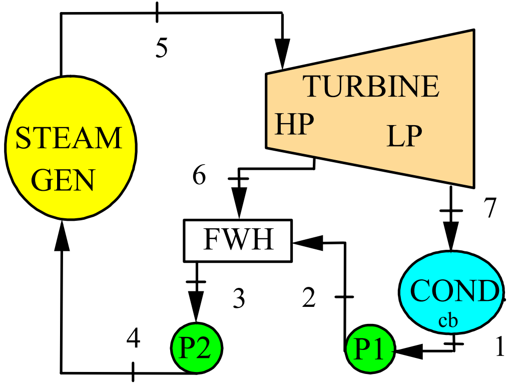
$$\dot{W}_T = \dot{m}_{TOT}h_5 - x\dot{m}_{TOT}h_6 - (1 - y)\dot{m}_{TOT}h_7$$

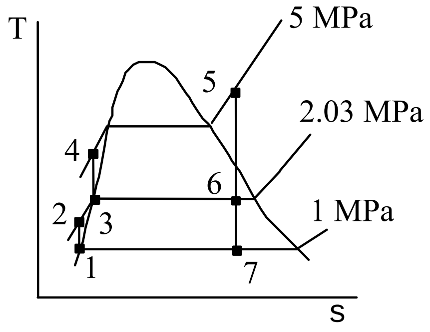
$$w_T = h_5 - y\, h_6 - (1 - y)\, h_7$$

$$= 329.1 - 0.1116 \times 290.1 - (1 - 0.1116) \times 271.5 = 55.52 \text{ kJ/kg}$$

11.42

A Rankine cycle operating with ammonia is heated by some low temperature source so the highest T is 120°C at a pressure of 5000 kPa. Its low pressure is 1003 kPa and it operates with one open feedwater heater at 2033 kPa. The total flow rate is 5 kg/s. Find the extraction flow rate to the feedwater heater assuming its outlet state is saturated liquid at 2033 kPa. Find the total power to the two pumps.





State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $x_3 = 0$, $h_3 = 421.48$ kJ/kg, $v_3 = 0.001777$ m^3/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5 \;\Rightarrow\; x_6 = (s_6 - s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(2033 - 1003) = 1.708 \text{ kJ/kg}$$

$$\Rightarrow h_2 = h_1 + w_{P1} = 298.25 + 1.708 = 299.96 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot m_6 / \dot m_{tot} = x$ (the extraction fraction)

Energy Eq.: $(1 - x)\, h_2 + x\, h_6 = 1\, h_3$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{421.48 - 299.96}{1461.53 - 299.96} = \mathbf{0.1046}$$

$$\dot m_{extr} = x\, \dot m_{tot} = 0.1046 \times 5 = \mathbf{0.523\ kg/s}$$

$$\dot m_1 = (1 - x)\, \dot m_{tot} = (1 - 0.1046)\, 5 = 4.477 \text{ kg/s}$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001777\,(5000 - 2033) = 5.272 \text{ kJ/kg}$$

Total pump work

$$\dot W_p = \dot m_1 w_{P1} + \dot m_{tot}\, w_{P2} = 4.477 \times 1.708 + 5 \times 5.272 = \mathbf{34\ kW}$$

11.43

A steam power plant has high and low pressures of 20 MPa and 10 kPa, and one open feedwater heater operating at 1 MPa with the exit as saturated liquid. The maximum temperature is 800°C and the turbine has a total power output of 5 MW. Find the fraction of the flow for extraction to the feedwater and the total condenser heat transfer rate.

The physical components and the T-s diagram is as shown in Fig. 11.10 in the main text for one open feedwater heater. The same state numbering is used. From the Steam Tables:

State 5: (P, T) $h_5 = 4069.8$ kJ/kg, $s_5 = 7.0544$ kJ/kg K,

State 1: (P, x = 0) $h_1 = 191.81$ kJ/kg, $v_1 = 0.00101$ m^3/kg

State 3: (P, x = 0) $h_3 = 762.8$ kJ/kg, $v_3 = 0.001127$ m^3/kg

Pump P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101 \times 990 = 1$ kJ/kg

$h_2 = h_1 + w_{P1} = 192.81$ kJ/kg

Turbine 5-6: $s_6 = s_5 \Rightarrow h_6 = 3013.7$ kJ/kg

$w_{T56} = h_5 - h_6 = 4069.8 – 3013.7 = 1056.1$ kJ/kg

Feedwater Heater ($\dot{m}_{TOT} = \dot{m}_5$): $x\dot{m}_5h_6 + (1 - x)\dot{m}_5h_2 = \dot{m}_5h_3$

$$\Rightarrow \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.8 - 192.81}{3013.7 - 192.81} = \mathbf{0.2021}$$

To get state 7 into condenser consider turbine.

$s_7 = s_6 = s_5 \quad \Rightarrow \quad x_7 = (7.0544 - 0.6493)/7.5009 = 0.85391$

$h_7 = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

Find specific turbine work to get total flow rate

$$\dot{W}_T = \dot{m}_{TOT}h_5 - x\dot{m}_{TOT}h_6 - (1 - x)\dot{m}_{TOT}h_7 =$$

$$= \dot{m}_{TOT} \times (h_5 - xh_6 - (1 - x)h_7) = \dot{m}_{TOT} \times 1677.3$$

$$\dot{m}_{TOT} = 5000/1677.3 = 2.98 \text{ kg/s}$$

$$\dot{Q}_L = \dot{m}_{TOT}\,(1\text{-}x)\,(h_7\text{-}h_1) = 2.98 \times 0.7979(2235.1 - 191.81) = \mathbf{4858\ kW}$$

11.44

Find the cycle efficiency for the cycle in Problem 11.39
CV Feedwater heater.

States given and fixed from knowing Fig. 11.10:

5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kgK

3: $h_3 = 171.226$ kJ/kg

1: $h_1 = 111.66$ kJ/kg, $v_1 = 0.001519$ m^3/kg

Analyze the pump: $w_{P1} = v_1 (P_2 - P_1) = 0.001519 \text{ m}^3\text{/kg} \times (400 - 236.3) \text{ kPa}$
$= 0.249$ kJ/kg
$h_2 = h_1 + w_{P1} = 111.66 + 0.249 = 111.909$ kJ/kg

Analyze the turbine: 6: 400 kPa, $s_6 = s_5$ => $h_6 = 1479.6$ kJ/kg
7: -15 C, $s_7 = s_5$ => $h_7 = 1413.56$ kJ/kg

Analyze the FWH leads to Eq.11.5:

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{171.226 - 111.909}{1479.6 - 11.909} = 0.0434$$

$w_T = h_5 - y\, h_6 - (1 - y)\, h_7$
$= 1614.6 - 0.0434 \times 1479.6 - (1 - 0.0434) \times 1413.56$
$= 198.15$ kJ/kg

Pump 2 gives: $w_{P2} = v_3 (P_4 - P_3) = 0.00156 \text{ m}^3\text{/kg} \times (1000 - 400) \text{ kPa}$
$= 0.936$ kJ/kg

Net work: $w_{net} = w_T - w_{P2} - (1 - y)\, w_{P1} = 198.15 - 0.936 - (1\text{-}0.0434)\, 0.249$
$= 196.98$ kJ/kg

Boiler: $q_H = h_5 - h_4 = h_5 - (h_3 + w_{P2}) = 1614.6 - 171.65 - 0.936$
$= 1442$ kJ/kg

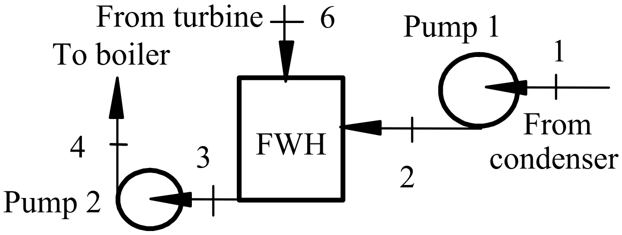
Cycle efficiency: $\eta = w_{net} / q_H = 196.98 / 1442 = \mathbf{0.1366}$

11.45

A power plant with one open feedwater heater has a condenser temperature of 45°C, a maximum pressure of 5 MPa, and boiler exit temperature of 900°C. Extraction steam at 1 MPa to the feedwater heater is mixed with the feedwater line so the exit is saturated liquid into the second pump. Find the fraction of extraction steam flow and the two specific pump work inputs.

Solution:

The complete diagram is as in Figure 11.10 in the main text.



State out of boiler 5: $h_5 = 4378.82$ kJ/kg, $s_5 = 7.9593$ kJ/kg K

C.V. Turbine reversible, adiabatic: $s_7 = s_6 = s_5$

State 6: P_6 , s_6 => $h_6 = 3640.6$ kJ/kg, $T_6 = 574^{\circ}C$

C.V Pump P1

$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(1000 - 9.6) =$ **1.0 kJ/kg**

=> $h_2 = h_1 + w_{P1} = 188.42 + 1.0 = 189.42$ kJ/kg

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

Energy Eq.: $(1 - x)\, h_2 + x\, h_6 = 1\, h_3$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3640.6 - 189.42} = \mathbf{0.1661}$$

C.V Pump P2

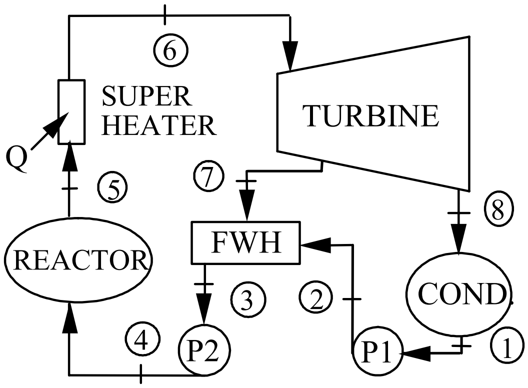
$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001127(5000 - 1000) =$ **4.5 kJ/kg**

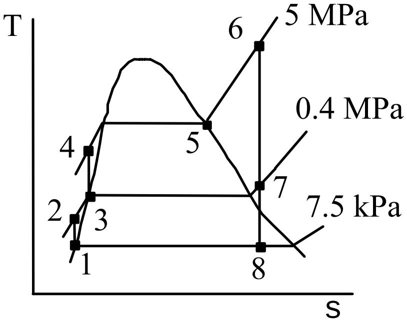
11.46

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 5 MPa exits this heat exchanger and is then superheated to 600°C in an external gas-fired superheater. The steam enters the turbine, which has one (open-type) feedwater extraction at 0.4 MPa. The condenser pressure is 7.5 kPa. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1 MW.

Solution:

The complete cycle diagram is similar to Figure 11.8 except the boiler is sparated into a section heated by the reactor and a super heater section.





CV. Pump P1

$$w_{P1} = 0.001008(400 - 7.5) = 0.4 \text{ kJ/kg} ;$$

$$h_2 = h_1 + w_{P1} = 168.8 + 0.4 = 169.2 \text{ kJ/kg}$$

CV. Pump P2

$$w_{P2} = 0.001084(5000 - 400) = 5.0 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{P2} = 604.7 + 5.0 = 609.7 \text{ kJ/kg}$$

C.V. Turbine (to get exit state properties)

$$s_7 = s_6 = 7.2589, \quad P_7 = 0.4 \text{ MPa} \;\Rightarrow\; T_7 = 221.2^{\circ}\text{C}, \quad h_7 = 2904.5 \text{ kJ/kg}$$

$$s_8 = s_6 = 7.2589 = 0.5764 + x_8 \times 7.6750 \qquad x_8 = 0.8707$$

$$h_8 = 168.8 + 0.8707 \times 2406.0 = 2263.7 \text{ kJ/kg}$$

CV: Feedwater heater FWH (to get the extraction fraction x_7)

Divide the equations with the total mass flow rate $\dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_6$

Continuity: $x_2 + x_7 = x_3 = 1.0$, Energy Eq.: $x_2h_2 + x_7h_7 = h_3$

$$x_7 = (604.7\text{-}169.2)/(2904.5\text{-}169.2) = 0.1592$$

CV: Turbine (to get the total specific work)

Full flow from 6 to 7 and the fraction $(1 - x_7)$ from 7 to 8.

$$w_T = (h_6 - h_7) + (1 - x_7)(h_7 - h_8)$$

$$= 3666.5\text{-}2904.5 + 0.8408(2904.5\text{-}2263.7) = 1300.8 \text{ kJ/kg}$$

CV: Pumps (P1 has $x_1 = 1 - x_7$, P2 has the full flow $x_3 = 1$)

$$w_P = x_1 w_{P1} + x_3 w_{P2} = 0.8408 \times 0.4 + 1 \times 5.0 = 5.3 \text{ kJ/kg}$$

$$w_{NET} = 1300.8 - 5.3 = 1295.5 \quad \Rightarrow \quad \dot{m} = 1000/1295.5 = 0.772 \text{ kg/s}$$

CV: Reactor (this has the full flow)

$$\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 0.772(2794.3 - 609.7) = \mathbf{1686\ kW}$$

CV: Superheater (this has the full flow)

$$\dot{Q}_{SUP} = \dot{m}(h_6 - h_5) = 0.772\,(3666.5 - 2794.3) = \mathbf{673\ kW}$$

11.47

Consider an ideal steam regenerative cycle in which steam enters the turbine at 3.0 MPa, 400°C, and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 0.8 MPa for an open feedwater heater. The feedwater leaves the heater as saturated liquid. The appropriate pumps are used for the water leaving the condenser and the feedwater heater. Calculate the thermal efficiency of the cycle and the net work per kilogram of steam.

Solution:

This is a standard Rankine cycle with an open FWH as shown in Fig.11.10

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$\Rightarrow\ h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$

C.V. FWH Call $\dot{m}_6 / \dot{m}_{tot} = x$ (the extraction fraction)

$$(1 - x)\, h_2 + x\, h_6 = 1\, h_3$$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

CV Boiler: $q_H = h_5 - h_4 = 3230.82 - 723.55 = 2507.3 \text{ kJ/kg}$

CV Turbine

2nd Law $s_7 = s_6 = s_5 = 6.9211 \text{ kJ/kg K}$

$P_6, s_6 \Rightarrow h_6 = 2891.6 \text{ kJ/kg}$ (superheated vapor)

$$s_7 = s_6 = s_5 = 6.9211 \quad \Rightarrow \quad x_7 = \frac{6.9211 - 0.6492}{7.501} = 0.83614$$

$$\Rightarrow \quad h_7 = 191.81 + x_7\, 2392.82 = 2192.55 \text{ kJ/kg}$$

Turbine has full flow in HP section and fraction 1-x in LP section

$$\dot{W}_T / \dot{m}_5 = h_5 - h_6 + (1 - x)(h_6 - h_7)$$

$$w_T = 3230.82 – 2891.6 + (1 - 0.1988)(2891.6 – 2192.55) = 899.3 \text{ kJ/kg}$$

P2 has the full flow and P1 has the fraction 1-x of the flow

$$w_{net} = w_T - (1 - x)\, w_{P1} - w_{P2}$$

$$= 899.3 - (1 - 0.1988)0.798 – 2.45 = \mathbf{896.2\ kJ/kg}$$

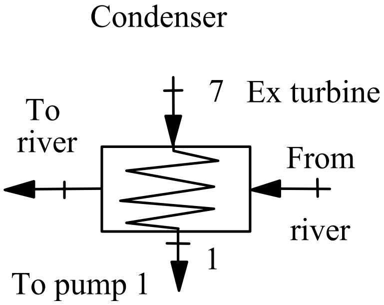
$$\eta_{cycle} = w_{net} / q_H = 896.2 / 2507.3 = \mathbf{0.357}$$

11.48

A steam power plant operates with a boiler output of 20 kg/s steam at 2 MPa, 600°C. The condenser operates at 50°C dumping energy to a river that has an average temperature of 20°C. There is one open feedwater heater with extraction from the turbine at 600 kPa and its exit is saturated liquid. Find the mass flow rate of the extraction flow. If the river water should not be heated more than 5°C how much water should be pumped from the river to the heat exchanger (condenser)?

Solution:

The setup is as shown in Fig. 11.10.



1: 50^oC sat liq. $v_1 = 0.001012\ m^3/kg$,

$h_1 = 209.31$ kJ/kg

2: 600 kPa $s_2 = s_1$

3: 600 kPa, sat liq. $h_3 = h_f = 670.54$ kJ/kg

5: (P, T) $h_5 = 3690.1$ kJ/kg,

$s_5 = 7.7023$ kJ/kg K

6: 600 kPa, $s_6 = s_5$ => $h_6 = 3270.0$ kJ/kg

CV P1

$$w_{P1} = v_1(P_2 - P_1) = 0.001012\ (600 - 12.35) = 0.595 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{P1} = 209.9 \text{ kJ/kg}$$

C.V FWH

$$x\ h_6 + (1 - x)\ h_2 = h_3$$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{670.54 - 209.9}{3270.0 - 209.9} = 0.1505$$

$$\dot{m}_6 = x\ \dot{m}_5 = 0.1505 \times 20 = \mathbf{3\ kg/s}$$

CV Turbine: $s_7 = s_6 = s_5$ => $x_7 = 0.9493$, $h_7 = 2471.17$ kJ/kg

CV Condenser

$$q_L = h_7 - h_1 = 2471.17 - 209.31 = 2261.86 \text{ kJ/kg}$$

The heat transfer out of the water from 7 to 1 goes into the river water

$$\dot{Q}_L = (1 - x)\ \dot{m}q_L = 0.85 \times 20 \times 2261.86 = 38\ 429 \text{ kW}$$

$$= \dot{m}_{H2O}\ \Delta h_{H2O} = \dot{m}_{H2O}\ (h_{f25} - h_{f20}) = \dot{m}\ (20.93)$$

$$\dot{m} = 38\ 429 / 20.93 = \mathbf{1836\ kg/s}$$

Closed Feedwater Heaters

11.49

Write the analysis (continuity and energy equations) for the closed feed water heater with a drip pump as shown in Fig.11.13. Take the control volume to have state 4 out so it includes the drip pump. Find the equation for the extraction fraction.

CV Feedwater heater plus drip pump.

Continuity Eq.: $\dot{m}_6 + \dot{m}_2 = \dot{m}_4$

Energy Eq.: $\dot{m}_6 h_6 + \dot{m}_2 h_2 + \dot{m}_6\, w_{P\ drip} = \dot{m}_4 h_4$

CV drip pump

$$w_{P\ drip} = v_6\,(P_4 - P_6) = h_{6b} - h_{6a}\ ; \quad h_{6a} = h_{f\ at\ P6}$$

Divide the energy equation with the full flow rate ($\dot{m}_4$) to get

Energy Eq.: $y\,h_6 + (1 - y)\,h_2 + y\,w_{P\ drip} = h_4$

Now solve for the fraction $y = \dot{m}_6 / \dot{m}_4$

$$y = \frac{h_4 - h_2}{h_6 - h_2 + w_{P\ drip}}$$

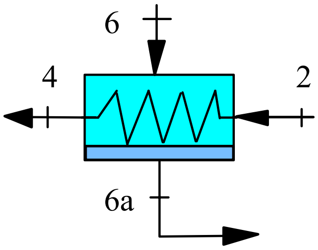
So to use this expression we assume we know states 2, 4 and 6 and have analyzed the drip pump.

11.50

A closed feedwater heater in a regenerative steam power cycle heats 20 kg/s of water from 100°C, 20 MPa to 250°C, 20 MPa. The extraction steam from the turbine enters the heater at 4 MPa, 275°C, and leaves as saturated liquid. What is the required mass flow rate of the extraction steam?

Solution:

The schematic from Figure 11.11 has the feedwater from the pump coming at state 2 being heated by the extraction flow coming from the turbine state 6 so the feedwater leaves as saturated liquid state 4 and the extraction flow leaves as condensate state 6a.



From table B.1	h kJ/kg
B.1.4: 100°C, 20 MPa	$h_2 = 434.06$
B.1.4: 250°C, 20 MPa	$h_4 = 1086.75$
B.1.3: 4 MPa, 275°C	$h_6 = 2886.2$
B.1.2: 4 MPa, sat. liq.	$h_{6a} = 1087.31$

C.V. Feedwater Heater

Energy Eq.: $\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_2 h_4 + \dot{m}_6 h_{6a}$

Since all four states are known we can solve for the extraction flow rate

$$\dot{m}_6 = \dot{m}_2 \frac{h_2 - h_4}{h_{6a} - h_6}$$

$$= 20 \frac{434.06 \ - 1086.75}{1087.31 - 2886.2} \text{ kg/s} = \mathbf{7.257\ kg/s}$$

11.51

A power plant with one closed feedwater heater has a condenser temperature of 45°C, a maximum pressure of 5 MPa, and boiler exit temperature of 900°C. Extraction steam at 1 MPa to the feedwater heater condenses and is pumped up to the 5 MPa feedwater line where all the water goes to the boiler at 200°C. Find the fraction of extraction steam flow and the two specific pump work inputs.

Solution:

$s_1 = 0.6387$ kJ/kg K,

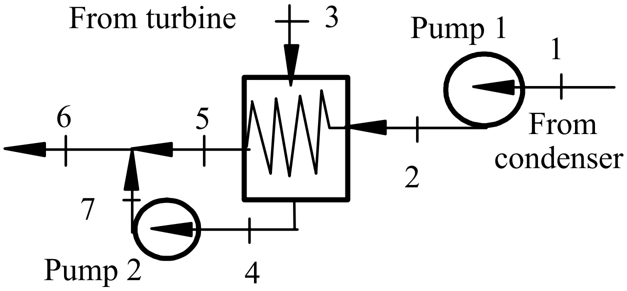
$h_1 = 188.45$ kJ/kg

$v_1 = 0.00101$ m^3/kg,

$s_4 = 2.1387$ kJ/kg K,

$h_4 = 762.81$ kJ/kg

$T_6 \Rightarrow\ h_6 = 853.9$ kJ/kg



C.V. Turbine: Reversible, adiabatic so constant s from inlet to extraction point

$s_3 = s_{IN} = 7.9593$ kJ/kg K $\Rightarrow$ $T_3 = \mathbf{573.8}$, $h_3 = 3640.6$ kJ/kg

C.V. P1: $w_{P1} = v_1(P_2 - P_1) = \mathbf{5.04\ kJ/kg}$ $\Rightarrow$ $h_2 = h_1 + w_{P1} = 193.49$ kJ/kg

C.V. P2: $w_{P2} = v_4(P_7 - P_4) = \mathbf{4.508\ kJ/kg}$ $\Rightarrow$ $h_7 = h_4 + w_{P2} = 767.31$ kJ/kg

C.V. Total FWH and pumps:

The extraction fraction is: $x = \dot{m}_3/\dot{m}_6$

Continuity Eq.: $\dot{m}_6 = \dot{m}_1 + \dot{m}_3$, $1 = (1 - y) + y$

Energy: $(1 - y)h_2 + y\,h_3 + y\,w_{P2} = h_6$

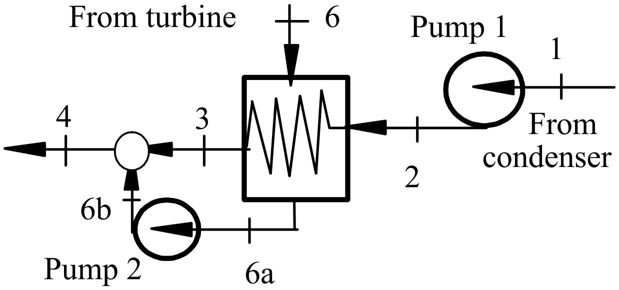
$$y = \frac{h_6 - h_2}{h_3 + w_{P2} - h_2} = \frac{853.9 - 193.49}{3640.6 + 4.508 - 193.49} = 0.1913$$

$$\mathbf{\dot{m}_3/\dot{m}_6 = y = 0.1913}$$

11.52

A Rankine cycle feeds 5 kg/s ammonia at 2 MPa, 140°C to the turbine, which has an extraction point at 800 kPa. The condenser is at -20°C and a closed feed water heater has an exit state (3) at the temperature of the condensing extraction flow and it has a drip pump. The source for the boiler is at constant 180°C. Find the extraction flow rate and state 4 into the boiler.

$P_1 = 190.2$ kPa,
$h_1 = 89.05$ kJ/kg
$v_1 = 0.001504\ m^3/kg$,
$s_5 = 5.5022$ kJ/kg K,
$h_5 = 1738.2$ kJ/kg
$T_{6a} = T_{sat\ 800\ kPa} = 17.85°C$
=> $h_{6a} = 264.18$ kJ/kg



C.V. Turbine: Reversible, adiabatic so constant s from inlet to extraction point

$s_6 = s_{IN} = 5.5022$ kJ/kg K => $T_6 = 63.4°C$, $h_6 = 1580.89$ kJ/kg

C.V. P1: $w_{P1} = v_1(P_2 – P_1) = 0.001504(2000 – 190.2) = 2.722$ kJ/kg

$\Rightarrow\ h_2 = h_1 + w_{P1} = 91.772$ kJ/kg

C.V. P2: $w_{P2} = v_{6a} (P_4 – P_6) = 0.0016108 (2000 – 800) = 1.933$ kJ/kg

$\Rightarrow\ h_{6b} = h_{6a} + w_{P2} = 266.11$ kJ/kg

C.V. Total FWH and pump (notice $h_3 = h_{6a}$ as we do not have table for this state)

The extraction fraction is: $y = \dot m_6/\dot m_4$

Energy: $(1 - y)h_2 + yh_6 = (1 - y)h_3 + yh_{6a}$

$$y = \frac{h_3 - h_2}{h_3 - h_2 + h_6 - h_{6a}} = \frac{264.18 - 91.772}{264.18 -91.772 + 1580.89 - 264.18} = 0.1158$$

$$\dot m_6 = y\ \dot m_4 = 0.1158 \times 5 = \mathbf{0.5789\ kg/s}$$

C.V. The junction after FWH and pump 2.

$$h_4 = (1–y)h_3 + y\ h_{6b} = (1– 0.1158) \times 264.18 + 0.1158 \times 266.11 = 264.4 \text{ kJ/kg}$$

11.53

Assume the powerplant in Problem 11.42 has one closed feedwater heater instead of the open FWH. The extraction flow out of the FWH is saturated liquid at 2033 kPa being dumped into the condenser and the feedwater is heated to 50°C. Find the extraction flow rate and the total turbine power output.

State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $h_3 = h_f + (P_3 – P_{sat})v_f = 421.48 + (5000–2033)0.001777 = 426.75$ kJ/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5 \Rightarrow x_6 = (s_6 – s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

State 6a: $x_{6a} = 0 \Rightarrow h_{6a} = 421.48$ kJ/kg

State 7: $s_7 = s_5 \Rightarrow x_7 = (s_7 – s_f)/s_{fg} = 0.9236$, $h_7 = 1374.43$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 – P_1) = 0.001658(5000 - 1003) = 6.627 \text{ kJ/kg}$$

$$\Rightarrow h_2 = h_1 + w_{P1} = 298.25 + 6.627 = 304.88 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

Energy Eq.: $h_2 + y\, h_6 = 1\, h_3 + y\, h_{6a}$

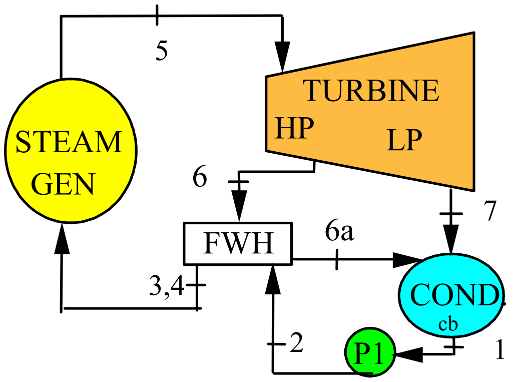
$$y = \frac{h_3 - h_2}{h_6 - h_{6a}} = \frac{426.75 - 304.88}{1461.53 - 421.48} = \mathbf{0.1172}$$

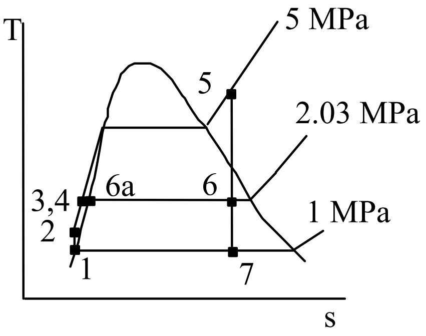
$$\dot{m}_{extr} = y\, \dot{m}_{tot} = 0.1172 \times 5 = \mathbf{0.586\ kg/s}$$

Total turbine work

$$\dot{W}_T = \dot{m}_{tot}(h_5 – h_6) + (1 – y)\dot{m}_{tot}\,(h_6 – h_7)$$

$$= 5(1586.3 – 1461.53) + (5 – 0.586)(1461.53 – 1374.43) = \mathbf{1008\ kW}$$





11.54

Do Problem 11.43 with a closed feedwater heater instead of an open and a drip pump to add the extraction flow to the feed water line at 20 MPa. Assume the temperature is 175°C after the drip pump flow is added to the line. One main pump brings the water to 20 MPa from the condenser.

Solution:

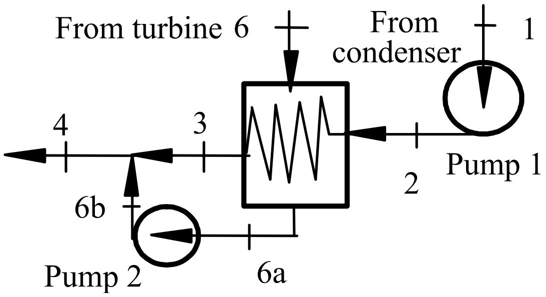
$v_1 = 0.00101 \text{ m}^3/\text{kg}$,

$h_1 = 191.81 \text{ kJ/kg}$

$T_4 = 175^{\circ}\text{C}; \; h_4 = 751.66 \text{ kJ/kg}$

$h_{6a} = h_{f\,1\text{MPa}} = 762.79 \text{ kJ/kg}$,

$v_{6a} = 0.001127 \text{ m}^3/\text{kg}$



Turbine section 1: $s_6 = s_5 = 7.0544 \text{ kJ/kg K}$

$P_6 = 1 \text{ MPa} \;\; \Rightarrow \;\; h_6 = 3013.7 \text{ kJ/kg}$

C.V Pump 1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(20\,000 - 10) = 20.19 \text{ kJ/kg}$$

$$\Rightarrow \;\; h_2 = h_1 + w_{P1} = 191.81 + 20.19 = 212.0 \text{ kJ/kg}$$

C.V Pump 2

$$w_{P2} = h_{6b} - h_{6a} = v_{6a}(P_{6b} - P_{6a}) = 0.001127(20\,000 - 1000) = 21.41 \text{ kJ/kg}$$

C.V FWH + P2 select the extraction fraction to be $y = \dot{m}_6 / \dot{m}_4$

$$y\, h_6 + (1 - y)\, h_2 + y\,(w_{P2}) = h_4$$

$$y = \frac{h_4 - h_2}{h_6 - h_2 + w_{P2}} = \frac{751.66 - 212.0}{3013.7 - 212.0 + 21.41} = \mathbf{0.191}$$

Turbine: $s_7 = s_6 = s_5 \;\; \& \;\; P_7 = 10 \text{ kPa}$

$$\Rightarrow \;\; x_7 = \frac{7.0544 - 0.6493}{7.5009} = 0.85391$$

$$h_7 = 191.81 + 0.85391 \times 2392.82 = 2235.1 \text{ kJ/kg}$$

$$w_T = [\, h_5 - h_6 + (1 - y)\,(h_6 - h_7)\,]$$

$$= [\,4069.8 - 3013.7 + 0.809\,(3013.7 - 2235.1)] = 1686 \text{ kJ/kg}$$

$$\dot{W}_T = 5000 \text{ kW} = \dot{m}_5 \times w_T = \dot{m}_5 \times 1686 \text{ kJ/kg} \quad \Rightarrow \quad \dot{m}_5 = 2.966 \text{ kg/s}$$

$$\dot{Q}_L = \dot{m}_5(1 - y)\,(h_7 - h_1) = 2.966 \times 0.809\,(2235.1 - 191.81) = \mathbf{4903\ kW}$$

11.55

Repeat Problem 11.47, but assume a closed instead of an open feedwater heater. A single pump is used to pump the water leaving the condenser up to the boiler pressure of 3.0 MPa. Condensate from the feedwater heater is drained through a trap to the condenser.

Solution:

C.V. Turbine, 2nd law:

$s_4 = s_5 = s_6 = 6.9211$ kJ/kg K

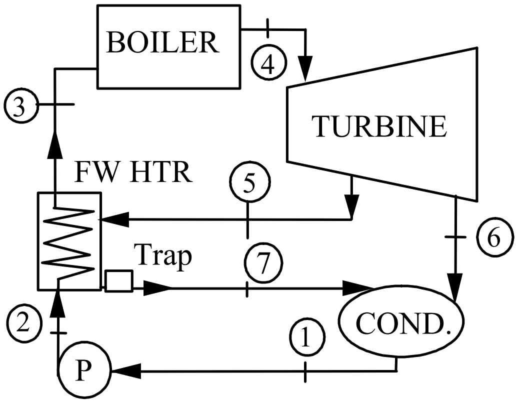
$h_4 = 3230.82$, $h_5 = 2891.6$

$\Rightarrow\ x_6 = (6.9211 - 0.6492)/7.501$

$= 0.83614$

$h_6 = 191.81 + x_6\, 2392.82$

$=2192.55$ kJ/kg



Assume feedwater heater exit at the T of the condensing steam

C.V Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.83$ kJ/kg

$T_3 = T_{sat}\,(P_5) = 170.43°C, \quad h_3 = h_f = h_7 = 721.1$ kJ/kg

C.V FWH

$\dot{m}_5 / \dot{m}_3 = y$, Energy Eq.: $h_2 + y\, h_5 = h_3 + h_7\, y$

$$y = \frac{h_3 - h_2}{h_5 - h_{f\,800}} = \frac{721.1 - 194.83}{2891.6 - 721.1} = 0.2425$$

Turbine work with full flow from 4 to 5 fraction 1-y flows from 5 to 6

$w_T = h_4 - h_5 + (1 - y)(h_5 - h_6)$

$= 3230.82 - 2891.6 + 0.7575\,(2891.6 - 2192.55)$

$= 868.75$ kJ/kg

$w_{net} = w_T - w_P = 868.75 - 3.02 =$ **865.7 kJ/kg**

$q_H = h_4 - h_3 = 3230.82 - 721.1 = 2509.7$ kJ/kg

$\eta_{cycle} = w_{net} / q_H = 865.7 / 2509.7 =$ **0.345**

11.56

Repeat Problem 11.47, but assume a closed instead of an open feedwater heater. A single pump is used to pump the water leaving the condenser up to the boiler pressure of 3.0 MPa. Condensate from the feedwater heater is going through a drip pump and added to the feedwater line so state 4 is at T_6.

Solution:

C.V. Turbine, 2nd law:

$s_5 = s_6 = s_7 = 6.9211$ kJ/kg K

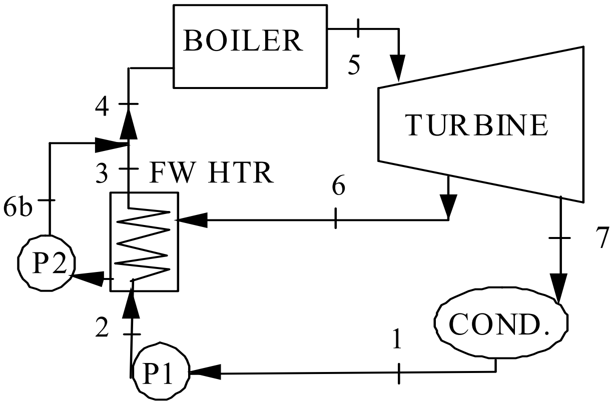
$h_5 = 3230.82$, $h_6 = 2891.6$

=> $x_7 = (6.9211 - 0.6492)/7.501$

$= 0.83614$

$h_7 = 191.81 + x_7\, 2392.82$

$=2192.55$ kJ/kg



Assume feedwater heater exit state 4 at the T of the condensing steam

C.V Pump 1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{P1} = 191.81 + 3.02 = 194.83 \text{ kJ/kg}$$

$$T_4 = T_{sat}(P_6) = 170.43°\text{C}, \quad h_4 \approx h_f = h_{6a} = 721.1 \text{ kJ/kg}$$

C.V Pump 2 (the drip pump)

$$w_{P2} = h_{6b} - h_{6a} = v_{6a}(P_{6b} - P_{6a}) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

C.V FWH + P2 select the extraction fraction to be $y = \dot{m}_6 / \dot{m}_4$

$$y\, h_6 + (1 - y)\, h_2 + y\, (w_{P2}) = h_4$$

$$y = \frac{h_4 - h_2}{h_6 - h_2 + w_{P2}} = \frac{721.1 - 194.83}{2891.6 - 194.83 + 2.45} = \mathbf{0.195}$$

Turbine work with full flow from 5 to 6 fraction 1– y flows from 6 to 7

$$w_T = [\, h_5 - h_6 + (1 - y)\, (h_6 - h_7)\,]$$

$$= [\, 3230.82 - 2891.6 + 0.805\, (2891.6 - 2192.55)] = 901.95 \text{ kJ/kg}$$

$$w_{net} = w_T - (1-y)w_{P1} - xw_{P2} = 901.95 - 0.805 \times 3.02 - 0.195 \times 2.45$$

$$= \mathbf{899.0 \text{ kJ/kg}}$$

$$q_H = h_5 - h_4 = 3230.82 - 721.1 = 2509.7 \text{ kJ/kg}$$

$$\eta_{cycle} = w_{net} / q_H = 899.0 / 2509.7 = \mathbf{0.358}$$

Nonideal Cycles

11.57

A Rankine cycle with water superheats to 500°C at 3 MPa in the boiler and the condenser operates at 100°C. All components are ideal except the turbine which has an exit state measured to be saturated vapor at 100°C. Find the cycle efficiency with a) an ideal turbine and b) the actual turbine.

Standard Rankine cycle 1-2-3-4s for ideal turbine.
Modified Rankine cycle 1-2-3-4ac for actual turbine

Table B.1.1: $v_1 = 0.001044 \text{ m}^3/\text{kg}, \quad h_1 = 419.02 \text{ kJ/kg}$

$h_3 = 3456.48 \text{ kJ/kg}, \quad s_3 = 7.2337 \text{ kJ/kg-K}$

State 4s: $s_{4s} = s_3 = 7.2337 = s_f + x_{4s}\, s_{fg} = 1.3068 + x_{4s}\, 6.0480 \Rightarrow x_{4s} = 0.97998$

$\Rightarrow h_{4s} = h_f + x_{4s} h_{fg} = 419.02 + x_{4s}\, 2257.03 = 2630.9 \text{ kJ/kg}$

State 4ac: $h_{4ac} = h_g = 2676.05 \text{ kJ/kg}$

C.V. Pump: Assume adiabatic, reversible and incompressible flow

$w_{ps} = \int v\, dP = v_1(P_2 - P_1) = 3.03 \text{ kJ/kg}$

$h_2 = h_1 + w_p = 419.02 + 3.03 = 422.05 \text{ kJ/kg}$

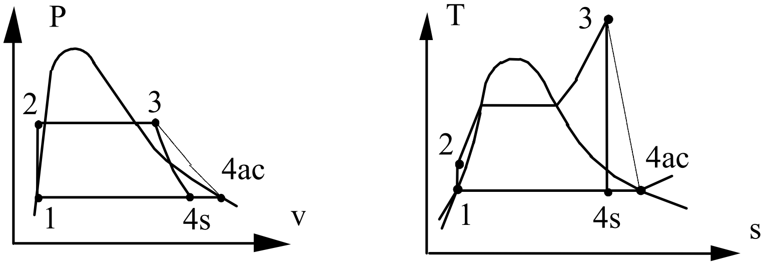
C.V. Boiler $q_B = h_3 - h_2 = 3456.48 - 422.05 = 3034.4 \text{ kJ/kg}$

C.V. Turbine; $w_{Ts} = h_3 - h_{4s} = 3456.48 - 2630.9 = 825.58 \text{ kJ/kg}$

Efficiency: $\eta_{th} = w_{net} / q_B = (825.58 - 3.03)/3034.4 = \mathbf{0.271}$

Actual turbine: $w_{Tac} = h_3 - h_{4ac} = 3456.48 - 2676.05 = 780.43 \text{ kJ/kg}$

Efficiency: $\eta_{th} = w_{net} / q_B = (780.43 - 3.03)/3034.4 = \mathbf{0.256}$



11.58

Steam enters the turbine of a power plant at 5 MPa and 400°C, and exhausts to the condenser at 10 kPa. The turbine produces a power output of 20 000 kW with an isentropic efficiency of 85%. What is the mass flow rate of steam around the cycle and the rate of heat rejection in the condenser? Find the thermal efficiency of the power plant and how does this compare with a Carnot cycle.

Solution: $\dot{W}_T = 20\,000$ kW and $\eta_{Ts} = 85$ %

State 3: $h_3 = 3195.6$ kJ/kg , $s_3 = 6.6458$ kJ/kgK

State 1: $P_1 = P_4 = 10$ kPa , sat liq , $x_1 = 0$

$T_1 = 45.8^{o}C$, $h_1 = h_f = 191.8$ kJ/kg , $v_1 = v_f = 0.00101$ m^3/kg

C.V Turbine energy Eq.: $q_T + h_3 = h_4 + w_T$; $q_T = 0$

$w_T = h_3 - h_4$, Assume Turbine is isentropic

$s_{4s} = s_3 = 6.6458$ kJ/kgK , $s_{4s} = s_f + x_{4s}\, s_{fg}$, solve for $x_{4s} = 0.7994$

$h_{4s} = h_f + x_{4s}h_{fg} = 191.81 + x_{4s}\, 2392.82 = 2104.6$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 1091$ kJ/kg , $w_T = \eta_{Ts}w_{Ts} = 927.3$ kJ/kg

$$\dot{m} = \frac{\dot{W}_T}{w_T} = \mathbf{21.568\ kg/s}, \qquad h_4 = h_3 - w_T = 2268.3 \text{ kJ/kg}$$

C.V. Condenser: 1st Law : $h_4 = h_1 + q_c + w_c$; $w_c = 0$

$q_c = h_4 - h_1 = 2076.5$ kJ/kg , $\dot{Q}_c = \dot{m}\, q_c =$ **44 786 kW**

C.V. Pump: Assume adiabatic, reversible and incompressible flow

$w_{ps} = \int v\, dP = v_1(P_2 - P_1) = 5.04$ kJ/kg

1st Law : $h_2 = h_1 + w_p = 196.8$ kJ/kg

C.V Boiler : 1st Law : $q_B + h_2 = h_3 + w_B$; $w_B = 0$

$q_B = h_3 - h_2 = 2998.8$ kJ/kg

$w_{net} = w_T - w_P = 922.3$ kJ/kg

$\eta_{th} = w_{net} / q_B =$ **0.307**

Carnot cycle : $T_H = T_3 = 400^{o}C$, $T_L = T_1 = 45.8^{o}C$

$$\eta_{th} = \frac{T_H - T_L}{T_H} = \mathbf{0.526}$$

11.59

A steam power cycle has a high pressure of 3.0 MPa and a condenser exit temperature of 45°C. The turbine efficiency is 85%, and other cycle components are ideal. If the boiler superheats to 800°C, find the cycle thermal efficiency.

Solution:

Basic Rankine cycle as shown in Figure 11.3 in the main text.

C.V. Turbine: $w_T = h_3 - h_4$, $s_4 = s_3 + s_{T,GEN}$

Ideal: Table B.1.3: $s_4 = s_3 = 7.9862$ kJ/kg K

$\Rightarrow x_{4s} = (7.9862 – 0.6386)/7.5261 = 0.9763$

$h_{4s} = h_f + x\, h_{fg} = 188.42 + 0.9763 \times 2394.77 = 2526.4$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 4146 – 2526.4 = 1619.6$ kJ/kg

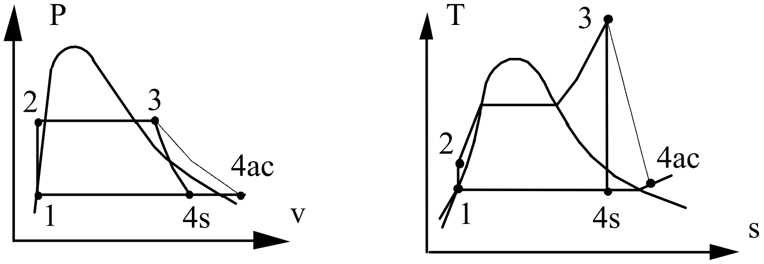
Actual: $w_{T,AC} = \eta \times w_{T,S} = 0.85 \times 1619.6 = 1376.66$ kJ/kg

C.V. Pump: $w_P = \int v\, dP \approx v_1(P_2 – P_1) = 0.00101\ (3000 – 9.6) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 188.42 + 3.02 = 191.44$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 4146 – 191.44 = 3954.6$ kJ/kg

$\eta = (w_{T,AC} – w_P)/q_H = (1376.66 – 3.02)/3954.6 = \mathbf{0.347}$



11.60

For the steam power plant described in Problem 11.13, assume the isentropic efficiencies of the turbine and pump are 85% and 80%, respectively. Find the component specific work and heat transfers and the cycle efficiency.

Solution:

This is a standard Rankine cycle with actual non-ideal turbine and pump.

CV Pump, Rev & Adiabatic:

$$w_{Ps} = h_{2s} - h_1 = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{ kJ/kg}; \quad s_{2s} = s_1$$

$$w_{Pac} = w_{Ps} / \eta_P = 3.02/0.8 = \mathbf{3.775} \text{ kJ/kg} = h_{2a} - h_1$$

$$h_{2a} = w_{Pac} + h_1 = 3.775 + 191.81 = 195.58 \text{ kJ/kg}$$

CV Boiler: $q_H = h_3 - h_{2a} = 2804.14 - 195.58 = \mathbf{2608.56\ kJ/kg}$

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 6.1869 = 0.6492 + x_4 (7.501) \quad \Rightarrow \quad x_4 = 0.7383$$

$$\Rightarrow \quad h_4 = 191.81 + 0.7383 (2392.82) = 1958.34 \text{ kJ/kg}$$

$$w_{Ts} = 2804.14 - 1958.34 = 845.8 \text{ kJ/kg}$$

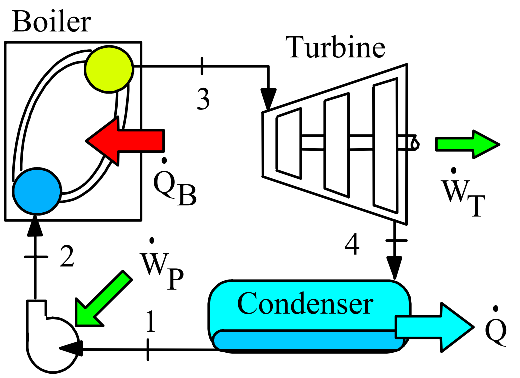
$$w_{Tac} = w_{Ts} \times \eta_T = \mathbf{718.9} = h_3 - h_{4a}$$

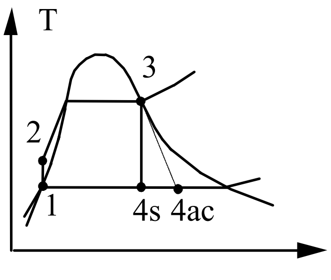
$$h_{4a} = h_3 - w_{Tac} = 2804.14 - 718.9 = 2085.24 \text{ kJ/kg}$$

CV Condenser: $q_L = h_{4a} - h_1 = 2085.24 - 191.81 = \mathbf{1893.4\ kJ/kg}$

$$\eta_{cycle} = \frac{w_{Tac} - w_{Pac}}{q_H} = \frac{718.9 - 3.78}{2608.56} = \mathbf{0.274}$$

This compares to 0.32 for the ideal case.





state 2s and 2ac nearly the same

11.61

A steam power plant operates with with a high pressure of 5 MPa and has a boiler exit temperature of of 600°C receiving heat from a 700°C source. The ambient at 20°C provides cooling for the condenser so it can maintain 45°C inside. All the components are ideal except for the turbine which has an exit state with a quality of 97%. Find the work and heat transfer in all components per kg water and the turbine isentropic efficiency. Find the rate of entropy generation per kg water in the boiler/heat source setup.

Solution:

Take CV around each component steady state in standard Rankine Cycle.

1: $v = 0.00101$; $h = 188.42$, $s = 0.6386$ (saturated liquid at 45°C).

3: $h = 3666.5$ kJ/kg, $s = 7.2588$ kJ/kg K superheated vapor

4_{ac}: $h = 188.42 + 0.97 \times 2394.8 = 2511.4$ kJ/kg

CV Turbine: no heat transfer $q = 0$

$$w_{ac} = h_3 - h_{4ac} = 3666.5 - 2511.4 = \mathbf{1155.1\ kJ/kg}$$

Ideal turbine: $s_4 = s_3 = 7.2588 \Rightarrow x_{4s} = 0.88, \quad h_{4s} = 2295$ kJ/kg

$$w_s = h_3 - h_{4s} = 3666.5 - 2295 = 1371.5 \text{ kJ/kg},$$

$$\text{Eff} = w_{ac} / w_s = 1155.1 / 1371.5 = \mathbf{0.842}$$

CV Condenser: no shaft work $w = 0$

$$q_{out} = h_{4ac} - h_1 = 2511.4 - 188.42 = \mathbf{2323\ kJ/kg}$$

CV Pump: no heat transfer, $q = 0$ incompressible flow so v = constant

$$w = v(P_2 - P_1) = 0.00101(5000 - 9.59) = \mathbf{5.04\ kJ/kg}$$

CV Boiler: no shaft work, $w = 0$

$$q_H = h_3 - h_2 = h_3 - h_1 - w_P = 3666.5 - 188.42 - 5.04 = \mathbf{3473\ kJ/kg}$$

$$s_2 + (q_H / T_H) + s_{Gen} = s_3 \text{ and } s_2 = s_1 \text{ (from pump analysis)}$$

$$s_{gen} = 7.2588 - 0.6386 - \frac{3473}{700 + 273} = \mathbf{3.05\ kJ/kg\ K}$$

11.62

Consider the power plant in Problem 11.39. Assume the high temperature source is a flow of liquid water at 120°C into a heat exchanger at constant pressure 300 kPa and that the water leaves at 90°C. Assume the condenser rejects heat to the ambient which is at -20°C. List all the places that have entropy generation and find the entropy generated in the boiler heat exchanger per kg ammonia flowing.

Solution:

a) The hot water/ammonia boiler.

b) The condensing ammonia (-15°C) to the ambient (-20°C) heat transfer.

c) The "feedwater" heater has mixing of a flow at state 6 with a flow at state 2.

State 3: $x_3 = 0$, $h_3 = 171.65$ kJ/kg, $v_3 = 0.00156$ m^3/kg, $s_3 = 0.6793$ kJ/kgK

State 5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kg K

C.V Pump P2 (rev. and adiabatic so $s_4 = s_3$).

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.00156(1000 - 400) = 0.936 \text{ kJ/kg}$$

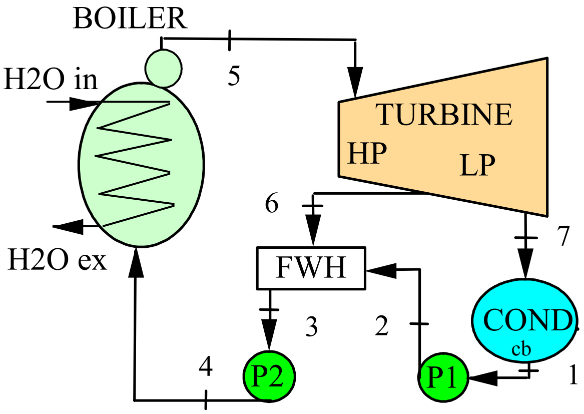
$$\Rightarrow\ h_5 - h_4 = h_5 - h_3 - w_{P2} = 1614.6 - 171.65 - 0.936 = 1442 \text{ kJ/kg}$$

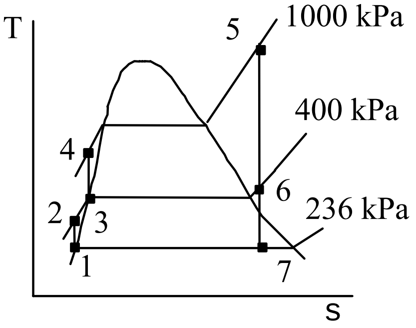
C.V. Boiler: Energy Eq.: $0 = \dot{m}_{amm}\,(h_5 - h_4) + \dot{m}_{H2O}\,(h_{in} - h_{ex})$

$$\dot{m}_{H2O} / \dot{m}_{amm} = \frac{h_5 - h_4}{h_{in} - h_{ex}} = \frac{1442}{503.69 - 376.9} = 11.373$$

Entropy Eq.: $0 = \dot{m}_{amm}\,(s_4 - s_5) + \dot{m}_{H2O}\,(s_{in} - s_{ex}) + \dot{S}_{gen}$

$$s_{amm} = (s_5 - s_4) - (\dot{m}_{H2O} / \dot{m}_{amm})(s_{in} - s_{ex})$$
$$= 5.4971 - 0.6793 - 11.373(1.5275 - 1.1924) = \mathbf{1.007\ kJ/kgK}$$





11.63

A small steam power plant has a boiler exit of 3 MPa, 400°C while it maintains 50 kPa in the condenser. All the components are ideal except the turbine which has an isentropic efficiency of 80% and it should deliver a shaft power of 9.0 MW to an electric generator. Find the specific turbine work , the needed flow rate of steam and the cycle efficiency.

Solution:

This is a standard Rankine cycle with an actual non-ideal turbine.

CV Turbine (Ideal):

$s_{4s} = s_3 = 6.9211$ kJ/kg K, $x_{4s} = (6.9211 - 1.091)/6.5029 = 0.8965$

$h_{4s} = 2407.35$ kJ/kg, $h_3 = 3230.8$ kJ/kg

$\Rightarrow\ w_{Ts} = h_3 - h_{4s} = 823.45$ kJ/kg

CV Turbine (Actual):

$w_{Tac} = \eta_T \times w_{Ts} = \mathbf{658.76} = h_3 - h_{4ac}, \quad \Rightarrow \quad h_{4ac} = 2572$ kJ/kg

$\dot{m} = \dot{W} / w_{Tac} = 9000/658.76 = \mathbf{13.66\ kg/s}$

C.V. Pump:

$w_P = h_2 - h_1 = v_1(P_2 - P_1) = 0.00103\ (3000 - 50) = 3.04$ kJ/kg

$\Rightarrow\ h_2 = h_1 + w_P = 340.47 + 3.04 = 343.51$ kJ/kg

C.V. Boiler: $q_H = h_3 - h_2 = 3230.8 - 343.51 = 2887.3$ kJ/kg

$\eta_{cycle} = (w_{Tac} - w_P) / q_H = (658.76 - 3.04) / 2887.3 = \mathbf{0.227}$

11.64

A steam power plant has a high pressure of 5 MPa and maintains 50°C in the condenser. The boiler exit temperature is 600°C. All the components are ideal except the turbine which has an actual exit state of saturated vapor at 50°C. Find the cycle efficiency with the actual turbine and the turbine isentropic efficiency.

Solution:

A standard Rankine cycle with an actual non-ideal turbine.

Boiler exit: $h_3 = 3666.5$ kJ/kg, $s_3 = 7.2588$ kJ/kg K

Ideal Turbine: 4s: 50°C, $s = s_3 \Rightarrow x = (7.2588 - 0.7037)/7.3725 = 0.88913$,

$$h_{4s} = 209.31 + 0.88913 \times 2382.75 = 2327.88 \text{ kJ/kg}$$

$$\Rightarrow \quad w_{Ts} = h_3 - h_{4s} = 1338.62 \text{ kJ/kg}$$

Condenser exit: $h_1 = 209.31$, Actual turbine exit: $h_{4ac} = h_g = 2592.1$

Actual turbine: $w_{Tac} = h_3 - h_{4ac} = 1074.4$ kJ/kg

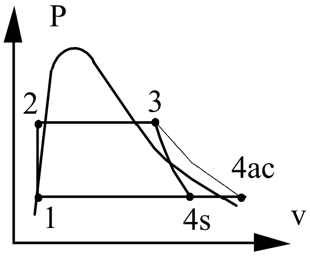
$\eta_T = w_{Tac} / w_{Ts} =$ **0.803: Isentropic Efficiency**

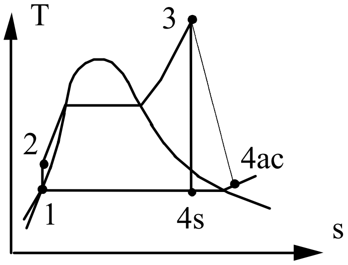
Pump: $w_P = v_1(P_2 - P_1) = 0.001012(5000-12.35) = 5.05$ kJ/kg

$h_2 = h_1 + w_P = 209.31 + 5.05 = 214.36$ kJ/kg

$q_H = h_3 - h_2 = 3666.5 - 214.36 = 3452.14$ kJ/kg

$\eta_{cycle} = (w_{Tac} - w_P) / q_H =$ **0.31: Cycle Efficiency**





11.65

A steam power plant operates with a high pressure of 4 MPa and has a boiler exit of 600°C receiving heat from a 700°C source. The ambient at 20°C provides cooling to maintain the condenser at 60°C, all components are ideal except for the turbine which has an isentropic efficiency of 92%. Find the ideal and the actual turbine exit qualities. Find the actual specific work and specific heat transfer in all four components.

Solution:

A standard Rankine cycle with an actual non-ideal turbine.

Boiler exit: $h_3 = 3674.44$ kJ/kg, $s_3 = 7.3688$ kJ/kg K

Condenser exit: $h_1 = 251.11$ kJ/kg,

Ideal Turbine: 4s: 50°C, $s = s_3 \Rightarrow x_{4s} = (7.3688 - 0.8311)/7.0784 = \mathbf{0.9236}$,

$h_{4s} = 251.11 + 0.9236 \times 2358.48 = 2429.43$ kJ/kg

$\Rightarrow \quad w_{Ts} = h_3 - h_{4s} = 1245.01$ kJ/kg

Actual turbine: $w_{Tac} = \eta_{T\,s} w_{Ts} = 0.92 \times 1245.01 = \mathbf{1074.4\ kJ/kg} = h_3 - h_{4ac}$

$h_{4ac} = h_3 - w_{Tac} = 3674.44 - 1145.4 = 2529.04$ kJ/kg

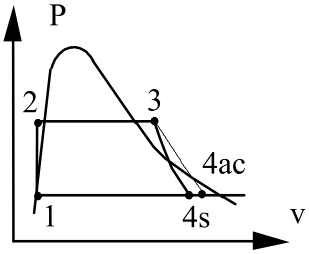
$x_{4ac} = (2529.04 - 251.11)/2358.48 = \mathbf{0.96585}$

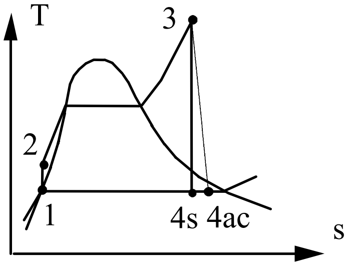
Pump: $w_P = v_1(P_2 - P_1) = 0.001017(4000 - 19.94) = \mathbf{4.05\ kJ/kg}$

$h_2 = h_1 + w_P = 251.11 + 4.05 = 255.16$ kJ/kg

$q_H = h_3 - h_2 = 3674.44 - 255.16 = \mathbf{3419.3\ kJ/kg}$

$q_L = h_{4ac} - h_1 = 2529.04 - 251.11 = \mathbf{2277.9\ kJ/kg}$





11.66

For the previous Problem find also the specific entropy generation in the boiler heat source setup.

CV. Boiler out to the source.

Entropy Eq.: $s_2 + \frac{q_H}{T_{source}} + s_{gen} = s_3$

State 1: Sat. liquid $h_1 = 251.11$ kJ/kg, $s_1 = 0.8311$ kJ/kgK

Pump: $w_P = v_1(P_2 - P_1) = 0.001017(4000 – 19.94) = 4.05$ kJ/kg

State 2: $h_2 = h_1 + w_P = 251.11 + 4.05 = 255.16$ kJ/kg, $s_2 = s_1$

State 3: $h_3 = 3674.44$ kJ/kg, $s_3 = 7.3688$ kJ/kgK

Boiler: $q_H = h_3 – h_2 = 3674.44 – 255.16 = 3419.3$ kJ/kg

$$s_{gen} = s_3 - s_2 - \frac{q_H}{T_{source}} = 7.3688 - 0.8311 - \frac{3419.3}{700 + 273} = \mathbf{3.023\ kJ/kgK}$$

11.67

Repeat Problem 11.43 assuming the turbine has an isentropic efficiency of 85%.

The physical components and the T-s diagram is as shown in Fig. 11.10 in the main text for one open feedwater heater. The same state numbering is used. From the Steam Tables:

State 5: (P, T) $h_5 = 4069.8$ kJ/kg, $s_5 = 7.0544$ kJ/kg K,

State 1: (P, x = 0) $h_1 = 191.81$ kJ/kg, $v_1 = 0.00101\ \mathrm{m^3/kg}$

State 3: (P, x = 0) $h_3 = 762.8$ kJ/kg, $v_3 = 0.001127\ \mathrm{m^3/kg}$

Pump P1: $w_{P1} = v_1(P_2 - P_1) = 0.00101 \times 990 = 1$ kJ/kg

$h_2 = h_1 + w_{P1} = 192.81$ kJ/kg

Turbine 5-6: $s_6 = s_5 \;\Rightarrow\; h_6 = 3013.7$ kJ/kg

$w_{T56,s} = h_5 - h_6 = 4069.8 – 3013.7 = 1056.1$ kJ/kg

$\Rightarrow\; w_{T56,AC} = 1056.1 \times 0.85 = 897.69$ kJ/kg

$w_{T56,AC} = h_5 - h_{6AC} \;\Rightarrow\; h_{6AC} = h_5 - w_{T56,AC}$

$= 4069.8 - 897.69 = 3172.11$ kJ/kg

Feedwater Heater ($\dot{m}_{TOT} = \dot{m}_5$): $x\dot{m}_5h_{6AC} + (1 - x)\dot{m}_5h_2 = \dot{m}_5h_3$

$$\Rightarrow \quad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.8 - 192.81}{3172.11 - 192.81} = \mathbf{0.1913}$$

To get the turbine work apply the efficiency to the whole turbine. (i.e. the first section should be slightly different).

$s_{7s} = s_{6s} = s_5 \;\Rightarrow\; x_{7s} = (7.0544 – 0.6493)/7.5009 = 0.85391,$

$h_{7s} = 191.81 + 0.85391 \times 2392.82 = 2235.1$ kJ/kg

$w_{T57,s} = h_5 - h_{7s} = 4069.8 - 2235.1 = 1834.7$ kJ/kg

$w_{T57,AC} = w_{T57,s}\eta_T = 1559.5 = h_5 - h_{7AC} \quad => \quad h_{7AC} = 2510.3$ kJ/kg

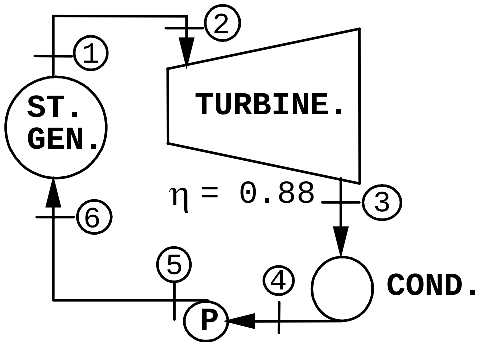
Find specific turbine work to get total flow rate

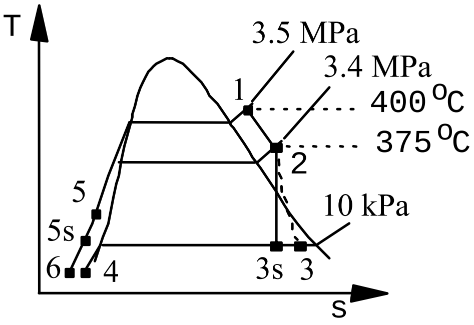
$$\dot{m}_{TOT} = \frac{\dot{W}_T}{xw_{T56} + (1\text{-}x)w_{T57}} = \frac{5000}{0.1913\times 897.69 + 0.8087\times 1559.5} = \mathbf{3.489\ kg/s}$$

$$\dot{Q}_L = \dot{m}_{TOT}(1 - x)(h_7 - h_1) = 3.489 \times 0.8087(2510.3 - 191.81) = \mathbf{6542\ kW}$$

11.68

Steam leaves a power plant steam generator at 3.5 MPa, 400°C, and enters the turbine at 3.4 MPa, 375°C. The isentropic turbine efficiency is 88%, and the turbine exhaust pressure is 10 kPa. Condensate leaves the condenser and enters the pump at 35°C, 10 kPa. The isentropic pump efficiency is 80%, and the discharge pressure is 3.7 MPa. The feedwater enters the steam generator at 3.6 MPa, 30°C. Calculate the thermal efficiency of the cycle and the entropy generation for the process in the line between the steam generator exit and the turbine inlet, assuming an ambient temperature of 25°C.





1: $h_1 = 3222.3$ kJ/kg, $s_1 = 6.8405$ kJ/kg K,

2: $h_2 = 3165.7$ kJ/kg, $s_2 = 6.7675$ kJ/kg K

3s: $s_{3S} = s_2 \Rightarrow x_{3S} = 0.8157, \quad h_{3S} = 2143.6$ kJ/kg

$w_{T,S} = h_2 - h_{3S} = 3165.7 - 2143.6 = 1022.1$ kJ/kg

$w_{T,AC} = \eta w_{T,S} = 899.4$ kJ/kg, 3ac: $h_3 = h_2 - w_{T,AC} = 2266.3$ kJ/kg

$-w_{P,S} = v_f(P_5 - P_4) = 0.001006(3700 - 10) = 3.7$ kJ/kg

$-w_{P,AC} = -w_{P,S}/\eta_P = 4.6$ kJ/kg

$q_H = h_1 - h_6 = 3222.3 - 129.0 = 3093.3$ kJ/kg

$\eta = w_{NET}/q_H = (899.4 - 4.6)/3093.3 =$ **0.289**

C.V. Line from 1 to 2: $w = 0$,

Energy Eq.: $q = h_2 - h_1 = 3165.7 - 3222.3 = -56.6$ kJ/kg

Entropy Eq.: $s_1 + s_{gen} + q/T_0 = s_2 \quad =>$

$s_{gen} = s_2 - s_1 - q/T_0 = 6.7675 - 6.8405 - (-56.6/298.15) =$ **0.117 kJ/kg K**

Cogeneration

11.69

A cogenerating steam power plant, as in Fig. 11.13, operates with a boiler output of 25 kg/s steam at 7 MPa, 500°C. The condenser operates at 7.5 kPa and the process heat is extracted as 5 kg/s from the turbine at 500 kPa, state 6 and after use is returned as saturated liquid at 100 kPa, state 8. Assume all components are ideal and find the temperature after pump 1, the total turbine output and the total process heat transfer.

Solution:

Pump 1: Inlet state is saturated liquid: $h_1 = 168.79$ kJ/kg, $v_1 = 0.001008$ m^3/kg

$$w_{P1} = \int v\, dP = v_1 (P_2 - P_1) = 0.001008(100 - 7.5) = 0.093 \text{ kJ/kg}$$

$$w_{P1} = h_2 - h_1 \Rightarrow h_2 = h_1 + w_{P1} = 168.88 \text{ kJ/kg}, \quad T_2 = \mathbf{40.3°C}$$

Turbine: $h_5 = 3410.3$ kJ/kg, $s_5 = 6.7974$ kJ/kg K

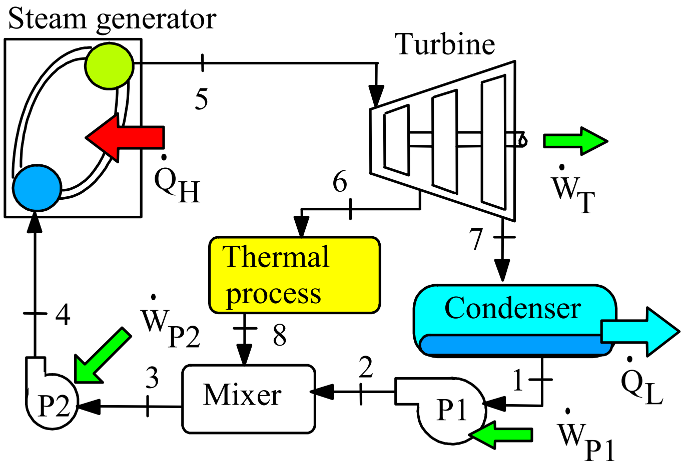
$$P_6,\ s_6 = s_5 \ \Rightarrow\ x_6 = 0.9952, \quad h_6 = 2738.6 \text{ kJ/kg}$$

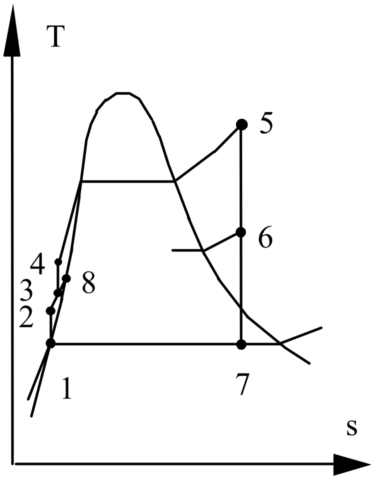
$$P_7,\ s_7 = s_5 \ \Rightarrow\ x_7 = 0.8106, \quad h_7 = 2119.0 \text{ kJ/kg}$$

From the continuity equation we have the full flow from 5 to 6 and the remainder after the extraction flow is taken out flows from 6 to 7.

$$\dot{W}_T = \dot{m}_5 (h_5 - h_6) + 0.80\dot{m}_5 (h_6 - h_7) = 25 (3410.3 - 2738.6)$$
$$+ 20 (2738.6 - 2119) = 16\,792.5 + 12\,392 = \mathbf{29.185\ MW}$$

$$\dot{Q}_{proc} = \dot{m}_6(h_6 - h_8) = 5(2738.6 - 417.46) = \mathbf{11.606\ MW}$$





11.70

A steam power plant has 4 MPa, 500°C into the turbine and to have the condenser itself deliver the process heat it is run at 101 kPa. How much net power as work is produced for a process heat of 10 MW.

Solution:

From the Rankine cycle we have the states:

1: 101 kPa, $x = 0$, $v_1 = 0.001043\ \text{m}^3/\text{kg}$, $h_1 = 418.6$ kJ/kg

3: 4 MPa, 500°C , $h_3 = 3445.2$ kJ/kg, $s_3 = 7.090$ kJ/kg K

C.V. Turbine: $s_4 = s_3 \;\Rightarrow\; x_4 = (7.090 - 1.3068)/6.048 = 0.9562,$

$$h_4 = 419.02 + 0.9562 \times 2257.03 = 2577.2 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 3445.2 – 2577.2 = 868 \text{ kJ/kg}$$

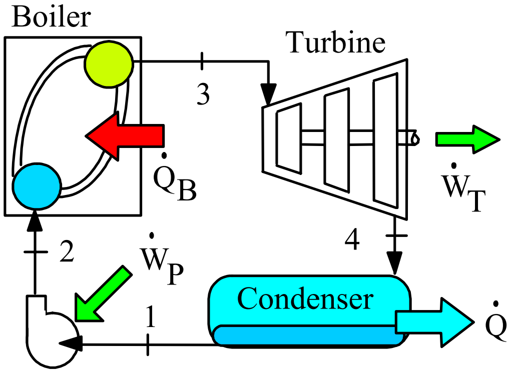
C.V. Pump: $w_P = v_1(P_2 - P_1) = 0.001043(4000 - 101) = 4.07$ kJ/kg

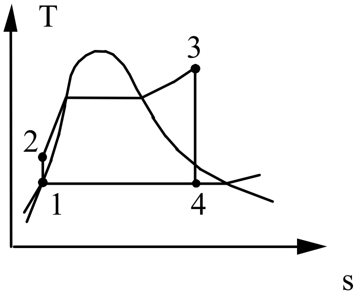
$$w_P = h_2 - h_1 \;\Rightarrow\; h_2 = 419.02 + 4.07 = 423.09 \text{ kJ/kg}$$

C.V. Condenser: $q_{L,out} = h_4 - h_1 = 2577.2 - 419.02 = 2158.2$ kJ/kg

$$\dot{m} = \dot{Q}_{proc} / q_{L,out} = 10\,000 \text{ kW} / 2158.2 \text{ kJ/kg} = 4.633 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}\,(w_T - w_P) = 4.633\,(868 – 4.07) = \mathbf{4002\ kW}$$





11.71

A 10 kg/s steady supply of saturated-vapor steam at 500 kPa is required for drying a wood pulp slurry in a paper mill. It is decided to supply this steam by cogeneration, that is, the steam supply will be the exhaust from a steam turbine. Water at 20°C, 100 kPa, is pumped to a pressure of 5 MPa and then fed to a steam generator with an exit at 400°C. What is the additional heat transfer rate to the steam generator beyond what would have been required to produce only the desired steam supply? What is the difference in net power?

Solution:

Desired exit State 4: $P_4 = 500$ kPa, sat. vap. => $x_4 = 1.0$, $T_4 = 151.9°C$

$$h_4 = h_g = 2748.7 \text{ kJ/kg}, \quad s_4 = s_g = 6.8212 \text{ kJ/kg-K}$$

Inlet State: 20°C, 100 kPa $h_1 = h_f = 83.94$ kJ/kg, $v_1 = v_f = 0.001002\ m^3/kg$

Without Cogeneration; The water is pumped up to 500 kPa and then heated in the steam generator to the desired exit T.

C.V. Pump: $w_{Pw/o} = v_1(P_4 - P_1) = 0.4$ kJ/kg

$$h_2 = h_1 + w_{Pw/o} = 84.3 \text{ kJ/kg}$$

C.V. Steam Generator: $q_{w/o} = h_4 - h_2 = 2664.4$ kJ/kg

With Cogeneration; The water is pumped to 5 MPa, heated in the steam generator to 400°C and then flows through the turbine with desired exit state.

C.V. Pump: $w_{Pw} = \int vdP = v_1(P_2 - P_1) = 4.91$ kJ/kg

$$h_2 = h_1 + w_{Pw} = 88.85 \text{ kJ/kg}$$

C.V. Steam Generator: Exit 400°C, 5 MPa => $h_3 = 3195.64$ kJ/kg

$$q_w = h_3 - h_2 = 3195.64 - 88.85 = 3106.8 \text{ kJ/kg}$$

C.V.: Turbine, Inlet and exit states given

$$w_t = h_3 - h_4 = 3195.64 - 2748.7 = 446.94 \text{ kJ/kg}$$

Comparison

Additional Heat Transfer: $q_w - q_{w/o} = 3106.8 - 2664.4 = 442.4$ kJ/kg

$$\dot{Q}_{extra} = \dot{m}(q_w - q_{w/o}) = \mathbf{4424\ kW}$$

Difference in Net Power: $w_{diff} = (w_t - w_{Pw}) + w_{Pw/o}$,

$$w_{diff} = 446.94 - 4.91 + 0.4 = 442.4 \text{ kJ/kg}$$

$$\dot{W}_{diff} = \dot{m}w_{diff} = \mathbf{4424\ kW}$$

By adding the extra heat transfer at the higher pressure and a turbine all the extra heat transfer can come out as work (it appears as a 100% efficiency)

11.72

A boiler delivers steam at 10 MPa, 550°C to a two-stage turbine as shown in Fig. 11.17. After the first stage, 25% of the steam is extracted at 1.4 MPa for a process application and returned at 1 MPa, 90°C to the feedwater line. The remainder of the steam continues through the low-pressure turbine stage, which exhausts to the condenser at 10 kPa. One pump brings the feedwater to 1 MPa and a second pump brings it to 10 MPa. Assume the first and second stages in the steam turbine have isentropic efficiencies of 85% and 80% and that both pumps are ideal. If the process application requires 5 MW of power, how much power can then be cogenerated by the turbine?

Solution:

5: $h_5 = 3500.9$, $s_5 = 6.7561$ kJ/kg K

First ideal turbine T1

6: $s_6 = s_5 \Rightarrow h_6 = 2932.4$ kJ/kg

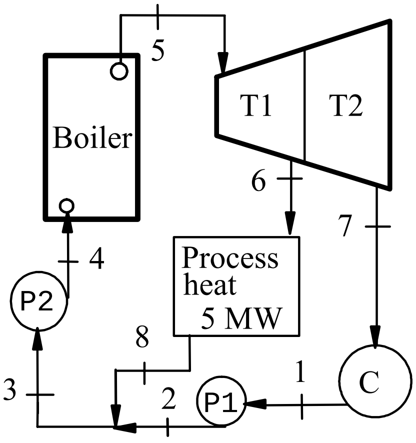
$$w_{T1} = h_5 - h_6 = 568.5 \text{ kJ/kg}$$

Ideal turbine T2

State 7: $s_7 = s_6 = s_5$

$$x = \frac{6.7561 - 0.6492}{7.501} = 0.8141$$

$$\Rightarrow \quad h_7 = 2139.9 \text{ kJ/kg}$$



$$w_{T2} = h_6 - h_7 = 2932.4 - 2139.9 = 792.5 \text{ kJ/kg}$$

Now do the process heat requirement

8: $h_8 = 377.3$ kJ/kg, approx. from the compressed liq. Table at 500 kPa

$$q_{PROC} = h_6 - h_8 = 2932.4 - 377.3 = 2555.1 \text{ kJ/kg}$$

$$\dot{m}_6 = \dot{Q}/q_{PROC} = 5000 / 2555.1 = 1.9569 \text{ kg/s} = 0.25\ \dot{m}_{TOT}$$

$$\Rightarrow \quad \dot{m}_{TOT} = \dot{m}_5 = 7.8275 \text{ kg/s},\ \ \dot{m}_7 = \dot{m}_5 - \dot{m}_6 = 5.8706 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}_5 h_5 - \dot{m}_6 h_6 - \dot{m}_7 h_7$$

$$= 7.8275 \times 3500.9 - 1.9569 \times 2932.4 - 5.8706 \times 2139.9$$

$$= \mathbf{9102\ kW}$$

11.73

In a cogenerating steam power plant the turbine receives steam from a high-pressure steam drum and a low-pressure steam drum as shown in Fig. P11.65. The condenser is made as two closed heat exchangers used to heat water running in a separate loop for district heating. The high-temperature heater adds 30 MW and the low-temperature heaters adds 31 MW to the district heating water flow. Find the power cogenerated by the turbine and the temperature in the return line to the deaerator.

Solution:

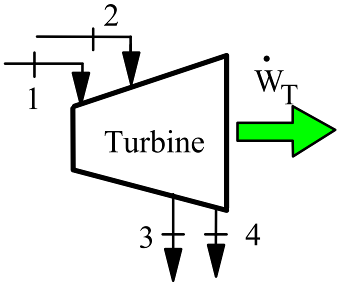
Inlet states from Table B.1.3

$h_1 = 3445.9$ kJ/kg, $s_1 = 6.9108$ kJ/kg K

$h_2 = 2855.4$ kJ/kg, $s_2 = 7.0592$ kJ/kg K

$\dot{m}_{TOT} = \dot{m}_1 + \dot{m}_2 = 27$ kg/s

Assume a reversible turbine and the two flows can mix without s generation.



Energy Eq.6.10: $\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3h_3 + \dot{m}_4h_4 + \dot{W}_T$

Entropy Eq.9.7: $\dot{m}_1s_1 + \dot{m}_2s_2 = \dot{m}_{TOT}s_{mix} \Rightarrow s_{MIX} = 6.9383$ kJ/kg K

State 3: $s_3 = s_{MIX} \Rightarrow h_3 = 2632.4$ kJ/kg, $x_3 = 0.966$

State 4: $s_4 = s_{MIX} \Rightarrow h_4 = 2413.5$ kJ/kg, $x_4 = 0.899$

$$\dot{W}_T = 22 \times 3445.9 + 5 \times 2855.4 - 13 \times 2632.4 - 14 \times 2413.5$$
$$= 22\ 077 \text{ kW} = \mathbf{22\ MW}$$

District heating line $\dot{Q}_{TOT} = \dot{m}(h_{95} - h_{60}) = 60\ 935$ kW

OK, this matches close enough

C.V. Both heaters: $\dot{m}_3h_3 + \dot{m}_4h_4 - \dot{Q}_{TOT} = \dot{m}_{TOT}h_{EX}$

$13 \times 2632.4 - 14 \times 2413.5 – 60\ 935 = 7075.2 = 27 \times h_{EX}$

$h_{EX} = 262 \approx h_f \Rightarrow T_{EX} = \mathbf{62.5°C}$

Remark: We could have computed the expansion from state 1 to P_2 followed by a mixing process to find a proper state 2a from which we expand down to P_3 and P_4

11.74

A smaller power plant produces 25 kg/s steam at 3 MPa, 600°C, in the boiler. It cools the condenser to an exit of 45°C and the cycle is shown in Fig. P11.67. There is an extraction done at 500 kPa to an open feedwater heater, and in addition a steam supply of 5 kg/s is taken out and not returned. The missing 5 kg/s water is added to the feedwater heater from a 20°C, 500 kPa source. Find the needed extraction flow rate to cover both the feedwater heater and the steam supply. Find the total turbine power output.

Solution:

The states properties from Tables B.1.1 and B.1.3

1: 45^oC, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m^3/kg, $P_{sat} = 9.59$ kPa

5: 3.0 MPa, 600^oC: $h_5 = 3682.34$ kJ/kg, $s_5 = 7.5084$ kJ/kg K

3: 500 kPa, $x = 0$: $h_3 = 640.21$ kJ/kg 8: $h_8 = 84.41$ kJ/kg

6: 500 kPa, $s_6 = s_5$ from HP turbine, $h_6 = 3093.26$ kJ/kg

C.V. Pump 1. Reversible and adiabatic. Incompressible so v = constant

Energy: $w_{p1} = h_2 - h_1 = \int v\, dP = v_1(P_2 - P_1)$

$= 0.00101\ (500 - 9.6) = 0.495$ kJ/kg

$h_2 = h_1 + w_{p1} = 188.42 + 0.495 = 188.915$ kJ/kg

C.V. Turbine sections

Entropy Eq.: $s_7 = s_5 = 7.5084$ kJ/kg K => two-phase state

$s_7 = 7.5084 = 0.6386 + x_7 \times 7.5261 \;\Rightarrow\; x_7 = 0.9128$

$h_7 = 188.42 + 0.9128 \times 2394.77 = 2374.4$ kJ/kg

C.V. Feedwater heater, including the make-up water flow, $x = \dot{m}_6/\dot{m}_5$.

Energy eq.: $\dot{m}_8h_8 + (\dot{m}_5 - \dot{m}_6)h_2 + (\dot{m}_6 - \dot{m}_8)h_6 = \dot{m}_5h_3$

Divide by $\dot{m}_5$ and solve for x

$$x = \frac{h_3 - h_2 + (h_6 - h_8)\,\dot{m}_8/\,\dot{m}_5}{h_6 - h_2} = \frac{640.21 - 188.915 + (3093.26 - 84.41)5/25}{3093.26 - 188.915}$$

$= 0.3626$

$\dot{m}_6 = x\ \dot{m}_5 = 0.3626 \times 25 =$ **9.065 kg/s**

C.V. Turbine energy equation

$\dot{W}_T = \dot{m}_5h_5 - \dot{m}_6h_6 - \dot{m}_7h_7$

$= 25 \times 3682.34 - 9.065 \times 3093.26 - 16.935 \times 2374.4$

= 26 182 kW

Refrigeration cycles

11.75

A refrigerator with R-134a as the working fluid has a minimum temperature of −10°C and a maximum pressure of 1 MPa. Assume an ideal refrigeration cycle as in Fig. 11.21. Find the specific heat transfer from the cold space and that to the hot space, and the coefficient of performance.

Solution:

Exit evaporator sat. vapor −10°C from B.5.1: $h_1 = 392.28$, $s_1 = 1.7319$ kJ/kgK

Exit condenser sat. liquid 1 MPa from B.5.1: $h_3 = 255.60$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.5.2 $\Rightarrow$ $h_2 \approx 425.68$ kJ/kg

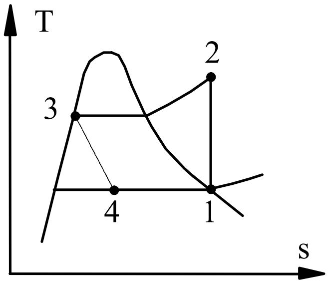
Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 392.28 - 255.60 =$ **136.7 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 425.68 - 255.60 =$ **170.1 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **4.09**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from Table B.5



11.76

Repeat the previous Problem with R-410a as the working fluid. Will that work in an ordinary kitchen?

Solution:

Exit evaporator sat. vapor −10°C from B.4.1: $h_1 = 275.78$, $s_1 = 1.0567$ kJ/kgK

Exit condenser sat. liquid 1 MPa from B.4.1: $h_3 = 68.92$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.4.2 $\Rightarrow$ $h_2 \approx 290.81$ kJ/kg

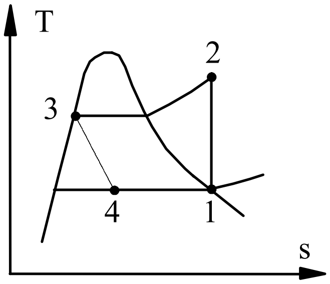
Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 275.78 - 68.92 =$ **206.9 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 290.81 - 68.92 =$ **221.9 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **13.8**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from Table B.4



The 1 MPa is too small, the condensing temperature is 7.25°C and the q_H in the condenser can not be rejected to a kitchen normally at 20°C.

11.77

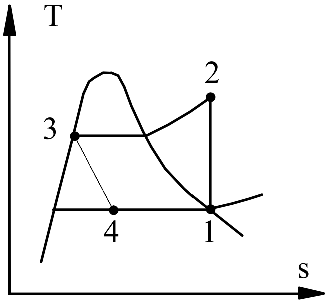
Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluids R-134a and R-410a.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$



	Property for:	R-134a, B.5	R-410a, B.4
Compressor	h_1, kJ/kg	389.2	273.9
	$s_2 = s_1$, kJ/kg K	1.7354	1.0671
	P_2, MPa	1.16	2.7283
	T_2, ^oC	51.8*	71.5**
	h_2, kJ/kg	429.9*	322.72**
	$w_C = h_2 - h_1$	40.7	48.82
Exp. valve	$h_3 = h_4$, kJ/kg	264.11	133.61
Evaporator	$q_L = h_1 - h_4$	125.1	140.29
	$\beta = q_L/w_C$	**3.07**	**2.87**

* For state 2 an interpolation between 1 and 1.2 MPa is needed for 1.16 MPa:

At 1 MPa, s = 1.7354 : T = 45.9 °C and h = 426.8 kJ/kg

At 1.2 MPa, s = 1.7354 : T = 53.3 °C and h = 430.7 kJ/kg

** For state 2 an interpolation between 2 and 3 MPa is needed for 2.728 MPa:

At 2 MPa, s = 1.0671 : T = 54.68 °C and h = 313.942 kJ/kg

At 3 MPa, s = 1.0671 : T = 77.80 °C and h = 326.0 kJ/kg

It would make more sense to use the CATT3 program.

11.78

The natural refrigerant carbon dioxide has a fairly low critical temperature. Find the high temperature, the condensing temperature and the COP if it is used in a standard cycle with high and low pressures of 6 MPa and 3 MPa.

Exit evaporator x = 1 and 3 MPa from B.3.2:

$$h_1 = 320.71 \text{ kJ/kg}, \quad s_1 = 1.2098 \text{ kJ/kgK}$$

Exit condenser saturated liquid 6 MPa from B.3.1:

$$T_3 = \mathbf{22°C}, \quad h_3 = 150 \text{ kJ/kg}$$

Exit compressor: 6 MPa, $s = s_1$, so interpolate in B.3.2

$$T_2 = \mathbf{45.9°C}, \quad h_2 = 348.24 \text{ kJ/kg}$$

COP: $$\beta = \frac{q_L}{w_c} = \frac{h_1 - h_3}{h_2 - h_1} = \mathbf{6.2}$$

Remark: The condensing T is too low for a standard refrigerator.

11.79

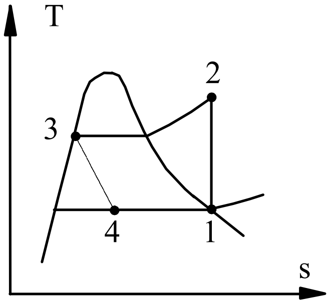
Do problem 11.77 with ammonia as the working fluid.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$



State 1: $h_1 = 1424.6$ kJ/kg, $s_1 = 5.5397$ kJ/kg-K

State 2: $s = s_1$, $P_2 = 1782$ kPa, $T_2 = 135.1^oC$, $h_2 = 1731.3$ kJ/kg

For state 2 an interpolation between 1.6 and 2 MPa is needed for 1.782 MPa:

At 1.6 MPa, s = 5.5397 : T = 126.0 °C and h = 1712.2 kJ/kg

At 2.0 MPa, s = 5.5397 : T = 146.1°C and h = 1754.1 kJ/kg

It would make more sense to use the CATT3 program.

$w_C = h_2 - h_1 = 1731.3 – 1424.6 = 306.7$ kJ/kg

State 3-4: $h_4 = h_3 = 396.3$ kJ/kg $q_L = h_1 - h_4 = 1028.3$ kJ/kg

$\beta = q_L/w_C$ **= 3.353**

11.80

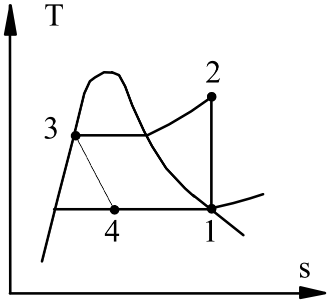
A refrigerator receives 500 W of electrical power to the compressor driving the cycle flow of R-134a. The refrigerator operates with a condensing temperature of 40°C and a low temperature of -5°C. Find the COP for the cycle.

Solution:

Ideal refrigeration cycle

$T_{cond} = 40^oC = T_3$

$T_{evap} = -5^oC = T_1$



State 1: $h_1 = 395.3$ kJ/kg, $s_1 = 1.7288$ kJ/kg-K

State 2: $s = s_1$, $P_2 = P_3 = 1017$ kPa, $T_2 = 45^oC$, $h_2 = 425$ kJ/kg

$w_C = h_2 - h_1 = 425 – 395.3 = 29.7$ kJ/kg

State 3-4: $h_4 = h_3 = 256.5$ kJ/kg $q_L = h_1 - h_4 = 138.8$ kJ/kg

$\beta = q_L/w_C$ **= 4.67**

11.81

A heat pump for heat upgrade uses ammonia with a low temperature of 25°C and a high pressure of 5000 kPa. If it receives 1 MW of shaft work what is the rate of heat transfer at the high temperature?

State 1: $h_1 = 1463.5$ kJ/kg, $s_1 = 5.0293$ kJ/kgK

State 3: $h_3 = h_f = 631.9$ kJ/kg

Entropy compressor: $s_2 = s_1 \;\; \Rightarrow \;\; T_2 = 156^{o}C, \;\; h_2 = 1709.1$ kJ/kg

Energy eq. compressor: $w_C = h_2 - h_1 = 245.6$ kJ/kg

Energy condenser: $q_H = h_2 - h_3 = 1077.2$ kJ/kg

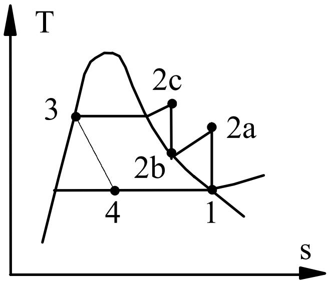
Scaling to power input: $\dot{Q}_H = q_H \frac{\dot{W}_{IN}}{w_C} = 1077.2 \; \frac{1000}{245.6} = \mathbf{4386\ kW}$

11.82

Reconsider the heat pump in the previous problem. Assume the compressor is split into two, first compress to 2000 kPa, then take heat transfer out at constant P to reach saturated vapor then compress to the 5000 kPa. Find the two rates of heat transfer, at 2000 kPa and at 5000 kPa for a total of 1 MW shaft work input.

Ideal heat pump

$T_{evap} = 25^oC = T_1$



State 1: $h_1 = 1463.5$ kJ/kg, $s_1 = 5.0293$ kJ/kgK

State 3: $h_3 = h_f = 631.9$ kJ/kg

Entropy compressor 1: $s_{2a} = s_1 \;\Rightarrow\; T_{2a} = 75.3^oC, \; h_{2a} = 1559.1$ kJ/kg

Energy eq. compressor 1: $w_{C1} = h_{2a} - h_1 = 1559.1 – 1463.5 = 95.6$ kJ/kg

Exit heat exchanger 1: $h_{2b} = 1471.5$ kJ/kg, $s_{2b} = 4.768$ kJ/kgK

Entropy compressor 2: $s_{2c} = s_{2b} \;\Rightarrow\; T_{2c} = 124.3^oC, \; h_{2c} = 1601.37$ kJ/kg

Energy eq. compressor 2: $w_{C2} = h_{2c} - h_{2b} = 1601.37 – 1471.5 = 129.87$ kJ/kg

Total power input: $\dot{W}_{IN} = \dot{m}\,(w_{C1} + w_{C2}) \;\Rightarrow$

$$\dot{m} = \frac{\dot{W}_{IN}}{w_{C1} + w_{C2}} = \frac{1000}{95.6 + 129.87} = 4.4352 \text{ kg/s}$$

Heat exchanger 1: $\dot{Q}_{H1} = \dot{m}(h_{2a} – h_{2b}) = 4.4352(1559.1-1471.5) =$ **388.5 kW**

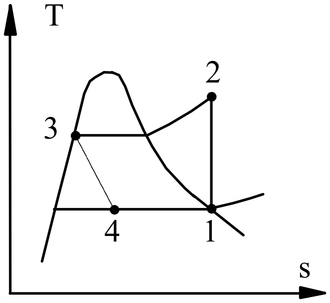
Heat exchanger 2: $\dot{Q}_{H2} = \dot{m}(h_{2c} – h_3) = 4.4352(1601.37-631.9) =$ **4299.8 kW**

11.83

An air-conditioner in the airport of Timbuktu runs a cooling system using R-410a with a high pressure of 1500 kPa and a low pressure of 200 kPa. It should cool the desert air at 45°C down to 15°C. Find the cycle COP. Will the system work?

Solution:

Ideal refrigeration cycle
$P_{cond} = P_2 = P_3 = 1500$ kPa
$P_{evap} = P_1$



State 1: $T_1 = -37.0^oC$, $h_1 = 264.3$ kJ/kg, $s_1 = 1.119$ kJ/kg-K

State 2: $P_2 = 1500$ kPa, $s_2 = s_1$: $T_2 = 53.8^oC$, $h_2 = 322$ kJ/kg

$w_C = h_2 - h_1 = 322 – 264.3 = 57.7$ kJ/kg

State 3-4: $P_4 = P_3$, $x_3 = 0$: $T_3 = 21.4^oC$, $h_4 = h_3 = 91.55$ kJ/kg

$q_L = h_1 - h_4 = 264.3 – 91.55 = 172.75$ kJ/kg

$\beta = q_L/w_C$ **= 3.0**

The heat rejection from 2-3 to ambient at 45°C has $T_3 = 21.4^oC$ not hot enough so the system will not work. The high pressure must be much higher so $T_3 > 45^oC$.

11.84

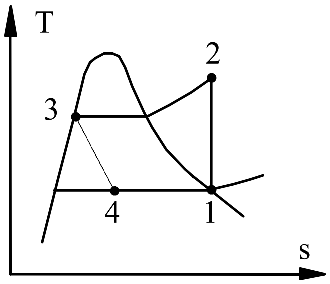
Consider an ideal heat pump that has a condenser temperature of 50°C and an evaporator temperature of 0°C. Determine the coefficient of performance of this heat pump for the working fluids R-134a and ammonia.

Solution:

Ideal heat pump

$T_{cond} = 50^oC = T_3$

$T_{evap} = 0^oC = T_1$



C.V.	Property for: From Table:	R-134a B.5	NH_3 B.2
	h_1, kJ/kg	398.36	1442.32
Compressor	$s_2 = s_1$, kJ/kgK	1.7262	5.3313
	P_2, MPa	1.3181	2.0333
	T_2, ^oC	55.1	115.6
	h_2, kJ/kg	429.55	1672.84
	$w_C = h_2 - h_1$	31.19	230.52
Exp. valve	$h_3 = h_4$, kJ/kg	271.83	421.58
Condenser	$q_H = h_2 - h_3$	157.72	1251.26
	$\beta' = q_H/w_C$	**5.06**	**5.428**

11.85

A refrigerator with R-134a as the working fluid has a minimum temperature of −10°C and a maximum pressure of 1 MPa. The actual adiabatic compressor exit temperature is 60°C. Assume no pressure loss in the heat exchangers. Find the specific heat transfer from the cold space and that to the hot space, the coefficient of performance and the isentropic efficiency of the compressor.

Solution:

State 1: Inlet to compressor, sat. vapor -10°C,

$h_1 = 392.28, \quad s_1 = 1.7319$ kJ/kgK

State 2: Actual compressor exit, $h_{2AC} = 441.89$ kJ/kg

State 3: Exit condenser, sat. liquid 1MPa, $h_3 = 255.60$ kJ/kg

State 4: Exit valve, $h_4 = h_3$

C.V. Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 392.28 - 255.60 =$ **136.7 kJ/kg**

C.V. Ideal Compressor: $w_{C,S} = h_{2,S} - h_1, \quad s_{2,S} = s_1$

State 2s: 1 MPa, s = 1.7319 kJ/kg K; $T_{2,S} = 44.9°C$, $h_{2,S} = 425.7$ kJ /kg

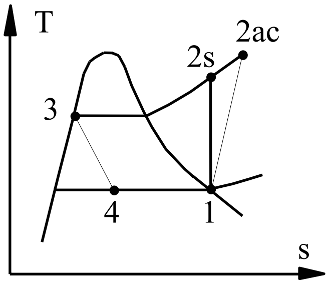
$w_{C,S} = h_{2,S} - h_1 = 33.42$ kJ/kg

C.V. Actual Compressor: $w_C = h_{2,AC} - h_1 = 49.61$ kJ/kg

$$\beta = \frac{q_L}{w_C} = \mathbf{2.76}, \qquad \eta_C = w_{C,S}/w_C = \mathbf{0.674}$$

C.V. Condenser: $q_H = h_{2,AC} - h_3 =$ **186.3 kJ/kg**

Ideal refrigeration cycle
with actual compressor
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_2 = 60^oC$
$T_{evap} = -10^oC = T_1$
Properties from Table B.5



11.86

A refrigerator in a meat warehouse must keep a low temperature of -15°C and the outside temperature is 20°C. It uses ammonia as the refrigerant which must remove 5 kW from the cold space. Find the flow rate of the ammonia needed assuming a standard vapor compression refrigeration cycle with a condenser at 20°C.

Solution:

Basic refrigeration cycle: $T_1 = T_4 = -15°C$, $T_3 = 20°C$

Table B.3: $h_4 = h_3 = 274.3$ kJ/kg; $h_1 = h_g = 1424.6$ kJ/kg

$$\dot{Q}_L = \dot{m}_{amm} \times q_L = \dot{m}_{amm} (h_1 - h_4)$$

$$q_L = 1424.6 - 274.3 = 1150.3 \text{ kJ/kg}$$

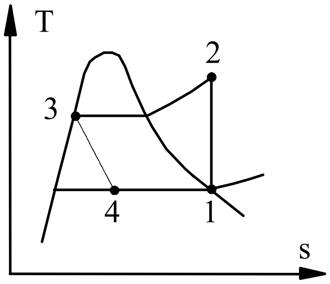
$$\dot{m}_{amm} = 5.0 / 1150.3 = \mathbf{0.00435\ kg/s}$$

Ideal refrigeration cycle

$T_{cond} = 20^oC$

$T_{evap} = -15^oC = T_1$

Properties from Table B.2



11.87

A refrigerator has a steady flow of R-410a as saturated vapor at –20°C into the adiabatic compressor that brings it to 1400 kPa. After the compressor, the temperature is measured to be 60°C. Find the actual compressor work and the actual cycle coefficient of performance.

Solution:

Table B.4.1: $h_1 = 271.89$ kJ/kg, $s_1 = 1.0779$ kJ/kg K

$P_2 = P_3 = 1400$ kPa, 18.88°C, $h_4 = h_3 = h_f = 87.45$ kJ/kg

$h_{2\,ac} = 330.07$ kJ/kg

C.V. Compressor (actual)

Energy Eq.: $w_{C\,ac} = h_{2\,ac} - h_1 = 330.07 - 271.89 = \mathbf{58.18\ kJ/kg}$

C.V. Evaporator

Energy Eq.: $q_L = h_1 - h_4 = h_1 - h_3 = 271.89 - 87.45 = 184.44$ kJ/kg

$$\beta = \frac{q_L}{w_{C\,ac}} = \frac{184.44}{58.18} = \mathbf{3.17}$$

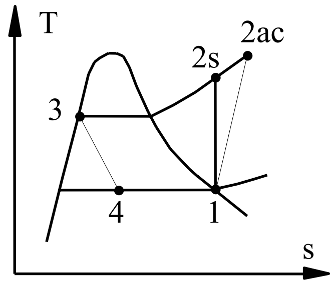
Ideal refrigeration cycle
with actual compressor

$T_{cond} = 18.88^oC = T_{sat\ 1400\ kPa}$

$T_2 = 60^oC$

$T_{evap} = -20^oC = T_1$

Properties from Table B.4

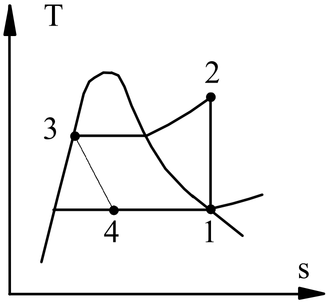


11.88

A heat pump uses R410a with a high pressure of 3000 kPa and an evaporator operating at 0^oC so it can absorb energy from underground water layers at 8^oC. Find the COP and the temperature it can deliver energy at.

Solution:

Ideal refrigeration cycle
R-401a
$P_{cond} = P_2 = P_3 = 3000$ kPa
$T_{evap} = 0^oC = T_1$



State 1: $h_1 = 279.1$ kJ/kg, $s_1 = 1.037$ kJ/kg-K

State 2: $P_2, s_2 = s_1$: $T_2 = 69.8^oC$, $h_2 = 315.4$ kJ/kg

$$w_C = h_2 - h_1 = 315.4 – 279.1 = 36.3 \text{ kJ/kg}$$

State 3-4: $h_3 = h_4, x_3 = 0$: $T_3 = 49.1^oC$, $h_4 = h_3 = 141.7$ kJ/kg

$$q_H = h_2 - h_3 = 315.4 – 141.7 = 173.7 \text{ kJ/kg}$$

$$\beta = q_H/w_C = \frac{173.7}{36.3} = \mathbf{4.785}$$

It can deliver heat at about $49^oC = T_3$ (minus a ΔT for the heat transfer rate)

11.89

The air conditioner in a car uses R-134a and the compressor power input is 1.5 kW bringing the R-134a from 201.7 kPa to 1200 kPa by compression. The cold space is a heat exchanger that cools atmospheric air from the outside 30°C down to 10°C and blows it into the car. What is the mass flow rate of the R-134a and what is the low temperature heat transfer rate. How much is the mass flow rate of air at 10°C?

Standard Refrigeration Cycle

Table B.5: $h_1 = 392.28$ kJ/kg; $s_1 = 1.7319$ kJ/kg K; $h_4 = h_3 = 266$ kJ/kg

C.V. Compressor (assume ideal)

$$\dot{m}_1 = \dot{m}_2 \qquad w_C = h_2 - h_1; \quad s_2 = s_1 + s_{gen}$$

$$P_2,\ s = s_1 \;\Rightarrow\; h_2 = 429.5 \text{ kJ/kg} \;\Rightarrow\; w_C = 37.2 \text{ kJ/kg}$$

$$\dot{m}\, w_C = \dot{W}_C \;\Rightarrow\; \dot{m} = 1.5 / 37.2 = \mathbf{0.0403\ kg/s}$$

C.V. Evaporator

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.0405(392.28 - 266) = \mathbf{5.21\ kW}$$

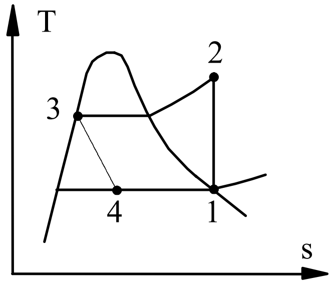
C.V. Air Cooler

$$\dot{m}_{air}\Delta h_{air} = \dot{Q}_L \approx \dot{m}_{air}C_p\Delta T$$

$$\dot{m}_{air} = \dot{Q}_L / (C_p\Delta T) = 5.21 / (1.004\times 20) = \mathbf{0.26\ kg / s}$$

Ideal refrigeration cycle

$P_{cond} = 1200 \text{ kPa} = P_3$
$P_{evap} = 201.7 \text{ kPa} = P_1$



11.90

A refrigerator using R-134a is located in a 20°C room. Consider the cycle to be ideal, except that the compressor is neither adiabatic nor reversible. Saturated vapor at -20°C enters the compressor, and the R-134a exits the compressor at 50°C. The condenser temperature is 40°C. The mass flow rate of refrigerant around the cycle is 0.2 kg/s, and the coefficient of performance is measured and found to be 2.3. Find the power input to the compressor and the rate of entropy generation in the compressor process.

Solution:

Table B.5: $P_2 = P_3 = P_{sat\ 40C} = 1017$ kPa, $h_4 = h_3 = 256.54$ kJ/kg

$s_2 \approx 1.7472$ kJ/kg K, $h_2 \approx 430.87$ kJ/kg;

$s_1 = 1.7395$ kJ/kg K, $h_1 = 386.08$ kJ/kg

$\beta = q_L / w_C \text{ -> } w_C = q_L / \beta = (h_1 - h_4) / \beta = (386.08 - 256.54) / 2.3 = 56.32$

$\dot{W}_C = \dot{m}\, w_C =$ **11.26 kW**

C.V. Compressor $h_1 + w_C + q = h_2$ ->

$q_{in} = h_2 - h_1 - w_C = 430.87 - 386.08 - 56.32 = -11.53$ kJ/kg i.e. a heat loss

$s_1 + \int dQ/T + s_{gen} = s_2$

$s_{gen} = s_2 - s_1 - q / T_o = 1.7472 - 1.7395 + (11.53 / 293.15) = 0.047$ kJ/kg K

$\dot{S}_{gen} = \dot{m}\, s_{gen} = 0.2 \times 0.047 =$ **0.0094 kW / K**

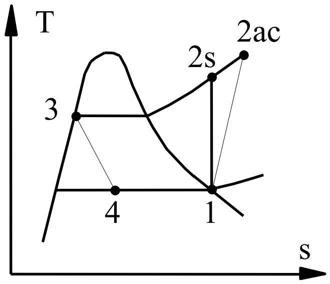
Ideal refrigeration cycle with actual compressor

$T_{cond} = 40^oC$

$T_2 = 50^oC$

$T_{evap} = -20^oC = T_1$

Properties from Table B.5



11.91

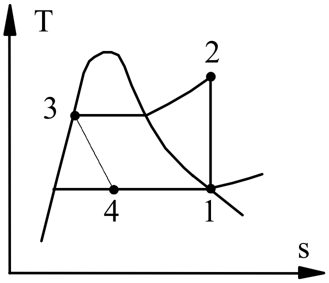
A small heat pump unit is used to heat water for a hot-water supply. Assume that the unit uses ammonia and operates on the ideal refrigeration cycle. The evaporator temperature is 15°C and the condenser temperature is 60°C. If the amount of hot water needed is 0.1 kg/s, determine the amount of energy saved by using the heat pump instead of directly heating the water from 15 to 60°C.

Solution:

Ideal ammonia heat pump

$T_1 = 15^oC, \quad T_3 = 60^oC$
From Table B.2.1

$h_1 = 1456.3$ kJ/kg, $\quad s_2 = s_1 = 5.1444$ kJ/kg K
$P_2 = P_3 = 2.614$ MPa, $\quad h_3 = 472.8$ kJ/kg



Entropy compressor: $s_2 = s_1 \;\; => \;\; T_2 = 111.6^oC, \;\; h_2 = 1643$ kJ/kg

Energy eq. compressor: $w_C = h_2 - h_1 = 186.7$ kJ/kg

Energy condenser: $q_H = h_2 - h_3 = 1170.2$ kJ/kg

To heat 0.1 kg/s of water from 15°C to 60°C,

$$\dot{Q}_{H2O} = \dot{m}(\Delta h) = 0.1(251.11 - 62.98) = 18.81 \text{ kW}$$

Using the heat pump

$$\dot{W}_{IN} = \dot{Q}_{H2O}(w_C/q_H) = 18.81(186.7 / 1170.2) = 3.0 \text{ kW}$$

a saving of **15.8 kW**

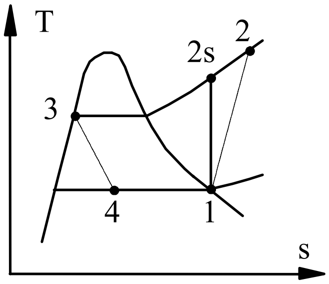
11.92

The refrigerant R-134a is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 10°C; its exit temperature from the compressor is measured and found to be 85°C. If the compressor exit is at 2 MPa what is the compressor isentropic efficiency and the cycle COP?

Solution:

R-134a heat pump:
Table B.4
State 1: $T_{EVAP} = 10^oC$, $x = 1$
$h_1 = 404.2$ kJ/kg, $s_1 = 1.7218$ kJ/kg K

State 2: T_2, P_2: $h_2 = 452.2$ kJ/kg



C.V. Compressor

Energy Eq.: $w_{C\ ac} = h_2 - h_1 = 452.2 - 404.2 = \mathbf{48.0\ kJ/kg}$

State 2s: 2 MPa , $s_{2S} = s_1 = 1.7218$ kJ/kg $T_{2S} = 73.2^oC$, $h_{2S} = 436.6$ kJ/kg

Efficiency: $$\eta = \frac{w_{C\ s}}{w_{C\ ac}} = \frac{h_{2S} - h_1}{h_2 - h_1} = \frac{436.6 - 404.2}{452.2 - 404.2} = \mathbf{0.675}$$

C.V. Condenser $T_3 = 67.5°C$, $h_3 = 300$ kJ/kg

Energy Eq.: $q_H = h_2 - h_3 = 452.2 - 300 = 152.2$ kJ/kg

COP Heat pump: $$\beta = \frac{q_H}{w_{C\ ac}} = \frac{152.2}{48.0} = \mathbf{3.17}$$

11.93

A refrigerator in a laboratory uses R-134a as the working substance. The high pressure is 1200 kPa, the low pressure is 101.3 kPa, and the compressor is reversible. It should remove 500 W from a specimen currently at –20°C (not equal to T_L in the cycle) that is inside the refrigerated space. Find the cycle COP and the electrical power required.

Solution:

State 1: 101.3 kPa, $x = 1$, Table B.5.1: $h_1 = 382.16$ kJ/kg, $s_1 = 1.7453$ kJ/kg K

State 3: 1200 kPa, 46.31°C, $x = 0$, Table B.5.1: $h_3 = 266.13$ kJ/kg

C.V. Compressor

Energy Eq.: $w_C = h_2 - h_1$

Entropy Eq.: $s_2 = s_1 + s_{gen} = s_1$

State 2: 1.2 MPa , $s_2 = s_1 = 1.7453$ kJ/kg, $T_2 \approx 56^{o}C$, $h_2 = 433.92$ kJ/kg

$w_C = h_2 - h_1 = 433.92 - 382.16 = 51.76$ kJ/kg

Energy Eq. evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 382.16 - 266.13 = 116.03$ kJ/kg

COP Refrigerator: $\beta = \frac{q_L}{w_C} = \frac{116.03}{51.76} = \mathbf{2.24}$

Power: $\dot{W}_{IN} = \dot{Q}_L / \beta = 500\text{ W}/ 2.24 = \mathbf{223\ W}$

11.94

Consider the previous problem and find the two rates of entropy generation in the process and where they occur.

Solution:

From the basic cycle we know that entropy is generated in the valve as the throttle process is irreversible.

State 1: 101.3 kPa, $x = 1$, Table B.5.1: $h_1 = 382.16$ kJ/kg, $s_1 = 1.7453$ kJ/kg K

State 3: 1200 kPa, $x = 0$, Table B.5.1: $h_3 = 266.13$ kJ/kg, $s_3 = 1.2204$ kJ/kg K

Energy Eq. evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 382.16 - 266.13 = 116.0$ kJ/kg

Mass flow rate: $\dot{m} = \dot{Q}_L / q_L = 0.5 / 116.0 = 0.00431$ kg/s

C.V. Valve

Energy Eq.: $h_4 = h_3 = 266.13$ kJ/kg $\Rightarrow$ $x_4 = (h_4 - h_f)/h_{fg}$

$$x_4 = \frac{266.13 - 165.8}{216.36} = 0.4637$$

$$s_4 = s_f + x_4\, s_{fg} = 0.869 + x_4 \times 0.8763 = 1.2754 \text{ kJ/kg K}$$

Entropy Eq.: $s_{gen} = s_4 - s_3 = 1.2754 - 1.2204 = 0.055$ kJ/kg K

$$\dot{S}_{gen\ valve} = \dot{m}s_{gen} = 0.00431 \times 0.055 \times 1000 = \mathbf{0.237\ W/K}$$

There is also entropy generation in the heat transfer process from the specimen at –20°C to the refrigerant $T_1 = -26.3°C = T_{sat}$ (101.3 kPa).

$$\dot{S}_{gen\ inside} = \dot{Q}_L \left[\frac{1}{T_{specimen}} - \frac{1}{T_L} \right] = 500 \left(\frac{1}{246.9} - \frac{1}{253.2}\right) = \mathbf{0.0504\ W/K}$$

11.95

In an actual refrigeration cycle using R-134a as the working fluid, the refrigerant flow rate is 0.05 kg/s. Vapor enters the compressor at 150 kPa, −10°C, and leaves at 1.2 MPa, 75°C. The power input to the non-adiabatic compressor is measured and found be 2.4 kW. The refrigerant enters the expansion valve at 1.15 MPa, 40°C, and leaves the evaporator at 160 kPa, −15°C. Determine the entropy generation in the compression process, the refrigeration capacity and the coefficient of performance for this cycle.

Solution:

Actual refrigeration cycle

1: compressor inlet $T_1 = -10^oC$, $P_1 = 150$ kPa

2: compressor exit $T_2 = 75^oC$, $P_2 = 1.2$ MPa

3: Expansion valve inlet $T_3 = 40^oC$
$P_3 = 1.15$ MPa

5: evaporator exit $T_5 = -15^oC$, $P_5 = 160$ kPa

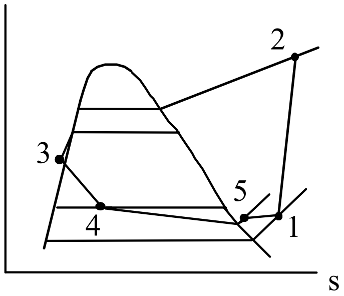


Table B.5 (CATT3) $h_1 = 394.2$ kJ/kg, $s_1 = 1.739$ kJ/kg-K,

$h_2 = 454.2$ kJ/kg, $s_2 = 1.805$ kJ/kg-K

CV Compressor: $h_1 + q_{COMP} + w_{COMP} = h_2$; $s_1 + \int dq/T + s_{gen} = s_2$

$$w_{COMP} = \dot{W}_{COMP}/\dot{m} = 2.4/0.05 = 48.0 \text{ kJ/kg}$$

$$q_{COMP} = h_2 - w_{COMP} - h_1 = 454.2 - 48.0 - 394.2 = 12 \text{ kJ/kg}$$

$$s_{gen} = s_2 - s_1 - q / T_o = 1.805 - 1.739 - 12/298.15 = \mathbf{0.0258\ kJ/kg\text{-}K}$$

C.V. Evaporator

$$q_L = h_5 - h_4 = 389.8 - 256.4 = 133.4 \text{ kJ/kg}$$

$$\Rightarrow \dot{Q}_L = \dot{m}q_L = 0.05 \times 133.4 = \mathbf{6.67\ kW}$$

COP: $$\beta = q\frac{q_L}{w_{COMP}} = \frac{133.4}{48.0} = \mathbf{2.78}$$

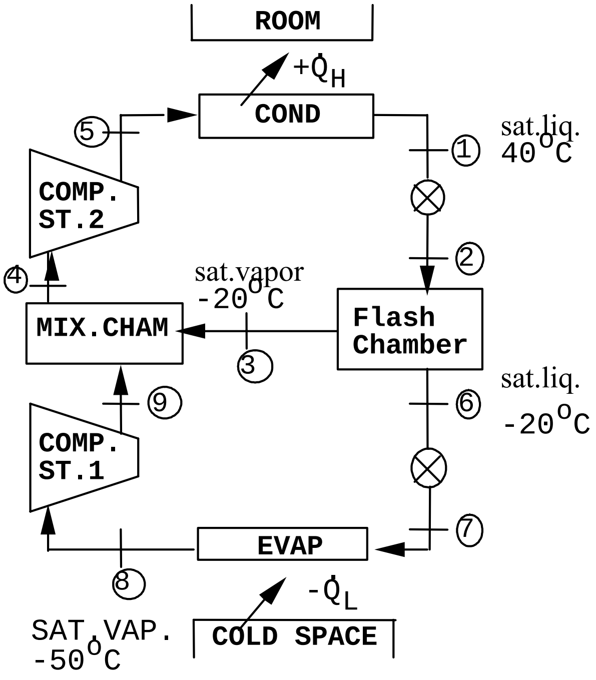
Extended refrigeration cycles

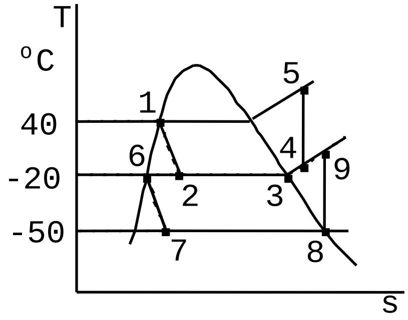
11.96

One means of improving the performance of a refrigeration system that operates over a wide temperature range is to use a two-stage compressor. Consider an ideal refrigeration system of this type that uses R-410a as the working fluid, as shown in Fig. 11.23. Saturated liquid leaves the condenser at 40°C and is throttled to −20°C. The liquid and vapor at this temperature are separated, and the liquid is throttled to the evaporator temperature, −50°C. Vapor leaving the evaporator is compressed to the saturation pressure corresponding to −20°C, after which it is mixed with the vapor leaving the flash chamber. It may be assumed that both the flash chamber and the mixing chamber are well insulated to prevent heat transfer from the ambient. Vapor leaving the mixing chamber is compressed in the second stage of the compressor to the saturation pressure corresponding to the condenser temperature, 40°C. Determine

a. The coefficient of performance of the system.

b. The coefficient of performance of a simple ideal refrigeration cycle operating over the same condenser and evaporator ranges as those of the two-stage compressor unit studied in this problem.





R-410a refrigerator with 2-stage compression

CV: expansion valve, upper loop

$h_2 = h_1 = 124.09 = 28.24 + x_2 \times 243.65; \quad x_2 = 0.3934$

$m_3 = x_2 m_2 = x_2 m_1 = 0.3934$ kg (for m_1=1 kg)

$m_6 = m_1 - m_3 = 0.6066$ kg

CV: expansion valve, lower loop

$h_7 = h_6 = 28.24 = -13.8 + x_7 \times 271.6, \quad x_7 = 0.15478$

$Q_L = m_6\,(h_8 - h_7) = 0.6066\,(257.80 - 28.24) = 139.25$ kJ

$q_L = Q_L / m_1 = 139.25$ kJ/kg-m$_1$

CV: 1st stage compressor

$s_8 = s_9 = 1.1568$ kJ/kg-K, $P_9 = P_{SAT\,-20\,^oC} = 0.3996$ MPa

$\Rightarrow\ T_9 = 2.7\ ^oC, \quad h_9 = 292.82$ kJ/kg

CV: mixing chamber (assume constant pressure mixing)

Energy Eq.: $m_6h_9 + m_3h_3 = m_1h_4$

or $h_4 = 0.6066 \times 292.82 + 0.3934 \times 271.89 = 284.6$ kJ/kg

$h_4, P_4 \approx 400$ kPa $\Rightarrow\ T_4 = -6.3\ ^oC, \ s_4 = 1.1261$ kJ/kg K

CV: 2nd stage compressor $P_4 = 400\text{ kPa} = P_9 = P_3$

$P_5 = P_{sat\ 40^oC} = 2.4207$ MPa, $s_5 = s_4 \Rightarrow T_5 = 81.3^oC$, $h_5 = 339.42$ kJ/kg

CV: condenser

Energy Eq.: $q_H = h_5 - h_1 = 339.42 - 124.09 = 215.33$ kJ/kg

$\beta_{2\ stage} = q_L/(q_H - q_L) = 139.25/(215.33 - 139.25) = \mathbf{1.83}$

b) 1 stage compression

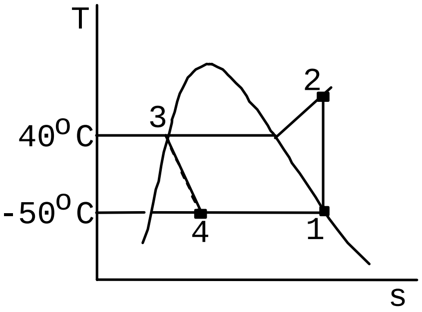
$h_3 = h_4 = 124.09$ kJ/kg

$h_1 = 257.80$ kJ/kg

$q_L = h_1 - h_4 = 133.7$ kJ/kg

$$\left.\begin{aligned} s_1 = s_2 &= 1.1568 \\ P_2 &= 2.4207\text{ MPa} \end{aligned}\right\}$$

$\Rightarrow\ T_2 = 90.7\ ^oC, h_2 = 350.35$

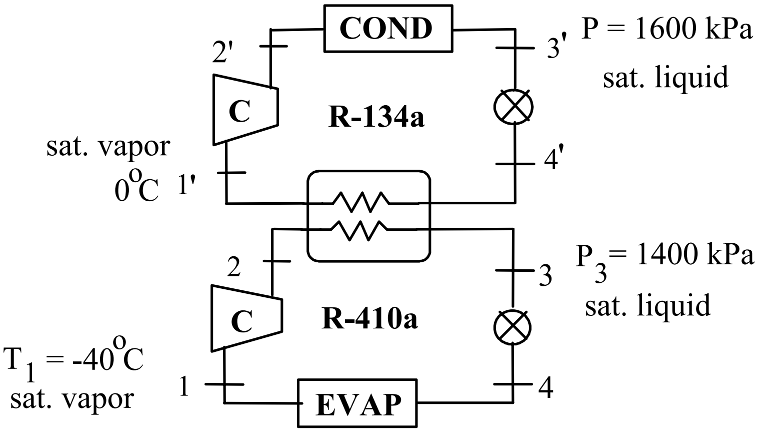


$q_H = h_2 - h_3 = 350.35 - 124.09 = 226.26$ kJ/kg

$\beta_{1\ stage} = q_L/(q_H - q_L) = 133.7/(226.26 - 133.7) = \mathbf{1.44}$

11.97

A cascade system with one refrigeration cycle operating with R-410a has an evaporator at -40°C and a high pressure of 1400 kPa. The high temperature cycle uses R-134a with an evaporator at 0°C and a high pressure of 1600 kPa. Find the ratio of the two cycles mass flow rates and the overall COP.



R-134a cycle **R-410a cycle**

	T,°C	P	h	s		T,°C	P	h	s
1′	0	293	398.6	1.727	1	-40	175	262.8	1.127
2′	63.9	1600	433.9	1.727	2	52.7	1400	322.4	1.127
3′	57.9	1600	284.4		3	18.9	1400	87.4	
4′	0		284.4		4	-40	175	87.4	

$$\dot{m}/\dot{m}' = \frac{h_1' - h_4'}{h_2 - h_3} = \frac{398.6 - 284.1}{322.4 - 87.4} = \mathbf{0.4872}$$

$$q_L = h_1 - h_4 = 262.8 - 87.4 = 175.4 \text{ kJ/kg}$$

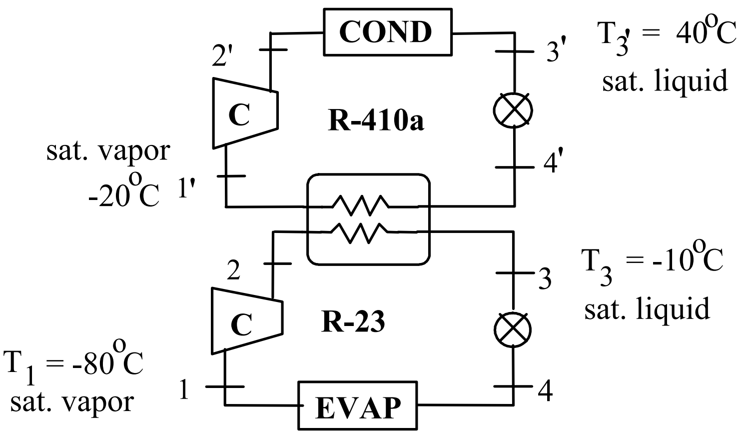
$$-\dot{W}_{TOT}/\dot{m} = (h_2 - h_1) + (\dot{m}'/\dot{m})(h_2' - h_1')$$

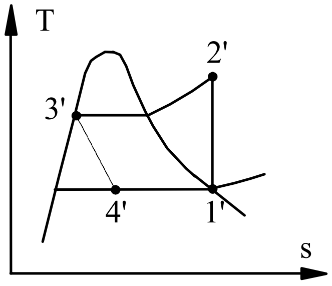
$$= 322.4 - 262.8 + \frac{1}{0.4872}(433.9 - 398.6) = 132.1\ \frac{\text{kJ}}{\text{kg}}$$

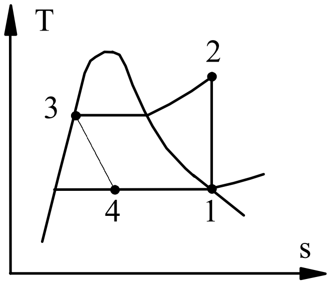
$$\beta = \dot{Q}_L/(-\dot{W}_{TOT}) = 175.4/132.1 = \mathbf{1.328}$$

11.98

A cascade system is composed of two ideal refrigeration cycles, as shown in Fig. 11.25. The high-temperature cycle uses R-410a. Saturated liquid leaves the condenser at 40°C, and saturated vapor leaves the heat exchanger at –20°C. The low-temperature cycle uses a different refrigerant, R-23. Saturated vapor leaves the evaporator at –80°C, h = 330 kJ/kg, and saturated liquid leaves the heat exchanger at –10°C, h = 185 kJ/kg. R-23 out of the compressor has h = 405 kJ/kg. Calculate the ratio of the mass flow rates through the two cycles and the coefficient of performance of the system.







	T,°C	P	h	s		T,°C	P	h	s
1′	-20	0.400	271.89	1.0779	1	-80	0.12	330	1.76
2′	71	2.421	322.61	1.0779	2	50	1.90	405	1.76
3′	40	2.421	124.09		3	-10	1.90	185	
4′	-20		124.09		4	-80	0.12	185	

$$\dot{m}/\dot{m}' = \frac{h_1' - h_4'}{h_2 - h_3} = \frac{271.89 - 124.09}{405 - 185} = \mathbf{0.6718}$$

$$q_L = h_1 - h_4 = 330 - 185 = 145 \text{ kJ/kg}$$

$$-\dot{W}_{TOT}/\dot{m} = (h_2 - h_1) + (\dot{m}'/\dot{m})(h_2' - h_1') = (405\text{-}330) + \frac{1}{0.6718}(322.61–271.89) = 150.5\ \frac{\text{kJ}}{\text{kg}}$$

$$\beta = \dot{Q}_L/(-\dot{W}_{TOT}) = 145/150.5 = \mathbf{0.96}$$

11.99

A split evaporator is used to provide cooling of the refrigerator section and separate cooling of the freezer section as shown in Fig. P11.99. Assume constant pressure in the two evaporators. How does the COP = $(Q_{L1} + Q_{L2})/W$ compare to a refrigerator with a single evaporator at the lowest temperature?

Throttle processes: $h_4 = h_3$; $h_5 = h_6$

Refrigerator: $q_{L\,R} = h_5 - h_4$

freezer: $q_{L\,F} = h_1 - h_6$

Add the two heat transfers: $q_{L\,R} + q_{L\,F} = h_5 - h_4 + h_1 - h_6 = h_1 - h_4$
which is the **same** as for the standard cycle expanding to the lowest pressure.

$$COP_{split} = COP_{std} = (h_1 - h_4) / (h_2 - h_1)$$

11.100

A refrigerator using R-410a is powered by a small natural gas fired heat engine with a thermal efficiency of 25%, as shown in Fig. P11.100. The R-410a condenses at 40°C and it evaporates at −20°C and the cycle is standard. Find the two specific heat transfers in the refrigeration cycle. What is the overall coefficient of performance as Q_L/Q_1?

Solution:

Evaporator: Inlet State is saturated liq-vap with $h_4 = h_3 = 124.09$ kJ/kg

The exit state is saturated vapor with $h_1 = 271.89$ kJ/kg

$q_L = h_1 - h_4 = h_1 - h_3 =$ **147.8 kJ/kg**

Compressor: Inlet State 1 and Exit State 2 about 2.42 MPa

$w_C = h_2 - h_1$; $s_2 = s_1 = 1.0779$ kJ/kgK

2: $T_2 \approx 70°C$ $h_2 = 322.6$ kJ/kg

$w_C = h_2 - h_1 = 50.71$ kJ/kg

Condenser: Brings it to saturated liquid at state 3

$q_H = h_2 - h_3 = 322.6 - 124.09 =$ **198.5 kJ/kg**

Overall Refrigerator:

$$\beta = q_L / w_C = 147.8 / 50.71 = 2.915$$

Heat Engine:

$$\dot{W}_{HE} = \eta_{HE}\dot{Q}_1 = \dot{W}_C = \dot{Q}_L / \beta$$

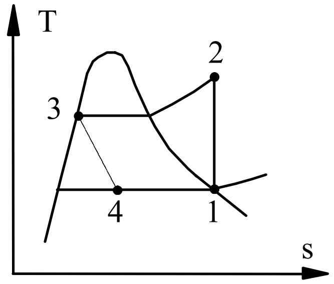
$$\dot{Q}_L / \dot{Q}_1 = \eta\beta = 0.25 \times 2.915 = \mathbf{0.729}$$

Ideal refrigeration cycle

$T_{cond} = 40^oC = T_3$

$T_{evap} = -20^oC = T_1$

Properties from Table B.4



Ammonia absorption cycles

11.101

Notice the configuration in Fig.11.26 has the left hand side column of devices substitute for a compressor in the standard cycle. What is an expression for the equivalent work output from the left hand side devices, assuming they are reversible and the high and low temperatures are constant, as a function of the pump work W and the two temperatures.

The left hand side devices works like a combination of a heat engine with some additional shaft work input. We can analyze this with a control volume around all the devices that substitute for the compressor in the standard cycle.

C.V. Pump, absorber, heat exchanger and generator.
This C.V. has an inlet flow at state 1 and exit flow at state 2 with numbers as in the standard cycle.

Energy Eq.: $0 = h_1 + w + q'_H - q'_L - h_2$ all per mass flow at 1 and 2.

Entropy Eq.: $0 = s_1 + \frac{q'_H}{T_{H'}} - \frac{q'_L}{T_{L'}} - s_2 + 0$

Now solve for q'_L from the entropy equation and substitute into the energy equation

$$q'_L = T_{L'} (s_1 - s_2) + (T_{L'} / T_{H'})\, q'_H$$

$$w + [1 - \frac{T_{L'}}{T_{H'}}]\, q'_H = (h_2 - h_1) - T_{L'} (s_2 - s_1)$$

The high T heat transfer acts as if it was delivered to a Carnot heat engine and the Carnot heat engine work output was added to the shaft work w. That sum gives the increase in exergy from state 1 to state 2. Notice in the standard cycle, $s_2 = s_1$, and the last term is zero.

11.102

As explained in the previous Problem the ammonia absorption cycle is very similar to the set-up sketched in Problem 11.100. Assume the heat engine has an efficiency of 30% and the COP of the refrigeration cycle is 3.0 what is then the ratio of the cooling to the heating heat transfer Q_L/Q_1?

Heat Engine: $W = \eta Q_1 = 0.3\ Q_1$

Refrigerator.: $\beta = Q_L\ /W \quad ⇨ \quad Q_L = \beta\ W = \beta\ \eta Q_1$

So now

$$Q_L/Q_1 = \beta\ \eta = 3 \times 0.3 = \mathbf{0.9}$$

11.103

Consider a small ammonia absorption refrigeration cycle that is powered by solar energy and is to be used as an air conditioner. Saturated vapor ammonia leaves the generator at 50°C, and saturated vapor leaves the evaporator at 10°C. If 7000 kJ of heat is required in the generator (solar collector) per kilogram of ammonia vapor generated, determine the overall performance of this system.

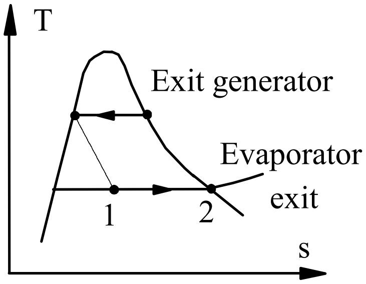
Solution;

NH_3 absorption cycle:

sat. vapor at 50°C exits the generator

sat. vapor at 10°C exits the evaporator

$q_H = q_{GEN} = 7000$ kJ/kg NH_3 out of gen.



C.V. Evaporator

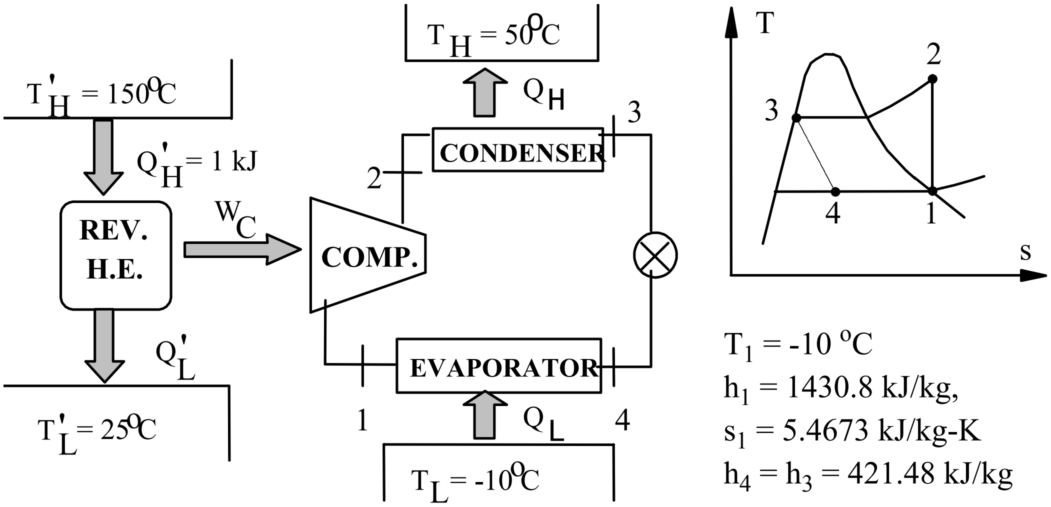
$$q_L = h_2 - h_1 = h_{g\ 10^oC} - h_{f\ 50^oC} = 1452.2 - 421.6 = 1030.6 \text{ kJ/kg}$$

COP $\Rightarrow$ $q_L/q_H = 1030.6/7000 = \mathbf{0.147}$

11.104

The performance of an ammonia absorption cycle refrigerator is to be compared with that of a similar vapor-compression system. Consider an absorption system having an evaporator temperature of −10°C and a condenser temperature of 50°C. The generator temperature in this system is 150°C. In this cycle 0.42 kJ is transferred to the ammonia in the evaporator for each kilojoule transferred from the high-temperature source to the ammonia solution in the generator. To make the comparison, assume that a reservoir is available at 150°C, and that heat is transferred from this reservoir to a reversible engine that rejects heat to the surroundings at 25°C. This work is then used to drive an ideal vapor-compression system with ammonia as the refrigerant. Compare the amount of refrigeration that can be achieved per kilojoule from the high-temperature source with the 0.42 kJ that can be achieved in the absorption system.

Solution:



$T_1 = -10\ ^oC$
$h_1 = 1430.8$ kJ/kg,
$s_1 = 5.4673$ kJ/kg-K
$h_4 = h_3 = 421.48$ kJ/kg

For the rev. heat engine: $\eta_{TH} = 1 - T_L'/T_H' = 1 - \frac{298.2}{423.2} = 0.295$

$\Rightarrow \qquad W_C = \eta_{TH}\, Q_H' = 0.295$ kJ

For the NH_3 refrig. cycle: $P_2 = P_3 = 2033$ kPa , Use 2000 kPa Table

$s_2 = s_1 = 5.4673$ kJ/kg-K => $T_2 \approx 135°C$, $h_2 \approx 1724$ kJ/kg

$w_C = h_2 - h_1 = 1724 - 1430.8 = 293.2$ kJ/kg

$q_L = h_1 - h_4 = 1430.8 - 421.48 = 1009.3$ kJ/kg

$\beta = q_L/w_C = 1009.3 / 293.2 = 3.44$

$\Rightarrow\ Q_L = \beta w_C = 3.44 \times 0.295 =$ **1.015 kJ**

This is based on the assumption of an ideal heat engine & refrigeration cycle.

Availabilty or Exergy Concepts

11.105

Find the availability of the water at all four states in the Rankine cycle described in Problem 11.30. Assume that the high-temperature source is 500°C and the low-temperature reservoir is at 25°C. Determine the flow of availability in or out of the reservoirs per kilogram of steam flowing in the cycle. What is the overall cycle second law efficiency?

Solution:

Reference State: 100 kPa, 25°C, $s_o = 0.3673$ kJ/kg K, $h_o = 104.87$ kJ/kg

$$\psi_1 = h_1 - h_o - T_o(s_1 - s_o)$$

$$= 191.81 - 104.87 - 298.15(0.6492 - 0.3673) = 2.89 \text{ kJ/kg}$$

$$\psi_2 = 194.83 - 104.87 - 298.15(0.6492 - 0.3673) = \psi_1 + 3.02 = 5.91 \text{ kJ/kg}$$

$$\psi_3 = 3230.82 - 104.87 - 298.15(6.9211 - 0.3673) = 1171.93 \text{ kJ/kg}$$

$$\psi_4 = \psi_3 - w_{T,s} = 1171.93 - 1038.3 = 133.6 \text{ kJ/kg}$$

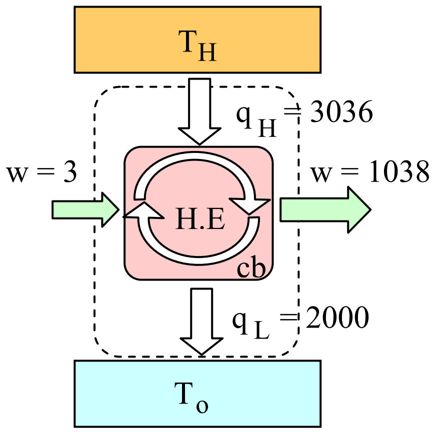
$$\Delta\psi_H = (1 - T_o/T_H)q_H = 0.6144 \times 3036 = 1865.3 \text{ kJ/kg}$$

$$\Delta\psi_L = (1 - T_o/T_o)q_C = 0 \text{ kJ/kg}$$

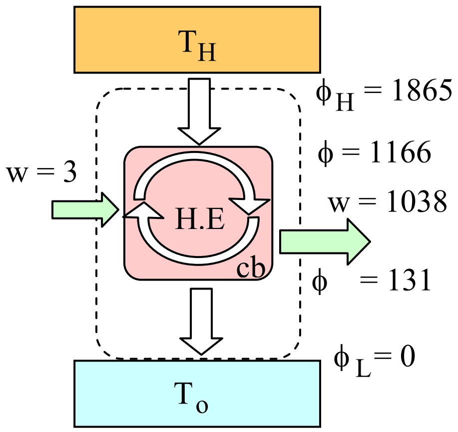
$$\eta_{II} = w_{NET}/\Delta\psi_H = (1038.3 - 3.02)/1865.3 = \mathbf{0.5657}$$

Notice— $T_H > T_3$, $T_L < T_4 = T_1$ so cycle is externally irreversible. Both q_H and q_C are over finite ΔT.

Energy Transfers (kJ/kg):



Exergy Transfers (kJ/kg):



11.106

If we neglect the external irreversibilities due to the heat transfers over finite temperature differences in a power plant how would you define its second law efficiency?

The first law efficiency is a conversion efficiency as

$$\eta_I = \frac{w_{net}}{q_H} = \frac{w_{net}}{h_3 - h_2}$$

The second law efficiency is the same ratio but expressed in availability (exergy)

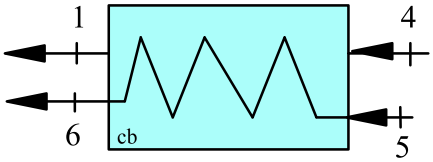
$$\eta_{II} = \frac{\text{output}}{\text{source}} = \frac{w_{net}}{\varphi_H} = \frac{w_{net}}{\varphi_3 - \varphi_2} \quad \text{or} \quad = \frac{w_{net}}{\varphi_H - \varphi_L}$$

The last expression must be used if the heat rejection at the low T is assigned any exergy value (normally not).

11.107

Find the flows and fluxes of exergy in the condenser of Problem 11.29. Use those to determine the second law efficiency.

For this case we select $T_o = 12°C = 285$ K, the ocean water temperature.



The states properties from Tables B.1.1 and B.1.3

1: 45^oC, $x = 0$: $h_1 = 188.42$ kJ/kg,

3: 3.0 MPa, 600^oC: $s_3 = 7.5084$ kJ/kg K

C.V. Turbine : $w_T = h_3 - h_4$; $s_4 = s_3$

$$s_4 = s_3 = 7.5084 = 0.6386 + x_4\ (7.5261) \quad => \quad x_4 = 0.9128$$

$$=> h_4 = 188.42 + 0.9128\ (2394.77) = 2374.4 \text{ kJ/kg}$$

C.V. Condenser : $q_L = h_4 - h_1 = 2374.4 - 188.42 = 2186$ kJ/kg

$$\dot{Q}_L = \dot{m}q_L = 25 \times 2186 = 54.65 \text{ MW} = \dot{m}_{ocean}\ C_p\ \Delta T$$

$$\dot{m}_{ocean} = \dot{Q}_L\ /\ C_p\ \Delta T = 54\ 650\ /\ (4.18 \times 3) = 4358 \text{ kg/s}$$

The net drop in exergy of the water is

$$\dot{\Phi}_{water} = \dot{m}_{water}\ [h_4 - h_1 - T_o(s_4 - s_1)]$$

$$= 25\ [\ 2374.4 - 188.4 - 285\ (7.5084 - 0.6386)]$$

$$= 54\ 650 - 48\ 947 = \mathbf{5703\ kW}$$

The net gain in exergy of the ocean water is

$$\dot{\Phi}_{ocean} = \dot{m}_{ocean}[h_6 - h_5 - T_o(s_6 - s_5)]$$

$$= \dot{m}_{ocean}[C_p(T_6 - T_5) - T_oC_p \ln\left(\frac{T_6}{T_5}\right)\ ]$$

$$= 4358\ [\ 4.18(15 - 12) - 285 \times 4.18 \ln \frac{273 + 15}{273 + 12}\ ]$$

$$= 54\ 650 - 54\ 364 = \mathbf{286\ kW}$$

The second law efficiency is

$$\eta_{II} = \dot{\Phi}_{ocean}\ /\ \dot{\Phi}_{water} = \frac{286}{5703} = \mathbf{0.05}$$

In reality all the exergy in the ocean water is destroyed as the 15°C water mixes with the ocean water at 12°C after it flows back out into the ocean and the efficiency does not have any significance. Notice the small rate of exergy relative to the large rates of energy being transferred.

11.108

Find the flows of exergy into and out of the feedwater heater in Problem 11.42.

State 1: $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m^3/kg

State 3: $x_3 = 0$, $h_3 = 421.48$ kJ/kg, $v_3 = 0.001777$ m^3/kg

State 5: $h_5 = 421.48$ kJ/kg, $s_5 = 4.7306$ kJ/kg K

State 6: $s_6 = s_5 \;\Rightarrow\; x_6 = (s_6 - s_f)/s_{fg} = 0.99052$, $h_6 = 1461.53$ kJ/kg

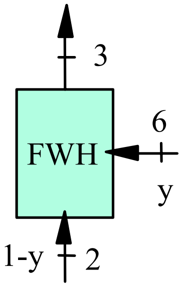
C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001658(2033 - 1003) = 1.708 \text{ kJ/kg}$$

$$\Rightarrow h_2 = h_1 + w_{P1} = 298.25 + 1.708 = 299.96 \text{ kJ/kg}$$

C.V. Feedwater heater: Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

Energy Eq.: $(1 - y)\, h_2 + y\, h_6 = 1\, h_3$



$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3640.6 - 189.42} = 0.1046$$

$$\dot{m}_{extr} = y\, \dot{m}_{tot} = 0.1046 \times 5 = 0.523 \text{ kg/s}$$

$$\dot{m}_2 = (1\text{-}y)\, \dot{m}_{tot} = (1 - 0.1046)\, 5 = 4.477 \text{ kg/s}$$

Reference State: 100 kPa, 20°C, $s_o = 6.2826$ kJ/kg K, $h_o = 1516.1$ kJ/kg

$$\psi_2 = h_2 - h_o - T_o(s_2 - s_o)$$

$$= 299.96 - 1516.1 - 293.15(1.121 - 6.2826) = 296.21 \text{ kJ/kg}$$

$$\psi_6 = 1461.53 - 1516.1 - 293.15(4.7306 - 6.2826) = 400.17 \text{ kJ/kg}$$

$$\psi_3 = 421.48 - 1516.1 - 293.15(1.5121 - 6.2826) = 303.14 \text{ kJ/kg}$$

The rate of exergy flow is then

$$\dot{\Phi}_2 = \dot{m}_2\psi_2 = 4.477 \times 296.21 = \mathbf{1326\ kW}$$

$$\dot{\Phi}_6 = \dot{m}_6\psi_6 = 0.523 \times 400.17 = \mathbf{209.3\ kW}$$

$$\dot{\Phi}_3 = \dot{m}_3\psi_3 = 5.0 \times 303.14 = \mathbf{1516\ kW}$$

The mixing is destroying 1326 + 209 – 1516 = 19 kW of exergy

11.109

The power plant using ammonia in Problem 11.62 has a flow of liquid water at 120°C, 300 kPa as a heat source, the water leaves the heat exchanger at 90°C. Find the second law efficiency of this heat exchanger.

C.V. The liquid water source - ammonia boiler heat exchanger.

$\Delta h_{liq} = 503.69 - 376.9 = 126.79$ kJ/kg (sat. liquid at 120°C and 90°C)

$\Delta s_{liq} = 1.5275 - 1.1924 = 0.3351$ kJ/kgK

The energy equation establishes the ratio of the mass flow rates

$$\dot{Q}_H = \dot{m}_{water}\, \Delta h_{liq} = \dot{m}_{NH3}\, q_H$$

$$=> \quad \dot{m}_{water} / \dot{m}_{NH3} = q_H / \Delta h_{liq} = 1442/126.79 = 11.373$$

Now the second law efficiency is the ratio of exergy pick-up over exergy source

$$\eta_{II} = \frac{\dot{m}_{NH3}\, \Delta\psi}{\dot{m}_{water}\, \Delta\psi} = \frac{\dot{m}_{NH3}\, [q_H - T_o(s_5 - s_3)]}{\dot{m}_{water}\, (\Delta h_{liq} - T_o \Delta s_{liq})}$$

From the power plant cycle we have state 3:

State 3: $x_3 = 0$, $h_3 = 171.65$ kJ/kg, $s_3 = 0.6793$ kJ/kgK, $v_3 = 0.00156$ m^3/kg

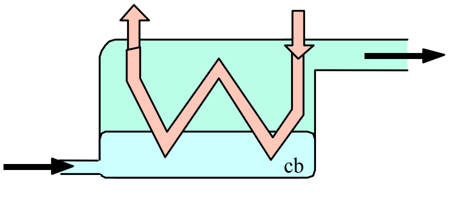
State 5: $h_5 = 1614.6$ kJ/kg, $s_5 = 5.4971$ kJ/kg K

C.V Pump P2: $w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.00156(1000 - 400) = 0.936$ kJ/kg

$$q_H = h_5 - h_4 = 1614.6 - (171.65 + 0.936) = 1442 \text{ kJ/kg}$$

$$s_5 - s_3 = 5.4971 - 0.6793 = 4.81775 \text{ kJ/kgK}$$

$$\eta_{II} = \frac{1}{11.373}\, \frac{1442 - 253.15 \times 4.81775}{126.79 - 253.15 \times 0.3351} = \mathbf{0.466}$$



11.110

For problem 11.52 consider the boiler/super-heater. Find the exergy destruction in this setup and the second law efficiency for the boiler-source set-up.

A Rankine cycle feeds 5 kg/s ammonia at 2 MPa, 140°C to the turbine, which has an extraction point at 800 kPa. The condenser is at -20°C and a closed feed water heater has an exit state (3) at the temperature of the condensing extraction flow and it has a drip pump. The source for the boiler is at constant 180°C. Find the extraction flow rate and state 4 into the boiler.

The boiler has flow in at state 4 and out at state 5 with the source providing a q at 180°C.

Assume state 4 is saturated liquid at T_4 so $h_4 = h_{f\,4}$ => $T_4 = 17.92^\circ C$

State 4: $h_4 = 264.4$ kJ/kg $= h_{f\,4}$, $s_4 = s_{f\,4} = 1.00705$ kJ/kgK

State 5: $h_5 = 1738.2$ kJ/kg, $s_5 = 5.5022$ kJ/kg K

Energy Eq. $q_H = h_5 - h_4 = 1738.2 - 264.4 = 1473.8$ kJ/kg

Entropy Eq. $s_4 + q_H/T_H + s_{gen} = s_5$ => $s_{gen} = s_5 - s_4 - q_H/T_H$

$$\dot{S}_{gen} = \dot{m}s_{gen} = 5\,[5.5022 - 1.00705 - \frac{1473.8}{180 + 273.15}]$$

$$= 5 \times 1.2428 = \mathbf{6.21\ kW/K}$$

The flow increase in exergy is:

$$\psi_5 - \psi_4 = h_5 - h_4 - T_o\,(s_5 - s_4) = 1473.8 - 298.15\,(5.5022 - 1.00705)$$

$$= 133.57 \text{ kJ/kg}$$

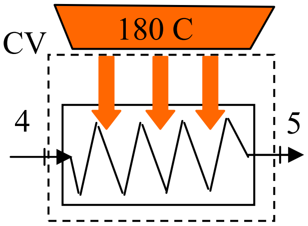
The exergy provided by the source is:

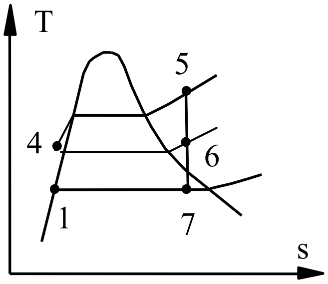
$$\phi_H = (\,1 - T_o/\,T_H)\,q_H = (1 - \frac{298.15}{180 + 273.15})\,1473.8 = 504.1 \text{ kJ/kg}$$

Second law efficiency is

$$\eta_{II} = \frac{\text{gain in exergy}}{\text{source exergy input}} = \frac{133.57}{504.1} = \mathbf{0.265}$$

$$(\,= 1 - T_o s_{gen}/\phi_H = 1 - 298.15 \times 1.2428 \,/\, 504.1\,)$$





11.111

Steam is supplied in a line at 3 MPa, 700°C. A turbine with an isentropic efficiency of 85% is connected to the line by a valve and it exhausts to the atmosphere at 100 kPa. If the steam is throttled down to 2 MPa before entering the turbine find the actual turbine specific work. Find the change in availability through the valve and the second law efficiency of the turbine.

Take C.V. as valve and a C.V. as the turbine.

Valve: $h_2 = h_1 = 3911.7$ kJ/kg, $s_2 > s_1 = 7.7571$ kJ/kg K,

$h_2, P_2 \Rightarrow s_2 = 7.9425$ kJ/kg K

$\psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2) = 0 - 298.15(7.7571 - 7.9425) = \mathbf{55.3\ kJ/kg}$

So some potential work is lost in the throttling process.

Ideal turbine: $s_3 = s_2 \Rightarrow h_{3s} = 2929.13$ kJ/kg $w_{T,s} = 982.57$ kJ/kg

$w_{T,ac} = h_2 - h_{3ac} = \eta w_{T,s} = \mathbf{835.2\ kJ/kg}$

$h_{3ac} = 3911.7 - 835.2 = 3076.5 \Rightarrow s_{3ac} = 8.219$ kJ/kg K

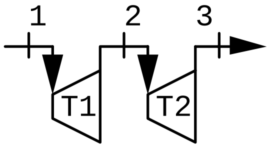
$w^{rev} = \psi_2 - \psi_{3ac} = h_2 - h_{3ac} - T_0(s_2 - s_{3ac}) = 835.2 - 298.15(7.9425 - 8.219)$

$= 917.63$ kJ/kg $\Rightarrow$

$\eta_{II} = w_{T,ac} / w^{rev} = 835.2/917.63 = \mathbf{0.91}$

11.112

A flow of steam at 10 MPa, 550°C goes through a two-stage turbine. The pressure between the stages is 2 MPa and the second stage has an exit at 50 kPa. Assume both stages have an isentropic efficiency of 85%. Find the second law efficiencies for both stages of the turbine.



CV: T1, $h_1 = 3500.9$ kJ/kg, $s_1 = 6.7561$ kJ/kg K

Isentropic $s_{2s} = s_1 \Rightarrow h_{2s} = 3017.9$ kJ/kg

$w_{T1,s} = h_1 - h_{2s} = 483$ kJ/kg

Actual T1: $w_{T1,ac} = \eta_{T1}\, w_{T1,s} = 410.55 = h_1 - h_{2ac}$

$h_{2ac} = h_1 - w_{T1,ac} = 3090.35$ kJ/kg, $s_{2ac} = 6.8782$ kJ/kg K

CV: T2, $s_{3s} = s_{2ac} = 6.8782 \Rightarrow x_{3s} = (6.8782-1.091)/6.5029 = 0.8899$,

$h_{3s} = 340.47 + 0.8899 \times 2305.4 = 2392.2$ kJ/kg

$w_{T2,s} = h_{2ac} - h_{3s} = 698.15 \Rightarrow w_{T2,ac} = \eta_{T2}\, w_{T2,s} = 593.4$ kJ/kg

$\Rightarrow h_{3ac} = 2496.9$, $x_{3ac} = (2496.9 - 340.47)/2305.4 = 0.9354$,

$s_{3ac} = 1.091 + 0.9354 \times 6.5029 = 7.1736$ kJ/kg K

Actual T1: $i_{T1,ac} = T_0(s_{2ac}-s_1) = 298.15(6.8782 - 6.7561) = 36.4$ kJ/kg

$\Rightarrow w^R_{T1} = w_{T1,ac} + i = 447$ kJ/kg, $\eta_{II} = w_{T1,ac}/w^R_{T1} = \mathbf{0.918}$

Actual T2: $i_{T2,ac} = T_0(s_{3ac}-s_{2ac}) = 298.15(7.1736 - 6.8782) = 88.07$ kJ/kg

$\Rightarrow w^R_{T2} = w_{T2,ac} + i_{T2,ac} = 681.5$, $\eta_{II} = w_{T2,ac}/w^R_{T2} = \mathbf{0.871}$

11.113

The simple steam power plant shown in Problem 6.103 has a turbine with given inlet and exit states. Find the availability at the turbine exit, state 6. Find the second law efficiency for the turbine, neglecting kinetic energy at state 5.

Solution:

interpolation or software: $h_5 = 3404.3$ kJ/kg, $s_5 = 6.8953$ kJ/kg K

Table B.1.2: $x_6 = 0.92$ so $h_6 = 2393.2$ kJ/kg, $s_6 = 7.5501$ kJ/kg K

Flow availability (exergy) from Eq.10.24

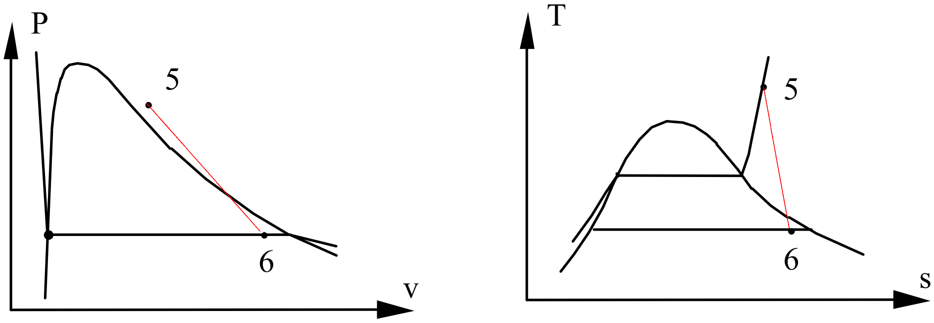
$$\psi_6 = h_6 - h_0 - T_0(s_6 - s_0)$$

$$= 2393.2 - 104.89 - 298.15(6.8953 - 0.3674) = \mathbf{146.79\ kJ/kg}$$

In the absence of heat transfer the work is form Eq.10.9 or 10.39

$$w^{rev} = \psi_5 - \psi_6 = h_5 - h_6 - T_0(s_5 - s_6) = 1206.3 \text{ kJ/kg}$$

$$w_{ac} = h_5 - h_6 = 1011.1 \text{ kJ/kg}; \qquad \eta_{II} = w_{ac}/w^{rev} = \mathbf{0.838}$$



11.114

Consider the high-pressure closed feedwater heater in the nuclear power plant described in Problem 6.106. Determine its second law efficiency.

For this case with no work the second law efficiency is from Eq. 10.32:

$$\eta_{II} = \dot{m}_{16}(\psi_{18} - \psi_{16})/\dot{m}_{17}(\psi_{17} - \psi_{15})$$

Properties (taken from computer software):

h [kJ/kg]	$h_{15} = 585$	$h_{16} = 565$	$h_{17} = 2593$	$h_{18} = 688$
s [kJ/kgK]	$s_{15} = 1.728$	$s_{16} = 1.6603$	$s_{17} = 6.1918$	$s_{18} = 1.954$

The change in specific flow availability becomes

$$\psi_{18} - \psi_{16} = h_{18} - h_{16} - T_0(s_{18} - s_{16}) = 35.433 \text{ kJ/kg}$$

$$\psi_{17} - \psi_{15} = h_{17} - h_{15} - T_0(s_{17} - s_{15}) = 677.12 \text{ kJ/kg}$$

$$\eta_{II} = (75.6 \times 35.433)/(4.662 \times 677.12) = \mathbf{0.85}$$

11.115

Find the availability of the water at all the states in the steam power plant described in Problem 11.60. Assume the heat source in the boiler is at 600°C and the low-temperature reservoir is at 25°C. Give the second law efficiency of all the components.

From solution to 11.13 and 11.60:

States	0	1 sat liq.	2a	3	4a (x = 0.7913)
h [kJ/kg]	104.89	191.81	195.58	2804.14	2085.24
s [kJ/kg K]	0.3674	0.6492	0.6529	6.1869	6.5847

The entropy for state 2a was done using the compressed liquid entry at 2MPa at the given h. You could interpolate in the compressed liquid tables to get at 3 MPa or use the computer tables to be more accurate.

Definition of flow exergy: $\psi = h - h_o - T_o(s - s_o)$

$$\psi_1 = 191.81 - 104.89 - 298.15(0.6492 - 0.3674) = 2.90 \text{ kJ/kg}$$

$$\psi_{2a} = 195.58 - 104.89 - 298.15(0.6529 - 0.3674) = 5.57 \text{ kJ/kg}$$

$$\psi_3 = 2804.14 - 104.89 - 298.15(6.1869 - 0.3674) = 964.17 \text{ kJ/kg}$$

$$\psi_{4a} = 2085.24 - 104.89 - 298.15(6.5847 - 0.3674) = 126.66 \text{ kJ/kg}$$

$$\eta_{II\ Pump} = (\psi_{2a} - \psi_1) / w_{p\ ac} = (5.57 - 2.9) / 3.775 = \mathbf{0.707}$$

$$\eta_{II\ Boiler} = (\psi_3 - \psi_{2a}) / [(1 - T_o/T_H)\, q_H]$$

$$= (964.17 - 3.18) / [0.658 \times 2608.6] = \mathbf{0.56}$$

$$\eta_{II\ Turbine} = w_{T\ ac} / (\psi_3 - \psi_{4a}) = 718.9 / (964.17 - 126.66) = \mathbf{0.858}$$

$$\eta_{II\ Cond} = \Delta\psi_{amb} / (\psi_{4a} - \psi_1) = \mathbf{0}$$

Remark: Due to the interpolation the efficiency for the pump is not quite correct. It should have a second law efficiency greater than the isentropic efficiency.

11.116

Find two heat transfer rates, the total cycle exergy destruction and a second law efficiency for the refrigerator in Problem 11.80.

A refrigerator receives 500 W of electrical power to the compressor driving the cycle flow of R-134a. The refrigerator operates with a condensing temperature of 40^oC and a low temperature of -5^oC. Find the COP for the cycle.

State 1: $h_1 = 395.3$ kJ/kg, $s_1 = 1.7288$ kJ/kg-K

State 2: $s_2 = s_1$, $P_2 = P_3 = 1017$ kPa, $T_2 = 45^oC$, $h_2 = 425$ kJ/kg

$w_C = h_2 - h_1 = 425 - 395.3 = 29.7$ kJ/kg

State 3: $h_3 = 256.5$ kJ/kg, $s_3 = 1.1909$ kJ/kg-K

State 4: $h_4 = h_3 = 256.5$ kJ/kg,

$q_H = h_2 - h_3 = 425 - 256.5 = 168.5$ kJ/kg

$q_L = h_1 - h_4 = 138.8$ kJ/kg

Entropy Eq. for C.V. from -5^oC to $+40^oC$: $\quad 0 = \frac{q_L}{T_L} - \frac{q_H}{T_H} + s_{gen}$

$$i = T_o\, s_{gen} = T_o\,(\frac{q_H}{T_H} - \frac{q_L}{T_L}) = 298.15\,(\frac{168.5}{313.2} - \frac{138.8}{268.15}) = 6.1 \text{ kJ/kg}$$

$I = \dot{m}\, i = (\dot{W}/w_C)\, i = (0.5 \text{ kW}/ 29.7 \text{ kJ/kg})\ 6.1 \text{ kJ/kg} = \mathbf{0.103\ kW}$

For this CV the 2nd law efficiency is

$$\eta_{II} = \frac{\text{output}}{\text{source}} = \frac{w_C - i}{w_C} = \frac{29.7 - 6.1}{29.7} = \mathbf{0.795}$$

Remark: The cold space gain in availability

$$\Delta\psi_L = (1 - \frac{T_o}{T_L})\,(-q_L) = (1 - \frac{298.15}{268.15})\,(-138.8) = 15.5 \text{ kJ/kg}$$

The high temperature reservoir also gains availability so

$$\Delta\psi_H = (1 - \frac{T_o}{T_H})\, q_H = (1 - \frac{298.15}{313.15})\ 168.5 = 8.07 \text{ kJ/kg}$$

For the refrigerator second law efficiency we neglect the exergy to T_H space as being useful so (this i-ref = $i + \Delta\psi_H$) and then

$$\eta_{II} = \frac{\text{output}}{\text{source}} = \frac{\Delta\psi_L}{w_C} = \frac{15.5}{29.7} = \mathbf{0.52}$$

The total balance is : $w_C = \Delta\psi_H + \Delta\psi_L + i = 8.07 + 15.5 + 6.1 = 29.67$ OK.

11.117

In a refrigerator saturated vapor R-134a at -20°C from the evaporator goes into a compressor that has a high pressure of 1000 kPa. After the compressor the actual temperature is measured to be 60°C. Find the actual specific work and the compressor 2nd law efficiency, using $T_o = 298$ K.

Inlet state: $h_1 = 386.08$ kJ/kg, $s_1 = 1.7395$ kJ/kg-K

Exit state: $h_{2a} = 441.89$ kJ/kg, $s_{2a} = 1.7818$ kJ/kg-K

Actual compressor: $w_{C\,ac} = h_{2a} - h_1 = 441.89 - 386.08 =$ **55.81 kJ/kg**

Rev. work:
$$-w^{rev} = \psi_{2a} - \psi_1 = h_{2a} - h_1 - T_0(s_{2a} - s_1)$$
$$= 55.81 - 298\ (1.7818 - 1.7395) = 43.205 \text{ kJ/kg}$$

$$\eta_{II} = \frac{\text{output}}{\text{source}} = \frac{-w^{rev}}{w_{C\,ac}} = \frac{43.205}{55.81} = \mathbf{0.774}$$

11.118

What is the second law efficiency of the heat pump in Problem 11.81?

A heat pump for heat upgrade uses ammonia with a low temperature of 25°C and a high pressure of 5000 kPa. If it receives 1 MW of shaft work what is the rate of heat transfer at the high temperature?

State 1: $h_1 = 1463.5$ kJ/kg, $s_1 = 5.0293$ kJ/kgK

State 3: $h_3 = h_f = 631.9$ kJ/kg, $s_3 = 2.1100$ kJ/kg-K

Entropy compressor: $s_2 = s_1$ => $T_2 = 156^{\circ}$C, $h_2 = 1709.1$ kJ/kg

Energy eq. compressor: $w_C = h_2 - h_1 = 245.6$ kJ/kg

Energy condenser: $q_H = h_2 - h_3 = 1077.2$ kJ/kg

Exergy output: $\Delta\psi_H = \psi_2 - \psi_3 = h_2 - h_3 - T_0(s_2 - s_3)$

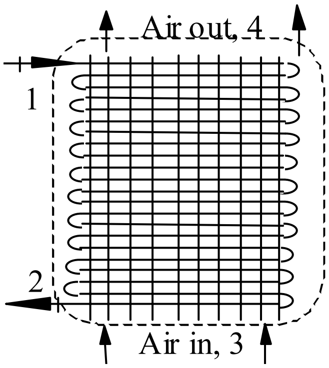
$= 1077.2 - 298\ (5.0293 - 2.1100)$

$= 207.25$ kJ/kg

$$\eta_{II} = \frac{\text{output}}{\text{source}} = \frac{\Delta\psi_H}{w_{C\ ac}} = \frac{207.25}{245.6} = \mathbf{0.844}$$

11.119

The condenser in a refrigerator receives R-134a at 700 kPa, 50°C and it exits as saturated liquid at 25°C. The flowrate is 0.1 kg/s and the condenser has air flowing in at ambient 15°C and leaving at 35°C. Find the minimum flow rate of air and the heat exchanger second-law efficiency.



C.V. Total heat exchanger.
Energy Eq.6.10

$$\dot{m}_1 h_1 + \dot{m}_a h_3 = \dot{m}_1 h_2 + \dot{m}_a h_4$$

$$\Rightarrow \quad \dot{m}_a = \dot{m}_1 \times \frac{h_1 - h_2}{h_4 - h_3}$$

$$= 0.1 \times \frac{436.89 - 234.59}{1.004(35 - 15)} = \mathbf{1.007\ kg/s}$$

Availability from Eq.10.24

$$\psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2) = 436.89 - 234.59$$

$$- 288.15(1.7919 - 1.1201) = 8.7208 \text{ kJ/kg}$$

$$\psi_4 - \psi_3 = h_4 - h_3 - T_0(s_4 - s_3)$$

$$= 1.004(35 - 15) - 288.15 \times 1.004 \times \ln \frac{308.15}{288.15} = +0.666 \text{ kJ/kg}$$

Efficiency from Eq.10.30

$$\eta_{II} = \frac{\dot{m}_a (\psi_4 - \psi_3)}{\dot{m}_1 (\psi_1 - \psi_2)} = \frac{1.007 \times 0.666}{0.1 \times 8.7208} = \mathbf{0.77}$$

Combined Cycles

11.120

A binary system power plant uses mercury for the high-temperature cycle and water for the low-temperature cycle, as shown in Fig. 12.20. The temperatures and pressures are shown in the corresponding *T–s* diagram. The maximum temperature in the steam cycle is where the steam leaves the superheater at point 4 where it is 500°C. Determine the ratio of the mass flow rate of mercury to the mass flow rate of water in the heat exchanger that condenses mercury and boils the water and the thermal efficiency of this ideal cycle.

The following saturation properties for mercury are known

P, MPa	T_g, °C	h_f, kJ/kg	h_g, kJ/kg	s_f kJ/kgK	s_g, kJ/kgK
0.04	309	42.21	335.64	0.1034	0.6073
1.60	562	75.37	364.04	0.1498	0.4954

Solution:

For the mercury cycle:

$s_d = s_c = 0.4954 = 0.1034 + x_d \times 0.5039, \quad x_d = 0.7779$

$h_b = h_a - w_{P\ HG} \approx h_a$ (since v_F is very small)

$q_H = h_c - h_a = 364.04 - 42.21 = 321.83$ kJ/kg

$q_L = h_d - h_a = 270.48 - 42.21 = 228.27$ kJ/kg

For the steam cycle:

$s_5 = s_4 = 7.0097 = 0.6493 + x_5 \times 7.5009, \quad x_5 = 0.8480$

$h_5 = 191.83 + 0.848 \times 2392.8 = 2220.8$

$w_P \approx v_1(P_2 - P_1) = 0.00101(4688 - 10) = 4.7$ kJ/kg

$h_2 = h_1 + w_P = 191.8 + 4.7 = 196.5$

q_H (from Hg) $= h_3 - h_2 = 2769.9 - 196.5 = 2600.4$

q_H (ext. source) $= h_4 - h_3 = 3437.4 - 2796.9 = 640.5$

CV: Hg condenser - H_2O boiler: 1st law: $m_{Hg}(h_d - h_a) = m_{H2O}(h_3 - h_2)$

$$m_{Hg}/m_{H2O} = \frac{2796.9 - 196.5}{270.48 - 42.21} = \mathbf{11.392}$$

$q_{H\ TOTAL} = (m_{Hg}/m_{H2O})(h_c - h_b) + (h_4 - h_3)$ (for 1 kg H_2O)

$= 11.392 \times 321.83 + 640.5 = 4306.8$ kJ

All q_L is from the H_2O condenser:

$q_L = h_5 - h_1 = 2220.8 - 191.8 = 2029.0$ kJ

$w_{NET} = q_H - q_L = 4306.8 - 2029.0 = 2277.8$ kJ

$\eta_{TH} = w_{NET}/q_H = 2277.8/4306.8 = \mathbf{0.529}$

11.121

A Rankine steam power plant should operate with a high pressure of 3 MPa, a low pressure of 10 kPa, and the boiler exit temperature should be 500°C. The available high-temperature source is the exhaust of 175 kg/s air at 600°C from a gas turbine. If the boiler operates as a counterflowing heat exchanger where the temperature difference at the pinch point is 20°C, find the maximum water mass flow rate possible and the air exit temperature.

Solution:

C.V. Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1)$

$= 0.00101(3000 - 10) = 3.02$ kJ/kg

$h_2 = h_1 + w_P = 191.83 + 3.02 = 194.85$ kJ/kg

Heat exchanger water states

State 2a: $T_{2a} = T_{SAT} = 233.9$ °C

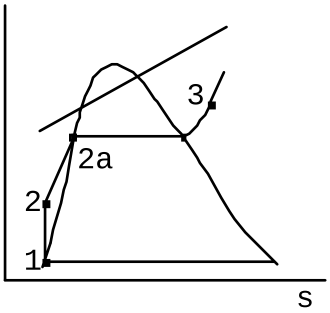
$h_{2a} = 1008.42$ kJ/kg

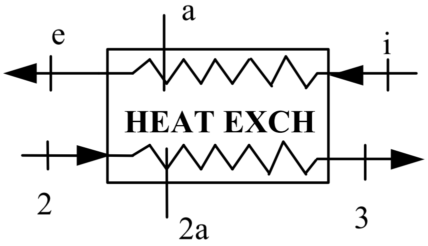
State 3: $h_3 = 3456.5$ kJ/kg

Heat exchanger air states

inlet: $h_{air,in} = 903.16$ kJ/kg

State 2a: $h_{air}(T_{2a} + 20) = 531.28$ kJ/kg





Air temperature should be 253.9°C at the point where the water is at state 2a.

C.V. Section 2a-3, i-a

$$\dot{m}_{H_2O}(h_3 - h_{2a}) = \dot{m}_{air}(h_i - h_a)$$

$$\dot{m}_{H_2O} = 175\,\frac{903.16 - 531.28}{3456.5 - 1008.42} = \mathbf{26.584\ kg/s}$$

Take C.V. Total: $\dot{m}_{H_2O}(h_3 - h_2) = \dot{m}_{air}(h_i - h_e)$

$\Rightarrow$ $h_e = h_i - \dot{m}_{H_2O}(h_3 - h_2)/\dot{m}_{air}$

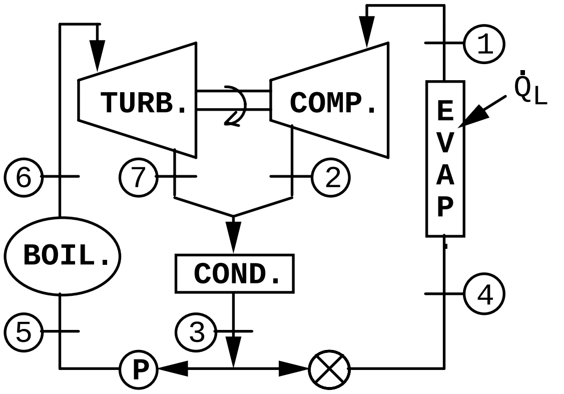
$= 903.6 - 26.584(3456.5 - 194.85)/175 = 408.13$ kJ/kg

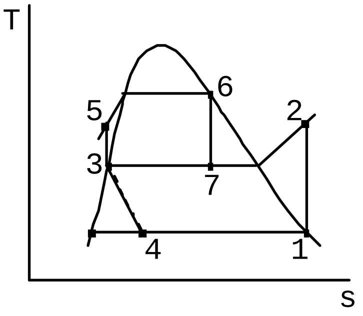
$\Rightarrow$ $T_e = 406.7$ K = **133.6 °C**, $T_e > T_2 = 46.5$ °C OK.

11.122

Consider an ideal dual-loop heat-powered refrigeration cycle using R-12 as the working fluid, as shown in Fig. P11.122. Saturated vapor at 90°C leaves the boiler and expands in the turbine to the condenser pressure. Saturated vapor at −15°C leaves the evaporator and is compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two exiting streams mix together and enter the condenser. Saturated liquid leaving the condenser at 45°C is then separated into two streams in the necessary proportions. Determine the ratio of mass flow rate through the power loop to that through the refrigeration loop. Find also the performance of the cycle, in terms of the ratio $\dot{Q}_L / \dot{Q}_H$.

Solution:





T_1 = -15 °C sat. vap.

Table B.3.1 $T_6 = 105^{\circ}C$ sat. vapor => $P_5 = P_6 = 3.6509$ MPa

Table B.3.1 $T_3 = 45^{\circ}C$ sat. liquid => $P_2 = P_3 = P_7 = 1.0843$ MPa

$h_1 = 180.97; \quad h_3 = h_4 = 79.71; \quad h_6 = 206.57$

C.V. Turbine

$s_7 = s_6 = 0.6325 = 0.2877 + x_7 \times 0.3934; \qquad x_7 = 0.8765$

$h_7 = 79.71 + 0.8765 \times 125.16 = 189.41$

C.V. Compressor (computer tables are used for this due to value of P)

$s_2 = s_1 = 0.7051,\ P_2$ => $T_2 = 54.7^{\circ}C, \quad h_2 = 212.6$ kJ/kg

CV: turbine + compressor

Continuity Eq.: $\dot{m}_1 = \dot{m}_2, \quad \dot{m}_6 = \dot{m}_7$;

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_7 h_7$

$$\dot{m}_6/\dot{m}_1 = (212.6 - 180.97)/(206.57 - 189.41) = \mathbf{1.843}$$

CV: pump

$$w_P = v_3(P_5 - P_3) = 0.000811(3651 - 1084) = 2.082 \text{ kJ/kg}$$

$$h_5 = h_3 + w_P = 81.79 \text{ kJ/kg}$$

CV: evaporator $\Rightarrow$ $\dot{Q}_L = \dot{m}_1(h_1 - h_4)$

CV: boiler $\Rightarrow$ $\dot{Q}_H = \dot{m}_6(h_6 - h_5)$

$$\beta = \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{m}_1(h_1 - h_4)}{\dot{m}_6(h_6 - h_5)} = \frac{180.97 - 79.71}{1.843(206.57 - 81.79)} = \mathbf{0.44}$$

11.123

For a cryogenic experiment heat should be removed from a space at 75 K to a reservoir at 180 K. A heat pump is designed to use nitrogen and methane in a cascade arrangement (see Fig. 11.25), where the high temperature of the nitrogen condensation is at 10 K higher than the low-temperature evaporation of the methane. The two other phase changes take place at the listed reservoir temperatures. Find the saturation temperatures in the heat exchanger between the two cycles that gives the best coefficient of performance for the overall system.

The nitrogen cycle is the bottom cycle and the methane cycle is the top cycle. Both are standard refrigeration cycles.

$T_{Hm} = 180 \text{ K} = T_{3m}$, $T_{LN} = 75 \text{ K} = T_{4N} = T_{1N}$

$T_{Lm} = T_{4m} = T_{1m} = T_{3N} - 10$, Trial and error on T_{3N} or T_{Lm}.

For each cycle we have,

$-w_C = h_2 - h_1$, $s_2 = s_1$, $-q_H = h_2 - h_3$, $q_L = h_1 - h_4 = h_1 - h_3$

Nitrogen: $T_4 = T_1 = 75$ K ⇒ $h_1 = 74.867$ kJ/kg, $s_1 = 5.4609$ kJ/kg K

N_2	T_3	h_3	P_2	h_2	$-w_c$	$-q_H$	q_L
a)	120	-17.605	2.5125	202.96	128.1	220.57	92.47
b)	115	-34.308	1.9388	188.35	113.5	222.66	109.18
c)	110	-48.446	1.4672	173.88	99.0	222.33	123.31

Methane: $T_3 = 180$ K ⇒ $h_3 = -0.5$ kJ/kg, $P_2 = 3.28655$ MPa

CH_4	T_4	h_1	s_1	h_2	$-w_c$	$-q_H$	q_L
a)	110	221	9.548	540.3	319.3	540.8	221.5
b)	105	212.2	9.691	581.1	368.9	581.6	212.7
c)	100	202.9	9.851	629.7	426.8	630.2	203.4

The heat exchanger that connects the cycles transfers a Q

$$\dot{Q}_{Hn} = q_{Hn}\,\dot{m}_n = \dot{Q}_{Lm} = q_{Lm}\,\dot{m}_m \quad => \quad \dot{m}_m/\dot{m}_n = q_{Hn}/q_{Lm}$$

The overall unit then has

$$\dot{Q}_{L\ 75\ K} = \dot{m}_n\, q_{Ln}\ ; \quad \dot{W}_{tot\ in} = -\ (\dot{m}_n w_{cn} + \dot{m}_m w_{cm})$$

$$\beta = \dot{Q}_{L\ 75\ K}/\dot{W}_{tot\ in} = q_{Ln}/[-w_{cn} - (\dot{m}_m/\dot{m}_n)w_{cm}]$$

Case	$\dot{m}_m/\dot{m}_n$	$w_{cn}+(\dot{m}_m/\dot{m}_n)w_{cm}$	β
a)	0.996	446.06	0.207
b)	1.047	499.65	0.219
c)	1.093	565.49	0.218

A maximum coefficient of performance is between case b) and c).

11.124

For Problem 11.121, determine the change of availability of the water flow and that of the air flow. Use these to determine a second law efficiency for the boiler heat exchanger.

From solution to 11.121:

$\dot{m}_{H2O} = 26.584$ kg/s, $h_2 = 194.85$ kJ/kg, $s_2 = 0.6587$ kJ/kg K

$h_3 = 3456.5$ kJ/kg, $s_3 = 7.2338$, $s^{\circ}_{Ti} = 7.9820$, $s^{\circ}_{Te} = 7.1762$ kJ/kg K

$h_i = 903.16$ kJ/kg, $h_e = 408.13$ kJ/kg

$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = 1301.28$ kJ/kg

$\psi_i - \psi_e = h_i - h_e - T_0(s^{\circ}_{Ti} - s^{\circ}_{Te}) = 254.78$ kJ/kg

$$\eta_{II} = \frac{(\psi_3 - \psi_2)\dot{m}_{H2O}}{(\psi_i - \psi_e)\dot{m}_{air}} = \frac{1301.28 \times 26.584}{254.78 \times 175} = \mathbf{0.776}$$

Review Problems

11.125

Do Problem 11.27 with R-134a as the working fluid in the Rankine cycle.

Consider the ammonia Rankine-cycle power plant shown in Fig. P11.27, a plant that was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s.

a. Determine the turbine power output and the pump power input for the cycle.
b. Determine the mass flow rate of water through each heat exchanger.
c. What is the thermal efficiency of this power plant?

Solution:

a) Turbine

$s_2 = s_1 = 1.7183 = 1.0485 + x_2 \times 0.6733 \quad => \quad x_2 = 0.9948$

$h_2 = 213.58 + 0.9948 \times 190.65 = 403.24$ kJ/kg

$w_T = h_1 - h_2 = 409.84 - 403.24 = 6.6$ kJ/kg

$\dot{W}_T = \dot{m}w_T = 6600$ kW

Pump: $w_P \approx v_3(P_4 - P_3) = 0.000794(572.8 - 415.8) = 0.125$ kJ/kg

$w_P = w_P/\eta_S = 0.125 \quad => \quad \dot{W}_P = \dot{m}w_P =$ **125 kW**

b) Consider the condenser heat transfer to the low T water

$\dot{Q}_{\text{to low T H2O}} = 1000(403.24 - 213.58) = 189\,660$ kW

$\dot{m}_{\text{low T H2O}} = \dfrac{189660}{29.38 - 20.98} =$ **22 579 kg/s**

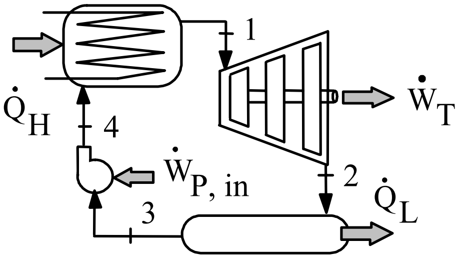
$h_4 = h_3 - w_P = 213.58 + 0.125 = 213.71$ kJ/kg

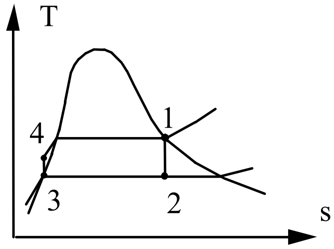
Now consider the boiler heat transfer from the high T water

$\dot{Q}_{\text{from high T H2O}} = 1000(409.84 - 213.71) = 196\,130$ kW

$\dot{m}_{\text{high T H2O}} = \dfrac{196130}{104.87 - 96.50} =$ **23 432 kg/s**

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \dfrac{6600 - 125}{196130} =$ **0.033**





11.126

A simple steam power plant is said to have the four states as listed: 1: (20°C, 100 kPa), 2: (25°C, 1 MPa), 3: (1000°C, 1 MPa), 4: (250°C, 100 kPa) with an energy source at 1100°C and it rejects energy to a 0°C ambient. Is this cycle possible? Are any of the devices impossible?

Solution:

The cycle should be like Figure 11.3 for an ideal or Fig.11.9 for an actual pump and turbine in the cycle. We look the properties up in Table B.1:

State 1: $h_1 = 83.94$, $s_1 = 0.2966$ State 2: $h_2 = 104.87$, $s_2 = 0.3673$

State 3: $h_3 = 4637.6$, $s_3 = 8.9119$ State 4: $h_4 = 2974.3$, $s_4 = 8.0332$

We may check the overall cycle performance

Boiler: $q_H = h_3 - h_2 = 4637.6 - 104.87 = 4532.7$ kJ/kg

Condenser: $q_L = h_4 - h_1 = 2974.3 - 83.94 = 2890.4$ kJ/kg

$$\eta_{cycle} = q_{net} / q_H = (q_H - q_L) / q_H = 1642.3 / 4532.7 = 0.362$$

$$\eta_{carnot} = 1 - T_L / T_H = 1 - \frac{273.15}{273.15 + 1100} = \mathbf{0.80} > \eta_{cycle} \quad \textbf{OK}$$

Check the second law for the individual devices:

C.V. Boiler plus wall to reservoir

$$s_{gen} = s_3 - s_2 - \frac{q_H}{T_{res}} = 8.9119 - 0.3673 - \frac{4532.7}{1373} = 5.24 \text{ kJ/kg K } > 0 \quad \textbf{OK}$$

C.V. Condenser plus wall to reservoir

$$s_{gen} = s_1 - s_4 + \frac{q_L}{T_{res}} = 0.2966 - 8.0332 + \frac{2890.4}{273} = 2.845 \text{ kJ/kg K } > 0 \quad \textbf{OK}$$

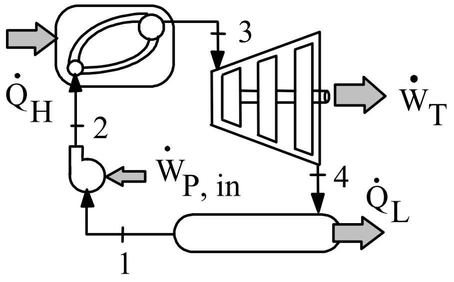
C.V. Pump: $w_p = h_2 - h_1 = 20.93$ kJ/kg ;

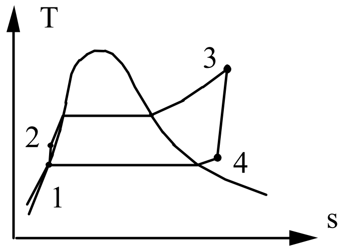
$$s_{gen} = s_2 - s_1 = 0.3673 - 0.2966 = 0.0707 \text{ kJ/kg K } > 0 \quad \textbf{OK}$$

C.V. Turbine: $w_T = h_3 - h_4 = 4637.6 - 2974.3 = 1663.3$ kJ/kg

$$s_{gen} = s_4 - s_3 = 8.0332 - 8.9119 = - 0.8787 \text{ kJ/kg K}$$

$s_{gen} < 0$ **NOT POSSIBLE**


$\dot{Q}_H$
3
$\dot{W}_T$
2
$\dot{W}_{P,\ in}$
4
$\dot{Q}_L$
1


T
3
2
4
1
s

11.127

Consider an ideal combined reheat and regenerative cycle in which steam enters the high-pressure turbine at 3.0 MPa, 400°C, and is extracted to an open feedwater heater at 0.8 MPa with exit as saturated liquid. The remainder of the steam is reheated to 400°C at this pressure, 0.8 MPa, and is fed to the low-pressure turbine. The condenser pressure is 10 kPa. Calculate the thermal efficiency of the cycle and the net work per kilogram of steam.

Solution:

In this setup the flow is separated into fractions x and 1-x after coming out of T1. The two flows are recombined in the FWH.

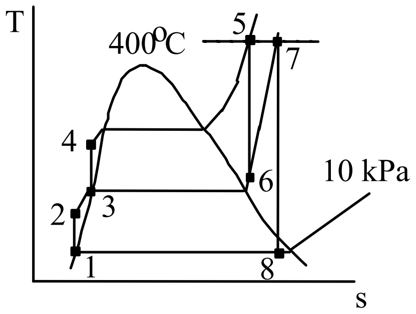
C.V. T1 $\quad s_6 = s_5 = 6.9211$ kJ/kg K $\;\Rightarrow\;$ $h_6 = 2891.6$ kJ/kg

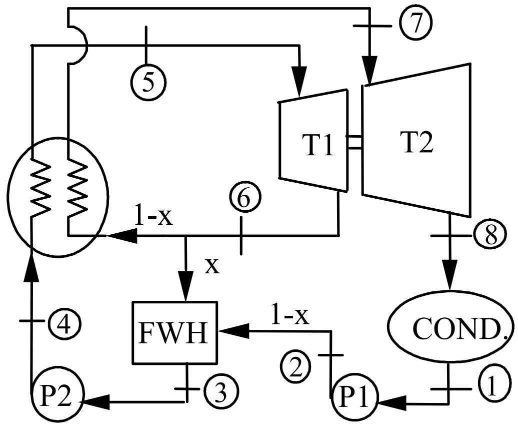
$$w_{T1} = h_5 - h_6 = 3230.82 - 2891.6 = 339.22 \text{ kJ/kg}$$

C.V. Pump 1:

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$\Rightarrow \quad h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$





C.V. FWH, $\quad h_3 = h_f = 721.1$

Energy equation per unit mass flow exit at 3:

$$x\,h_6 + (1 - x)\,h_2 = h_3 \qquad \Rightarrow \qquad x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V. Pump 2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$\Rightarrow \quad h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

C.V. Boiler/steam generator including reheater.

Total flow from 4 to 5 only fraction 1-x from 6 to 7

$q_H = h_5 - h_4 + (1 - x)(h_7 - h_6\,) = 2507.3 + 301.95 = 2809.3$ kJ/kg

C.V. Turbine 2

$s_8 = s_7 = 7.5715$ kJ/kg K => $x_8 = (7.5715 - 0.6492)/7.501 = 0.92285$

$h_8 = h_f + x_8\, h_{fg} = 191.81 + 0.92285 \times 2392.82 = 2400.0$ kJ/kg

$w_{T2} = h_7 - h_8 = 3267.07 - 2400.02 = 867.05$ kJ/kg

Sum the work terms to get net work. Total flow through T1 only fraction 1-x through T2 and P1 and after FWH we have the total flow through P2.

$w_{net} = w_{T1} + (1 - x)\, w_{T2} - (1 - x)\, w_{P1} - w_{P2}$

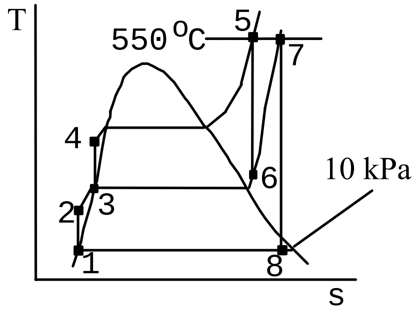
$= 339.2 + 697.3 - 0.64 – 2.45 =$ **1033.41 kJ/kg**

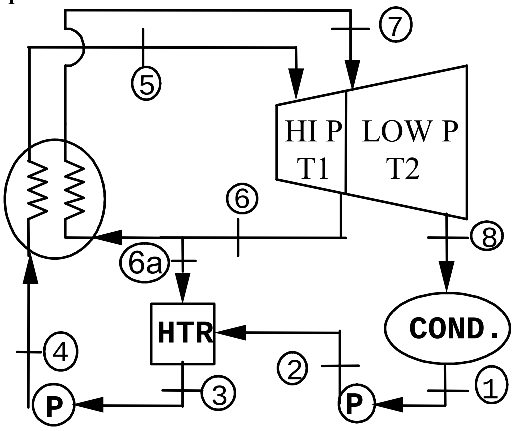
$\eta_{cycle} = w_{net} / q_H = 1033.41 / 2809.3 =$ **0.368**

11.128

An ideal steam power plant is designed to operate on the combined reheat and regenerative cycle and to produce a net power output of 10 MW. Steam enters the high-pressure turbine at 8 MPa, 550°C, and is expanded to 0.6 MPa, at which pressure some of the steam is fed to an open feedwater heater, and the remainder is reheated to 550°C. The reheated steam is then expanded in the low-pressure turbine to 10 kPa. Determine the steam flow rate to the high-pressure turbine and the power required to drive each of the pumps.

a)





b) $-w_{P12} = 0.00101(600 - 10) = 0.6$ kJ/kg

$h_2 = h_1 - w_{P12} = 191.8 + 0.6 = 192.4$ kJ/kg

$-w_{P34} = 0.00101(8000 - 600) = 8.1$ kJ/kg

$h_4 = h_3 - w_{P34} = 670.6 + 8.1 = 678.7$; $h_5 = 3521.0$ kJ/kg,

$s_6 = s_5 = 6.8778 \Rightarrow T_6 = 182.32\ ^oC \quad h_6 = 2810.0$ kJ/kg,

$h_7 = 3591.9, \quad s_8 = s_7 = 8.1348 = 0.6493 + x_8 \times 7.5009 \Rightarrow x_8 = 0.9979$

$h_8 = 191.83 + 0.9979 \times 2392.8 = 2579.7$ kJ/kg

CV: heater

Cont: $m_{6a} + m_2 = m_3 = 1$ kg, Energy Eq: $m_{6a}h_6 + m_2h_2 = m_3h_3$

$$m_{6a} = \frac{670.6 - 192.4}{2810.0 - 192.4} = 0.1827, \quad m_2 = m_7 = 1 - m_{6a} = 0.8173$$

CV: turbine $\quad w_T = (h_5 - h_6) + (1 - m_{6a})(h_7 - h_8)$

$= 3521 - 2810 + 0.8173(3591.9 - 2579.7) = 1538.2$ kJ/kg

CV: pumps

$w_P = m_2w_{P12} + m_4w_{P34} = 0.8214\times(-0.6) + 1\times(-8.1) = -8.6$ kJ/kg

$w_{Net} = 1538.2 - 8.6 = 1529.6$ kJ/kg (m_5)

$\dot{m}_5 = \dot{W}_{Net}/w_{Net} = 10000/1529.6 =$ **6.53 kg/s**

11.129

Steam enters the turbine of a power plant at 5 MPa and 400°C, and exhausts to the condenser at 10 kPa. The turbine produces a power output of 20 000 kW with an isentropic efficiency of 85%. What is the mass flow rate of steam around the cycle and the rate of heat rejection in the condenser? Find the thermal efficiency of the power plant.

Solution: $\dot{W}_T = 20\ 000$ kW and $\eta_{Ts} = 85$ %

State 3: $h_3 = 3195.6$ kJ/kg , $s_3 = 6.6458$ kJ/kgK

State 1: $P_1 = P_4 = 10$ kPa , sat liq , $x_1 = 0$

$T_1 = 45.8^oC$, $h_1 = h_f = 191.8$ kJ/kg , $v_1 = v_f = 0.00101$ m^3/kg

C.V Turbine : Energy Eq.: $q_T + h_3 = h_4 + w_T$; $q_T = 0$

$w_T = h_3 - h_4$, Assume Turbine is isentropic

$s_{4s} = s_3 = 6.6458$ kJ/kgK , $s_{4s} = s_f + x_{4s}\, s_{fg}$, solve for $x_{4s} = 0.7994$

$h_{4s} = h_f + x_{4s}h_{fg} = 1091.0$ kJ/kg

$w_{Ts} = h_3 - h_{4s} = 1091$ kJ/kg , $w_T = \eta_{Ts}w_{Ts} = 927.3$ kJ/kg

$\dot{m} = \frac{\dot{W}_T}{w_T} =$ **21.568 kg/s** , $h_4 = h_3 - w_T = 2268.3$ kJ/kg

C.V. Condenser: Energy Eq.: $h_4 = h_1 + q_c + w_c$; $w_c = 0$

$q_c = h_4 - h_1 = 2076.5$ kJ/kg , $\dot{Q}_c = \dot{m}\, q_c =$ **44786 kW**

C.V. Pump: Assume adiabatic, reversible and incompressible flow

$w_{ps} = \int v\, dP = v_1(P_2 - P_1) = 5.04$ kJ/kg

Energy Eq.: $h_2 = h_1 + w_p = 196.8$ kJ/kg

C.V Boiler : Energy Eq.: $q_B + h_2 = h_3 + w_B$; $w_B = 0$

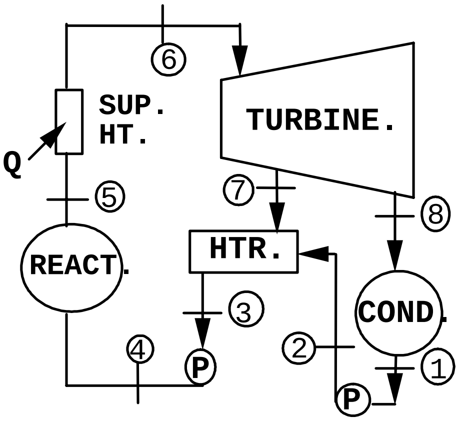
$q_B = h_3 - h_2 = 2998.8$ kJ/kg

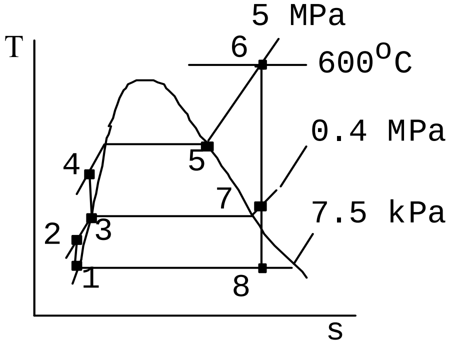
$w_{net} = w_T - w_P = 922.3$ kJ/kg

$\eta_{th} = w_{net} / q_B =$ **0.307**

11.130

In one type of nuclear power plant, heat is transferred in the nuclear reactor to liquid sodium. The liquid sodium is then pumped through a heat exchanger where heat is transferred to boiling water. Saturated vapor steam at 5 MPa exits this heat exchanger and is then superheated to 600°C in an external gas-fired superheater. The steam enters the reversible turbine, which has one (open-type) feedwater extraction at 0.4 MPa, and the condenser pressure is 7.5 kPa. Determine the heat transfer in the reactor and in the superheater to produce a net power output of 1 MW.





$\dot{W}_{NET} = 1$ MW

$-w_{P12} = 0.001008(400 - 7.5) = 0.4$ kJ/kg

$h_2 = h_1 - w_{P12} = 168.8 + 0.4 = 169.2$ kJ/kg

$-w_{P34} = 0.001084(5000 - 400) = 5.0$ kJ/kg

$h_4 = h_3 - w_{P34} = 604.7 + 5.0 = 609.7$ kJ/kg

$s_7 = s_6 = 7.2589, \quad P_7 = 0.4 \text{ MPa} \Rightarrow \quad T_7 = 221.2\ ^oC, \quad h_7 = 2904.5$ kJ/kg

$s_8 = s_6 = 7.2589 = 0.5764 + x_8 \times 7.6750 ; \quad x_8 = 0.8707$

$h_8 = 168.8 + 0.8707 \times 2406.0 = 2263.7$ kJ/kg

CV: heater

cont: $\dot{m}_2 + \dot{m}_7 = \dot{m}_3$, Energy Eq.: $(1 - y)\ h_2 + y\ h_7 = h_3$

$y = \dot{m}_7 / \dot{m}_3 = (604.7 - 169.2)/(2904.5 - 169.2) = 0.1592$

CV: turbine

$w_T = (h_6 - h_7) + (1 - y)(h_7 - h_8)$

$= 3666.5 - 2904.5 + 0.8408\ (2904.5 - 2263.7) = 1300.8$ kJ/kg

CV: pumps

$w_P = (1 - y)\, w_{P12} + w_{P34} = 0.8408\ (-0.4) + 1(-5.0) = -5.33$ kJ/kg

$w_{NET} = 1300.8 - 5.3 = 1295.5 \quad => \quad \dot{m} = 1000/1295.5 = 0.7719$ kg/s

CV: reactor

$\dot{Q}_{REACT} = \dot{m}(h_5 - h_4) = 0.7719\ (2794.3 - 609.7) =$ **1686 kW**

CV: superheater

$\dot{Q}_{SUP} = 0.7719\ (h_6 - h_5) = 0.7719\ (3666.5 - 2794.3) =$ **673 kW**

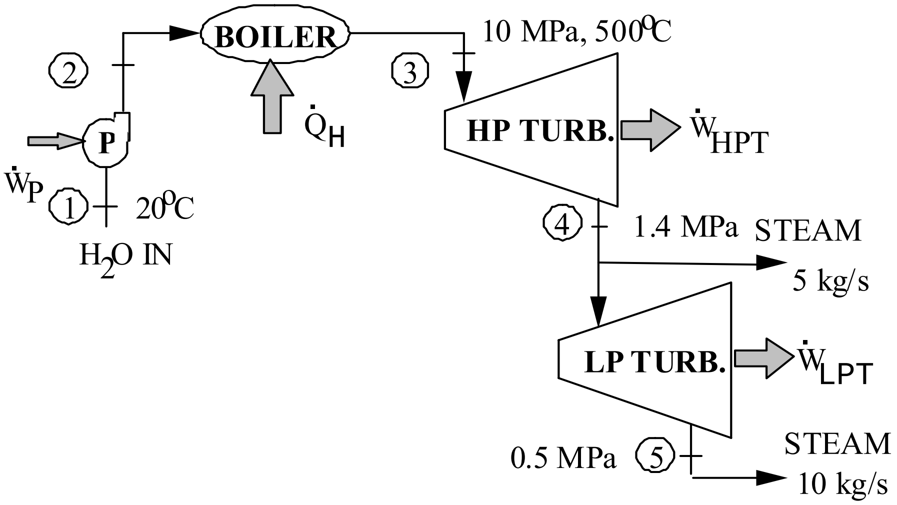
11.131

An industrial application has the following steam requirement: one 10-kg/s stream at a pressure of 0.5 MPa and one 5-kg/s stream at 1.4 MPa (both saturated or slightly superheated vapor). It is obtained by cogeneration, whereby a high-pressure boiler supplies steam at 10 MPa, 500°C to a reversible turbine. The required amount is withdrawn at 1.4 MPa, and the remainder is expanded in the low-pressure end of the turbine to 0.5 MPa providing the second required steam flow.

a. Determine the power output of the turbine and the heat transfer rate in the boiler.

b. Compute the rates needed were the steam generated in a low-pressure boiler without cogeneration. Assume that for each, 20°C liquid water is pumped to the required pressure and fed to a boiler.

Solution:



a) **With cogeneration**

high-pressure turbine.

$s_4 = s_3 = 6.5966 \text{ kJ/kg K} \quad \Rightarrow \quad T_4 = 219.9\ ^{o}\text{C}, \; h_4 = 2852.6 \text{ kJ/kg}$

$w_{S\,HPT} = h_3 - h_4 = 3373.7 - 2852.6 = 521.1 \text{ kJ/kg}$

low-pressure turbine

$s_5 = s_4 = 6.5966 = 1.8607 + x_5 \times 4.9606, \; x_5 = 0.9547$

$h_5 = 640.23 + 0.9547 \times 2108.5 = 2653.2 \text{ kJ/kg}$

$w_{S\,LPT} = h_4 - h_5 = 2852.6 - 2653.2 = 199.4 \text{ kJ/kg}$

$\dot{W}_{TURB} = 15 \times 521.1 + 10 \times 199.4 = \mathbf{9810\ kW}$

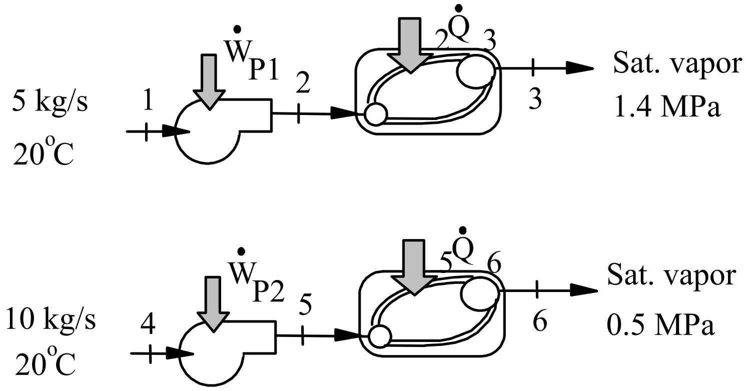
$\dot{W}_P = 15 \times 0.001002\ (10\,000 - 2.3) = 150.3 \text{ kW}$

$h_2 = h_1 + w_P = 83.96 + 10.02 = 94.0 \text{ kJ/kg}$

$\dot{Q}_H = \dot{m}_1(h_3 - h_2) = 15(3373.7 - 94.0) =$ **49 196 kW**

b) **Without cogeneration**

This is to be compared to the amount of heat required to supply 5 kg/s of 1.4 MPa sat. vap. plus 10 kg/s of 0.5 MPa sat. vap. from 20°C water.



Pump 1 and boiler 1

$w_P = 0.001002(1400 - 2.3) = 14.0$ kJ/kg,

$h_2 = h_1 + w_P = 83.96 + 14.0 = 85.4$ kJ/kg

$_2\dot{Q}_3 = \dot{m}_1(h_3 - h_2) = 5(2790.0 - 85.4) = 13\ 523$ kW

$\dot{W}_{P1} = 5 \times 14.0 = 7$ kW

Pump 2 and boiler 2

$h_5 = h_4 + w_{P2} = 83.96 + 0.001002(500 - 2.3) = 84.5$ kJ/kg

$_5\dot{Q}_6 = \dot{m}_4(h_6 - h_5) = 10(2748.7 - 84.5) = 26\ 642$ kW

$\dot{W}_{P2} = 10 \times 0.5 = 5$ kW

Total $\dot{Q}_H = 13523 + 26642 =$ **40 165 kW**

Notice here that the extra heat transfer is about 9000 kW to run the turbines but that provides 9800 kW of work for electricity (a 100% conversion of the extra Q to W).

11.132

The effect of a number of open feedwater heaters on the thermal efficiency of an ideal cycle is to be studied. Steam leaves the steam generator at 20 MPa, 600°C, and the cycle has a condenser pressure of 10 kPa. Determine the thermal efficiency for each of the following cases. **A:** No feedwater heater. **B**: One feedwater heater operating at 1 MPa. **C:** Two feedwater heaters, one operating at 3 MPa and the other at 0.2 MPa.

a) no feed water heater

$$w_P = \int_1^2 v dP$$

$\approx 0.00101(20000 - 10)$

$= 20.2$ kJ/kg

$h_2 = h_1 + w_P = 191.8 + 20.2 = 212.0$

$s_4 = s_3 = 6.5048$

$= 0.6493 + x_4 \times 7.5009$

$x_4 = 0.78064$

$h_4 = 191.83 + 0.780\,64 \times 2392.8$

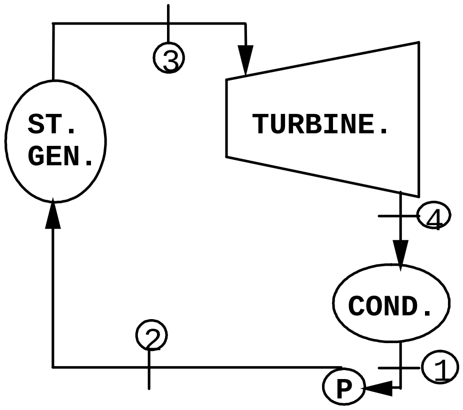
$= 2059.7$

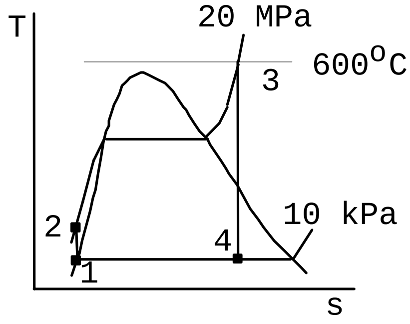
$w_T = h_3 - h_4 = 3537.6 - 2059.7$

$= 1477.9$ kJ/kg

$w_N = w_T - w_P = 1477.9 - 20.2 = 1457.7$

$q_H = h_3 - h_2 = 3537.6 - 212.0 = 3325.6$





$$\eta_{TH} = \frac{w_N}{q_H} = \frac{1457.7}{3325.6} = \mathbf{0.438}$$

b) one feedwater heater

$w_{P12} = 0.00101(1000 - 10)$

$= 1.0$ kJ/kg

$h_2 = h_1 + w_{P12} = 191.8 + 1.0 = 192.8$

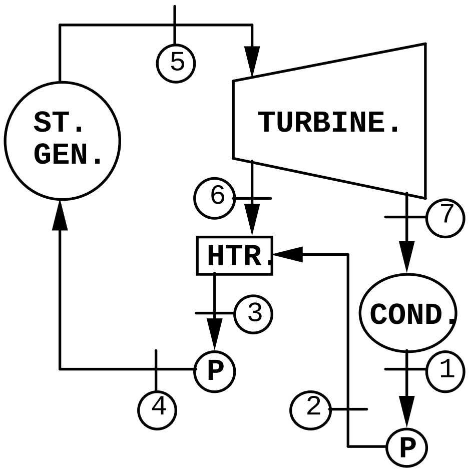
$w_{P34} = 0.001127\ (20000 - 1000)$

$= 21.4$ kJ/kg

$h_4 = h_3 + w_{P34} = 762.8 + 21.4 = 784.2$

$s_6 = s_5 = 6.5048$

$= 2.1387 + x_6 \times 4.4478$



$x_6 = 0.9816$

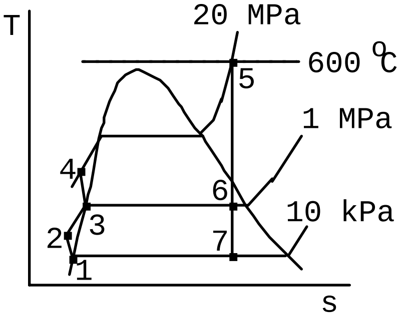
$h_6 = 762.8 + 0.9816 \times 2015.3 = 2741.1$

CV: heater

const: $m_3 = m_6 + m_2 = 1.0$ kg

1st law: $m_6h_6 + m_2h_2 = m_3h_3$

$$m_6 = \frac{762.8 - 192.8}{2741.1 - 192.8} = 0.2237$$



$m_2 = 0.7763, \quad h_7 = 2059.7$ ($= h_4$ of part a))

CV: turbine $\quad w_T = (h_5 - h_6) + m_2(h_6 - h_7)$

$= (3537.6 - 2741.1) + 0.7763(2741.1 - 2059.7) = 1325.5$ kJ/kg

CV: pumps

$w_P = m_1w_{P12} + m_3w_{P34} = 0.7763(1.0) + 1(21.4) = 22.2$ kJ/kg

$w_N = 1325.5 - 22.2 = 1303.3$ kJ/kg

CV: steam generator

$q_H = h_5 - h_4 = 3537.6 - 784.2 = 2753.4$ kJ/kg

$\eta_{TH} = w_N/q_H = 1303.3/2753.4 =$ **0.473**

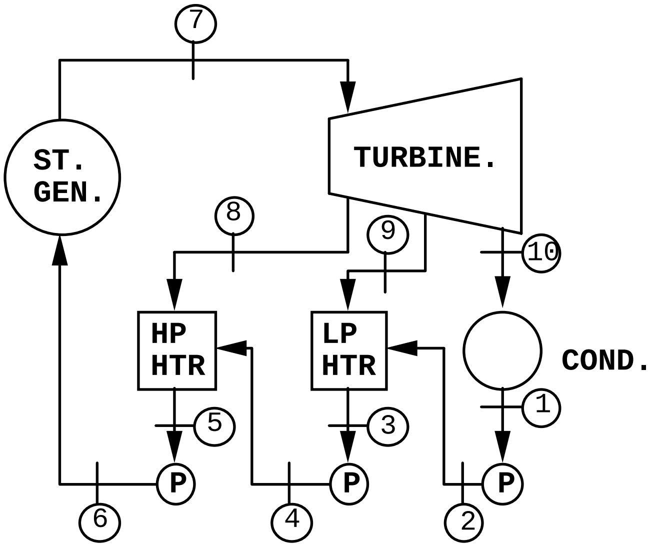
c) two feedwater heaters

$w_{P12} = 0.00101 \times (200 - 10) = 0.2$ kJ/kg

$h_2 = w_{P12} + h_1 = 191.8 + 0.2 = 192.0$

$w_{P34} = 0.001061 \times (3000 - 200) = 3.0$ kJ/kg

$h_4 = h_3 + w_{P34} = 504.7 + 3.0 = 507.7$



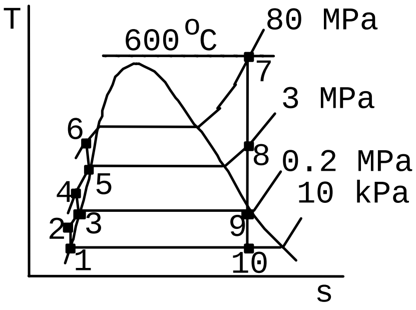
$w_{P56} = 0.001217(20000 - 3000)$

$= 20.7$ kJ/kg

$h_6 = h_5 + w_{P56} = 1008.4 + 20.7 = 1029.1$

$$\left.\begin{array}{l} s_8 = s_7 = 6.5048 \\ \text{at } P_8 = 3 \text{ MPa} \end{array}\right\} \begin{array}{l} T_8 = 293.2\ ^oC \\ h_8 = 2974.8 \end{array}$$

$s_9 = s_8 = 6.5048 = 1.5301 + x_9 \times 5.5970$



$x_9 = 0.8888 \;\Rightarrow\; h_9 = 504.7 + 0.888 \times 2201.9 = 2461.8$ kJ/kg

CV: high pressure heater

cont: $m_5 = m_4 + m_8 = 1.0$ kg ; 1st law: $m_5h_5 = m_4h_4 + m_8h_8$

$$m_8 = \frac{1008.4 - 507.7}{2974.8 - 507.7} = 0.2030 \qquad m_4 = 0.7970$$

CV: low pressure heater

cont: $m_9 + m_2 = m_3 = m_4$; 1st law: $m_9h_9 + m_2h_2 = m_3h_3$

$$m_9 = \frac{0.7970(504.7 - 192.0)}{2461.8 - 192.0} = 0.1098$$

$m_2 = 0.7970 - 0.1098 = 0.6872$

CV: turbine

$w_T = (h_7 - h_8) + (1 - m_8)(h_8 - h_9) + (1 - m_8 - m_9)(h_9 - h_{10})$

$= (3537.6 - 2974.8) + 0.797(2974.8 - 2461.8)$

$+ 0.6872(2461.8 - 2059.7) = 1248.0$ kJ/kg

CV: pumps

$w_P = m_1w_{P12} + m_3w_{P34} + m_5w_{P56}$

$= 0.6872(0.2) + 0.797(3.0) + 1(20.7) = 23.2$ kJ/kg

$w_N = 1248.0 - 23.2 = 1224.8$ kJ/kg

CV: steam generator

$q_H = h_7 - h_6 = 3537.6 - 1029.1 = 2508.5$ kJ/kg

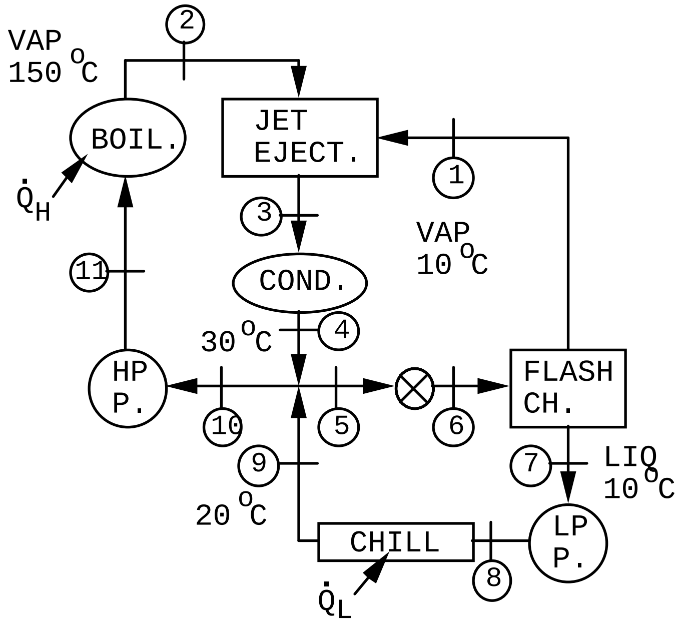
$\eta_{TH} = w_N/q_H = 1224.8/2508.5 = \mathbf{0.488}$

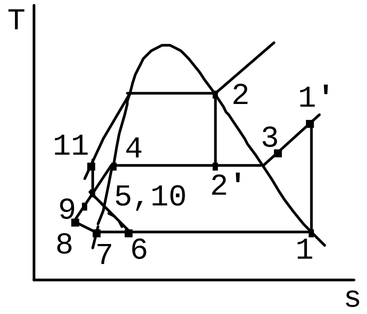
11.133

A jet ejector, a device with no moving parts, functions as the equivalent of a coupled turbine-compressor unit (see Problems 9.157 and 9.168). Thus, the turbine-compressor in the dual-loop cycle of Fig. P11.122 could be replaced by a jet ejector. The primary stream of the jet ejector enters from the boiler, the secondary stream enters from the evaporator, and the discharge flows to the condenser. Alternatively, a jet ejector may be used with water as the working fluid. The purpose of the device is to chill water, usually for an air-conditioning system. In this application the physical setup is as shown in Fig. P11.133. Using the data given on the diagram, evaluate the performance of this cycle in terms of the ratio Q_L/Q_H.

a. Assume an ideal cycle.

b. Assume an ejector efficiency of 20% (see Problem 9.168).





$T_1 = T_7 = 10$ °C

$T_2 = 150$ °C

$T_4 = 30$ °C

$T_9 = 20$ °C

Assume $T_5 = T_{10}$

(from mixing streams 4 & 9).

$P_3 = P_4 = P_5 = P_8 = P_9 = P_{10} = P_{G\,30\,^oC} = 4.246$ kPa

$P_{11} = P_2 = P_{G\,150^oC} = 475.8$ kPa, $\quad P_1 = P_6 = P_7 = P_{G\,10^oC} = 1.2276$ kPa

Cont: $\dot{m}_1 + \dot{m}_9 = \dot{m}_5 + \dot{m}_{10}$, $\quad \dot{m}_5 = \dot{m}_6 = \dot{m}_7 + \dot{m}_1$

$\dot{m}_7 = \dot{m}_8 = \dot{m}_9$, $\quad \dot{m}_{10} = \dot{m}_{11} = \dot{m}_2$, $\quad \dot{m}_3 = \dot{m}_4$

a) $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$; ideal jet ejector

$s_1' = s_1$ & $s_2' = s_2$ (1' & 2' at $P_3 = P_4$)

then, $\dot{m}_1(h_1' - h_1) = \dot{m}_2(h_2 - h_2')$

From $s'_2 = s_2 = 0.4369 + x'_2 \times 8.0164$; $x'_2 = 0.7985$

$h'_2 = 125.79 + 0.7985 \times 2430.5 = 2066.5$ kJ/kg

From $s'_1 = s_1 = 8.9008 \Rightarrow T'_1 = 112$ °C, $h'_1 = 2710.4$ kJ/kg

$$\Rightarrow \dot{m}_1/\dot{m}_2 = \frac{2746.5 - 2066.5}{2710.4 - 2519.8} = 3.5677$$

Also $h_4 = 125.79$ kJ/kg, $h_7 = 42.01$ kJ/kg, $h_9 = 83.96$ kJ/kg

Mixing of streams 4 & 9 ⇒ 5 & 10:

$$(\dot{m}_1 + \dot{m}_2)h_4 + \dot{m}_7h_9 = (\dot{m}_7 + \dot{m}_1 + \dot{m}_2)h_{5=10}$$

Flash chamber (since $h_6 = h_5$) : $(\dot{m}_7+\dot{m}_1)h_{5=10} = \dot{m}_1h_1 + \dot{m}_7h_1$

⇒ using the primary stream $\dot{m}_2 = 1$ kg/s:

$4.5677 \times 125.79 + \dot{m}_7 \times 83.96 = (\dot{m}_7 + 4.5677)h_5$

& $(\dot{m}_7 + 3.5677)h_5 = 3.5677 \times 2519.8 + \dot{m}_7 \times 42.01$

Solving, $\dot{m}_7 = 202.627$ & $h_5 = 84.88$ kJ/kg

LP pump: $-w_{LP\,P} = 0.0010(4.246 - 1.2276) = 0.003$ kJ/kg

$h_8 = h_7 - w_{LP\,P} = 42.01 + 0.003 = 42.01$ kJ/kg

Chiller: $\dot{Q}_L = \dot{m}_7(h_9-h_8) = 202.627(83.96 - 42.01) = 8500$ kW (for $\dot{m}_2 = 1$)

HP pump: $-w_{HP\,P} = 0.001002(475.8 - 4.246) = 0.47$ kJ/kg

$h_{11} = h_{10} - w_{HP\,P} = 84.88 + 0.47 = 85.35$ kJ/kg

Boiler: $\dot{Q}_{11} = \dot{m}_{11}(h_2 - h_{11}) = 1(2746.5 - 85.35) = 2661.1$ kW

⇒ $\dot{Q}_L/\dot{Q}_H = 8500/2661.1 = \mathbf{3.194}$

b) Jet eject. eff. $= (\dot{m}_1/\dot{m}_2)_{ACT}/(\dot{m}_1/\dot{m}_2)_{IDEAL} = 0.20$

$\Rightarrow \left(\dot{m}_1/\dot{m}_2\right)_{ACT} = 0.2 \times 3.5677 = 0.7135$

using $\dot{m}_2 = 1$ kg/s: $1.7135 \times 125.79 + \dot{m}_7 \times 83.96 = (\dot{m}_7 + 1.7135)h_5$

& $(\dot{m}_7 + 0.7135)h_5 = 0.7135 \times 2519.8 + \dot{m}_7 \times 42.01$

Solving, $\dot{m}_7 = 39.762$ & $h_5 = h_{10} = 85.69$ kJ/kg

Then, $\dot{Q}_L = 39.762(83.96 - 42.01) = 1668$ kW

$h_{11} = 85.69 + 0.47 = 86.16$ kJ/kg

$\dot{Q}_H = 1(2746.5 - 86.16) = 2660.3$ kW

& $\dot{Q}_L/\dot{Q}_H = 1668/2660.3 =$ **0.627**

Computer Problems

11.179 a

A refrigerator with R-12 as the working fluid has a minimum temperature of −10°C and a maximum pressure of 1 MPa. Assume an ideal refrigeration cycle as in Fig. 11.21. Find the specific heat transfer from the cold space and that to the hot space, and the coefficient of performance.

Solution:

Exit evaporator sat. vapor −10°C from B.3.1: $h_1 = 183.19$, $s_1 = 0.7019$ kJ/kgK

Exit condenser sat. liquid 1 MPa from B.3.1: $h_3 = 76.22$ kJ/kg

Compressor: $s_2 = s_1$ & P_2 from B.3.2 $\Rightarrow$ $h_2 \approx 210.1$ kJ/kg

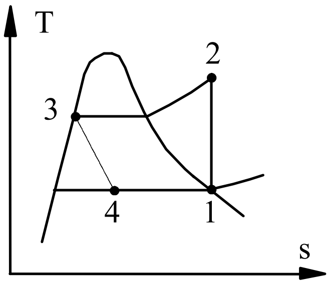
Evaporator: $q_L = h_1 - h_4 = h_1 - h_3 = 183.19 - 76.22 =$ **107 kJ/kg**

Condenser: $q_H = h_2 - h_3 = 210.1 - 76.22 =$ **133.9 kJ/kg**

COP: $\beta = q_L/w_c = q_L/(q_H - q_L) =$ **3.98**

Ideal refrigeration cycle
$P_{cond} = P_3 = P_2 = 1$ MPa
$T_{evap} = -10^oC = T_1$

Properties from Table B.3



11.179 b

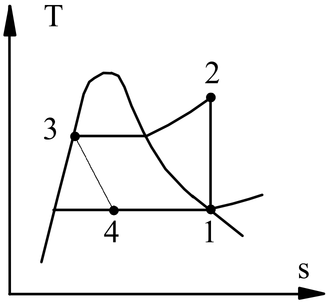
Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluid R-12.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$



	Property for:	**R-12**
Compressor	h_1, kJ/kg	180.97
	$s_2 = s_1$, kJ/kg K	0.7051
	P_2, MPa	1.0843
	T_2, °C	54.7
	h_2, kJ/kg	212.63
	$w_C = h_2 - h_1$	31.66
Exp. valve	$h_3 = h_4$, kJ/kg	79.71
Evaporator	$q_L = h_1 - h_4$	101.26
	$\beta = q_L/w_C$	**3.198**

The value of h_2 is taken from the computer program as it otherwise will be a double interpolation due to the value of P_2.

11.179 c

A refrigerator in a meat warehouse must keep a low temperature of -15°C and the outside temperature is 20°C. It uses R-12 as the refrigerant which must remove 5 kW from the cold space. Find the flow rate of the R-12 needed assuming a standard vapor compression refrigeration cycle with a condenser at 20°C.

Solution:

Basic refrigeration cycle: $T_1 = T_4 = -15°C$, $T_3 = 20°C$

Computer Tables: $h_4 = h_3 = 54.87$ kJ/kg; $h_1 = h_g = 180.97$ kJ/kg

$$\dot{Q}_L = \dot{m}_{R\text{-}12} \times q_L = \dot{m}_{R\text{-}12}(h_1 - h_4)$$

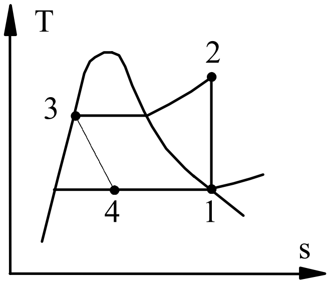
$$q_L = 180.97 - 54.87 = 126.1 \text{ kJ/kg}$$

$$\dot{m}_{R\text{-}12} = 5.0 / 126.1 = \mathbf{0.03965\ kg/s}$$

Ideal refrigeration cycle

$T_{cond} = 20^oC$

$T_{evap} = -15^oC = T_1$

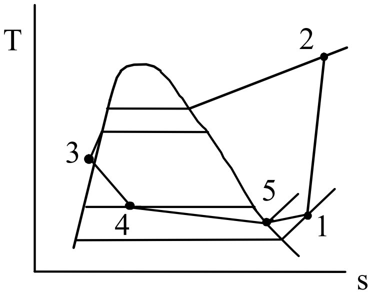


11.179 d

In an actual refrigeration cycle using R-12 as the working fluid, the refrigerant flow rate is 0.05 kg/s. Vapor enters the compressor at 150 kPa, –10°C, and leaves at 1.2 MPa, 75°C. The power input to the compressor is measured and found be 2.4 kW. The refrigerant enters the expansion valve at 1.15 MPa, 40°C, and leaves the evaporator at 175 kPa, –15°C. Determine the entropy generation in the compression process, the refrigeration capacity and the coefficient of performance for this cycle.

Solution:

Actual refrigeration cycle



1: compressor inlet $T_1 = -10^oC$, $P_1 = 150$ kPa

2: compressor exit $T_2 = 75^oC$, $P_2 = 1.2$ MPa

3: Expansion valve inlet $T_3 = 40^oC$
$P_3 = 1.15$ MPa

5: evaporator exit $T_5 = -15^oC$, $P_5 = 175$ kPa

Table B.3 $h_1 = 184.8$, $s_1 = 0.7324$, $h_2 = 226.7$, $s_2 = 0.741$

CV Compressor: $h_1 + q_{COMP} + w_{COMP} = h_2$; $s_1 + \int dq/T + s_{gen} = s_2$

$$w_{COMP} = \dot{W}_{COMP}/\dot{m} = 2.4/0.05 = 48.0 \text{ kJ/kg}$$

$$q_{COMP} = h_2 - w_{COMP} - h_1 = 226.7 - 48.0 - 184.8 = -6.1 \text{ kJ/kg}$$

$$s_{gen} = s_2 - s_1 - q / T_o = 0.741 - 0.7324 + 6.1/298.15 = \mathbf{0.029 \text{ kJ / kg K}}$$

C.V. Evaporator

$$q_L = h_5 - h_4 = 181.5 - 74.59 = 106.9 \text{ kJ/kg}$$

$$\Rightarrow \dot{Q}_L = \dot{m}q_L = 0.05 \times 106.9 = \mathbf{5.346 \text{ kW}}$$

COP: $\beta = q_L/w_{COMP} = 106.9/48.0 = \mathbf{2.23}$

11.180 a

Do Problem 11.21 with R-22 as the working fluid.

A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

Solution:

CV: Pump (use R-22 Computer Table)

$$w_P = h_2 - h_1 = \int_1^2 v dP \approx v_1(P_2-P_1) = 0.000884(4037 - 1534) = 2.21 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 94.27 + 2.21 = 96.48 \text{ kJ/kg}$$

CV: Boiler: $q_H = h_3 - h_2 = 253.69 - 96.48 = 157.21$ kJ/kg

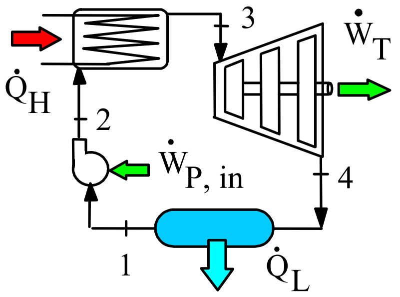
CV: Turbine

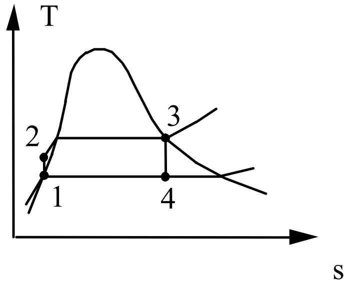
$$s_4 = s_3 = 0.7918 = 0.3417 + x_4 \times 0.5329, \quad => \quad x_4 = 0.8446$$

$$h_4 = 94.27 + 0.8446 \times 166.88 = 235.22 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 253.69 - 235.22 = 18.47 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (18.47 - 2.21)/157.21 = \mathbf{0.1034}$$

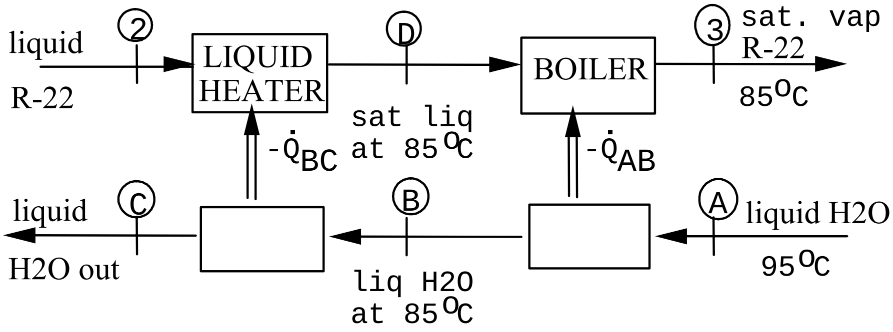




11.180 b

Do problem 11.24 with R-22 as the working fluid.

A flow with 2 kg/s of water is available at 95°C for the boiler. The restrictive factor is the boiling temperature of 85°C. Therefore, break the process up from 2-3 into two parts as shown in the diagram.



$-\dot{Q}_{AB} = \dot{m}_{H2O}(h_A - h_B) = 2(397.94 - 355.88) = 84.12$ kW

$= \dot{m}_{R\text{-}22}(253.69 - 165.09) \;\Rightarrow\; \dot{m}_{R\text{-}22} = 0.949$ kg/s

To verify that $T_D = T_3$ is the restrictive factor, find T_C.

$-\dot{Q}_{AC} = 0.949(165.09 - 96.48) = 65.11 = 2.0(355.88 - h_C)$

$h_C = 323.32$ kJ/kg $\Rightarrow$ $T_C = 77.2°C$ **OK**

State 1: 40°C, 1533.5 kPa, $v_1 = 0.000884\ m^3/kg$

CV Pump: $w_P = v_1(P_2 - P_1) = 0.000884(4036.8 - 1533.5) = 2.21$ kJ/kg

CV: Turbine

$s_4 = s_3 = 0.7918 = 0.3417 + x_4 \times 0.5329 \;\Rightarrow\; x_4 = 0.8446$

$h_4 = 94.27 + 0.8446 \times 166.88 = 235.22$ kJ/kg

Energy Eq.: $w_T = h_3 - h_4 = 253.69 - 235.22 = 18.47$ kJ/kg

Cycle: $w_{NET} = w_T - w_P = 18.47 - 2.21 = 16.26$ kJ/kg

$\dot{W}_{NET} = \dot{m}_{R22} w_{NET} = 0.949 \times 16.26 =$ **15.43 kW**

11.180 c

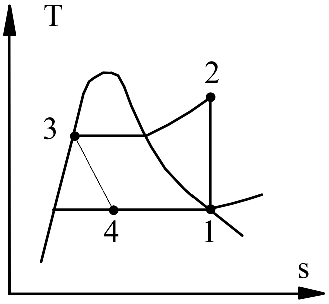
Consider an ideal refrigeration cycle that has a condenser temperature of 45°C and an evaporator temperature of −15°C. Determine the coefficient of performance of this refrigerator for the working fluid R-22.

Solution:

Ideal refrigeration cycle

$T_{cond} = 45^oC = T_3$

$T_{evap} = -15^oC = T_1$



	Property for:	**R-22**
Compressor	h_1, kJ/kg	244.13
	$s_2 = s_1$, kJ/kg K	0.9505
	P_2, MPa	1.729
	T_2, ^oC	74.4
	h_2, kJ/kg	289.26
	$w_C = h_2 - h_1$	45.13
Exp. valve	$h_3 = h_4$, kJ/kg	100.98
Evaporator	$q_L = h_1 - h_4$	143.15
	$\beta = q_L/w_C$	**3.172**

The value of h_2 is taken from the computer program as it otherwise will be a double interpolation due to the value of P_2.

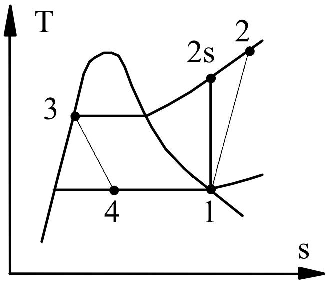
11.180 d

The refrigerant R-22 is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 10°C; its exit temperature from the compressor is measured and found to be 85°C. If the compressor exit is at 2 MPa what is the compressor isentropic efficiency and the cycle COP?

Solution:

R-22 heat pump:
Computer Table
State 1: $T_{EVAP} = 10^oC$, $x = 1$
$h_1 = 253.42$ kJ/kg, $s_1 = 0.9129$ kJ/kg K

State 2: T_2, P_2: $h_2 = 295.17$ kJ/kg



C.V. Compressor

Energy Eq.: $w_{C\,ac} = h_2 - h_1 = 295.17 – 253.42 = \mathbf{41.75\ kJ/kg}$

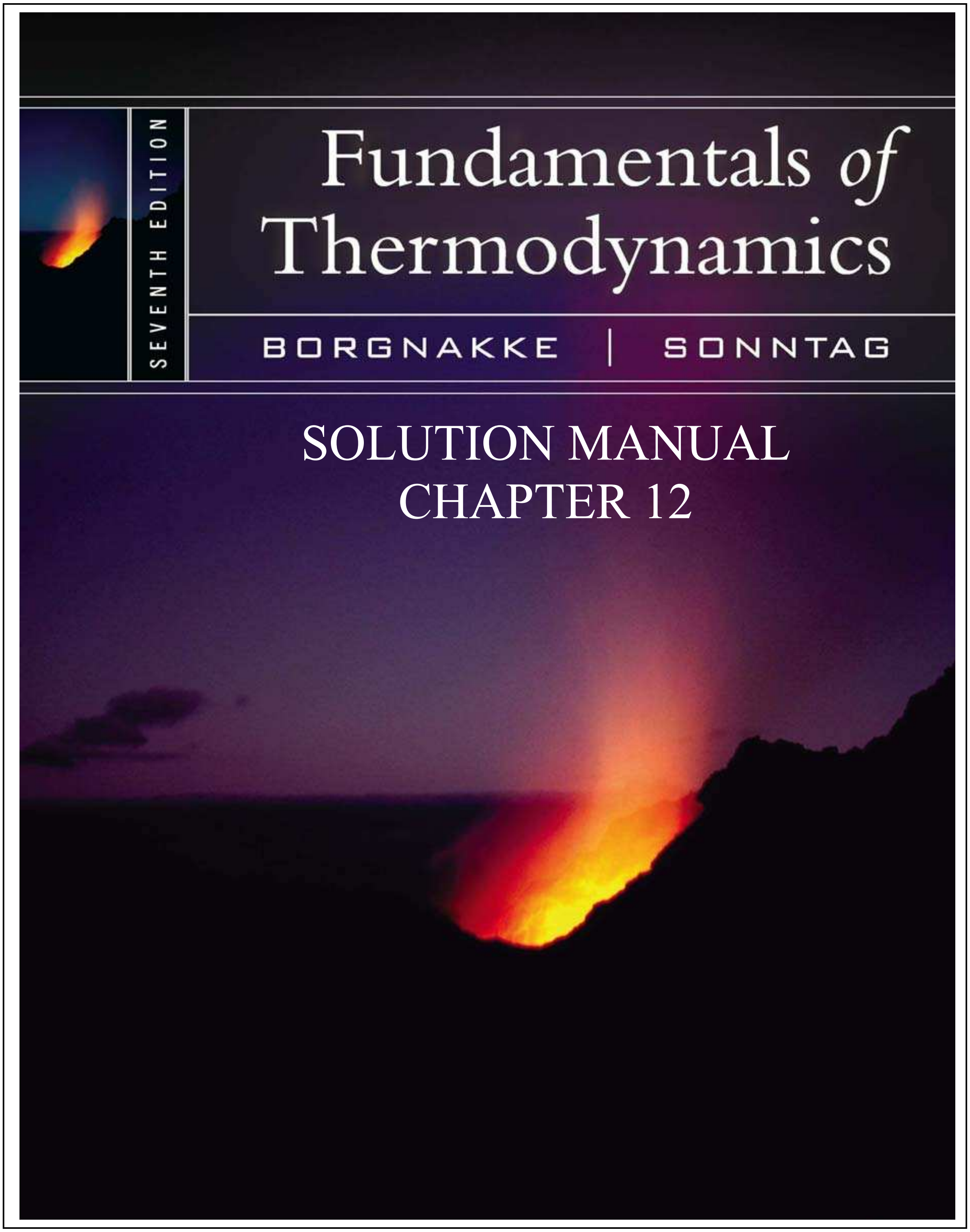
State 2s: 2 MPa , $s_{2S} = s_1 = 0.9129$ kJ/kg $T_{2S} = 69^oC$, $h_{2S} = 280.2$ kJ/kg

Efficiency: $$\eta = \frac{w_{C\,s}}{w_{C\,ac}} = \frac{h_{2S} - h_1}{h_2 - h_1} = \frac{280.2 - 253.42}{295.17 - 253.42} = \mathbf{0.6414}$$

C.V. Condenser

Energy Eq.: $q_H = h_2 - h_3 = 295.17 – 109.6 = 185.57$ kJ/kg

COP Heat pump: $$\beta = \frac{q_H}{w_{C\,ac}} = \frac{185.57}{41.75} = \mathbf{4.44}$$



SEVENTH EDITION

Fundamentals *of* Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 12

CONTENT

Problems re-solved with the Pr, vr functions from A.7.2:
32, 37, 74 see also 79, 80, 91, 101 and 127

In-Text Concept Questions

12.a

The Brayton cycle has the same 4 processes as the Rankine cycle, but the T-s and P-v diagrams look very different; why is that?

The Brayton cycle have all processes in the superheated vapor (close to ideal gas) region. The Rankine cycle crosses in over the two-phase region.

12.b

Is it always possible to add a regenerator to the Brayton cycle? What happens when the pressure ratio is increased?

No. When the pressure ratio is high, the temperature after compression is higher than the temperature after expansion. The exhaust flow can then not heat the flow into the combustor.

12.c

Why would you use an intercooler between compressor stages?

The cooler provides two effects. It reduces the specific volume and thus reduces the work in the following compressor stage. It also reduces the temperature into the combustor and thus lowers the peak temperature. This makes the control of the combustion process easier (no autoignition or uncontrollable flame spread), it reduces the formation of NOx that takes place at high temperatures and lowers the cooling requirements for the chamber walls.

12.d

The jet engine does not produce shaft work; how is power produced?

The turbine produces just enough shaft work to drive the compressor and it makes a little electric power for the aircraft. The power is produced as thrust of the engine. In order to exhaust the gases at high speed they must be accelerated so the high pressure in the turbine exit provides that force (high P relative to ambient). The high P into the turbine is made by the compressor, which pushes the flow backwards, and thus has a net resulting force forwards on the blades transmitted to the shaft and the aircraft. The outer housing also has a higher pressure inside that gives a net component in the forward direction.

12.e

How is the compression in the Otto cycle different from the Brayton cycle?

The compression in an Otto cycle is a volume reduction dictated by the piston motion. The physical handles are the volumes V_1 and V_2.

The compression in a Brayton cycle is the compressor pushing on the flow so it determines the pressure. The physical control is the pressure P_2 determined by how much torque you drive the shaft with.

12.f

How many parameters do you need to know to completely describe the Otto cycle? How about the Diesel cycle?

Otto cycle. State 1 (2 parameters) and the compression ratio CR and the energy release per unit mass in the combustion, a total of **4 parameters**. With that information you can draw the diagrams in Figure 11.28. Another way of looking at it is four states (8 properties) minus the four process equations ($s_2 = s_1$, $v_3 = v_2$, $s_4 = s_3$ and $v_4 = v_1$) gives 4 unknowns.

Diesel cycle. Same as for the Otto cycle namely **4 parameters**. The only difference is that one constant v process is changed to a constant P process.

12.g

The exhaust and inlet flow processes are not included in the Otto or Diesel cycles. How do these necessary processes affect the cycle performance?

Due to the pressure loss in the intake system and the dynamic flow process we will not have as much mass in the cylinder or as high a P as in a reversible process. The exhaust flow requires a slightly higher pressure to push the flow out through the catalytic converter and the muffler (higher back pressure) and the pressure loss in the valve so again there is a loss relative to a reversible process. Both of these processes subtract a pumping work from the net work out of the engine and a lower charge mass gives less power (not necessarily lower efficiency) than otherwise could be obtained.

Concept-Study Guide Problems

12.1

Is a Brayton cycle the same as a Carnot cycle? Name the four processes.

No. The Brayton cycle approximates a gas turbine.

1-2:	An isentropic compression (constant s)	Compressor
2-3:	An isobaric heating (constant P)	Combustor
3-4:	An isentropic expansion (constant s)	Turbine
4-1:	An isobaric cooling, heat rejection (constant P)	Heat exchanger

Comment: This cycle is the same as the Rankine cycle 4 processes, but it takes place in the ideal gas region of states. The last process does not exist in the real gas turbine which is an open cycle.

12.2

Why is the back work ratio in the Brayton cycle much higher than in the Rankine cycle?

Recall the expression for shaft work in a steady flow device

$$w = -\int v\, dP$$

The specific volume in the compressor is not so much smaller than the specific volume in the turbine of the Brayton cycle as it is in the pump (liquid) compared to turbine (superheated vapor) in the Rankine cycle.

12.3

For a given Brayton cycle the cold air approximation gave a formula for the efficiency. If we use the specific heats at the average temperature for each change in enthalpy will that give a higher or lower efficiency?

The specific heats are increasing functions of temperature. As the expression for the efficiency is from p.477

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{C_{P\,4\text{-}1}}{C_{P\,3\text{-}2}} \frac{T_4 - T_1}{T_3 - T_2}$$

The average T from 4 to 1 is lower than the average T from 2 to 3 and therefore the ratio of the two specific heats is lower than one yielding a **higher efficiency** than the cold air approximation gives.

12.4

Does the efficiency of a jet engine change with altitude since the density varies?

No, just like the standard Brayton cycle the simple model performance depends only on the compression ratio.

12.5

Why are the two turbines in Figures 12.7-8 not connected to the same shaft?

Such a configuration gives a little more flexibility in the control of the cycle under varying loads. The two turbines would then not have to run at the same speed for various power output levels.

12.6

Why is an air refrigeration cycle not common for a household refrigerator?

The capacity of the air cycle per mass flowing through the system is very small compared with the vapor compression cycle. The cycle also includes the expander which is one more piece of equipment that will add cost and maintenance requirements to the system.

12.7

Does the inlet state (P_1, T_1) have any influence on the Otto cycle efficiency? How about the power produced by a real car engine?

Very little. The efficiency for the ideal cycle only depends on compression ratio when we assume cold air properties. The u's are slightly non-linear in T so there will be a small effect.

In a real engine there are several effects. The inlet state determines the density and thus the total mass in the chamber. The more mass the more energy is released when the fuel burns, the peak P and T will also change which affects the heat transfer loss to the walls and the formation of NOx (sensitive to T). The combustion process may become uncontrollable if T is too high (knocking). Some increase in P_1 like that done by a turbo-charger or super-charger increases the power output and if high, it must be followed by an intercooler to reduce T_1. If P_1 is too high the losses starts to be more than the gain so there is an optimum level.

12.8

For a given compression ratio does an Otto cycle have higher or lower efficiency than a diesel cycle?

This is actually not clear from the formula's so we need to refer to the T-s diagram where we see that the average T at which the heat is added is higher for the Otto cycle than the diesel cycle for the same compression ratio. However since the diesel cycle runs with much higher compressions ratios than the Otto cycle most typical diesel cycles have higher efficiency than typical Otto cycles.

12.9

How many parameters do you need to know to completely describe the Atkinson cycle? How about the Miller cycle?

Four parameters for the Atkinson cycle. A total of 8 properties minus four known process equations give 4 unknowns.

The Miller cycle has one additional process so that requires one more parameter for a total of **five**. That is 10 properties with 5 known process equations leaving 5 unknowns.

12.10

Why would one consider a combined cycle system for a power plant? For a heat pump or refrigerator?

Dual cycle or combined cycle systems have the advantage of a smaller difference between the high and low ranges for P and T. The heat can be added at several different temperatures reducing the difference between the energy source T and the working substance T. The working substance vapor pressure at the desired T can be reduced from a high value by adding a topping cycle with a different substance or have a higher low pressure at very low temperatures.

12.11

Can the exhaust flow from a gas turbine be useful?

Usually the temperature in the exhaust flow is fairly high compared to ambient conditions so we could use the hot exhaust flow energy. It could be used directly for heating purposes in process applications or heating of buildings. As a topping cycle it can be used to heat/boil water for use in a Rankine cycle.

12.12

Where may a heat engine driven refrigerator be useful?

Any remote location where electricity is not available. Since a rotating shaft is available in a car engine the car A/C unit is driven by a belt engaged with a magnetic clutch so you do not have to have an electric motor.

12.13

Since any heat transfer is driven by a temperature difference, how does that affect all the real cycles relative to the ideal cycles?

Heat transfers are given as $\dot{Q} = CA\ \Delta T$ so to have a reasonable rate the area and the temperature difference must be large. The working substance then must have a different temperature than the ambient it exchanges energy with. This gives a smaller temperature difference for a heat engine with a lower efficiency as a result. The refrigerator or heat pump must have the working substance with a higher temperature difference than the reservoirs and thus a lower coefficient of performance (COP).

The smaller CA is, the larger ΔT must be for a certain magnitude of the heat transfer rate. This can be a design problem, think about the front end air intake grill for a modern car which is very small compared to a car 20 years ago.

Brayton Cycles, Gas Turbines

12.14

In a Brayton cycle the inlet is at 300 K, 100 kPa and the combustion adds 670 kJ/kg. The maximum temperature is 1200 K due to material considerations. Find the maximum permissible compression ratio and for that the cycle efficiency using cold air properties.

Solution:

Combustion: $h_3 = h_2 + q_H$; $\quad {}_2w_3 = 0$ and $T_{max} = T_3 = 1200$ K

$$T_2 = T_3 - q_H/C_P = 1200 - 670/1.004 = 532.7 \text{ K}$$

Reversible adiabatic compression leads to constant s, from Eq.8.28:

$$P_2 / P_1 = (T_2/T_1)^{\frac{k}{k-1}} = (532.7/300)^{3.5} = \mathbf{7.46}$$

Reversible adiabatic expansion leads to constant s, from Eq.8.28

$$T_4 = T_3 \, (P_4/P_3)^{\frac{k-1}{k}} = T_3 \, T_1/T_2 = 1200 \frac{300}{532.7} = 675.8 \text{ K}$$

For net work we get

$$w_T = C_P \, (T_3 - T_4) = 1.004(1200 - 675.8) = 526.3 \text{ kJ/kg}$$

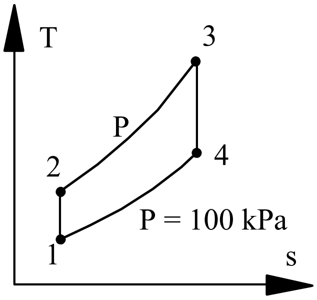
$$w_C = C_P \, (T_2 - T_1) = 1.004(532.7 - 300) = 233.6 \text{ kJ/kg}$$

$$w_{net} = w_T - w_C = 526.3 - 233.6 = 292.7 \text{ kJ/kg}$$

$$\eta = w_{net} / q_H = 292.7 / 670 = \mathbf{0.437}$$

12.15

A Brayton cycle has compression ratio of 15:1 with a high temperature of 1600 K and the inlet at 290 K, 100 kPa. Use cold air properties and find the specific heat addition and specific net work output.



Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Compression in compressor: $s_2 = s_1$

⇒ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$q_H = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 628.65) =$ **975.2 kJ/kg**

Do the overall cycle efficiency and the net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = 0.5387$$

$w_{NET} = \eta\ q_H = 0.5387 \times 975.2 =$ **525.34 kJ/kg**

12.16

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

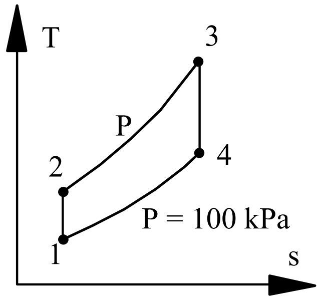
Solution:

Brayton cycle so this means:

Minimum T: $T_1 = 300$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 14$



Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Expansion in turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.8.32

$$T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

$$w_{NET} = 851.2 - 339.5 = 511.7 \text{ kJ/kg}$$

Do the overall net and cycle efficiency

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/511.7 = 195.4 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 = \mathbf{166.32\ MW}$$

$$w_C/w_T = 339.5/851.2 = \mathbf{0.399}$$

Energy input is from the combustor

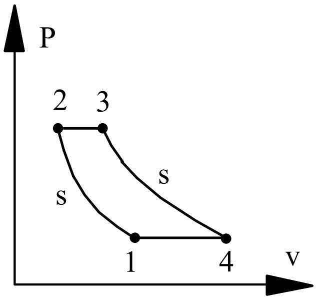
$$q_H = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 638.1) = 965.7 \text{ kJ/kg}$$

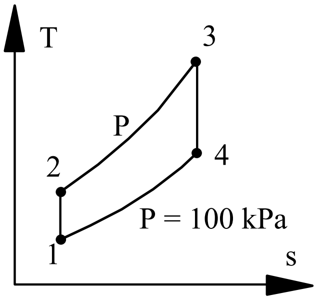
$$\eta_{TH} = w_{NET}/q_H = 511.7/965.7 = \mathbf{0.530}$$

12.17

Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:





Compression ratio $\frac{P_2}{P_1} = 12$

Max temperature $T_3 = 1100^oC$

$\dot{m} = 10$ kg/s

The compression is reversible and adiabatic so constant s. From Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2(12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.32

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2\left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\dot{W}_C = \dot{m}w_C = \mathbf{3048\ kW}, \qquad \dot{W}_T = \dot{m}w_T = \mathbf{7013\ kW}$$

Energy added by the combustion process

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1373.2 - 596.8) = 779.5 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/779.5 = \mathbf{0.509}$$

12.18

Repeat Problem 12.17, but assume variable specific heat for the air, table A.7. Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:

From A.7: $h_1 = 293.6$ kJ/kg, $s^o_{T1} = 6.84597$ kJ/kg K

The compression is reversible and adiabatic so constant s. From Eq.8.28

$$s_2 = s_1 \;\Rightarrow\; s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.84597 + 0.287\ln 12 = 7.55914$$

$$\Rightarrow\; T_2 = 590 \text{ K},\quad h_2 = 597.2 \text{ kJ/kg}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = h_2 - h_1 = 597.2 - 293.6 = 303.6 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.28

From A.7: $h_3 = 1483.1$ kJ/kg, $s^o_{T3} = 8.50554$ kJ/kg K

$$s_4 = s_3 \;\Rightarrow\; s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.50554 + 0.287\ln(1/12) = 7.79237$$

$$\Rightarrow\; T_4 = 734.8 \text{ K},\quad h_4 = 751.1 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1483.1 - 751.1 = 732 \text{ kJ/kg}$$

Scale the work with the mass flow rate

$$\Rightarrow\; \dot{W}_C = \dot{m}w_C = \mathbf{3036\ kW},\qquad \dot{W}_T = \dot{m}w_T = \mathbf{7320\ kW}$$

Energy added by the combustion process

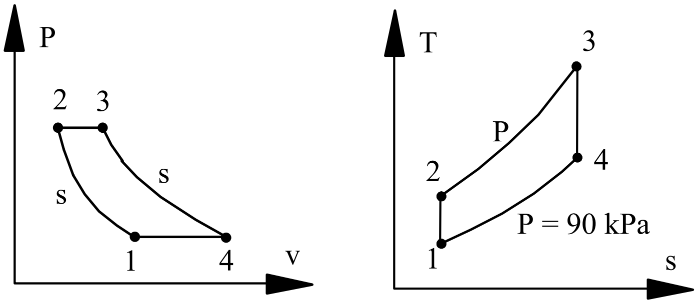
$$q_H = h_3 - h_2 = 1483.1 - 597.2 = 885.9 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_C = 732 - 303.6 = 428.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 428.4/885.9 = \mathbf{0.484}$$

12.19

A Brayton cycle has inlet at 290 K, 90 kPa and the combustion adds 1000 kJ/kg. How high can the compression ratio be so the highest temperature is below 1700 K? Use cold air properties to solve.



Compression ratio
P_2 / P_1
Max temperature
$T_3 = 1700$ K

Combustion adds
$_2q_3 = h_3 - h_2$

Let us work back from state 3 to 2 as

$$_2q_3 = h_3 - h_2 = C_{P0}(T_3 - T_2) \quad \Rightarrow$$

$$T_2 = T_3 - {}_2q_3/ C_{P0} = 1700 - 1000/1.004 = 704 \text{ K}$$

The compression is reversible and adiabatic so constant s. From Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \Rightarrow \qquad \left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{704}{290}\right)^{3.5} = \mathbf{22.29}$$

12.20

A Brayton cycle produces net 50 MW with an inlet state of 17°C, 100 kPa and the pressure ratio is 14:1. The highest cycle temperature is 1600 K. Find the thermal efficiency of the cycle and the mass flow rate of air using cold air properties.

Inlet state is state 1, $P_2/P_1 = 14$ and $T_3 = 1600$ K.

Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 290 \times (14)^{0.2857} = 616.38 \text{ K}$$

$$\eta_{CYCLE} = w_{NET}/q_H = 1 - T_1/T_2 = 1 - \frac{290}{616.38} = \mathbf{0.5295}$$

Combustion: constant pressure

$$q_H = h_3 - h_2 = C_{P0}\,(T_3 - T_2) = 1.004(1600 - 616.38) = 987.56 \text{ K}$$

$$w_{NET} = \eta_{CYCLE} q_H = 0.5295 \times 987.55 = 522.9 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}/\,w_{NET} = 50\,000 \text{ kW} / 522.9 \text{ kJ/kg} = \mathbf{95.62\ kg/s}$$

Expansion: isentropic

$$T_4 = T_3 / (P_3/P_4)^{(k-1)/k} = 1600/14^{0.2857} \mathbf{= 752.8\ K\ \ > T_2}$$

Since $T_4 > T_2$ some regeneration can be done. We could find (not needed)

$$\eta_{CYCLE} = 1 - \frac{T_1}{T_3}\,(P_2/P_1)^{(k-1)/k} = 1 - T_2 / T_3 = 0.615$$

12.21

A Brayton cycle produces 14 MW with an inlet state of 17°C, 100 kPa, and a compression ratio of 16:1. The heat added in the combustion is 960 kJ/kg. What are the highest temperature and the mass flow rate of air, assuming cold air properties?

Solution:

Temperature after compression is

$$T_2 = T_1\, r_p^{(k-1)/k} = 290 \times 16^{0.4/1.4} = 640.35 \text{ K}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 640.35 + \frac{960}{1.004} = \mathbf{1596.5\ K}$$

Efficiency is from Eq.12.1

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

From the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\ 000}{0.547} = 25\ 594 \text{ kW}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\ 594 \text{ kW}/\ 960 \text{ kJ/kg} = \mathbf{26.66\ kg/s}$$

12.22

Do Problem 12.21 with properties from table A.7.1 instead of cold air properties.

Solution:

With the variable specific heat we must go through the processes one by one to get net work and the highest temperature T_3.

From A.7.1: $h_1 = 290.43$ kJ/kg, $s^o_{T1} = 6.83521$ kJ/kg K

The compression is reversible and adiabatic so constant s. From Eq.8.28

$$s_2 = s_1 \;\Rightarrow\; s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.83521 + 0.287 \ln 16 = 7.63094$$

$$\Rightarrow\; T_2 = 631.9 \text{ K},\quad h_2 = 641 \text{ kJ/kg}$$

Energy equation with compressor work in

$$w_C = -{}_1w_2 = h_2 - h_1 = 641 - 290.43 = 350.57 \text{ kJ/kg}$$

Energy Eq. combustor: $h_3 = h_2 + q_H = 641 + 960 = 1601$ kJ/kg

State 3: (P, h): $\mathbf{T_3 = 1471\ K}$, $s^o_{T3} = 8.58811$ kJ/kg K

The expansion is reversible and adiabatic so constant s. From Eq.8.28

$$s_4 = s_3 \;\Rightarrow\; s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.58811 + 0.287\ln(1/16) = 7.79238$$

$$\Rightarrow\; T_4 = 734.8 \text{ K},\quad h_4 = 751.11 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1601 - 751.11 = 849.89 \text{ kJ/kg}$$

Now the net work is

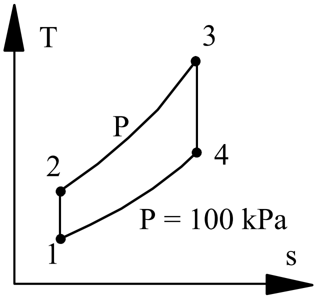
$$w_{net} = w_T - w_C = 849.89 - 350.57 = 499.32 \text{ kJ/kg}$$

The total required power requires a mass flow rate as

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{14\,000}{499.32} \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{28.04\ kg/s}$$

12.23

Solve Problem 12.15 using the air tables A.7 instead of cold air properties.



Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Compression in compressor: $s_2 = s_1$

⇒ Implemented in Eq.8.28

$$s_2 = s_1 \;\Rightarrow\; s^o_{T2} = s^o_{T1} + R\ln(P_2/P_1) = 6.83521 + 0.287 \ln 15 = 7.61242$$

$$\Rightarrow T_2 = 620.9 \text{ K}, \quad h_2 = 629.34 \text{ kJ/kg}$$

$$w_C = h_2 - h_1 = 629.34 - 290.43 = 338.91 \text{ kJ/kg}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = 1757.33 - 629.34 = \mathbf{1127.99\ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.28

$$s_4 = s_3 \;\Rightarrow\; s^o_{T4} = s^o_{T3} + R\ln(P_4/P_3) = 8.69051 + 0.287 \ln(1/15) = 7.9133$$

$$\Rightarrow T_4 = 821.0 \text{ K}, \quad h_4 = 845.42 \text{ kJ/kg}$$

Energy equation with turbine work out

$$w_T = h_3 - h_4 = 1757.33 - 845.42 = 911.91 \text{ kJ/kg}$$

Now the net work is

$$w_{net} = w_T - w_C = 911.91 - 338.91 = \mathbf{573\ kJ/kg}$$

[Comment: Cycle efficiency $\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = \frac{573}{1127.99} = 0.508$]

12.24

Solve Problem 12.14 with variable specific heats using Table A.7.

A Brayton cycle inlet is at 300 K, 100 kPa and the combustion adds 670 kJ/kg. The maximum temperature is 1200 K due to material considerations. What is the maximum allowed compression ratio? For this compression ratio calculate the net work and cycle efficiency using cold air properties.

Solution:

Combustion: $h_3 = h_2 + q_H$; ${}_2w_3 = 0$ and $T_{max} = T_3 = 1200$ K

$$h_2 = h_3 - q_H = 1277.8 - 670 = 607.8 \text{ kJ/kg}$$

From Table A.7.1

$$T_2 \approx 600 \text{ K}; \quad s^o_{T2} = 7.57638 \text{ ; } T_1 = 300 \text{ K}; \quad s^o_{T1} = 6.86926 \text{ kJ/kg K}$$

Reversible adiabatic compression leads to constant s, from Eq.8.28:

$$P_2 / P_1 = \exp[\ (s^o_{T2} - s^o_{T1})/R\ ] = \exp(2.4638) = \mathbf{11.75}$$

Reversible adiabatic expansion leads to constant s, from Eq.8.28

$$s^o_{T4} = s^o_{T3} + R \ln(P_4 / P_3) = 8.34596 + 0.287 \ln(1 / 11.75) = 7.6388 \text{ kJ/kgK}$$

From Table A.7.1 by linear interpolation $T_4 \approx 636.6$ K, $h_4 = 645.97$ kJ/kg

$$w_T = h_3 - h_4 = 1277.8 - 645.97 = 631.8 \text{ kJ/kg}$$

$$w_C = h_2 - h_1 = 607.8 - 300.47 = 307.3 \text{ kJ/kg}$$

$$w_{net} = w_T - w_C = 631.8 - 307.3 = \mathbf{324.5\ kJ/kg}$$

$$\eta = w_{net} / q_H = 324.5 / 670 = \mathbf{0.484}$$

Regenerators, Intercoolers, and Non-ideal Cycles

12.25

Would it be better to add an ideal regenerator to the Brayton cycle in problem 12.20?

Inlet state is state 1, $P_2/P_1 = 14$ and $T_3 = 1600$ K.

Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 290 \times (14)^{0.2857} = 616.38 \text{ K}$$

Expansion: isentropic

$$T_4 = T_3 / (P_3/P_4)^{(k-1)/k} = 1600/14^{0.2857} \mathbf{= 752.8 \text{ K} \;\; > T_2}$$

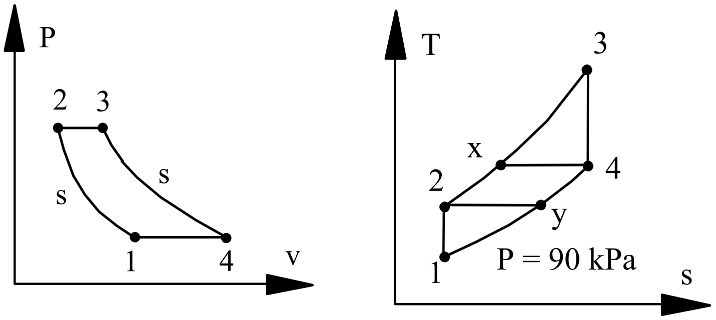
Since $T_4 > T_2$ some regeneration can be done.

We could find (not needed)

$$\eta_{CYCLE} = 1 - \frac{T_1}{T_3}(P_2/P_1)^{(k-1)/k} = 1 - T_2 / T_3 = 0.615$$

12.26

A Brayton cycle with an ideal regenerator has inlet at 290 K, 90 kPa with the highest P, T as 1170 kPa, 1700 K. Find the specific heat transfer and the cycle efficiency using cold air properties.



Compression ratio
P_2 / P_1
Max temperature
$T_3 = 1700$ K
Combustion adds
$q_H = h_3 - h_x$

Let us work from state 1 to 2 as

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \times (1170/90)^{0.2857} = 603.5 \text{ K}$$

Energy equation with compressor work in

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004(603.5 - 290) = 313.5 \text{ kJ/kg} = q_L$$

The expansion is reversible and adiabatic so constant s. From Eq.8.32

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1700\left(\frac{90}{1170}\right)^{0.2857} = \mathbf{816.9\ K} > T_2$$

Since the exhaust $T_4 > T_2$ a regenerator can be used. Now the heat transfer added is

$$q_H = h_3 - h_x = C_{P0}(T_3 - T_x) = 1.004(1700 - 816.9) = \mathbf{886.6\ kJ/kg} = w_T$$

$$\eta_{TH} = w_{NET}/q_H = (886.6 - 313.5)/\ 886.6 = \mathbf{0.646}$$

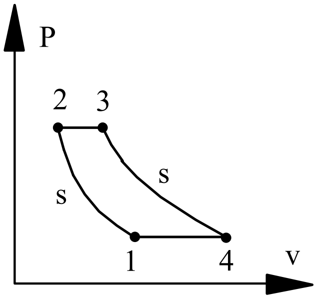
We could also have found efficiency as

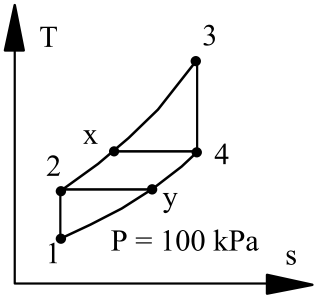
$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{T_2}{T_3} = 1 - \frac{603.5}{1700} = 0.645$$

12.27

An ideal regenerator is incorporated into the ideal air-standard Brayton cycle of Problem 12.17. Find the thermal efficiency of the cycle with this modification. Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20°C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100°C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Solution:





Compression ratio

$$\frac{P_2}{P_1} = 12$$

Max temperature

$T_3 = 1100^oC$

$\dot{m} = 10$ kg/s

The compression is reversible and adiabatic so constant s. From Eq.8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.2(12)^{0.286} = 596.8 \text{ K}$$

Energy equation with compressor work in

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004(596.8 - 293.2) = 304.8 \text{ kJ/kg}$$

The expansion is reversible and adiabatic so constant s. From Eq.8.32

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373.2\left(\frac{1}{12}\right)^{0.286} = 674.7 \text{ K}$$

Energy equation with turbine work out

$$w_T = C_{P0}(T_3 - T_4) = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg}$$

Ideal regenerator: $T_X = T_4 = 674.7$ K

$$q_H = h_3 - h_X = 1.004(1373.2 - 674.7) = 701.3 \text{ kJ/kg} = w_T$$

$$\eta_{TH} = w_{NET}/q_H = (701.3 - 304.8)/701.3 = \mathbf{0.565}$$

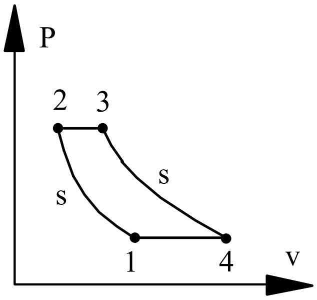
We could also have found efficiency as

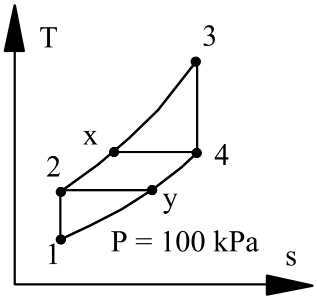
$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{T_2}{T_3}$$

12.28

Consider an ideal gas-turbine cycle with a pressure ratio across the compressor of 12 to 1. The compressor inlet is at 300 K and 100 kPa, and the cycle has a maximum temperature of 1600 K. An ideal regenerator is also incorporated into the cycle. Find the thermal efficiency of the cycle using cold air (298 K) properties.

Solution:





Compression ratio

$$\frac{P_2}{P_1} = 12$$

Max temperature

$T_3 = 1600$ K

The efficiency with an ideal regenerator ($T_4 = T_x$) is from page 485

$$\eta_{TH} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \frac{300}{1600} \times 12^{0.2857} = \mathbf{0.619}$$

12.29

A two-stage air compressor has an intercooler between the two stages as shown in Fig. P12.29. The inlet state is 100 kPa, 290 K, and the final exit pressure is 1.6 MPa. Assume that the constant pressure intercooler cools the air to the inlet temperature, $T_3 = T_1$. It can be shown that the optimal pressure, $P_2 = (P_1P_4)^{1/2}$, for minimum total compressor work. Find the specific compressor works and the intercooler heat transfer for the optimal P_2.

Solution:

Optimal intercooler pressure $P_2 = \sqrt{100 \times 1600} = 400$ kPa

1: $h_1 = 290.43$ kJ/kg, $s^o_{T1} = 6.83521$ kJ/kg K

C.V. C1: $w_{C1} = h_2 - h_1$, $s_2 = s_1$ leading to Eq.8.28

$\Rightarrow$ $s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 4 = 7.2331$ kJ/kg K

$\Rightarrow$ $T_2 = 430.3$ K, $h_2 = 432.05$ kJ/kg

$w_{C1} = 432.05 - 290.43 =$ **141.6 kJ/kg**

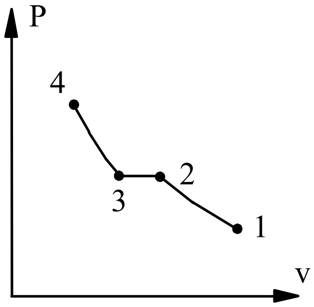
C.V. Cooler: $T_3 = T_1$ $\Rightarrow$ $h_3 = h_1$

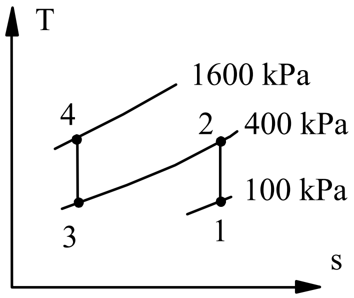
$q_{OUT} = h_2 - h_3 = h_2 - h_1 = w_{C1} =$ **141.6 kJ/kg**

C.V. C2: $T_3 = T_1$, $s_4 = s_3$ and since $s^o_{T3} = s^o_{T1}$, $P_4/P_3 = P_2/P_1$

$\Rightarrow$ $s^o_{T4} = s^o_{T3} + R \ln(P_4/P_3) = s^o_{T2}$, so we have $T_4 = T_2$

Thus we get $w_{C2} = w_{C1} =$ **141.6 kJ/kg**





12.30

Assume the compressor in Problem 12.21 has an intercooler that cools the air to 330 K operating at 500 kPa followed by a second stage of compression to 1600 kPa. Find the specific heat transfer in the intercooler and the total combined work required.

Solution:

C.V. Stage 1: 1 => 2

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_2 = T_1\ (P_2/P_1)^{(k-1)/k} = 290\ (500/100)^{\ 0.2857} = 459.3 \text{ K}$$

$$w_{c1in} = C_P(\ T_2 - T_1) = 1.004(459.3 - 290) = 187.0 \text{ kJ/kg}$$

CV. Intercooler 2 => 3

Constant pressure cooling

$$q_{out} = h_2 - h_3 = C_P\ (T_2 - T_3) = 1.004\ (459.3 - 330) = \mathbf{129.8\ kJ/kg}$$

C.V. Stage 2: 3 => 4

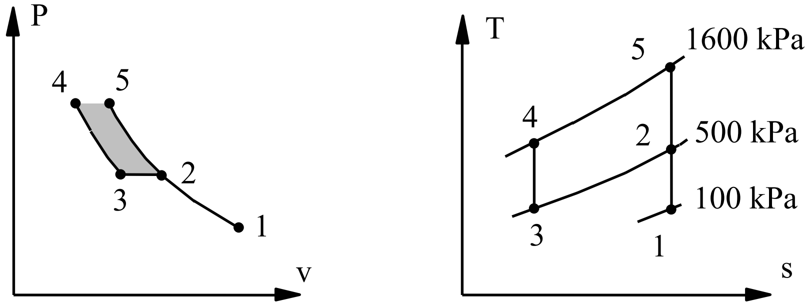
Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_4 = T_3\ (P_4/P_3)^{(k-1)/k} = 330\ (1600/500)^{\ 0.2857} = 460.1 \text{ K}$$

$$w_{c2in} = C_P(\ T_4 - T_3) = 1.004(460.1 - 330) = 130.6 \text{ kJ/kg}$$

$$w_{tot} = w_{c1} + w_{c2} = 187 + 130.6 = \mathbf{318\ kJ/kg}$$

The intercooler reduces the work for stage 2 as T is lower and so is specific volume. The reduction in work due to the intercooler is shaded in the P-v diagram.



12.31

The gas-turbine cycle shown in Fig. P12.31 is used as an automotive engine. In the first turbine, the gas expands to pressure P_5, just low enough for this turbine to drive the compressor. The gas is then expanded through the second turbine connected to the drive wheels. The data for the engine are shown in the figure and assume that all processes are ideal. Determine the intermediate pressure P_5, the net specific work output of the engine, and the mass flow rate through the engine. Find also the air temperature entering the burner T_3, and the thermal efficiency of the engine.

a) Consider the compressor

$$s_2 = s_1 \;\Rightarrow\; T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8\text{ K}$$

$$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6\text{ kJ/kg}$$

Consider then the first turbine work

$$w_{T1} = -w_C = 201.6 = C_{P0}(T_4 - T_5) = 1.004(1600 - T_5)$$

$$\Rightarrow\; T_5 = 1399.2\text{ K}$$

$$s_5 = s_4 \;\Rightarrow\; P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = 600\left(\frac{1399.2}{1600}\right)^{3.5} = \mathbf{375\ kPa}$$

b) $$s_6 = s_5 \;\Rightarrow\; T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\left(\frac{100}{375}\right)^{0.286} = 958.8\text{ K}$$

The second turbine gives the net work out

$$w_{T2} = C_{P0}(T_5 - T_6) = 1.004(1399.2 - 958.8) = 442.2\text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{T2} = 150/442.2 = \mathbf{0.339\ kg/s}$$

c) Ideal regenerator $\Rightarrow$ $T_3 = T_6 = \mathbf{958.8\ K}$

$$q_H = C_{P0}(T_4 - T_3) = 1.004(1600 - 958.8) = 643.8\text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 442.2/643.8 = \mathbf{0.687}$$

12.32

Repeat Problem 12.29 when the intercooler brings the air to $T_3 = 320$ K. The corrected formula for the optimal pressure is $P_2 = [\ P_1P_4\ (T_3/T_1)^{n/(n-1)}]^{1/2}$ see Problem 9.241, where *n* is the exponent in the assumed polytropic process.

Solution:

The polytropic process has n = k (isentropic) so $n/(n - 1) = 1.4/0.4 = 3.5$

$$P_2 = 400\sqrt{(320/290)^{3.5}} = 475.2 \text{ kPa}$$

C.V. C1: $s_2 = s_1 \Rightarrow T_2 = T_1\ (P_2/P_1)^{\frac{k-1}{k}} = 290\ (475.2/100)^{0.2857} = 452.67$ K

$$w_{C1} = h_2 - h_1 = C_p(T_2 - T_1) = 1.004(452.67 - 290) = \mathbf{163.3\ kJ/kg}$$

C.V. Cooler: $q_{OUT} = h_2 - h_3 = 1.004(452.67 - 320) = \mathbf{133.2\ kJ/kg}$

C.V. C2: $s_4 = s_3 \Rightarrow T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 320\ (1600/475.2)^{0.2857} = 452.67$ K

$$w_{C2} = h_4 - h_3 = C_p(T_2 - T_1) = 1.004(452.67 - 320) = \mathbf{133.2\ kJ/kg}$$

12.33

Repeat Problem 12.16, but include a regenerator with 75% efficiency in the cycle. A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Both compressor and turbine are reversible and adiabatic so constant s, Eq.8.32 relates then T to P assuming constant heat capacity.

Compressor: $\Rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1$ K

$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5$ kJ/kg

Turbine $s_4 = s_3 \Rightarrow T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2$ K

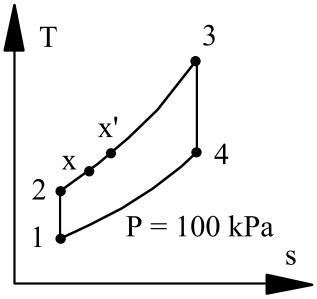
$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2$ kJ/kg

$w_{NET} = 851.2 - 339.5 = 511.7$ kJ/kg

$\dot{m} = \dot{W}_{NET}/w_{NET} = 100\ 000/511.7 = 195.4$ kg/s

$\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 =$ **166.32 MW**

$w_C/w_T = 339.5/851.2 =$ **0.399**



For the regenerator

$$\eta_{REG} = 0.75 = \frac{h_X - h_2}{h_{X'} - h_2} = \frac{T_X - T_2}{T_4 - T_2} = \frac{T_X - 638.1}{752.2 - 638.1}$$

$\Rightarrow T_X = 723.7$ K

Turbine and compressor work not affected by regenerator.

Combustor needs to add less energy with the regenerator as

$q_H = C_{P0}(T_3 - T_X) = 1.004(1600 - 723.7) = 879.8$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 511.7/879.8 =$ **0.582**

12.34

An air compressor has inlet of 100 kPa, 290 K and brings it to 500 kPa after which the air is cooled in an intercooler to 340 K by heat transfer to the ambient 290 K. Assume this first compressor stage has an isentropic efficiency of 85% and it is adiabatic. Using constant specific heat and find the compressor exit temperature and the specific entropy generation in the process.

C.V.: Stage 1 air, Steady flow

Isentropic compressor is done first.

Process: adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.6.13: $-w_{C1} = h_2 - h_1$, Entropy Eq.9.8: $s_2 = s_1$

Assume constant $C_{P0} = 1.004$ from A.5 and isentropic leads to Eq.8.32

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(500/100)^{0.286} = 459.3 \text{ K}$$

$$w_{C1s} = h_1 - h_{2s} = C_{Po}(T_1 - T_2) = 1.004(290 - 459.3) = -170 \text{ kJ/kg}$$

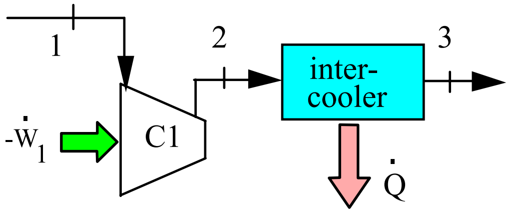
Now the **actual compressor** work becomes

$$w_{C1\ ac} = w_{C1s}/\eta_{C1s} = -170/0.85 = -200 \text{ kJ/kg} = h_1 - h_{2ac} = C_{Po}(T_1 - T_{2ac})$$

$$T_{2ac} = T_1 - w_{C1\ ac}/C_{Po} = 290 + 200/1.004 = \mathbf{489.2\ K}$$

$$s_{gen} = s_{2ac} - s_1 = C_{Po} \ln(\frac{T_{2ac}}{T_1}) - R\ln(\frac{P_2}{P_1})$$

$$= 1.004 \ln(489.2/290) - 0.287(500/100) = \mathbf{0.0631\ kJ/kg\text{-}K}$$



12.35

A two-stage compressor in a gas turbine brings atmospheric air at 100 kPa, 17°C to 500 kPa, then cools it in an intercooler to 27°C at constant P. The second stage brings the air to 1000 kPa. Assume both stages are adiabatic and reversible. Find the combined specific work to the compressor stages. Compare that to the specific work for the case of no intercooler (i.e. one compressor from 100 to 1000 kPa).

Solution:

C.V. Stage 1: 1 => 2

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_2 = T_1 (P_2/P_1)^{(k-1)/k} = 290 (500/100)^{0.2857} = 459.3 \text{ K}$$

$$w_{c1in} = C_P(T_2 - T_1) = 1.004(459.3 – 290) = 187.0 \text{ kJ/kg}$$

C.V. Stage 2: 3 => 4

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_4 = T_3 (P_4/P_3)^{(k-1)/k} = 300 (1000/500)^{0.2857} = 365.7 \text{ K}$$

$$w_{c2in} = C_P(T_4 - T_3) = 1.004(365.7 – 300) = 65.96 \text{ kJ/kg}$$

$$w_{tot} = w_{c1} + w_{c2} = 187 + 65.96 = \mathbf{253\ kJ/kg}$$

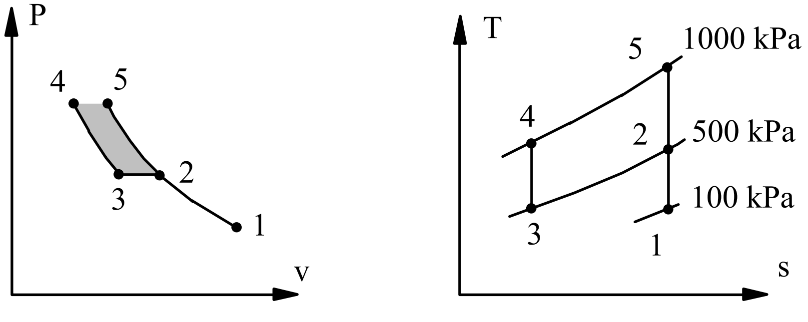
The intercooler reduces the work for stage 2 as T is lower and so is specific volume.

C.V. One compressor 1 => 5

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_5 = T_1 (P_5/P_1)^{(k-1)/k} = 290 (1000/100)^{0.2857} = 559.88 \text{ K}$$

$$w_{in} = C_P(T_5 - T_1) = 1.004(559.88 –290) = \mathbf{271\ kJ/kg}$$

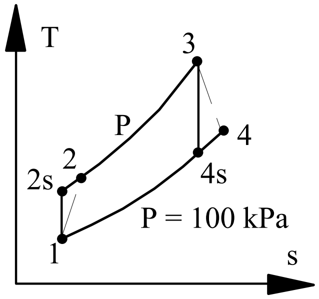


The reduction in work due to the intercooler is shaded in the P-v diagram.

12.36

Repeat Problem 12.16, but assume that the compressor has an isentropic efficiency of 85% and the turbine an isentropic efficiency of 88%.

Solution:



Brayton cycle so this means:

Minimum T: $T_1 = 300$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}

Ideal compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_{Cs} = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Actual compressor

$$\Rightarrow\ w_C = w_{SC}/\eta_{SC} = 339.5/0.85 = 399.4 \text{ kJ/kg} = C_{P0}(T_2\text{-}T_1)$$

$$\Rightarrow\ T_2 = T_1 + w_c/C_{P0} = 300 + 399.4/1.004 = 697.8 \text{ K}$$

Ideal turbine: $s_4 = s_3 \Rightarrow$ Implemented in Eq.8.32

$$T_{4s} = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$w_{Ts} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

Actual turbine

$$\Rightarrow w_T = \eta_{ST}\ \ w_{ST} = 0.88 \times 851.2 = 749.1 \text{ kJ/kg} = C_{P0}(T_3\text{-}T_4)$$

$$\Rightarrow T_4 = T_3 - w_T/C_{P0} = 1600 - 749.1/1.004 = 853.9 \text{ K}$$

Do the overall net and cycle efficiency

$$w_{NET} = w_T - w_C = 749.1 - 399.4 = 349.7 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/349.7 = 286.0 \text{ kg/s}$$

$$\dot{W}_T = \dot{m}w_T = 286.0{\times}749.1 = \mathbf{214.2\ MW}$$

$$w_C/w_T = 399.4/749.1 = \mathbf{0.533}$$

Energy input is from the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1600 - 697.8) = 905.8 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 349.7/905.8 = \mathbf{0.386}$$

12.37

A gas turbine with air as the working fluid has two ideal turbine sections, as shown in Fig. P12.37, the first of which drives the ideal compressor, with the second producing the power output. The compressor input is at 290 K, 100 kPa, and the exit is at 450 kPa. A fraction of flow, *x*, bypasses the burner and the rest (1 – *x*) goes through the burner where 1200 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine, with exhaust at 100 kPa. If the mixing should result in a temperature of 1000 K into the first turbine find the fraction *x*. Find the required pressure and temperature into the second turbine and its specific power output.

C.V.Comp.: $-w_C = h_2 - h_1;\ \ s_2 = s_1$

Reversible and adiabatic gives constant s which from Eq.8.32 gives:

$$T_2 = T_1\,(P_2/P_1)^{(k-1)/k} = 290\,(450/100)^{\,0.2857} = 445.7\text{ K}$$

$$h_2 = 447.75\text{ kJ/kg},\quad -w_C = 447.75 - 290.43 = 157.3\text{ kJ/kg}$$

C.V.Burner: $h_3 = h_2 + q_H = 447.75 + 1200 = 1647.75\text{ kJ/kg}$

$$\Rightarrow\ T_3 = 1510\text{ K}$$

C.V.Mixing chamber: $(1 - x)h_3 + xh_2 = h_{MIX} = 1046.22\text{ kJ/kg}$

$$x = \frac{h_3 - h_{MIX}}{h_3 - h_2} = \frac{1647.75 - 1046.22}{1647.75 - 447.75} = \mathbf{0.5013}$$

$$\dot W_{T1} = \dot W_{C,in}\ \Rightarrow\ \dot w_{T1} = -w_C = 157.3 = h_3 - h_4$$

$$h_4 = 1046.22 - 157.3 = 888.9\text{ kJ/kg}\ \Rightarrow\ T_4 = \mathbf{860\ K}$$

$$P_4 = P_{MIX}(T_4/T_{MIX})^{k/(k-1)} = 450 \times (860/1000)^{3.5} = \mathbf{265\ kPa}$$

$$s_4 = s_5\ \Rightarrow\ T_5 = T_4\,(P_5/P_4)^{(k-1)/k} = 860\,(100/265)^{0.2857} = 651\text{ K}$$

$$h_5 = 661.2\text{ kJ/kg}$$

$$w_{T2} = h_4 - h_5 = 888.9 - 661.2 = \mathbf{227.7\ kJ/kg}$$

12.38

A gas turbine cycle has two stages of compression, with an intercooler between the stages. Air enters the first stage at 100 kPa, 300 K. The pressure ratio across each compressor stage is 5 to 1, and each stage has an isentropic efficiency of 82%. Air exits the intercooler at 330 K. Calculate the temperature at the exit of each compressor stage and the total specific work required.

Solution:

State 1: $P_1 = 100$ kPa, $T_1 = 300$ K

State 3: $T_3 = 330$ K

$P_2 = 5\ P_1 = 500$ kPa; $\quad P_4 = 5\ P_3 = 2500$ kPa

Energy Eq.: $w_{c1} + h_1 = h_2 \quad \Rightarrow \quad w_{c1} = h_2 - h_1 \approx C_P(T_2 - T_1)$

Ideal C1 constant s, Eq.8.32: $\quad T_{2s} = T_1\ (P_2/P_1)^{(k-1)/k} = 475.4$ K

$w_{c1\ s} = h_{2s} - h_1 \approx C_P(T_{2s} - T_1) = 176.0$ kJ/kg,

Actual Eq.9.28: $\quad w_{c1} = w_{c1\ s}/\eta = 176/0.82 = 214.6$ kJ/kg

$T_2 = T_1 + w_{c1}/C_P =$ **513.7 K**

Ideal C2 constant s, Eq.8.32: $\quad T_{4s} = T_3\ (P_4/P_3)^{(k-1)/k} = 552.6$ K

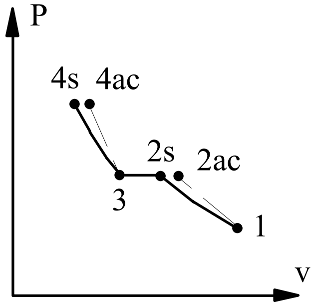
$w_{c2\ s} = h_{4s} - h_3 \approx C_P(T_{4s} - T_3\ ) = 193.4$ kJ/kg;

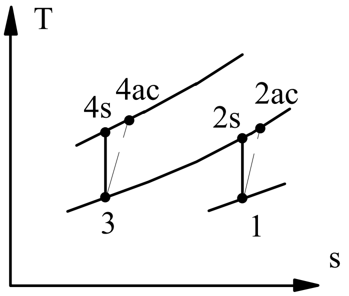
Actual Eq.9.28: $\quad w_{c2} = w_{c2\ s}/\eta = 235.9$ kJ/kg

$T_4 = T_3 + w_{c2}\ /\ C_P =$ **565 K**

Total work in:

$w = w_{c1} + w_{c2} = 214.6 + 235.9 =$ **450.5 kJ/kg**





12.39

Repeat the questions in Problem 12.31 when we assume that friction causes pressure drops in the burner and on both sides of the regenerator. In each case, the pressure drop is estimated to be 2% of the inlet pressure to that component of the system, so P_3 = 588 kPa, $P_4 = 0.98\ P_3$ and P_6 = 102 kPa.

Solution:

a) From solution 12.31: $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8 \text{ K}$

$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6 \text{ kJ/kg}$

$P_3 = 0.98 \times 600 = 588 \text{ kPa}, \quad P_4 = 0.98 \times 588 = 576.2 \text{ kPa}$

$s_5 = s_4 \Rightarrow \quad P_5 = P_4(T_{5S}/T_4)^{\frac{k}{k-1}} = 576.2\left(\frac{1399.2}{1600}\right)^{3.5} = \mathbf{360.4\ kPa}$

b) $P_6 = 100/0.98 = 102 \text{ kPa}, \quad s_{6S} = s_5$

$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\left(\frac{102}{292.8}\right)^{0.286} = 975.2 \text{ K}$

$w_{ST2} = C_{P0}(T_5\text{-}T_6) = 1.004(1399.2 - 975.2) = 425.7 \text{ kJ/kg}$

$\dot{m} = \dot{W}_{NET}/w_{NET} = 150/425.7 = \mathbf{0.352\ kg/s}$

c) $T_3 = T_6 = 975.2 \text{ K}$

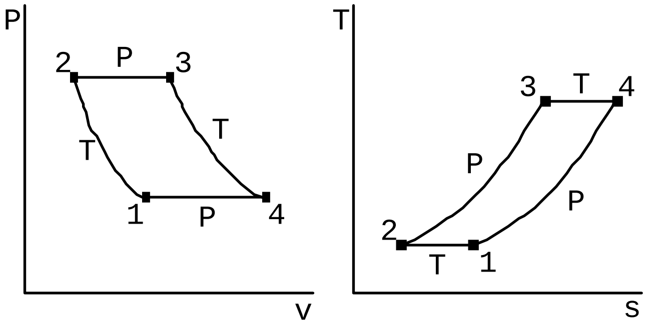
$q_H = C_{P0}(T_4 - T_3) = 1.004\ (1600 - 975.2) = 627.3 \text{ kJ/kg}$

$\eta_{TH} = w_{NET}/q_H = 425.7/627.3 = \mathbf{0.678}$

Ericsson Cycles

12.40

Consider an ideal air-standard Ericsson cycle that has an ideal regenerator as shown in Fig. P12.40. The high pressure is 1 MPa and the cycle efficiency is 70%. Heat is rejected in the cycle at a temperature of 350 K, and the cycle pressure at the beginning of the isothermal compression process is 150 kPa. Determine the high temperature, the compressor work, and the turbine work per kilogram of air.



$P_2 = P_3 = 1$ MPa

$T_1 = T_2 = 350$ K

$P_1 = 150$ kPa

${}_2q_3 = -{}_4q_1$ (ideal reg.)

$\Rightarrow q_H = {}_3q_4$ & $w_T = q_H$

$r_p = P_2/P_1 = 10$

$$\eta_{TH} = \eta_{\text{CARNOT TH.}} = 1 - T_L/T_H = 0.7 \;\Rightarrow\; T_3 = T_4 = T_H = \mathbf{1167\ K}$$

$$q_L = w_C = \int v\, dP = RT_1 \ln\left(\frac{P_2}{P_1}\right) = 0.287 \times 350 \times \ln\left(\frac{1000}{150}\right) = \mathbf{190.56\ kJ/kg}$$

$$w_T = q_H = -\int v\, dP = -RT_3 \ln(P_4/P_3) = \mathbf{635.2\ kJ/kg}$$

12.41

An air-standard Ericsson cycle has an ideal regenerator. Heat is supplied at 1000°C and heat is rejected at 80°C. Pressure at the beginning of the isothermal compression process is 70 kPa. The heat added is 700 kJ/kg. Find the compressor work, the turbine work, and the cycle efficiency.

Solution:

Identify the states

Heat supplied at high temperature $T_3 = T_4 = 1000°C = 1273.15 \text{ K}$

Heat rejected at low temperature $T_1 = T_2 = 80°C = 353.15 \text{ K}$

Beginning of the compression: $P_1 = 70 \text{ kPa}$

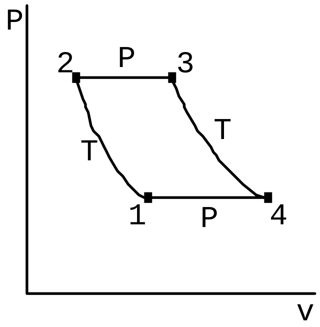
Ideal regenerator: ${}_2q_3 = -{}_4q_1 \quad \Rightarrow \quad q_H = {}_3q_4 = 700 \text{ kJ/kg}$

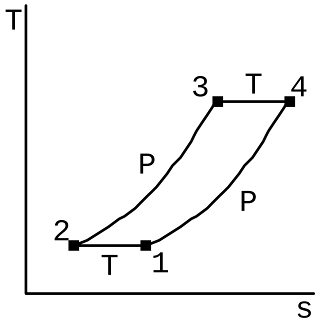
$\Rightarrow \quad w_T = q_H = \mathbf{700\ kJ/kg}$

$$\eta_{TH} = \eta_{CARNOT} = 1 - \frac{353.15}{1273.15} = \mathbf{0.7226}$$

$$w_{NET} = \eta_{TH}\, q_H = 0.7226 \times 700 = 505.8 \text{ kJ/kg}$$

$$w_C = q_L = q_H - w_{NET} = 700 - 505.8 = \mathbf{194.2\ kJ/kg}$$





Jet Engine Cycles

12.42

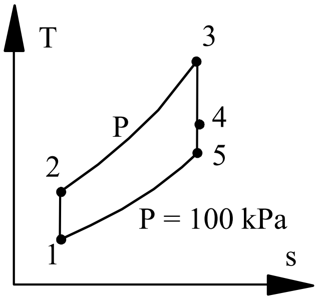
The Brayton cycle in Problem 12.16 is changed to be a jet engine. Find the exit velocity using cold air properties.

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Brayton cycle so this means:
Minimum T: $T_1 = 300$ K
Maximum T: $T_3 = 1600$ K
Pressure ratio: $P_2/P_1 = 14$

Solve using constant C_{P0}



Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$w_C = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Turbine ($w_T = w_C$) and nozzle: $s_5 = s_4 = s_3 \Rightarrow$ Implemented in Eq.8.32

$$T_5 = T_3(P_5/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2 \text{ K}$$

$$h_3 - h_5 = C_{P0}(T_3 - T_5) = 1.004\ (1600 - 752.2) = 851.2 \text{ kJ/kg}$$

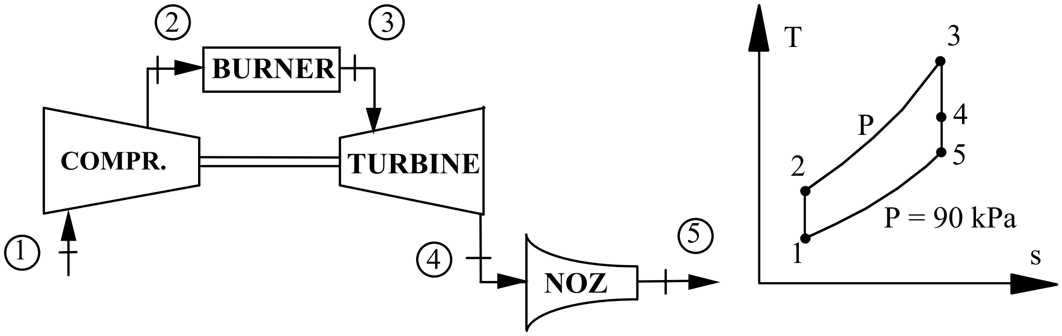
$$1/2\mathbf{V}_5^2 = h_3 - h_5 - w_C = 851.2 - 339.5 = 511.7 \text{ kJ/kg}$$

$$\mathbf{V}_5 = \sqrt{2 \times 1000 \times 511.7} = \mathbf{1012\ m/s}$$

12.43

Consider an ideal air-standard cycle for a gas-turbine, jet propulsion unit, such as that shown in Fig. 12.9. The pressure and temperature entering the compressor are 90 kPa, 290 K. The pressure ratio across the compressor is 14 to 1, and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. Determine the velocity of the air leaving the nozzle.

Solution:



C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.25, 8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290\ (14)^{0.2857} = 616.4\text{ K}$$

$$w_C = h_2 - h_1 \approx C_{P0}\,(T_2 - T_1) = 1.004\ (616.4 - 290) = 327.7\text{ kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C$ and $s_4 = s_3 \Rightarrow$

$$T_4 = T_3 - w_C/C_{P0} = 1500 - 327.7/1.004 = 1173.6\text{ K}$$

C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.32

$$T_5 = T_3\left(\frac{P_5}{P_3}\right)^{\frac{k-1}{k}} = 1500\left(\frac{90}{1260}\right)^{0.2857} = 705.7\text{ K}$$

Now the energy equation

$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 \approx C_{P0}\,(T_4 - T_5) = 1.004\ (1173.6 - 705.7) = 469.77\text{ kJ/kg}$$

$$\Rightarrow\quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 469.77} = \mathbf{969\ m/s}$$

12.44

Solve the previous problem using the air tables.

Consider an ideal air-standard cycle for a gas-turbine, jet propulsion unit, such as that shown in Fig. 12.9. The pressure and temperature entering the compressor are 90 kPa, 290 K. The pressure ratio across the compressor is 14 to 1, and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. Determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle.

C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.28

$$\Rightarrow \quad s^o_{T2} = s^o_{T1} + R \ln(P_2/P_1) = 6.83521 + 0.287 \ln 14 = 7.59262 \text{ kJ/kg K}$$

From A.7 $h_2 = 617.2$ kJ/kg, $T_2 = 609.4$ K

$$w_C = h_2 - h_1 = 617.2 - 290.43 = 326.8 \text{ kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C$ and $s_4 = s_3 \Rightarrow$

$$h_4 = h_3 - w_C = 1635.8 - 326.8 = 1309 \text{ kJ/kg}$$

$$\Rightarrow \quad s^o_{T4} = 8.37142 \text{ kJ/kg K}, \quad T_4 = 1227 \text{ K}$$

$$P_4 = P_3 \exp[(s^o_{T4} - s^o_{T3})/R] = 1260 \exp[\,(8.37142 - 8.61208)/0.287\,]$$

$$= 1260 \exp(-0.83854) = \mathbf{544.8\ kPa}$$

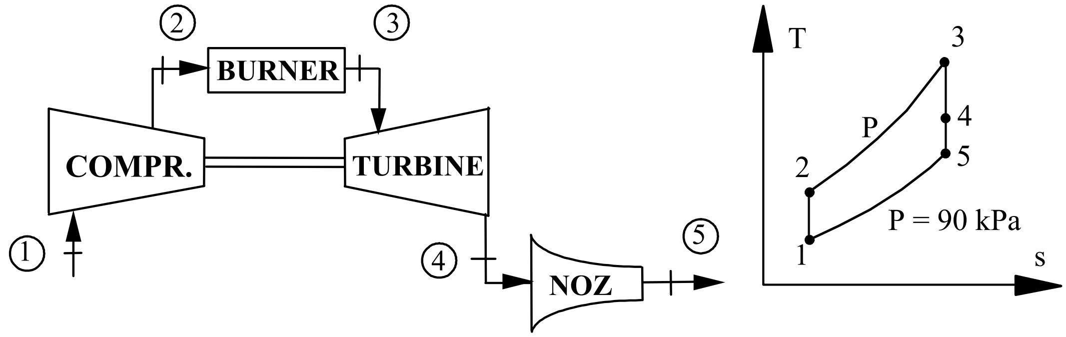
C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.28

$$\Rightarrow \quad s^o_{T5} = s^o_{T3} + R \ln(P_5/P_3) = 8.61208 + 0.287 \ln(1/14) = 7.85467 \text{ kJ/kgK}$$

=> From A.7 $T_5 = 778$ K, $h_5 = 798.2$ kJ/kg

Now the energy equation

$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 = 510.8 \quad \Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 510.8} = \mathbf{1011\ m/s}$$



12.45

The turbine section in a jet engine receives gas (assume air) at 1200 K, 800 kPa with an ambient atmosphere at 80 kPa. The turbine is followed by a nozzle open to the atmosphere and all the turbine work drives a compressor. Find the turbine exit pressure so the nozzle has an exit velocity of 800 m/s. Hint: take the CV around both turbine and nozzle.

Solution:

C.V. Reversible and adiabatic turbine and nozzle. This gives constant s, from Eq.8.32 we can relate the T's and P's

State 3: 1200 K, 800 kPa State 5: 80 kPa; $s_5 = s_3$

Eq.8.32: $T_5 = T_3 (P_5/P_3)^{(k-1)/k} = 1200(80/800)^{0.2857} = 621.56$ K

Energy: $h_3 + 0 = h_5 + (1/2)\mathbf{V}_5^2 + w_T = h_4 + w_T$

$$w_T = h_3 - h_5 - (1/2)\mathbf{V}_5^2 \cong C_P(T_3 - T_5) - (1/2)\mathbf{V}_3^2$$

$$= 1.004(1200 - 621.56) - (1/2) \times 800^2/1000$$

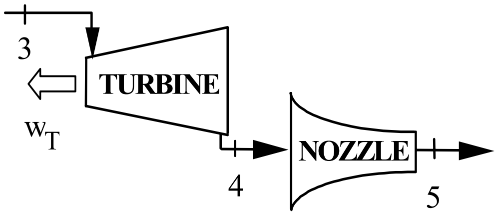
$$= 580.75 - 320 = 260.75 \text{ kJ/kg}$$

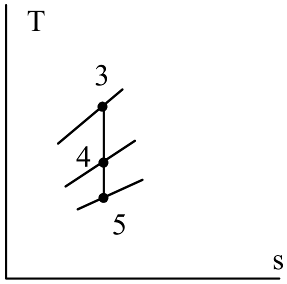
C.V. Nozzle alone to establish state 4 (same s as state 5 and 3).

$$h_4 = h_5 + (1/2)\mathbf{V}_5^2 = h_3 - w_T$$

$$T_4 = T_5 + (1/2)\mathbf{V}_5^2/C_P = 621.56 + 320/1.004 = 940.29 \text{ K}$$

$$P_4 = P_3 (T_4/T_3)^{k/(k-1)} = 800 \times (940.29/1200)^{3.5} = \mathbf{340.7\ kPa}$$





12.46

Given the conditions in the previous problem what pressure could an ideal compressor generate (not the 800 kPa but higher).

C.V. Reversible and adiabatic turbine and nozzle. This gives constant s, from Eq.8.32 we can relate the T's and P's

State 3: 1200 K, 800 kPa State 5: 80 kPa; $s_5 = s_3$

Eq.8.32: $T_5 = T_3\ (P_5/P_3)^{(k-1)/k} = 1200(80/800)^{\ 0.2857} = 621.56$ K

Energy: $h_3 + 0 = h_5 + (1/2)\mathbf{V}_5^2 + w_T = h_4 + w_T$

$$w_T = h_3 - h_5 - (1/2)\mathbf{V}_5^2 \cong C_P(T_3 - T_5) - (1/2)\mathbf{V}_3^2$$

$$= 1.004(1200 - 621.56) - (1/2) \times 800^2/1000$$

$$= 580.75 - 320 = 260.75 \text{ kJ/kg}$$

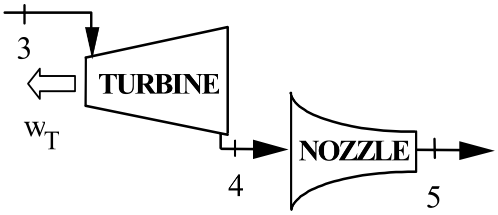
C.V. Compressor

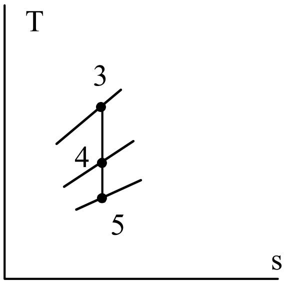
$$w_c = h_2 - h_1 = w_T = 260.75 \text{ kJ/kg}$$

$$T_2 = T_1 + w_c/\ C_P = 270 + 260.75/1.004 = 529.71 \text{ K}$$

Reversible adiabatic compressor, constant s gives relation in Eq.8.32

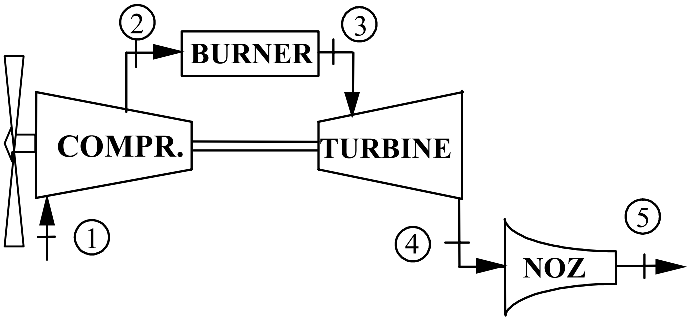
$$P_2 = P_1 \times (T_2/T_1)^{k/(k-1)} = 85 \times (529.71/270)^{3.5} = \mathbf{899\ kPa}$$

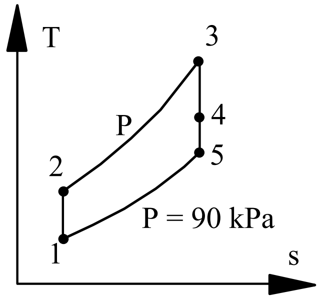




12.47

Consider a turboprop engine where the turbine powers the compressor and a propeller. Assume the same cycle as in Problem 12.43 with a turbine exit temperature of 900 K. Find the specific work to the propeller and the exit velocity.





C.V. Compressor: Reversible and adiabatic $s_2 = s_1$ From Eq.8.25, 8.32

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290\ (14)^{0.2857} = 616.4\text{ K}$$

$$w_C = h_2 - h_1 \approx C_{P0}\,(T_2 - T_1) = 1.004\ (616.4 - 290) = 327.7\text{ kJ/kg}$$

C.V. Turbine: $w_T = h_3 - h_4 = w_C + w_{prop}$ and $s_4 = s_3 \Rightarrow$

$$w_{prop} = C_{P0}\,(T_3 - T_4) - w_C = 1.004(1500 - 900) - 327.7 = 274.7\text{ kJ/kg}$$

C.V. Nozzle: $s_5 = s_4 = s_3$ so from Eq.8.32

$$T_5 = T_3\left(\frac{P_5}{P_3}\right)^{\frac{k-1}{k}} = 1500\left(\frac{90}{1260}\right)^{0.2857} = 705.7\text{ K}$$

Now the energy equation

$$(1/2)\mathbf{V}_5^2 = h_4 - h_5 \approx C_{P0}\,(T_4 - T_5) = 1.004\ (900 - 705.7) = 195.08\text{ kJ/kg}$$

$$\Rightarrow \quad \mathbf{V}_5 = \sqrt{2 \times 1000 \times 195.08} = \mathbf{625\ m/s}$$

12.48

Consider an air standard jet engine cycle operating in a 280 K, 100 kPa environment. The compressor requires a shaft power input of 4000 kW. Air enters the turbine state 3 at 1600 K, 2 MPa, at the rate of 9 kg/s, and the isentropic efficiency of the turbine is 85%. Determine the pressure and temperature entering the nozzle at state 4. If the nozzle efficiency is 95%, determine the temperature and velocity exiting the nozzle at state 5.

Solution:

C.V. Shaft: $\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{W}_C$

CV Turbine: $h_3 - h_4 = \dot{W}_C / \dot{m} = 4000/9 = 444.4$ kJ/kg

$T_4 = T_3 - w_{Ta}/ C_p = 1600 - 444.4/1.004 = 1157.4$ K

Work back to the ideal turbine conditions

Eq.9.27: $w_{Ts} = w_{Ta} / \eta = 444.4/0.85 = 522.82 = h_3 - h_{4s} = C_p(T_3 - T_{4s})$

$\Rightarrow \quad T_{4s} = 1600 - 522.82/1.004 = 1079.3$ K,

$P_4 = P_3 (T_{4s}/T_3)^{k/(k-1)} = 2000 \times (1079.3/1600)^{3.5} =$ **504.2 kPa**

C.V. Nozzle: First the ideal (reversible and adiabatic i.e. constant s, Eq.8.32)

$\Rightarrow T_{5s} = T_4 (P_5/P_4)^{(k-1)/k} = 1157.4 (100/504.2)^{(k-1)/k} = 729$ K

Energy Eq.: $(1/2)\mathbf{V}_{5s}^2 = h_4 - h_{5s} = C_p(T_4 - T_{5s})$

$= 1.004(1157.4 - 729) = 430.1$ kJ/kg

Now consider the actual nozzle

Eq.9.30: $0.5\mathbf{V}_{5a}^2 = \eta(0.5\mathbf{V}_{5s}^2) = 408.6$ kJ/kg

$\Rightarrow \mathbf{V}_{5a} = \sqrt{2 \times 1000 \times 408.6} =$ **904 m/s**

$T_{5a} = T_4 - 0.5\mathbf{V}_{5a}^2/C_p = 1157.4 - 408.6 / 1.004 =$ **750 K**

12.49

Solve the previous problem using the air tables.

Solution done with the air tables A.7:

C.V. Shaft: $\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{W}_C$

CV Turbine: $h_3 - h_4 = \dot{W}_C / \dot{m} = 4000/9 = 444.4$ kJ/kg

$h_4 = 1757.3 – 444.4 = 1312.9$ kJ/kg

Work back to the ideal turbine conditions

Eq.9.27: $w_{Ta} = w_C = 444.4 \Rightarrow w_{Ts} = w_{Ta} / \eta = 522.82 = h_3 - h_{4s}$

$h_{4s} = 1234.5 \Rightarrow T_{4s} \approx 1163$ K, $s^o_{T4s} = 8.3091$ kJ/kg K

$s_{4s} - s_3 = 0 = s^o_{T4s} - s^o_{T3} - R \ln(P_4/P_3)$

$0 = 8.3091 - 8.6905 - 0.287 \ln(P_4/2000)$ => $\mathbf{P_4 = 530}$ **kPa**

State 4 from A.7.1: $h_4 = 1312.9$, $T_4 = 1229.8$ K, $s^o_{T4} = 8.3746$ kJ/kg K

First consider the reversible adiabatic (isentropic) nozzle so from Eq.8.28

$s_{5s} - s_4 = 0 = s^o_{T5s} - s^o_{T4} - R \ln(P_5/P_4)$

$s^o_{T5s} = 8.3746 + 0.287 \ln(100/530) = 7.8960$ kJ/kg K

Table A.7.1: $T_{5s} = 808.1$ K, $h_{5s} = 831.0$ kJ/kg

$\Rightarrow 0.5\mathbf{V}^2_{5s} = h_4 - h_{5s} = 1312.9 - 831.0 = 481.9$ kJ/kg

Now consider the actual nozzle

Eq.9.30: $0.5\mathbf{V}^2_{5a} = \eta(0.5\mathbf{V}^2_{5s}) = 457.81$ kJ/kg $\Rightarrow$ $\mathbf{V}_{5a}$= **957 m/s**

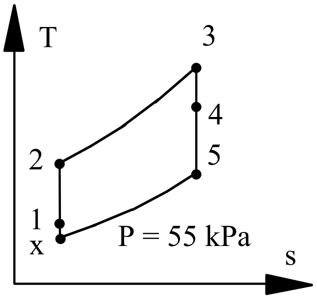
$h_{5a} = h_4 - 0.5\mathbf{V}^2_{5a} = 1312.9 – 457.81 = 855.1$ kJ/kg

$\Rightarrow T_{5a} \approx$ **830 K**

12.50

A jet aircraft is flying at an altitude of 4900 m, where the ambient pressure is approximately 55 kPa and the ambient temperature is −18°C. The velocity of the aircraft is 280 m/s, the pressure ratio across the compressor is 14:1 and the cycle maximum temperature is 1450 K. Assume the inlet flow goes through a diffuser to zero relative velocity at state 1. Find the temperature and pressure at state 1.

Solution:



Ambient

$T_X = -18^oC = 255.2$ K, $P_X = 55$ kPa $= P_5$

also $\mathbf{V}_X = 280$ m/s

Assume that the air at this state is reversibly decelerated to zero velocity and then enters the compressor at 1.

$P_2/P_1 = 14$ & $T_3 = 1450$ K

C.V. Diffuser section, state 1 is the stagnation state.

EnergyEq.: $$T_1 = T_X + \frac{V_X^2}{2 \times 1000 \times C_p} = 255.2 + \frac{(280)^2}{2 \times 1000 \times 1.004} = \mathbf{294.3\ K}$$

Eq.8.32: $$P_1 = P_X\left(\frac{T_1}{T_X}\right)^{\frac{k}{k-1}} = 55\left(\frac{294.3}{255.2}\right)^{3.5} = \mathbf{90.5\ kPa}$$

12.51

The turbine in a jet engine receives air at 1250 K, 1.5 MPa. It exhausts to a nozzle at 250 kPa, which in turn exhausts to the atmosphere at 100 kPa. The isentropic efficiency of the turbine is 85% and the nozzle efficiency is 95%. Find the nozzle inlet temperature and the nozzle exit velocity. Assume negligible kinetic energy out of the turbine.

C.V. Turbine: First the ideal (reversible and adiabatic i.e. constant s, Eq.8.32)

$$T_{es} = T_i\ (P_e/P_i) = 1250\ (250/1500)^{(k-1)/k} = 749.2\text{ K}$$

Energy Eq.: $w_{T,s} = h_i - h_{es} = C_p\ (T_i - T_{es}) = 1.004\ (1250 - 749.2) = 502.8\text{ kJ/kg}$

Eq.9.27: $w_{T,AC} = w_{T,s} \times \eta_T = 427.4\text{ kJ/kg} = h_i - h_{e,AC} = C_p\ (T_i - T_{e,AC})$

$\Rightarrow\ T_{e,AC} = 1250 - 427.4 / 1.004 = \mathbf{824.3\ K},$

C.V. Nozzle: First the ideal (reversible and adiabatic i.e. constant s, Eq.8.32)

$\Rightarrow\ T_{es} = T_i\ (P_e/P_i)^{(k-1)/k} = 824.3\ (100/250)^{(k-1)/k} = 634.4\text{ K}$

Energy Eq.: $(1/2)\mathbf{V}_{e,s}^2 = h_i - h_{e,s} = 1.004(824.3 - 634.4) = 190.66\text{ kJ/kg}$

Eq.9.30: $(1/2)\mathbf{V}_{e,AC}^2 = (1/2)\mathbf{V}_{e,s}^2 \times \eta_{NOZ} = 181.13\text{ kJ/kg}$

$\mathbf{V}_{e,AC} = \sqrt{2 \times 1000 \times 181.13} = \mathbf{602\ m/s}$

12.52

Solve the previous problem using the air tables.

Solution using air tables A.7:

C.V. Turbine: $h_i = 1336.7$, $s^o_{Ti} = 8.3940$, $s_{es} = s_i$ then from Eq.8.28

$\Rightarrow s^o_{Tes} = s^o_{Ti} + R\ \ln(P_e/P_i) = 8.3940 + 0.287\ \ln\ (250/1500) = 7.8798$ kJ/kg K

Table A.7.1 $T_{es} = 796$ K, $h_{es} = 817.9$ kJ/kg,

Energy Eq.: $w_{T,s} = h_i - h_{es} = 1336.7 - 817.9 = 518.8$ kJ/kg

Eq.9.27: $w_{T,AC} = w_{T,s} \times \eta_T = 441$ kJ/kg $= h_i - h_{e,AC}$

$\Rightarrow h_{e,AC} = 895.7 \Rightarrow \mathbf{T_{e,AC} = 866\ K}$, $s^o_{Te} = 7.9730$ kJ/kg K

C.V. Nozzle: $h_i = 895.7$ kJ/kg, $s^o_{Ti} = 7.9730$ kJ/kgK, $s_{es} = s_i$ then from Eq.8.28

$\Rightarrow s^o_{Tes} = s^o_{Ti} + R\ \ln(P_e/P_i) = 7.9730 + 0.287\ \ln\ (100/250) = 7.7100$ kJ/kgK

Table A.7.1 $\Rightarrow T_{e,s} = 681$ K, $h_{e,s} = 693.1$ kJ/kg

Energy Eq.: $(1/2)\mathbf{V}^2_{e,s} = h_i - h_{e,s} = 895.7 - 693.1 = 202.6$ kJ/kg

Eq.9.30: $(1/2)\mathbf{V}^2_{e,AC} = (1/2)\mathbf{V}^2_{e,s} \times \eta_{NOZ} = 192.47$ kJ/kg

$\mathbf{V}_{e,AC} = \sqrt{2 \times 1000 \times 192.47} = \mathbf{620\ m/s}$

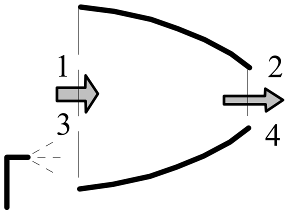
12.53

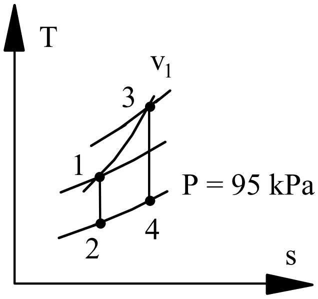
An afterburner in a jet engine adds fuel after the turbine thus raising the pressure and temperature due to the energy of combustion. Assume a standard condition of 800 K, 250 kPa after the turbine into the nozzle that exhausts at 95 kPa. Assume the afterburner adds 450 kJ/kg to that state with a rise in pressure for same specific volume, and neglect any upstream effects on the turbine. Find the nozzle exit velocity before and after the afterburner is turned on.

Solution:

Before afterburner is on: 1: 800 K; 250 kPa and 2: 95 kPa

After afterburner is on: 3: $v = v_1$ and 4: 95 kPa





Assume reversible adiabatic nozzle flow, then constant s from Eq.8.32

$$T_2 = T_1\,(P_2/P_1)^{(k-1)/k} = 800 \times (95/250)^{\,0.2857} = 606.8\text{ K}$$

Energy Eq.: $(1/2)\mathbf{V}_2^2 = C_P(T_1 - T_2)$

$$\mathbf{V}_2 = \sqrt{2\ C_P(T_1 - T_2)} = \sqrt{2 \times 1004(800 - 606.8)} = \mathbf{622.8\ m/s}$$

Add the q_{AB} at assumed constant volume then energy equation gives

$$T_3 = T_1 + q_{AB}/C_v = 800 + 450/0.717 = 1427.6\text{ K}$$

$$v_3 = v_1 \;\Rightarrow\; P_3 = P_1(\,T_3/T_1) = 250 \times 1427.6/800 = 446.1\text{ kPa}$$

Reversible adiabatic expansion, again from Eq.8.32

$$T_4 = T_3\,(P_4/P_3)^{(k-1)/k} = 1427.6 \times (95/446.1)^{\,0.2857} = 917.7\text{ K}$$

$$\mathbf{V}_2 = \sqrt{2\ C_P(T_3 - T_4)} = \sqrt{2 \times 1004(1427.6 - 917.7)} = \mathbf{1012\ m/s}$$

Air-standard refrigeration cycles

12.54

An air standard refrigeration cycle has air into the compressor at 100 kPa, 270 K with a compression ratio of 3:1. The temperature after heat rejection is 300 K. Find the COP and the lowest cycle temperature.

From the isentropic compression/expansion processes

$$T_2 / T_1 = (P_2/P_1)^{\frac{k-1}{k}} = 3^{0.2857} = 1.36874 = T_3/T_4$$

$$T_4 = T_3 / 1.36874 = \mathbf{219.2\ K}$$

The COP, Eq.12.5, is

$$\beta = \frac{1}{\frac{T_2}{T_1} - 1} = [\ 1.36874 - 1]^{-1} = \mathbf{2.712}$$

12.55

A standard air refrigeration cycle has -10^{o}C, 100 kPa into the compressor, and the ambient cools the air down to 35^{o}C at 400 kPa. Find the lowest temperature in the cycle, the low T specific heat transfer and the specific compressor work.

Solution:

State 3: 35^{o}C = 308.2 K, 400 kPa

The lowest T is at state 4 which we can relate to state 3

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 308.2\left(\frac{100}{400}\right)^{0.2857} = 204.9 \text{ k} = \mathbf{-68.3^{o}C}$$

$$_4q_1 = h_1 - h_4 = C_p\,(T_1 - T_4) = 1.004\,(-10 + 68.3) = \mathbf{58.5\ kJ/kg}$$

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 263.2\left(\frac{400}{100}\right)^{0.2857} = 391.3 \text{ K}$$

$$w_c = h_1 - h_2 = C_p\,(T_1 - T_2) = 1.004\,(263.2 - 391.3) = \mathbf{-128.6\ kJ/kg}$$

12.56

The formula for the COP assuming cold air properties is given for the standard refrigeration cycle in Eq.12.5. Develop the similar formula for the cycle variation with a heat exchanger as shown in Fig. 12.12.

Definition of COP: $\beta = \dfrac{q_L}{w_{net}} = \dfrac{q_L}{q_H - q_L} = \dfrac{1}{\dfrac{q_H}{q_L} - 1}$

Assuming an ideal heat exchanger $h_4 = h_6$ and $h_1 = h_3$ so $q_L = w_E$

From the refrigeration cycle we get the ratio of the heat transfers as

$$\frac{q_H}{q_L} = \frac{C_p(T_2 - T_3)}{C_p(T_6 - T_5)} = \frac{T_2 - T_1}{T_4 - T_5} = \frac{T_1}{T_5}\,\frac{(T_2/T_1) - 1}{(T_4/T_5) - 1}$$

The pressure ratios are the same and we have isentropic compression/expansion

$$\frac{P_2}{P_1} = \frac{P_4}{P_5} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{T_4}{T_5}\right)^{k/(k-1)}$$

so now we get

$$\frac{T_2}{T_1} = \frac{T_4}{T_5} \qquad \text{so} \qquad \frac{q_H}{q_L} = \frac{T_1}{T_5} = \frac{T_1}{T_4}\frac{T_4}{T_5} = \frac{T_1}{T_4}\, r_P^{(k-1)/k}$$

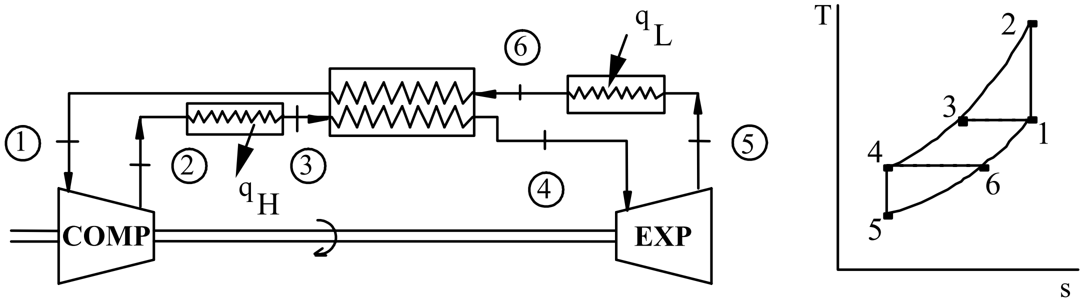
and the COP reduces to

$$\beta = \frac{1}{\dfrac{T_1}{T_5} - 1} = \frac{1}{\dfrac{T_1}{T_4}\, r_P^{(k-1)/k} - 1}$$

12.57

Assume a refrigeration cycle as shown in Fig. 12.12 with reversible adiabatic compressor and expander. For this cycle the low pressure is 100 kPa and the high pressure is 1.4 MPa with constant pressure heat exchangers, see Fig. 12.12 T-s diagram. The temperatures are $T_4 = T_6 = -50^oC$ and $T_1 = T_3 = 15^oC$. Find the COP for this refrigeration cycle.

Solution:



Standard air refrigeration cycle with $P_1 = 100$ kPa, $P_2 = 1.4$ MPa

We will solve the problem with cold air properties.

Compressor, isentropic $s_2 = s_1$ so from Eq.8.32

$$\Rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2(1400/100)^{0.286} = 613 \text{ K}$$

$$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(613 - 288.2) = 326 \text{ kJ/kg}$$

Expansion in expander (turbine)

$$s_5 = s_4 \Rightarrow T_5 = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2(100/1400)^{0.286} = 104.9 \text{ K}$$

$$w_E = C_{P0}(T_4 - T_5) = 1.004(223.2 - 104.9) = 118.7 \text{ kJ/kg}$$

Net cycle work

$$w_{NET} = w_E - w_C = 118.7 - 326.0 = -207.3 \text{ kJ/kg}$$

$$q_L = C_{P0}(T_6 - T_5) = w_E = 118.7 \text{ kJ/kg}$$

Overall cycle performance, COP

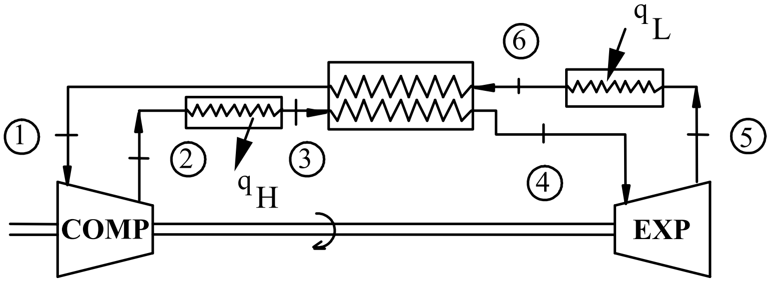
$$\beta = q_L/w_{NET} = 118.7 / 207.3 = \mathbf{0.573}$$

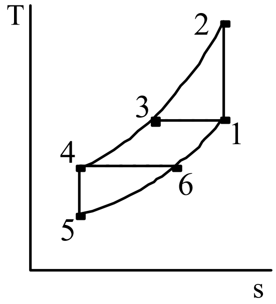
12.58

Repeat Problems 12.57, but assume that helium is the cycle working fluid instead of air. Discuss the significance of the results.

A heat exchanger is incorporated into an ideal air-standard refrigeration cycle, as shown in Fig. P12.57. It may be assumed that both the compression and the expansion are reversible adiabatic processes in this ideal case. Determine the coefficient of performance for the cycle.

Solution:





Standard air refrigeration cycle with helium and states as

$T_1 = T_3 = 15\ ^oC = 288.2$ K, $P_1 = 100$ kPa, $P_2 = 1.4$ MPa

$T_4 = T_6 = -50\ ^oC = 223.2$ K

Compressor, isentropic $s_2 = s_1$ so from Eq.8.32

$$\Rightarrow\ T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2\left(\frac{1400}{100}\right)^{0.40} = 828.2 \text{ K}$$

$$w_C = -w_{12} = C_{P0}(T_2 - T_1) = 5.193(828.2 - 288.2) = 2804.1 \text{ kJ/kg}$$

Expansion in expander (turbine)

$$s_5 = s_4 \quad \Rightarrow \quad T_5 = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2\left(\frac{100}{1400}\right)^{0.40} = 77.7 \text{ K}$$

$$w_E = C_{P0}(T_4 - T_5) = 15.193(223.2 - 77.7) = 755.5 \text{ kJ/kg}$$

Net cycle work

$$w_{NET} = 755.5 - 2804.1 = -2048.6 \text{ kJ/kg}$$

$$q_L = C_{P0}(T_6 - T_5) = 5.193(223.2 - 77.7) = 755.5 \text{ kJ/kg}$$

Overall cycle performance, COP

$$\beta = q_L/w_{NET} = 755.5/2048.6 = \mathbf{0.369}$$

Notice that the low temperature is lower and work terms higher than with air. It is due to the higher heat capacity C_{P0} and ratio of specific heats (k = 1 2/3). The expense is a lower COP requiring more work input per kJ cooling.

12.59

Repeat Problem 12.57, but assume an isentropic efficiency of 75% for both the compressor and the expander.

Standard air refrigeration cycle with

$T_1 = T_3 = 15\ ^oC = 288.2\text{ K},\quad P_1 = 100\text{ kPa},\quad P_2 = 1.4\text{ MPa}$

$T_4 = T_6 = -50\ ^oC = 223.2\text{ K}$

We will solve the problem with cold air properties.

Ideal compressor, isentropic $s_{2S} = s_1$ so from Eq.8.32

$$\Rightarrow\ T_{2S} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 288.2(1400/100)^{0.286} = 613\text{ K}$$

$$w_{SC} = -w_{12} = C_{P0}(T_{2S} - T_1) = 1.004(613 - 288.2) = 326\text{ kJ/kg}$$

The actual compressor

$$w_C = w_{SC} / \eta_{SC} = 326/0.75 = 434.6\text{ kJ/kg}$$

Expansion in ideal expander (turbine)

$$s_5 = s_4 \Rightarrow\ T_{5S} = T_4(P_5/P_4)^{\frac{k-1}{k}} = 223.2(100/1400)^{0.286} = 104.9\text{ K}$$

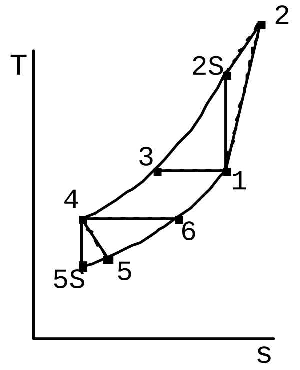
$$w_E = C_{P0}(T_4 - T_5) = 1.004(223.2 - 104.9) = 118.7\text{ kJ/kg}$$

The actual expander (turbine)

$$w_E = \eta_{SE} \times w_{SE} = 0.75 \times 118.7 = 89.0\text{ kJ/kg}$$
$$= C_{P0}(T_4 - T_5) = 1.004(223.2 - T_5)$$

$$\Rightarrow\ T_5 = 134.5\text{ K}$$

$$w_{NET} = 89.0 - 434.6 = -345.6\text{ kJ/kg}$$



$$q_L = C_{P0}(T_6 - T_5) = 1.004(223.2 - 134.5) = 89.0\text{ kJ/kg}$$

$$\beta = q_L/(-w_{NET}) = 89.0/345.6 = \mathbf{0.258}$$

Otto Cycles

12.60

A 4 stroke gasoline engine runs at 1800 RPM with a total displacement of 2.4L and a compression ratio of 10:1. The intake is at 290 K, 75 kPa with a mean effective pressure of 600 kPa. Find the cycle efficiency and power output.

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 10^{-0.4} = \mathbf{0.60}$$

The power output comes from speed and displacement in Eq. 12.11

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 600 \times 0.0024 \times \frac{1800}{60} \times \frac{1}{2} = \mathbf{21.6\ kW}$$

12.61

A 4 stroke gasoline 4.2 L engine running at 2000 RPM has inlet state of 85 kPa, 280 K and after combustion it is 2000 K and the highest pressure is 5 MPa. Find the compression ratio, the cycle efficiency and the exhaust temperature.

Solution:

Combustion $v_3 = v_2$. Highest T and P are after combustion.

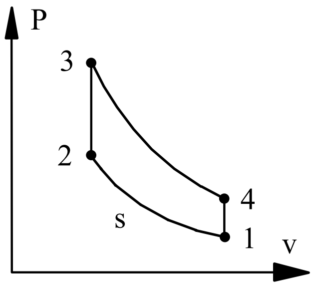
$$\text{CR} = v_1/v_2 = v_1/v_3 = (T_1P_3/T_3P_1) = \frac{280 \times 5000}{2000 \times 85} = \mathbf{8.235}$$

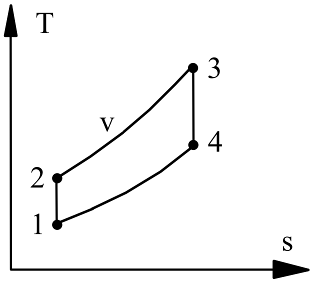
Efficiency from the compression ratio

$$\eta = 1 - \text{CR}^{1-k} = 1 - 8.235^{-0.4} = \mathbf{0.57}$$

Expansion

$$T_4 = T_3\ (v_3/v_4)^{k-1} = T_3\ [\ 1/\ \text{CR}\ ]^{k-1} = 2000 \times (1/\ 8.235)^{\ 0.4} = \mathbf{860.5\ K}$$





12.62

Find the power from the engine in Problem 12.61

Solution:

Combustion $v_3 = v_2$. Highest T and P are after combustion.

$$CR = v_1/v_2 = v_1/v_3 = (T_1P_3/T_3P_1) = \frac{280 \times 5000}{2000 \times 85} = 8.235$$

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 8.235^{-0.4} = 0.57$$

Compression: $T_2 = T_1(CR)^{k-1} = 280 \times (8.235)^{0.4} = 650.8\text{ K}$

$$q_H = u_3 - u_2 = C_v(T_3 - T_2) = 0.717\,(2000 - 650.8) = 967.4\text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.287 \times 280/85 = 0.9454\text{ m}^3\text{/kg}$$

Displacement and then mep from net work

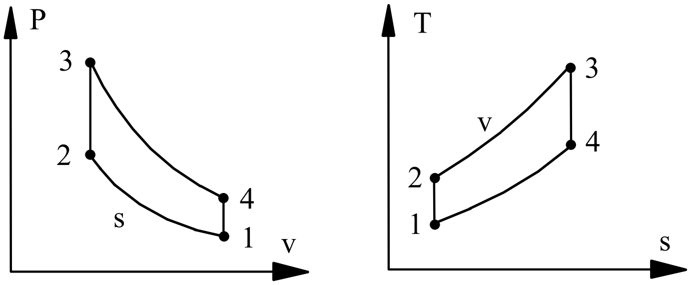
$$v_1 - v_2 = v_1 - v_1/CR = v_1[1 - (1/CR)] = 0.8306\text{ m}^3\text{/kg}$$

$$P_{meff} = w_{net}/(v_1 - v_2) = \eta\, q_H/(v_1 - v_2)$$

$$= 0.57 \times 967.4/0.8306 = 663.9\text{ kPa}$$

Total power from Eq. 12.11 and 4 stroke cycle

$$\dot{W} = P_{meff}V_{displ}\frac{RPM}{60}\frac{1}{2} = 663.9 \times 0.0042 \times \frac{2000}{60} \times \frac{1}{2} = \mathbf{46.45\ kW}$$



12.63

Air flows into a gasoline engine at 95 kPa, 300 K. The air is then compressed with a volumetric compression ratio of 8:1. In the combustion process 1300 kJ/kg of energy is released as the fuel burns. Find the temperature and pressure after combustion using cold air properties.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.33 and Eq.8.34

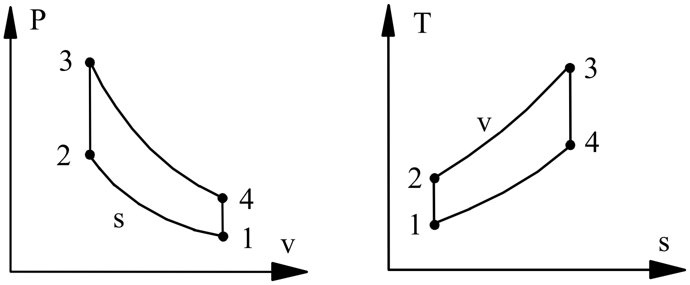
$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 8^{0.4} = 689.2 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 95 \times 8^{1.4} = 1746 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 689.2 + 1300/0.717 = \mathbf{2502\ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1746\ (2502 / 689.2) = \mathbf{6338\ kPa}$$



12.64

A 2.4 L gasoline engine runs at 2500 RPM with a compression ratio of 9:1. The state before compression is 40 kPa, 280 K and after combustion it is at 2000 K. Find the highest T and P in the cycle, the specific heat transfer added, the cycle efficiency and the exhaust temperature.

This is a basic 4-stroke Otto cycle.

Compression: $T_2 = T_1(CR)^{k-1} = 280 \times (9)^{0.4} = 674.3$ K

$$P_2 = P_1\ CR^k = 40 \times 9^{1.4} = 866.96 \text{ kPa}$$

Combustion $v_3 = v_2$. Highest T and P are after combustion.

$$T_3 = \mathbf{2000\ K},\quad P_3 = P_2\ T_3 / T_2 = \frac{866.96 \times 2000}{674.3} = \mathbf{2571.4\ kPa}$$

$$q_H = u_3 - u_2 = C_v(T_3 - T_2) = 0.717\ (2000 - 674.3) = \mathbf{950.5\ kJ/kg}$$

Efficiency from the compression ratio

$$\eta = 1 - CR^{1-k} = 1 - 9^{-0.4} = \mathbf{0.585}$$

Expansion with volume ratio equals compression ratio

$$T_4 = T_3\ CR^{1-k} = 2000 \times 9^{-0.4} = \mathbf{830.5\ K}$$

12.65

Suppose we reconsider the previous problem and instead of the standard ideal cycle we assume the expansion is a polytropic process with n = 1.5. What are the exhaust temperature and the expansion specific work?

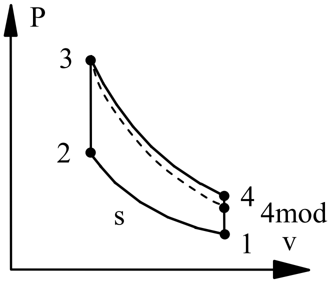
This is a modified 4-stroke Otto cycle.

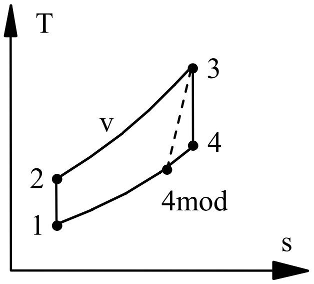
If a polytropic expansion with n = 1.5 instead then

$$T_{4mod} = T_3 \, CR^{1-n} = 2000 \times 9^{-0.5} = 667 \text{ K}$$

This means a heat loss out and thus less work (see diagrams)

$$w = \frac{R}{1-n}(T_{4mod} - T_3) = \frac{0.287}{1-1.5}(667 - 2000) = \mathbf{765.1\ kJ/kg}$$





12.66

A gasoline engine has a volumetric compression ratio of 8 and before compression has air at 280 K, 85 kPa. The combustion generates a peak pressure of 6500 kPa. Find the peak temperature, the energy added by the combustion process and the exhaust temperature.

Solution:

Solve the problem with cold air properties.

Compression. Isentropic so we use Eqs.8.33-8.34

$$P_2 = P_1(v_1/v_2)^k = 85(8)^{1.4} = 1562 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 280(8)^{0.4} = 643.3 \text{ K}$$

Combustion. Constant volume

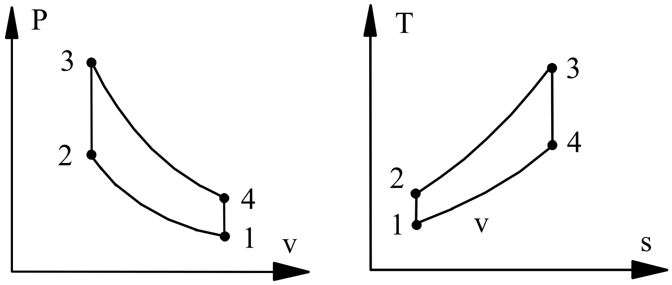
$$T_3 = T_2\,(P_3/P_2) = 643.3 \times 6500/1562 = \mathbf{2677\ K}$$

$$q_H = u_3 - u_2 \approx C_v(T_3 - T_2)$$

$$= 0.717\,(2677 - 643.3) = \mathbf{1458\ kJ/kg}$$

Exhaust. Isentropic expansion so from Eq.8.33

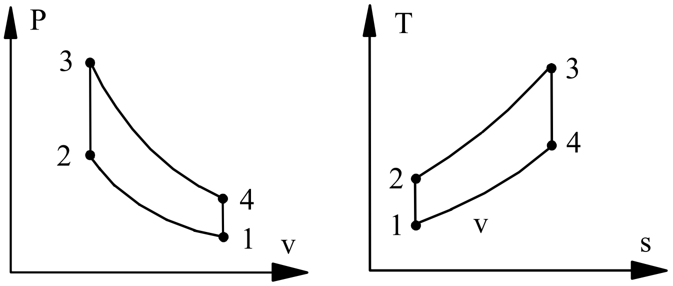
$$T_4 = T_3/8^{0.4} = 2677/2.2974 = \mathbf{1165\ K}$$



12.67

To approximate an actual spark-ignition engine consider an air-standard Otto cycle that has a heat addition of 1800 kJ/kg of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 90 kPa, 10°C. Assuming constant specific heat, with the value from Table A.5, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle and the mean effective pressure.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$P_2 = P_1(v_1/v_2)^k = 90(7)^{1.4} = 1372$ kPa

$T_2 = T_1(v_1/v_2)^{k-1} = 283.2 \times (7)^{0.4} = 616.6$ K

Combustion: constant volume

$T_3 = T_2 + q_H/C_{V0} = 616.6 + 1800/0.717 =$ **3127 K**

$P_3 = P_2T_3/T_2 = 1372 \times 3127 / 616.6 =$ **6958 kPa**

Efficiency and net work

$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.2/616.5 =$ **0.541**

$w_{net} = \eta_{TH} \times q_H = 0.541 \times 1800 = 973.8$ kJ/kg

Displacement and P_{meff}

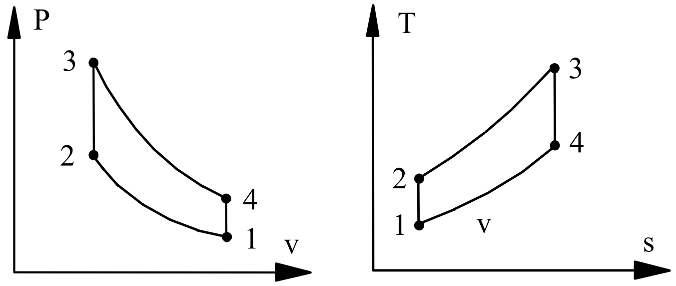
$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029$ m^3/kg

$v_2 = (1/7)\, v_1 = 0.1290$ m^3/kg

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{973.8}{0.9029 - 0.129} = \mathbf{1258\ kPa}$$

12.68

A 3.3 L minivan engine runs at 2000 RPM with a compression ratio of 10:1. The intake is at 50 kPa, 280 K and after expansion it is at 750 K. Find the highest T in the cycle, the specific heat transfer added by combustion and the mean effective pressure.



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$T_2 = T_1(v_1/v_2)^{k-1} = 280 \times (10)^{0.4} = 703.3 \text{ K}$$

Expansion: isentropic, T_3 is the highest T and T_4 was given

$$T_3 = T_4 \, CR^{k-1} = 750 \times 10^{0.4} = \mathbf{1883.9\ K}$$

$$q_H = u_3 - u_2 = C_{V0}\,(T_3 - T_2) = 0.717\,(1883.9 - 703.3) = \mathbf{846.5\ kJ/kg}$$

Efficiency and net work

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 280/703.3 = 0.602$$

$$w_{net} = \eta_{TH} \times q_H = 0.602 \times 846.5 = 509.6 \text{ kJ/kg}$$

Displacement and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 280)/50 = 1.6072 \text{ m}^3\text{/kg}$$

$$v_2 = (1/10)\, v_1 = 0.16072 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{509.6}{1.6072 - 0.16072} = \mathbf{352.3\ kPa}$$

12.69

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto Cycle

Combustion process: $T_3 = 2050$ K; $u_2 = u_3 - q_H$

$$T_2 = T_3 - q_H / C_{vo} = 2050 - 1000 / 0.717 = 655.3 \text{ K}$$

Compression process

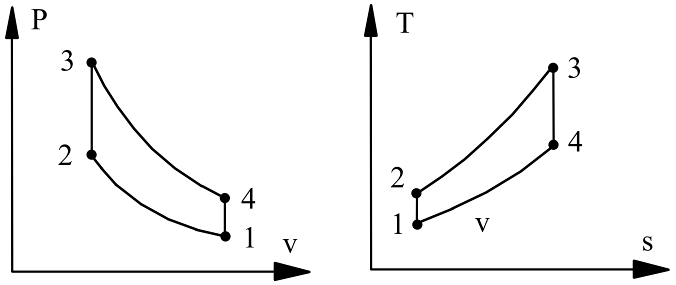
$$P_2 = P_1(T_2 / T_1)^{k/(k-1)} = 90(655.3/290)^{3.5} = 1561 \text{ kPa}$$

$$CR = v_1 / v_2 = (T_2 / T_1)^{1/(k-1)} = (655.3 / 290)^{2.5} = \mathbf{7.67}$$

$$-{}_1w_2 = u_2 - u_1 = C_{vo}(T_2 - T_1) = 0.717(655.3 - 290) = \mathbf{262\ kJ / kg}$$

Highest pressure is after the combustion where $v_3 = v_2$ so we get

$$P_3 = P_2T_3 / T_2 = 1561 \times 2050 / 655.3 = \mathbf{4883\ kPa}$$



12.70

Answer the same three questions for the previous problem, but use variable heat capacities (use table A.7).

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto cycle, solve using Table A.7.1

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1000 = 725.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 960.5 \text{ K} ; \qquad s^o_{T2} = 8.0889 \text{ kJ/kg K}$$

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow$ From Eq.8.28

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2v_1/T_1v_2)$$

$$= 8.0889 - 6.8352 - 0.287 \ln(960.5/290) - 0.287 \ln(v_1/v_2)$$

Solving => $v_1 / v_2 =$ **23.78**

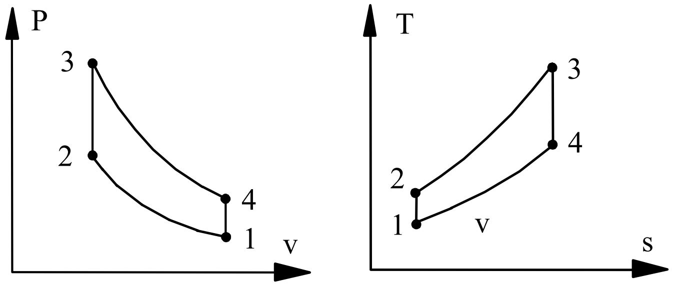
Comment: This is much too high for an actual Otto cycle.

$$-_1w_2 = u_2 - u_1 = 725.7 - 207.2 = \mathbf{518.5 \text{ kJ/kg}}$$

Highest pressure is after combustion where $v_3 = v_2$ so we get

$$P_3 = P_2T_3 / T_2 = P_1(T_3 / T_1)(v_1 / v_3)$$

$$= 90 \times (2050 / 290) \times 23.78 = \mathbf{15\ 129 \text{ kPa}}$$



12.71

A four stroke gasoline engine has a compression ratio of 10:1 with 4 cylinders of total displacement 2.3 L. the inlet state is 280 K, 70 kPa and the engine is running at 2100 RPM with the fuel adding 1800 kJ/kg in the combustion process. What is the net work in the cycle and how much power is produced?

Solution:

Overall cycle efficiency is from Eq.12.12, $r_v = v_1/v_2 = 10$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - 10^{-0.4} = 0.602$$

$$w_{net} = \eta_{TH} \times q_H = 0.602 \times 1800 = \mathbf{1083.6\ kJ/kg}$$

We also need specific volume to evaluate Eqs.12.9 to 12.11

$$v_1 = RT_1 / P_1 = 0.287 \times 280 / 70 = 1.148\ \mathrm{m^3/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1\,(1 - \frac{1}{r_v})} = \frac{1083.6}{1.148 \times 0.9} = 1048.8\ \mathrm{kPa}$$

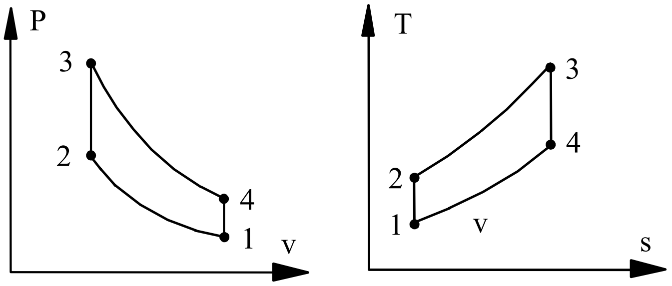
Now we can find the power from Eq.12.11

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 1048.8 \times 0.0023 \times \frac{2100}{60} \times \frac{1}{2} = \mathbf{42.2\ kW}$$

12.72

A gasoline engine receives air at 10 C, 100 kPa, having a compression ratio of 9:1 by volume. The heat addition by combustion gives the highest temperature as 2500 K. use cold air properties to find the highest cycle pressure, the specific energy added by combustion, and the mean effective pressure.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$P_2 = P_1(v_1/v_2)^k = 100\ (9)^{1.4} = 2167.4 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.15\ (9)^{0.4} = 681.89 \text{ K}$$

Combustion: constant volume

$$P_3 = P_2(T_3\ /\ T_2) = 2167.4 \times 2500\ /\ 681.89 = \mathbf{7946.3\ kPa}$$

$$q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 0.717\ (2500 - 681.89) = \mathbf{1303.6\ kJ/kg}$$

Efficiency, net work, displacement and P_{meff}

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.15/681.89 = 0.5847$$

$$w_{net} = \eta_{TH} \times q_H = 0.5847 \times 1303.6 = 762.29 \text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.287 \times 283.15\ /\ 100 = 0.81264 \text{ m}^3\text{/kg},$$

$$v_2 = v_1/9 = 0.090293 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{762.29}{0.81264 - 0.090293} = \mathbf{1055\ kPa}$$

12.73

A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 290 K, 85 kPa in the cylinder. The combustion peak pressure is 6000 kPa. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.

Solution:

Compression. Isentropic so we use Eqs.8.33-8.34

$$P_2 = P_1(v_1/v_2)^k = 85\ (10)^{1.4} = 2135.1\text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 290\ (10)^{0.4} = 728.45\text{ K}$$

Combustion. Constant volume

$$T_3 = T_2\ (P_3/P_2) = 728.45 \times 6000/2135.1 = \mathbf{2047\ K}$$

Exhaust. Isentropic expansion so from Eq.8.33

$$T_4 = T_3\ /\ (v_1/v_2)^{k-1} = T_3\ /\ 10^{0.4} = 2047\ /\ 2.5119 = \mathbf{814.9\ K}$$

Overall cycle efficiency is from Eq.12.9, $r_v = v_1/v_2$

$$\eta = 1 - r_v^{1-k} = 1 - 10^{-0.4} = \mathbf{0.602}$$

Comment: No actual gasoline engine has an efficiency that high, maybe 35%.

12.74

Repeat Problem 12.67, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (or the specific heat from Fig. 5.10 at high temperature).

Solution:

Table A.7 is used with interpolation.

$T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $s^o_{T1} = 6.8113$ kJ/kg K

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.28

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2v_1/T_1v_2)$$

$$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.8113 + 0.287 \ln 7 = 7.3698$$

This becomes trial and error so estimate first at 600 K and use A.7.1.

$LHS_{600} = 7.5764 - 0.287 \ln(600/283.2) = 7.3609$ (too low)

$LHS_{620} = 7.6109 - 0.287 \ln(620/283.2) = 7.3860$ (too high)

Interpolate to get: $T_2 = 607.1$ K, $u_2 = 440.5$ kJ/kg

$\Rightarrow\ -{}_1w_2 = u_2 - u_1 = 238.2$ kJ/kg,

$u_3 = 440.5 + 1800 = 2240.5 \Rightarrow T_3 =$ **2575.8 K** , $s^o_{T3} = 9.2859$ kJ/kgK

$P_3 = 90 \times 7 \times 2575.8 / 283.2 =$ **5730 kPa**

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From Eq.8.28 as before

$$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.2859 + 0.287 \ln(1/7) = 8.7274$$

This becomes trial and error so estimate first at 1400 K and use A.7.1.

$LHS_{1400} = 8.5289 - 0.287 \ln(1400/2575.8) = 8.7039$ (too low)

$LHS_{1450} = 8.5711 - 0.287 \ln(1450/2575.8) = 8.7360$ (too high)

Interpolation $\Rightarrow T_4 = 1436.6$ K, $u_4 = 1146.9$ kJ/kg

${}_3w_4 = u_3 - u_4 = 2240.5 - 1146.9 = 1093.6$ kJ/kg

Net work, efficiency and mep

➔ $w_{net} = {}_3w_4 + {}_1w_2 = 1093.6 - 238.2 = 855.4$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 855.4 / 1800 =$ **0.475**

$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029\ m^3/kg$

$v_2 = (1/7)\, v_1 = 0.1290\ m^3/kg$

$P_{meff} = \dfrac{w_{net}}{v_1 - v_2} = 855.4 / (0.9029 - 0.129) =$ **1105 kPa**

12.75

An Otto cycle has the lowest T as 290 K and the lowest P as 85 kPa, the highest T is 2400 K and combustion adds 1200 kJ/kg as heat transfer. Find the compression ratio and the mean effective pressure.

Solution:

Identify states: $T_1 = 290$ K, $P_1 = 85$ kPa, $T_3 = 2300$ K, $q_H = 1200$ kJ/kg

Combustion: $q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 1200$ kJ/kg

$$T_2 = T_3 - q_H / C_{vo} = 2400 - 1200/0.717 = 726.36 \text{ K}$$

Compression: $\text{CR} = r_v = v_1/v_2 = (T_2/T_1)^{1/(k-1)} = \left(\frac{726.36}{290}\right)^{2.5} = \mathbf{9.93}$

Overall cycle efficiency is from Eq.12.12, $r_v = v_1/v_2 = 9.93$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - T_1/T_2 = 1 - 290/726.36 = 0.601$$

$$w_{net} = \eta_{TH} \times q_H = 0.601 \times 1200 = 721.2 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.12.9 to 12.11

$$v_1 = RT_1 / P_1 = 0.287 \times 290 / 85 = 0.9792 \text{ m}^3\text{/kg}$$

$$v_2 = v_1/\text{CR} = 0.09861 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{721.2}{0.9792 - 0.09861} \frac{\text{kJ/kg}}{\text{m}^3\text{/kg}} = \mathbf{819\ kPa}$$

12.76

The cycle in the previous problem is used in a 2.4 L engine running 1800 RPM. How much power does it produce?

Identify states: $T_1 = 290$ K, $P_1 = 85$ kPa, $T_3 = 2300$ K, $q_H = 1200$ kJ/kg

Combustion: $q_H = u_3 - u_2 = C_{vo}(T_3 - T_2) = 1200$ kJ/kg

$$T_2 = T_3 - q_H / C_{vo} = 2400 - 1200/0.717 = 726.36 \text{ K}$$

Compression: $CR = r_v = v_1/v_2 = (T_2/T_1)^{1/(k-1)} = \left(\frac{726.36}{290}\right)^{2.5} = 9.93$

Overall cycle efficiency is from Eq.12.12, $r_v = v_1/v_2 = 9.93$

$$\eta_{TH} = 1 - r_v^{1-k} = 1 - T_1/T_2 = 1 - 290/726.36 = 0.601$$

$$w_{net} = \eta_{TH} \times q_H = 0.601 \times 1200 = 721.2 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.12.9 to 12.11

$$v_1 = RT_1 / P_1 = 0.287 \times 290 / 85 = 0.9792 \text{ m}^3\text{/kg}$$

$$v_2 = v_1/CR = 0.09861 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{721.2}{0.9792 - 0.09861} \frac{\text{kJ/kg}}{\text{m}^3\text{/kg}} = 819 \text{ kPa}$$

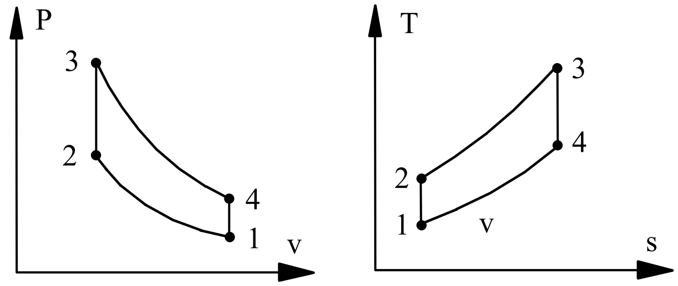
Now we can find the power from Eq.12.11

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 819 \times 0.0024 \times \frac{1800}{60} \times \frac{1}{2} = \mathbf{29.5\ kW}$$

12.77

When methanol produced from coal is considered as an alternative fuel to gasoline for automotive engines, it is recognized that the engine can be designed with a higher compression ratio, say 10 instead of 7, but that the energy release with combustion for a stoichiometric mixture with air is slightly smaller, about 1700 kJ/kg. Repeat Problem 12.67 using these values.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$P_2 = P_1(v_1/v_2)^k = 90(10)^{1.4} = 2260.7 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.15(10)^{0.4} = 711.2 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H / C_{vo} = 711.2 + 1700 / 0.717 = \mathbf{3082 \text{ K}}$$

$$P_3 = P_2(T_3 / T_2) = 2260.7 \times 3082 / 711.2 = \mathbf{9797 \text{ kPa}}$$

Efficiency, net work, displacement and P_{meff}

$$\eta_{TH} = 1 - T_1/T_2 = 1 - 283.15/711.2 = \mathbf{0.602}$$

$$w_{net} = \eta_{TH} \times q_H = 0.6 \times 1700 = 1023.4 \text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.287 \times 283.15/90 = 0.9029 \text{ m}^3\text{/kg},$$

$$v_2 = v_1/10 = 0.0903 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 1023.4 / (0.9029 - 0.0903) = \mathbf{1255 \text{ kPa}}$$

12.78

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.7.2.

Compression 1 to 2: $s_2 = s_1 \Rightarrow v_{r2} = v_{r1}(v_2/v_1)$

$v_{r2} = 196.37/9 = 21.819 \Rightarrow T_2 \cong 680$ K, $P_{r2} \cong 20.784$, $u_2 = 496.94$

$P_2 = P_1(P_{r2}/P_{r1}) = 90\ (20.784 / 0.995) = 1880$ kPa

${}_1w_2 = u_1 - u_2 = 207.19 - 496.94 = -289.75$ kJ/kg

Combustion 2 to 3:

$q_H = u_3 - u_2 = 1486.33 - 496.94 = 989.39$ kJ/kg

$P_3 = P_2(T_3/T_2) = 1880\ (1800 / 680) = 4976$ kPa

Expansion 3 to 4:

$s_4 = s_3 \Rightarrow v_{r4} = v_{r3} \times 9 = 1.143 \times 9 = 10.278$

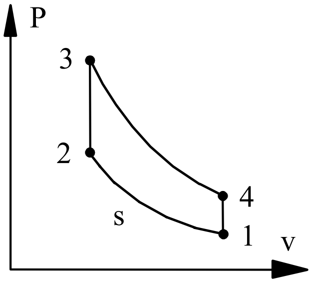
$\Rightarrow T_4 = 889$ K, $P_{r4} = 57.773$, $u_4 = 665.8$ kJ/kg

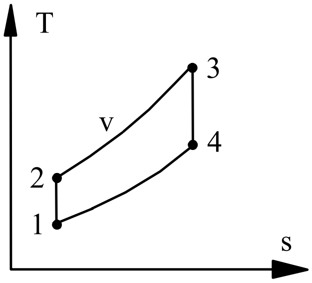
$P_4 = P_3(P_{r4}/P_{r3}) = 4976\ (57.773 / 1051) =$ **273.5 kPa**

${}_3w_4 = u_3 - u_4 = 1486.33 - 665.8 = 820.5$ kJ/kg

$w_{NET} = {}_3w_4 + {}_1w_2 = 820.5 - 289.75 =$ **530.8 kJ/kg**

$\eta = w_{NET}/q_H = 530.8/989.39 =$ **0.536**



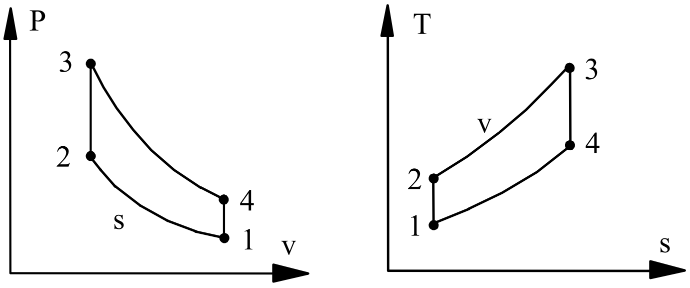


12.79

Air flows into a gasoline engine at 95 kPa, 300 K. The air is then compressed with a volumetric compression ratio of 8:1. In the combustion process 1300 kJ/kg of energy is released as the fuel burns. Find the temperature and pressure after combustion using cold air properties.

Solution:

Solve the problem with variable heat capacity, use A.7.1 and A.7.2.



Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From A.7.2

$$v_{r2} = \frac{v_{r1}}{8} = \frac{179.49}{8} = 22.436,$$

$$T_2 = 673 \text{ K}, \quad u_2 = 491.5 \text{ kJ/kg}, \quad P_{r2} = 20$$

$$P_2 = P_1 \times \frac{P_{r2}}{P_{r1}} = 95 \times \frac{20}{1.1146} = 1705 \text{ kPa}$$

Compression 2 to 3:

$$u_3 = u_2 + q_H = 491.5 + 1300 = 1791.5 \text{ kJ/kg}$$

$$T_3 = \mathbf{2118\ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1705 \times \frac{2118}{673} = \mathbf{5366\ kPa}$$

12.80

Solve Problem 12.70 using the P_r and v_r functions from Table A7.2

A gasoline engine takes air in at 290 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use the cold air properties (i.e. constant heat capacities at 300 K) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Solution:

Standard Otto cycle, solve using Table A.7.1 and Table A.7.2

Combustion process: $T_3 = 2050$ K ; $u_3 = 1725.7$ kJ/kg

$$u_2 = u_3 - q_H = 1725.7 - 1000 = 725.7 \text{ kJ/kg}$$

$$\Rightarrow T_2 = 960.5 \text{ K} ; \qquad v_{r2} = 8.2166$$

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow$ From the v_r function

$$v_1/v_2 = v_{r1}/v_{r2} = \frac{195.36}{8.2166} = \mathbf{23.78}$$

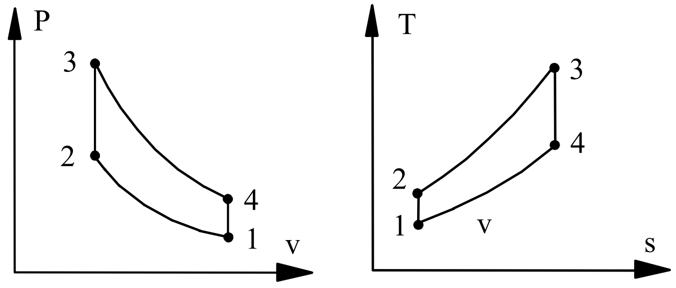
Comment: This is much too high for an actual Otto cycle.

$$-{}_1w_2 = u_2 - u_1 = 725.7 - 207.2 = \mathbf{518.5\ kJ/kg}$$

Highest pressure is after combustion

$$P_3 = P_2T_3 / T_2 = P_1(T_3 / T_1)(v_1 / v_3)$$

$$= 90 \times (2050 / 290) \times 23.78 = \mathbf{15\ 129\ kPa}$$



12.81

It is found experimentally that the power stroke expansion in an internal combustion engine can be approximated with a polytropic process with a value of the polytropic exponent *n* somewhat larger than the specific heat ratio *k*. Repeat Problem 12.67 but assume that the expansion process is reversible and polytropic (instead of the isentropic expansion in the Otto cycle) with *n* equal to 1.50.

See solution to 12.67 except for process 3 to 4.

$T_3 = \mathbf{3127\ K}$, $P_3 = \mathbf{6.958\ MPa}$

$v_3 = RT_3/P_3 = v_2 = 0.129\ m^3/kg$, $v_4 = v_1 = 0.9029\ m^3/kg$

Process: $Pv^{1.5}$ = constant.

$P_4 = P_3(v_3/v_4)^{1.5} = 6958\ (1/7)^{1.5} = 375.7$ kPa

$T_4 = T_3(v_3/v_4)^{0.5} = 3127(1/7)^{0.5} = 1181.9$ K

$${}_1w_2 = \int Pdv = \frac{R}{1\text{-}1.4}(T_2 - T_1) = \frac{0.287}{-0.4}(606.6 - 283.15) = -239.3\ \text{kJ/kg}$$

$${}_3w_4 = \int Pdv = R(T_4 - T_3)/(1 - 1.5)$$

$$= -0.287(1181.9 - 3127)/0.5 = 1116.5\ \text{kJ/kg}$$

$w_{NET} = 1116.5 - 239.3 = 877.2$ kJ/kg

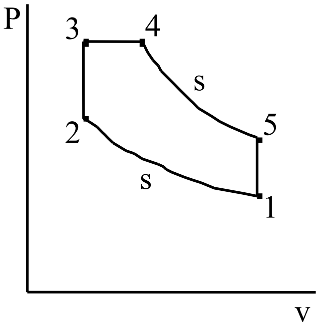
$\eta_{CYCLE} = w_{NET}/q_H = 877.2/1800 = \mathbf{0.487}$

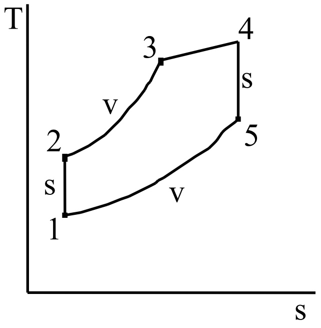
$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 877.2/(0.9029 - 0.129) = \mathbf{1133\ kPa}$$

Note a smaller w_{NET}, η_{CYCLE}, P_{meff} compared to an ideal cycle.

12.82

In the Otto cycle all the heat transfer q_H occurs at constant volume. It is more realistic to assume that part of q_H occurs after the piston has started its downward motion in the expansion stroke. Therefore, consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume that the total q_H is 2100 kJ/kg, that the state at the beginning of the compression process is 90 kPa, 20°C, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.





$P_1 = 90$ kPa, $T_1 = 20^oC$

$r_V = v_1/v_2 = 7$

a) $q_{23} = (2/3) \times 2100$

$= 1400$ kJ/kg;

$q_{34} = 2100/3 = 700$ kJ/kg

b) $P_2 = P_1(v_1/v_2)^k = 90(9)^{1.4} = 1951$ kPa

$T_2 = T_1(v_1/v_2)^{k-1} = 293.15(9)^{0.4} = 706$ K

$T_3 = T_2 + q_{23}/C_{V0} = 706 + 1400/0.717 = \mathbf{2660\ K}$

$P_3 = P_2T_3/T_2 = 1951(2660/706) = \mathbf{7350.8\ kPa} = P_4$

$T_4 = T_3 + q_{34}/C_{P0} = 2660 + 700/1.004 = 3357$ K

$$\frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{P_4}{P_1} \times \frac{T_1}{T_4} = \frac{7350.8}{90} \times \frac{293.15}{3357} = 7.131$$

$T_5 = T_4(v_4/v_5)^{k-1} = 3357(1/7.131)^{0.4} = 1530$ K

$q_L = C_{V0}(T_5\text{-}T_1) = 0.717(1530 - 293.15) = 886.2$ kJ/kg

$\eta_{TH} = 1 - q_L/q_H = 1 - 886.2/2100 = \mathbf{0.578}$

Std. Otto Cycle: $\eta_{TH} = 1 - (9)^{-0.4} = \mathbf{0.585}$, small difference

Diesel Cycles

12.83

A diesel engine has an inlet at 95 kPa, 300 K and a compression ratio of 20:1. The combustion releases 1300 kJ/kg. Find the temperature after combustion using cold air properties.

Solution:

Compression process (isentropic) from Eqs.8.33-34

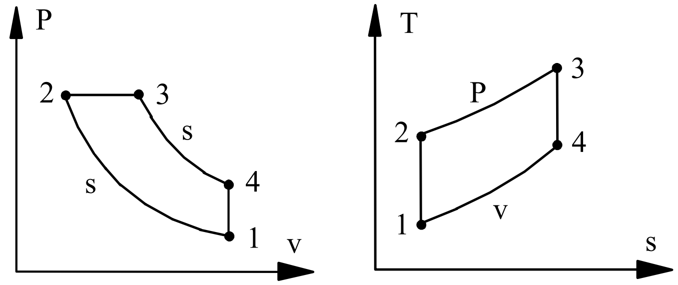
$$T_2 = T_1(v_1 / v_2)^{k-1} = 300 \times 20^{0.4} = 994.3 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$_2w_3 = P\,(v_3 - v_2)$$

$$_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$$

$$T_3 = T_2 + {}_2q_3 / C_{po} = 994.3 + 1300 / 1.004 = \mathbf{2289\ K}$$



12.84

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Standard Diesel cycle and we will use cold air properties.

Compression process (isentropic) from Eqs.8.32-8.34:

$$(P_2/P_1) = (v_1/v_2)^k = CR^{1.4}$$

$$CR = v_1/v_2 = (P_2/P_1)^{1/k} = (6000/95)^{1/1.4} = \mathbf{19.32}$$

$$T_2 = T_1(P_2/P_1)^{k-1/k} = 290 \times (6000/95)^{\,0.2857} = 947.9\text{ K}$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1\ T_3/(T_2 \times CR)\ ; \quad v_4 = v_1$$

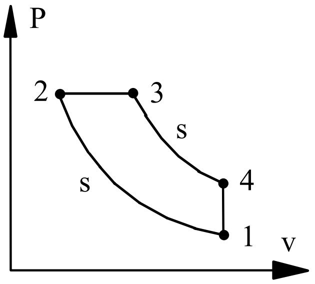
Expansion process, isentropic from Eq.8.32

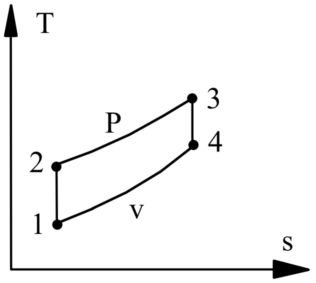
$$T_4 = T_3\ (v_3/v_4)^{k-1} = T_3\ [T_3/\ (CR \times T_2)]^{k-1}$$

$$= 2400 \times [\ 2400/(19.32 \times 947.9)\ ]^{0.4} = 1064.6\text{ K}$$

Efficiency from Eq.12.13

$$\eta = 1 - \frac{1}{k}\frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{1.4}\frac{1064.6 - 290}{2400 - 947.9} = \mathbf{0.619}$$





12.85

Find the cycle efficiency and mean effective pressure for the cycle in Problem 12.83

Solution:

Compression process (isentropic) from Eqs.8.33-34

$$T_2 = T_1(v_1 / v_2)^{k-1} = 300 \times 20^{0.4} = 994.3 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$_2w_3 = P\,(v_3 - v_2)$$

$$_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$$

$$T_3 = T_2 + {}_2q_3 / C_{po} = 994.3 + 1300 / 1.004 = 2289 \text{ K}$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1\, T_3/(T_2 \times CR)\ ; \quad v_4 = v_1$$

Expansion process, isentropic from Eq.8.32

$$T_4 = T_3\,(v_3/v_4)^{k-1} = T_3\,[T_3/(CR \times T_2)]^{k-1}$$

$$= 2289 \times [\,2289/(20 \times 994.3)\,]^{0.4} = 964 \text{ K}$$

Efficiency from Eq.12.13

$$\eta = 1 - \frac{1}{k}\frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{1.4}\,\frac{964 - 300}{2289 - 994.3} = \mathbf{0.634}$$

$$w_{net} = \eta\, q_H = 0.634 \times 1300 = 824.2 \text{ kJ/kg}$$

$$v_1 = RT_1/P_1 = 0.9063 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = \frac{w_{net}}{v_1 - v_1/CR}$$

$$= \frac{824.2}{0.9063\,(1 - 1/20)}\ \frac{\text{kJ/kg}}{\text{m}^3\text{/kg}} = \mathbf{957\ kPa}$$

12.86

A diesel engine has a compression ratio of 20:1 with an inlet of 95 kPa, 290 K, state 1, with volume 0.5 L. The maximum cycle temperature is 1800 K. Find the maximum pressure, the net specific work and the thermal efficiency.

Solution:

Compression process (isentropic) from Eqs.8.33-34

$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961\text{ K}$

$P_2 = 95 \times (20)^{1.4} = 6297.5\text{ kPa}$;

$v_2 = v_1/20 = RT_1/(20\,P_1) = 0.043805\text{ m}^3/\text{kg}$

$-{}_1w_2 = u_2 - u_1 \approx C_{vo}(T_2 - T_1) = 0.717\,(961 - 290) = 481.1\text{ kJ/kg}$

Combustion at constant P which is the maximum presssure

$P_3 = P_2 = \mathbf{6298\ kPa}$;

$v_3 = v_2\,T_3 / T_2 = 0.043805 \times 1800/961 = 0.08205\text{ m}^3/\text{kg}$

${}_2w_3 = P\,(v_3 - v_2) = 6298 \times (0.08215 - 0.043805) = 241.5\text{ kJ/kg}$

${}_2q_3 = u_3 - u_2 + {}_2w_3 = h_3 - h_2 = C_{po}(T_3 - T_2)$

$= 1.004(1800 - 961) = 842.4\text{ kJ/kg}$

Expansion process (isentropic) from Eq.8.33

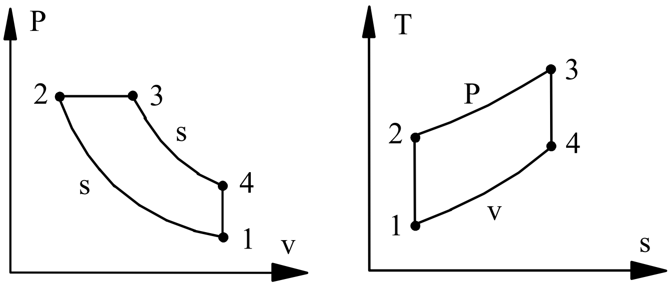
$T_4 = T_3(v_3 / v_4)^{0.4} = 1800\,(0.08205 / 0.8761)^{0.4} = 698\text{ K}$

${}_3w_4 = u_3 - u_4 \approx C_{vo}(T_3 - T_4) = 0.717\,(1800 - 698) = 790.1\text{ kJ/kg}$

Cycle net work and efficiency

$w_{net} = {}_2w_3 + {}_3w_4 + {}_1w_2 = 241.5 + 790.1 - 481.1 = \mathbf{550.5\ kJ/kg}$

$\eta = w_{net} / q_H = 550.5/ 842.4 = \mathbf{0.653}$



12.87

A diesel engine has a bore of 0.1 m, a stroke of 0.11 m and a compression ratio of 19:1 running at 2000 RPM (revolutions per minute). Each cycle takes two revolutions and has a mean effective pressure of 1400 kPa. With a total of 6 cylinders find the engine power in kW and horsepower, hp.

Solution:

Work from mean effective pressure, Eq.12.9.

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} \quad => \quad w_{net} = P_{meff}\,(v_{max} - v_{min})$$

The displacement is

$$\Delta V = \pi \text{Bore}^2 \times 0.25 \times S = \pi \times 0.1^2 \times 0.25 \times 0.11 = 0.000864 \text{ m}^3$$

Work per cylinder per power stroke, Eq.12.10

$$W = P_{meff}(V_{max} - V_{min}) = 1400 \times 0.000864 \text{ kPa m}^3 = 1.2096 \text{ kJ/cycle}$$

Only every second revolution has a power stroke so we can find the power, see also Eq.12.11

$$\dot{W} = W \times N_{cyl} \times \text{RPM} \times 0.5 \text{ (cycles / min)}\times\text{(min / 60 s)}\times\text{(kJ / cycle)}$$

$$= 1.2096 \times 6 \times 2000 \times 0.5 \times (1/60) = \mathbf{121\ kW} = \mathbf{162\ \ hp}$$

The conversion factor from kW to hp is from Table A.1 under power.

12.88

A super charger is used for a two stroke 10 L diesel engine so intake is 200 kPa, 320 K and the cycle has compression ratio of 18:1 and mean effective pressure of 830 kPa. The engine is 10 L running at 200 RPM find the power output.

Solution:

The power is from Eq.12.11

$$\dot{W} = m\, w_{net} \times N_{cyl} \times RPM \times 1/60$$
$$= m\, P_{meff}\, (v_1 - v_2) \times N_{cyl} \times RPM/60 = P_{meff}\, V_{displ}\, RPM/60$$
$$= 830 \times 0.010 \times 200 / 60 = \mathbf{27.7\ kW}$$

12.89

At the beginning of compression in a diesel cycle $T = 300$ K, $P = 200$ kPa and after combustion (heat addition) is complete $T = 1500$ K and $P = 7.0$ MPa. Find the compression ratio, the thermal efficiency and the mean effective pressure.

Solution:

Standard Diesel cycle. See P-v and T-s diagrams for state numbers.

Compression process (isentropic) from Eqs.8.33-8.34

$$P_2 = P_3 = 7000 \text{ kPa} \;\Rightarrow\; v_1 / v_2 = (P_2/P_1)^{1/k} = (7000 / 200)^{0.7143} = \mathbf{12.67}$$

$$T_2 = T_1(P_2 / P_1)^{(k-1)/k} = 300(7000 / 200)^{0.2857} = 828.4 \text{ K}$$

Expansion process (isentropic) first get the volume ratios

$$v_3 / v_2 = T_3 / T_2 = 1500 / 828.4 = 1.81$$

$$v_4 / v_3 = v_1 / v_3 = (v_1 / v_2)(v_2 / v_3) = 12.67 / 1.81 = 7$$

The exhaust temperature follows from Eq.8.33

$$T_4 = T_3(v_3 / v_4)^{k-1} = 1500\ (1 / 7)^{0.4} = 688.7 \text{ K}$$

$$q_L = C_{vo}(T_4 - T_1) = 0.717(688.7 - 300) = 278.5 \text{ kJ/kg}$$

$$q_H = h_3 - h_2 \approx C_{po}(T_3 - T_2) = 1.004(1500 - 828.4) = 674 \text{ kJ/kg}$$

Overall performance

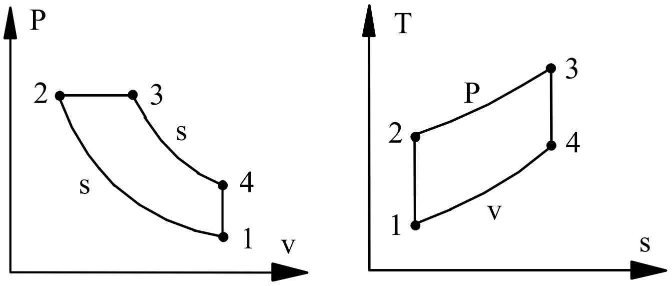
$$\eta = 1 - q_L / q_H = 1 - 278.5 / 674 = \mathbf{0.587}$$

$$w_{net} = q_{net} = q_H - q_L = 674 - 278.5 = 395.5 \text{ kJ/kg}$$

$$v_{max} = v_1 = R\ T_1 / P_1 = 0.287 \times 300 / 200 = 0.4305 \text{ m}^3\text{/kg}$$

$$v_{min} = v_{max} / (v_1 / v_2) = 0.4305 / 12.67 = 0.034 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_{max} - v_{min}} = 395.5 / (0.4305 - 0.034) = \mathbf{997\ kPa}$$



Remark: This is a too low compression ratio for a practical diesel cycle.

12.90

Do problem 12.84, but use the properties from A.7 and not the cold air properties. A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Compression: $s_2 = s_1$ => from Eq.8.28

$$s^o_{T2} = s^o_{T1} + R \ln(P_2 / P_1) = 6.8352 + 0.287 \ln(6000/95) = 8.025 \text{ kJ/kg K}$$

A.7.1 => $T_2 = 907.6$ K; $h_2 = 941.16$ kJ/kg;

$h_3 = 2755.8$ kJ/kg; $s^o_{T3} = 9.19586$ kJ/kg K

$q_H = h_3 - h_2 = 2755.8 – 941.16 = 1814.2$ kJ/kg

$CR = v_1/v_2 = (T_1/T_2)(P_2/P_1) = (290/907.6) \times (6000/ 95) = \mathbf{20.18}$

Expansion process

$$s^o_{T4} = s^o_{T3} + R \ln(P_4 / P_3) = s^o_{T3} + R \ln(T_4 / T_3) + R \ln(v_3/v_4)$$

$$v_3/v_4 = v_3/v_1 = (v_2/v_1) \times (T_3/T_2) = (T_3/T_2)\ (1/CR)$$

$$= (2400/907.6)\ (1/20.18) = 0.13104$$

$$s^o_{T4} - R \ln(T_4 / T_3) = s^o_{T3} + R \ln(v_3/v_4) = 9.1958 + 0.287 \ln 0.13104 = 8.61254$$

Trial and error on T_4 since it appears both in s^o_{T4} and the ln function

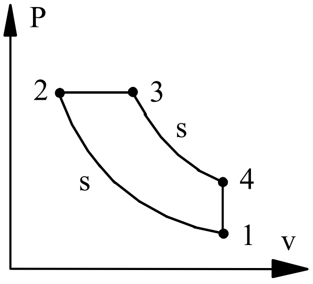
$T_4 = 1300$ K LHS $= 8.4405 – 0.287 \ln (1300/2400) = 8.616$

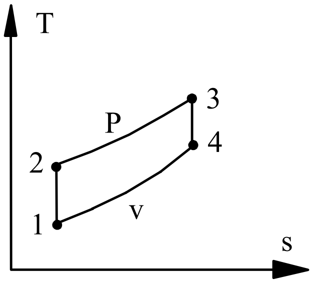
$T_4 = 1250$ K LHS $= 8.3940 – 0.287 \ln (1250/2400) = 8.5812$

Now Linear interpolation $T_4 = 1295$ K, $u_4 = 1018.26$ kJ/kg

$q_L = u_4 - u_1 = 1018.26 – 207.19 = 811.08$ kJ/kg

$\eta = 1 – (q_L/ q_H) = 1 – (811.08/1814.2) = \mathbf{0.553}$





12.91

Solve Problem 12.84 using the P_r and v_r functions from Table A7.2

A diesel engine has a state before compression of 95 kPa, 290 K, and a peak pressure of 6000 kPa, a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency.

Solution:

Compression: $s_2 = s_1$ => From definition of the P_r function

$$P_{r2} = P_{r1}\ (P_2/P_1) = 0.9899\ (6000/95) = 62.52$$

A.7.1 => $T_2 = 907$ K; $h_2 = 941.0$ kJ/kg;

$h_3 = 2755.8$ kJ/kg; $v_{r3} = 0.43338$

$q_H = h_3 - h_2 = 2755.8 – 941.0 = 1814.8$ kJ/kg

$CR = v_1/v_2 = (T_1/T_2)(P_2/P_1) = (290/907) \times (6000/\ 95) = \mathbf{20.19}$

Expansion process

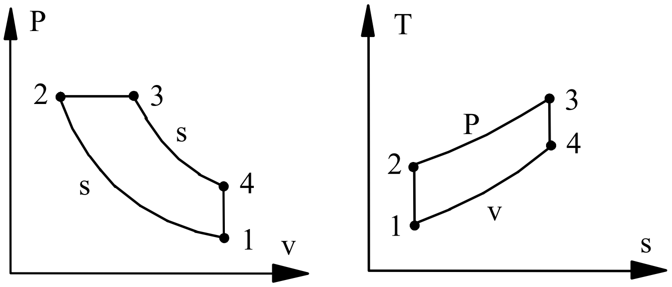
$$v_{r4} = v_{r3}\ (v_4\ /\ v_3) = v_{r3}\ (v_1\ /\ v_3) = v_{r3}\ (v_1\ /\ v_2) \times (T_2/T_3)$$

$$= v_{r3}\ CR \times (T_2/T_3) = 0.43338 \times 20.19 \times (907/2400) = 3.30675$$

Linear interpolation $T_4 = 1294.8$ K, $u_4 = 1018.1$ kJ/kg

$q_L = u_4 - u_1 = 1018.1 – 207.2 = 810.9$ kJ/kg

$\eta = 1 – (q_L/\ q_H) = 1 – (810.9/1814.8) = \mathbf{0.553}$



12.92

The worlds largest diesel engine has displacement of 25 m^3 running at 200 RPM in a two stroke cycle producing 100 000 hp. Assume an inlet state of 200 kPa, 300 K and a compression ratio of 20:1. What is the mean effective pressure?

We have 3 parameters for the cycle: T_1, P_1 and CR we need one more, so this comes from the total rate of work.

$$\dot{W} = m\, w_{net} \times N_{cyl} \times RPM \times 1/60$$
$$= m\, P_{meff}\,(v_1 - v_2) \times N_{cyl} \times RPM/60 = P_{meff}\, V_{displ}\, RPM/60$$

$$P_{meff} = \dot{W}\, 60 / V_{displ}\, RPM = \frac{100\,000 \times 0.746 \text{ kW} \times 60 \text{ s}}{25 \text{ m}^3 \times 200} = \mathbf{895\ kPa}$$

With this information we could now get all the cycle states etc.

A similar engine is in this container ship shown at dock. The engine was build up inside the ship as it is too large to put in finished. The cranes are on the port dock.

12.93

A diesel engine has air before compression at 280 K, 85 kPa. The highest temperature is 2200 K and the highest pressure is 6 MPa. Find the volumetric compression ratio and the mean effective pressure using cold air properties at 300 K.

Solution:

Compression $(P_2/P_1) = (v_1/v_2)^k = CR^k$

$$CR = v_1/v_2 = (P_2/P_1)^{1/k} = (6000/85)^{1/1.4} = \mathbf{20.92}$$

$$T_2 = T_1(P_2/P_1)^{k-1/k} = 280 \times (6000/85)^{0.2857} = 944.8 \text{ K}$$

Combustion. Highest temperature is after combustion.

$$h_2 = C_P(T_3 - T_2) = 1.004(2200 - 944.8) = 1260.2 \text{ kJ/kg}$$

Expansion ($v_4 = CR \times v_2$ and $P_2 = P_3$)

$$T_4 = T_3 (v_3/v_4)^{k-1} = T_3 [v_3 / (CR \times v_2)]^{k-1} = T_3 [T_3/ (CR \times T_2)]^{k-1}$$

$$= 2200 \times (2200/20.92 \times 944.8)^{0.4} = 914.2 \text{ K}$$

$$q_L = u_4 - u_1 = C_V(T_4 - T_1) = 0.717(914.2 - 280) = 454.7 \text{ kJ/kg}$$

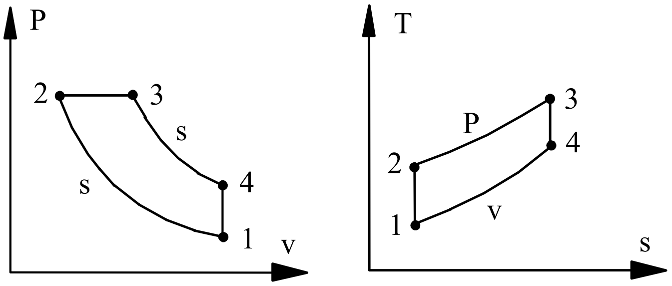
$$v_1 = RT_1/P_1 = 0.287 \times 280/85 = 0.9454 \text{ m}^3\text{/kg}$$

Displacement and mep from net work

$$v_1 - v_2 = v_1 - v_1/CR = v_1[1 - (1/CR)] = 0.9002 \text{ m}^3\text{/kg}$$

$$P_{meff} = w_{net}/(v_1 - v_2) = (q_H - q_L)/(v_1 - v_2)$$

$$= (1260.2 - 454.7)/0.9002 = \mathbf{894.8\ kPa}$$



12.94

Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the thermal efficiency for a maximum temperature of 2200 K

Solution:

Diesel cycle: $P_1 = 95$ kPa, $T_1 = 290$ K, $CR = v_1/v_2 = 20$,

$$v_1 = 0.287 \times 290/95 = 0.8761 \text{ m}^3/\text{kg} = v_4 = CR\ v_2,$$

Compression process (isentropic) from Eqs.8.33-34

$$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961.2 \text{ K}$$

Combustion at constant P which is the maximum pressure

$$v_3 = v_2\ T_3 / T_2 = \frac{0.8761}{20} \times \frac{2200}{961.2} = 0.10026 \text{ m}^3/\text{kg}$$

Expansion process (isentropic) from Eq.8.33

$$T_4 = T_3(v_3 / v_4)^{0.4} = 2200\ (0.10026 / 0.8761)^{0.4} = 924.4 \text{ K}$$

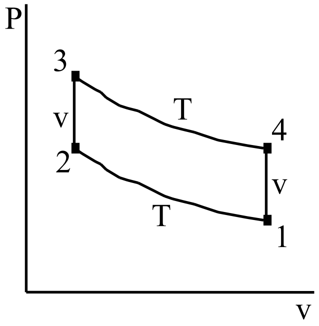
Cycle net work and efficiency

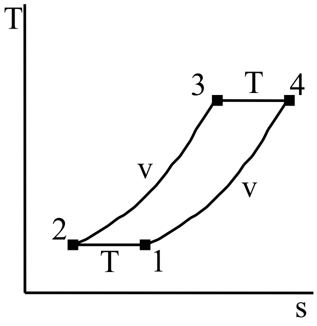
$$\eta_{TH} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{924.4 - 290}{1.4\ (2200 - 961.2)} = \mathbf{0.634}$$

Stirling and Carnot Cycles

12.95

Consider an ideal Stirling-cycle engine in which the state at the beginning of the isothermal compression process is 100 kPa, 25°C, the compression ratio is 6, and the maximum temperature in the cycle is 1100°C. Calculate the maximum cycle pressure and the thermal efficiency of the cycle with and without regenerators.





Ideal Stirling cycle
$T_1 = T_2 = 25\ ^oC$
$P_1 = 100$ kPa
$CR = v_1/v_2 = 6$
$T_3 = T_4 = 1100\ ^oC$

Isothermal compression (heat goes out)

$$T_1 = T_2 \ \Rightarrow\ P_2 = P_1(v_1/v_2) = 100 \times 6 = 600 \text{ kPa}$$

$$_1w_2 = {}_1q_2 = -RT_1 \ln(v_1/v_2) = -0.287\times 298.2 \ln(6) = -153.3 \text{ kJ/kg}$$

Constant volume heat addition

$$V_2 = V_3 \ \Rightarrow\ P_3 = P_2T_3/T_2 = 600\times1373.2/298.2 = \mathbf{2763\ kPa}$$

$$q_{23} = u_3 - u_2 = C_{v\,o}(T_3 - T_2) = 0.717\ (1100 - 25) = 770.8 \text{ kJ/kg}$$

Isothermal expansion (heat comes in)

$$w_{34} = q_{34} = RT_3 \ln(v_4/v_3) = 0.287 \times 1373.2 \times \ln 6 = 706.1 \text{ kJ/kg}$$

$$w_{net} = 706.1 - 153.3 = 552.8 \text{ kJ/kg}$$

Efficiency without regenerator, (q_{23} and q_{34} are coming in from source)

$$\eta_{\text{NO REGEN}} = \frac{w_{net}}{q_{23} + q_{34}} = \frac{552.8}{770.8 + 706.1} = \mathbf{0.374},$$

Efficiency with regenerator, (Now only q_{34} is coming in from source)

$$\eta_{\text{WITH REGEN}} = \frac{w_{net}}{q_{34}} = \frac{552.8}{706.1} = \mathbf{0.783}$$

12.96

An air-standard Stirling cycle uses helium as the working fluid. The isothermal compression brings helium from 100 kPa, 37°C to 600 kPa. The expansion takes place at 1200 K and there is no regenerator. Find the work and heat transfer in all of the 4 processes per kg helium and the thermal cycle efficiency.

Helium table A.5: $R = 2.077$ kJ/kg K, $C_{vo} = 3.1156$ kJ/kg K

Compression/expansion: $v_4 / v_3 = v_1 / v_2 = P_2 / P_1 = 600 / 100 = 6$

1 -> 2 $-{}_1w_2 = -q_{12} = \int P\,dv = R\,T_1 \ln(v_1 / v_2) = RT_1 \ln(P_2 / P_1)$

$= 2.077 \times 310 \times \ln 6 = \mathbf{1153.7\ kJ/kg}$

2 -> 3 : ${}_2w_3 = \mathbf{0}$; $q_{23} = C_{vo}(T_3 - T_2) = 3.1156(1200 - 310) = \mathbf{2773\ kJ/kg}$

3 -> 4: ${}_3w_4 = q_{34} = R\,T_3 \ln\frac{v_4}{v_3} = 2.077 \times 1200 \ln 6 = \mathbf{4465.8\ kJ/kg}$

4 -> 1 ${}_4w_1 = \mathbf{0}$; $q_{41} = C_{vo}(T_4 - T_1) = \mathbf{-2773\ kJ/kg}$

$$\eta_{cycle} = \frac{{}_1w_2 + {}_3w_4}{q_{23} + q_{34}} = \frac{-1153.7 + 4465.8}{2773 + 4465.8} = \mathbf{0.458}$$

12.97

Consider an ideal air-standard Stirling cycle with an ideal regenerator. The minimum pressure and temperature in the cycle are 100 kPa, 25°C, the compression ratio is 10, and the maximum temperature in the cycle is 1000°C. Analyze each of the four processes in this cycle for work and heat transfer, and determine the overall performance of the engine.

Ideal Stirling cycle diagram as in Fig. 12.17, with

$P_1 = 100$ kPa, $T_1 = T_2 = 25^oC$, $v_1/v_2 = 10$, $T_3 = T_4 = 1000^oC$

From 1-2 at const T: ${}_1w_2 = {}_1q_2 = T_1(s_2 - s_1)$

$$= -RT_1 \ln(v_1/v_2) = -0.287 \times 298.2 \times \ln(10) = -197.1 \text{ kJ/kg}$$

From 2-3 at const V: ${}_2w_3 = \mathbf{0}$

$q_{23} = C_{V0}(T_3 - T_2) = 0.717\ (1000 - 25) = \mathbf{699\ kJ/kg}$

From 3-4 at const T; ${}_3w_4 = {}_3q_4 = T_3(s_4 - s_3)$

$$= +RT_3 \times \ln\frac{v_4}{v_3} = 0.287 \times 1237.2 \times \ln(10) = \mathbf{841.4\ kJ/kg}$$

From 4-1 at const V; ${}_4w_1 = \mathbf{0}$

$q_{41} = C_{V0}(T_1 - T_4) = 0.717\ (25 - 1000) = \mathbf{-699\ kJ/kg}$

$w_{NET} = -197.1 + 0 + 841.4 + 0 = 644.3$ kJ/kg

Since q_{23} is supplied by $-q_{41}$ (regenerator)

$$q_H = q_{34} = 841.4 \text{ kJ/kg}, \quad \eta_{TH} = \frac{w_{NET}}{q_H} = \frac{644.3}{841.4} = \mathbf{0.766}$$

NOTE: $q_H = q_{34} = RT_3 \times \ln(10)$, $q_L = -{}_1q_2 = RT_1 \times \ln(10)$

$$\eta_{TH} = \frac{q_H - q_L}{q_H} = \frac{T_3 - T_1}{T_3} = \frac{975}{1273.2} = 0.766 = \text{Carnot efficiency}$$

12.98

The air-standard Carnot cycle was not shown in the text; show the *T–s* diagram for this cycle. In an air-standard Carnot cycle the low temperature is 280 K and the efficiency is 60%. If the pressure before compression and after heat rejection is 100 kPa, find the high temperature and the pressure just before heat addition.

Solution:

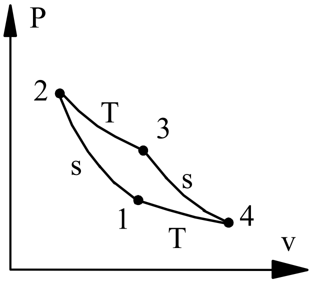
Carnot cycle efficiency from Eq.7.5

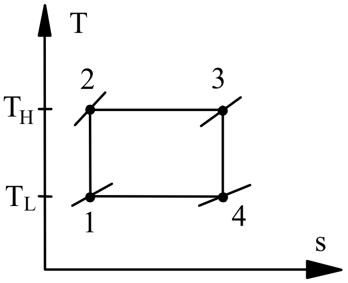
$$\eta = 0.6 = 1 - T_H/T_L$$

$$\Rightarrow\ T_H = T_L/0.4 = \mathbf{700\ K}$$

Just before heat addition is state 2 and after heat rejection is state 1 so $P_1 = 100$ kPa and the isentropic compression is from Eq.8.32

$$P_2 = P_1(T_H/T_L)^{\frac{1}{k-1}} = \mathbf{2.47\ MPa}$$





12.99

Air in a piston/cylinder goes through a Carnot cycle in which $T_L = 26.8°C$ and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion for constant C_P, C_v.

Solution:

Carnot cycle efficiency Eq.7.5:

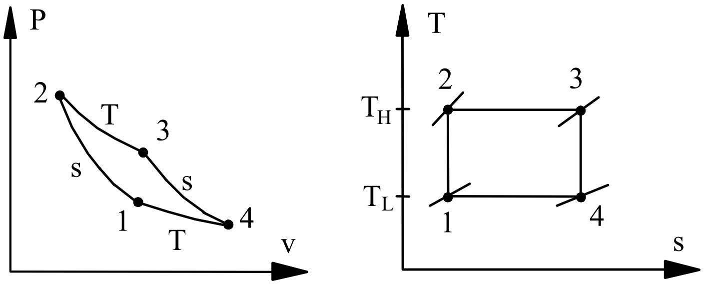
$$\eta = 1 - T_L/T_H = 2/3 \;\Rightarrow\; T_H = 3 \times T_L = 3 \times 300 = \mathbf{900\ K}$$

Adiabatic expansion 3 to 4: Pv^k = constant, work from Eq.8.38 ($n = k$)

$$_3w_4 = (P_4v_4 - P_3v_3)/(1 - k) = \frac{R}{1 - k}(T_4 - T_3) = u_3 - u_4$$

$$= C_v(T_3 - T_4) = 0.717(900 - 300) = \mathbf{429.9\ kJ/kg}$$

$$v_4/v_3 = (T_3/T_4)^{1/(k - 1)} = 3^{2.5} = \mathbf{15.6}$$



12.100

Do the previous problem 12.99 using values from Table A.7.1.

Air in a piston/cylinder goes through a Carnot cycle in which T_L = 26.8°C and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion.

Solution:

Carnot cycle efficiency Eq.7.5:

$$\eta = 1 - T_L/T_H = 2/3 \;\Rightarrow\; T_H = 3 \times T_L = 3 \times 300 = \mathbf{900\ K}$$

From A.7.1: u_3 = 674.82 kJ/kg, s^{o}_{T3} = 8.0158 kJ/kg K

u_4 = 214.36 kJ/kg, s^{o}_{T4} = 6.8693 kJ/kg K

Energy equation with q = 0

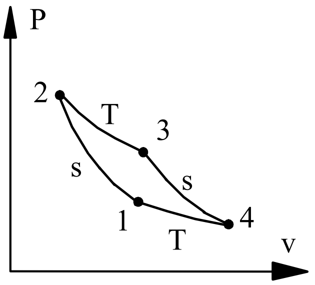
$$_3w_4 = u_3 - u_4 = 674.82 - 214.36 = \mathbf{460.5\ kJ/kg}$$

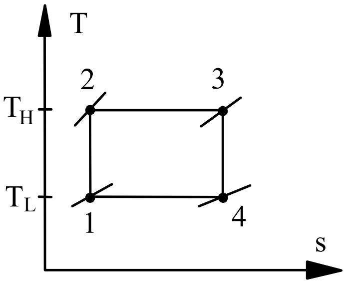
Entropy equation, constant s

$$s^{o}_{T4} = s^{o}_{T3} + R \ln(P_4 / P_3) = s^{o}_{T3} + R \ln(T_4 / T_3) + R \ln(v_3/v_4)$$

$$=> \quad 6.8693 = 8.0158 + 0.287 \ln(300/900) + 0.287 \ln(v_3/v_4)$$

$$=> \quad v_4/v_3 = \mathbf{18.1}$$





12.101

Do Problem 12.99 using the P_r, v_r functions in Table A.7.2

Air in a piston/cylinder goes through a Carnot cycle in which T_L = 26.8°C and the total cycle efficiency is $\eta = 2/3$. Find T_H, the specific work and volume ratio in the adiabatic expansion.

Solution:

Carnot cycle efficiency Eq.7.5:

$$\eta = 1 - T_L/T_H = 2/3 \;\Rightarrow\; T_H = 3 \times T_L = 3 \times 300 = \mathbf{900\ K}$$

From A.7.1: $u_3 = 674.82$ kJ/kg, $v_{r3} = 9.9169$

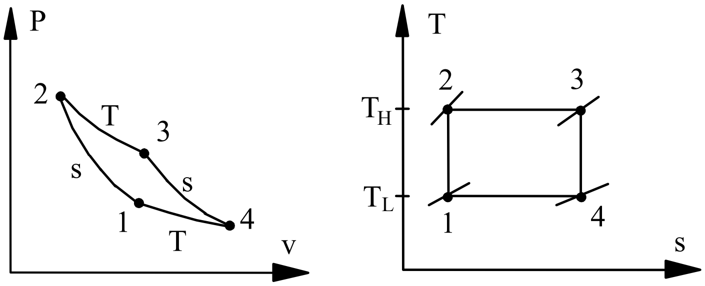
$u_4 = 214.36$ kJ/kg, $v_{r4} = 179.49$

Energy equation with q = 0

$${}_3w_4 = u_3 - u_4 = 674.82 - 214.36 = \mathbf{460.5\ kJ/kg}$$

Entropy equation, constant s expressed with the v_r function

$$v_4/v_3 = v_{r4}/v_{r3} = 179.49 / 9.9169 = \mathbf{18.1}$$



Atkinson and Miller cycles

12.102

An Atkinson cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and a heat release of 1000 kJ/kg. Find the needed expansion ratio.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow\;$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 9^{\,0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \times 9^{\,1.4} = 3251 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 722.5 + 1000/0.717 = 2117 \text{ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 3251\ (2117 / 722.5) = 9526 \text{ kPa}$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \;\Rightarrow\; CR = (P_3 / P_4)^{\,1/k}$$

$$CR = (9526 / 150)^{0.71429} = \mathbf{19.4}$$

12.103

An Atkinson cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and expansion ratio of 14. Find the needed heat release in the combustion.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 9^{0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \times 9^{1.4} = 3251 \text{ kPa}$$

The expansion backwards from $P_4 = P_1$ gives

$$P_3 = P_4 \times (v_4/v_3)^k = P_4 \times CR^k = 150 \times 14^{1.4} = 6034.9 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2 \Rightarrow P_3 = P_2 \times (T_3/T_2)$

Energy Eq.: $u_3 = u_2 + q_H \quad \Rightarrow \quad q_H = C_v (T_3 - T_2)$

Solve for T_3 from process equation

$$T_3 = T_2 \times (P_3 / P_2) = 722.5 \times (6034.9 / 3251) = 1341.2 \text{ K}$$

Solve for q_H from the energy equation

$$q_H = C_v (T_3 - T_2) = 0.717 (1341.2 - 722.5) = \mathbf{443.6 \text{ kJ/kg}}$$

12.104

Assume we change the Otto cycle in Problem 12.63 to an Atkinson cycle by keeping the same conditions and only increase the expansion to give a different state 4. Find the expansion ratio and the cycle efficiency.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1\,(v_1/v_2)^{k-1} = 300 \times 8^{0.4} = 689.2\text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^{k} = 95 \times 8^{1.4} = 1746\text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 689.2 + 1300/0.717 = 2502\text{ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 1746\,(2502 / 689.2) = 6338\text{ kPa}$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^{k} = P_3 \times CR^{-k} \;=>\; CR = (P_3\,/P_4)^{1/k}$$

$$CR = (6338 / 95)^{0.71429} = \mathbf{20.09}$$

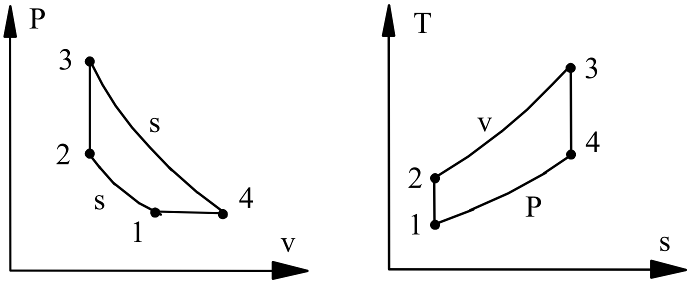
So now the efficiency becomes

$$\eta = 1 - k\,\frac{CR - CR_1}{CR^k - CR_1^k} = 1 - 1.4\,\frac{20.09 - 8}{20.09^{\,k} - 8^{\,k}} = \mathbf{0.65}$$

12.105

Repeat Problem 12.67 assuming we change the Otto cycle to an Atkinson cycle by keeping the same conditions and only increase the expansion to give a different state 4.

Solution:



Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$P_2 = P_1(v_1/v_2)^k = 90\ (7)^{1.4} = 1372 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 283.2 \times (7)^{0.4} = 616.6 \text{ K}$$

Combustion: constant volume

$$T_3 = T_2 + q_H/C_v = 616.6 + 1800/0.717 = \mathbf{3127\ K}$$

$$P_3 = P_2T_3/T_2 = 1372 \times 3127 / 616.6 = \mathbf{6958\ kPa}$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \;\; => \;\; CR = (P_3 /P_4)^{1/k}$$

$$CR = (6958 / 90)^{0.71429} = \mathbf{22.32}$$

So now the efficiency and net work become

$$\eta_{TH} = 1 - k\frac{CR - CR_1}{CR^k - CR_1^k} = 1 - 1.4\frac{22.32 - 7}{22.32^{\,k} - 7^{\,k}} = \mathbf{0.654}$$

$$w_{net} = \eta_{TH} \times q_H = 0.654 \times 1800 = 1177 \text{ kJ/kg}$$

Displacement (we take this as $v_4 - v_2$) and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029 \text{ m}^3\text{/kg}$$

$$v_2 = (1/7)\, v_1 = 0.1290 \text{ m}^3\text{/kg};\;\; v_4 = 22.32\ v_2 = 2.879 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{NET}}{v_4 - v_2} = \frac{1177}{2.879 - 0.129} = \mathbf{428\ kPa}$$

Comment: The ratio $CR / CR_1 = 22.32/7$ is unrealistic large.

12.106

An Atkinson cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and expansion ratio of 14. Find the mean effective pressure.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 9^{\,0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^k = 150 \times 9^{\,1.4} = 3251 \text{ kPa}$$

The expansion backwards from $P_4 = P_1$ gives

$$P_3 = P_4 \times (v_4/v_3)^k = P_4 \times CR^k = 150 \times 14^{1.4} = 6034.9 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $v_3 = v_2 \Rightarrow P_3 = P_2 \times (T_3/T_2)$

Energy Eq.: $u_3 = u_2 + q_H \quad \Rightarrow \quad q_H = C_v\,(T_3 - T_2)$

Solve for T_3 from process equation

$$T_3 = T_2 \times (P_3 / P_2) = 722.5 \times (6034.9 / 3251) = 1341.2 \text{ K}$$

Solve for q_H from the energy equation

$$q_H = C_v\,(T_3 - T_2) = 0.717\,(1341.2 - 722.5) = 443.6 \text{ kJ/kg}$$

The efficiency and net work become

$$\eta_{TH} = 1 - k\,\frac{CR - CR_1}{CR^k - CR_1^k} = 1 - 1.4\,\frac{14 - 9}{14^{\,k} - 9^{\,k}} = 0.623$$

$$w_{net} = \eta_{TH} \times q_H = 0.623 \times 443.6 = 276.4 \text{ kJ/kg}$$

Displacement (we take this as $v_4 - v_2$) and P_{meff}

$$v_1 = RT_1/P_1 = (0.287 \times 300)/150 = 0.574 \text{ m}^3\text{/kg}$$

$$v_2 = (1/9)\,v_1 = 0.06378 \text{ m}^3\text{/kg};\;\; v_4 = 14\,v_2 = 0.8929 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{NET}}{v_4 - v_2} = \frac{276.4}{0.8929 - 0.06378} = \mathbf{333\ kPa}$$

12.107

A Miller cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and expansion ratio of 14. If P_4 is 250 kPa find the heat release in the combustion.

Solution:

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 9^{\,0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^{k} = 150 \times 9^{\,1.4} = 3251 \text{ kPa}$$

The expansion backwards from P_4 gives

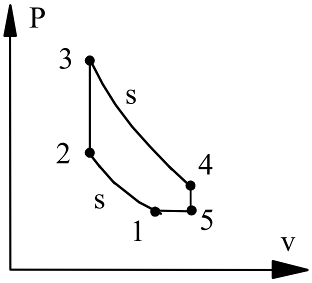
$$P_3 = P_4 \times (v_4/v_3)^{k} = P_4 \times CR^{k} = 250 \times 14^{1.4} = 10\,058 \text{ kPa}$$

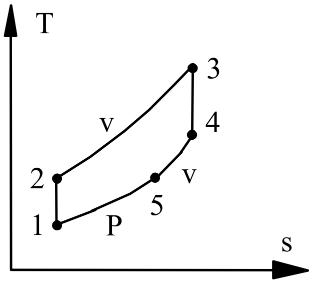
Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v \quad \text{and} \quad P_3 = P_2 \times (T_3/T_2)$$

$$T_3 = T_2 \times (P_3 / P_2) = 722.5 \times (10\,058 / 3251) = 2235.3 \text{ K}$$

$$q_H = C_v\,(T_3 - T_2) = 0.717\,(2235.3 - 722.5) = \mathbf{1085\ kJ/kg}$$





12.108

A Miller cycle has state 1 as 150 kPa, 300 K, compression ratio of 9 and a heat release of 1000 kJ/kg. Find the needed expansion ratio so P_4 is 250 kPa.

Solve the problem with constant heat capacity.

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.33 and Eq.8.34

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 9^{0.4} = 722.5 \text{ K}$$

$$P_2 = P_1 \times (v_1/v_2)^{k} = 150 \times 9^{1.4} = 3251 \text{ kPa}$$

Combustion 2 to 3 at constant volume: $u_3 = u_2 + q_H$

$$T_3 = T_2 + q_H/C_v = 722.5 + 1000/0.717 = 2117 \text{ K}$$

$$P_3 = P_2 \times (T_3/T_2) = 3251\ (2117 / 722.5) = 9526 \text{ kPa}$$

The expansion should now bring pressure down to P_4

$$P_4 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k} \Rightarrow CR = (P_3 / P_4)^{1/k}$$

$$CR = (9526 / 250)^{0.71429} = \mathbf{13.5}$$

12.109

In a Miller cycle assume we know state 1 (intake state) compression ratios CR_1 and CR. Find an expression for the minimum allowable heat release so $P_4 = P_5$ that is, it becomes an Atkinson cycle.

Compression: Reversible and adiabatic so constant s from Eq.8.33-34

$$T_2 = T_1(v_1/v_2)^{k-1} = T_1 CR_1^{k-1}$$

Combustion: constant volume

$$T_3 = T_2 + q_H/C_v \quad \text{and} \quad P_3 = P_2 T_3/T_2$$

The expansion should now bring pressure down so $P_4 = P_1$

$$P_4 = P_1 = P_3 \times (v_3/v_4)^k = P_3 \times CR^{-k}$$

From the ideal gas law and the above pressure relation we have

$$P_3 v_3 / P_1 v_1 = T_3 / T_1 = (P_3 / P_1) CR_1 = CR^k\, CR_1$$

so now substitute

$$T_3 = T_2 + q_H/C_v = T_1 CR_1^{k-1} + q_H/C_v$$

to get

$$T_1 CR_1^{k-1} + q_H/C_v = T_1\, CR^k\, CR_1$$

and now solve for q_H

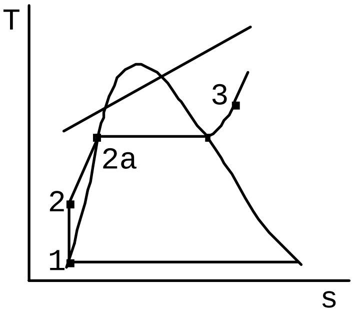
$$q_H = C_v\, T_1\, [CR^k\, CR_1 - CR_1^{k-1}] = \mathbf{C_v\, T_1\, [\, CR^k - CR_1^{\,k}\,] / CR_1}$$

Combined Cycles

12.110

A Rankine steam power plant should operate with a high pressure of 3 MPa, a low pressure of 10 kPa, and the boiler exit temperature should be 500°C. The available high-temperature source is the exhaust of 175 kg/s air at 600°C from a gas turbine. If the boiler operates as a counterflowing heat exchanger where the temperature difference at the pinch point is 20°C, find the maximum water mass flow rate possible and the air exit temperature.

Solution:



C.V. Pump

$w_P = h_2 - h_1 = v_1(P_2 - P_1)$

$= 0.00101(3000 - 10) = 3.02$ kJ/kg

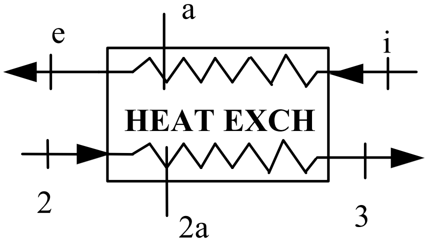
$h_2 = h_1 + w_P = 191.83 + 3.02 = 194.85$ kJ/kg

Heat exchanger water states

State 2a: $T_{2a} = T_{SAT} = 233.9$ °C

$h_{2a} = 1008.42$ kJ/kg

State 3: $h_3 = 3456.5$ kJ/kg



Heat exchanger air states

inlet: $h_{air,in} = 903.16$ kJ/kg

State 2a: $h_{air}(T_{2a} + 20) = 531.28$ kJ/kg

Air temperature should be 253.9°C at the point where the water is at state 2a.

C.V. Section 2a-3, i-a

$$\dot{m}_{H_2O}(h_3 - h_{2a}) = \dot{m}_{air}(h_i - h_a)$$

$$\dot{m}_{H_2O} = 175\,\frac{903.16 - 531.28}{3456.5 - 1008.42} = \mathbf{26.584\ kg/s}$$

Take C.V. Total: $\dot{m}_{H_2O}(h_3 - h_2) = \dot{m}_{air}(h_i - h_e)$

$\Rightarrow\ h_e = h_i - \dot{m}_{H_2O}(h_3 - h_2)/\dot{m}_{air}$

$= 903.6 - 26.584(3456.5 - 194.85)/175 = 408.13$ kJ/kg

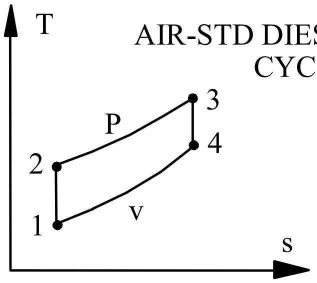
$\Rightarrow\ T_e = 406.7$ K = **133.6 °C**, $T_e > T_2 = 46.5$ °C OK.

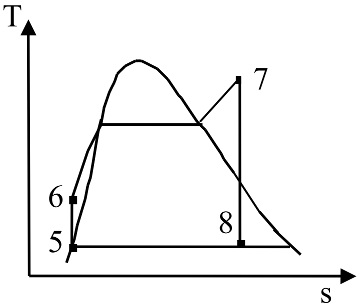
12.111

A simple Rankine cycle with R-410a as the working fluid is to be used as a bottoming cycle for an electrical generating facility driven by the exhaust gas from a Diesel engine as the high temperature energy source in the R-410a boiler. Diesel inlet conditions are 100 kPa, 20°C, the compression ratio is 20, and the maximum temperature in the cycle is 2800°C. The R-410a leaves the bottoming cycle boiler at 80°C, 4 MPa and the condenser pressure is 1800 kPa. The power output of the Diesel engine is 1 MW. Assuming ideal cycles throughout, determine

a. The flow rate required in the diesel engine.

b. The power output of the bottoming cycle, assuming that the diesel exhaust is cooled to 200°C in the R-410a boiler.





Diesel cycle information given means: $\dot{W}_{DIESEL} = 1$ MW

$P_1 = 100$ kPa, $T_1 = 20\ ^oC$, $CR = v_1/v_2 = 20$, $T_3 = 2800^oC$

Consider the Diesel cycle

$$T_2 = T_1(v_1/v_2)^{k-1} = 293.2(20)^{0.4} = 971.8\ K$$

$$P_2 = P_1(v_1/v_2)^k = 100(20)^{1.4} = 6629\ kPa$$

$$q_H = C_{P0}(T_3 - T_2) = 1.004(3073.2 - 971.8) = 2109.8\ kJ/kg$$

$$v_1 = \frac{0.287 \times 293.2}{100} = 0.8415, \quad v_2 = \frac{0.8415}{20} = 0.04208$$

$$v_3 = v_2(T_3/T_2) = 0.04208(3073.2/971.8) = 0.13307$$

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = 3073.2\left(\frac{0.133\ 07}{0.8415}\right)^{0.4} = 1469.6\ K$$

$$q_L = 0.717(293.2 - 1469.6) = -843.5\ kJ/kg$$

$$w_{NET} = 2109.8 - 843.5 = 1266.3\ kJ/kg$$

$$\dot{m}_{AIR} = \dot{W}_{NET}/w_{NET} = 1000/1266.3 = \mathbf{0.79\ kg/s}$$

Rankine cycle information given means:

Boiler exit state: $T_7 = 80^oC$, $P_7 = 4$ MPa,

Condenser: $P_8 = P_5 = 1800$ kPa, $T_5 = 28.22^oC$, $h_5 = h_f = 103.1$ kJ/kg

Consider the Rankine cycle

$s_8 = s_7 = 1.0028$ => Sup. vapor $T_8 = 32.92^oC$, $h_8 = 290.6$ kJ/kg

$w_T = h_7 - h_8 = 311.48 - 290.6 = 20.88$ kJ/kg

$-w_P = v_5(P_6 - P_5) = 0.0009595(4000 - 1800) = 2.11$ kJ/kg

$h_6 = h_5 - w_P = 103.1 + 2.11 = 105.21$ kJ/kg

$q_H = h_7 - h_6 = 311.48 – 105.21 = 206.27$ kJ/kg

Connecting the two cycles.

$\dot{Q}_H$ available from Diesel exhaust cooled to 200 ^oC:

$$\dot{Q}_H = 0.79 \times 0.717(1469.6 - 473.2) = 564 \text{ kW}$$

$$\Rightarrow \dot{m}_{R\text{-}410a} = \dot{Q}_H/q_H = 564/206.27 = 2.734 \text{ kg/s}$$

$$\dot{W}_{R\text{-}410a} = 2.734\ (20.88 - 2.11) = \mathbf{51.3\ kW}$$

12.112

A small utility gasoline engine of 250 cc runs at 1500 RPM with a compression ratio of 7:1. The inlet state is 75 kPa, 17°C and the combustion adds 1500 kJ/kg to the charge. This engine runs a heat pump using R-410a with high pressure of 4 MPa and an evaporator operating at 0°C. Find the rate of heating the heat pump can deliver.

Overall cycle efficiency is from Eq.12.12, $r_v = v_1/v_2 = 7$

$$\eta_{TH} = 1 - r_{1-k} = 1 - 7^{-0.4} = 0.5408$$

$$w_{net} = \eta_{TH} \times q_H = 0.5408 \times 1500 = 811.27 \text{ kJ/kg}$$

We also need specific volume to evaluate Eqs.12.9 to 12.11

$$v_1 = RT_1 / P_1 = 0.287 \times 290 / 75 = 1.1097 \text{ m}^3\text{/kg}$$

$$v_2 = v_1/CR = 0.15853 \text{ m}^3\text{/kg}$$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = \frac{811.27}{1.097 - 0.15853} \frac{\text{kJ/kg}}{\text{m}^3\text{/kg}} = 852.9 \text{ kPa}$$

Now we can find the power from Eq.12.11 (assume 4 stroke engine)

$$\dot{W} = P_{meff} V_{displ} \frac{RPM}{60} \frac{1}{2} = 852.9 \times 2.5 \times 10^{-4} \times \frac{1500}{60} \times \frac{1}{2} = 2.665 \text{ kW}$$

For the refrigeration cycle we have:

State 1: $h_1 = 279.12$ kJ/kg, $s_1 = 1.0368$ kJ/kgK

State 2: 4 MPa, $s_2 = s_1$ interpolate => $h_2 = 323.81$ kJ/kg

State 3: 4 MPa interpolate $h_3 = 171.62$ kJ/kg

$$\beta_{HP} = \frac{q_H}{w_C} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{323.81 - 171.62}{323.81 - 279.12} = 3.405$$

The work out of the heat engine equals the input to the heat pump

$$\dot{Q}_H = \beta_{HP} \dot{W} = 3.405 \times 2.665 \text{ kW} = \mathbf{9.07 \text{ kW}}$$

12.113

Can the combined cycles in the previous problem deliver more heat than what comes from the R-410a? Find any amounts if so by assuming some conditions.

The hot exhaust gases from the gasoline engine can also be used and as the heat pump delivers heat at a temperature that equals the saturation T at 4 MPa which is 61.9°C, the exhaust gas can be cooled to that. Not all the heat out of the heat engine is exhaust gas, part of $\dot{Q}_L$ is heat transfer rejected by the cooling (air or coolant). So as the engine produces 2.66 kW with an efficiency of 54% we must have:

$$\dot{Q}_{H\ fuel} = \dot{W} / \eta_{TH} = 2.66 / 0.54 = 4.93 \text{ kW} \quad \text{input so}$$

$$\dot{Q}_{L\ engine} = \dot{Q}_{H\ fuel} - \dot{W} = 4.93 - 2.66 = 2.3 \text{ kW}$$

Generally half of this is exhaust gas and the other half heat transfer. So we can potentially get an additional 1 kW to heat with from the exhaust gas alone in addition to the heat pump output.

12.114

The power plant shown in Fig. 12.21 combines a gas-turbine cycle and a steam-turbine cycle. The following data are known for the gas-turbine cycle. Air enters the compressor at 100 kPa, 25°C, the compressor pressure ratio is 14, the heater input rate is 60 MW; the turbine inlet temperature is 1250°C, the exhaust pressure is 100 kPa; the cycle exhaust temperature from the heat exchanger is 200°C. The following data are known for the steam-turbine cycle. The pump inlet state is saturated liquid at 10 kPa, the pump exit pressure is 12.5 MPa; turbine inlet temperature is 500°C. Determine

a. The mass flow rate of air in the gas-turbine cycle.
b. The mass flow rate of water in the steam cycle.
c. The overall thermal efficiency of the combined cycle.

a) From Air Tables, A.7: $P_{r1} = 1.0913$, $h_1 = 298.66$, $h_5 = 475.84$ kJ/kg

$s_2 = s_1 \Rightarrow P_{r2S} = P_{r1}(P_2/P_1) = 1.0913 \times 14 = 15.2782$

$T_2 = 629$ K, $h_2 = 634.48$

$w_C = h_1 - h_2 = 298.66 - 634.48 = -335.82$ kJ/kg

At $T_3 = 1523.2$ K: $P_{r3} = 515.493$, $h_3 = 1663.91$ kJ/kg

$$\dot{m}_{AIR} = \dot{Q}_H/(h_3 - h_2) = \frac{60\,000}{1663.91 - 634.48} = \textbf{58.28 kg/s}$$

b) $P_{r4S} = P_{r3}(P_4/P_3) = 515.493(1/14) = 36.8209$

$\Rightarrow T_4 = 791$ K, $h_4 = 812.68$ kJ/kg

$w_{T\ gas} = h_3 - h_4 = 1663.91 - 812.68 = 851.23$ kJ/kg

Steam cycle: $-w_P \approx 0.00101(12500 - 10) = 12.615$ kJ/kg

$h_6 = h_9 - w_P = 191.83 + 12.615 = 204.45$ kJ/kg

At 12.5 MPa, 500 °C: $h_7 = 3341.1$ kJ/kg, $s_7 = 6.4704$ kJ/kg K

$$\dot{m}_{H_2O} = \dot{m}_{AIR}\frac{h_4 - h_5}{h_7 - h_6} = 58.28\,\frac{812.68 - 475.84}{3341.1 - 204.45} = \textbf{6.259 kg/s}$$

c) $s_8 = s_7 = 6.4704 = 0.6492 + x_8 \times 7.501$, $x_8 = 0.7761$

$h_8 = 191.81 + 0.7761 \times 2392.8 = 2048.9$ kJ/kg

$w_{T\ steam} = h_7 - h_8 = 3341.1 - 2048.9 = 1292.2$ kJ/kg

$$\dot{W}_{NET} = \left[\dot{m}(w_T + w_C)\right]_{AIR} + \left[\dot{m}(w_T + w_P)\right]_{H_2O}$$

$$= 58.28\ (851.23 - 335.82) + 6.259\ (1292.2 - 12.615)$$

$$= 30\ 038 + 8009 = 38\ 047\ \text{kW} = 38.05\ \text{MW}$$

$$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 38.047/60 = \mathbf{0.634}$$

Availabilty or Exergy Concepts

12.115

Consider the Brayton cycle in Problem 12.21. Find all the flows and fluxes of exergy and find the overall cycle second-law efficiency. Assume the heat transfers are internally reversible processes, and we neglect any external irreversibility.

Solution:

Efficiency is from Eq.12.1

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 16^{-0.4/1.4} = 0.547$$

from the required power we can find the needed heat transfer

$$\dot{Q}_H = \dot{W}_{net} / \eta = \frac{14\,000}{0.547} = 25\,594 \text{ kW}$$

$$\dot{m} = \dot{Q}_H / q_H = 25\,594 \text{ kW}/ 960 \text{ kJ/kg} = \mathbf{26.66 \text{ kg/s}}$$

Temperature after compression is

$$T_2 = T_1\, r_p^{(k-1)/k} = 290 \times 16^{0.4/1.4} = 640.35 \text{ K}$$

The highest temperature is after combustion

$$T_3 = T_2 + q_H/C_p = 640.35 + \frac{960}{1.004} = 1596.5 \text{ K}$$

For the exit flow I need the exhaust temperature

$$T_4 = T_3\ \ r_p^{-\frac{k-1}{k}} = 1596.5 \times 16^{-0.2857} = 723 \text{ K}$$

$$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H \qquad \text{since the low T exergy flow out is lost}$$

The high T exergy input from combustion is

$$\dot{\Phi}_H = \dot{m}(\psi_3 - \psi_2) = \dot{m}[h_3 - h_2 - T(s_3 - s_2)]$$

$$= 26.66\, [960 - 298 \times 1.004 \ln (\frac{1596.5}{640.35})] = \mathbf{18\,303 \text{ kW}}$$

$$\eta_{II} = \dot{W}_{NET}/\dot{\Phi}_H = 14\,000 / 18\,303 = \mathbf{0.765}$$

$$\dot{\Phi}_{flow\ in} = \dot{m}(\psi_1 - \psi_o) = \dot{m}[h_1 - h_o - T_o(s_1 - s_o)]$$

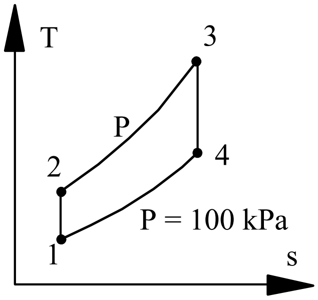
$$= 26.66\, [\ 1.004(17 - 25) - 298 \times 1.004 \ln (\frac{290}{298})\ ] = \mathbf{2.0 \text{ kW}}$$

$$\dot{\Phi}_{flow\ out} = \dot{m}(\psi_4 - \psi_o) = \dot{m}[h_4 - h_o - T(s_4 - s_o)]$$

$$= 26.66\, [\ 1.004(723 - 298) - 298 \times 1.004 \ln (\frac{723}{298})\ ] = \mathbf{4302 \text{ kW}}$$

12.116

A Brayton cycle has a compression ratio of 15:1 with a high temperature of 1600 K and an inlet state of 290 K, 100 kPa. Use cold air properties to find the specific net work output and the second law efficiency if we neglect the "value" of the exhaust flow.



Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Solve using constant C_{P0}

Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 628.65) = 975.2 \text{ kJ/kg}$$

Do the overall cycle efficiency and net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = \mathbf{0.5387}$$

$$w_{NET} = \eta\ q_H = 0.5387 \times 975.2 = 525.34 \text{ kJ/kg}$$

Notice the q_H does not come at a single T so neglecting external irreversibility we get

$$\Phi_{qH} = \text{increase in flow exergy} = \psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$

$$= q_H - T_o\ C_P \ln(T_3/T_2) = 975.2 - 298.15 \times 1.004 \ln\left(\frac{1600}{628.65}\right)$$

$$= 695.56 \text{ kJ/kg}$$

$$\eta_{II} = \frac{w_{net}}{\psi_3 - \psi_2} = \frac{525.34}{695.56} = \mathbf{0.755}$$

12.117

Reconsider the previous problem and find the second law efficiency if you do consider the “value” of the exhaust flow.

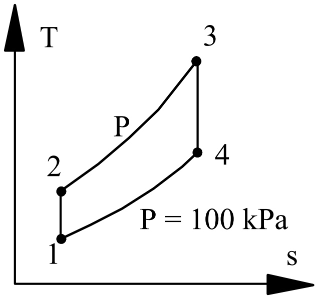
Brayton cycle so this means:

Minimum T: $T_1 = 290$ K

Maximum T: $T_3 = 1600$ K

Pressure ratio: $P_2/P_1 = 15$

Solve using constant C_{P0}



Compression in compressor: $s_2 = s_1 \Rightarrow$ Implemented in Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 290(15)^{0.286} = 628.65 \text{ K}$$

Energy input is from the combustor

$$q_H = h_3 - h_2 = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 628.65) = 975.2 \text{ kJ/kg}$$

Do the overall cycle thermal efficiency and net work

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k} = 1 - 15^{-0.4/1.4} = \mathbf{0.5387}$$

$$w_{NET} = \eta\ q_H = 0.5387 \times 975.2 = 525.34 \text{ kJ/kg}$$

Notice the q_H does not come at a single T so neglecting external irreversibility we get

$$\Phi_{qH} = \text{increase in flow exergy} = \psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$$

$$= q_H - T_o\ C_P \ln(T_3/T_2) = 975.2 - 298.15 \times 1.004 \ln\left(\frac{1600}{628.65}\right)$$

$$= 695.56 \text{ kJ/kg}$$

$$T_4 = T_3\ (P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/15)^{0.286} = 738 \text{ K}$$

$$\Phi_{qL} = \psi_4 - \psi_1 = h_4 - h_1 - T_o(s_4 - s_1) = C_P(T_4 - T_1) - T_o\ C_P \ln\frac{T_4}{T_1}$$

$$= 1.004\ [\ 738 - 290 - 298 \ln(\frac{738}{290})\ ] = 170.33 \text{ kJ/kg}$$

$$\eta_{II} = \frac{w_{net}}{\Phi_{qH} - \Phi_{qL}} = \frac{525.34}{695.56 - 170.33} = \mathbf{1}$$

Why is it 1?, the cycle is reversible so we could have said that right away.

12.118

For Problem 12.110, determine the change of availability of the water flow and that of the air flow. Use these to determine a second law efficiency for the boiler heat exchanger.

From solution to 12.110:

$\dot{m}_{H2O} = 26.584$ kg/s, $h_2 = 194.85$ kJ/kg, $s_2 = 0.6587$ kJ/kg K

$h_3 = 3456.5$ kJ/kg, $s_3 = 7.2338$, $s^o_{Ti} = 7.9820$, $s^o_{Te} = 7.1762$ kJ/kg K

$h_i = 903.16$ kJ/kg, $h_e = 408.13$ kJ/kg

$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = 1301.28$ kJ/kg

$\psi_i - \psi_e = h_i - h_e - T_0(s^o_{Ti} - s^o_{Te}) = 254.78$ kJ/kg

$$\eta_{II} = \frac{(\psi_3 - \psi_2)\dot{m}_{H2O}}{(\psi_i - \psi_e)\dot{m}_{air}} = \frac{1301.28 \times 26.584}{254.78 \times 175} = \mathbf{0.776}$$

12.119

Determine the second law efficiency of an ideal regenerator in the Brayton cycle.

The ideal regenerator has no temperature difference between the two flows and is therefore reversible. Its second law efficiency is **100%**.

12.120

Assume a regenerator in a Brayton cycle has an efficiency of 75%. Find an expression for the second law efficiency.

To do this, assume the pressure is constant in the two flows so we do not have any pressure loss. The change in enthalpy and entropy is then only due to changes in temperature for the two flows.

$$\eta_{II} = \frac{\psi_x - \psi_2}{\psi_4 - \psi_2} = \frac{h_x - h_2 - T_o(s_x - s_2)}{h_4 - h_2 - T_o(s_4 - s_2)} = \frac{C_P\,(T_x - T_2) - T_o\,C_P \ln(T_x/T_2)}{C_P\,(T_4 - T_2) - T_o\,C_P \ln(T_4/T_2)}$$

$$= \frac{T_x - T_2 - T_o\ \ln(T_x/T_2)}{T_4 - T_2 - T_o\ \ln(T_4/T_2)}$$

Finally we have

$$\eta_{II} = \frac{\eta_{reg} - a \ln(T_x/T_2)}{1 - a \ln(T_4/T_2)}$$

where a is defined as

$$a = \frac{T_o}{T_4 - T_2}$$

12.121

The Brayton cycle in Problem 12.14 had a heat addition of 670 kJ/kg. What is the exergy increase in the heat addition process?

Solution:

Combustion: $h_3 = h_2 + q_H$; ${}_2w_3 = 0$ and $T_{max} = T_3 = 1200$ K

$T_2 = T_3 - q_H/C_P = 1200 – 670/1.004 = 532.7$ K

Now the exergy increase from 2 to 3 becomes ($P_3 = P_2$)

$$\psi_3 - \psi_2 = h_3 - h_2 - T_0(s_3 - s_2) = q_H - T_0\, C_P \ln(T_3/T_2)$$

$$= 670 – 298.15 \times 1.004 \ln\left(\frac{1200}{532.7}\right)$$

$$= \mathbf{426.9\ kJ/kg}$$

12.122

The conversion efficiency of the Brayton cycle in Eq.12.1 was done with cold air properties. Find a similar formula for the second law efficiency assuming the low T heat rejection is assigned zero exergy value.

The thermal efficiency (first law) is Eq.12.1

$$\eta_{TH} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k}$$

The corresponding 2nd law efficiency is

$$\eta_{II} = \frac{w_{net}}{\Phi_H} = \frac{h_3 - h_2 - (h_4 - h_1)}{h_3 - h_2 - T_o(s_3 - s_2)}$$

where we used Φ_H = increase in flow exergy = $\psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$. Now divide the difference $h_3 - h_2$ out to get

$$\eta_{II} = \frac{1 - \dfrac{h_4 - h_1}{h_3 - h_2}}{1 - T_o(s_3 - s_2)/(h_3 - h_2)} = \frac{\eta_{TH}}{1 - T_o(s_3 - s_2)/(h_3 - h_2)}$$

$$= \frac{\eta_{TH}}{1 - T_o \dfrac{C_P \ln(T_3 / T_2)}{C_P (T_3 - T_2)}}$$

$$= \frac{\eta_{TH}}{1 - T_o \dfrac{\ln(T_3 / T_2)}{T_3 - T_2}}$$

Comment: Due to the temperature sensitivity of Φ_H the temperatures do not reduce out from the expression.

12.123

Redo the previous problem for a large stationary Brayton cycle where the low T heat rejection is used in a process application and thus has a non-zero exergy.

The thermal efficiency (first law) is Eq.12.1

$$\eta_{TH} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = 1 - r_p^{-(k-1)/k}$$

The corresponding 2nd law efficiency is

$$\eta_{II} = \frac{w_{net}}{\Phi_H - \Phi_L} = \frac{h_3 - h_2 - (h_4 - h_1)}{h_3 - h_2 - T_o(s_3 - s_2)}$$

where we used

Φ_H = increase in flow exergy = $\psi_3 - \psi_2 = h_3 - h_2 - T_o(s_3 - s_2)$.

Φ_L = rejection of flow exergy = $\psi_4 - \psi_1 = h_4 - h_1 - T_o(s_4 - s_1)$.

Now divide all terms with the difference $h_3 - h_2$ to get

$w_{net} /(h_3 - h_2) = \eta_{TH}$;

$\Phi_H/(h_3 - h_2) = 1 - T_o(s_3 - s_2)(h_3 - h_2) = 1 - T_o \ln(T_3 /T_2)\ (T_3 - T_2)^{-1}$

$$\Phi_L/(h_3 - h_2) = \frac{h_4 - h_1}{h_3 - h_2} [\ 1 - T_o \frac{s_4 - s_1}{h_4 - h_1}\ ] = (1 - \eta_{TH})\Big[1 - T_o \frac{\ln T_4/T_1}{T_4 - T_1}\Big]$$

Substitute all terms to get

$$\eta_{II} = \frac{\eta_{TH}}{1 - T_o \frac{\ln(T_3 / T_2)}{T_3 - T_2} - (1 - \eta_{TH})[1 - T_o \frac{\ln(T_4 / T_1)}{T_4 - T_1}]}$$

Comment: Due to the temperature sensitivity of Φ_H and Φ_L the temperatures do not reduce out from the expression.

Review problems

12.124

Repeat Problem 12.31, but assume that the compressor has an efficiency of 82%, that both turbines have efficiencies of 87%, and that the regenerator efficiency is 70%.

a) From solution 12.31: $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8\text{ K}$

$$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6\text{ kJ/kg}$$

$$-w_C = -w_{SC}/\eta_{SC} = 201.6/0.82 = 245.8\text{ kJ/kg} = w_{T1}$$

$$= C_{P0}(T_4 - T_5) = 1.004(1600 - T_5) \quad\Rightarrow\quad T_5 = 1355.2\text{ K}$$

$$w_{ST1} = w_{T1}/\eta_{ST1} = 245.8/0.87 = 282.5\text{ kJ/kg}$$

$$= C_{P0}(T_4 - T_{5S}) = 1.004(1600 - T_{5S}) \quad\Rightarrow\quad T_{5S} = 1318.6\text{ K}$$

$$s_{5S} = s_4 \Rightarrow P_5 = P_4(T_{5S}/T_4)^{\frac{k}{k-1}} = 600\left(\frac{1318.6}{1600}\right)^{3.5} = \mathbf{304.9\ kPa}$$

b) $P_6 = 100$ kPa, $s_{6S} = s_5$

$$T_{6S} = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1355.2\left(\frac{100}{304.9}\right)^{0.286} = 985.2\text{K}$$

$$w_{ST2} = C_{P0}(T_5 - T_{6S}) = 1.004(1355.2 - 985.2) = 371.5\text{ kJ/kg}$$

$$w_{T2} = \eta_{ST2} \times w_{ST2} = 0.87 \times 371.5 = \mathbf{323.2\ kJ/kg}$$

$$323.2 = C_{P0}(T_5 - T_6) = 1.004(1355.2 - T_6) \Rightarrow T_6 = 1033.3\text{K}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 150/323.2 = \mathbf{0.464\ kg/s}$$

c) $w_C = 245.8 = C_{P0}(T_2 - T_1) = 1.004(T_2 - 300) \Rightarrow T_2 = 544.8\text{ K}$

$$\eta_{REG} = \frac{h_3 - h_2}{h_6 - h_2} = \frac{T_3 - T_2}{T_6 - T_2} = \frac{T_3 - 544.8}{1033.3 - 544.8} = 0.7$$

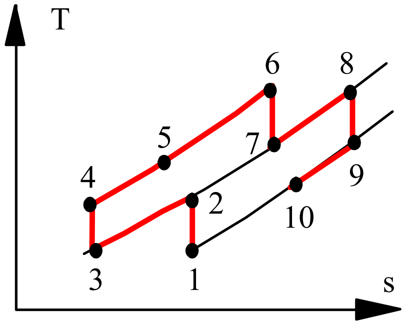
$$\Rightarrow T_3 = 886.8\text{ K}$$

$$q_H = C_{P0}(T_4 - T_3) = 1.004(1600 - 886.8) = 716\text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 323.2/716 = \mathbf{0.451}$$

12.125

Consider a gas turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each compressor stage and each turbine stage is 8 to 1. The pressure at the entrance of the first compressor is 100 kPa, the temperature entering each compressor is 20°C, and the temperature entering each turbine is 1100°C. A regenerator is also incorporated into the cycle and it has an efficiency of 70%. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.



$P_2/P_1 = P_4/P_3 = P_6/P_7 = P_8/P_9 = 8.0$

$P_1 = 100$ kPa

$T_1 = T_3 = 20\ ^oC, \quad T_6 = T_8 = 1100\ ^oC$

Cold air and $s_2 = s_1$ and $s_4 = s_3$

$$T_4 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 293.15(8)^{0.286} = 531 \text{ K}$$

Total $-w_C = 2 \times (-w_{12}) = 2C_{P0}(T_2 - T_1) = 2 \times 1.004(531 - 293.15) =$ **477.6 kJ/kg**

Also $s_6 = s_7$ and $s_8 = s_9$: $\Rightarrow \quad T_7 = T_9 = T_6\left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 1373.15\left(\frac{1}{8}\right)^{0.286} = 758 \text{ K}$

Total $w_T = 2 \times w_{67} = 2C_{P0}(T_6 - T_7) = 2 \times 1.004(1373.15 - 758) = 1235.2$ kJ/kg

$w_{NET} = 1235.2 - 477.6 = 757.6$ kJ/kg

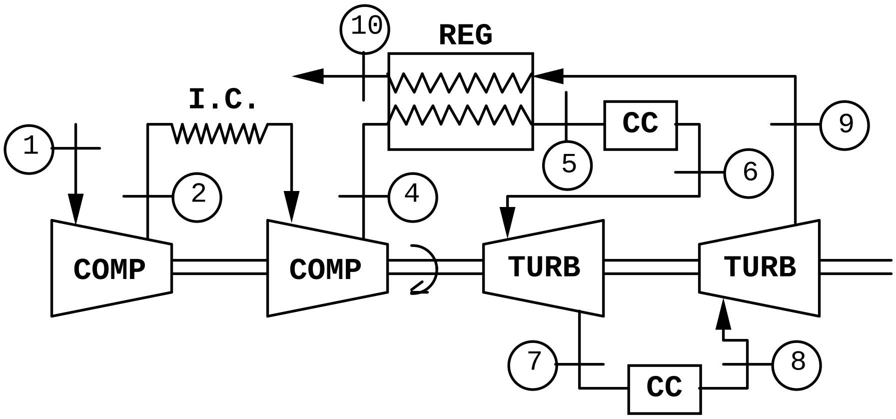
Ideal regenerator: $T_5 = T_9$, $T_{10} = T_4$ so the actual one has

$$\eta_{REG} = \frac{h_5 - h_4}{h_9 - h_4} = \frac{T_5 - T_4}{T_9 - T_4} = \frac{T_5 - 531}{758 - 531} = 0.7 \qquad \Rightarrow T_5 = 689.9 \text{ K}$$

$\Rightarrow q_H = (h_6 - h_5) + (h_8 - h_7) = C_{P0}(T_6 - T_5) + C_{P0}(T_8 - T_7)$

$= 1.004(1373.15 – 689.9) + 1.004\ (1373.15 – 758) = 1303.6$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 757.6/1303.6 =$ **0.581**


10
REG
I.C.
1
2
4
CC
5
6
9
COMP
COMP
TURB
TURB
7
CC
8

12.126

A gas turbine cycle has two stages of compression, with an intercooler between the stages. Air enters the first stage at 100 kPa, 300 K. The pressure ratio across each compressor stage is 5 to 1, and each stage has an isentropic efficiency of 82%. Air exits the intercooler at 330 K. The maximum cycle temperature is 1500 K, and the cycle has a single turbine stage with an isentropic efficiency of 86%. The cycle also includes a regenerator with an efficiency of 80%. Calculate the temperature at the exit of each compressor stage, the second-law efficiency of the turbine and the cycle thermal efficiency.

State 1: $P_1 = 100$ kPa, $T_1 = 300$ K State 7: $P_7 = P_o = 100$ kPa

State 3: $T_3 = 330$ K; State 6: $T_6 = 1500$ K, $P_6 = P_4$

$P_2 = 5\ P_1 = 500$ kPa; $P_4 = 5\ P_3 = 2500$ kPa

Ideal compression $T_{2s} = T_1\ (P_2/P_1)^{(k-1)/k} = 475.4$ K

1st Law: $q + h_i = h_e + w;\ q = 0 \ \Rightarrow\ w_{c1} = h_1 - h_2 = C_P(T_1 - T_2)$

$w_{c1\ s} = C_P(T_1 - T_{2s}) = -176.0$ kJ/kg, $w_{c1} = w_{c1\ s}/\ \eta = -214.6$

$T_2 = T_1 - w_{c1}/C_P =$ **513.9 K**

$T_{4s} = T_3\ (P_4/P_3)^{(k-1)/k} = 475.4$ K

$w_{c2\ s} = C_P(T_3 - T_{4s}) = -193.6$ kJ/kg; $w_{c2} = -236.1$ kJ/kg

$T_4 = T_3 - w_{c2}\ /\ C_P =$ **565.2 K**

Ideal Turbine (reversible and adiabatic)

$T_{7s} = T_6(P_7/P_6)^{(k-1)/k} = 597.4$ K $\ \Rightarrow\ w_{Ts} = C_P(T_6 - T_{7s}) = 905.8$ kJ/kg

1st Law Turbine: $q + h_6 = h_7 + w;\quad q = 0$

$w_T = h_6 - h_7 = C_P(T_6 - T_7) = \eta_{Ts}\ w_{Ts} = 0.86 \times 905.8 = 779.0$ kJ/kg

$T_7 = T_6 - w_T/\ C_P = 1500 - 779/1.004 = 723.7$ K

$$s_6 - s_7 = C_P \ln \frac{T_6}{T_7} - R \ln \frac{P_6}{P_7} = -0.1925 \text{ kJ/kg K}$$

$$\psi_6 - \psi_7 = (h_6 - h_7) - T_o(s_6 - s_7) = 779.0 - 298.15(-0.1925) = 836.8 \text{ kJ/kg}$$

$$\eta_{2nd\ Law} = \frac{w_T}{\psi_6 - \psi_7} = 779.0\ /\ 836.8 = \mathbf{0.931}$$

d) $\eta_{th} = q_H\ /\ w_{net}$; $w_{net} = w_T + w_{c1} + w_{c2} = 328.3$ kJ/kg

1st Law Combustor: $q + h_i = h_e + w;\quad w = 0$

$q_c = h_6 - h_5 = C_P(T_6 - T_5)$

Regenerator: $\eta_{reg} = \dfrac{T_5 - T_4}{T_7 - T_4} = 0.8$ -> $T_5 = 692.1$ K

$q_H = q_c = 810.7$ kJ/kg; $\eta_{th} =$ **0.405**

12.127

A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency using properties from Table A.7.

Use table A.7.1 and interpolation. (See problem 12.78 for use of TBL A.7.2)

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow$ From Eq.8.28

$$0 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = s^o_{T2} - s^o_{T1} - R \ln(T_2v_1/T_1v_2)$$

$$s^o_{T2} - R \ln(T_2/T_1) = s^o_{T1} + R \ln(v_1/v_2) = 6.83521 + 0.287 \ln 9 = 7.4658$$

This becomes trial and error so estimate first at 680 K and use A.7.1.

$LHS_{680} = 7.7090 - 0.287 \ln(680/290) = 7.4644$ (too low)

$LHS_{700} = 7.7401 - 0.287 \ln(700/290) = 7.4872$ (too high)

Interpolate to get: $T_2 = 681.23$ K, $u_2 = 497.9$ kJ/kg

$P_2 = P_1 (T_2/T_1) (v_1/v_2) = 90 (681.23 / 290) \times 9 = 1902.7$ kPa

${}_1w_2 = u_1 - u_2 = 207.2 - 497.9 = -290.7$ kJ/kg

Combustion 2 to 3: constant volume $v_3 = v_2$

$q_H = u_3 - u_2 = 1486.3 - 497.9 = 988.4$ kJ/kg

$P_3 = P_2(T_3/T_2) = 1902.7 (1800/681.2) = 5028$ kPa

Expansion 3 to 4: $s_4 = s_3 \;\Rightarrow$ From Eq.8.28 as before

$$s^o_{T4} - R \ln(T_4/T_3) = s^o_{T3} + R \ln(v_3/v_4) = 8.8352 + 0.287 \ln(1/9) = 8.2046$$

This becomes trial and error so estimate first at 850 K and use A.7.1.

$LHS_{850} = 7.7090 - 0.287 \ln(850/1800) = 8.1674$ (too low)

$LHS_{900} = 7.7401 - 0.287 \ln(900/1800) = 8.2147$ (too high)

Interpolation $\Rightarrow$ $T_4 = 889.3$ K, $u_4 = 666$ kJ/kg

$P_4 = P_3(T_4/T_3)(v_3/v_4) = 5028 (889.3/1800) (1/9) =$ **276 kPa**

${}_3w_4 = u_3 - u_4 = 1486.3 - 666.0 = 820.3$ kJ/kg

Net work and overall efficiency

$w_{NET} = {}_3w_4 + {}_1w_2 = 820.3 - 290.7 =$ **529.6 kJ/kg**

$\eta = w_{NET}/q_H = 529.6/988.4 =$ **0.536**

12.128

Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the maximum temperature (by iteration) in the cycle to have a thermal efficiency of 60%?

Solution:

Diesel cycle: $P_1 = 95$ kPa, $T_1 = 290$ K, $v_1/v_2 = 20$, $\eta_{TH} = 0.6$

Since the efficiency depends on T_3 and T_4, which are connected through the expansion process in a nonlinear manner we have an iterative problem.

$$T_2 = T_1(v_1/v_2)^{k-1} = 290(20)^{0.4} = 961.2 \text{ K}$$

$$v_1 = 0.287 \times 290/95 = 0.876 \text{ m}^3\text{/kg} = v_4,$$

$$v_2 = v_1/CR = 0.876 / 20 = 0.0438 \text{ m}^3\text{/kg}$$

$$v_3 = v_2(T_3/T_2) = 0.0438\ (T_3/961.2) = 0.0000456\ T_3$$

$$T_3 = T_4\ (v_4/v_3)^{k-1} = \left(\frac{0.876}{0.0000456\ T_3}\right)^{0.4} \Rightarrow\ T_4 = 0.019345\ T_3^{1.4}$$

Now substitute this into the formula for the efficiency

$$\eta_{TH} = 0.60 = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{0.019345 \times T_3^{1.4} - 290}{1.4(T_3 - 961.2)}$$

$$\Rightarrow\ 0.019345 \times T_3^{1.4} - 0.56 \times T_3 + 248.272 = 0$$

Trial and error on this non-linear equation in T_3

3050 K: LHS = +1.06 3040 K: LHS = -0.036,

Linear interpolation $T_3 =$ **3040 K**

12.129

Find the temperature after combustion and the specific energy release by combustion in Problem 12.92 using cold air properties. This is a difficult problem and it requires iterations.

The worlds largest diesel engine has displacement of 25 m^3 running at 200 RPM in a two stroke cycle producing 100 000 hp. Assume an inlet state of 200 kPa, 300 K and a compression ratio of 20:1. What is the mean effective pressure?

We have 3 parameters for the cycle: T_1, P_1 and CR we need one more, so this comes from the total rate of work Eq.12.11.

$$\dot{W} = m\, P_{meff}\, (v_1 - v_2) \times N_{cyl} \times \text{RPM}/60 = P_{meff}\, V_{displ}\, \text{RPM}/60$$

$$P_{meff} = \dot{W}\, 60 / V_{displ}\, \text{RPM} = \frac{100\,000 \times 0.746 \text{ kW} \times 60 \text{ s}}{25 \text{ m}^3 \times 200} = 895 \text{ kPa}$$

$$v_1 = RT/P = 0.287 \times 300 / 200 = 0.4305 \text{ m}^3\text{/kg};$$

$$w_{net} = P_{meff}\, (v_1 - v_2) = 895 \times 0.4305 \left(1 - \frac{1}{20} \right) = 366.03 \text{ kJ/kg}$$

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300\, (20)^{0.4} = 994.3 \text{ K}$$

Combustion and expansion volumes

$$v_3 = v_2 \times T_3/T_2 = v_1\, T_3/(T_2 \times CR)\ ; \quad v_4 = v_1$$

Expansion process, isentropic from Eq.8.32

$$T_4 = T_3\, (v_3/v_4)^{k-1} = T_3\, [T_3/\, (CR{\times}T_2)]^{k-1} = T_3^k\, (CR{\times}T_2)^{1-k} = 0.01908\, T_3^k$$

The net work is also given by the heat transfers

$$w_{net} = q_H - q_L = C_P(T_3 - T_2) - C_v(T_4 - T_1)$$

Substitute w_{net} and T_4 into this equation and we get

$$366.03 = 1.004(T_3 - 994.3) - 0.717(0.01908\, T_3^k - 300)$$

divide by 1.004 and combine terms to get

$$1144.63 = T_3 - 0.013626\, T_3^k$$

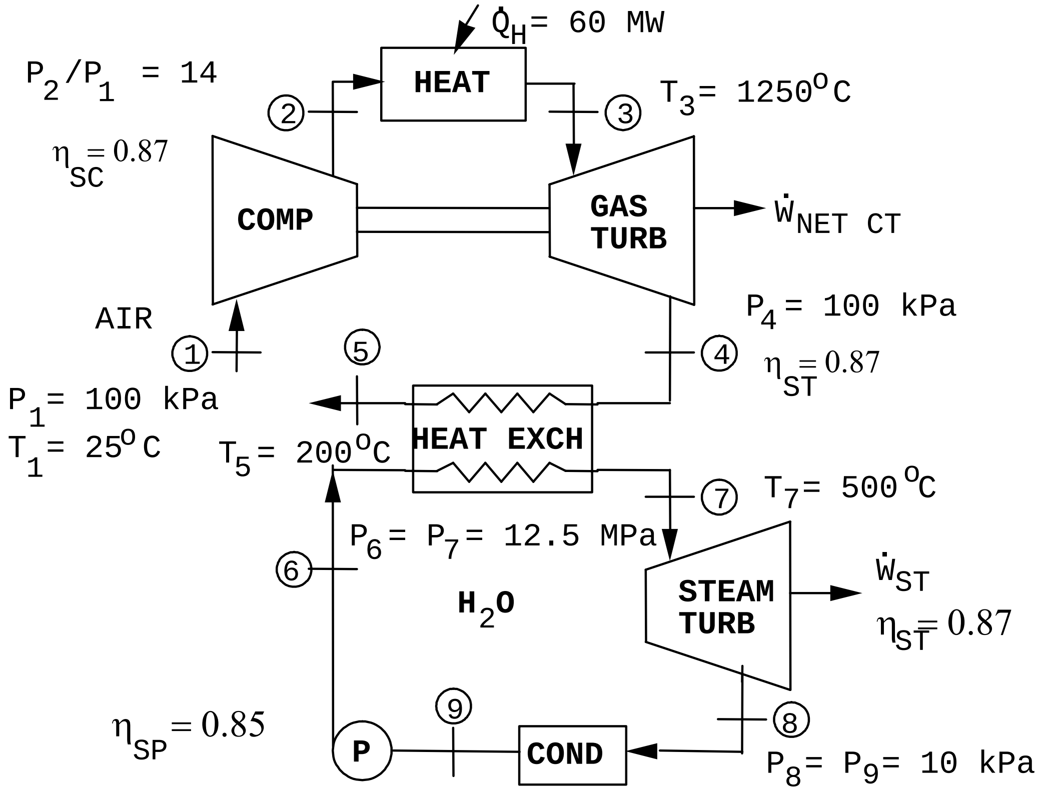
Trial and error, guess T_3 ⇨ $RHS_{1400} = 1054.1;\ RHS_{1600} = 1183$

Interpolate: T_3 = **1540.5 K**

$q_H = C_P(T_3 - T_2) = 1.004(1540.5 - 994.3) =$ **548 kJ/kg**

12.130

Reevaluate the combined Brayton and Rankine cycles in Problem 12.114. For a more realistic case assume the air compressor, the air turbine, the steam turbine and the pump all have an isentropic efficiency of 87%.



a) From Air Tables, A.7: $P_{r1} = 1.0913$, $h_1 = 298.66$, $h_5 = 475.84$ kJ/kg

$s_2 = s_1 \Rightarrow P_{r2S} = P_{r1}(P_2/P_1) = 1.0913 \times 14 = 15.2782$

$T_{2S} = 629$ K, $h_{2S} = 634.48$

$w_{SC} = h_1 - h_{2S} = 298.66 - 634.48 = -335.82$ kJ/kg

$w_C = w_{SC}/\eta_{SC} = -335.82/0.87 = -386 = h_1 - h_2 \quad \Rightarrow \quad h_2 = 684.66$ kJ/kg

At $T_3 = 1523.2$ K: $P_{r3} = 515.493$, $h_3 = 1663.91$ kJ/kg

$$\dot{m}_{AIR} = \dot{Q}_H/(h_3 - h_2) = \frac{60\ 000}{1663.91 - 684.66} = \mathbf{61.27\ kg/s}$$

b) $P_{r4S} = P_{r3}(P_4/P_3) = 515.493(1/14) = 36.8209$

=> $T_{4S} = 791$ K, $h_{4S} = 812.68$ kJ/kg

$w_{ST} = h_3 - h_{4S} = 1663.91 - 812.68 = 851.23$ kJ/kg

$w_T = \eta_{ST} \times w_{ST} = 0.87 \times 851.23 = 740.57 = h_3 - h_4 \;\; => \;\; h_4 = 923.34 \text{ kJ/kg}$

Steam cycle: $-w_{SP} \approx 0.00101(12500 - 10) = 12.615 \text{ kJ/kg}$

$$-w_P = - w_{SP}/\eta_{SP} = 12.615/0.85 = 14.84 \text{ kJ/kg}$$

$$h_6 = h_9 - w_P = 191.83 + 14.84 = 206.67 \text{ kJ/kg}$$

At 12.5 MPa, 500 ^{o}C: $h_7 = 3341.7$ kJ/kg, $s_7 = 6.4617$ kJ/kg K

$$\dot{m}_{H_2O} = \dot{m}_{AIR} \frac{h_4 - h_5}{h_7 - h_6} = 61.27 \frac{923.34 - 475.84}{3341.7 - 206.67} = \mathbf{8.746 \text{ kg/s}}$$

c) $s_{8S} = s_7 = 6.4617 = 0.6492 + x_{8S} \times 7.501, \quad x_{8S} = 0.7749$

$h_{8S} = 191.81 + 0.7749 \times 2392.8 = 2046.0 \text{ kJ/kg}$

$w_{ST} = h_7 - h_{8S} = 3341.7 - 2046.0 = 1295.7 \text{ kJ/kg}$

$w_T = \eta_{ST} \times w_{ST} = 0.87 \times 1295.7 = 1127.3 \text{ kJ/kg}$

$$\dot{W}_{NET} = \left[\dot{m}(w_T+w_C)\right]_{AIR} + \left[\dot{m}(w_T+w_P)\right]_{H_2O}$$

$$= 61.27(740.57 - 386.0) + 8.746(1127.3 - 14.84)$$

$$= 21725 + 9730 = 31455 \text{ kW} = 31.455 \text{ MW}$$

$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 31.455/60 = \mathbf{0.524}$

Problems solved using Table A.7.2

12.32

Repeat Problem 12.29 when the intercooler brings the air to $T_3 = 320$ K. The corrected formula for the optimal pressure is $P_2 = [\, P_1P_4\, (T_3/T_1)^{n/(n-1)}]^{1/2}$ see Problem 9.241, where *n* is the exponent in the assumed polytropic process.

Solution:

The polytropic process has n = k (isentropic) so $n/(n - 1) = 1.4/0.4 = 3.5$

$$P_2 = 400 \sqrt{(320/290)^{3.5}} = 475.2 \text{ kPa}$$

C.V. C1: $s_2 = s_1 \Rightarrow P_{r2} = P_{r1}(P_2/P_1) = 0.9899 \times (475.2/100)$

$$= 4.704 \;\Rightarrow\; T_2 = 452 \text{ K},\; h_2 = 453.75$$

$$-w_{C1} = h_2 - h_1 = 453.75 - 290.43 = \mathbf{163.3\ kJ/kg}$$

C.V. Cooler: $q_{OUT} = h_2 - h_3 = 453.75 - 320.576 = \mathbf{133.2\ kJ/kg}$

C.V. C2: $s_4 = s_3 \Rightarrow P_{r4} = P_{r3}(P_4/P_3) = 1.3972 \times (1600/475.2) = 4.704$

$$\Rightarrow\; T_4 = T_2 = 452 \text{ K},\quad h_4 = 453.75$$

$$-w_{C2} = h_4 - h_3 = 453.75 - 320.576 = \mathbf{133.2\ kJ/kg}$$

12.37

A gas turbine with air as the working fluid has two ideal turbine sections, as shown in Fig. P12.37, the first of which drives the ideal compressor, with the second producing the power output. The compressor input is at 290 K, 100 kPa, and the exit is at 450 kPa. A fraction of flow, *x*, bypasses the burner and the rest (1 − *x*) goes through the burner where 1200 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine, with exhaust at 100 kPa. If the mixing should result in a temperature of 1000 K into the first turbine find the fraction *x*. Find the required pressure and temperature into the second turbine and its specific power output.

C.V.Comp.: $-w_C = h_2 - h_1;\ \ s_2 = s_1$

$$P_{r2} = P_{r1}\,(P_2/P_1) = 0.9899 \times (450/100) = 4.4545,\ \ T_2 = 445\text{ K}$$

$$h_2 = 446.74, \quad -w_C = 446.74 - 290.43 = 156.3\text{ kJ/kg}$$

C.V.Burner: $h_3 = h_2 + q_H = 446.74 + 1200 = 1646.74$ kJ/kg

$$\Rightarrow\ T_3 = 1509\text{ K}$$

C.V.Mixing chamber: $(1 - x)h_3 + xh_2 = h_{MIX} = 1046.22$ kJ/kg

$$x = \frac{h_3 - h_{MIX}}{h_3 - h_2} = \frac{1646.74 - 1046.22}{1646.74 - 446.74} = \mathbf{0.50}$$

$$\dot{W}_{T1} = \dot{W}_{C,in}\ \Rightarrow\ \dot{w}_{T1} = -w_C = 156.3 = h_3 - h_4$$

$$h_4 = 1046.22 - 156.3 = 889.9\ \Rightarrow\ T_4 = \mathbf{861\ K}$$

$$P_4 = (P_{r4}/P_{rMIX})\,P_{MIX} = (51/91.65) \times 450 = \mathbf{250.4\ kPa}$$

$$s_4 = s_5\ \Rightarrow\ P_{r5} = P_{r4}\,(P_5/P_4) = 51 \times (100/250.4) = 20.367$$

$$h_5 = 688.2 \quad T_5 = 676\text{ K}$$

$$w_{T2} = h_4 - h_5 = 889.9 - 688.2 = \mathbf{201.7\ kJ/kg}$$

12.74

Repeat Problem 12.67, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (and the specific heat from Fig. 5.10 at high temperature).

Solution:

Table A.7 is used with interpolation.

$T_1 = 283.2$ K, $u_1 = 202.3$ kJ/kg, $v_{r1} = 210.44$

Compression 1 to 2: $s_2 = s_1 \;\Rightarrow$ From definition of the v_r function

$v_{r2} = v_{r1}\,(v_2/v_1) = 210.4\ (1/7) = 30.063$

Interpolate to get: $T_2 = 603.9$ K, $u_2 = 438.1$ kJ/kg

$\Rightarrow\ -{}_1w_2 = u_2 - u_1 = 235.8$ kJ/kg,

$u_3 = 438.1 + 1800 = 2238.1 \;\Rightarrow\; T_3 = \mathbf{2573.4\ K}$, $v_{r3} = 0.34118$

$P_3 = 90 \times 7 \times 2573.4 / 283.2 = \mathbf{5725\ kPa}$

Expansion 3 to 4: $s_4 = s_3 \;\Rightarrow$ From the v_r function as before

$v_{r4} = v_{r3}\,(v_4/v_3) = 0.34118\ (7) = 2.3883$

Interpolation $\Rightarrow$ $T_4 = 1435.4$ K, $u_4 = 1145.8$ kJ/kg

${}_3w_4 = u_3 - u_4 = 2238.1 - 1145.8 = 1092.3$ kJ/kg

Net work, efficiency and mep

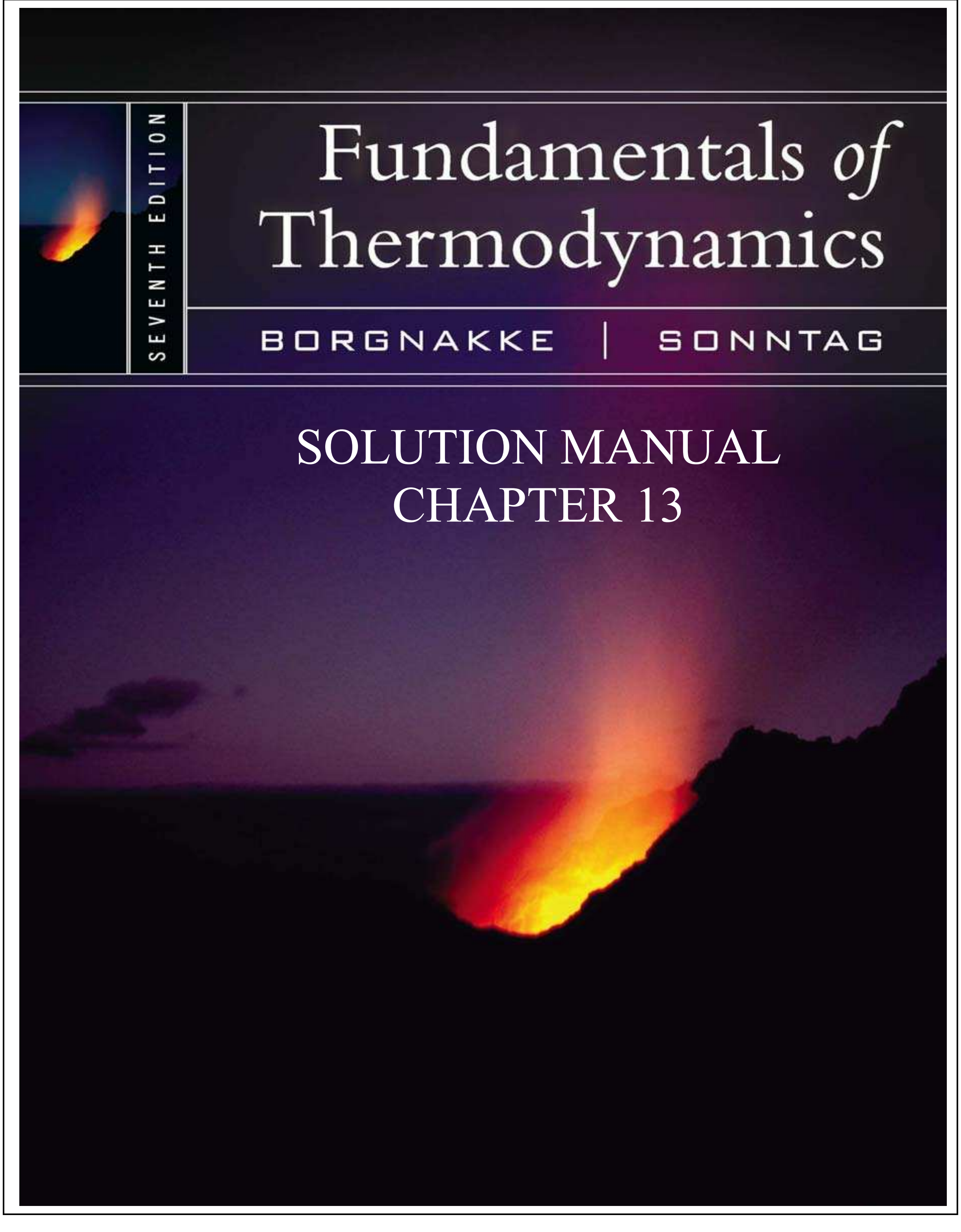
➔ $w_{net} = {}_3w_4 + {}_1w_2 = 1092.3 - 235.8 = 856.5$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 856.5 / 1800 = \mathbf{0.476}$

$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029\ m^3/kg$

$v_2 = (1/7)\,v_1 = 0.1290\ m^3/kg$

$$P_{meff} = \frac{w_{net}}{v_1 - v_2} = 856.5 / (0.9029 - 0.129) = \mathbf{1107\ kPa}$$



SEVENTH EDITION

Fundamentals of Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 13

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In-Text Concept Questions

13.a

Are the mass and mole fractions for a mixture ever the same?

Generally not. If the components all had the same molecular weight the mass and mole fractions would be the same.

13.b

For a mixture how many component concentrations are needed?

A total of N-1 concentrations are needed, N equals total number of components, whether mass or mole fractions. They must sum up to one so the last one is by default.

13.c

Are any of the properties (P, T, v) for oxygen and nitrogen in air the same?

In any mixture under equilibrium T is the same for all components.
Each species has its own pressure equal to its partial pressure P_i.
The partial volume for a component is: $v_i = V/m_i$ and V is the same for all components so v_i is not.

13.d

If I want to heat a flow of a 4 component mixture from 300 to 310 K at constant P, how many properties and which ones do I need to know to find the heat transfer?

You need to know the flow rate, the four mass fractions, and the component specific heat values (or the h values at both temperatures).

13.e

To evaluate the change in entropy between two states at different T and P values for a given mixture, do I need to find the partial pressures?

Not necessarily provided it is an ideal gas. If the mixture composition does not change then the mixture can be treated as a pure substance where each of the partial pressures is a constant fraction of the total pressure, Eq.13.10 and the changes in u, h and s can be evaluated with the mixture properties as in Eqs. 13.20-24.
If constant specific heat is an inappropriate model to use then u, h and a standard entropy must be evaluated from expressions as in Eqs.13.11-12 and 13.16, this is precisely what is done to make the air tables A.7 from the nitrogen, oxygen and argon properties.

If the substance is not an ideal gas mixture then the properties will depend on the partial pressures.

13.f

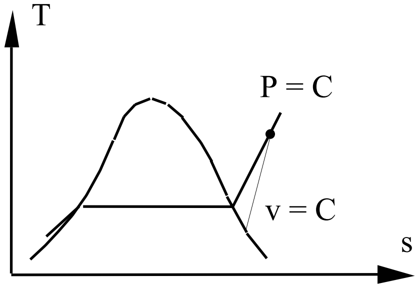
What happens to relative and absolute humidity when moist air is heated?

Relative humidity decreases, while absolute humidity remains constant.
See Figs. 13.8 and 13.9.

13.g

If I cool moist air, do I reach the dew first in a constant-P or constant-V process?

The constant-volume line is steeper than the constant-pressure line, see Fig. 13.3. Saturation in the constant-P process is at a higher T.



13.h

What happens to relative and absolute humidity when moist air is cooled?

Relative humidity increase, while absolute humidity remains constant until we reach the dew point. See Figs. 13.8 and 13.9. If we cool below the dew point the relative humidity stays at 100% and the absolute humidity (humidity ratio) drops as water condenses to liquid (or freezes to solid) and drops out of the gas mixture.

13.i

Explain in words what the absolute and relative humidity expresses?

Absolute humidity is the ratio of the mass of vapor to the mass of dry air. It says how much water is there per unit mass of dry air.
Relative humidity is the ratio of the mole fraction of vapor to that in a saturated mixture at the same T and P. It expresses how close to the saturated state the water is.

13.j

An adiabatic saturation process changes Φ, ω and T. In which direction?

Relative humidity and absolute humidity increase, and temperature decreases. Why does the temperature decrease? The energy to evaporate some liquid water to go into the gas mixture comes from the immediate surroundings to the liquid water surface where water evaporates, look at the dashed curve in Fig. 13.9. The moist air and the liquid water both cool down.

Concept-study Guide Problems

13.1

Equal masses of argon and helium are mixed. Is the molecular mass of the mixture the linear average of the two individual ones?

No. The individual molecular masses must be combined using the mole fractions as in:

$$M_{mix} = \sum y_j M_j$$

13.2

A constant flow of pure argon and pure helium are mixed to produce a flow of mixture mole fractions 0.25 and 0.75 respectively. Explain how to meter the inlet flows to ensure the proper ratio assuming inlet pressures are equal to the total exit pressure and all temperatures are the same.

The inlet flow rate in terms of mass or moles is the same as the exit rate for each component in the mixture. Since the inlet P for each component is the same as the total exit P (which is the sum of the partial pressures if ideal gas) then the volume flow rates in and out are different for each species.

$$P\,\dot{V}_i = \dot{m}_i\,R_iT = \dot{n}_i\,\bar{R}T$$

$$P\,\dot{V}_{tot} = \dot{n}_{tot}\,\bar{R}T$$

We can therefore meter the volume flow rate $\dot{V}_i$ to be proportional to $\dot{n}_i$ for each line of the inlet flows.

13.3

For a gas mixture in a tank are the partial pressures important?

Yes. The sum of the partial pressures equals the total pressure and if they are ideal gases the partial pressures are equal to the mole fraction times the total pressure so

$$P_i = y_i\, P \qquad \text{and} \qquad \Sigma\, P_i = \Sigma\, y_i\, P = P$$

13.4

An ideal mixture at T, P is made from ideal gases at T, P by charging them into a steel tank. Assume heat is transferred so T stays the same as the supply. How do the properties (P, v and u) for each component change up, down or constant?

Solution:

Ideal gas: u = u(T) so **constant**
P **drops** from P to partial P_i
v **increases** from v at P to v at P_i same T

13.5

An ideal mixture at T, P is made from ideal gases at T, P by flow into a mixing chamber without any external heat transfer and an exit at P. How do the properties (P, v and h) for each component change up, down or constant?

Solution:

Ideal gas: $h_{mix} = \Sigma (c_i h_i)_{out} = \Sigma (c_i h_i)_{in}$ same function of T
so **constant** T and then also **constant** h_i
P **drops** from P to partial P_i
v **increases** from v at P to v at P_i same T

13.6

If a certain mixture is used in a number of different processes do I need to consider partial pressures?

No. If the mixture composition stays the same the pressure for each component, which is a partial pressure, is the same fraction of the total pressure, thus any variation follows the total pressure. Recall air is a mixture and we can deal with most processes involving air without knowledge about its composition. However, to make the air properties we do need to deal with the composition but only once.

13.7

Why is it that I can use a set of tables for air, which is a mixture, without dealing with its composition?

As long as the composition is fixed any property is a fixed weighted average of the components properties and thus only varies with T and total P.

A process that will cool air to saturation and condensation can not be handled by the air tables. In such a process the composition of the liquid and vapor mixtures are different.

13.8

Develop a formula to show how the mass fraction of water vapor is connected to the humidity ratio.

By definition the mass concentration is

$$c = \frac{m_v}{m_a + m_v} = \frac{m_v/m_a}{1 + m_v/\,m_a} = \frac{\omega}{1 + \omega}$$

and since ω is small then $1 + \omega \approx 1$ and c is close to ω (but not equal to).

13.9

For air at 110°C and 100 kPa is there any limit on the amount of water it can hold?

No. Since $P_g = 143.3$ kPa at 110°C and $P_v < 100$ kPa ω can be infinity.

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v}$$

As P_v approaches P, w goes towards infinity.

13.10

Can moist air below the freezing point, say -5^{o}C, have a dew point?

Yes. At the dew point, water would begin to appear as a solid. It snows. Since it is frost forming on surfaces rather than dew, you can call it frost point.

13.11

Why does a car with an air-conditioner running often have water dripping out?

The cold evaporator that cools down an air flow brings it below the dew point temperature and thus condenses out water.

13.12

Moist air at 35°C, $\omega = 0.0175$ and $\Phi = 50\%$ should be brought to a state of 20°C, $\omega = 0.01$ and $\Phi = 70\%$. Do I need to add or subtract water?

The humidity ratio (absolute humidity) expresses how much water vapor is present in the mixture

$$\omega = m_v / m_a$$

so to decrease ω we must **subtract water** from the mixture.
The relative humidity expresses how close to the saturated state the vapor is as

$$\Phi = P_v / P_g$$

and not about how much water there is.

Mixture composition and properties

13.13

A 3 L liquid mixture is 1/3 of each of water, ammonia and ethanol by volume. Find the mass fractions and total mass of the mixture.

Each component has a partial volume of 1 L = 0.001 m^3

$m_{water} = V/v_f = 0.001\ m^3/\ (0.001\ m^3/kg) = 1\ kg$

$m_{amm} = V/v_f = 0.001\ m^3 \times 604\ kg/m^3 = 0.604\ kg$

$m_{ethanol} = V/v_f = 0.001\ m^3 \times 783\ kg/m^3 = 0.783\ kg$

Total mass is: $m = 1 + 0.604 + 0.783 = 2.387\ kg$

$c_{water} = m_{water}/m = 1\ /\ 2.387 = 0.419$
$c_{amm} = m_{amm}/m = 0.604\ /\ 2.387 = 0.253$
$c_{ethanol} = m_{ethanol}/m = 0.783\ /2.387 = 0.328$

13.14

If oxygen is 21% by mole of air, what is the oxygen state (P, T, v) in a room at 300 K, 100 kPa of total volume 60 m^3?

The temperature is 300 K,
The partial pressure is $P = yP_{tot} = 0.21 \times 100 = 21$ kPa.
At this T, P: $v_{O2} = RT/P_{O2} = 0.2598 \times 300/21 = 3.711\ m^3/kg$

Remark: If we found the oxygen mass then $m_{O2}v_{O2} = V = 60\ m^3$

13.15

A gas mixture at 20°C, 125 kPa is 50% N_2, 30% H_2O and 20% O_2 on a mole basis. Find the mass fractions, the mixture gas constant and the volume for 5 kg of mixture.

Solution:

The conversion follows the definitions and identities:

From Eq.13.3: $c_i = y_i M_i / \sum y_j M_j$

From Eq.13.5:

$$M_{mix} = \sum y_j M_j = 0.5 \times 28.013 + 0.3 \times 18.015 + 0.2 \times 31.999$$

$$= 14.0065 + 5.4045 + 6.3998 = 25.811$$

$c_{N2} = 14.0065 / 25.811 = 0.5427$, $c_{H2O} = 5.4045 / 25.811 = 0.2094$

$c_{O2} = 6.3998 / 25.811 = 0.2479$, sums to 1 OK

From Eq.13.14:

$R_{mix} = \bar{R}/M_{mix} = 8.3145 / 25.811 =$ **0.3221 kJ/kg K**

$V = mR_{mix}\, T/P = 5 \times 0.3221 \times 393.15/125 =$ **5.065 m^3**

13.16

A mixture of 60% N_2, 30% Ar and 10% O_2 on a mass basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m^3. Find the mole fractions and the mass of argon.

Solution:

Total mixture $PV = m R_{mix} T$

From Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.6 \times 0.2968 + 0.3 \times 0.2081 + 0.1 \times 0.2598$$
$$= 0.26629 \text{ kJ/kg K}$$

$$m = PV/R_{mix}T = 250 \times 0.5 / 0.26649 \times 310 = 1.513 \text{ kg}$$

$m_{ar} = 0.3\ m =$ **0.454 kg**

From Eq.13.4: $y_i = (c_i / M_i) / \sum c_j/M_j$

	c_i	M_i	c_i/M_i	y_i	
N_2	0.6	28.013	0.02141	**0.668**	
Ar	0.3	39.948	0.00751	**0.234**	
O_2	0.1	31.999	<u>0.003125</u>	**0.098**	round up
			0.032055		

13.17

A mixture of 60% N_2, 30% Ar and 10% O_2 on a mole basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m^3. Find the mass fractions and the mass of argon.

Solution:

From Eq. 13.3: $c_i = y_i\ M_i / \sum y_j M_j$

Eq.13.5:

$M_{mix} = \sum y_j M_j = 0.6 \times 28.013 + 0.3 \times 39.948 + 0.1 \times 31.999 = 31.992$

$c_{N2} = (0.6 \times 28.013) / 31.992 = 0.5254$

$c_{Ar} = (0.3 \times 39.948) / 31.992 = 0.3746$

$c_{O2} = (0.1 \times 31.999) / 31.992 = 0.1$, sums to 1 OK

From Eq.13.14:

$R_{mix} = \bar{R}/M_{MIX} = 8.3145 / 31.992 = 0.260$ kJ/kg K

$m_{mix} = PV/(R_{mix}\ T) = 250 \times 0.5 / 0.26 \times 310 = 1.551$ kg

$m_{Ar} = c_{Ar} \times m_{mix} = 0.3746 \times 1.551 =$ **0.581 kg**

13.18

A flow of oxygen and one of nitrogen, both 300 K, are mixed to produce 1 kg/s air at 300 K, 100 kPa. What are the mass and volume flow rates of each line?

For the mixture, $M = 0.21 \times 32 + 0.79 \times 28.013 = 28.85$
For O_2 , $c = 0.21 \times 32 / 28.85 = 0.2329$
For N_2 , $c = 0.79 \times 28.013 / 28.85 = 0.7671$

Since the total flow out is 1 kg/s, these are the component flows in kg/s.
Volume flow of O_2 in is

$$\dot{V} = c\dot{m}\, v = c\dot{m}\,\frac{RT}{P} = 0.2329 \times 0.2598 \times 300/100 = \mathbf{0.1815\ m^3/s}$$

Volume flow of N_2 in is

$$\dot{V} = c\dot{m}\, v = c\dot{m}\,\frac{RT}{P} = 0.7671 \times 0.2968 \times 300/100 = \mathbf{0.6830\ m^3/s}$$

13.19

A new refrigerant R-407 is a mixture of 23% R-32, 25% R-125 and 52% R-134a on a mass basis. Find the mole fractions, the mixture gas constant and the mixture heat capacities for this new refrigerant.

Solution:

From the conversion in Eq.13.4 we get:

	c_i	M_i	c_i/M_i	y_i
R-32	0.23	52.024	0.004421	0.381
R-125	0.25	120.022	0.002083	0.180
R-134a	0.52	102.03	$\underline{0.0050965}$	0.439
			0.0116005	

Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.23 \times 0.1598 + 0.25 \times 0.06927 + 0.52 \times 0.08149$$

$$= \mathbf{0.09645\ kJ/kg\ K}$$

Eq.13.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.23 \times 0.822 + 0.25 \times 0.791 + 0.52 \times 0.852$$

$$= \mathbf{0.8298\ kJ/kg\ K}$$

Eq.13.21:

$$C_{v\ mix} = \sum c_i C_{v\ i} = 0.23 \times 0.662 + 0.25 \times 0.721 + 0.52 \times 0.771$$

$$= \mathbf{0.7334\ kJ/kg\ K} \ \ (= C_{P\ MIX} - R_{MIX})$$

13.20

A 100 m^3 storage tank with fuel gases is at 20°C, 100 kPa containing a mixture of acetylene C_2H_2, propane C_3H_8 and butane C_4H_{10}. A test shows the partial pressure of the C_2H_2 is 15 kPa and that of C_3H_8 is 65 kPa. How much mass is there of each component?

Solution:

Assume ideal gases, then the ratio of partial to total pressure is the mole fraction, $y = P/P_{tot}$

$y_{C2H2} = 15/100 = 0.15, \quad y_{C3H8} = 65/100 = 0.65, \quad y_{C4H10} = 20/100 = 0.20$

$$n_{tot} = \frac{PV}{\bar{R}T} = \frac{100 \times 100}{8.31451 \times 293.15} = 4.1027 \text{ kmoles}$$

$$m_{C2H2} = (nM)_{C2H2} = y_{C2H2}\, n_{tot}\, M_{C2H2}$$

$$= 0.15 \times 4.1027 \times 26.038 = \mathbf{16.024\ kg}$$

$$m_{C3H8} = (nM)_{C3H8} = y_{C3H8}\, n_{tot}\, M_{C3H8}$$

$$= 0.65 \times 4.1027 \times 44.097 = \mathbf{117.597\ kg}$$

$$m_{C4H10} = (nM)_{C4H10} = y_{C4H10}\, n_{tot}\, M_{C4H10}$$

$$= 0.20 \times 4.1027 \times 58.124 = \mathbf{47.693\ kg}$$

13.21

A 2 kg mixture of 25% N_2, 50% O_2 and 25% CO_2 by mass is at 150 kPa and 300 K. Find the mixture gas constant and the total volume.

Solution:

From Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.25 \times 0.2968 + 0.5 \times 0.2598 + 0.25 \times 0.1889$$

$$= \mathbf{0.2513\ kJ/kg\ K}$$

Ideal gas law: $PV = mR_{mix}T$

$$V = mR_{mix}T/P = 2 \times 0.2513 \times 300/150 = \mathbf{1.005\ m^3}$$

13.22

A new refrigerant R-410a is a mixture of R-32 and R-125 in a 1:1 mass ratio. What are the overall molecular weight, the gas constant and the ratio of specific heats for such a mixture?

Eq.13.5:

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\frac{0.5}{52.024} + \frac{0.5}{120.022}} = \mathbf{72.586}$$

Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.1598 + 0.5 \times 0.06927 = \mathbf{0.1145\ kJ/kg\ K}$$

$$= \bar{R}/M_{MIX} = 8.3145 / 72.586 = \text{same} \quad \text{(this is from Eq.13.14)}$$

Eq.13.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.5 \times 0.822 + 0.5 \times 0.791 = 0.8065\ \text{kJ/kg K}$$

Eq.13.21:

$$C_{V\ mix} = \sum c_i C_{V\ i} = 0.5 \times 0.662 + 0.5 \times 0.722 = 0.692\ \text{kJ/kg K}$$

$$(= C_{P\ mix} - R_{mix})$$

$$k_{mix} = C_{P\ mix} / C_{V\ mix} = 0.8065 / 0.692 = \mathbf{1.1655}$$

13.23

Do Problem 13.22 for R-507a which is 1:1 mass ratio of R-125 and R-143a. The refrigerant R-143a has molecular mass of 84.041 and $C_p = 0.929$ kJ/kg-K.

Refrigerant R-143a is not in Table A.5 so:

$$R = \bar{R}/M = 8.3145 / 84.041 = 0.098934 \text{ kJ/kg-K}$$
$$C_V = C_p - R = 0.929 - 0.098934 = 0.8301 \text{ kJ/kg-K}$$

Eq.13.5:

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\frac{0.5}{120.022} + \frac{0.5}{84.041}} = \mathbf{98.859}$$

Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.06927 + 0.5 \times 0.098934 = \mathbf{0.0841\ kJ/kg\ K}$$
$$= \bar{R}/M_{MIX} = 8.3145 / 98.859 = \text{same} \quad \text{(this is from Eq.13.14)}$$

Eq.13.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.5 \times 0.791 + 0.5 \times 0.929 = 0.86 \text{ kJ/kg K}$$

Eq.13.21:

$$C_{V\ mix} = \sum c_i C_{V\ i} = 0.5 \times 0.722 + 0.5 \times 0.8301 = 0.776 \text{ kJ/kg K}$$
$$(= C_{P\ mix} - R_{mix})$$

$$k_{mix} = C_{P\ mix} / C_{V\ mix} = 0.86 / 0.776 = \mathbf{1.108}$$

Simple processes

13.24

A rigid container has 1 kg CO_2 at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any heat transfer. What is final T, P?

No Q, No W so the energy equation gives constant U

Energy Eq.: $U_2 – U_1 = 0 = m_{CO2}(u_2 – u_1)_{CO2} + m_{Ar}(u_2 – u_1)_{Ar}$

$$= m_{CO2}C_{v\ CO2}(T_2 – T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 – T_1)_{Ar}$$

$$= (1×0.653 + 1×0.312) × T_2 - 1×0.653×300 - 1×0.312×400$$

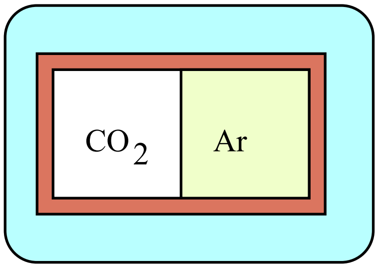
$T_2 =$ **332.3 K**,

$$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$$

$$= 1×0.1889×300/150 + 1×0.2081×400/150 = 0.932\ 73\ m^3$$

Pressure from ideal gas law and Eq.13.15 for R

$P_2 = (1×0.1889 + 1×0.2081) ×332.3/0.932\ 73 =$ **141.4 kPa**



13.25

At a certain point in a coal gasification process, a sample of the gas is taken and stored in a 1-L cylinder. An analysis of the mixture yields the following results:

Component	H_2	CO	CO_2	N_2
Percent by mass	2	45	28	25

Determine the mole fractions and total mass in the cylinder at 100 kPa, 20°C. How much heat transfer must be transferred to heat the sample at constant volume from the initial state to 100°C?

Solution:

Determine mole fractions from Eq.13.4: $y_i = (c_i / M_i) / \sum c_j/M_j$

$$\sum c_j / M_j = 0.02 / 2.016 + 0.45 / 28.01 + 0.28 / 44.01 + 0.25 / 28.013$$

$$= 0.009921 + 0.016065 + 0.006362 + 0.00892 = 0.041268$$

$$M_{mix} = 1 / \sum c_j/M_j = 1/0.041268 = 24.232$$

From Eq.13.4

$y_{H2} = 0.009921 \times 24.232 = 0.2404$ $\quad$ $y_{CO} = 0.016065 \times 24.232 = 0.3893$

$y_{CO2} = 0.006362 \times 24.232 = 0.1542$ $\quad$ $y_{N2} = 0.00892 \times 24.232 = 0.2161$

$$R_{mix} = \bar{R}/M_{mix} = 8.3145/24.232 = 0.34312 \text{ kJ/kg/K}$$

$$m = PV/RT = 100\times10^{-3}/0.34312 \times 293.15 = \mathbf{9.942\times10^{-4}\ kg}$$

$$C_{V0\ MIX} = \sum c_i\, C_{V0\ i} = 0.02 \times 10.085 + 0.45 \times 0.744$$

$$+ 0.28 \times 0.653 + 0.25 \times 0.745 = 0.9056 \text{ kJ/kg K}$$

$${}_1Q_2 = U_2 - U_1 = mC_{V0}(T_2-T_1) = 9.942\times10^{-4}\times 0.9056\times(100-20) = \mathbf{0.0720\ kJ}$$

13.26

The mixture in Problem 13.21 is heated to 500 K with constant volume. Find the final pressure and the total heat transfer needed using Table A.5.

Solution:

C.V. Mixture of constant volume.

Process: V = constant => ${}_1W_2 = \int P\,dV = 0$

Energy Eq.: ${}_1Q_2 = m(u_2 - u_1) \cong m\,C_{V\,mix}\,(T_2 - T_1)$

Ideal gas: $PV = mRT$ => $P_2 = P_1(T_2 / T_1)(V_1/V_2)$

$P_2 = P_1T_2/T_1 = 150 \times 500/300 = \mathbf{250\ kPa}$

From Eq.13.21:

$$C_{V\,mix} = \sum c_iC_{V\,i} = 0.25 \times 0.745 + 0.5 \times 0.662 + 0.25 \times 0.653$$
$$= 0.6805\ \text{kJ/kg K}$$

$${}_1Q_2 = 2 \times 0.6805(500 - 300) = \mathbf{272.2\ kJ}$$

13.27

The mixture in Problem 13.21 is heated up to 500 K in a constant pressure process. Find the final volume and the total heat transfer using Table A.5.

Solution:

C.V. Mixture

Process: P = constant => ${}_1W_2 = \int P\,dV = P(V_2 - V_1)$

Energy Eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1)$

$= m(h_2 - h_1) \cong m\,C_{P\,mix}(T_2 - T_1)$

From Eq.13.15:

$R_{mix} = \sum c_i R_i = 0.25 \times 0.2968 + 0.5 \times 0.2598 + 0.25 \times 0.1889$

$= 0.2513$ kJ/kg K

From Eq.13.23:

$C_{P\,mix} = \sum c_i\,C_{P\,i} = 0.25 \times 1.042 + 0.5 \times 0.922 + 0.25 \times 0.842$

$= 0.932$ kJ/kg K

$V_2 = m\,R_{mix}\,T_2/P_2$

$= 2 \times 0.2513 \times 500/150 = \mathbf{1.675\ m^3}$

${}_1Q_2 = 2 \times 0.932(500 - 300) = \mathbf{372.8\ kJ}$

13.28

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO_2 at 1600 K both at 150 kPa are mixed without any heat transfer. What is the exit T, P?

No work implies no pressure change for a simple flow.

$P_e = \mathbf{150\ kPa}$

The energy equation becomes

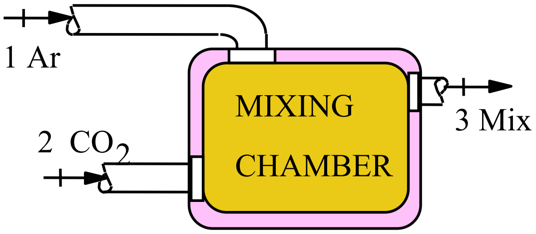
$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$$\Rightarrow \quad \dot{m}_{CO2}C_{p\ CO2}(T_e - T_i)_{CO2} + \dot{m}_{Ar}C_{p\ Ar}(T_e - T_i)_{Ar} = 0$$

$$\Rightarrow \quad \dot{m}_{Ar}C_{p\ Ar}T_i + \dot{m}_{CO2}C_{p\ CO2}T_i = [\dot{m}_{Ar}C_{p\ Ar} + \dot{m}_{CO2}C_{p\ CO2}]\ T_e$$

$$1\times0.520\times300 + 1\times0.842\times1600 = (1\times0.520 + 1\times0.842) \times T_e$$

$T_e = \mathbf{1103.7\ K}$,



13.29

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO_2 at 1600 K both at 150 kPa are mixed without any heat transfer. Find the exit T, P using variable specific heats.

No work implies no pressure change for a simple flow.

P_e = **150 kPa**

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO2}$$

$$\Rightarrow \quad \dot{m}_{CO2}\,(h_e - h_i)_{CO2} + \dot{m}_{Ar}C_{p\,Ar}(T_e - T_i)_{Ar} = 0$$

$$\Rightarrow \quad 1\times (h_e - 1748.12) + 1\times 0.52 \times (T_e - 300) = 0$$

$$h_{e\;CO2} + 0.52\;T_e = 1748.12 + 0.52 \times 300 = 1904.12 \text{ kJ/kg}$$

Trial and error on T_e

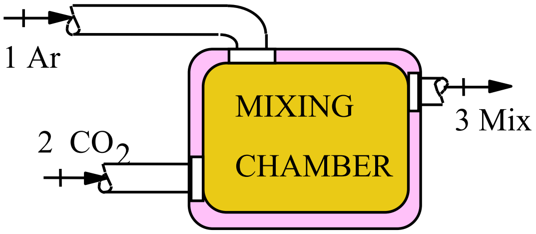
T_e = 1100 K: LHS = 1096.36 + 0.52 × 1100 = 1668.36 too small

T_e = 1300 K: LHS = 1352.28 + 0.52 × 1300 = 2028.28 too large

T_e = 1200 K: LHS = 1223.34 + 0.52 × 1200 = 1847.34 too small

Final interpolation

$$T_e = 1200 + 100\,\frac{1904.12 - 1847.34}{2028.28 - 1847.34} = \mathbf{1231.4\ K},$$



13.30

A rigid insulated vessel contains 12 kg of oxygen at 200 kPa, 280 K separated by a membrane from 26 kg carbon dioxide at 400 kPa, 360 K. The membrane is removed and the mixture comes to a uniform state. Find the final temperature and pressure of the mixture.

Solution:

C.V. Total vessel. Control mass with two different initial states.

Mass: $m = m_{O2} + m_{CO2} = 12 + 26 = 38$ kg

Process: V = constant (rigid) => W = 0, insulated => Q = 0

Energy: $U_2 - U_1 = 0 - 0 = m_{O2}\, C_{V\,O2}(T_2 - T_{1\,O2}) + m_{CO2}C_{V\,CO2}(T_2 - T_{1\,CO2})$

Initial state from ideal gas Table A.5

$C_{V\,O2} = 0.662$ kJ/kg, $C_{V\,CO2} = 0.653$ kJ/kg K

O_2 : $V_{O2} = mRT_1/P = 12 \times 0.2598 \times 280/200 = 4.3646\ m^3$,

CO_2 : $V_{CO2} = mRT_1/P = 26 \times 0.1889 \times 360/400 = 4.4203\ m^3$

Final state mixture

$R_{MIX} = \sum c_i R_i = [12 \times 0.2598 + 26 \times 0.1889\,]/38 = 0.2113$ kJ/kg K

The energy equation becomes

$$m_{O2}\, C_{V\,O2}\, T_2 + m_{CO2}C_{V\,CO2}\, T_2$$
$$= m_{O2}\, C_{V\,O2}\, T_{1\,O2} + m_{CO2}C_{V\,CO2}\, T_{1\,CO2}$$
$$(7.944 + 16.978\,)\, T_2 = 2224.32 + 6112.08 = 8336.4 \text{ kJ}$$
$$\Rightarrow\ T_2 = \mathbf{334.5\ K}$$

From mixture gas constant and total volume

$P_2 = mR_{mix}T_2/V = 38 \times 0.2113 \times 334.5 / 8.7849 =$ **305.7 kPa**

13.31

A steady flow of 0.1 kg/s carbon dioxide at 1000 K in one line is mixed with 0.2 kg/s of nitrogen at 400 K from another line, both at 100 kPa. The mixing chamber is insulated and has constant pressure of 100 kPa. Use constant heat capacity to find the mixing chamber exit temperature.

Take CV around the mixing chamber

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Concentrations: $c_{CO_2} = \dot{m}_1/\dot{m}_3 = 1/3; \quad c_{N_2} = \dot{m}_2/\dot{m}_3 = 2/3$

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = (1/3) \times 0.842 + (2/3) \times 1.042 = 0.97533 \text{ kJ/kg}$$

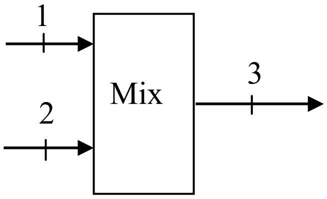
$$R_{mix} = \sum c_i R_i = (1/3) \times 0.1889 + (2/3) \times 0.2968 = 0.2608 \text{ kJ/kg}$$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = \dot{m}_1 h_{3\ CO_2} + \dot{m}_2 h_{3\ N_2}$

Divide this equation with $\dot{m}_3$ and take differences in h as $C_P\ \Delta T$

$$\frac{1}{3} C_{P\ CO_2} T_1 + \frac{2}{3} C_{P\ N_2} T_2 = [\frac{1}{3} C_{P\ CO_2} + \frac{2}{3} C_{P\ N_2}]\ T_3 = C_{P\ mix} T_3$$

$$T_3 = [\frac{1}{3} \times 0.842 \times 1000 + \frac{2}{3} \times 1.042 \times 400\ ] / 0.97533 = \mathbf{572.7\ K}$$



13.32

A pipe flows 1.5 kg/s of a mixture with mass fractions of 40% CO_2 and 60% N_2 at 400 kPa, 300 K. Heating tape is wrapped around a section of pipe with insulation added and 2 kW electrical power is heating the pipe flow. Find the mixture exit temperature.

Solution:

C.V. Pipe heating section. Assume no heat loss to the outside, ideal gases.

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}C_{P\ mix}(T_e - T_i)$

From Eq.13.23

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.4 \times 0.842 + 0.6 \times 1.042 = 0.962 \text{ kJ/kg K}$$

Substitute into energy equation and solve for exit temperature

$$T_e = T_i + \dot{Q} / \dot{m}C_{P\ mix} = 300 + 2 / (1.5 \times 0.962) = \mathbf{301.3\ K}$$

13.33

An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mass basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

Solution:

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{m}C_{P\ mix}(T_i - T_e)$

Properties: From Eqs.13.15 and 13.23

$R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$

$= 0.30248$ kJ/kg K

$C_{P\ mix} = \sum c_i\ C_{P\ i} = 0.1 \times 0.842 + 0.1 \times 1.872 + 0.8 \times 1.042$

$= 1.105$ kJ/kg K

$PV = mR_{mix}T \Rightarrow \dot{m} = P\dot{V} / R_{mix}T$

$\dot{m} = 500\times2/(0.30248\times1000) = 3.306$ kg/s

$\dot{W}_T = 3.306 \times 1.105\ (1000 - 700) =$ **1096 kW**

13.34

Solve Problem 13.33 using the values of enthalpy from Table A.8.
An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mass basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

Solution:

C.V. Turbine, Steady, 1 inlet, 1 exit flow with an ideal gas mixture, q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}(h_i - h_e) = \dot{m} \sum c_j (h_i - h_e)_j$

Properties: From Eqs.13.15 and 13.23

$$R_{mix} = \sum c_i R_i = 0.1 \times 0.1889 + 0.1 \times 0.4615 + 0.8 \times 0.2968$$
$$= 0.30248 \text{ kJ/kg K}$$

$$PV = mR_{mix}T \quad => \quad \dot{m} = P\dot{V} / R_{mix}T$$

$$\dot{m} = 500 \times 2/(0.30248 \times 1000) = 3.306 \text{ kg/s}$$

Now get the h values from Table A.8 (all in kJ/kg)

$$\dot{W}_T = 3.306 \, [\, 0.1 \, (971.67 - 616.22) + 0.1 \, (1994.13 - 1338.56)$$
$$+ 0.8 \, (1075.91 - 735.86) \,]$$
$$= \mathbf{1234 \text{ kW}}$$

13.35

Solve Problem 13.33 with the percentages on a mole basis An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mole basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

C.V. Turbine, Steady flow, 1 inlet, 1 exit flow with an ideal gas mixture, and no heat transfer so $q = 0$.

Energy Eq.: $\dot{W}_T = \dot{m}\,(h_i - h_e) = \dot{n}\,(\bar{h}_i - \bar{h}_e) = \dot{n}\,\bar{C}_{P\ mix}\,(T_i - T_e)$

Ideal gas law: $PV = n\bar{R}T$ =>

$$\dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{500 \times 2}{8.3145 \times 1000} = 0.1203 \text{ kmole/s}$$

The mixture heat capacity becomes

$$\bar{C}_{P\ mix} = \sum y_i\, \bar{C}_i = 0.1 \times 44.01 \times 0.842 + 0.1 \times 18.015 \times 1.872 + 0.8 \times 28.013 \times 1.042 = 30.43 \text{ kJ/kmol K}$$

$$\dot{W}_T = 0.1203 \times 30.43\,(1000 - 700) = \mathbf{1098\ kW}$$

13.36

Solve Problem 13.33 with the percentages on a mole basis and use Table A.9. An insulated gas turbine receives a mixture of 10% CO_2, 10% H_2O and 80% N_2 on a mole basis at 1000 K, 500 kPa. The volume flow rate is 2 m^3/s and its exhaust is at 700 K, 100 kPa. Find the power output in kW using constant specific heat from A.5 at 300 K.

C.V. Turbine, Steady flow, 1 inlet, 1 exit flow with an ideal gas mixture, and no heat transfer so q = 0.

Energy Eq.: $\dot{W}_T = \dot{m}\,(h_i - h_e) = \dot{n}\,(\bar{h}_i - \bar{h}_e) = \dot{n}\,\left[\,\sum y_j\,(\bar{h}_i - \bar{h}_e)_j\,\right]$

Ideal gas law: $PV = n\bar{R}T$ =>

$$\dot{n} = \frac{P\dot{V}}{\bar{R}T} = \frac{500 \times 2}{8.3145 \times 1000} = 0.1203 \text{ kmol/s}$$

Read the enthalpies from Table A.9 (they are all in kJ/kmol)

$$\dot{W}_T = 0.1203\big[0.1(33397 - 17754) + 0.1(26000 - 14190) + 0.8(21463 - 11937)\big]$$

$= \mathbf{1247\ kW}$

13.37

A mixture of 0.5 kg nitrogen and 0.5 kg oxygen is at 100 kPa, 300 K in a piston cylinder keeping constant pressure. Now 800 kJ is added by heating. Find the final temperature and the increase in entropy of the mixture using Table A.5 values.

Solution:

C.V. Mixture in the piston cylinder.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = \text{constant} \quad \Rightarrow \quad {}_1W_2 = \int P\, dV = P(V_2 - V_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1)$$

$$h_2 - h_1 = {}_1Q_2/m \cong C_{P\ mix}(T_2 - T_1)$$

From Eq.13.23 and Table A.5:

$$C_{P\ mix} = (1/2) \times 0.922 + (1/2) \times 1.042 = 0.982 \text{ kJ/kg K}$$

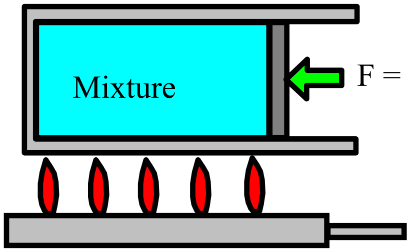
$$T_2 = T_1 + {}_1Q_2/mC_{P\ mix}$$

$$= 300 + 800/(1 \times 0.982) = \mathbf{1114.7\ K}$$

From Eq.13.24:

$$m(s_2 - s_1) = m[C_{P\ mix} \ln(T_2 / T_1) - R \ln(P_2 / P_1)]$$

$$= 1 \times 0.982 \times \ln(1114.7/300) = \mathbf{1.29\ kJ/K}$$



13.38

A mixture of 0.5 kg nitrogen and 0.5 kg oxygen is at 100 kPa, 300 K in a piston cylinder keeping constant pressure. Now 800 kJ is added by heating. Find the final temperature and the increase in entropy of the mixture using Table A.8 values.

Solution:

C.V. Mixture in the piston cylinder.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant => ${}_1W_2 = \int P\,dV = P\,(V_2 - V_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + mP(v_2 - v_1)$$

$$= m(h_2 - h_1)$$

$$h_2 - h_1 = {}_1Q_2/m = 800/1 = 800 \text{ kJ/kg}$$

Since T_2 is so high we use Table A.8 values guessing a T_2

$$(h_2 - h_1)_{1100K} = \frac{1}{2}(1193.62 - 311.67) + \frac{1}{2}(1090.62 - 273.15)$$

$$= 849.71 \text{ kJ/kg} \qquad \text{too high}$$

$$(h_2 - h_1)_{1000K} = \frac{1}{2}(1075.91 - 311.67) + \frac{1}{2}(980.95 - 273.15)$$

$$= 736.02 \text{ kJ/kg} \qquad \text{too low}$$

$$T_2 = 1000 + 100[(800 - 736.02)/(849.71 - 736.02)] = \mathbf{1056.3\ K}$$

From Eqs.13.16 and 13.18:

$$s_2 - s_1 = \frac{1}{2}(s^o_{T2} - s^o_{T1})_{N2} + \frac{1}{2}(s^o_{T2} - s^o_{T1})_{O2}$$

$$= \frac{1}{2}(8.2082 - 6.8463) + \frac{1}{2}(7.6709 - 6.4168)$$

$$= \mathbf{1.308\ kJ/kg\ K}$$

13.39

Natural gas as a mixture of 75% methane and 25% ethane by mass is flowing to a compressor at 17°C, 100 kPa. The reversible adiabatic compressor brings the flow to 250 kPa. Find the exit temperature and the needed work per kg flow.

Solution:

C.V. Compressor. Steady, adiabatic $q = 0$, reversible $s_{gen} = 0$

Energy Eq.6.13: $-w = h_{ex} - h_{in}$; Entropy Eq.9.8: $s_i + s_{gen} = s_e$

Process: reversible => $s_{gen} = 0$ => $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.13.15 and 13.23:

$$R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785 \text{ kJ/kg K}$$

$$C_{P\ mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132 \text{ kJ/kg K}$$

$$C_V = C_{P\ mix} - R_{mix} = 2.132 - 0.45785 = 1.6742 \text{ kJ/kg K}$$

Ratio of specific heats: $k = C_p / C_v = 1.2734$

The isentropic process gives Eq.8.23

$$T_e = T_i\,(P_e / P_i)^{(k-1)/k} = 290\,(250/100)^{0.2147} = \mathbf{353\ K}$$

Work from the energy equation:

$$w_{c\ in} = C_P\,(T_e - T_i) = 2.132\,(353 - 290) = \mathbf{134.3\ kJ/kg}$$

13.40

New refrigerant R-410a is a mixture of R-32 and R-125 in a 1:1 mass ratio. A process brings 0.5 kg R-410a from 270 K to 320 K at a constant pressure 250 kPa in a piston cylinder. Find the work and heat transfer.

Solution:

C.V. R-410a

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1)$

Process: $P = \text{constant}$ $\quad {}_1W_2 = P(V_2 - V_1) = mR(T_2 - T_1)$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

From Eq.13.15:

$$R_{mix} = \sum c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145 \text{ kJ/kg K}$$

FromEq.13.23:

$$C_{P\ mix} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065 \text{ kJ/kg K}$$

From the process equation

$${}_1W_2 = 0.5 \times 0.1145\ (320 - 270) = \mathbf{2.863\ kJ}$$

From the energy equation

$${}_1Q_2 = m\, C_{P\ mix}\ (T_2 - T_1) = 0.5 \times 0.8065\ (320 - 270) = \mathbf{20.16\ kJ}$$

13.41

A piston/cylinder device contains 0.1 kg of a mixture of 40 % methane and 60 % propane gases by mass at 300 K and 100 kPa. The gas is now slowly compressed in an isothermal (T = constant) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution:

C.V. Mixture of methane and propane, this is a control mass.

Assume methane & propane are ideal gases at these conditions.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Property from Eq.13.15

$$R_{mix} = 0.4\ R_{CH4} + 0.6\ R_{C3H8}$$

$$= 0.4 \times 0.5183 + 0.6 \times 0.1886 = 0.3205 \text{ kJ/kg K}$$

Process: T = constant & ideal gas =>

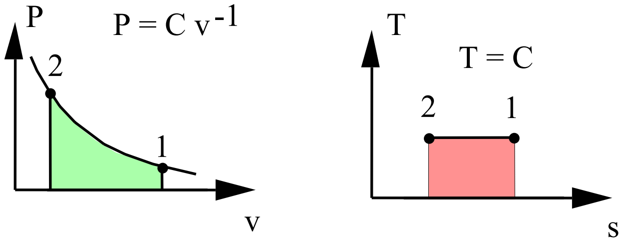
$${}_1W_2 = \int P\,dV = mR_{mix}T \int (1/V)dV = mR_{mix}T \ln (V_2/V_1)$$

$$= mR_{mix}T \ln (P_1/P_2)$$

$$= 0.1 \times 0.3205 \times 300 \ln (100/250) = \mathbf{-8.81\ kJ}$$

Now heat transfer from the energy equation where we notice that u is a constant (ideal gas and constant T) so

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = {}_1W_2 = \mathbf{-8.81\ kJ}$$



13.42

The substance R-410a, see Problem 13.40 is at 100 kPa, 290 K. It is now brought to 250 kPa, 400 K in a reversible polytropic process. Find the change in specific volume, specific enthalpy and specific entropy for the process.

Solution:

Eq.13.15: $R_{mix} = \Sigma\, c_iR_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145$ kJ/kg K

Eq.13.23: $C_{Pmix} = \Sigma\, c_iC_{Pi} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065$ kJ/kg K

$v_1 = RT_1/P_1 = 0.1145 \times 290/100 = 0.33205\ \text{m}^3\text{/kg}$

$v_2 = RT_2/P_2 = 0.1145 \times 400/250 = 0.1832\ \text{m}^3\text{/kg}$

$v_2 - v_1 = 0.1832 - 0.33205 = \mathbf{-0.14885\ m^3/kg}$

$h_2 - h_1 = C_{Pmix}\,(T_2 - T_1) = 0.8065\,(400 - 290) = \mathbf{88.72\ kJ/kg}$

From Eq.13.24

$s_2 - s_1 = C_{Pmix} \ln(T_2 / T_1) - R_{mix} \ln(P_2 / P_1)$

$= 0.8065 \ln(400/290) - 0.1145 \ln(250/100) = \mathbf{0.154\ kJ/kg\ K}$

13.43

A compressor brings R-410a (see problem 13.40) from –10 ^{o}C, 125 kPa up to 500 kPa in an adiabatic reversible compression. Assume ideal gas behavior and find the exit temperature and the specific work.

Solution:

C.V. Compressor

Process: $q = 0$; adiabatic and reversible.

Energy Eq.6.13: $w = h_i - h_e$;

Entropy Eq.9.8: $s_e = s_i + s_{gen} + \int dq/T = s_i + 0 + 0 = s_i$

From Eq.13.15:

$$R_{mix} = \sum c_i R_i = \frac{1}{2} \times 0.1598 + \frac{1}{2} \times 0.06927 = 0.1145 \text{ kJ/kg K}$$

FromEq.13.23:

$$C_{P\ mix} = \frac{1}{2} \times 0.822 + \frac{1}{2} \times 0.791 = 0.8065 \text{ kJ/kg K}$$

$$R_{mix}/ C_{P\ mix} = 0.1145/0.8065 = 0.14197$$

For constant s, ideal gas and use constant specific heat as in Eq.8.23

$$T_e/T_i = (P_e/P_i)^{R/Cp}$$

$$T_e = 263.15 \times (500/125)^{0.14197} = \mathbf{320.39\ K}$$

$$w \cong C_{P\ mix}(T_i - T_e) = 0.8065\ (263.15 – 320.39)$$

$$= \mathbf{-46.164\ kJ/kg}$$

13.44

Two insulated tanks A and B are connected by a valve. Tank A has a volume of 1 m^3 and initially contains argon at 300 kPa, 10°C. Tank B has a volume of 2 m^3 and initially contains ethane at 200 kPa, 50°C. The valve is opened and remains open until the resulting gas mixture comes to a uniform state. Determine the final pressure and temperature.

Solution:

C.V. Tanks A + B. Control mass no W, no Q.

Energy Eq.5.11: $U_2 - U_1 = 0 = m_{Ar}C_{V0}(T_2 - T_{A1}) + m_{C_2H_6}C_{VO}(T_2 - T_{B1})$

$$m_{Ar} = P_{A1}V_A/RT_{A1} = (300 \times 1) / (0.2081 \times 283.15) = 5.0913 \text{ kg}$$

$$m_{C_2H_6} = P_{B1}V_B/RT_{B1} = (200 \times 2) / (0.2765 \times 323.15) = 4.4767 \text{ kg}$$

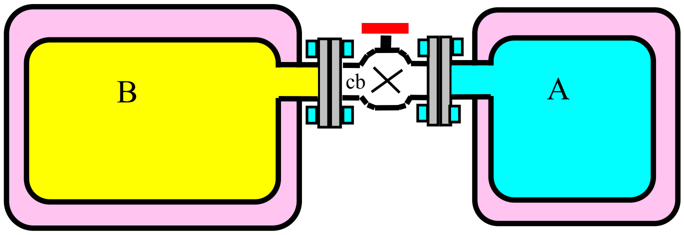
Continuity Eq.: $m_2 = m_{Ar} + m_{C_2H_6} = 9.568$ kg

Energy Eq.: $5.0913 \times 0.312\ (T_2 - 283.2)$
$+ 4.4767 \times 1.490\ (T_2 - 323.2) = 0$

Solving, $T_2 =$ **315.5 K**

$$R_{mix} = \Sigma\, c_i R_i = \frac{5.0913}{9.568} \times 0.2081 + \frac{4.4767}{9.568} \times 0.2765 = 0.2401 \text{ kJ/kg K}$$

$P_2 = m_2RT_2/(V_A+V_B) = 9.568 \times 0.2401 \times 315.5/3 =$ **242 kPa**



13.45

A steady flow of 0.1 kg/s carbon dioxide at 1000 K in one line is mixed with 0.2 kg/s of nitrogen at 400 K from another line, both at 100 kPa. The exit mixture at 100 kPa is compressed by a reversible adiabatic compressor to 500 kPa. Use constant heat capacity to find the mixing chamber exit temperature and the needed compressor power.

Take CV around the mixing chamber

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$;

Concentrations: $c_{CO_2} = \dot{m}_1/\dot{m}_3 = 1/3;\quad c_{N_2} = \dot{m}_2/\dot{m}_3 = 2/3$

$$C_{P\,mix} = \sum c_i\, C_{P\,i} = (1/3) \times 0.842 + (2/3) \times 1.042 = 0.97533 \text{ kJ/kg}$$

$$R_{mix} = \sum c_i R_i = (1/3) \times 0.1889 + (2/3) \times 0.2968 = 0.2608 \text{ kJ/kg}$$

Energy Eq.: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = \dot{m}_1 h_{3\,CO_2} + \dot{m}_2 h_{3\,N_2}$

Divide this equation with $\dot{m}_3$ and take differences in h as $C_P\, \Delta T$

$$\frac{1}{3} C_{P\,CO_2} T_1 + \frac{2}{3} C_{P\,N_2} T_2 = [\frac{1}{3} C_{P\,CO_2} + \frac{2}{3} C_{P\,N_2}]\, T_3 = C_{P\,mix} T_3$$

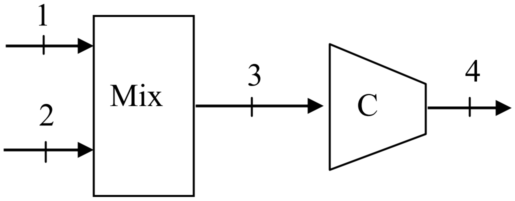
$$T_3 = [\frac{1}{3} \times 0.842 \times 1000 + \frac{2}{3} \times 1.042 \times 400\ ] / 0.97533 = \mathbf{572.7\ K}$$

Now we can do the adiabatic compression

$$T_4 = T_3\, (P_4 / P_3)^{R/Cp} = 572.66 \times 5^{\frac{0.2608}{0.97533}} = 880.6 \text{ K}$$

$$w_C = C_{Pmix}\, (T_4 - T_3) = 0.97533\, (880.6 - 572.66) = 300.34 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}\, w_C = 0.3 \times 300.34 = \mathbf{90.1\ kW}$$



13.46

A mixture of 2 kg oxygen and 2 kg of argon is in an insulated piston cylinder arrangement at 100 kPa, 300 K. The piston now compresses the mixture to half its initial volume. Find the final pressure, temperature and the piston work.

Solution:

C.V. Mixture. Control mass, boundary work and no Q, assume reversible.

Energy Eq.5.11: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = - {}_1w_2$;

Entropy Eq.8.37: $s_2 - s_1 = 0 + 0 = 0$

Process: constant s => $Pv^k = \text{constant}$, $v_2 = v_1/2$,

Assume ideal gases ($T_1 >> T_C$) and use k_{mix} and $C_{v\,mix}$ for properties.

Eq.13.15: $R_{mix} = \Sigma\, c_iR_i = 0.5 \times 0.25983 + 0.5 \times 0.20813 = 0.234$ kJ/kg K

Eq.13.23 $C_{Pmix} = \Sigma\, c_iC_{Pi} = 0.5 \times 0.9216 + 0.5 \times 0.5203 = 0.721$ kJ/kg K

$C_{vmix} = C_{Pmix} - R_{mix} = 0.487$ kJ/kg K

Ratio of specific heats: $k_{mix} = C_{Pmix}/C_{vmix} = 1.4805$

The relations for the polytropic process

Eq.8.25: $P_2 = P_1(v_1/v_2)^k = P_1(2)^k = 100(2)^{1.4805} =$ **279 kPa**

Eq.8.24: $T_2 = T_1(v_1/v_2)^{k-1} = T_1(2)^{k-1} = 300(2)^{0.4805} =$ **418.6 K**

Work from the energy equation

${}_1W_2 = m_{tot}\,(u_1 - u_2) = m_{tot}\,C_v(T_1 - T_2) = 4{\times}0.487\,(300 - 418.6) =$ **-231 kJ**

13.47

A piston cylinder has 0.1 kg mixture of 25% argon, 25% nitrogen and 50% carbon dioxide by mass at total pressure 100 kPa and 290 K. Now the piston compresses the gases to a volume 7 times smaller in a polytropic process with n = 1.3. Find the final pressure and temperature, the work, and the heat transfer for the process.

Solution:

Expansion ratio: $v_2/ v_1 = 1/7$

Mixture properties:

$$R_{mix} = \Sigma\, c_iR_i = 0.25 \times 0.2081 + 0.25 \times 0.2968 + 0.5 \times 0.1889$$
$$= 0.220675 \text{ kJ/kg K}$$

$$C_{v\ mix} = \sum c_i\, C_{vi} = 0.25 \times 0.312 + 0.25 \times 0.745 + 0.5 \times 0.653$$
$$= 0.59075 \text{ kJ/kg K}$$

Process eq.: Rev. adiabatic and ideal gas gives $Pv^n = C$, with n = 1.3

$$P_2 = P_1\, (v_1/v_2)^n = 100 \times 7^{1.3} = \mathbf{1254.9\ kPa}$$

$$T_2 = T_1\, (v_1/v_2)^{n-1} = 290 \times 7^{0.3} = \mathbf{519.9\ K}$$

Polytropic process work term from Eq.4.5 and ideal gas law

$$_1W_2 = \frac{mR}{1 - n}\,(T_2 - T_1) = \frac{0.1 \times 0.220675}{-0.3}\,(519.9 - 290) = \mathbf{-16.91\ kJ}$$

Energy Eq.: $_1Q_2 = U_2 - U_1 + {}_1W_2 = m\, C_{v\ mix}\, (T_2 - T_1) + {}_1W_2$

$$= 0.1 \times 0.59075\, (519.9 - 290) - 16.91$$
$$= \mathbf{-3.33\ kJ}$$

13.48

The gas mixture from Problem 13.25 is compressed in a reversible adiabatic process from the initial state in the sample cylinder to a volume of 0.2 L. Determine the final temperature of the mixture and the work done during the process.

Solution:

From Eq.13.15

$$R_{mix} = \sum c_i R_i = 0.02 \times 4.1243 + 0.45 \times 0.2968 + 0.28 \times 0.1889$$

$$+ 0.25 \times 0.2968 = 0.34314 \text{ kJ/kg K}$$

$$m = PV/R_{mix}T = 100 \times 10^{-3}/(0.34314 \times 293.15) = \mathbf{9.941 \times 10^{-4}\ kg}$$

$$C_{V0\,MIX} = \sum c_i\, C_{V0\,i} = 0.02 \times 10.085 + 0.45 \times 0.744$$

$$+ 0.28 \times 0.653 + 0.25 \times 0.745 = 0.9056 \text{ kJ/kg K}$$

$$C_{P0\,MIX} = C_{V0\,MIX} + R_{mix} = 0.9056 + 0.34314 = 1.2487 \text{ kJ/kg K}$$

$$\rightarrow k = C_{P0}/C_{V0} = 1.2487/0.9056 = 1.379$$

The process (adiabatic and reversible) is isentropic expressed in Eq.8.32

$$\rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1} = 293.15 \left(\frac{1}{0.2}\right)^{0.379} = \mathbf{539.5\ K}$$

$${}_1W_2 = -\Delta U_{12} = -mC_{V0}(T_2 - T_1)$$

$$= -9.941 \times 10^{-4} \times 0.9056 \times (539.5 - 293.15) = \mathbf{-\ 0.22\ kJ}$$

Entropy generation

13.49

A flow of gas A and a flow of gas B are mixed in a 1:1 mole ratio with the same T. What is the entropy generation per kmole flow out?

For this case each component has a mole fraction of one half so,

$$y_A = y_B = n_A/n_{tot} = 0.5$$

Eq. 13.19: $\Delta S = -\bar{R}(0.5 \ln 0.5 + 0.5 \ln 0.5) = +\, 0.6931\, \bar{R}$

$= \mathbf{5.763\ kJ/kmol\ K}$

13.50

A rigid container has 1 kg argon at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any external heat transfer. What is final T, P? Is any s generated?

Energy Eq.: $U_2 - U_1 = 0 = 2mu_2 - mu_{1a} - mu_{1b} = mC_v(2T_2 - T_{1a} - T_{1b})$

$$T_2 = (T_{1a} + T_{1b})/2 = \mathbf{350\ K},$$

Process Eq.: V = constant =>

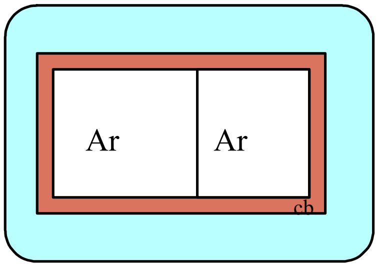
$$P_2V = 2mRT_2 = mR(T_{1a} + T_{1b}) = P_1V_{1a} + P_1V_{1b} = P_1V$$

$$P_2 = P_1 = \mathbf{150\ kPa},$$

ΔS due to temperature changes only, not P, internally we have a Q over a ΔT

$$\Delta S = m\,(s_2 - s_{1a}) + m\,(s_2 - s_{1b}) = mC_p\,[\,\ln (T_2/T_{1a}) + \ln (T_2/T_{1b})\,]$$

$$= 1 \times 0.520\left[\ln\frac{350}{300} + \ln\frac{350}{400}\right] = \mathbf{0.0107\ kJ/K}$$



13.51

What is the entropy generation in problem 13.24?

No Q, No W so the energy equation gives constant U

Energy Eq.: $U_2 - U_1 = 0 = m_{CO2}(u_2 - u_1)_{CO2} + m_{Ar}(u_2 - u_1)_{Ar}$

$$= m_{CO2}C_{v\ CO2}(T_2 - T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 - T_1)_{Ar}$$

$$= (1\times0.653 + 1\times0.312) \times T_2 - 1\times0.653\times300 - 1\times0.312\times400$$

$T_2 = \mathbf{332.3\ K}$,

$$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$$

$$= 1\times0.1889\times300/150 + 1\times0.2081\times400/150 = 0.932\ 73\ m^3$$

Pressure from ideal gas law and Eq.13.15 for R

$$P_2 = (1\times0.1889 + 1\times0.2081) \times332.3/0.932\ 73 = \mathbf{141.4\ kPa}$$

$$S_2 - S_1 = 0 + {}_1S_{2\ gen} = m_{CO2}(s_2 - s_1)_{CO2} + m_{Ar}(s_2 - s_1)_{Ar}$$

For each component: $s_2 - s_1 = C_P \ln\frac{T_2}{T_1} - R \ln\frac{yP_2}{P_1}$ [P's are total pressure]

$$y_{CO2} = \frac{c_{CO2}/M_{CO2}}{c_{CO2}/M_{CO2} + c_{Ar}/M_{Ar}} = \frac{0.5 / 44.01}{0.5 / 39.948 + 0.5 / 44.01} = 0.4758$$

$$y_{Ar} = 1 - y_{CO2} = 0.5242$$

$${}_1S_{2\ gen} = 1\times [0.520 \ln(\frac{332.3}{400}) - 0.2081 \ln (\frac{0.5242 \times 141.4}{150})]$$

$$+ 1\times [0.842 \ln(\frac{332.3}{300}) - 0.1889 \ln (\frac{0.4758 \times 141.4}{150})]$$

$$= 0.05027 + 0.23756 = \mathbf{0.2878\ kJ/K}$$

13.52

A flow of 2 kg/s mixture of 50% CO_2 and 50% O_2 by mass is heated in a constant pressure heat exchanger from 400 K to 1000 K by a radiation source at 1400 K. Find the rate of heat transfer and the entropy generation in the process shown in Fig. P13.52.

Solution:

C.V. Heat exchanger $\quad w = 0$

Energy Eq.6.12: $\dot{Q}_{in} = \dot{m}(h_e - h_i)$

Values from Table A.8 due to the high T.

$$\dot{Q}_{in} = 2\,[\frac{1}{2} \times (971.67 - 303.76) + \frac{1}{2} \times (980.95 - 366.03)] = \mathbf{1282.8\ kW}$$

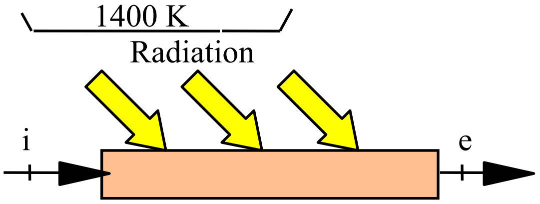
Entropy Eq.9.8: $\dot{m}_e s_e = \dot{m}_i s_i + \dot{Q}/T_s + \dot{S}_{gen}$

As the pressure is constant the pressure correction in Eq.8.28 drops out to give the generation as

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}/T_s$$

$$= 2\,[\frac{1}{2} \times (6.119 - 5.1196) + \frac{1}{2} \times (7.6121 - 6.6838)] - 1282.8/1400$$

$$= \mathbf{1.01\ kW/K}$$



13.53

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1 \text{ kmol/s}; \quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1 \text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O}\, h_1 + \dot{m}_{O2}\, h_2 = \dot{m}_{H2O}\, h_{3\ H2O} + \dot{m}_{O2}\, h_{3\ O2}$

Entropy Eq.: $\dot{m}_{H2O}\, s_1 + \dot{m}_{O2}\, s_2 + \dot{S}_{gen} = \dot{m}_{H2O}\, s_{3\ H2O} + \dot{m}_{O2}\, s_{3\ O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O}\, (h_{3\ H2O} - h_1) + \dot{m}_{O2}\, (h_{3\ O2} - h_2) = 0$$

$$\dot{m}_{H2O}\, C_{P\ H2O}(T_3 - T_1) + \dot{m}_{O2}\, C_{P\ O2}(T_3 - T_2) = 0$$

$$1.8 \times 1.872\, (T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \mathbf{545.6\ K}$$

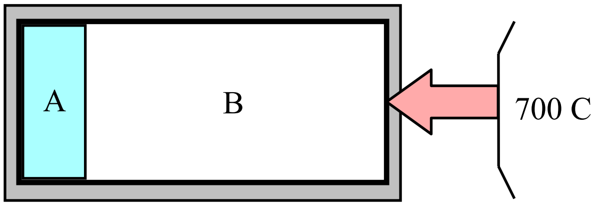
$$\dot{S}_{gen} = \dot{m}_{H2O}\, (s_{3\ H2O} - s_1) + \dot{m}_{O2}\, (s_{3\ O2} - s_2)$$

$$= \dot{m}_{H2O} \left[C_{P\ H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O} \right] + \dot{m}_{O2} \left[C_{P\ O2} \ln \frac{T_3}{T_2} - R \ln y_{O2} \right]$$

$$= 1.8 \left[1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5 \right]$$

$$+ 3.2 \left[0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5 \right]$$

$$= -\,0.132 + 1.492 = \mathbf{1.36\ kW/K}$$



13.54

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are at 100 kPa and the mass ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kg of the exit mixture.

Solution:

CV mixing chamber. The inlet ratio is so $\dot{m}_{CO_2} = 2\,\dot{m}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity Eq.6.9: $\dot{m}_{N_2} + 2\dot{m}_{N_2} = \dot{m}_{ex} = 3\dot{m}_{N_2}$;

Energy Eq.6.10: $\dot{m}_{N_2}(h_{N_2} + 2\,h_{CO_2}) = 3\dot{m}_{N_2}h_{mix\ ex}$

Take 300 K as reference and write $h = h_{300} + C_{Pmix}(T - 300)$.

$$C_{P\,N_2}(T_{i\,N_2} - 300) + 2C_{P\,CO_2}(T_{i\,CO_2} - 300) = 3C_{P\,mix}(T_{mix\ ex} - 300)$$

$$C_{P\,mix} = \sum c_i C_{P\,i} = \frac{2}{3} \times 0.842 + \frac{1}{3} \times 1.042 = 0.9087 \text{ kJ/kg K}$$

$$3C_{P\,mix}T_{mix\ ex} = C_{P\,N_2}T_{i\,N_2} + 2C_{P\,CO_2}T_{i\,CO_2} = 830.64 \text{ kJ/kg}$$

$T_{mix\ ex}$ = **304.7 K**;

To find the entropies we need the partial pressures, which assuming ideal gas are equal to the mole fractions times the total pressure:

$$y_i = [c_i/\,M_i] \,/ \sum c_j/M_j$$

$$y_{N_2} = [0.3333\,/\,28.013]\,/\,[\frac{0.3333}{28.013} + \frac{0.6666}{44.01}] = 0.44$$

$$y_{CO_2} = 1 - y_{N_2} = 0.56$$

$$\dot{S}_{gen} = \dot{m}_{ex}s_{ex} - (\dot{m}s)_{iCO_2} - (\dot{m}s)_{iN_2} = \dot{m}_{N_2}(s_e - s_i)_{N_2} + 2\dot{m}_{N_2}(s_e - s_i)_{CO_2}$$

$$\frac{\dot{S}_{gen}}{3\dot{m}_{N_2}} = \frac{1}{3}\,[C_{PN_2}\ln\frac{T_{ex}}{T_{iN_2}} - R_{N_2}\ln y_{N_2}\,] + \frac{2}{3}\,[C_{PCO_2}\ln\frac{T_{ex}}{T_{iCO_2}} - R_{CO_2}\ln y_{CO_2}]$$

$$= \frac{1}{3}\,[\,1.042\,\ln(\frac{304.7}{280}) - 0.2968\,\ln 0.44\,]$$

$$+ \frac{2}{3}\,[\,0.842\,\ln(\frac{304.7}{320}) - 0.1889\,\ln 0.56\,]$$

$$= 0.110585 + 0.068275$$

= **0.1789 kJ/kg mix K**

13.55

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are coming in at 100 kPa and the mole ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kmole of the exit mixture.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\,\dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

Continuity: $\dot{n}_{CO_2} + 2\dot{n}_{N_2} = \dot{n}_{ex} = 3\dot{n}_{N_2}$;

Energy Eq.: $\dot{n}_{N_2}(\bar{h}_{N_2} + 2\bar{h}_{CO_2}) = 3\dot{n}_{N_2}\bar{h}_{mix\ ex}$

Take 300 K as reference and write $\bar{h} = \bar{h}_{300} + \bar{C}_{Pmix}(T - 300)$.

$$\bar{C}_{P\ N_2}(T_{i\ N_2} - 300) + 2\bar{C}_{P\ CO_2}(T_{i\ CO_2} - 300) = 3\bar{C}_{P\ mix}(T_{mix\ ex} - 300)$$

Find the specific heats in Table A.5 to get

$$\bar{C}_{P\ mix} = \sum y_i\bar{C}_{P\ i} = (1.042 \times 28.013 + 2 \times 0.842 \times 44.01)/3$$

$$= 34.43 \text{ kJ/kmol K}$$

$$3\bar{C}_{P\ mix}T_{mix\ ex} = \bar{C}_{P\ N_2}T_{i\ N_2} + 2\bar{C}_{P\ CO_2}T_{i\ CO_2} = 31889 \text{ kJ/kmol}$$

$$T_{mix\ ex} = \mathbf{308.7\ K}$$

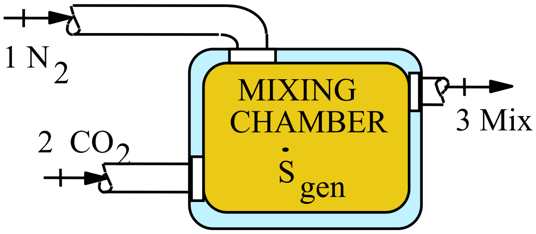
Partial pressures are total pressure times molefraction

$$P_{ex\ N_2} = P_{tot}/3; \quad P_{ex\ CO_2} = 2P_{tot}/3$$

$$\dot{S}_{gen} = \dot{n}_{ex}\bar{s}_{ex} - (\dot{n}\bar{s})_{iCO_2} - (\dot{n}\bar{s})_{iN_2} = \dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{N_2} + 2\dot{n}_{N_2}(\bar{s}_e - \bar{s}_i)_{CO_2}$$

$$\dot{S}_{gen}/3\dot{n}_{N_2} = [\bar{C}_{PN_2}\ln\frac{T_{ex}}{T_{iN_2}} - \bar{R}\ln y_{N_2} + 2\bar{C}_{PCO_2}\ln\frac{T_{ex}}{T_{iCO_2}} - 2\,\bar{R}\ln y_{CO_2}]/3$$

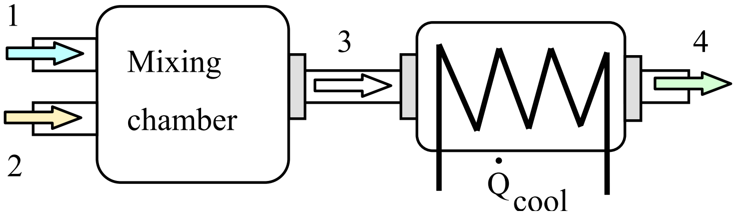
$$= [2.8485 + 9.1343 - 2.6607 + 6.742]/3 = \mathbf{5.35\ kJ/kmol\ mix\ K}$$



13.56

A flow of 1 kg/s carbon dioxide at 1600 K, 100 kPa is mixed with a flow of 2 kg/s water at 800 K, 100 kPa and after the mixing it goes through a heat exchanger where it is cooled to 500 K by a 400 K ambient. How much heat transfer is taken out in the heat exchanger? What is the entropy generation rate for the whole process?

Solution:



C.V. Total mixing section and heat exchanger. Steady flow and no work. To do the entropy at the partial pressures we need the mole fractions.

$$\dot{n}_{H2O} = \dot{m}_{H2O}/M_{H2O} = 2 / 18.015 = 0.11102 \text{ kmol/s}$$

$$\dot{n}_{CO2} = \dot{m}_{CO2}/M_{CO2} = 1 / 44.01 = 0.022722 \text{ kmol/s}$$

$$y_{H2O} = \frac{0.11102}{0.11102 + 0.022722} = 0.8301, \quad y_{CO2} = 1 - y_{H2O} = 0.1699$$

Energy Eq.: $\dot{m}_{H2O}\, h_1 + \dot{m}_{CO2}\, h_2 = \dot{Q}_{cool} + \dot{m}_{H2O}\, h_{4\ H2O} + \dot{m}_{CO2}\, h_{4\ CO2}$

Entropy Eq.: $\dot{m}_{H2O}\, s_1 + \dot{m}_{CO2}\, s_2 + \dot{S}_{gen} = \dfrac{\dot{Q}_{cool}}{T_{amb}} + \dot{m}_{H2O}\, s_{4\ H2O} + \dot{m}_{CO2}\, s_{4\ CO2}$

As T is fairly high we use Table A.8 for properties on a mass basis.

	1	2	4 H2O	4 CO2
h [kJ/kg]	1550.13	1748.12	935.12	401.52
s^o_T [kJ/kg K]	12.4244	6.7254	11.4644	5.3375

$$\dot{Q}_{cool} = \dot{m}_{H2O}\,(h_1 - h_{4\ H2O}) + \dot{m}_{CO2}\,(h_2 - h_{4\ CO2})$$

$$= 2\,(1550.13 - 935.12) + 1\,(1748.12 - 401.52) = \mathbf{2577\ kW}$$

$$\dot{S}_{gen} = \dot{m}_{H2O}\,(s_{4\ H2O} - s_1) + \dot{m}_{CO2}\,(s_{4\ CO2} - s_2) + \frac{\dot{Q}_{cool}}{T_{amb}}$$

$$= 2\,[11.4644 - 12.4244 - 0.4615 \ln(0.8301)\,]$$

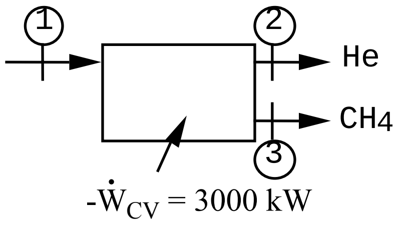
$$+ 1\,[5.3375 - 6.7254 - 0.1889 \ln(0.1699)\,] + \frac{2577}{400}$$

$$= -1.74813 - 1.05307 + 6.4415 = \mathbf{3.64\ kW/K}$$

13.57

The only known sources of helium are the atmosphere (mole fraction approximately 5×10^{-6}) and natural gas. A large unit is being constructed to separate 100 m^3/s of natural gas, assumed to be 0.001 He mole fraction and 0.999 CH_4. The gas enters the unit at 150 kPa, 10°C. Pure helium exits at 100 kPa, 20°C, and pure methane exits at 150 kPa, 30°C. Any heat transfer is with the surroundings at 20°C. Is an electrical power input of 3000 kW sufficient to drive this unit?

0.999 CH_4
0.001 He
at 150 kPa, 10 °C
$\dot{V}_1 = 100\ m^3/s$



$P_2 = 100$ kPa
$T_2 = 20$ °C

$P_3 = 140$ kPa
$T_3 = 30$ °C

$$\dot{n}_1 = P_1\dot{V}_1/RT_1 = 150\times100/(8.3145\times283.2) = 6.37 \text{ kmol/s}$$

$$=> \quad \dot{n}_2 = 0.001; \quad \dot{n}_1 = 0.006\,37; \quad \dot{n}_3 = 6.3636 \text{ kmol/s}$$

$$\bar{C}_{P\,He} = 4.003\times5.193 = 20.7876 \text{ kJ/kmol K},$$

$$\bar{C}_{P\,CH_4} = 16.043\times2.254 = 36.1609 \text{ kJ/kmol K}$$

Energy Eq.:

$$\dot{Q}_{CV} = \dot{n}_2\bar{h}_2 + \dot{n}_3\bar{h}_3 - \dot{n}_1\bar{h}_1 + \dot{W}_{CV} = \dot{n}_2\bar{C}_{P0\,He}(T_2-T_1) + \dot{n}_3\bar{C}_{P0\,CH_4}(T_3-T_1) + \dot{W}_{CV}$$

$$= 0.00637\times20.7876(20 - 10) + 6.3636\times36.1609(30 - 10) + (-3000)$$

$$= +1600 \text{ kW}$$

Entropy Eq.:

$$\dot{S}_{gen} = \dot{n}_2\bar{s}_2 + \dot{n}_3\bar{s}_3 - \dot{n}_1\bar{s}_1 - \dot{Q}_{CV}/T_0$$

$$= 0.00637\left[20.7876 \ln\frac{293.2}{283.2} - 8.3145 \ln\frac{100}{0.001\times150}\right]$$

$$+ 6.3636\left[36.1609 \ln\frac{303.2}{283.2} - 8.3145 \ln\frac{140}{0.999\times150}\right] - 1600/293.2$$

$$= \mathbf{+13.5\ kW/K\ > 0}$$ Since positive, this is possible

13.58

Repeat Problem 13.39 for an isentropic compressor efficiency of 82%.

Solution:

C.V. Compressor. Steady, adiabatic q = 0, reversible $s_{gen} = 0$

Energy Eq.6.13: $-w = h_{ex} - h_{in}$; Entropy Eq.9.8: $s_i + s_{gen} = s_i = s_e$

Process: reversible => $s_{gen} = 0$ => $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.13.15 and 13.23:

$$R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785 \text{ kJ/kg K}$$

$$C_{P\,mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132 \text{ kJ/kg K}$$

$$C_V = C_{P\,mix} - R_{mix} = 2.132 - 0.45785 = 1.6742 \text{ kJ/kg K}$$

Ratio of specific heats: $k = C_p / C_v = 1.2734$

The isentropic process gives Eq.8.32

$$T_e = T_i (P_e / P_i)^{(k-1)/k} = 290 (250/100)^{0.2147} = 353 \text{ K}$$

Work from the energy equation:

$$w_{c\,in} = C_P (T_e - T_i) = 2.132 (353 – 290) = 134.3 \text{ kJ/kg}$$

The actual compressor requires more work

$w_{c\,actual} = w_{c\,in}/\eta = 134.3/0.82 = \mathbf{163.8\ kJ/kg} = C_p (T_{e\,actual} - T_i)$

=> $T_{e\,actual} = T + w_{c\,actual}/C_P = 290 + 163.8 / 2.132 = \mathbf{366.8\ K}$

13.59

A steady flow 0.3 kg/s of 50% carbon dioxide and 50% water mixture by mass at 1200 K and 200 kPa is used in a constant pressure heat exchanger where 300 kW is extracted from the flow. Find the exit temperature and rate of change in entropy using Table A.5

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $c_{CO2} = c_{H2O} = 0.5$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i) \approx \dot{m}\, C_P\, (T_e - T_i) \;\;\Rightarrow\;\; T_e = T_i + \dot{Q}/\dot{m}\, C_P$

Inlet state: Table A.5

$$C_P = 0.5 \times 0.842 + 0.5 \times 1.872 = 1.357 \text{ kJ/kg-K}$$

Exit state: $T_e = T_i + \dot{Q}/\dot{m}\, C_P = 1200 \text{ K} - 300 \text{ kW}/(0.3 \times 1.357 \text{ kW/K})$

$= \mathbf{463\ K}$

The rate of change of entropy for the flow is (P is assumed constant)

$$\dot{m}(s_e - s_i) = \dot{m}[s^o_{Te} - s^o_{Ti} - R \ln(P_e/P_i)] = \dot{m}[s^o_{Te} - s^o_{Ti}) \approx \dot{m}\, C_P \ln(T_e/T_i)$$

$$= 0.3 \times 1.357 \ln(463 / 1200) = \mathbf{-0.388\ kW/K}$$

The entropy generation rate cannot be estimated unless the average T at which the heat transfer leaves the control volume is known.

13.60

A steady flow of 0.3 kg/s of 50% carbon dioxide and 50% water by mass at 1200K and 200 kPa is used in a heat exchanger where 300 kW is extracted from the flow. Find the flow exit temperature and the rate of change of entropy using Table A.8.

Solution:

C.V. Heat exchanger, Steady, 1 inlet, 1 exit, no work.

Continuity Eq.: $c_{CO2} = c_{H2O} = 0.5$

Energy Eq.: $\dot{Q} = \dot{m}(h_e - h_i)$ => $h_e = h_i + \dot{Q}/\dot{m}$

Inlet state: Table A.8 $h_i = 0.5 \times 1223.34 + 0.5 \times 2466.25 = 1844.8$ kJ/kg

Exit state: $h_e = h_i + \dot{Q}/\dot{m} = 1844.8 - 300/0.3 = 844.8$ kJ/kg

Trial and error for T with h values from Table A.8

@500 K $h_e = 0.5(401.52 + 935.12) = 668.32$ kJ/kg

@600 K $h_e = 0.5(506.07 + 1133.67) = 819.87$ kJ/kg

@650 K $h_e = 0.5(560.51 + 1235.30) = 897.905$ kJ/kg

Interpolate to have the right h: **T = 616 K**

Entropy Eq.9.8: $\dot{m}s_e = \dot{m}s_i + \dot{Q}/T + \dot{S}_{gen}$

The rate of change of entropy for the flow is (P is assumed constant)

$$\dot{m}(s_e - s_i) = \dot{m}(s^o_{Te} - s^o_{Ti})$$

$$= 0.3[\ 0.5(5.5558 - 6.3483) + 0.5(11.8784 - 13.3492)\ ]$$

$$= \mathbf{-0.339\ kW/K}$$

The entropy generation rate cannot be estimated unless the average T at which the heat transfer leaves the control volume is known.

13.61

A mixture of 60% helium and 40% nitrogen by mass enters a turbine at 1 MPa, 800 K at a rate of 2 kg/s. The adiabatic turbine has an exit pressure of 100 kPa and an isentropic efficiency of 85%. Find the turbine work.

Solution:

Assume ideal gas mixture and take CV as turbine.

Energy Eq.6.13: $w_{T\,s} = h_i - h_{es}$,

Entropy Eq.9.8: $s_{es} = s_i$, adiabatic and reversible

Process Eq.8.23: $T_{es} = T_i(P_e/P_i)^{(k-1)/k}$

Properties from Eq.13.23, 13.15 and 8.23

$C_{P\ mix} = 0.6\times 5.193 + 0.4\times 1.042 = 3.5326$ kJ/kg K

$R_{mix} = 0.6\times 2.0771 + 0.4\times 0.2968 = 1.365$ kJ/kg K

$(k-1)/k = R/C_{P\ mix} = 1.365/3.5326 = 0.3864$

$T_{es} = 800(100/1000)^{0.3864} = 328.6$ K

$w_{Ts} = C_P(T_i - T_{es}) = 3.5326(800 - 328.6) = 1665$ kJ/kg

$w_{T\ ac} = \eta w_{Ts} = 1415.5$ kJ/kg

$\dot{W}_{T\ ac} = \dot{m} w_{T\ ac} =$ **2831 kW**

13.62

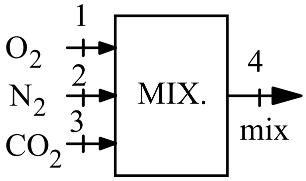
Three steady flows are mixed in an adiabatic chamber at 150 kPa. Flow one is 2 kg/s of O_2 at 340 K, flow two is 4 kg/s of N_2 at 280 K and flow three is 3 kg/s of CO_2 at 310 K. All flows are at 150 kPa the same as the total exit pressure. Find the exit temperature and the rate of entropy generation in the process.

Solution:

C.V. Mixing chamber, no heat transfer, no work.

Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3 = \dot{m}_4 h_4$



Entropy Eq.9.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_4 s_4$

Assume ideal gases and since T is close to 300 K use heat capacity from A.5 in the energy equation as

$$\dot{m}_1 C_{P\,O2}(T_1 - T_4) + \dot{m}_2 C_{P\,N2}(T_2 - T_4) + \dot{m}_3 C_{P\,CO2}(T_3 - T_4) = 0$$

$$2 \times 0.922 \times 340 + 4 \times 1.042 \times 280 + 3 \times 0.842 \times 310$$

$$= (2 \times 0.922 + 4 \times 1.042 + 3 \times 0.842\,)\,T_4$$

$$\Rightarrow \quad 2577.06 = 8.538\,T_4 \qquad \Rightarrow \qquad \mathbf{T_4 = 301.83\ K}$$

State 4 is a mixture so the component exit pressure is the partial pressure. For each component $s_e - s_i = C_P \ln(T_e / T_i) - R \ln(P_e / P_i)$ and the pressure ratio is $P_e / P_i = y\,P_4 / P_i = y$ for each.

$$n = \sum \frac{m}{M} = \frac{2}{32} + \frac{4}{28.013} + \frac{3}{44.01} = 0.0625 + 0.1428 + 0.06817 = 0.2735$$

$$y_{O2} = \frac{0.0625}{0.2735} = 0.2285, \quad y_{N2} = \frac{0.1428}{0.2735} = 0.5222, \quad y_{CO2} = \frac{0.06817}{0.2735} = 0.2493$$

The entropy generation becomes

$$\dot{S}_{gen} = \dot{m}_1(s_4 - s_1) + \dot{m}_2(s_4 - s_2) + \dot{m}_3(s_4 - s_3)$$

$$= \quad 2\,[\,0.922 \ln(301.83/340) - 0.2598 \ln(0.2285)]$$

$$+ 4\,[\,1.042 \ln(301.83/280) - 0.2968 \ln(0.5222)]$$

$$+ 3\,[\,0.842 \ln(301.83/310) - 0.1889 \ln(0.2493)]$$

$$= 0.5475 + 1.084 + 0.2399 = \mathbf{1.871\ kW/K}$$

13.63

A tank has two sides initially separated by a diaphragm. Side A contains 1 kg of water and side B contains 1.2 kg of air, both at 20°C, 100 kPa. The diaphragm is now broken and the whole tank is heated to 600°C by a 700°C reservoir. Find the final total pressure, heat transfer and total entropy generation.

C.V. Total tank out to reservoir.

Energy Eq.5.11: $U_2 - U_1 = m_a(u_2 - u_1)_a + m_v(u_2 - u_1)_v = {}_1Q_2$

Entropy Eq.8.14 and 8.18:

$$S_2 - S_1 = m_a(s_2 - s_1)_a + m_v(s_2 - s_1)_v = {}_1Q_2/T_{res} + S_{gen}$$

Volume: $V_2 = V_A + V_B = m_v v_{v1} + m_a v_{a1} = 0.001 + 1.009 = 1.01\ m^3$

$v_{v2} = V_2/m_v = 1.01\ m^3/kg,\ T_2 \ \Rightarrow\ P_{2v} = 400\ kPa$

$v_{a2} = V_2/m_a = 0.8417\ m^3/kg,\ T_2 \ \Rightarrow\ P_{2a} = mRT_2/V_2 = 297.7\ kPa$

$P_{2tot} = P_{2v} + P_{2a} = \mathbf{697.7\ kPa}$

Water table B.1: $u_1 = 83.95$ kJ/kg, $u_2 = 3300$ kJ/kg,

$s_1 = 0.2966$ kJ/kg K, $s_2 = 8.4558$ kJ/kg K

Air table A.7: $u_1 = 293$ kJ/kg, $u_2 = 652.3$ kJ/kg,

$s_{T1} = 2.492$ kJ/kg K, $s_{T2} = 3.628$ kJ/kg K

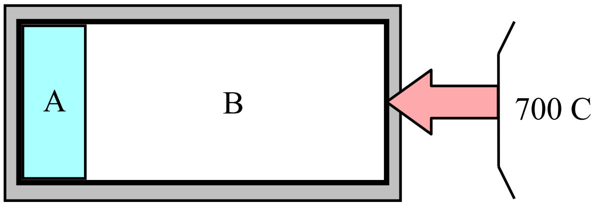
From energy equation we have

${}_1Q_2 = 1(3300 - 83.95) + 1.2(652.6 – 209.4) = \mathbf{3747.9\ kJ}$

From the entropy equation we have

$S_{gen} = 1(8.4557 - 0.2966) + 1.2\ [7.9816 - 6.846 - 0.287 \times \ln(297.7/100)]$

$- 3747.9 / 973.2 = \mathbf{5.3\ kJ/K}$



13.64

Reconsider the Problem 13.44, but let the tanks have a small amount of heat transfer so the final mixture is at 400 K. Find the final pressure, the heat transfer and the entropy change for the process.

C.V. Both tanks. Control mass with mixing and heating of two ideal gases.

$$n_{Ar} = P_{A1}V_A/\bar{R}T_{A1} = \frac{300\times1}{8.3145\times283.2} = 0.1274 \text{ kmol}$$

$$n_{C_2H_6} = P_{B1}V_B/\bar{R}T_{B1} = \frac{200\times2}{8.3145\times323.2} = 0.1489 \text{ kmol}$$

Continuity Eq.: $n_2 = n_{Ar} + n_{C_2H_6} = 0.2763$ kmol

Energy Eq.: $U_2\text{-}U_1 = n_{Ar}\bar{C}_{V0}(T_2\text{-}T_{A1}) + n_{C_2H_6}\bar{C}_{VO}(T_2\text{-}T_{B1}) = {}_1Q_2$

$P_2 = n_2\bar{R}T_2/(V_A+V_B) = 0.2763\times8.3145\times400 / 3 =$ **306.3 kPa**

${}_1Q_2 = 0.1274\times39.948\times0.312(400 - 283.15)$

$+ 0.1489\times30.07\times1.49(400 - 323.15) =$ **698.3 kJ**

$\Delta S_{SURR} = -{}_1Q_2/T_{SURR}$; $\Delta S_{SYS} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6}$

$y_{Ar} = 0.1274/0.2763 = 0.4611$

$$\Delta\bar{S}_{Ar} = \bar{C}_{P\ Ar} \ln\frac{T_2}{T_{A1}} - \bar{R}\ln\frac{y_{Ar}P_2}{P_{A1}}$$

$$= 39.948\times0.520 \ln\frac{400}{283.15} - 8.3145 \ln\frac{0.4611\times306.3}{300}$$

$= 13.445$ kJ/kmol K

$$\Delta\bar{S}_{C_2H_6} = \bar{C}_{C_2H_6} \ln\frac{T_2}{T_{B1}} - \bar{R}\ln\frac{y_{C_2H_6}P_2}{P_{B1}}$$

$$= 30.07\times1.766 \ln\frac{400}{323.15} - 8.3145 \ln\frac{0.5389\times306.3}{200}$$

$= 12.9270$ kJ/kmol K

Assume the surroundings are at 400 K (it heats the gas)

$\Delta S_{NET} = n_{Ar}\Delta\bar{S}_{Ar} + n_{C_2H_6}\Delta\bar{S}_{C_2H_6} + \Delta S_{SURR}$

$= 0.1274\times13.445 + 0.1489\times12.9270 - 698.3/400$

= **1.892 kJ/K**

Air- water vapor mixtures

13.65

Atmospheric air is at 100 kPa, 25^{o}C and relative humidity 75%. Find the absolute humidity and the dew point of the mixture. If the mixture is heated to 30^{o}C what is the new relative humidity?

Solution:

Eq.13.25: $P_v = \phi\ P_g = 0.75 \times 3.169 = 2.377$ kPa

Eq.13.28: $w = 0.622\ P_v/(P_{tot} - P_v) = 0.622 \times 2.377/(100 - 2.377) = 0.01514$

T_{dew} is the T such that $P_g(T) = P_v = 2.377$ kPa

B.1.1 => $T \cong 20.2$ °C

Heating => w is constant => P_v is constant

From Table B.1.1: $P_g(30°C) = 4.246$ kPa

$\phi = P_v/P_g = 2.377/4.246 =$ **0.56 or 56 %**

13.66

A flow of 1 kg/s saturated moist air (relative humidity 100%) at 100 kPa, 10^oC goes through a heat exchanger and comes out at 25^oC. What is the exit relative humidity and how much power is needed?

Solution:

State 1 : $\phi_1 = 1$; $P_v = P_g = 1.2276$

Eq.13.28: $w = 0.622\ P_v/P_a = 0.622 \times 1.2276/(100 - 1.2276) = 0.00773$

State 2 : No water added => $w_2 = w_1$ => $P_{v2} = P_{v1}$

$\phi_2 = P_{v2}/P_{g2} = 1.2276/3.169 = 0.387$ or 39 %

Energy Eq.6.10

$\dot{Q} = \dot{m}_2 h_2 - \dot{m}_1 h_1 = \dot{m}_a(h_2 - h_1)_{air} + w\dot{m}_a(h_2 - h_1)_{vapor}$

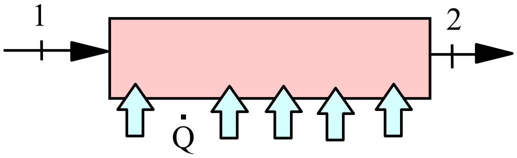
$\dot{m}_{tot} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1 + w_1)$

Energy equation with $C_{P\ air}$ from A.5 and h's from B.1.1

$$\dot{Q} = \frac{\dot{m}_{tot}}{1 + w_1} C_{P\ air}\ (25 - 10) + \frac{\dot{m}_{tot}}{1 + w_1} w\ (h_{g2} - h_{g1})$$

$$= \frac{1}{1.00773} \times 1.004(25 - 10) + \frac{1 \times 0.00773}{1.00773} (2547.17 - 2519.74)$$

$$= 14.9445 + 0.210407 = \mathbf{15.15\ kW}$$



13.67

If I have air at 100 kPa and a) –10°C b) 45°C and c) 110°C what is the maximum absolute humidity I can have?

Humidity is related to relative humidity (max 100%) and the pressures as in Eq.13.28 where from Eq.13.25 $P_v = \Phi\ P_g$ and $P_a = P_{tot} - P_v$.

$$\omega = 0.622\,\frac{P_v}{P_a} = 0.622\,\frac{\Phi\ P_g}{P_{tot} - \Phi P_g}$$

a) $P_g = 0.2601$ kPa, $\omega = 0.622 \times \frac{0.2601}{100 - 0.26} =$ **0.001 62**

b) $P_g = 9.593$ kPa, $\omega = 0.622 \times \frac{9.593}{100 - 9.593} =$ **0.0660**

c) $P_g = 143.3$ kPa, **no limit** on ω for $P_{tot} = 100$ kPa

13.68

A new high-efficiency home heating system includes an air-to-air heat exchanger which uses energy from outgoing stale air to heat the fresh incoming air. If the outside ambient temperature is −10°C and the relative humidity is 30%, how much water will have to be added to the incoming air, if it flows in at the rate of 1 m^3/s and must eventually be conditioned to 20°C and 40% relative humidity?

Solution:

Outside ambient air: $P_{V1} = \phi_1 P_{G1} = 0.30\times 0.2602 = 0.078$ kPa

Assuming $P_1 = P_2 = 100$ kPa, => $P_{A1} = 100 - 0.078 = 99.922$ kPa

$$\dot{m}_A = \frac{P_{A1}\dot{V}_1}{R_A T_1} = \frac{99.922\times 1}{0.287\times 263.2} = 1.3228 \text{ kg/s}$$

From Eq.13.28: $w_1 = 0.622 \times \frac{0.078}{99.922} = 0.00049$

Conditioned to : $T_2 = 20\ ^oC$, $\phi_2 = 0.40$

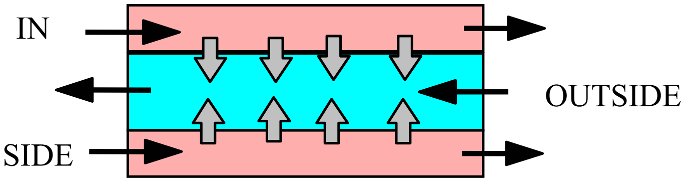
Eq.13.25: $P_{V2} = \phi_2 P_{G2} = 0.40 \times 2.339 = 0.9356$ =>

Eq.13.28: $w_2 = 0.622 \times \frac{0.9356}{99.064} = 0.00587$

Continuity equation for water,

$$\dot{m}_{LIQ\ IN} = \dot{m}_A(w_2 - w_1) = 1.3228(0.00587 - 0.00049)$$

$$= 0.00712 \text{ kg/s} = \mathbf{25.6\ kg/h}$$



13.69

Consider 100 m^3 of atmospheric air which is an air–water vapor mixture at 100 kPa, 15°C, and 40% relative humidity. Find the mass of water and the humidity ratio. What is the dew point of the mixture?

Solution:

Air-vapor P = 100 kPa, T = 15 ^{o}C, $\phi = 40\%$

Use Table B.1.1 and then Eq.13.25

$P_g = P_{sat15} = 1.705$ kPa => $P_v = \phi\ P_g = 0.4 \times 1.705 = 0.682$ kPa

$$m_v = \frac{P_v V}{R_v T} = \frac{0.682 \times 100}{0.461 \times 288.15} = \mathbf{0.513\ kg}$$

$P_a = P_{tot} - P_{v1} = 100 - 0.682 = 99.32$ kPa

$$m_a = \frac{P_a V}{R_a T} = \frac{99.32 \times 100}{0.287 \times 288.15} = 120.1\ \text{kg}$$

$$w_1 = \frac{m_v}{m_a} = \frac{0.513}{120.1} = \mathbf{0.0043}$$

T_{dew} is T when $P_v = P_g = 0.682$ kPa;

Table B.1.2 gives T = **1.4 ^{o}C**

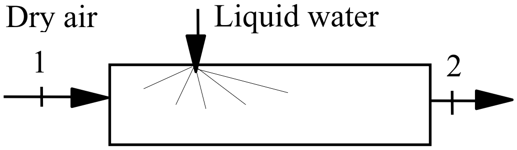
13.70

A flow of 2 kg/s completely dry air at T_1, 100 kPa is cooled down to 10°C by spraying liquid water at 10°C, 100 kPa into it so it becomes saturated moist air at 10°C. The process is steady state with no external heat transfer or work. Find the exit moist air humidity ratio and the flow rate of liquid water. Find also the dry air inlet temperature T_1.

Solution:

2: saturated $P_v = P_g = 1.2276$ kPa and h_{fg} (10°C)= 2477.7 kJ/kg

Eq.13.25: $w_2 = 0.622 \times 1.2276/ (100 - 1.2276) =$ **0.00773**



C.V. Box

Continuity Eq.: $\dot{m}_a + \dot{m}_{liq} = \dot{m}_a(1 + w_2)$ =>

$\dot{m}_{liq} = w_2\, \dot{m}_a =$ **0.0155 kg/s**

Energy Eq.: $\dot{m}_a\, h_{a1} + \dot{m}_{liq}\, h_f = \dot{m}_a\, (h_{a2} + w_2\, h_{g2})$

$h_{a1} - h_{a2} = C_{pa}\, (T_1 - T_2) = w_2\, h_{g2} - w_2\, h_f = w_2\, h_{fg}$

$= 0.0073 \times 2477.75 = 9.15$ kJ/kg dry air

$T_1 = T_2 + (h_{a1} - h_{a2})/C_{pa} = 10 + 9.15/1.004 =$ **29.1°C**

13.71

The products of combustion are flowing through a heat exchanger with 12% CO_2, 13% H_2O and 75% N_2 on a volume basis at the rate 0.1 kg/s and 100 kPa. What is the dew-point temperature? If the mixture is cooled 10°C below the dew-point temperature, how long will it take to collect 10 kg of liquid water?

Solution:

Volume basis is the same as mole fraction

$y_{H_2O} = 0.13; \quad P_{H_2O} = 0.13 \times 100 = 13$ kPa,

Table B.1.2 $\quad T_{DEW} = \mathbf{50.95\ ^oC}$

Cool to 40.95 $^oC < T_{DEW}$ so saturated $\quad \rightarrow P_G = 7.805$ kPa

$y_{H_2O} = 7.805/100 = n_{H_2O(v)}/(n_{H_2O(v)} + 0.87)$

$n_{H_2O(v)} = 0.07365$ per kmol mix in

$\rightarrow n_{LIQ} = 0.13 - 0.07365 = 0.05635$

Eq.13.5: $\quad M_{MIX\ IN} = 0.12 \times 44.01 + 0.13 \times 18.015 + 0.75 \times 28.013 = 28.63$ kg/kmol

$\dot{n}_{MIX\ IN} = \dot{m}_{TOTAL}/M_{MIX\ IN} = 0.1/28.63 = 0.003493$ kmol/s

$\dot{n}_{LIQ\ COND} = 0.003\ 493 \times 0.05635 = 0.000\ 197$ kmol/s

or $\dot{m}_{LIQ\ COND} = 0.000\ 197 \times 18.015 = 0.003\ 55$ kg/s

For 10 kg, takes ~ **47 minutes**

13.72

Consider a 1 m^3/s flow of atmospheric air at 100 kPa, 25°C, and 80% relative humidity. Assume this flows into a basement room where it cools to 15°C, 100 kPa. How much liquid water will condense out?

Solution:

State 1: $P_g = P_{sat25} = 3.169$ kPa => $P_v = \phi\, P_g = 0.8 \times 3.169 = 2.535$ kPa

$$\dot{m}_{v1} = \frac{P_v\dot{V}}{R_vT} = \frac{2.535 \times 1}{0.461 \times 298.15} = 0.0184 \text{ kg/s}$$

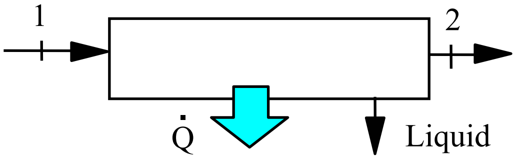
$$w_1 = \frac{\dot{m}_{v1}}{\dot{m}_{A1}} = 0.622\,\frac{P_{v1}}{P_{A1}} = 0.622\,\frac{2.535}{100 - 2.535} = 0.0162$$

$$\dot{m}_{A1} = \frac{\dot{m}_{v1}}{w_1} = \frac{0.0184}{0.0162} = 1.136 \text{ kg/s} = \dot{m}_{A2} \qquad \text{(continuity for air)}$$

Check for state 2:

$P_{g15°C} = 1.705$ kPa $< P_{v1}$

so liquid water out.



State 2 is saturated $\phi_2 = 100\%$, $P_{v2} = P_{g2} = 1.705$ kPa

$$w_2 = 0.622\,\frac{P_{v2}}{P_{A2}} = 0.622\,\frac{1.705}{100 - 1.705} = 0.0108$$

$$\dot{m}_{v2} = w_2\dot{m}_{A2} = 0.0108 \times 1.136 = 0.0123 \text{ kg/s}$$

$$\dot{m}_{liq} = \dot{m}_{v1} - \dot{m}_{v2} = 0.0184 - 0.0123 = \mathbf{0.0061\ kg/s}$$

Note that the given volume flow rate at the inlet is not that at the exit. The mass flow rate of dry air is the quantity that is the same at the inlet and exit.

13.73

Ambient moist air enters a steady-flow air-conditioning unit at 102 kPa, 30°C, with a 60% relative humidity. The volume flow rate entering the unit is 100 L/s. The moist air leaves the unit at 95 kPa, 15°C, with a relative humidity of 100%. Liquid condensate also leaves the unit at 15°C. Determine the rate of heat transfer for this process.

Solution:

State 1: $P_{V1} = \phi_1 P_{G1} = 0.60 \times 4.246 = 2.5476$

$w_1 = 0.622 \times 2.5476/(102 - 2.5476) = 0.01593$

$$\dot{m}_A = \frac{P_{A1}\dot{V}_1}{R_A T_1} = \frac{99.45 \times 0.1}{0.287 \times 303.2} = 0.1143 \text{ kg/s}$$

$P_{V2} = P_{G2} = 1.705$ kPa, $\quad w_2 = 0.622 \times 1.705/(95 - 1.705) = 0.01137$

Energy Eq.6.10: $\dot{Q}_{CV} + \dot{m}_A h_{A1} + \dot{m}_{V1} h_{V1} = \dot{m}_A h_{A2} + \dot{m}_{V2} h_{A2} + \dot{m}_3 h_{L3}$

$\dot{Q}_{CV}/\dot{m}_A = C_{P0A}(T_2 - T_1) + w_2 h_{V2} - w_1 h_{V1} + (w_1 - w_2) h_{L3}$

$= 1.004(15-30) + 0.01137 \times 2528.9 - 0.01593 \times 2556.2$

$+ 0.00456 \times 63.0 = -26.732$ kJ/kg air

$\dot{Q}_{CV} = 0.1143(-26.73) = \mathbf{-3.055\ kW}$

13.74

A room with air at 40% relative humidity, 20°C having 50 kg of dry air is made moist by boiling water to a final state of 20°C and 100% humidity. How much water was added to the air?

The water content is expressed by the absolute humidity (humidity ratio) from Eq.13.28 and 13.25

$$w_1 = 0.622 \times \frac{0.4 \times 2.339}{101.325 - 0.9356} = 0.005797$$

$$w_2 = 0.622 \times \frac{2.339}{101.325 - 2.339} = 0.014697$$

$$m_{water} = m_a\,(w_2 - w_1) = 50\,(0.014697 - 0.005797) = \mathbf{0.445\ kg}$$

13.75

Consider a 500-L rigid tank containing an air–water vapor mixture at 100 kPa, 35°C, with a 70% relative humidity. The system is cooled until the water just begins to condense. Determine the final temperature in the tank and the heat transfer for the process.

Solution:

$P_{v1} = \phi P_{G1} = 0.7 \times 5.628 = 3.9396$ kPa

Since m_v = const & V = const & also $P_v = P_{G2}$:

$P_{G2} = P_{v1} \times T_2/T_1 = 3.9396 \times T_2/308.2 = 0.01278\ T_2$

Assume $T_2 = 30^oC$: $0.01278 \times 303.2 = 3.875 \neq 4.246 = P_{G\ 30C}$

Assume $T_2 = 25^oC$: $0.01278 \times 298.2 = 3.811 \neq 3.169 = P_{G\ 25C}$

interpolating → $T_2 =$ **28.2 °C**

$$w_2 = w_1 = 0.622 \frac{3.9396}{(100\text{-}3.9369)} = 0.025\ 51$$

$m_a = P_{a1}V/R_aT_1 = (100\text{-}3.94) \times 0.5/(0.287 \times 308.2) = 0.543$ kg

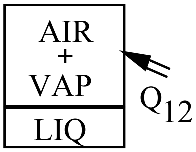
Energy Eq.5.11: $_1Q_2 = U_2 - U_1 = m_a(u_{a2} - u_{a1}) + m_v(u_{v2} - u_{v1})$

$= 0.717(28.2 - 35) + 0.02551\ (2414.2 - 2423.4) = -5.11$ kJ/kg

→ $_1Q_2 = 0.543(-5.11) =$ **-2.77 kJ**

13.76

A saturated air-water vapor mixture at 20 °C, 100 kPa, is contained in a 5-m^3 closed tank in equilibrium with 1 kg of liquid water. The tank is heated to 80°C. Is there any liquid water in the final state? Find the heat transfer for the process.



a) Since $V_{liq} = m_{liq}v_F \approx 0.001 \text{ m}^3$, $V_{GAS} \approx V$

$$\phi_1 = 1.00 \rightarrow P_{v1} = P_{G1} = 2.339 \text{ kPa}$$

$$w_1 = 0.622\times 2.339 /(100 - 2.339) = 0.0149$$

$$m_a = \frac{P_{a1}V}{R_aT_1} = \frac{97.661\times4.999}{0.287\times293.2} = 5.802 \text{ kg} \quad => \quad m_{v1} = w_1m_a = 0.086 \text{ kg}$$

At state 2: $P_{a2} = 97.661\times\frac{353.2}{293.2}\times\frac{4.999}{5} = 117.623 \text{ kPa}$

$$w_{MAX\,2} = 0.622 \times 47.39 / 117.623 = 0.2506$$

But $w_{2\,ACTUAL} = \frac{0.086 + 1.0}{5.802} = 0.1872 < w_{MAX\,2}$ → **No liquid at 2**

$$Q_{12} = m_a (u_{a2} - u_{a1}) + m_{v2}\, u_{v2} - m_{v1}u_{v1} - m_{liq\,1}u_{liq\,1}$$

$$= 5.802 \times 0.717(80 - 20) + 1.086 \times 2482.2 - 0.086 \times 2402.9 - 1 \times 84.0$$

$$= 249.6 + 2695.7 - 206.65 - 84 = \mathbf{2655\ kJ}$$

13.77

A flow of 0.2 kg/s liquid water at 80°C is sprayed into a chamber together with 16 kg/s dry air at 80°C. All the water evaporates and the air leaves at 40°C. What is the exit relative humidity and the heat transfer.

CV. Chamber.

Continuity Eq. water: $\dot{m}_{liq} = w_{ex}\, \dot{m}_a$

Energy Eq.: $\dot{m}_{liq}\, h_{liq} + \dot{m}_a\, h_{a\,i} + \dot{Q} = \dot{m}_a\, (wh_v + h_a)_{ex}$

$$w_{ex} = \dot{m}_{liq} / \dot{m}_a = 0.2 / 16 = 0.0125$$

From Eq.13.25 and 13.28 you can get

$$\phi_{ex} = P_v / P_g = \frac{w}{0.622 + w} \frac{P}{P_g} = \frac{0.0125}{0.622 + 0.0125} \times \frac{100}{7.384} = \mathbf{0.267 = 27\%}$$

$$\dot{Q} = \dot{m}_a\, (wh_v + h_a)_{ex} - \dot{m}_{liq}\, h_{liq} + \dot{m}_a\, h_{a\,i} = \dot{m}_a(\, h_{a\,ex} - h_{a\,i}) + \dot{m}_{liq}(h_v - h_{liq})$$

$$= \dot{m}_a\, C_{p\,a}\, (T_{ex} - T_{in}) + \dot{m}_{liq}\, (h_{v\,40} - h_{f\,80})$$

$$= 16 \times 1.004\, (40 - 80) + 0.2\, (2574.26 - 334.88)$$

$$= - 642.56 + 447.88 = \mathbf{-194.68\ kW}$$

13.78

A rigid container, 10 m^3 in volume, contains moist air at 45°C, 100 kPa, $\phi = 40\%$. The container is now cooled to 5°C. Neglect the volume of any liquid that might be present and find the final mass of water vapor, final total pressure and the heat transfer.

Solution:

CV container. $m_2 = m_1$; $m_2u_2 - m_1u_1 = {}_1Q_2$

State 1: 45°C, $\phi = 40\%$ => $w_1 = 0.0236$, $T_{dew} = 27.7°C$

Final state $T_2 < T_{dew}$ so condensation, $\phi_2 = 100\%$

$P_{v1} = 0.4\ P_g = 0.4 \times 9.593 = 3.837$ kPa, $P_{a1} = P_{tot} - P_{v1} = 96.163$ kPa

$m_a = P_{a1}V/RT_1 = 10.532$ kg, $m_{v1} = w_1\ m_a = 0.248$ kg

$P_{v2} = P_{g2} = 0.8721$ kPa, $P_{a2} = P_{a1}T_2/T_1 = 84.073$ kPa

$P_2 = P_{a2} + P_{v2} = \mathbf{84.95\ kPa}$

$m_{v2} = P_{v2}V/R_vT_2 = \mathbf{0.06794\ kg}$ (= $V/v_g = 0.06797$ steam table)

$m_{f2} = m_{v1} - m_{v2} = 0.180$ kg

The heat transfer from the energy equation becomes

$${}_1Q_2 = m_a(u_2-u_1)_a + m_{v2}u_{g2} + m_{f2}u_{f2} - m_{v1}u_{g1}$$

$$= m_a\ C_v(T_2 - T_1) + m_{v2}\ 2382.3 + m_{f2}\ 20.97 - m_{v1}\ 2436.8$$

$$= -302.06 + 161.853 + 3.775 - 604.33 = \mathbf{-\ 740.8\ kJ}$$

13.79

A water-filled reactor of 1 m^3 is at 20 MPa, 360°C and located inside an insulated containment room of 100 m^3 that contains air at 100 kPa and 25°C. Due to a failure the reactor ruptures and the water fills the containment room. Find the final pressure.

CV Total container.

$$m_v\,(u_2 - u_1) + m_a\,(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0$$

Initial water: $v_1 = 0.0018226$, $u_1 = 1702.8$ kJ/kg, $m_v = V/v = 548.67$ kg

Initial air: $m_a = \dfrac{PV}{RT} = \dfrac{100 \times 99}{0.287 \times 298.2} = 115.7$ kg

Substitute into energy equation

$$548.67\,(u_2 - 1702.8) + 115.7{\times}0.717\,(T_2 - 25) = 0$$

$u_2 + 0.1511\,T_2 = 1706.6$ kJ/kg & $v_2 = V_2/m_v = 0.18226\ m^3/kg$

Trial and error 2-phase (T_{guess}, v_2 => x_2 => u_2 => LHS)

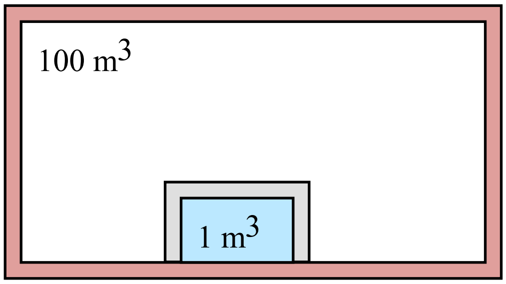
T = 150°C LHS = 1546 T = 160°C LHS = 1820.2

T = 155°C LHS = 1678.1 => T = 156°C LHS = 1705.7 OK

$x_2 = 0.5372$, $P_{sat} = 557.5$ kPa

$$P_{a2} = P_{a1}V_1T_2/V_2T_1 = 100{\times}\ 99 \times 429.15 / (100{\times}298.15) = 142.5\ \text{kPa}$$

$$\Rightarrow \quad P_2 = P_{a2} + P_{sat} = \mathbf{700\ kPa.}$$



Tables and formulas or psychrometric chart

13.80

I want to bring air at 35°C, $\Phi = 40\%$ to a state of 25°C, $\omega = 0.01$ do I need to add or subtract water?

The humidity ratio (absolute humidity) expresses how much water vapor is present in the mixture

$$\omega = m_v / m_a$$

Assuming P = 100 kPa,

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v} \quad \text{and} \quad P_v = \Phi\, P_g$$

At 35°C, 40 % :

$$\omega = 0.622 \times \frac{0.40 \times 5.628}{100 - 0.40 \times 5.628} = 0.014\ 32$$

To get to $\omega = 0.01$, it is necessary to **subtract water**.

13.81

A flow moist air at 100 kPa, 40°C, 40% relative humidity is cooled to 15°C in a constant pressure device. Find the humidity ratio of the inlet and the exit flow, and the heat transfer in the device per kg dry air.

Solution:

C.V. Cooler. $\dot{m}_{v1} = \dot{m}_{liq} + \dot{m}_{v2}$

Tables: $P_{g1} = 7.384$ kPa, $P_{v1} = \phi\, P_g = 0.4 \times 7.384 = 2.954$ kPa,

$\omega_1 = 0.622 \times 2.954 /(100 - 2.954) = 0.0189$

$T_2 < T_{dew}$ [from $P_g(T_{dew}) = 2.954$] => $P_{v2} = 1.705$ kPa $= P_{g2}$ =>

$\omega_2 = 0.622 \times 1.705 /(100 - 1.705) = 0.0108$

$h_{v1} = 2574.3$ kJ/kg, $h_{v2} = 2528.9$ kJ/kg, $h_f = 62.98$ kJ/kg

$\bar{q}_{out} = C_P(T_1 - T_2) + \omega_1 h_{v1} - \omega_2\, h_{v2} - (\omega_1 - \omega_2)\, h_f$

$= 1.004(40 - 15) + 0.0189 \times 2574.3 - 0.0108 \times 2528.9 - 0.0073 \times 62.98$

= **45.98 kJ/kg dry air**

Psychrometric chart: State 2: $T < T_{dew} = 23°C$ => $\phi_2 = 100\%$

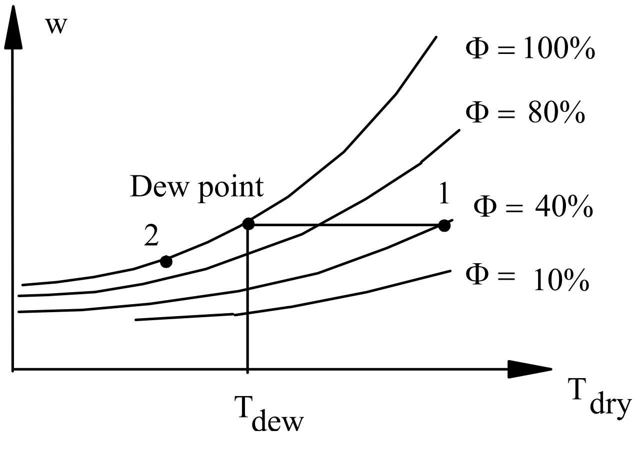
$\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.018$, $\tilde{h}_1 = 106$; $\dot{m}_{v2}/\dot{m}_a = \omega_2 = 0.0107$, $\tilde{h}_2 = 62$

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0073$, $h_f = 62.98$ kJ/kg

$\dot{m}_a\, \bar{q}_{out} = \dot{m}_a \tilde{h}_1 - \dot{m}_{liq}\, h_f - \dot{m}_a\, \tilde{h}_2$ =>

$\bar{q}_{out} = \tilde{h}_1 - (\omega_1 - \omega_2)\, h_f - \tilde{h}_2 = 106 - 0.0073 \times 62.98 - 62$

= **43.54 kJ/kg-dry air**



13.82

Use the formulas and the steam tables to find the missing property of: ϕ, ω, and T_{dry}, total pressure is 100 kPa; repeat the answers using the psychrometric chart

a. $\phi = 50\%$, $\omega = 0.010$ b. $T_{dry} = 25°C$, $T_{wet} = 21°C$

Solution:

a. From Eq.13.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P\,\omega/(0.622 + \omega) = 100 \times 0.01/0.632 = 1.582 \text{ kPa}$$

From Eq.13.25 $P_g = P_v/\phi = 1.582/0.5 = 3.165$ kPa => $\mathbf{T = 25°C}$

b. At 21°C:: $P_g = 2.505$ => $\omega_2 = 0.622 \times 2.505/(100 - 2.505) = 0.016$

From the steam tables B.1.1

$h_{f2} = 88.126$ and $h_{fg2} = 2451.76$ kJ/kg, $h_{v1} = 2547.17$

From Eq.13.30:

$$\omega_1 = [C_p(T_2\text{-}T_1) + \omega_2\, h_{fg2}]/(h_{v1} - h_{f2}) = \mathbf{0.0143}$$

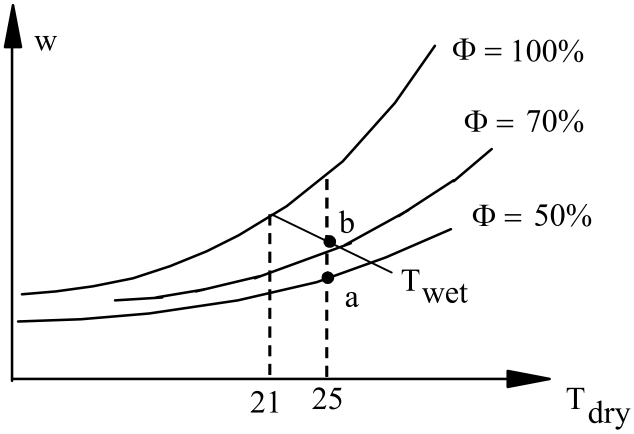
From Eq.13.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P\,\omega/(0.622 + \omega) = 2.247 \text{ kPa},$$

From Eq.13.25: $\phi = 2.247/3.169 = \mathbf{0.71}$

Using the psychrometric chart E.4:

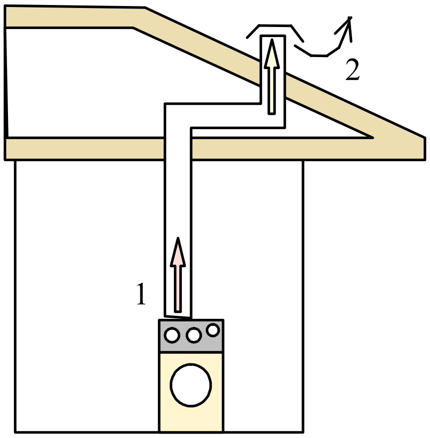
a: $\boldsymbol{T_{dry} = 25.3}$ **°C** b: $\boldsymbol{\omega = 0.0141}$**,** $\boldsymbol{\phi}$ **= 71-72%**



13.83

The discharge moist air from a clothes dryer is at 35°C, 80% relative humidity. The flow is guided through a pipe up through the roof and a vent to the atmosphere. Due to heat transfer in the pipe the flow is cooled to 24°C by the time it reaches the vent. Find the humidity ratio in the flow out of the clothes dryer and at the vent. Find the heat transfer and any amount of liquid that may be forming per kg dry air for the flow.

Solution:



State 1: $w = 0.0289$, $\tilde{h}_1 = 128$, $T_{dew} = 31^{o}C$

State 2: $24^{o}C < T_{dew}$ so it is saturated.

$w = 0.019$, $\tilde{h}_2 = 92$ kJ/kg air

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 =$ **0.0099 kg/kg dry air**

Energy Eq.:

$$\dot{Q}/\dot{m}_a = \tilde{h}_1 - \tilde{h}_2 - (\omega_1 - \omega_2)\, h_f$$

$$= 128 - 92 - 0.0099 \times 100.68$$

$$= \mathbf{35\ kJ/kg\ dry\ air}$$

13.84

A flow, 0.2 kg/s dry air, of moist air at 40°C, 50% relative humidity flows from the outside state 1 down into a basement where it cools to 16°C, state 2. Then it flows up to the living room where it is heated to 25°C, state 3. Find the dew point for state 1, any amount of liquid that may appear, the heat transfer that takes place in the basement and the relative humidity in the living room at state 3.

Solve using psychrometric chart:

a) $T_{dew} = 27.2$ $(w = w_1, \phi = 100\%)$ $w_1 = 0.0232,\ \tilde{h}_1 = 118.2$ kJ/kg air

b) $T_2 < T_{dew}$ so we have $\phi_2 = 100\%$ liquid water appear in the basement.

$\Rightarrow w_2 = 0.0114$ $\tilde{h}_2 = 64.4$ and from steam tbl. $h_f = 67.17$

$\dot{m}_{liq} = \dot{m}_{air}(w_1\text{-}w_2) = 0.2(0.0232\text{-}0.0114) = 0.00236$ kg/s

c) Energy equation: $\dot{m}_{air}\,\tilde{h}_1 = \dot{m}_{liq}\,h_f + \dot{m}_{air}\,\tilde{h}_2 + \dot{Q}_{out}$

$$\dot{Q}_{out} = 0.2[118.2 - 64.4 - 0.0118\times67.17] = 10.6 \text{ kW}$$

d) $w_3 = w_2 = 0.0114$ & 25°C $\Rightarrow$ $\phi_3 = 58\%$.

If you solve by the formulas and the tables the numbers are:

$P_{g40} = 7.384$ kPa; $P_{v1} = \phi\, P_{g40} = 0.5 \times 7.384 = 3.692$ kPa

$w_1 = 0.622 \times 3.692 / (100 - 3.692) = 0.02384$

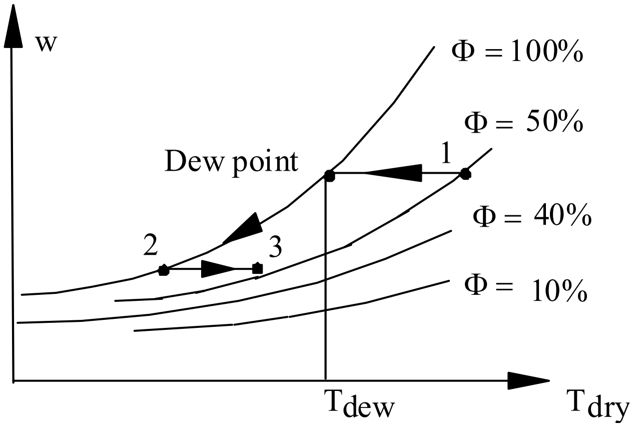
$P_{v1} = P_g\,(T_{dew})$ $\Rightarrow$ $T_{dew\ 1} = 27.5$ °C

2: $\phi = 100\%$, $P_{v2} = P_{g2} = 1.832$ kPa, $w_2 = 0.622\times1.832/98.168 = 0.0116$

$\dot{m}_{liq} = \dot{m}_{air}\,(w_1\text{-}w_2) = 0.2\times0.01223 = 0.00245$ kg/s

3: $w_3 = w_2$ $\Rightarrow$ $P_{v3} = P_{v2} = 1.832$ kPa & $P_{g3} = 3.169$ kPa

$\phi_3 = P_v/P_g = 1.832/3.169 = 57.8\%$



13.85

A steady supply of 1.0 m^3/s air at 25°C, 100 kPa, 50% relative humidity is needed to heat a building in the winter. The outdoor ambient is at 10°C, 100 kPa, 50% relative humidity. What are the required liquid water input and heat transfer rates for this purpose?

Solution:

Air: $R_a = 0.287$ kJ/kg K, $C_p = 1.004$ kJ/kg-K

State 1: $T_1 = 10°C$, $\phi_1 = 50\%$, $P_1 = 100$ kPa

$P_{g1} = 1.2276$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.6138$ kPa,

$P_{a1} = P_1 - P_{v1} = 99.39$ kPa => $\omega_1 = 0.622\ P_{v1}/P_{a1} = 0.003841$

State 2: $T_2 = 25°C$, $P_2 = 100$ kPa, $\phi_2 = 50\%$, $\dot{V}_2 = 1\ m^3/s$

$P_{g2} = 3.169$ kPa, $P_{v2} = \phi_2 P_{g2} = 1.5845$ kPa,

$P_{a2} = P_2 - P_{v2} = 98.415$ kPa, $\omega_2 = 0.622\ P_{v2}/P_{a2} = 0.010014$

$\dot{m}_{a2} = P_{a2} \dot{V}_2/R_a T_2 = 98.415 \times 1/(0.287 \times 298.15) = 1.15$ kg/s

Steam tables B.1.1: $h_{v1} = 2519.7$ kJ/kg, $h_{v2} = 2547.2$ kJ/kg

State 3: Assume: Liq. Water at $T_3 = 25°C$, $h_{f3} = 104.9$ kJ/kg

Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{f3} = \dot{m}_{v2} - \dot{m}_{v1}$

$\dot{m}_{f3} = \dot{m}_{a2}(\omega_2 - \omega_1) = 1.15 \times 0.006173 =$ **0.0071 kg/s**

1^{st}Law: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f3}h_{f3} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}$

$$\frac{\dot{Q}}{\dot{m}_a} = C_p(T_2 - T_1) + \omega_2 h_{v2} - \omega_1 h_{v1} - \frac{\dot{m}_{f3}}{\dot{m}_a} h_{f3} \quad => \quad \dot{Q} = \mathbf{34.76\ kW}$$

13.86

In a ventilation system inside air at 34°C and 70% relative humidity is blown through a channel where it cools to 25°C with a flow rate of 0.75 kg/s dry air. Find the dew point of the inside air, the relative humidity at the end of the channel, and the heat transfer in the channel.

C.V. Cooler. $\dot{m}_{v1} = \dot{m}_{liq} + \dot{m}_{v2}$

Tables: $P_{g1} = 5.352$ kPa, $P_{v1} = \phi\, P_g = 0.7 \times 5.352 = 3.75$ kPa,

$\omega_1 = 0.622 \times 3.75 /(101 - 3.75) = 0.024$

$T_2 < T_{dew} = 27C$ [from $P_g(T_{dew}) = 3.75$] => $P_{v2} = 3.169 \text{ kPa} = P_{g2}$ =>

$\omega_2 = 0.622 \times 3.169 /(101 - 3.169) = 0.02015$

$h_{v1} = 2547.17$ kJ/kg, $h_{v2} = 2563.47$ kJ/kg, $h_f = 104.87$ kJ/kg

$$\bar{q}_{out} = C_P(T_1 - T_2) + \omega_1 h_{v1} - \omega_2 h_{v2} - (\omega_1 - \omega_2)\, h_f$$

$$= 1.004(34 - 25) + 0.024 \times 2547.17 - 0.02015 \times 2563.47$$

$$- 0.00385 \times 104.87 = 18.11 \text{ kJ/kg dry air}$$

$\dot{Q} = \dot{m}_a\ \bar{q}_{out} =$ **13.58 kW**

Psychrometric chart: State 2: $T < T_{dew} = 27.5°C$ => $\phi_2 = 100\%$

$\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.0234$, $\tilde{h}_1 = 113.7$ kJ/kg air;

$\dot{m}_{v2}/\dot{m}_a = \omega_2 = 0.0202$, $\tilde{h}_2 = 96$ kJ/kg air

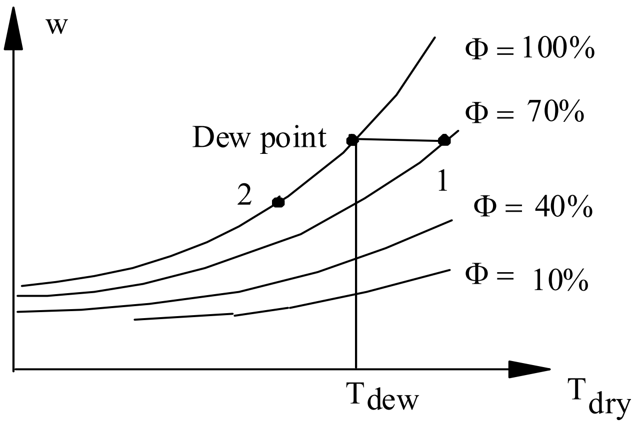
$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0032$, $h_f = 104.87$ kJ/kg

$\dot{m}_a\, \bar{q}_{out} = \dot{m}_a \tilde{h}_1 - \dot{m}_{liq}\, h_f - \dot{m}_a\, \tilde{h}_2$ =>

$$\bar{q}_{out} = \tilde{h}_1 - (\omega_1 - \omega_2)\, h_f - \tilde{h}_2 = 113.7 - 0.0032 \times 104.87 - 96$$

$$= 17.36 \text{ kJ/kg-dry air}$$

$\dot{Q} = \dot{m}_a\ \bar{q}_{out} =$ **13.0 kW**


w
Φ = 100%
Φ = 70%
Dew point
2
1
Φ = 40%
Φ = 10%
T_{dew}
T_{dry}

13.87

Two moist air streams with 85% relative humidity, both flowing at a rate of 0.1 kg/s of dry air are mixed in a steady setup. One inlet flowstream is at 32.5°C and the other at 16°C. Find the exit relative humidity.

Solution:

CV mixing chamber.

Continuity Eq. water: $\dot{m}_{air}\, w_1 + \dot{m}_{air}\, w_2 = 2\dot{m}_{air}\, w_{ex}$;

Energy Eq.: $\dot{m}_{air}\, \tilde{h}_1 + \dot{m}_{air}\, \tilde{h}_2 = 2\dot{m}_{air}\, \tilde{h}_{ex}$

Properties from the tables and formulas

$P_{g32.5} = 4.937$; $P_{v1} = 0.85\times4.937 = 4.196$ kPa

$w_1 = 0.622 \times 4.196 / (100 - 4.196) = 0.0272$

$P_{g16} = 1.831$; $P_{v2} = 0.85\times1.831 = 1.556$ kPa

$w_2 = 0.622 \times 1.556 / (100 - 1.556) = 0.00983$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.0185$;

For the energy equation we have $\tilde{h} = h_a + wh_v$ so:

$2\, \tilde{h}_{ex} - \tilde{h}_1 - \tilde{h}_2 = 0 = 2h_{a\ ex} - h_{a\ 1} - h_{a\ 2} + 2w_{ex}h_{v\ ex} - w_1h_{v\ 1} - wh_{v\ 2}$

we will use constant heat capacity to avoid an iteration on T_{ex}.

$$C_{p\ air}(2T_{ex} - T_1 - T_2) + C_{p\ H2O}(2w_{ex}T_{ex} - w_1T_1 - w_2T_2) = 0$$

$$T_{ex} = [\ C_{p\ air}(T_1 + T_2) + C_{p\ H2O}(w_1T_1 + w_2T_2)\ ]/\ [2C_{p\ air} + 2w_{ex}C_{p\ H2O}]$$

$$= [\ 1.004\ (32.5 + 16) + 1.872(0.0272 \times 32.5 + 0.00983 \times 16]/2.0773$$

$$= 24.4°\text{C}$$

$$P_{v\ ex} = \frac{w_{ex}}{0.622 + w_{ex}} P_{tot} = \frac{0.0185}{0.622 + 0.0185} 100 = 2.888 \text{ kPa},$$

$P_{g\ ex} = 3.069$ kPa => $\phi = 2.888 / 3.069 =$ **0.94 or 94%**

Properties taken from the psychrometric chart

State 1: $w_1 = 0.0266$, $\tilde{h}_1 = 120$ State 2: $w_2 = 0.0094$, $\tilde{h}_2 = 60$

Continuity Eq. water: $w_{ex} = (w_1 + w_2)/2 = 0.018$;

Energy Eq.: $\tilde{h}_{ex} = (\tilde{h}_1 + \tilde{h}_2)/2 = 90$ kJ/kg dry air

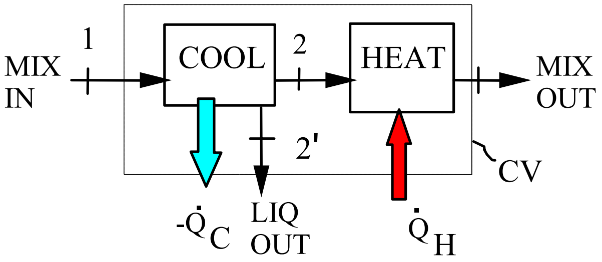
exit: w_{ex}, $\tilde{h}_{ex}$ => $T_{ex} =$ **24.5°C**, **ϕ = 94%**

Notice how the energy in terms of temperature is close to the average of the two flows but the relative humidity is not.

13.88

A combination air cooler and dehumidification unit receives outside ambient air at 35°C, 100 kPa, 90% relative humidity. The moist air is first cooled to a low temperature T_2 to condense the proper amount of water, assume all the liquid leaves at T_2. The moist air is then heated and leaves the unit at 20°C, 100 kPa, relative humidity 30% with volume flow rate of 0.01 m^3/s. Find the temperature T_2, the mass of liquid per kilogram of dry air and the overall heat transfer rate.

Solution:



$$P_{v1} = \phi_1 P_{G1} = 0.9 \times 5.628 = 5.0652 \text{ kPa}$$

$$w_1 = 0.622 \times \frac{5.0652}{100\text{-}5.0652} = 0.033\ 19$$

$$P_{v3} = \phi_3 P_{G3} = 0.3 \times 2.339 = 0.7017 \text{ kPa}$$

$$w_2 = w_3 = 0.622 \times \frac{0.7017}{100\text{-}0.7017} = 0.0044$$

$$\dot{m}_{LIQ\ 2'}/\dot{m}_a = w_1 - w_2 = 0.033\ 19 - 0.0044 = \mathbf{0.028\ 79\ kg/kg\ air}$$

$$P_{G2} = P_{v3} = 0.7017 \text{ kPa} \quad \rightarrow \quad T_2 = \mathbf{1.7\ {}^oC}$$

For a C.V. around the entire unit

$$\dot{Q}_{CV} = \dot{Q}_H + \dot{Q}_C$$

Net heat transfer, Energy Eq.:

$$\dot{Q}_{CV}/\dot{m}_a = (h_{a3}\text{-}h_{a1}) + w_3 h_{v3} - w_1 h_{v1} + \dot{m}_{L2'}\, h_{L2'}/\dot{m}_a$$

$$= 1.004(20\text{-}35) + 0.0044 \times 2538.1 - 0.033\ 19 \times 2565.3 + 0.028\ 79 \times 7.28$$

$$= -88.82 \text{ kJ/kg air}$$

$$\dot{m}_a = \frac{P_{a3}\dot{V}_3}{R_a T_3} = \frac{(100\text{-}0.7017) \times 0.01}{0.287 \times 293.2} = 0.0118 \text{ kg/s}$$

$$\dot{Q}_{CV} = 0.0118(-88.82) = \mathbf{-1.05\ kW}$$

13.89

To make dry coffee powder we spray 0.2 kg/s coffee (assume liquid water) at 80°C into a chamber where we add 8 kg/s dry air at T. All the water should evaporate and the air should leave with a minimum 40°C and we neglect the powder. How high should T in the inlet air flow be?

CV. Chamber. We assume it is adiabatic.

Continuity Eq. water alone: $\dot{m}_{liq} = w_{ex}\, \dot{m}_a$

Energy Eq.: $\dot{m}_{liq}\, h_{f\,80} + \dot{m}_a\, h_{a\,Ti} = \dot{m}_{liq}\, h_{v\,40} + \dot{m}_a\, h_{a\,40}$

$$w_{ex} = \dot{m}_{liq} / \dot{m}_a = 0.2 / 8 = 0.025$$

From the energy equation you get

$$h_{a\,Ti} - h_{a\,40} = C_{p\,a}\,(T_{in} - T_{ex}) = \dot{m}_{liq}\,(h_{v\,40} - h_{f\,80}) / \dot{m}_a$$

$$1.004\ \Delta T = 0.2\,(2574.26 - 334.88) / 8 = 55.985 \text{ kJ/kg}$$

$$\Delta T = 55.76^{o}\text{C} \quad \Rightarrow \quad T_{in} = 40 + 55.76 = 95.76^{o}\text{C} = \mathbf{96^{o}C}$$

13.90

An insulated tank has an air inlet, $\omega_1 = 0.0084$, and an outlet, $T_2 = 22°C$, $\phi_2 =$ 90% both at 100 kPa. A third line sprays 0.25 kg/s of water at 80°C, 100 kPa. For steady operation find the outlet specific humidity, the mass flow rate of air needed and the required air inlet temperature, T_1.

Solution:

Take CV tank in steady state. Continuity and energy equations are:

Continuity Eq. water: $\dot{m}_3 + \dot{m}_a w_1 = \dot{m}_a w_2$

Energy Eq.: $\dot{m}_3 h_f + \dot{m}_a \tilde{h}_1 = \dot{m}_a \tilde{h}_2$

All state properties are known except T_1.

From the psychrometric chart we get

State 2: $w_2 =$ **0.015**, $\tilde{h}_2 = 79.5$ State 3: $h_f = 334.91$ (steam tbl)

$\dot{m}_a = \dot{m}_3/(w_2 - w_1) = 0.25/(0.015-0.0084) =$ **37.88 kg/s**

$\tilde{h}_1 = \tilde{h}_2 - (w_2 - w_1)h_f = 79.5 - 0.0066 \times 334.91 = 77.3$

Chart $(w_1, \tilde{h}_1)$ => $T_1 =$ **36.5°C**

Using the tables and formulas we get

State 2: $P_{g22} = 2.671$; $P_{v2} = 0.9 \times 2.671 = 2.4039$ kPa

$w_2 = 0.622 \times 2.4039 / (100 - 2.4039) =$ **0.0153**

$\dot{m}_a = \dot{m}_3/(w_2 - w_1) = 0.25/(0.0153 - 0.0084) =$ **36.23 kg/s**

To avoid iterations on T_1 we use specific heat values also for water vapor by writing $h_{v1} = h_{v2} + C_{p\ h2o}(T_1 - T_2)$ so the energy equation is

$$C_{p\ a} T_1 + w_1 C_{p\ h2o}(T_1 - T_2) + w_1 h_{v2} = C_{p\ a} T_2 + w_2 h_{v2} - (w_2 - w_1)\ h_f$$

The equation now becomes

$$(1.004 + 0.0084 \times 1.872)T_1 = (0.0084 \times 1.872 + 1.004)\ 22$$
$$+ (0.0153 - 0.0084)(2541.7 - 334.91) = 37.219$$

$T_1 =$ **36.5°C**

13.91

A water-cooling tower for a power plant cools 45°C liquid water by evaporation. The tower receives air at 19.5°C, $\phi = 30\%$, 100 kPa that is blown through/over the water such that it leaves the tower at 25°C, $\phi = 70\%$. The remaining liquid water flows back to the condenser at 30°C having given off 1 MW. Find the mass flow rate of air, and the amount of water that evaporates.

Solution:

CV Total cooling tower, steady state.

Continuity Eq. for water in air: $w_{in} + \dot{m}_{evap}/\dot{m}_a = w_{ex}$

Energy Eq.: $\dot{m}_a \tilde{h}_{in} + \dot{m}_1 h_{45} = \dot{m}_a \tilde{h}_{ex} + (\dot{m}_1 - \dot{m}_{evap}) h_{30}$

Inlet: 19.5°C, 30% rel hum => $w_{in} = 0.0041$, $\tilde{h}_{in} = 50$ kJ/kg dry air

Exit : 25°C, 70% rel hum => $w_{ex} = 0.0138$, $\tilde{h}_{ex} = 80$ kJ/kg dry air

Take the two water flow difference to mean the 1 MW

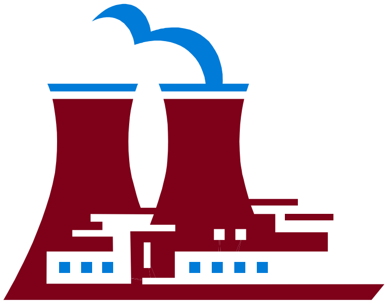
$$\dot{Q} = \dot{m}_1 h_{45} - (\dot{m}_1 - \dot{m}_{evap}) h_{30} = 1 \text{ MW}$$

Substitute this into the energy equation above and we get

$$\dot{m}_a(\tilde{h}_{ex} - \tilde{h}_{in}) = \dot{m}_a(80 - 50) = 1000 \text{ kW} \quad \Rightarrow \quad \dot{m}_a = \mathbf{33.33\ kg/s}$$

$$\dot{m}_{evap} = (w_{ex} - w_{in})\, \dot{m}_a = 0.0097 \times 33.33 = \mathbf{0.323\ kg/s}$$

The needed make-up water flow could be added to give a slightly different meaning to the 1 MW.



13.92

Moist air at 31°C and 50% relative humidity flows over a large surface of liquid water. Find the adiabatic saturation temperature by trial and error. Hint: it is around 22.5°C.

For adiabatic saturation ($\Phi_2 = 1$ and assume P = 100 kPa), energy Eq.13.30

$$\omega_1 (h_{v1} - h_{f2}) = C_p(T_2 - T_1) + \omega_2 h_{fg2}$$

State 1: $$\omega_1 = 0.622 \times \frac{\Phi P_{g1}}{P_1 - \Phi P_{g1}} = 0.622 \times \frac{0.5 \times 4.5}{100 - 0.5 \times 4.5} = 0.01432$$

$$\phi_2 = 1 \quad \& \quad \omega_2 = 0.622 \times P_{g2}/(P_2 - P_{g2})$$

Only one unknown in energy Eq.: T_2 . Trial and error on energy equation:

$$C_pT_2 + \omega_2 h_{fg2} + \omega_1 h_{f2} = C_pT_1 + \omega_1 h_{v1}$$

$$= 1.004 \times 31 + 0.01432 \times 2558 = 67.7546 \text{ kJ/kg}$$

$T_2 = 20$ °C: $P_{g2} = 2.339$ kPa, $h_{f2} = 83.94$ kJ/kg, $h_{fg2} = 2454.12$ kJ/kg

=> $\omega_2 = 0.622 \times 2.339/ 97.661 = 0.0149$

LHS = $1.004 \times 20 + 0.0149 \times 2454.12 + 0.01432 \times 83.94 = 57.848$

$T_2 = 25$ °C: $P_{g2} = 3.169$ kPa, $h_{f2} = 104.87$ kJ/kg, $h_{fg2} = 2442.3$ kJ/kg

=> $\omega_2 = 0.622 \times 3.169/ 96.831 = 0.02036$

LHS = $1.004 \times 25 + 0.02036 \times 2442.3 + 0.01432 \times 104.87 = 76.327$

Linear interpolation to match RHS = 67.7546:

$$T_2 = 20 + (25 - 20) \frac{67.7546 - 57.848}{76.327 - 57.848} = \mathbf{22.7\ ^oC}$$

13.93

A flow of air at 5°C, $\phi = 90\%$, is brought into a house, where it is conditioned to 25°C, 60% relative humidity. This is done with a combined heater-evaporator where any liquid water is at 10°C. Find any flow of liquid, and the necessary heat transfer, both per kilogram dry air flowing. Find the dew point for the final mixture.

CV heater and evaporator. Use psychrometric chart.

Inlet: $w_1 = 0.0048$, $\tilde{h}_1 = 37.5$ kJ/kg dry air, $h_f = 42.01$ kJ/kg

Exit: $w_2 = 0.0118$, $\tilde{h}_2 = 75$ kJ/kg dry air, $\mathbf{T_{dew} = 16.5°C}$

From these numbers we see that water and heat must be added.

Continuity eq. for water

$$\dot{m}_{LIQ\ IN}/\dot{m}_A = w_2 - w_1 = \mathbf{0.007\ kg/kg\ dry\ air}$$

Energy equation per kg dry air

$$q = \tilde{h}_2 - \tilde{h}_1 - (w_2 - w_1)h_f = \mathbf{37.3\ kJ/kg\ dry\ air}$$

13.94

An air conditioner for an airport receives desert air at 45°C, 10% relative humidity and must deliver this to the buildings at 20°C, 50% relative humidity. They have a cooling system with R-410a running with high pressure of 3000 kPa and low pressure of 1000 kPa and their tap water is 18°C. What should be done to the air? Find the needed heating/cooling per kg dry air.

Check out the psychrometric chart:

State 1: $w_1 = 0.0056$, $h_1 = 79$; State 2: $w_2 = 0.0072$, $h_2 = 58$ kJ/kg

Liquid tap: $h = 75.556$ kJ/kg from B.1.1

Now we know the following:

We must **add water** ($w_2 > w_1$) and then **cool** ($T_{wet\ 1} > 20^{o}C$)

Water continuity equation: $\dot{m}_{liq} = \dot{m}_{air} (w_2 - w_1)$

Energy equation: $h + (w_2 - w_1)h + q = h$

$$q = 58 - 79 - (0.0072 - 0.0056)\ 75.556$$
$$= -21.12 \text{ kJ/kg dry air}$$

For the refrigeration cycle we can find from table B.3.1

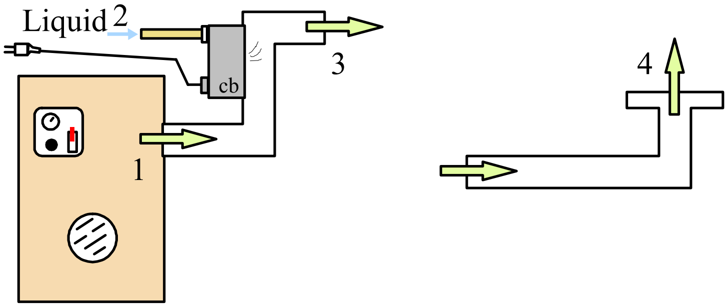
$P_{low} = 200$ kPa $\Rightarrow$ $T_{evaporator} = -12^{o}C$, which is cold enough

$P_{high} = 1500$ kPa $\Rightarrow$ $T_{condenser} = 59^{o}C > 45^{o}C$ so it is hot enough.

No absolute scaling was provided (the mass flow rates or $\dot{W}$) so we do not know if the motor/compressor combination is big enough.

13.95

A flow of moist air from a domestic furnace, state 1, is at 45°C, 10% relative humidity with a flow rate of 0.05 kg/s dry air. A small electric heater adds steam at 100°C, 100 kPa generated from tap water at 15°C. Up in the living room the flow comes out at state 4: 30°C, 60% relative humidity. Find the power needed for the electric heater and the heat transfer to the flow from state 1 to state 4.



Properties from the psychrometric chart

State 1: $w_1 = 0.0056$, $\tilde{h}_1 = 79$ kJ/kg dry air
State 4: $w_4 = 0.0160$, $\tilde{h}_4 = 90.5$ kJ/kg dry air

Continuity equation for water from 1 to 4

$$\dot{m}_{liq} = \dot{m}_a (\omega_4 - \omega_1) = 0.05\ (0.016 - 0.0056) = 0.00052 \text{ kg/s}$$

Energy Eq. for heater:

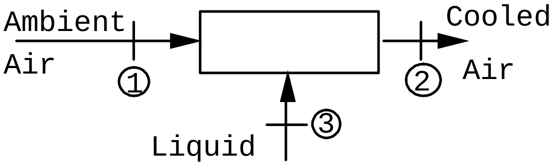
$$\dot{Q}_{heater} = \dot{m}_{liq} (h_{out} - h_{in}) = 0.00052\ (2676.05 - 62.98) = \mathbf{1.36\ kW}$$

Energy Eq. for line:

$$\dot{Q}_{line} = \dot{m}_a (\tilde{h}_4 - \tilde{h}_1) - \dot{m}_{liq}\, h_{vap}$$
$$= 0.05(90.5 - 79) - 0.00052 \times 2676.05$$
$$= \mathbf{-0.816\ kW}$$

13.96

One means of air-conditioning hot summer air is by evaporative cooling, which is a process similar to the adiabatic saturation process. Consider outdoor ambient air at 35°C, 100 kPa, 30% relative humidity. What is the maximum amount of cooling that can be achieved by such a technique? What disadvantage is there to this approach? Solve the problem using a first law analysis and repeat it using the psychrometric chart, Fig. E.4.



$P_1 = P_2 = 100$ kPa

$P_{v1} = \phi_1 P_{g1} = 0.30 \times 5.628 = 1.6884$

$\omega_1 = 0.622 \times 1.6884/98.31 = 0.01068$

For adiabatic saturation (Max. cooling is for $\phi_2 = 1$),

Energy Eq. Eq.13.30: $\omega_1 (h_{v1} - h_{f2}) = C_p(T_2 - T_1) + \omega_2 h_{fg2}$

$$\phi_2 = 1 \quad \& \quad \omega_2 = 0.622 \times P_{G2}/(P_2 - P_{G2})$$

Only one unknown: T_2 . Trial and error on energy equation:

$$C_pT_2 + \omega_2 h_{fg2} + \omega_1 h_{f2} = C_pT_1 + \omega_1 h_{v1}$$

$$= 1.004 \times 35 + 0.01068 \times 2565.3 = 62.537 \text{ kJ/kg}$$

$T_2 = 20$ °C: $P_{G2} = 2.339$ kPa, $h_{f2} = 83.94$, $h_{fg2} = 2454.12$ kJ/kg

=> $\omega_2 = 0.622 \times 2.339/ 97.661 = 0.0149$

LHS $= 1.004 \times 20 + 0.0149 \times 2454.1 + 0.01068 \times 83.94 = 57.543$ kJ/kg

$T_2 = 25$ °C: $P_{G2} = 3.169$ kPa, $h_{f2} = 104.87$, $h_{fg2} = 2442.3$ kJ/kg

=> $\omega_2 = 0.622 \times 3.169/ 96.831 = 0.02036$

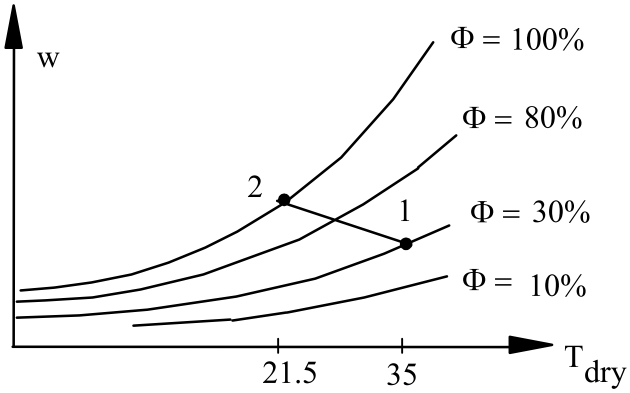
LHS $= 1.004 \times 25 + 0.02036 \times 2442.3 + 0.01068 \times 104.87 = 75.945$ kJ/kg

linear interpolation: $T_2 =$ **21.4 °C**

This method lowers the temperature but the relative and absolute humidity becomes very high and the slightest cooling like on a wall results in condensation.

Solution 13.96 Continued.

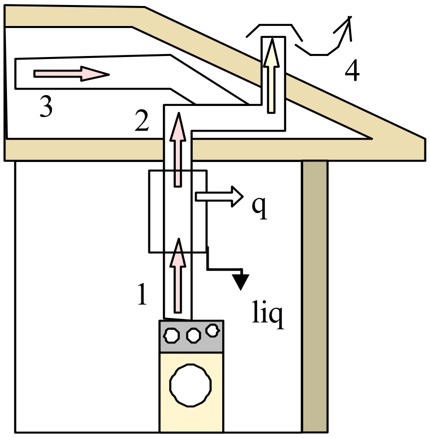
b) chart E.4 : Adiabatic saturation $T_2 \approx$ WetBulbTemperature $\approx$ **21.5 ^{o}C**



13.97

A flow out of a clothes dryer of 0.05 kg/s dry air is at 40°C and relative humidity 60%. It flows through a heat exchanger where it exits at 20°C. After heat exchanger the flow combines with another flow of 0.03 kg/s dry air at 30°C and relative humidity 30%. Find the dew point of state 1, see Fig. P13.97, the heat transfer per kg dry air and the final exit state humidity ratio and relative humidity.

Use the psychrometric chart to solve the problem.



State 1: $w = 0.0286$, $\tilde{h}_1 = 131.5$, $T_{dew} = 31^oC$

State 2: $20^oC < T_{dew}$ so it is saturated.

$w_2 = 0.0148$, $\tilde{h}_2 = 77.8$ kJ/kg air

State 3: $w_3 = 0.016$, $\tilde{h}_3 = 90.3$ kJ/kg air,

$$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0138 \text{ kg/kg dry air}$$

Energy Eq.:

$$\dot{Q}/\dot{m}_a = \tilde{h}_1 - \tilde{h}_2 - (\omega_1 - \omega_2)\, h_f$$
$$= 131.5 - 77.8 - 0.0138 \times 83.94$$
$$= 52.5 \text{ kJ/kg dry air}$$

Do CV around the junction where flow 2 and 3 combines to give exit at 4.

Continuity water: $\dot{m}_{a1}\, \omega_2 + \dot{m}_{a3}\, \omega_3 = \dot{m}_{a4}\, \omega_4$

$$\omega_4 = (\dot{m}_{a1}/\dot{m}_{a4})\, \omega_2 + (\dot{m}_{a3}/\dot{m}_{a4})\, \omega_3$$
$$= \frac{0.05}{0.08} \times 0.0148 + \frac{0.03}{0.08} \times 0.016 = 0.01525$$

Energy Equation: $\dot{m}_{a1}\, \tilde{h}_2 + \dot{m}_{a3}\, \tilde{h}_3 = \dot{m}_{a4}\, \tilde{h}_4$

$$\tilde{h}_4 = (\dot{m}_{a1}/\dot{m}_{a4})\, \tilde{h}_2 + (\dot{m}_{a3}/\dot{m}_{a4})\, \tilde{h}_3$$
$$= \frac{0.05}{0.08} \times 77.8 + \frac{0.03}{0.08} \times 90.3 = 82.49 \text{ kJ/kg air}$$

From chart given $(\omega_4, \tilde{h}_4)$ we get: $\Phi_4 = 80\%$ and $T_4 = 24^oC$

13.98

Atmospheric air at 35°C, relative humidity of 10%, is too warm and also too dry. An air conditioner should deliver air at 21°C and 50% relative humidity in the amount of 3600 m^3 per hour. Sketch a setup to accomplish this, find any amount of liquid (at 20°C) that is needed or discarded and any heat transfer.

Solution:

CV air conditioner. First we must check if water should be added or subtracted. We can know this from the absolute humidity ratio.

Properties from the tables and formulas

State 1: $P_{g35} = 5.628$; $P_{v1} = 0.10 \times 5.628 = 0.5628$ kPa

$w_1 = 0.622 \times 0.5628 / (101.325 - 0.5628) = 0.003474$

State 2: $P_{g21} = 2.505$; $P_{v2} = 0.5 \times 2.505 = 1.253$ kPa

$w_2 = 0.622 \times 1.253 / (101.325 - 1.253) = 0.007785$

As w goes up we must add liquid water. Now we get

Continuity Eq.: $\dot{m}_A(1 + w_1) + \dot{m}_{liq} = \dot{m}_A(1 + w_2)$

Energy Eq.: $\dot{m}_A \tilde{h}_{1mix} + \dot{m}_{liq} h_f + \dot{Q}_{CV} = \dot{m}_A \tilde{h}_{2mix}$

For the liquid flow we need the air mass flowrate out, 3600 m^3/h = 1 m^3/s

$\dot{m}_A = P_{a\,2}\dot{V}/RT = (101.325 - 1.253)1/0.287 \times 294.15 = 1.185$ kg/s

$\dot{m}_{liq} = \dot{m}_A(w_2 - w_1) = 0.00511$ kg/s = **18.4 kg/h**

$\dot{Q}_{CV} = \dot{m}_A[C_{p\,a}(T_2 - T_1) + w_2 h_{v2} - w_1 h_{v1}] - \dot{m}_{liq} h_f$

$= 1.185\ [\ 1.004\ (21 - 35) + 0.007785 \times 2539.9 - 0.003474 \times 2565.3]$

$- 0.00511 \times 83.96 =$ **- 4.21 kW**

If from psychrometric chart.

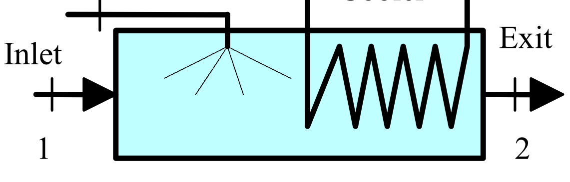
Inlet: $w_1 = 0.0030$, $\tilde{h}_{mix,1} = 63.0$, $h_{f,20} = 83.96$ kJ/kg

Exit: $w_2 = 0.0076$, $\tilde{h}_{mix,2} = 60.2$ kJ/kg dry air

P_{v2} and $\dot{m}_A = P_{a\,2}\dot{V}/RT$ same as above

$\dot{Q}_{CV} = \dot{m}_A(\tilde{h}_{2mix} - \tilde{h}_{1mix}) - \dot{m}_{liq} h_f = 1.185(60.2 - 63) - 0.00511 \times 83.96$

$= - 3.74$ kW

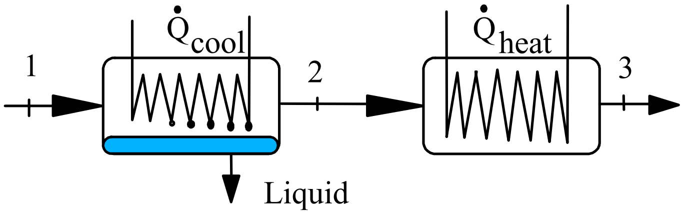


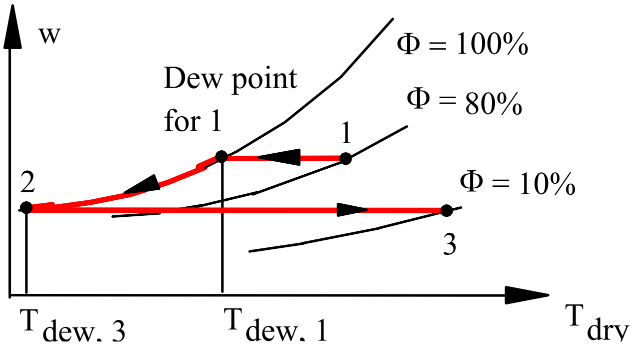
13.99

In a car’s defrost/defog system atmospheric air, 21°C, relative humidity 80%, is taken in and cooled such that liquid water drips out. The now dryer air is heated to 41°C and then blown onto the windshield, where it should have a maximum of 10% relative humidity to remove water from the windshield. Find the dew point of the atmospheric air, specific humidity of air onto the windshield, the lowest temperature and the specific heat transfer in the cooler.

Solution:

Solve using the psychrometric chart





Air inlet: 21°C, $\phi = 80\%$

=> $w_1 = 0.0124$,

$T_{dew} = \mathbf{17.3°C}$

$\tilde{h}_1 = 72$

Air exit: 41°C, $\phi = 10\%$

=> $w_3 = \mathbf{0.0044}$,

$T_{dew} = \mathbf{1.9°C}$

To remove enough water we must cool to the exit T_{dew}, followed by heating to T_{ex}. The enthalpy from chart $\tilde{h}_2 = 32.5$ and from B.1.1, $h_f(1.9°C) = 8$ kJ/kg

CV cooler:

$$\dot{m}_{liq}/\dot{m}_{air} = w_1 - w_3 = 0.0124 - 0.0044 = 0.008 \text{ kg liq/kg air}$$

$$q = \dot{Q}_{CV}/\dot{m}_{air} = \tilde{h}_2 + (w_1 - w_3)\, h_f - \tilde{h}_1$$

$$= 32.5 + 0.008 \times 8 - 72 = \mathbf{-39.4 \text{ kJ/kg dry air}}$$

If the steam and air tables are used the numbers are

State 1: $P_{g1} = 2.505$ kPa, $P_{v1} = 2.004$ kPa => $w_1 = 0.01259$

$h_{g1} = 2539.9$, $h_{a1} = 294.3$ => $\tilde{h}_1 = 326.3$ kJ/kg

State 3: $P_{g3} = 7.826$, $P_{v3} = 0.783$ => $w_3 = \mathbf{0.00486}$

State 2: $w_{g3} = w_3$ => $T_2 = T_{3dew} = 3.3°C$, $h_{f2} = 13.77$ kJ/kg

$h_{g2} = 2507.4$, $h_{a2} = 276.56$ => $\tilde{h}_2 = 288.75$ kJ/kg

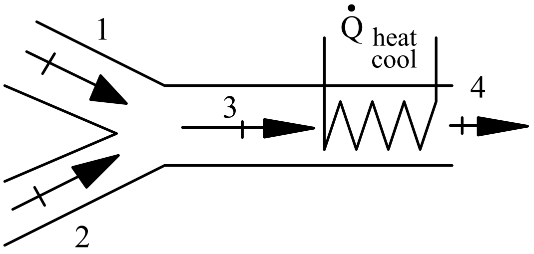
$\dot{m}_{liq}/\dot{m}_{air} = 0.00773$, $q = 288.75 + 0.00773 \times 13.77 - 326.3 = \mathbf{-37.45 \text{ kJ/kg air}}$

13.100

A flow of moist air at 45°C, 10% relative humidity with a flow rate of 0.2 kg/s dry air is mixed with a flow of moist air at 25°C, and absolute humidity of w = 0.018 with a rate of 0.3 kg/s dry air. The mixing takes place in an air duct at 100 kPa and there is no significant heat transfer. After the mixing there is heat transfer to a final temperature of 40°C. Find the temperature and relative humidity after mixing. Find the heat transfer and the final exit relative humidity.

Solution:

C.V : Total Setup
state 3 is internal to CV.



Continuity Eq.: $\dot{m}_{a1}\, w_1 + \dot{m}_{a2}\, w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\, w_3 = (\dot{m}_{a1} + \dot{m}_{a2})\, w_4$

Energy Eq. $\dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2 = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_3$

State 1: From Psychrometric chart $w_1 = 0.0056$, $\tilde{h}_1 = 79$ kJ/kg dry air

State 2: From Psychrometric chart $\Phi_2 = 90\%$, $\tilde{h}_2 = 90.5$ kJ/kg dry air

$$w_3 = w_4 = \frac{\dot{m}_{a1}w_1 + \dot{m}_{a2}w_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{0.2}{0.5}\,0.0056 + \frac{0.3}{0.5}\,0.018 = 0.01304$$

$$\tilde{h}_3 = \frac{\dot{m}_{a1}\tilde{h}_1 + \dot{m}_{a2}\tilde{h}_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{0.2}{0.5} \times 79 + \frac{0.3}{0.5} \times 90.5 = 85.9 \text{ kJ/kg dry air}$$

State 3: From Psychrometric chart $T_3 = 32.5°C$, $\Phi_3 = 45\%$

State 4: 40°C, $w_4 = 0.01304$ Read from Psychrometric chart

$\tilde{h}_4 = 94$, $\mathbf{\Phi_4 = 29\%}$

Now do the energy equation for the whole setup

Energy Eq. $\dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2 + \dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_4$

$$\dot{Q} = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_4 - \dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2$$

$$= 0.5 \times 94 - 0.2 \times 79 - 0.3 \times 90.5 = \mathbf{4.05\ kW}$$

13.101

An indoor pool evaporates 1.512 kg/h of water, which is removed by a dehumidifier to maintain 21°C, $\phi = 70\%$ in the room. The dehumidifier, shown in Fig. P13.101, is a refrigeration cycle in which air flowing over the evaporator cools such that liquid water drops out, and the air continues flowing over the condenser. For an air flow rate of 0.1 kg/s the unit requires 1.4 kW input to a motor driving a fan and the compressor and it has a coefficient of performance, $\beta = Q_L/W_c = 2.0$. Find the state of the air as it returns to the room and the compressor work input.

Solution:

The unit must remove 1.512 kg/h liquid to keep steady state in the room. As water condenses out state 2 is saturated.

State 1: 21°C, 70% => $w_1 = 0.0108$, $\tilde{h}_1 = 68.5$

CV 1 to 2: $\dot{m}_{liq} = \dot{m}_a(w_1 - w_2)$ => $w_2 = w_1 - \dot{m}_{liq}/\dot{m}_a$

$q_L = \tilde{h}_1 - \tilde{h}_2 - (w_1 - w_2)\, h_{f2}$

$w_2 = 0.0108 - 1.512/3600 \times 0.1 = 0.0066$

State 2: w_2, 100% => $T_2 = 8°C$, $\tilde{h}_2 = 45$, $h_{f2} = 33.6$

$q_L = 68.5 - 45 - 0.0042 \times 33.6 = 23.36$ kJ/kg dry air

CV Total system : $\tilde{h}_3 = \tilde{h}_1 + \dot{W}_{el}/\dot{m}_a - (w_1 - w_2)\, h_f$

$= 68.5 + 14 - 0.14 = 82.36$ kJ/kg dry air

State 3: $w_3 = w_2$, $\tilde{h}_3$ => $T_3 =$ **46°C**, $\phi_3 =$ **11-12%**

$\dot{W}_c = \dot{m}_a\, q_L / \beta =$ **1.165 kW**

13.102

A moist air flow of 5 kg/min at 30°C, $\Phi = 60\%$, 100 kPa goes through a dehumidifier in a setup shown in Problem 13.101. The air is cooled down to 15°C and then blown over the condenser. The refrigeration cycle runs with R-134a with a low pressure of 200 kPa and a high pressure of 1000 kPa. Find the COP of the refrigeration cycle, the ratio $\dot{m}_{R\text{-}134a}/\dot{m}_{air}$ and the outgoing T_3 and Φ_3.

Standard Refrigeration Cycle

Table B.5: $h_1 = 392.15$ kJ/kg; $s_1 = 1.732$ kJ/kg K; $h_4 = h_3 = 255.56$ kJ/kg

C.V. Compressor (assume ideal)

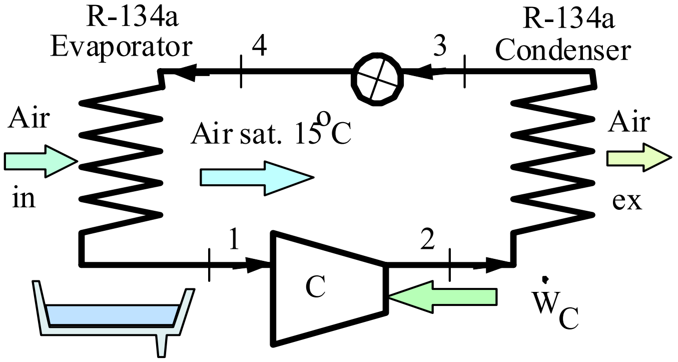
$\dot{m}_1 = \dot{m}_2 \qquad w_C = h_2 - h_1; \quad s_2 = s_1 + s_{gen}$

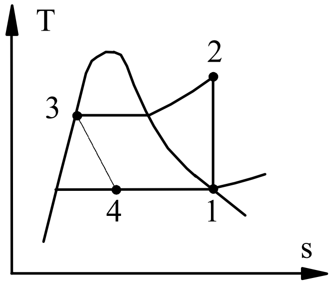
$P_2,\ s = s_1 \ \Rightarrow \ h_2 = 425.71 \text{ kJ/kg} \quad \Rightarrow \quad w_C = 33.56 \text{ kJ/kg}$

C.V. Evaporator: $q_L = h_1 - h_4 = 392.15 - 255.56 = 136.59$ kJ/kg

C.V. Condenser: $q_H = h_2 - h_3 = 425.71 - 255.56 = 170.15$ kJ/kg

$$\text{COP} = \beta = q_L / w_C = \frac{136.59}{33.56} = 4.07$$





$P_1 = 200$ kPa, $P_2 = 1000$ kPa

For the air processes let us use the psychrometric chart.

Air inlet: $w_{in} = 0.016$, $\tilde{h}_{in} = 90.5$ kJ/kg dry air, $T_{dew} = 21°\text{C} > 15°\text{C}$

Air 15°C: $\phi = 100\%$, $w_7 = 0.0107$, $\tilde{h}_7 = 62$, $h_f = 62.98$ (B.1.1)

Now do the continuity (for water) and energy equations for the cooling process

$\dot{m}_{liq}/\dot{m}_{air} = w_{in} - w_7 = 0.016 - 0.0107 = 0.0053$ kg/kg air

$q_{cool} = \tilde{h}_{in} - \tilde{h}_7 - \dot{m}_{liq}h_f/\dot{m}_{air} = 90.5 - 62 - 0.0053 \times 62.98 = 28.17$ kJ/kg air

Now the cooling of the air is done by the R-134a so

$$\dot{Q}_{cool} = \dot{m}_{air}\, q_{cool} = \dot{m}_{R134a}\, q_L \quad \Rightarrow \quad \dot{m}_{R134a}/\dot{m}_{air} = \frac{q_{cool}}{q_L} = \frac{28.17}{136.59} = \mathbf{0.2062}$$

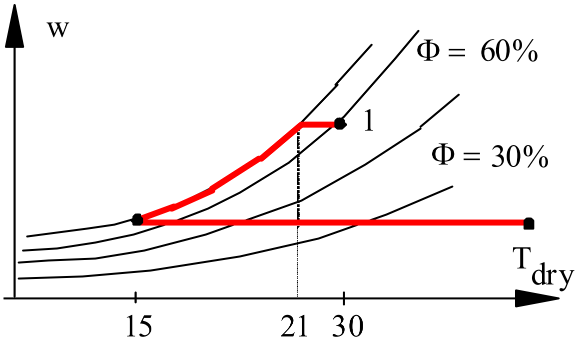
Energy eq. for the air flow being heated

$$\dot{Q}_{heat} = \dot{m}_{air}(\tilde{h}_{ex} - \tilde{h}_7) \quad \Rightarrow \quad \tilde{h}_{ex} = \tilde{h}_7 + \dot{Q}_{heat} / \dot{m}_{air} = \tilde{h}_7 + q_H \times \dot{m}_{R134a}/\dot{m}_{air}$$

$$\tilde{h}_{ex} = 62 + 170.15 \times 0.2062 = 97.08 \text{ kJ/kg dry air} \quad \text{and} \quad w_{ex} = w_7$$

Locate state in the psychrometric chart [just outside edge of chart]

$$T_{ex} = 49.3^{o}C \quad \text{and} \quad \phi_{ex} = 15\%$$



Psychrometric chart only

13.103

Use the psychrometric chart to find the missing property of: ϕ, ω, T_{wet}, T_{dry}

a. $T_{\text{dry}} = 25°\text{C}$, $\phi = 80\%$ b. $T_{\text{dry}} = 15°\text{C}$, $\phi = 100\%$

c. $T_{\text{dry}} = 20°\text{C}$, and $\omega = 0.008$ d. $T_{\text{dry}} = 25°\text{C}$, $T_{\text{wet}} = 23°\text{C}$

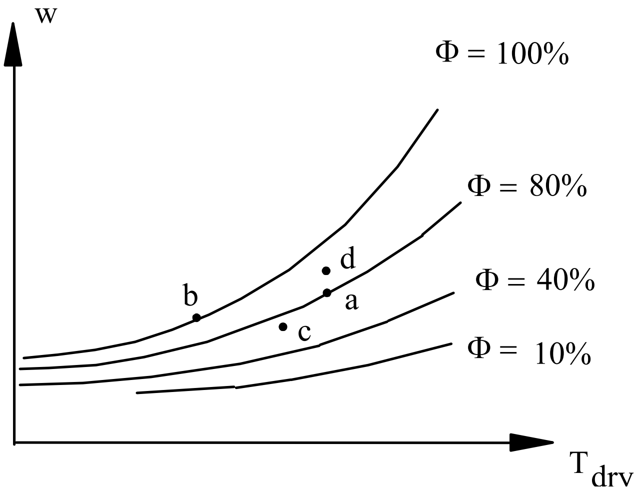
Solution:

a. 25°C, $\phi = 80\%$ => $\omega = 0.016$; $T_{\text{wet}} = 22.3°\text{C}$

b. 15°C, $\phi = 100\%$ => $\omega = 0.0106$; $T_{\text{wet}} = 15°\text{C}$

c. 20°C, $\omega = 0.008$ => $\phi = 57\%$; $T_{\text{wet}} = 14.4°\text{C}$

d. 25°C, $T_{\text{wet}} = 23°\text{C}$ => $\omega = 0.017$; $\phi = 86\%$



13.104

Use the psychrometric chart to find the missing property of: ϕ, ω, T_{wet}, T_{dry}

a. $\phi = 50\%$, $\omega = 0.012$ b. $T_{wet} = 15°C$, $\phi = 60\%$.

c. $\omega = 0.008$ and $T_{wet} = 17°C$ d. $T_{dry} = 10°C$, $\omega = 0.006$

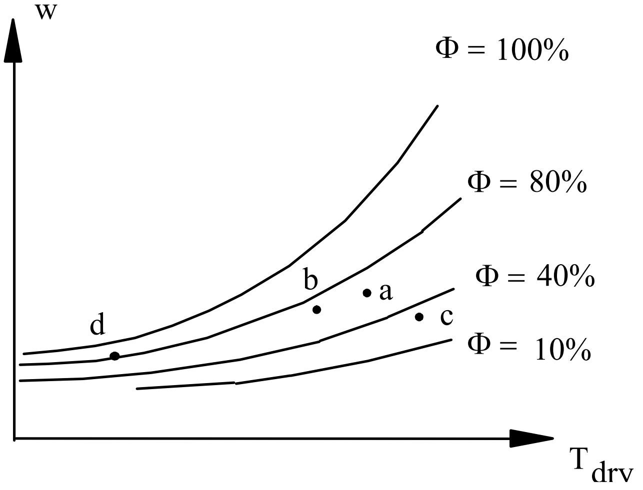
Solution:

a. $\phi = 50\%$, $\omega = 0.012$ => $T_{dry} = 28.5°C$, $T_{wet} = 20.6$ °C

b. $T_{wet} = 15°C$, $\phi = 60\%$ => $T_{dry} = 20.2°C$, $\omega = 0.0086$

c. $\omega = 0.008$, $T_{wet} = 17°C$ => $T_{dry} = 27.2°C$, $\phi = 37\%$

d. $T_{dry} = 10°C$, $\omega = 0.006$ => $\phi = 80\%$, $T_{wet} = 8.2°C$



13.105

For each of the states in Problem 13.104 find the dew point temperature.

Solution:

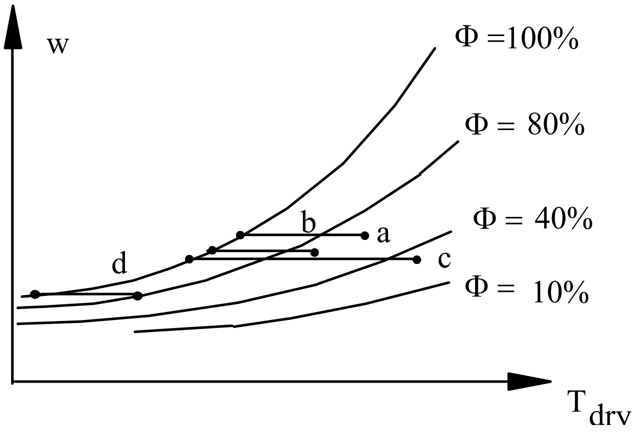
The dew point is the state with the same humidity ratio (abs humidity ω) and completely saturated $\phi = 100\%$. From psychrometric chart:

a.	$T_{dew} = 16.8°C$	c.	$T_{dew} = 10.9°C$
b.	$T_{dew} = 12°C$	d.	$T_{dew} = 6.5°C$

Finding the solution from the tables is done for cases a,c and d as

Eq.13.28 solve: $P_v = P_g = \omega P_{tot} /[\omega + 0.622] = P_g (T_{dew})$ in B.1.1

For case b use energy Eq. 13.30 to find ω_1 first from $T_{ad\ sat} = T_{wet}$.



13.106

Use the formulas and the steam tables to find the missing property of: ϕ, ω, and T_{dry}, total pressure is 100 kPa; repeat the answers using the psychrometric chart

a. $\phi = 50\%$, $\omega = 0.010$ b. $T_{wet} = 15°C$, $\phi = 50\%$ c. $T_{dry} = 25°C$, $T_{wet} = 21°C$

a. From Eq.12.28 with $P_a = P - P_v$ solve for P_v:

$$P_v = P\,\omega/(0.622 + \omega) = 100 \times 0.01/0.632 = 1.582 \text{ kPa}$$

From Eq.12.25: $P_g = P_v/\phi = 1.582/0.5 = 3.165$ kPa => $T = 25°C$

b. Assume T_{wet} is adiabatic saturation T and use energy Eq.12.30

At 15°C: $P_g = 1.705$ kPa => $\omega = 0.622 \times \dfrac{1.705}{100 - 1.705} = 0.01079$

$LHS = \omega_1 (h_{v1} - h_{f2}) + C_pT_1 = RHS = C_pT_2 + \omega_2 h_{fg2}$

$RHS = 1.004\times15 + 0.01079 \times 2465.93 = 41.667$ kJ/kg

$\omega_1 = 0.622\ \phi P_g/(100 - \phi P_g)$ where P_g is at T_1. Trial and error.

$LHS_{25C} = 49.98$, $LHS_{20C} = 38.3$ => $T = 21.4°C$, $\omega_1 = 0.008$

c. At 21°C: $P_g = 2.505$ kPa => $\omega_2 = 0.622 \times \dfrac{2.505}{100 - 2.505} = 0.016$

$h_{f2} = 88.126$ and $h_{fg2} = 2451.76$ kJ/kg, $h_{v1} = 2547.17$ kJ/kg

From Eq.12.30: $\omega_1 = [C_p(T_2-T_1) + \omega_2 h_{fg2}]/(h_{v1} - h_{f2}) = 0.0143$

$P_v = P\,\omega/(0.622 + \omega) = 2.247$, $\phi = 2.247/3.169 = 0.71$

Using the psychrometric chart E.4:

a: $T_{dry} = 25.3$ °C b. $T_{dry} = 21.6°C$, $\omega = 0.008$

c: $\omega = 0.0141$, $\phi = 71\text{-}72\%$

13.107

An air-conditioner should cool a flow of ambient moist air at 40°C, 40% relative humidity with 0.2 kg/s flow of dry air. The exit temperature should be 25°C and the pressure is 100 kPa. Find the rate of heat transfer needed and check for the formation of liquid water.

Solution:

Before we know if we should have a liquid water flow term we need to check for condensation (the dew point).

Using the psychrometric chart. i: $w_i = 0.018$, $\tilde{h}_i = 106$ kJ/kg air, $T_{dew} = 23°C$

Since $T_{dew} < T_e$ then **no condensation** occurs

CV heat exchanger: $\dot{m}_{Ai} = \dot{m}_{Ae}$, $\dot{m}_{vi} = \dot{m}_{ve}$, $w_e = w_i$

$$(h_a + wh_v)_i + q = (h_a + wh_v)_e = \tilde{h}_e,$$

$$\tilde{q} = \tilde{h}_e - \tilde{h}_i = 90.5 - 106 = -15.5 \text{ kJ/kg dry air}$$

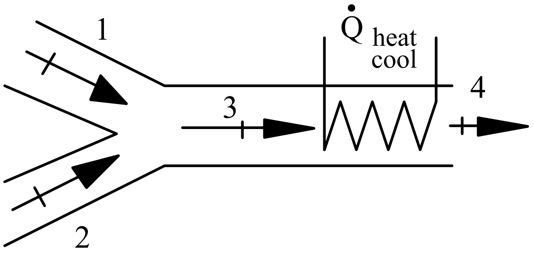
$$\dot{Q} = \dot{m}_{air}\, \tilde{q} = 0.2 \text{ kg/s} \times (-15.5 \text{ kJ/kg}) = \mathbf{-3.1\ kW} \quad \text{(it goes out)}$$

13.108

A flow of moist air at 21°C, 60% relative humidity should be produced from mixing of two different moist air flows. Flow 1 is at 10°C, relative humidity 80% and flow 2 is at 32°C and has $T_{\text{wet}} = 27°\text{C}$. The mixing chamber can be followed by a heater or a cooler. No liquid water is added and $P = 100$ kPa. Find the two controls one is the ratio of the two mass flow rates $\dot{m}_{a1}/\dot{m}_{a2}$ and the other is the heat transfer in the heater/cooler per kg dry air.

Solution:

C.V : Total Setup
state 3 is internal to CV.



Continuity Eq.: $\dot{m}_{a1}\, w_1 + \dot{m}_{a2}\, w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\, w_4$

Energy Eq. $\dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2 + \dot{Q}_{a1} = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_4$

Define $x = \dot{m}_{a1}/\dot{m}_{a2}$ and substitute into continuity equation

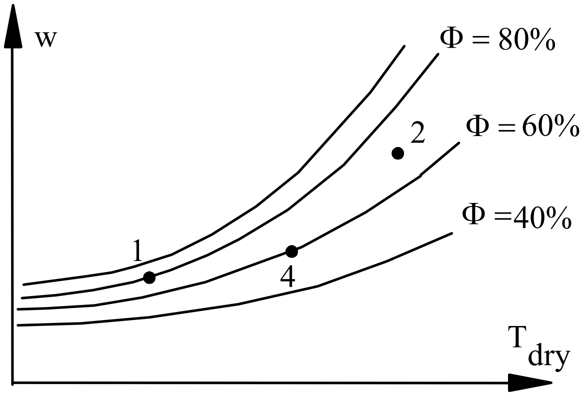
$$\Rightarrow \quad x\, w_1 + w_2 = (1+x)\, w_4 \quad \Rightarrow \quad x = \frac{w_4 - w_2}{w_1 - w_4} = \mathbf{3.773}$$

Energy equation scaled to total flow of dry air

$$\tilde{q} = \dot{Q}_{a1}/(\dot{m}_{a1} + \dot{m}_{a2}) = \tilde{h}_4 - [x/(1+x)]\, \tilde{h}_1 - [1/(1+x)]\, \tilde{h}_2$$

$$= 64 - 0.7905 \times 45 - 0.2095 \times 105$$

$$= \mathbf{6.43\ kJ/kg\text{-}dry\ air}$$



State 1:
$w_1 = 0.006$, $\tilde{h}_1 = 45$

State 2:
$w_2 = 0.0208$, $\tilde{h}_2 = 105$

State 4:
$w_4 = 0.0091$, $\tilde{h}_4 = 64$,
$T_{\text{dew 4}} = 12.5°\text{C}$

13.109

In a hot and dry climate, air enters an air-conditioner unit at 100 kPa, 40°C, and 5% relative humidity, at the steady rate of 1.0 m^3/s. Liquid water at 20°C is sprayed into the air in the AC unit at the rate 20 kg/hour, and heat is rejected from the unit at at the rate 20 kW. The exit pressure is 100 kPa. What are the exit temperature and relative humidity?

State 1: $T_1 = 40°C$, $P_1 = 100$ kPa, $\phi_1 = 5\%$, $\dot{V}_{a1} = 1\ m^3/s$

$P_{g1} = 7.3837$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.369$ kPa, $P_{a1} = P - P_{v1} = 99.63$ kPa

$$\omega_1 = 0.622 \frac{P_{v1}}{P_{a1}} = 0.0023,\ \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.108 \text{ kg/s},\quad h_{v1} = 2574.3 \text{ kJ/kg}$$

State 2 : Liq. Water. 20°C, $\dot{m}_{f2} = 20$ kg/hr $= 0.00556$ kg/s, $h_{f2} = 83.9$ kJ/kg

Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a3}$, $\dot{m}_{v1} + \dot{m}_{12} = \dot{m}_{v3}$

$\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.0023 = 0.0073$

State 3 : $P_3 = 100$ kPa and $P_{v3} = P_3\omega_3/(0.622 + \omega_3) = 1.16$ kPa

Energy Eq. with $\dot{Q} = -20$ kW :

$$\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f2}h_{f2} = \dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3};$$

$$(h_{a3} - h_{a1}) + \omega_3 h_{v3} = \omega_1 h_{v1} + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1}$$

$$= 0.0023 \times 2574.3 + (0.00556\times83.9 - 20)/1.108 = -11.7$$

Unknowns: h_{a3}, h_{v3} implicitly given by a single unknown: T_3

Trial and Error for T_3; $T_3 =$ **10°C**, $P_{g3} = 1.23$ kPa, $\phi_3 = \frac{P_{v3}}{P_{g3}} = \mathbf{0.94}$

If we solved with the psychrometric chart we would get:

State 1: $\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.002$, $\tilde{h}_1 = 65$ kJ/kg dry air;

State 3: $\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.002 = 0.007$

Now the energy equation becomes

$$\tilde{h}_3 = \tilde{h}_1 + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1} = 65 + (0.00556\times83.9 - 20)/1.108 = 47.4$$

Given ω_3 we find the state around 10°C and $\phi_3 = 90\%$

13.110

Compare the weather two places where it is cloudy and breezy. At beach A it is 20°C, 103.5 kPa, relative humidity 90% and beach B has 25°C, 99 kPa, relative humidity 20%. Suppose you just took a swim and came out of the water. Where would you feel more comfortable and why?

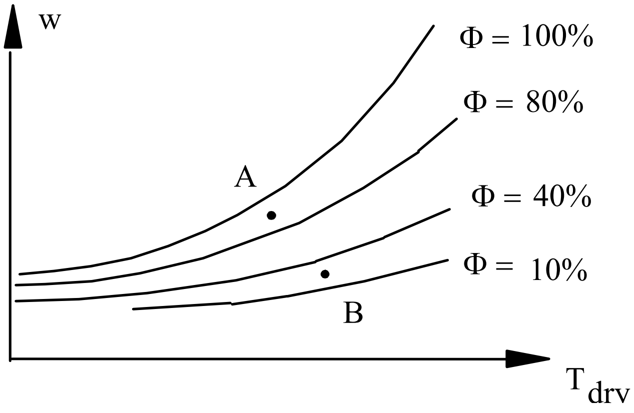
Solution:

As your skin is wet and air is flowing over it you will feel T_{wet}. With the small difference in pressure from 100 kPa we can use the psychrometric chart.

A: 20°C, $\phi = 90\%$ => $T_{wet} = 18.7°C$

B: 25°C, $\phi = 20\%$ => $T_{wet} = 12.3°C$

At beach A it is comfortable, at beach B it feels chilly.



13.111

Ambient air at 100 kPa, 30°C, 40% relative humidity goes through a constant pressure heat exchanger as a steady flow. In one case it is heated to 45°C and in another case it is cooled until it reaches saturation. For both cases find the exit relative humidity and the amount of heat transfer per kilogram dry air.

Solution:

CV heat exchanger: $\dot{m}_{Ai} = \dot{m}_{Ae}$, $\dot{m}_{vi} = \dot{m}_{ve}$, $w_e = w_i$

$$(h_a + wh_v)_i + q = (h_a + wh_v)_e = \tilde{h}_e, \quad q = \tilde{h}_e - \tilde{h}_i$$

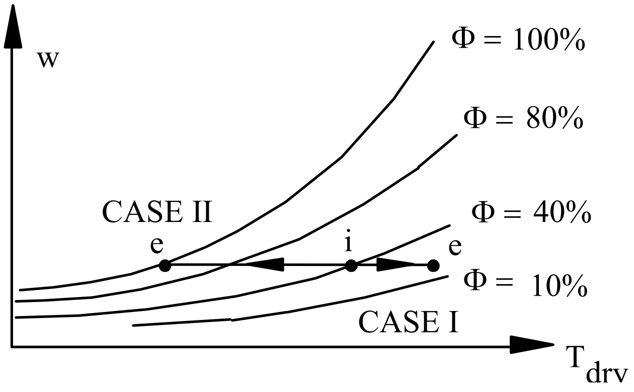
Using the psychrometric chart: i: $w_i = 0.0104$, $\tilde{h}_i = 76$

Case I) e: $T_e = 45\ ^oC$, $w_e = w_i \Rightarrow \tilde{h}_e = 92$,

$\boldsymbol{\phi_e = 17\%}$, $q = 92\text{-}76 =$ **16 kJ/kg dry air**

Case II) e: $w_e = w_i$, $\boldsymbol{\phi_e = 100\%} \Rightarrow \tilde{h}_e = 61$, $T_e = 14.5^oC$

$q = 61\text{-}76 =$ **-15 kJ/kg dry air**



13.112

A flow of moist air at 100 kPa, 35^{o}C, 40% relative humidity is cooled by adiabatic evaporation of liquid 20^{o}C water to reach a saturated state. Find the amount of water added per kg dry air and the exit temperature.

Since the liquid is not necessarily at the adiabatic saturation temperature the exit may be close to but not exactly the wet bulb temperature. We will use that as a good guess and check the energy equation.

Chart E4: $T_{wet} = 23.6^oC$, $w_1 = 0.0138$, $\tilde{h}_{1\,mix} = 90$ kJ/kg

$w_2 = 0.0186$, $\tilde{h}_{2mix} = 90.6$ kJ/kg

Continuity Eq.: $\dot{m}_A(1 + w_1) + \dot{m}_{liq} = \dot{m}_A(1 + w_2)$

Energy Eq. (q = 0): $\dot{m}_A\tilde{h}_{1mix} + \dot{m}_{liq}h_f = \dot{m}_A\tilde{h}_{2mix}$

Divide the energy equation by the mass flow rate of air

$\dot{m}_{liq}/\dot{m}_A = w_2 - w_1 = 0.0186 - 0.0138 = 0.0048$ kg water/kg air

so

$\tilde{h}_{1mix} + (w_2 - w_1)\, h_f = \tilde{h}_{2mix}$

LHS = $90 + 0.0048 \times 83.94 = 90.4$; RHS = 90.6

The temperature should be a little lower which will lower w_2 also so

$T_2 =$ **23.5^{o}C**, $w_2 = 0.0185$ => $\dot{m}_{liq}/\dot{m}_A =$ **0.0047 kg water/kg air**

This is close to the accuracy by which we can read the chart and the first answer is nearly as good.

13.113

Consider two states of atmospheric air. (1) 35°C, T_{wet} = 18°C and (2) 26.5°C, ϕ = 60%. Suggest a system of devices that will allow air in a steady flow process to change from (1) to (2) and from (2) to (1). Heaters, coolers (de)humidifiers, liquid traps etc. are available and any liquid/solid flowing is assumed to be at the lowest temperature seen in the process. Find the specific and relative humidity for state 1, dew point for state 2 and the heat transfer per kilogram dry air in each component in the systems.

Use the psychrometric chart E.4

1: w_1= 0.006, $\tilde{h}_1 = 70.5$, $\phi_1 = 18\%$, $T_{dew} = 6.5°C$, $\tilde{h}_{dew} = 42$

2: w_2= 0.013, $\tilde{h}_2 = 79.4$, $\phi_2 = 60\%$, $T_{dew} = 18°C$, $\tilde{h}_{dew} = 71$

Since $w_2 > w_1$ water must be added in process I to II and removed in the process II to I. Water can only be removed by cooling below dew point temperature so

I to II: Adiabatic sat. I to Dew,II, then heater from Dew,II to II

II to I: Cool to Dew,I then heat Dew,I to I

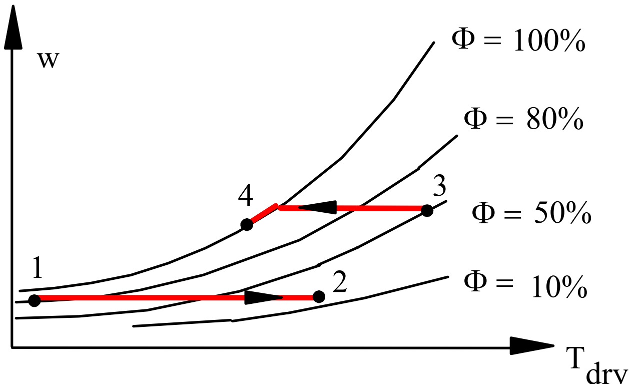
The first one can be done because $T_{dew\ II} = T_{ad\ sat\ I}$

I to II: $q = \tilde{h}_{II} - \tilde{h}_{dewII} = 79.4 - 71 =$ **8.4 kJ/kg air**

II to I: $q_{cool} = \tilde{h}_{II} - \tilde{h}_{dewI} - (w_2 - w_1)h_f(\text{at } T_{dewI})$

$= 79.4 - 0.007 \times 27.29 =$ **37.2 kJ/kg air**

$q_{heat} = \tilde{h}_I - \tilde{h}_{dewI} = 70.5 - 42 =$ **28.5 kJ/kg air**



13.114

To refresh air in a room, a counterflow heat exchanger, see Fig. P13.114, is mounted in the wall, drawing in outside air at 0.5°C, 80% relative humidity and pushing out room air, 40°C, 50% relative humidity. Assume an exchange of 3 kg/min dry air in a steady flow device, and also that the room air exits the heat exchanger at 23°C to the outside. Find the net amount of water removed from the room, any liquid flow in the heat exchanger and (T, ϕ) for the fresh air entering the room.

We will use the psychrometric chart to solve this problem.

State 3 (room): $w_3 = 0.0232$, $\tilde{h}_3 = 119.2$, $T_{dew\ 3} = 27°C$

State 1(outside): 0.5°C, $\phi = 80\%$ => $w_1 = 0.0032$, $\tilde{h}_1 = 29.2$ kJ/kg dry air

CV room: $\dot{m}_{v,out} = \dot{m}_a (w_3 - w_2) = \dot{m}_a (w_3 - w_1)$

$$= 3(0.0232\text{-}0.0032) = \mathbf{0.06\ kg/min}$$

The room air is cooled to 23°C $< T_{dew\ 3}$ so liquid will form in the exit flow channel and state 4 is saturated.

State 4: 23°C, $\phi = 100\%$ => $w_4 = 0.0178$, $\tilde{h}_4 = 88$, $h_{f4} = 96.52$ kJ/kg

CV 3 to 4 (flow cooled below $T_{dew\ 3}$ so liquid forms):

$$\dot{m}_{liq\ 4} = \dot{m}_a (w_3 - w_4) = 3\ (0.0232 - 0.0178) = \mathbf{0.0162\ kg/min}$$

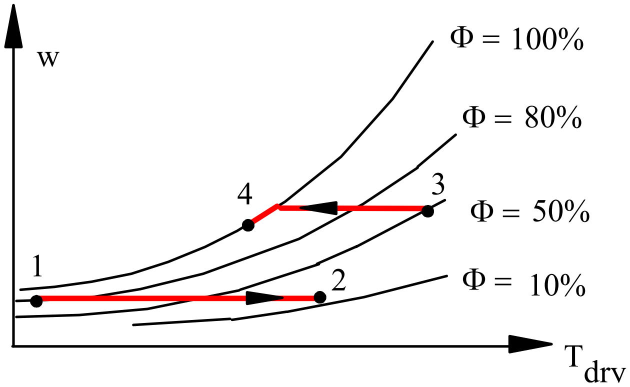
CV Heat exchanger with no external heat transfer:

$$m_a(\tilde{h}_2 - \tilde{h}_1) = m_a(\tilde{h}_3 - \tilde{h}_4) - m_{liq}h_{f4}$$

$$\tilde{h}_2 = \tilde{h}_1 + \tilde{h}_3 - \tilde{h}_4 - (w_3\text{-}w_4)\ h_{f4} = 29.2 + 119.2 - 88 - 0.0054\times96.52$$

$$= 59.9 \text{ kJ/kg dry air}$$

State 2: $w_2 = w_1$, $\tilde{h}_2$ => $T_2 = \mathbf{32.5°C}$, $\phi = \mathbf{12\%}$



Availability (exergy) in mixtures

13.115

Find the second law efficiency of the heat exchanger in Problem 13.52.
A flow of 2 kg/s mixture of 50% CO_2 and 50% O_2 by mass is heated in a constant pressure heat exchanger from 400 K to 1000 K by a radiation source at 1400 K. Find the rate of heat transfer and the entropy generation in the process.

Solution:

The second law efficiency follows Eq.13.32 where the wanted term is the flow increase of exergy and the source is the radiation.

$$\dot{\Phi}_{flow} = \dot{m}(\psi_{ex} - \psi_{in}); \qquad \dot{\Phi}_{source} = \dot{Q}_{in}\,(1 - \frac{T_o}{T_{source}})$$

Heat exchanger Energy Eq.6.12: $\dot{Q}_{in} = \dot{m}(h_e - h_i)$

Values from Table A.8 due to the high T.

$$\dot{Q}_{in} = 2\,[\frac{1}{2} \times (971.67 - 303.76) + \frac{1}{2} \times (980.95 - 366.03)] = 1282.8 \text{ kW}$$

$$\dot{\Phi}_{source} = \dot{Q}_{in}\,(1 - \frac{T_o}{T_{source}}) = 1282.8\,(\,1 - \frac{298.15}{1400}) = 1009.6 \text{ kW}$$

Entropy Eq.9.8: $\dot{m}_e s_e = \dot{m}_i s_i + \dot{Q}/T_s + \dot{S}_{gen}$

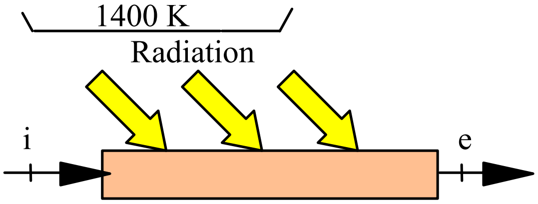
As P = C, the pressure correction in Eq.8.28 drops out to give generation as

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}/T_s$$
$$= 2\,[0.5 \times (6.119 - 5.1196) + 0.5 \times (7.6121 - 6.6838)] - 1282.8/1400$$
$$= 1.01 \text{ kW/K}$$

$$\dot{\Phi}_{flow} = \dot{\Phi}_{source} - \dot{\Phi}_{destruction} = \dot{\Phi}_{source} - T\,\dot{S}_{gen}$$
$$= 1009.6 - 298.15 \times 1.01 = 708.5$$

$$\eta = \frac{\dot{\Phi}_{flow}}{\dot{\Phi}_{source}} = \frac{708.5}{1009.6} = \mathbf{0.70}$$

Remark: We could also explicitly have found the flow exergy increase.



13.116

Consider the mixing of a steam flow with an oxygen flow in Problem 13.53. Find the rate of total inflowing availability and the rate of exergy destruction in the process.

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

Exergy Flow: $\dot{\Phi}_{in} = \dot{m}\,\psi_{in} = \dot{m}_{H2O}\,\psi_1 + \dot{m}_{O2}\,\psi_2$

$$\psi_1 = h_1 - h_o - T_o(s_1 - s_o)$$

$$= C_{P\,H2O}(T_1 - T_o) - T_o\,[C_{P\,H2O}\ln(T_1/T_o) - R\ln(P_1/P_o)]$$

$$= 1.872\,(400 - 25) - 298.15[\,1.872\ln\frac{673.15}{298.15} - 0.4615\ln\frac{400}{100}\,]$$

$$= 702 - 298.15\,(1.5245 - 0.63978) = 438.2\text{ kJ/kg}$$

$$\psi_2 = h_2 - h_o - T_o(s_2 - s_o)$$

$$= C_{P\,O2}(T_2 - T_o) - T_o\,[C_{P\,O2}\ln(T_2/T_o) - R\ln(P_2/P_o)]$$

$$= 0.922(126.85\text{–}25) - 298.15[0.922\ln\frac{400}{298.15} - 0.2598\ln\frac{400}{100}\,]$$

$$= 93.906 - 298.15\,(0.27095 - 0.36016) = 120.5\text{ kJ/kg}$$

$$\dot{\Phi}_{in} = \dot{m}_{H2O}\,\psi_1 + \dot{m}_{O2}\,\psi_2 = 1.8\ 438.2 + 3.2\ 120.5 = \mathbf{1174.4\ kW}$$

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1\text{ kmol/s};\quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1\text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O}\,h_1 + \dot{m}_{O2}\,h_2 = \dot{m}_{H2O}\,h_{3\,H2O} + \dot{m}_{O2}\,h_{3\,O2}$

Entropy Eq.: $\dot{m}_{H2O}\,s_1 + \dot{m}_{O2}\,s_2 + \dot{S}_{gen} = \dot{m}_{H2O}\,s_{3\,H2O} + \dot{m}_{O2}\,s_{3\,O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O}\,(h_{3\,H2O} - h_1) + \dot{m}_{O2}\,(h_{3\,O2} - h_2) = 0$$

$$\dot{m}_{H2O}\,C_{P\,H2O}(T_3 - T_1) + \dot{m}_{O2}\,C_{P\,O2}(T_3 - T_2) = 0$$

$$1.8 \times 1.872\,(T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \mathbf{545.6\ K}$$

$$\dot{S}_{gen} = \dot{m}_{H2O}\,(s_{3\ H2O} - s_1) + \dot{m}_{O2}\,(s_{3\ O2} - s_2)$$

$$= \dot{m}_{H2O}\,[\,C_{P\ H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O}\,] + \dot{m}_{O2}\,[\,C_{P\ O2} \ln \frac{T_3}{T_2} - R \ln y_{O2}\,]$$

$$= 1.8\,[\,1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5\,]$$

$$+\ 3.2\,[\,0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5\,]$$

$$= -\ 0.132 + 1.492 = \mathbf{1.36\ kW/K}$$

The exergy destruction is proportional to the entropy generation

$$\dot{\Phi}_{in} = T_o\,\dot{S}_{gen} = 298.15 \times 1.36 = \mathbf{405.5\ kW}$$

13.117

A mixture of 75% carbon dioxide and 25% water by mole is flowing at 1600 K, 100 kPa into a heat exchanger where it is used to deliver energy to a heat engine. The mixture leave the heat exchanger at 500 K with a mass flow rate of 2 kg/min. Find the rate of energy and the rate of exergy delivered to the heat engine.

C.V. Heat exchanger, steady flow and no work.

From Table A.8:

CO2: $h_{in} = 1748.12$ kJ/kg, $s^o_{T\ in} = 6.7254$ kJ/kg K

CO2: $h_{ex} = 401.52$ kJ/kg, $s^o_{T\ ex} = 5.3375$ kJ/kg K

H2O: $h_{in} = 3487.69$ kJ/kg, $s^o_{T\ in} = 14.0822$ kJ/kg K

H2O: $h_{ex} = 935.12$ kJ/kg, $s^o_{T\ ex} = 11.4644$ kJ/kg K

Energy Eq.: $\dot{Q} = \dot{m}\,(h_{in} - h_{ex}) = \dot{m} \sum c_i\,(h_{in} - h_{ex})_i$

$$= \frac{2}{60}\,[0.75\,(1748.12 - 401.52) + 0.25(3487.69 - 935.12)]$$

$$= \frac{1}{30}\,[\,1009.95 + 638.14\,] = \mathbf{54.94\ kW}$$

Entropy change (P does not change so partial pressures are constant):

$$s_{in} - s_{ex} = 0.75(6.7254 - 5.3375) + 0.25(14.0822 - 11.9644)$$
$$= 1.6954 \text{ kJ/kg K}$$

Exergy Flux:

$$\dot{\Phi} = \dot{m}\,(\psi_{in} - \psi_{ex}) = \dot{Q} - T_o\,\dot{m}\,(s_{in} - s_{ex})$$

$$= 54.94 - 298.15 \times \frac{1}{30} \times 1.6954$$

$$= \mathbf{38.09\ kW}$$

Review problems

13.118

Weighing of masses gives a mixture at 60°C, 225 kPa with 0.5 kg O_2, 1.5 kg N_2 and 0.5 kg CH_4. Find the partial pressures of each component, the mixture specific volume (mass basis), mixture molecular weight and the total volume.

Solution:

From Eq.12.4: $y_i = (m_i / M_i) / \sum m_j/M_j$

$n_{tot} = \sum m_j/M_j = (0.5/31.999) + (1.5/28.013) + (0.5/16.04)$

$= 0.015625 + 0.053546 + 0.031172 = 0.100343$

$y_{O2} = 0.015625/0.100343 = 0.1557$,

$y_{N2} = 0.053546/0.100343 = 0.5336$,

$y_{CH4} = 0.031172/0.100343 = 0.3107$

From Eq.12.10:

$P_{O2} = y_{O2}\ P_{tot} = 0.1557 \times 225 = 35$ kPa,

$P_{N2} = y_{N2}\ P_{tot} = 0.5336 \times 225 = 120$ kPa

$P_{CH4} = y_{CH4}\ P_{tot} = 0.3107 \times 225 = 70$ kPa

$V_{tot} = n_{tot}\ \bar{R}T/P = 0.100343 \times 8.31451 \times 333.15 / 225 = \mathbf{1.235\ m^3}$

$v = V_{tot}/m_{tot} = 1.235 / (0.5 + 1.5 + 0.5) = \mathbf{0.494\ m^3/kg}$

From Eq.12.5:

$M_{mix} = \sum y_j M_j = m_{tot}/n_{tot} = 2.5/0.100343 = \mathbf{24.914}$

13.119

A carbureted internal combustion engine is converted to run on methane gas (natural gas). The air-fuel ratio in the cylinder is to be 20 to 1 on a mass basis. How many moles of oxygen per mole of methane are there in the cylinder?

Solution:

The mass ratio $m_{AIR}/m_{CH_4} = 20$, so relate mass and mole $n = m/M$

$$\frac{n_{AIR}}{n_{CH_4}} = \left(\frac{m_{AIR}}{m_{CH_4}}\right) \times M_{CH_4}/M_{AIR} = 20 \times 16.04/28.97 = 11.0735$$

$$\rightarrow \frac{n_{O_2}}{n_{CH_4}} = \frac{n_{O_2}}{n_{AIR}} \times \frac{n_{AIR}}{n_{CH_4}} = 0.21 \times 11.0735 = \mathbf{2.325\ mole\ O_2/mole\ CH_4}$$

13.120

A mixture of 50% carbon dioxide and 50% water by mass is brought from 1500 K, 1 MPa to 500 K, 200 kPa in a polytropic process through a steady state device. Find the necessary heat transfer and work involved using values from Table A.5.

Solution:

Process Pv^n = constant leading to

$$n \ln(v_2/v_1) = \ln(P_1/P_2); \quad v = RT/P$$

$$n = \ln\left(\frac{1000}{200}\right) / \ln\left(\frac{500 \times 1000}{200 \times 1500}\right) = 3.1507$$

Eq.13.15: $R_{mix} = \Sigma\, c_iR_i = 0.5 \times 0.1889 + 0.5 \times 0.4615 = 0.3252$ kJ/kg K

Eq.13.23: $C_{P\,mix} = \Sigma\, c_iC_{Pi} = 0.5 \times 0.8418 + 0.5 \times 1.872 = 1.3569$ kJ/kg K

Work is from Eq.9.19:

$$w = -\int v dP = -\frac{n}{n-1}(P_ev_e - P_iv_i) = -\frac{nR}{n-1}(T_e - T_i) = \mathbf{476.4\ kJ/kg}$$

Heat transfer from the energy equation

$$q = h_e - h_i + w = C_P(T_e - T_i) + w = \mathbf{-880.5\ kJ/kg}$$

13.121

Solve Problem 13.120 using specific heat $C_P = \Delta h/\Delta T$, from Table A.8 at 1000 K. A mixture of 50% carbon dioxide and 50% water by mass is brought from 1500 K, 1 MPa to 500 K, 200 kPa in a polytropic process through a steady state device. Find the necessary heat transfer and work involved using values from Table A.5.

Solution:

Using values from Table A.8 we estimate the heat capacities

$$C_{P\,CO_2} = \frac{1096.36 - 849.72}{1100 - 900} = 1.2332 \text{ kJ/kg K}$$

$$C_{P\,H_2O} = \frac{2226.73 - 1768.6}{1100 - 900} = 2.2906 \text{ kJ/kg K}$$

Eq.12.23: $C_{P\,mix} = \Sigma\, c_i C_{Pi} = 0.5 \times 1.2332 + 0.5 \times 2.2906 = 1.7619$ kJ/kg K

Eq.12.15: $R_{mix} = \Sigma\, c_i R_i = 0.5 \times 0.1889 + 0.5 \times 0.4615 = 0.3252$ kJ/kg K

Process $Pv^n = C$ => $n = \ln(P_1/P_2) / \ln(v_2/v_1)$ and use $Pv = RT$

$$n = \ln\left(\frac{1000}{200}\right) / \ln\left(\frac{500 \times 1000}{200 \times 1500}\right) = 3.1507$$

Work is from Eq.9.19

$$w = -\int v dP = -\frac{n}{n-1}(P_e v_e - P_i v_i) = -\frac{nR}{n-1}(T_e - T_i) = \mathbf{476.4 \text{ kJ/kg}}$$

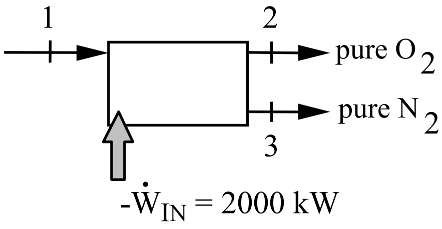
Heat transfer from energy equation

$$q = h_e - h_i + w = 1.7619(500 - 1500) + 476.4 = \mathbf{-1285.5 \text{ kJ/kg}}$$

13.122

A large air separation plant takes in ambient air (79% N_2, 21% O_2 by mole) at 100 kPa, 20°C, at a rate of 25 kg/s. It discharges a stream of pure O_2 gas at 200 kPa, 100°C, and a stream of pure N_2 gas at 100 kPa, 20°C. The plant operates on an electrical power input of 2000 kW. Calculate the net rate of entropy change for the process.

Air 79 % N_2
21 % O_2
$P_1 = 100$ kPa
$T_1 = 20\ ^oC$
$\dot{m}_1 = 25$ kg/s



$P_2 = 200$ kPa
$T_2 = 100\ ^oC$

$P_3 = 100$ kPa
$T_3 = 20\ ^oC$

Solution:

To have the flow terms on a mass basis let us find the mass fractions

From Eq. 10.3: $c_i = y_i\, M_i / \sum y_j M_j$

$c_{O_2} = 0.21 \times 32 / [0.21 \times 32 + 0.79 \times 28.013] = 0.23293$;

$c_{N_2} = 1 - c_{O_2} = 0.76707$

$\dot{m}_2 = c_{O_2}\dot{m}_1 = 5.823$ kg/s ; $\dot{m}_3 = c_{N_2}\dot{m}_1 = 19.177$ kg/s

The energy equation, Eq.6.10 gives the heat transfer rate as

$$\dot{Q}_{CV} = \Sigma\, \dot{m}\Delta h_i + \dot{W}_{CV} = \dot{m}_{O_2} C_{P0\,O_2}(T_2 - T_1) + \dot{m}_{N_2} C_{P0\,N_2}(T_3 - T_1) + \dot{W}_{CV}$$

$$= 5.823 \times 0.922 \times (100\text{-}20) + 0 - 2000 = -1570.5 \text{ kW}$$

The entropy equation, Eq.9.7 gives the generation rates as

$$\dot{S}_{gen} = \Sigma\, \dot{m}_i \Delta s_i - \dot{Q}_{CV}/T_0 = (\dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1) - \dot{Q}_{CV}/T_0$$

Use Eq.8.16 for the entropy change

$$\Sigma\, \dot{m}_i \Delta s_i = 5.823 \left[\ 0.922 \ln \frac{373.2}{293.2} - 0.2598 \ln \frac{200}{21}\right]$$

$$+\ 19.177\ [0 - 0.2968 \ln (100/79)] = -3.456 \text{ kW/K}$$

$\dot{S}_{gen} = 1570.5/293.2 - 3.456 =$ **1.90 kW/K**

13.123

Take Problem 13.55 with inlet temperature of 1400 K for the carbon dioxide and 300 K for the nitrogen. First estimate the exit temperature with the specific heats from Table A.5 and use this to start iterations using A.9 to find the exit temperature.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{CO_2} = 2\ \dot{n}_{N_2}$ and assume no external heat transfer, no work involved.

$\bar{C}_{P\ CO_2} = 44.01 \times 0.842 = 37.06$; $\bar{C}_{P\ N_2} = 28.013 \times 1.042 = 29.189$ kJ/kmol K

Continuity Equation: $0 = \Sigma\dot{n}_{in} - \Sigma\dot{n}_{ex}$;

Energy Equation: $0 = \Sigma\dot{n}_{in}\ \bar{h}_{in} - \Sigma\dot{n}_{ex}\ \bar{h}_{ex}$

$$0 = 2\dot{n}_{N_2}\ \bar{C}_{P\ CO_2}(T_{in} - T_{ex})_{\ CO_2} + \dot{n}_{N_2}\ \bar{C}_{P\ N_2}\ (T_{in} - T_{ex})_{\ N_2}$$

$$0 = 2 \times 37.06 \times (1400 - T_{ex}) + 29.189 \times (300 - T_{ex})$$

$$0 = 103768 + 8756.7 - 103.309\ T_{ex} \quad \Rightarrow \quad \mathbf{T_{ex} = 1089\ K}$$

From Table A.9: $\Sigma\dot{n}_{in}\ \bar{h}_{in} = \dot{n}_{N_2}\ [2 \times 55895 + 1 \times 54] = \dot{n}_{N_2} \times 111844$

@ 1000K : $\Sigma\dot{n}_{ex}\ \bar{h}_{ex} = \dot{n}_{N_2}\ [2 \times 33397 + 21463] = \dot{n}_{N_2} \times 88257$

@ 1100K : $\Sigma\dot{n}_{ex}\ \bar{h}_{ex} = \dot{n}_{N_2}\ [2 \times 38885 + 24760] = \dot{n}_{N_2} \times 102530$

@ 1200K : $\Sigma\dot{n}_{ex}\ \bar{h}_{ex} = \dot{n}_{N_2}\ [2 \times 44473 + 28109] = \dot{n}_{N_2} \times 117055$

Now linear interpolation between 1100 K and 1200 K

$$T_{ex} = 1100 + 100 \times \frac{111844-102530}{117055-102530} = \mathbf{1164\ K}$$

13.124

A piston/cylinder has 100 kg of saturated moist air at 100 kPa, 5°C. If it is heated to 45°C in an isobaric process, find ${}_1Q_2$ and the final relative humidity. If it is compressed from the initial state to 200 kPa in an isothermal process, find the mass of water condensing.

Solution:

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$,

Initial state 1: $\phi_1 = 100\%$, Table B.1.1: $P_{v1} = 0.8721$ kPa, $h_{v1} = 2510.54$

Eq.13.28 $$w_1 = 0.622 \frac{P_{v1}}{P_{tot} - P_{v1}} = \frac{0.8721}{100 - 0.8721} = 0.005472$$

Eq.13.26 with $m_a = m_{tot} - m_{v1} = m_{tot} - w_1 m_a$ gives

$$m_a = m_{tot}/(1+ w_1) = 99.456 \text{ kg},$$

Eq.13.26 $m_{v1} = w_1 m_a = 0.544$ kg

Case a: P = constant => ${}_1W_2 = mP(v_2-v_1)$ =>

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = m_aC_p(T_2 - T_1) + m(h_{v2} - h_{v1})$$

State 2: $w_2 = w_1$, T_2 => $P_{v2} = P_{v1}$ and

Table B.1.1: $h_{v2} = 2583.19$ kJ/kg, $P_{g2} = 9.593$ kPa

Eq.13.25 $$\phi_2 = \frac{P_{v2}}{P_{g2}} = \frac{0.8721}{9.593} = 0.091 \quad \text{or} \quad \phi_2 = \mathbf{9.1\%}$$

From the energy equation

$${}_1Q_2 = 99.456 \times 1.004(45 - 5) + 0.544(2583.19 - 2510.54) = \mathbf{4034\ kJ}$$

Case b: As P is raised $P_v = y_v P$ would be higher than P_g => condensation.

T = constant & $\phi_2 = 100\%$ => $P_v = P_g = 0.8721$ kPa

$$w_2 = 0.622 \frac{P_{v2}}{P_{a2}} = 0.622 \frac{P_{v2}}{P_{tot2} - P_{v2}} = \frac{0.8721}{200 - 0.8721} = 0.002724$$

$m_{v2} = w_2\, m_a = 0.271$ kg, $m_{liq} = m_{v1} - m_{v2} =$ **0.273 kg**

13.125

A piston/cylinder contains helium at 110 kPa at ambient temperature 20°C, and initial volume of 20 L as shown in Fig. P13.125. The stops are mounted to give a maximum volume of 25 L and the nitrogen line conditions are 300 kPa, 30°C. The valve is now opened which allows nitrogen to flow in and mix with the helium. The valve is closed when the pressure inside reaches 200 kPa, at which point the temperature inside is 40°C. Is this process consistent with the second law of thermodynamics?

$P_1 = 110\text{ kPa},\ T_1 = 20\ ^oC,\ V_1 = 20\text{ L},\ V_{max} = 25\text{ L} = V_2$

$P_2 = 200\text{ kPa},\ T_2 = 40\ ^oC,\ P_i = 300\text{ kPa},\ T_i = 30\ ^oC$

Constant P to stops, then constant V = V_{max} => $W_{cv} = P_1(V_2 - V_1)$

$Q_{cv} = U_2 - U_1 + W_{cv} - n_i\bar{h}_i,$

$= n_2\bar{h}_2 - n_1\bar{h}_1 - n_i\bar{h}_i - (P_2 - P_1)V_2$

$= n_A(\bar{h}_{A2} - \bar{h}_{Ai}) + n_B(\bar{h}_{B2} - \bar{h}_{B1}) - (P_2 - P_1)V_2$

$n_B = n_1 = P_1V_1/\bar{R}T_1 = 110\times0.02/8.3145\times293.2 = 0.0009\text{ kmol}$

$n_2 = n_A + n_B = P_2V_2/\bar{R}T_2 = 200\times0.025/8.3145\times313.2 = 0.00192\text{ kmol},$

$n_A = n_2 - n_B = 0.00102\text{ kmol}$

Mole fractions: $y_{A2} = 0.00102/0.00192 = 0.5313,\quad y_{B2} = 0.4687$

$Q_{cv} = 0.00102\times28.013\times1.042(40 - 30) + 0.0009\times4.003\times5.193(40 - 20)$

$- (200 - 110)\ 0.025$

$= 0.298 + 0.374 - 2.25 = -\ 1.578\text{ kJ}$

$S_{gen} = n_2\bar{s}_2 - n_1\bar{s}_1 - n_i\bar{s}_i - Q_{cv}/T_0$

$= n_A(\bar{s}_{A2} - \bar{s}_{Ai}) + n_B(\bar{s}_{B2} - \bar{s}_{B1}) - Q_{cv}/T_0$

$$\bar{s}_{A2} - \bar{s}_{Ai} = 29.189\ \ln\frac{313.2}{303.2} - \bar{R}\ \ln\frac{0.5313*200}{300} = 9.5763$$

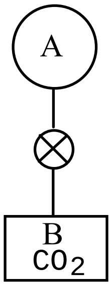
$$\bar{s}_{B2} - \bar{s}_{B1} = 20.7876\ \ln\frac{313.2}{293.2} - \bar{R}\ \ln\frac{0.4687*200}{110} = 2.7015$$

$S_{gen} = 0.00102\times9.5763 + 0.0009\times2.7015 + 1.578/293.2$

= **0.0176 kJ/K > 0** **Satisfies 2nd law.**

13.126

A spherical balloon has an initial diameter of 1 m and contains argon gas at 200 kPa, 40°C. The balloon is connected by a valve to a 500-L rigid tank containing carbon dioxide at 100 kPa, 100°C. The valve is opened, and eventually the balloon and tank reach a uniform state in which the pressure is 185 kPa. The balloon pressure is directly proportional to its diameter. Take the balloon and tank as a control volume, and calculate the final temperature and the heat transfer for the process.



$$V_{A1} = \frac{\pi}{6} 1^3 = 0.5236,$$

$$m_{A1} = \frac{P_{A1}V_{A1}}{RT_{A1}} = \frac{200 \times 0.5236}{0.208\ 13 \times 313.2} = 1.606 \text{ kg}$$

$m_{B1} = P_{B1}V_{B1}/RT_{B1} = 100 \times 0.50/0.18892 \times 373.2 = 0.709$ kg

Process: $P = C\,D = CV^{1/3}$ ⇨ polytropic $n = -1/3$ ⇨

$$V_{A2} = V_{A1}\left(\frac{P_2}{P_{A1}}\right)^3 = 0.5236\left(\frac{185}{200}\right)^3 = 0.4144 \text{ m}^3$$

2: Uniform ideal gas mixture :

$$P_2(V_{A2}+V_B) = (m_AR_A+m_BR_B)T_2$$

$T_2 = 185(0.4144+0.50) / (1.606 \times 0.20813 + 0.709 \times 0.18892) =$ **361.3 K**

$$W_{12} = \frac{P_2V_{A2}-P_{A1}V_{A1}}{1-(-1/3)} = \frac{185 \times 0.4144-200 \times 0.5236}{(4/3)} = -21.0 \text{ kJ}$$

$Q = m_AC_{V0A}(T_2-T_{A1}) + m_BC_{V0B}(T_2-T_{B1}) + W_{12}$

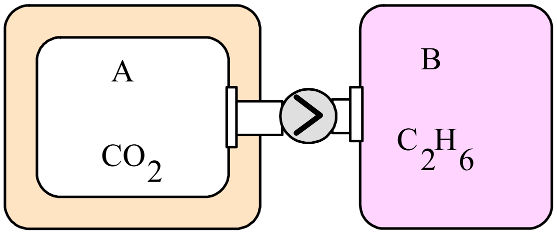
$= 1.606 \times 0.312(361.3 - 313.2) + 0.709 \times 0.653(361.3 - 373.2) - 21.0$

$= 18.6 - 21.0 =$ **-2.4 kJ**

13.127

An insulated rigid 2 m^3 tank A contains CO_2 gas at 200°C, 1MPa. An uninsulated rigid 1 m^3 tank B contains ethane, C_2H_6, gas at 200 kPa, room temperature 20°C. The two are connected by a one-way check valve that will allow gas from A to B, but not from B to A. The valve is opened and gas flows from A to B until the pressure in B reaches 500 kPa and the valve is closed. The mixture in B is kept at room temperature due to heat transfer. Find the total number of moles and the ethane mole fraction at the final state in B. Find the final temperature and pressure in tank A and the heat transfer to/from tank B.

Solution:



Tank A: $V_A = 2\ m^3$, state A_1 : CO_2, $T_{A1} = 200°C = 473.2$ K, $P_{A1} = 1$ MPa

$\bar{C}_{v0\ CO_2} = 0.653 \times 44.01 = 28.74$, $\bar{C}_{P0\ CO_2} = 0.842 \times 44.01 = 37.06$ kJ/kmol K

Tank B: $V_B = 1\ m^3$, state B_1: C_2H_6, $T_{B1} = 20°C = 293.2$ K, $P_{B1} = 200$ kPa

Slow Flow A to B to $P_{B2} = 500$ kPa and assume $T_{B2} = T_{B1} = T_0$

Total moles scales to pressure, so with same V and T we have

$$P_{B1}V_B = n_{B1}\bar{R}T_{B1}\ , \qquad P_{B2}V_B = n_{B2\ mix}\ \bar{R}T_{B2}$$

Mole fraction: $$y_{C_2H_6\ B_2} = \frac{n_{B1}}{n_{B2}} = \frac{P_{B1}}{P_{B2}} = \frac{200}{500} = \mathbf{0.400}$$

$$n_{B1} = \frac{P_{B1}V_B}{RT_{B1}} = \frac{200 \times 1}{R \times 293.2} = 0.08204 \text{ kmol}$$

$$n_{B2\ mix} = \frac{P_{B2}V_B}{RT_{B2}} = \frac{500 \times 1}{R \times 293.2} = 0.2051 \text{ kmol}$$

$$n_{CO_2\ B2} = 0.2051 - 0.08201 = 0.12306 \text{ kmol}$$

Now we can work backwards to final state in A

$$n_{A1} = \frac{P_{A1}V_A}{RT_{A1}} = \frac{1000 \times 2}{R \times 473.2} = 0.50833 \text{ kmol;}\ \ n_{A2} = n_{A1} - n_{CO_2\ B2} = 0.38527 \text{ kmol}$$

C.V. A: All CO_2 Transient with flow out and adiabatic.

Energy Eq.: $Q_{CV\ A} = 0 = n_{A2}\, \bar{u}_{A2} - n_{A1}\, \bar{u}_{A1} + n_{ave}\, \bar{h}_{ave}$

$0 = n_{A2}\, \bar{C}_{v0\ CO_2} T_{A2} - n_{A1}\, \bar{C}_{v0\ CO_2} T_{A1} + n_{ave}\, \bar{C}_{P0\ CO_2}\, (T_{A1} + T_{A2})/2$

$0 = 28.74(0.38527\times T_{A2} - 0.50833\times 473.2) + 0.12306 \times 37.06(473.2 + T_{A2})/2$

$\Rightarrow \quad T_{A2} = \mathbf{436.9\ K}$

$$P_{A2} = \frac{n_{A2}\, RT_{A2}}{V_A} = \frac{0.38527 \times R \times 436.9}{2} = \mathbf{700\ kPa}$$

C.V. B: Transient with flow in and non-adiabatic.

$Q_{CV\ B} + n_{Bi}\, \bar{h}_{Bi\ ave} = (n\bar{u})_{B2} - (n\bar{u})_{B1} = (n\bar{u})_{CO_2\ B2} + (n\bar{u})_{C_2H_6\ B2} - (n\bar{u})_{C_2H_6\ B1}$

$Q_{CV\ B} = 0.12306 \times 28.74 \times 293.2 + 0 - 0.12306 \times 37.06\ (473.2 + 436.9)/2$

$= \mathbf{-1038\ kJ}$

13.128

You have just washed your hair and now blow dry it in a room with 23°C, ϕ = 60%, (1). The dryer, 500 W, heats the air to 49°C, (2), blows it through your hair where the air becomes saturated (3), and then flows on to hit a window where it cools to 15°C (4). Find the relative humidity at state 2, the heat transfer per kilogram of dry air in the dryer, the air flow rate, and the amount of water condensed on the window, if any.

The blowdryer heats the air at constant specific humidity to 2 and it then goes through an adiabatic saturation process to state 3, finally cooling to 4.

1: 23°C, 60% rel hum => $w_1 = 0.0104$, $\tilde{h}_1 = 69$ kJ/kg dry air

2: $w_2 = w_1, T_2$ => $\boldsymbol{\phi_2 = 15\%}$, $\tilde{h}_2 = 95$ kJ/kg dry air

CV. 1 to 2:

$w_2 = w_1$; $q = \tilde{h}_2 - \tilde{h}_1 = 95 - 69 = \mathbf{26\ kJ/kg\ dry\ air}$

$\dot{m}_a = Q/q = 0.5/26 = \mathbf{0.01923\ kg/s}$

CV. 2 to 3: $w_3 - w_2 = \dot{m}_{liq}/\dot{m}_a$; $\dot{m}_a \tilde{h}_2 + \dot{m}_{liq} h_f = \dot{m}_a \tilde{h}_3$

3: $\phi = 100\%$ => $T_3 = \mathbf{T_{wet,2} = 24.8°C}$, $w_3 = \mathbf{0.0198}$

4: $\phi = 100\%$, T_4 => $w_4 = 0.01065$

$\dot{m}_{liq} = (w_3 - w_4)\dot{m}_a = (0.0198 - 0.01065)\times 0.01923 = \mathbf{0.176\ g/s}$

If the steam **tables and formula's** are used then we get

$h_{g1} = 2543.5$, $h_{g2} = 2590.3$, $P_{g1} = 2.837$ kPa, $P_{v1} = 1.7022$ kPa,

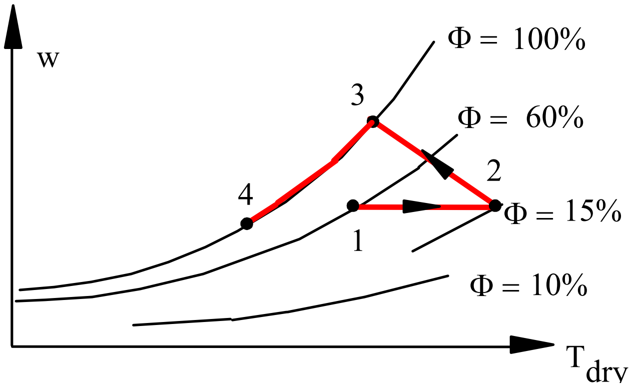
$P_{g2} = 11.8$ kPa, $w_1 = 0.01077$, $w_2 = w_1$, $P_{v2} = P_{v1}$

$\phi_2 = P_{v2}/P_{g2} = 14.4\%$, $h_{f3} = 114$ kJ/kg,

Trial and error for adiabatic saturation temperature.

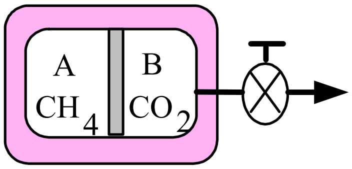
$T_3 = 25°C$, $w_3 = 0.02$, $P_{v4} = P_{g4} = 1.705$ kPa,

$w_4 = 0.622\times 1.705/(100 - 1.705) = 0.0108$



13.129

A 0.2 m^3 insulated, rigid vessel is divided into two equal parts A and B by an insulated partition, as shown in Fig. P13.129. The partition will support a pressure difference of 400 kPa before breaking. Side A contains methane and side B contains carbon dioxide. Both sides are initially at 1 MPa, 30°C. A valve on side B is opened, and carbon dioxide flows out. The carbon dioxide that remains in B is assumed to undergo a reversible adiabatic expansion while there is flow out. Eventually the partition breaks, and the valve is closed. Calculate the net entropy change for the process that begins when the valve is closed.



$\Delta P_{MAX} = 400$ kPa, $\quad P_{A1} = P_{B1} = 1$ MPa

$V_{A1} = V_{B1} = 0.1\ m^3$

$T_{A1} = T_{B1} = 30^oC = 303.2$ K

CO_2 inside B: $s_{B2} = s_{B1}$ to $P_{B2} = 600$ kPa ($P_{A2} = 1000$ kPa)

For CO_2, k = 1.289 => $T_{B2} = 303.2\left(\frac{600}{1000}\right)^{\frac{0.289}{1.289}} = 270.4$ K

$n_{B2} = P_{B2}V_{B2}/\bar{R}T_{B2} = 600\times0.1/8.3145\times270.4 = 0.026\ 688$

$n_{A2} = n_{A1} = 1000\times0.1/8.3145\times303.2 = 0.039\ 668$ kmol

The process 2 to 3 is adiabatic but irreversible with no work.

$$Q_{23} = 0 = n_3\bar{u}_3 - \sum_i n_{i2}\bar{u}_{i2} + 0 = n_{A2}\bar{C}_{vo\ A}(T_3\text{-}T_{A2}) + n_{B2}\bar{C}_{vo\ B}(T_3\text{-}T_{B2}) = 0$$

$$0.039\ 668\times16.04\times1.736(T_3\text{-}303.2) + 0.026\ 688\times44.01\times0.653(T_3\text{-}270.4) = 0$$

Solve $\quad T_3 = 289.8$ K

Get total and partial pressures for the entropy change

$P_3 = n\bar{R}T/V = 0.066356\times8.3145\times289.8/0.2 = 799.4$ kPa

$P_{A3} = 0.5978\times799.4 = 477.9$ kPa , $\quad P_{B3} = P_3 - P_{A3} = 321.5$ kPa

$$\bar{s}_{A3} - \bar{s}_{A2} = 16.04\times2.254\ \ln(\frac{289.8}{303.2}) - 8.3145\ \ln\frac{477.9}{1000} = 4.505 \text{ kJ/kmol K}$$

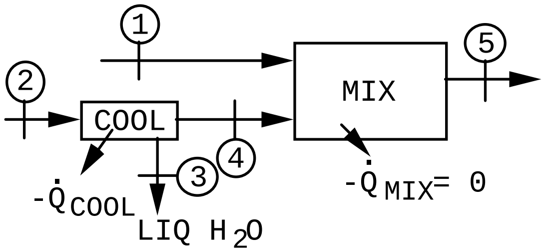
$$\bar{s}_{B3} - \bar{s}_{B2} = 44.01\times0.842\ \ln(\frac{289.8}{270.4}) - 8.3145\ \ln\frac{321.5}{600} = 7.7546 \text{ kJ/kmol K}$$

$$\Delta S_{NET} = n_{A2}(\bar{s}_{A3} - \bar{s}_{A2}) + n_{B2}(\bar{s}_{B3} - \bar{s}_{B2})$$

$$= 0.039668 \times 4.505 + 0.026688 \times 7.7546 = \mathbf{+0.3857\ kJ/K}$$

13.130

Ambient air is at a condition of 100 kPa, 35°C, 50% relative humidity. A steady stream of air at 100 kPa, 23°C, 70% relative humidity, is to be produced by first cooling one stream to an appropriate temperature to condense out the proper amount of water and then mix this stream adiabatically with the second one at ambient conditions. What is the ratio of the two flow rates? To what temperature must the first stream be cooled?



$P_1 = P_2 = 100$ kPa

$T_1 = T_2 = 35\ ^oC$

$\phi_1 = \phi_2 = 0.50,\ \phi_4 = 1.0$

$P_5 = 100,\ T_5 = 23\ ^oC$

$\phi_5 = 0.70$

$$P_{v1} = P_{v2} = 0.5\times5.628 = 2.814 \text{ kPa} \;\Rightarrow\; w_1 = w_2 = 0.622\times\frac{2.814}{100\text{-}2.814} = 0.0180$$

$$P_{v5} = 0.7\times2.837 = 1.9859 \text{ kPa} \;\Rightarrow\; w_5 = 0.622\times\frac{1.9859}{100\text{-}1.9859} = 0.0126$$

C.V.: Mixing chamber: Call the mass flow ratio $r = m_{a2}/m_{a1}$

cons. mass: $w_1 + r\,w_4 = (1+ r)w_5$

Energy Eq.: $h_{a1} + w_1h_{v1} + rh_{a4} + rw_4h_{v4} = (1+r)h_{a5} + (1+r)w_5h_{v5}$

$\rightarrow 0.018 + rw_4 = (1+r)\ 0.0126$

or $\quad r = \dfrac{m_{a2}}{m_{a1}} = \dfrac{0.018\text{-}0.0126}{0.0126\text{-}w_4}$, $\quad$ with $\quad w_4 = 0.622 \times \dfrac{P_{G4}}{100\text{-}P_{G4}}$

$$1.004\times308.2 + 0.018\times2565.3 + r\times1.004\times T_4 + r\,w_4h_{v4}$$
$$= (1+r)\times1.004 \times 296.2 + (1+r)\times0.0126\times2543.6$$

or $\quad r\left[1.004\times T_4 + w_4h_{G4} - 329.3\right] + 26.2 = 0$

Assume $T_4 = \mathbf{5\ ^oC}$ $\quad\rightarrow P_{G4} = 0.8721,\ \ h_{G4} = 2510.5$

$w_4 = 0.622\times0.8721/(100\text{-}0.8721) = 0.0055$

$$r = m_{a2}/m_{a1} = \frac{0.018\text{-}0.0126}{0.0126\text{-}0.0055} = \mathbf{0.7606}$$

$0.7606[1.004\times278.2 + 0.0055\times2510.5 - 329.6] + 26.2 = -1.42 \approx 0$ OK

13.131

An air-water vapor mixture enters a steady flow heater humidifier unit at state 1: 10°C, 10% relative humidity, at the rate of 1 m^3/s. A second air-vapor stream enters the unit at state 2: 20°C, 20% relative humidity, at the rate of 2 m^3/s. Liquid water enters at state 3: 10°C, at the rate of 400 kg per hour. A single air-vapor flow exits the unit at state 4: 40°C. Calculate the relative humidity of the exit flow and the rate of heat transfer to the unit.

Assume: P = 100 kPa

State 1: $T_1 = 10°C$, $\phi_1 = 10\%$, $\dot{V}_{a1} = 1\ m^3/s$

$P_{g1} = 1.2276$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.1228$ kPa,

$P_{a1} = P - P_{v1} = 99.877$ kPa

$$\omega_1 = 0.622 \frac{P_{v1}}{P_{a1}} = 0.000765, \quad \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.2288 \text{ kg/s}$$

$\dot{m}_{v1} = \omega_1 \dot{m}_{a1} = 0.00094$ kg/s, $h_{v1} = h_{g1} = 2519.7$ kJ/kg

State 2: $T_2 = 20°C$, $\phi_2 = 20\%$, $\dot{V}_{a2} = 2\ m^3/s$

$P_{g2} = 2.3385$ kPa, $P_{v2} = \phi\ P_{g2} = 0.4677$ kPa, $P_{a2} = P - P_{v2} = 99.532$ kPa

$$\omega_2 = 0.622 \frac{P_{v2}}{P_{a2}} = 0.002923, \quad \dot{m}_{a2} = \frac{P_{a2}\ \dot{V}_{a2}}{RT_{a2}} = 2.3656 \text{ kg/s}$$

$\dot{m}_{v2} = \omega_2 \dot{m}_{a2} = 0.00691$ kg/s, $h_{v2} = h_{g2} = 2538.1$ kJ/kg

State 3: Liquid. $T_3 = 10°C$, $\dot{m}_{f3} = 400$ kg/hr $= 0.1111$ kg/s, $h_{f3} = 42$ kJ/kg

State 4: $T_4 = 40°C$

Continuity Eq. air: $\dot{m}_{a4} = \dot{m}_{a2} + \dot{m}_{a1} = 3.5944$ kg/s,

Continuity Eq. water: $\dot{m}_{v4} = \dot{m}_{v1} + \dot{m}_{v2} + \dot{m}_{f3} = 0.11896$ kg/s

$$\omega_4 = \frac{\dot{m}_{v4}}{\dot{m}_{a4}} = 0.0331 = 0.622 \frac{P_{v4}}{P - P_{v4}} \quad \rightarrow P_{v4} = 5.052 \text{ kPa}$$

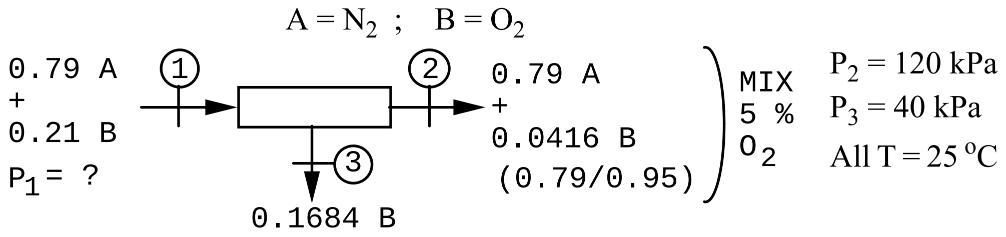
$$P_{g4} = 7.384 \text{ kPa}, \quad \phi_4 = \frac{P_{v4}}{P_{g4}} = \mathbf{0.684}, \quad h_{v4} = h_{g4} = 2574.3 \text{ kJ/kg}$$

1st Law: $\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} + \dot{m}_{f3}h_{f3} = \dot{m}_{a4}h_{a4} + \dot{m}_{v4}h_{v4}$

$$\dot{Q} = 1.004(3.5944 \times 40 - 1.2288 \times 10 - 2.3656 \times 20) + 0.11896 \times 2574.3$$
$$- 0.00094 \times 2519.7 - 0.00691 \times 2538.1 - 0.1111 \times 42.0$$
$$= \mathbf{366\ kW}$$

13.132

A semi-permeable membrane is used for the partial removal of oxygen from air that is blown through a grain elevator storage facility. Ambient air (79% nitrogen, 21% oxygen on a mole basis) is compressed to an appropriate pressure, cooled to ambient temperature 25°C, and then fed through a bundle of hollow polymer fibers that selectively absorb oxygen, so the mixture leaving at 120 kPa, 25°C, contains only 5% oxygen. The absorbed oxygen is bled off through the fiber walls at 40 kPa, 25°C, to a vacuum pump. Assume the process to be reversible and adiabatic and determine the minimum inlet air pressure to the fiber bundle.



Let $s_{A1} = s_{B1} = 0$ at $T = 25\ ^oC$ & P_1

$$\bar{s}_{MIX\,1} = 0 + 0 - y_{A1}\bar{R} \ln y_{A1} - y_{B1}\bar{R} \ln y_{B1}$$

$$\bar{s}_{MIX\,2} = 0 + 0 - \bar{R} \ln (P_2/P_1) - y_{A2}\bar{R} \ln y_{A2} - y_{B2}\bar{R} \ln y_{B2}$$

Pure B: $\bar{s}_3 = 0 - \bar{R} \ln (P_3/P_1)$

For $\dot{n}_1\bar{s}_1 = \dot{n}_2\bar{s}_2 + \dot{n}_3\bar{s}_3$ all T's the same, so only partial pressure terms

$$\bar{R}\Big[-0.8316 \ln (P_2/P_1) - 0.79 \ln 0.95 - 0.0416 \ln 0.05$$
$$- 0.1684 \ln (P_3/P_1) + 0.79 \ln 0.79 + 0.21 \ln 0.21\Big] = 0$$

$$0.8316 \ln (P_2/P_1) + 0.1684 \ln (P_3/P_1) = -0.3488 + 4.6025 - \ln P_1 = -0.3488$$

$P_{1\ min}$ = **141 kPa**

For $P_1 > P_{1\ min}$ we would have entropy generation $\Delta\dot{S} > 0$

13.133

A dehumidifier receives a flow of 0.25 kg/s dry air at 35°C, 90% relative humidity as shown in figure P13.101. It is cooled down to 20°C as it flows over the evaporator and then heated up again as it flows over the condenser. The standard refrigeration cycle uses R-410a with an evaporator temperature of –5°C and a condensation pressure of 3000 kPa. Find the amount of liquid water removed and the heat transfer in the cooling process. How much compressor work is needed? What is the final air exit temperature and relative humidity?

Solution:

This set-up has a standard refrigeration cycle with R-410a. This cycle and the air flow interacts through the two heat transfer processes. The cooling of the air is provided by the refrigeration cycle and thus requires an amount of work that depends on the cycle COP.

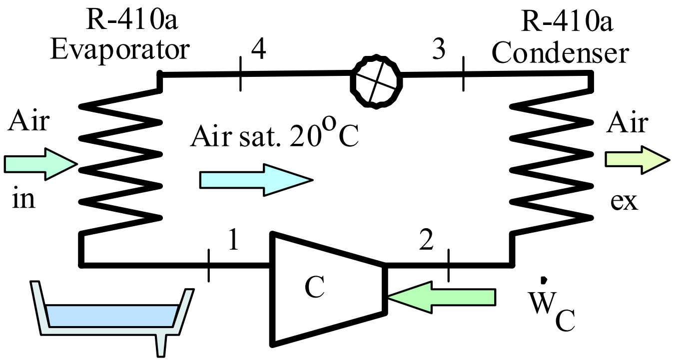
Refrigeration cycle:

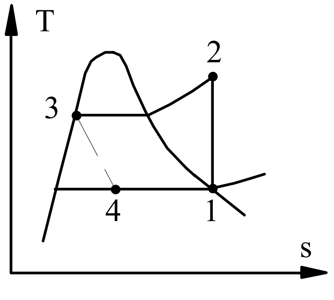
State 1: $x = 1$ $h_1 = 277.53$ kJ/kg, $s_1 = 1.0466$ kJ/kg K

State 2: $s_2 = s_1$, $h_2 = 318.97$ kJ/kg, $T_2 = 72.9^oC$

State 3: $x_3 = 0.0$, $h_3 = h_f = 141.78$ kJ/kg, ($T_3 = 49.07°C$)

State 4: $h_4 = h_3$ and $P_4 = P_1$





$T_1 = –5^oC$, $P_2 = 3000$ kPa

Now we get

$w_C = h_2 – h_1 = 318.97 – 277.53 = 41.44$ kJ/kg

$q_H = h_2 – h_3 = 318.97 – 141.78 = 177.19$ kJ/kg

$q_L = h_1 – h_4 = 277.53 – 141.78 = 135.75$ kJ/kg

For the air processes let us use the psychrometric chart.

Air inlet: $w_{in} = 0.019$, $\tilde{h}_{in} = 96$ kJ/kg dry air, $T_{dew} = 24^oC > 15^oC$

Air 15: $\phi = 100\%$, $w_{20} = 0.0148$, $\tilde{h}_{20} = 77.5$, $h_f = 83.94$ (B.1.1)

Now do the continuity (for water) and energy equations for the cooling process

$\dot{m}_{liq} = \dot{m}_{air}\,(w_{in} - w_{20}) = 0.25\ (0.019 - 0.0148) = \mathbf{0.00105\ kg/s}$

$\dot{Q}_{cool} = \dot{m}_{air}(\,\tilde{h}_{in} - \tilde{h}_{20}) - \dot{m}_{liq}h_f = 0.25(96 - 77.5) - 0.00105 \times 83.94$

$= \mathbf{4.537\ kW}$

Now the heater from the R-410a cycle has

$\dot{Q}_{heat} = \dot{Q}_{cool}\ (q_H / q_L) = 4.537\ (177.19 / 135.75) = 5.922$ kW

so the compressor work is the balance of the two

$\dot{W}_C = \dot{Q}_{heat} - \dot{Q}_{cool} = 5.922 - 4.537 = \mathbf{1.385\ kW}$

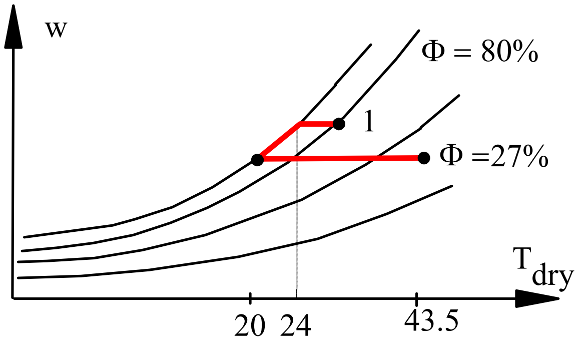
Energy eq. for the air flow being heated

$\dot{Q}_{heat} = \dot{m}_{air}(\,\tilde{h}_{ex} - \tilde{h}_{20}) \;\Rightarrow\; \tilde{h}_{ex} = \tilde{h}_{20} + \dot{Q}_{heat} / \dot{m}_{air}$

$\tilde{h}_{ex} = 77.5 + 5.922 / 0.25 = 101.2$ kJ/kg dry air and $w_{ex} = w_{20}$

Locate state in the psychrometric chart

$T = 43.5^oC$ and $\phi = 27\%$



13.134

The air-conditioning by evaporative cooling in Problem 13.96 is modified by adding a dehumidification process before the water spray cooling process. This dehumidification is achieved as shown in Fig. P13.134 by using a desiccant material, which absorbs water on one side of a rotating drum heat exchanger. The desiccant is regenerated by heating on the other side of the drum to drive the water out. The pressure is 100 kPa everywhere and other properties are on the diagram. Calculate the relative humidity of the cool air supplied to the room at state 4, and the heat transfer per unit mass of air that needs to be supplied to the heater unit.

States as noted on Fig. P13.134, text page 506.

At state 1, 35 °C: $P_{V1} = \phi_1 P_{G1} = 0.30 \times 5.628 = 1.6884$

$$w_1 = 0.622 \times 1.6884/98.31 = 0.010\ 68$$

At $T_3 = 25\ ^{o}C$: $w_3 = w_2 = w_1/2 = 0.00534$

Evaporative cooling process to state 4, where $T_4 = 20^{o}C$

As in Eq. 12.30: $w_3(h_{v3} - h_{f4}) = C_{P0A}(T_4 - T_3) + w_4\, h_{fg4}$

$$0.005\ 34\ (2547.2 - 83.9) = 1.004(20 - 25) + w_4 \times 2454.2$$

$$w_4 = 0.0074 = 0.622 \times P_{v4} / (100 - P_{v4})$$

$P_{V4} = 1.176$ kPa, $\phi_4 = 1.176 / 2.339 = \mathbf{0.503}$

Following now the flow back we have

At $T_5 = 25\ ^{o}C$, $w_5 = w_4 = 0.0074$

Evaporative cooling process to state 6, where $T_6 = 20^{o}C$

$$w_5(h_{v5} - h_{f6}) = C_{P0A}(T_6 - T_5) + w_6\, h_{fg6}$$

$$0.0074(2547.2 - 83.9) = 1.004(20 - 25) + w_6 \times 2454.2$$

$$\Rightarrow \quad w_6 = 0.009\ 47$$

For adiabatic heat exchanger,

$\dot{m}_{A2} = \dot{m}_{A3} = \dot{m}_{A6} = \dot{m}_{A7} = \dot{m}_A$, Also $w_2 = w_3$, $w_6 = w_7$

So now only T_7 is unknown in the energy equation

$$h_{A2} + w_2 h_{v2} + h_{A6} + w_6 h_{v6} = h_{A3} + w_3 h_{v3} + h_{A7} + w_7 h_{v7}$$

or

$$C_{P0A}T_7 + w_6(h_{v7} - h_{v6}) = C_{P0A}(T_2 + T_6 - T_3) + w_2(h_{v2} - h_{v3})$$

$$1.004\ T_7 + 0.009\ 47(h_{v7} - 2538.1) = 1.004(60 + 20 - 25)$$

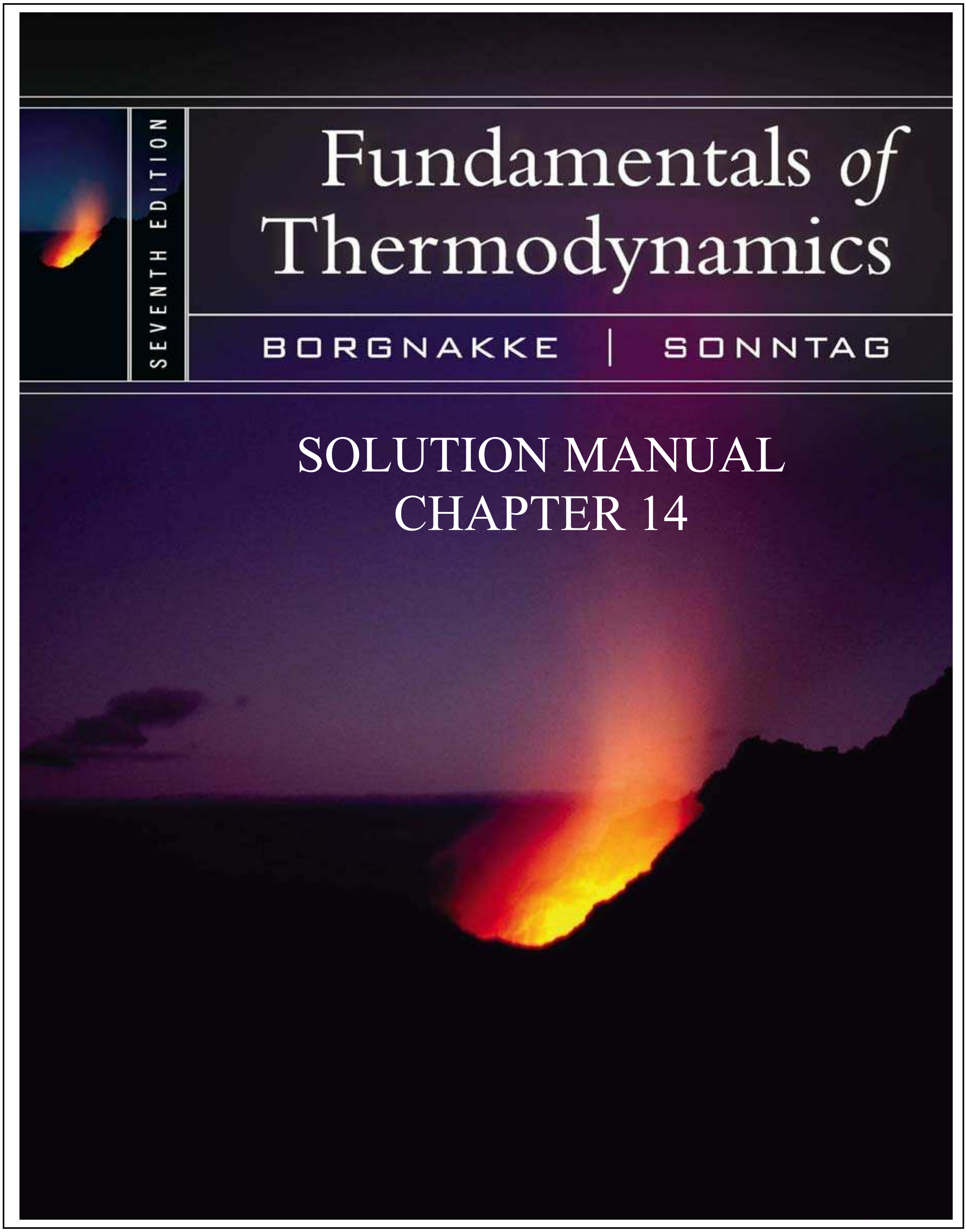
$$+ 0.005\ 34(2609.6 - 2547.2) = 55.526$$

By trial and error, $T_7 = 54.7\ ^{o}C$, $h_{v7} = 2600.3$ kJ/kg

For the heater 7-8, $w_8 = w_7$,

$$\dot{Q}/\dot{m}_A = C_{P0A}(T_8 - T_7) + w_7(h_{v8} - h_{v7})$$

$$= 1.004(80 - 54.7) + 0.009\,47(2643.7 - 2600.3) = \mathbf{25.8\ kJ/kg\ dry\ air}$$



Fundamentals of Thermodynamics

SEVENTH EDITION

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SOLUTION MANUAL
CHAPTER 14

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CONTENT CHAPTER 14

The following table gives the values for the compressibility, enthalpy departure and the entropy departure along the saturated liquid-vapor boundary. These are used for all the problems using generalized charts as the figures are very difficult to read accurately (consistently) along the saturated liquid line. It is suggested that the instructor hands out copies of this page or let the students use the computer for homework solutions.

T_r	P_r	Z_f	Z_g	$d(h/RT)_f$	$d(h/RT)_g$	$d(s/R)_f$	$d(s/R)_g$
0.96	0.78	0.14	0.54	3.65	1.39	3.45	1.10
0.94	0.69	0.12	0.59	3.81	1.19	3.74	0.94
0.92	0.61	0.10	0.64	3.95	1.03	4.00	0.82
0.90	0.53	0.09	0.67	4.07	0.90	4.25	0.72
0.88	0.46	0.08	0.70	4.17	0.78	4.49	0.64
0.86	0.40	0.07	0.73	4.26	0.69	4.73	0.57
0.84	0.35	0.06	0.76	4.35	0.60	4.97	0.50
0.82	0.30	0.05	0.79	4.43	0.52	5.22	0.45
0.80	0.25	0.04	0.81	4.51	0.46	5.46	0.39
0.78	0.21	0.035	0.83	4.58	0.40	5.72	0.35
0.76	0.18	0.03	0.85	4.65	0.34	5.98	0.31
0.74	0.15	0.025	0.87	4.72	0.29	6.26	0.27
0.72	0.12	0.02	0.88	4.79	0.25	6.54	0.23
0.70	0.10	0.017	0.90	4.85	0.21	6.83	0.20
0.68	0.08	0.014	0.91	4.92	0.18	7.14	0.17
0.66	0.06	0.01	0.92	4.98	0.15	7.47	0.15
0.64	0.05	0.009	0.94	5.04	0.12	7.81	0.12
0.60	0.03	0.005	0.95	5.16	0.08	8.56	0.08
0.58	0.02	0.004	0.96	5.22	0.06	8.97	0.07
0.54	0.01	0.002	0.98	5.34	0.03	9.87	0.04
0.52	0.0007	0.0014	0.98	5.41	0.02	10.38	0.03

In-Text Concept Questions

14.a

Mention two uses of the Clapeyron equation.

If you have experimental information about saturation properties down to a certain temperature Clapeyron equation will allow you to make an intelligent curve extrapolation of the saturated pressure versus temperature function $P_{sat}(T)$ for lower temperatures.

From Clapeyrons equation we can calculate a heat of evaporation, heat of sublimation or heat of fusion based on measurable properties P, T and v. The similar changes in entropy are also obtained since

$$h_{fg} = Ts_{fg}; \qquad h_{if} = Ts_{if}; \qquad h_{ig} = Ts_{ig}$$

14.b

If I raise the temperature in a constant pressure process, does *g* go up or down?

From the definition and variation in Gibbs function, see Eq.14.15 and Maxwells relation Eq.14.21 last one, we get

$$dg = -s\ dT$$

so Gibbs function decreases as temperature increases.

14.c

If I raise the pressure in an isentropic process, does *h* go up or down? Is that independent upon the phase?

$Tds = 0 = dh – vdP$, so h increases as P increases, for any phase. The magnitude is proportional to v (i.e. large for vapor and small for liquid and solid phases)

14.d

If I raise the pressure in a solid at constant *T*, does *s* go up or down?

In Example 14.4, it is found that change in s with P at constant T is negatively related to volume expansivity (a positive value for a solid),

$$ds_T = - v\ \alpha_P\ dP_T$$

so raising P decreases s.

14.e

What does it imply if the compressibility factor is larger than 1?

A compressibility factor that is greater than one comes from domination of intermolecular forces of repulsion (short range) over forces of attraction (long range) – either high temperature or very high density. This implies that the density is lower than what is predicted by the ideal gas law, the ideal gas law assumes the molecules (atoms) can be pressed closer together.

14.f

What is the benefit of the generalized charts? Which properties must be known besides the charts themselves?

The generalized charts allow for the approximate calculations of enthalpy and entropy changes (and P,v,T behavior), for processes in cases where specific data or equation of state are not known. They also allow for approximate phase boundary determinations. It is necessary to know the critical pressure and temperature, as well as ideal-gas specific heat.

Concept-Study Guide Problems

14.1

The slope dP/dT of the vaporization line is finite as you approach the critical point, yet h_{fg} and v_{fg} both approach zero. How can that be?

The slope is $\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}}$

Recall the math problem what is the limit of f(x)/g(x) when x goes towards a point where both functions f and g goes towards zero. A finite limit for the ratio is obtained if both first derivatives are different from zero so we have

$$dP/dT \rightarrow [dh_{fg}/dT] \;/\; d(Tv_{fg})/dT \quad \text{as} \quad T \rightarrow T_c$$

14.2

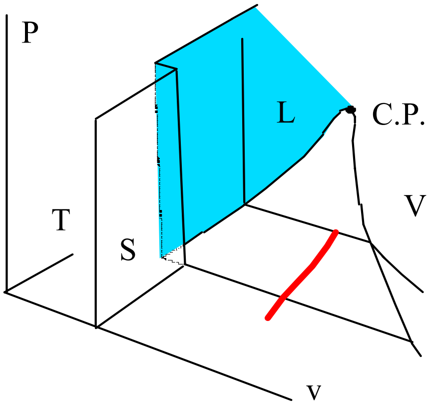
In view of Clapeyron’s equation and Fig. 3.7, is there something special about ice I versus the other forms of ice?

Yes. The slope of the phase boundary dP/dT is negative for ice I to liquid whereas it is positive for all the other ice to liquid interphases. This also means that these other forms of ice are all heavier than liquid water. The pressure must be more than 200 MPa ≈ 2000 atm so even the deepest ocean cannot reach that pressure (recall about 1 atm per 10 meters down).

14.3

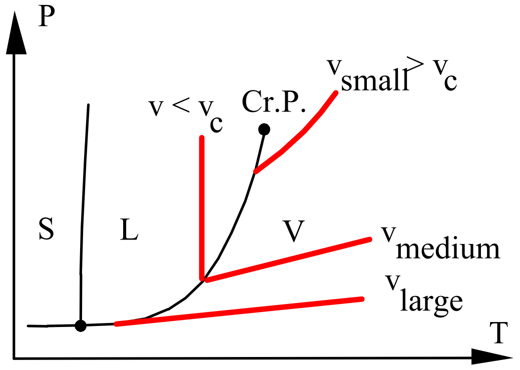
If we take a derivative as $(\partial P/\partial T)_v$ in the two-phase region, see Figs. 3.18 and 3.19, does it matter what v is? How about T?

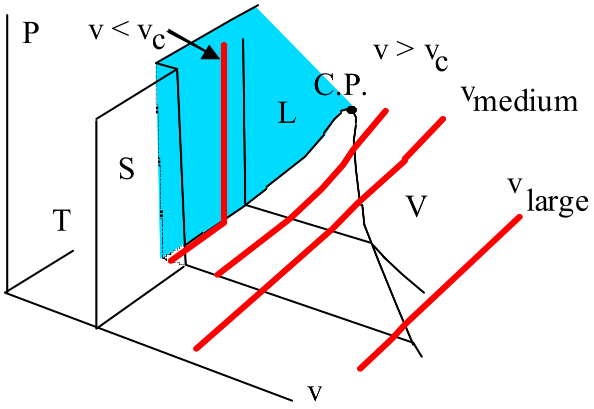
In the two-phase region, P is a function only of T, and not dependent on v. The slope is the same at a given T regardless of v. The slope becomes higher with higher T and generally is the highest near the critical point.



14.4

Sketch on a P-T diagram how a constant v line behaves in the compressed liquid region, the two-phase L-V region and the superheated vapor region?





14.5

If I raise the pressure in an isothermal process does h go up or down for a liquid or solid? What do you need to know if it is a gas phase?

Eq. 14.25: $\left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_P = v[1 - T\alpha_P]$

Liquid or solid, α_p is very small, h increases with P ;

For a gas, we need to know the equation of state.

14.6

The equation of state in Example 14.3 was used as explicit in v. Is it explicit in P?

Yes, the equation can be written explicitly in P.

$$P = RT / [v + C/T^3]$$

14.7

Over what range of states are the various coefficients in Section 14.5 most useful?

For solids or liquids, where the coefficients are essentially constant over a wide range of P's and T's.

14.8

For a liquid or a solid is v more sensitive to T or P? How about an ideal gas?

For a liquid or solid, v is much more sensitive to T than P.
For an ideal gas, $v = RT/P$, varies directly with T, inversely with P.

14.9

Most equations of state are developed to cover which range of states?

Most equations of state are developed to cover the gaseous phase, from low to moderate densities. Many cover high-density regions as well, including the compressed liquid region. To cover a wider region the EOS must be more complex and usually has many terms so it is only useful on a computer.

14.10

Is an equation of state valid in the two-phase regions?

No. In a two-phase region, P depends only on T. There is a discontinuity at each phase boundary. It is actually difficult to determine the phase boundary from the EOS.

14.11

As $P \rightarrow 0$, the specific volume $v \rightarrow \infty$. For $P \rightarrow \infty$, does $v \rightarrow 0$?

At very low P, the substance will be essentially an ideal gas, $Pv = RT$, so that v becomes very large. However at very high P, the substance eventually must become a solid, which cannot be compressed to a volume approaching zero.

14.12

Must an equation of state satisfy the two conditions in Eqs. 14.49 and 14.50?

It has been observed from experimental measurements that substances do behave in that manner. If an equation of state is to be accurate in the near-critical region, it would have to satisfy these two conditions.

If the equation is simple it may be overly restrictive to impose these as it may lead to larger inaccuracies in other regions.

14.13

At which states are the departure terms for h and s small? What is Z there?

Departure terms for h and s are small at very low pressure or at very high temperature. In both cases, Z is close to 1 and this is the ideal gas region.

14.14

The departure functions for h and s as defined are always positive. What does that imply for the real substance h and s values relative to ideal gas values?

Real-substance h and s are less than the corresponding ideal-gas values. This is true for the range shown in the figures, $P_r < 10$. For higher P the isotherms do bend down and negative values are possible.

14.15

What is the benefit of Kay’s rule versus a mixture equation of state?

Kay’s rule for a mixture is not nearly as accurate as an equation of state for the mixture, but it is very simple to use.

Clapeyron Equation

14.16

An approximation for the saturation pressure can be $\ln P_{sat} = A - B/T$, where A and B are constants. Which phase transition is that suitable for, and what kind of property variations are assumed?

Clapeyron Equation expressed for the three phase transitions are shown in Eqs. 14.5-14.7. The last two leads to a natural log function if integrated and ideal gas for the vapor is assumed.

$$\frac{dP_{sat}}{dT} = P_{sat} \frac{h_{evap}}{RT^2}$$

where h_{evap} is either h_{fg} or h_{ig}. Separate the variables and integrate

$$P^{-1}_{sat}\, dP_{sat} = h_{evap}\, R^{-1}\, T^{-2}\, dT$$

$$\ln P_{sat} = A - B/T \; ; \qquad B = h_{evap}\, R^{-1}$$

if we also assume h_{evap} is constant and A is an integration constant. The function then applies to the liquid-vapor and the solid-vapor interphases with different values of A and B. As h_{evap} is not excactly constant over a wide interval in T it means that the equation cannot be used for the total domain.

14.17

Verify that Clapeyron's equation is satisfied for R-410a at 0^{o}C in Table B.4.

Clapeyron Eq.: $$\frac{dP_{sat}}{dT} = \frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}}$$

B.4: $P = 798.7$ kPa, $h_{fg} = 221.37$ kJ/kg, $v_{fg} = 0.03182$ m^3/kg

Slope around 0^{o}C best approximated by cord from -5^{o}C to $+5^{o}$C

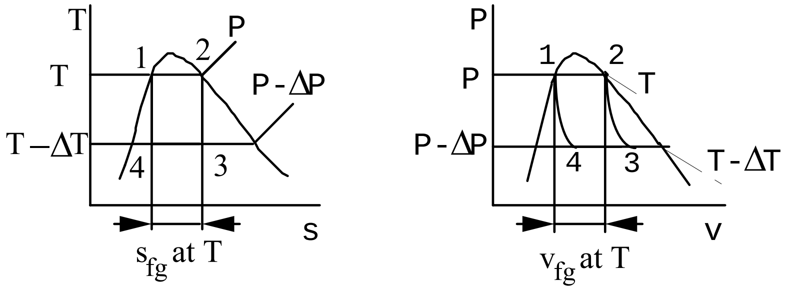
$$\frac{dP_g}{dT} = \frac{933.9 - 678.9}{5 - (-5)} = 25.5 \text{ kPa/K},$$

$$\frac{h_{fg}}{Tv_{fg}} = \frac{221.37}{273.15 \times 0.03182} = 25.47 \text{ kPa/K}$$

This fits very well. Use CATT3 to do from -1 to +1 for better approximation.

14.18

In a Carnot heat engine, the heat addition changes the working fluid from saturated liquid to saturated vapor at *T*, *P*. The heat rejection process occurs at lower temperature and pressure ($T - \Delta T$), ($P - \Delta P$). The cycle takes place in a piston cylinder arrangement where the work is boundary work. Apply both the first and second law with simple approximations for the integral equal to work. Then show that the relation between ΔP and ΔT results in the Clapeyron equation in the limit $\Delta T \rightarrow dT$.



$$q_H = Ts_{fg}; \quad q_L = (T-\Delta T)s_{fg} \; ; \qquad w_{net} = q_H - q_L = \Delta T s_{fg}$$

Problem similar to development in section 13.1 for shaft work, here boundary movement work, $w = \int P dv$

$$w_{NET} = P(v_2-v_1) + \int_2^3 P dv + (P - \Delta P)(v_4 - v_3) + \int_1^4 P dv$$

Approximating,

$$\int_2^3 P dv \approx \left(P - \frac{\Delta P}{2}\right)\left(v_3 - v_2\right); \qquad \int_1^4 P dv \approx \left(P - \frac{\Delta P}{2}\right)\left(v_1 - v_4\right)$$

Collecting terms: $w_{NET} \approx \Delta P\left[\left(\frac{v_2+v_3}{2}\right) - \left(\frac{v_1+v_4}{2}\right)\right]$

(the smaller the ΔP, the better the approximation)

$$\Rightarrow \frac{\Delta P}{\Delta T} \approx \frac{s_{fg}}{\frac{1}{2}(v_2 + v_3) - \frac{1}{2}(v_1 + v_4)}$$

In the limit as $\Delta T \rightarrow 0$: $v_3 \rightarrow v_2 = v_g$, $v_4 \rightarrow v_1 = v_f$

$$\& \lim_{\Delta T \rightarrow 0} \frac{\Delta P}{\Delta T} = \frac{dP_{sat}}{dT} = \frac{s_{fg}}{v_{fg}}$$

14.19

Verify that Clapeyrons equation is satisfied for carbon dioxide at 0°C in Table B.3.

Clapeyron Eq.: $$\frac{dP_{sat}}{dT} = \frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}}$$

B.3: $P = 3485.1$ kPa, $h_{fg} = 230.89$ kJ/kg, $v_{fg} = 0.00916\ m^3/kg$

Slope around 0°C best approximated by cord from -2°C to +2°C

$$\frac{dP_g}{dT} = \frac{3673.3 - 3304.2}{2 - (-2)} = 92.275 \text{ kPa/K},$$

$$\frac{h_{fg}}{Tv_{fg}} = \frac{230.89}{273.15 \times 0.00916} \frac{\text{kJ/kg}}{\text{K m}^3\text{/kg}} = 92.280 \text{ kPa/K}$$

This fits very well.

14.20

Use the approximation given in problem 14.16 and Table B.1 to determine A and B for steam from properties at 25°C only. Use the equation to predict the saturation pressure at 30°C and compare to table value.

$$\ln P_{sat} = A - B/T \qquad \Rightarrow \qquad \frac{dP_{sat}}{dT} = P_{sat}\,(-B)(-T^{-2})$$

so we notice from Eq.14.7 and Table values from B.1.1 and A.5 that

$$B = \frac{h_{fg}}{R} = \frac{2442.3}{0.4615}\,\frac{\text{kJ/kg}}{\text{kJ/kg-K}} = 5292 \text{ K}$$

Now the constant A comes from the saturation pressure as

$$A = \ln P_{sat} + B/T = \ln 3.169 + \frac{5292}{273.15 + 25} = 18.9032$$

Use the equation to predict the saturation pressure at 30°C as

$$\ln P_{sat} = A - B/T = 18.9032 - \frac{5292}{273.15 + 30} = 1.4462$$

$$P_{sat} = 4.2469 \text{ kPa}$$

compare this with the table value of $P_{sat} = 4.246$ kPa and we have a very close approximation.

14.21

A certain refrigerant vapor enters a steady flow constant pressure condenser at 150 kPa, 70°C, at a rate of 1.5 kg/s, and it exits as saturated liquid. Calculate the rate of heat transfer from the condenser. It may be assumed that the vapor is an ideal gas, and also that at saturation, $v_f << v_g$. The following quantities are known for this refrigerant:

$$\ln P_g = 8.15 - 1000/T\,; \qquad C_P = 0.7 \text{ kJ/kg K}$$

with pressure in kPa and temperature in K. The molecular weight is 100.

Refrigerant: State 1 $T_1 = 70^oC$ $P_1 = 150$ kPa

State 2 $P_2 = 150$ kPa $x_2 = 1.0$ State 3 $P_3 = 150$ kPa $x_3 = 0.0$

Get the saturation temperature at the given pressure

$$\ln (150) = 8.15 - 1000/T_2 \;\Rightarrow\; T_2 = 318.5 \text{ K} = 45.3^oC = T_3$$

$${}_1q_3 = h_3 - h_1 = (h_3 - h_2) + (h_2 - h_1) = - h_{fg\,T3} + C_{P0}(T_2 - T_1)$$

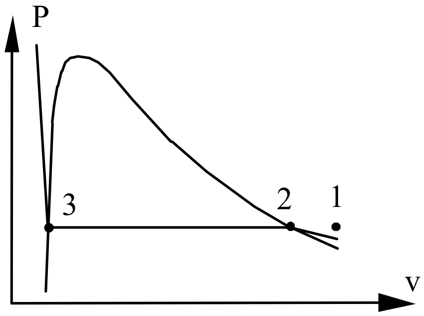
$$\frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}}, \qquad v_{fg} \approx v_g = \frac{RT}{P_g}, \qquad \frac{dP_g}{dT} = P_g \frac{d \ln P_g}{dT} = \frac{h_{fg}}{RT^2} P_g$$

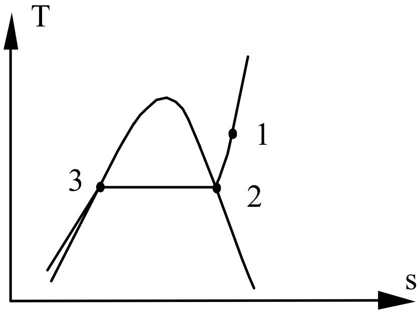
$$\frac{d \ln P_g}{dT} = +1000/T^2 = h_{fg}/RT^2$$

$$h_{fg} = 1000 \times R = 1000 \times 8.3145/100 = 83.15 \text{ kJ/kg}$$

$${}_1q_3 = -83.15 + 0.7(45.3 - 70) = -100.44 \text{ kJ/kg}$$

$$\dot{Q}_{COND} = 1.5(-100.44) = \mathbf{-150.6\ kW}$$





14.22

Calculate the values h_{fg} and s_{fg} for nitrogen at 70 K and at 110 K from the Clapeyron equation, using the necessary pressure and specific volume values from Table B.6.1.

Clapeyron equation Eq.14.7: $\frac{dP_g}{dT} = \frac{h_{fg}}{Tv_{fg}} = \frac{s_{fg}}{v_{fg}}$

For N_2 at 70 K, using values for P_g from Table B.6 at 75 K and 65 K, and also v_{fg} at 70 K,

$$h_{fg} \approx T(v_g\text{-}v_f)\frac{\Delta P_g}{\Delta T} = 70(0.525\ 015)\left(\frac{76.1\text{-}17.41}{75\text{-}65}\right) = 215.7 \text{ kJ/kg} \ \ (207.8)$$

$$s_{fg} = h_{fg}/T = 3.081 \text{ kJ/kg K} \ \ (2.97)$$

Comparison not very close because P_g not linear function of T. Using 71 K & 69 K from the software,

$$h_{fg} = 70(0.525\ 015)\left(\frac{44.56\text{-}33.24}{71\text{-}69}\right) = 208.0 \text{ kJ/kg}$$

At 110 K, $h_{fg} \approx 110(0.014\ 342)\left(\frac{1938.8\text{-}1084.2}{115\text{-}105}\right) = 134.82 \text{ kJ/kg} \ \ (134.17)$

$$s_{fg} = \frac{134.82}{110} = 1.226 \text{ kJ/kg K} \ \ (1.22)$$

14.23

Find the saturation pressure for refrigerant R-410a at –80^{o}C assuming it is higher than the triple point temperature.

The lowest temperature in Table B.4 for R-410a is -60^{o}C, so it must be extended to -80^{o}C using the Clapeyron Eq. 14.7 integrated as in example 14.1

Table B.4: at T_1 = -60^{o}C = 213.15 K, P_1 = 64.1 kPa, R = 0.1145 kJ/kg-K

$$\ln \frac{P}{P_1} = \frac{h_{fg}}{R} \frac{(T - T_1)}{T \times T_1} = \frac{279.96}{0.1145} \frac{(193.15 - 213.15)}{193.15 \times 213.15} = -1.1878$$

P = 64.1 exp(-1.1878) = **19.54 kPa**

14.24

Ammonia at –70°C is used in a special application at a quality of 50%. Assume the only table available is B.2 that goes down to –50°C. To size a tank to hold 0.5 kg with x = 0.5, give your best estimate for the saturated pressure and the tank volume.

To size the tank we need the volume and thus the specific volume. If we do not have the table values for v_f and v_g we must estimate those at the lower T. We therefore use Clapeyron equation to extrapolate from –50°C to –70°C to get the saturation pressure and thus v_g assuming ideal gas for the vapor.

The values for v_f and h_{fg} do not change significantly so we estimate

Between -50°C and –70°C: $h_{fg} = 1430$ kJ/kg

and at –70°C we get: $v_f = 0.001375 \text{ m}^3/\text{kg}$

The integration of Eq.14.7 is the same as in Example 13.1 so we get

$$\ln \frac{P_2}{P_1} = \frac{h_{fg}}{R} \left(\frac{T_2 - T_1}{T_2 T_1}\right) = \frac{1430}{0.4882} \frac{-70 + 50}{203.15 \times 223.15} = -1.2923$$

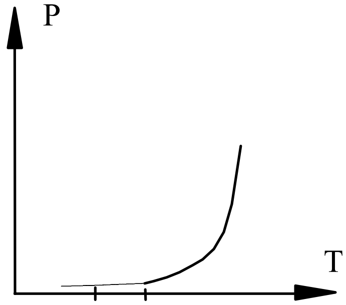
$$P_2 = P_1 \exp(-1.2923) = 40.9 \exp(-1.2923) = 11.2 \text{ kPa}$$

$$v_g = RT_2/P_2 = \frac{0.4882 \times 203.15}{11.2} = 8.855 \text{ m}^3/\text{kg}$$

$$v_2 = (1\text{-}x)\, v_f + x\, v_g = 0.5 \times 0.001375 + 0.5 \times 8.855 = 4.428 \text{ m}^3/\text{kg}$$

$$V_2 = mv_2 = \mathbf{2.214\ m^3}$$

A straight line extrapolation will give a negative pressure.



14.25

Use the approximation given in problem 14.16 and Table B.4 to determine A and B for refrigerant R-410a from properties at 0°C only. Use the equation to predict the saturation pressure at 5°C and compare to table value.

$$\ln P_{sat} = A - B/T \qquad \Rightarrow \qquad \frac{dP_{sat}}{dT} = P_{sat}\,(-B)(-T^{-2})$$

so we notice from Eq.14.7 and Table values from B.4.1 and A.5 that

$$B = \frac{h_{fg}}{R} = \frac{221.37}{0.1145}\ \frac{\text{kJ/kg}}{\text{kJ/kg-K}} = 1933.4 \text{ K}$$

Now the constant A comes from the saturation pressure as

$$A = \ln P_{sat} + B/T = \ln 798.7 + \frac{1933.4}{273.15} = 13.7611$$

Use the equation to predict the saturation pressure at 5°C as

$$\ln P_{sat} = A - B/T = 13.7611 - \frac{1933.4}{273.15 + 5} = 6.8102$$

$$P_{sat} = 907 \text{ kPa}$$

compare this with the table value of $P_{sat} = 933.9$ kPa and we have an approximation 3% low. Notice h_{fg} decreases so we could have used a lower value for the average in the interval.

14.26

The triple point of CO_2 is -56.4°C. Predict the saturation pressure at that point using Table B.3.

The lowest temperature in Table B.3 for CO_2 is -50°C, so it must be extended to -56.4°C = 216.75 K using the Clapeyron Eq. 14.7 integrated as in Ex. 14.1

Table B.3: at T_1 = -50°C = 223.15 K, P_1 = 682.3 kPa, h_{fg} = 339.73 kJ/kg

Table A.5: R = 0.1889 kJ/kg-K

$$\ln \frac{P}{P_1} = \frac{h_{fg}}{R} \frac{(T - T_1)}{T \times T_1} = \frac{339.73}{0.1889} \frac{(216.75 - 223.15)}{216.75 \times 223.15} = -0.23797$$

$$P = 682.3 \exp(-0.23797) = \mathbf{537.8\ kPa}$$

Notice from Table 3.2 P = 520.8 kPa so we are 3% high. As h_{fg} becomes larger for lower T's we could have estimated a more suitable value for the interval from -50 to -56.4°C

14.27

Helium boils at 4.22 K at atmospheric pressure, 101.3 kPa, with h_{fg} = 83.3 kJ/kmol. By pumping a vacuum over liquid helium, the pressure can be lowered, and it may then boil at a lower temperature. Estimate the necessary pressure to produce a boiling temperature of 1 K and one of 0.5 K.

Solution:

Helium at 4.22 K: $P_1 = 0.1013$ MPa, $\bar{h}_{FG} = 83.3$ kJ/kmol

$$\frac{dP_{SAT}}{dT} = \frac{h_{FG}}{Tv_{FG}} \approx \frac{h_{FG}P_{SAT}}{RT^2} \quad \Rightarrow \quad \ln\frac{P_2}{P_1} = \frac{h_{FG}}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

For $T_2 = 1.0$ K:

$$\ln\frac{P_2}{101.3} = \frac{83.3}{8.3145}\left[\frac{1}{4.22} - \frac{1}{1.0}\right] \quad => \quad P_2 = 0.048 \text{ kPa} = \mathbf{48\ Pa}$$

For $T_2 = 0.5$ K:

$$\ln\frac{P_2}{101.3} = \frac{83.3}{8.3145}\left[\frac{1}{4.22} - \frac{1}{0.5}\right]$$

$P_2 = 2.1601\times10^{-6}$ kPa = $\mathbf{2.1601 \times 10^{-3}}$ **Pa**

14.28

Using the properties of water at the triple point, develop an equation for the saturation pressure along the fusion line as a function of temperature.

Solution:

The fusion line is shown in Fig. 3.4 as the S-L interphase. From Eq.14.5 we have

$$\frac{dP_{fusion}}{dT} = \frac{h_{if}}{Tv_{if}}$$

Assume h_{if} and v_{if} are constant over a range of T’s. We do not have any simple models for these as function of T other than curve fitting. Then we can integrate the above equation from the triple point (T_1, P_1) to get the pressure P(T) as

$$P - P_1 = \frac{h_{if}}{v_{if}} \ln \frac{T}{T_1}$$

Now take the properties at the triple point from B.1.1 and B.1.5

$$P_1 = 0.6113 \text{ kPa}, \quad T_1 = 273.16 \text{ K}$$

$$v_{if} = v_f - v_i = 0.001 - 0.0010908 = - 9.08 \times 10^{-5} \text{ m}^3\text{/kg}$$

$$h_{if} = h_f - h_i = 0.0 - (-333.4) = 333.4 \text{ kJ/kg}$$

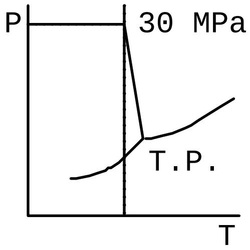
The function that approximates the pressure becomes

$$P = 0.6113 - 3.672 \times 10^6 \ln \frac{T}{T_1} \qquad [\text{kPa}]$$

14.29

Using thermodynamic data for water from Tables B.1.1 and B.1.5, estimate the freezing temperature of liquid water at a pressure of 30 MPa.

$$H_2O \quad \frac{dP_{if}}{dT} = \frac{h_{if}}{Tv_{if}} \approx \text{const}$$



At the triple point,

$$v_{if} = v_f - v_i = 0.001\,000 - 0.001\,090\,8 = -0.000\,090\,8 \text{ m}^3/\text{kg}$$

$$h_{if} = h_f - h_i = 0.01 - (-333.40) = 333.41 \text{ kJ/kg}$$

$$\frac{dP_{if}}{dT} = \frac{333.41}{273.16(-0.000\,090\,8)} = -13\,442 \text{ kPa/K}$$

$\Rightarrow$ at P = 30 MPa,

$$T \approx 0.01 + \frac{(30\,000-0.6)}{(-13\,442)} = \mathbf{-2.2\ ^oC}$$

14.30

Ice (solid water) at –3°C, 100 kPa, is compressed isothermally until it becomes liquid. Find the required pressure.

Water, triple point T = 0.01°C , P = 0.6113 kPa

Table B.1.1: $v_f = 0.001 \text{ m}^3/\text{kg}$, $h_f = 0.01 \text{ kJ/kg}$,

Tabel B.1.5: $v_i = 0.001\ 0908 \text{ m}^3/\text{kg}$, $h_i = -333.4 \text{ kJ/kg}$

Clapeyron $$\frac{dP_{if}}{dT} = \frac{h_f - h_i}{(v_f - v_i)T} = \frac{333.4}{-0.0000908 \times 273.16} = -13\ 442 \text{ kPa/K}$$

$$\Delta P \approx \frac{dP_{if}}{dT} \Delta T = -13\ 442(-3 - 0.01) = 40\ 460 \text{ kPa}$$

$$P = P_{tp} + \Delta P = \mathbf{40\ 461\ kPa}$$

14.31

From the phase diagram for carbon dioxide in Fig. 3.6 and 3.7 for water what can you infer for the specific volume change during melting assuming the liquid has a higher h than the solid phase for those two substances.

The saturated pressure versus temperature has a positive slope for carbon dioxide and a negative slope for water.

$$\text{Clapeyron} \qquad \frac{dP_{if}}{dT} = \frac{h_f - h_i}{(v_f - v_i)T}$$

So if we assume $h_f - h_i > 0$ then we notice that the volume change in the melting gives

Water: $\quad v_f - v_i < 0 \quad$ so $\quad v_f < v_i$

Carbon dioxide: $\quad v_f - v_i > 0 \quad$ so $\quad v_f > v_i$

14.32

A container has a double wall where the wall cavity is filled with carbon dioxide at room temperature and pressure. When the container is filled with a cryogenic liquid at 100 K the carbon dioxide will freeze so that the wall cavity has a mixture of solid and vapor carbon dioxide at the sublimation pressure. Assume that we do not have data for CO_2 at 100 K, but it is known that at –90°C: $P_{sat} = 38.1$ kPa, $h_{IG} = 574.5$ kJ/kg. Estimate the pressure in the wall cavity at 100 K.

Solution:

For CO_2 space: at $T_1 = -90\ ^oC = 183.2$ K , $P_1 = 38.1$ kPa, $h_{IG} = 574.5$ kJ/kg

For $T_2 = T_{CO2} = 100$ K: Clapeyron $\frac{dP_{SUB}}{dT} = \frac{h_{IG}}{Tv_{IG}} \approx \frac{h_{IG}P_{SUB}}{RT^2}$

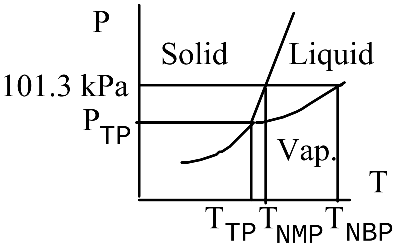
$$\ln\frac{P_2}{P_1} = \frac{h_{IG}}{R}\left[\frac{1}{183.2} - \frac{1}{100}\right] = \frac{574.5}{0.188\ 92}\left[\frac{1}{183.2} - \frac{1}{100}\right] = -13.81$$

or $P_2 = P_1 \times 1.005\times10^{-6} \Rightarrow P_2 = 3.83\times10^{-5}$ kPa = $\mathbf{3.83\times10^{-2}}$ **Pa**

14.33

Small solid particles formed in combustion should be investigated. We would like to know the sublimation pressure as a function of temperature. The only information available is *T*, h_{FG} for boiling at 101.3 kPa and *T*, h_{IF} for melting at 101.3 kPa. Develop a procedure that will allow a determination of the sublimation pressure, $P_{sat}(T)$.

T_{NBP} = normal boiling pt T.
T_{NMP} = normal melting pt T.
T_{TP} = triple point T.



1) $T_{TP} \approx T_{NMP}$

2) $$\int_{0.1013\text{ MPa}}^{P_{TP}} (1/P_{SAT})\, dP_{SAT} \approx \int_{T_{NMP}}^{T_{TP}} \frac{h_{FG}}{RT^2}\, dT$$

Since $h_{FG} \approx \text{const} \approx h_{FG\,NBP}$ the integral over temperature becomes

$$\ln\frac{P_{TP}}{0.1013} \approx \frac{h_{FG\,NBP}}{R}\left[\frac{1}{T_{NBP}} - \frac{1}{T_{TP}}\right] \quad \rightarrow \quad \text{get } P_{TP}$$

3) $h_{IG\text{ at }TP} = h_G - h_I = (h_G - h_F) + (h_F - h_I) \approx h_{FG\,NBP} + h_{IF\,NMP}$

Assume $h_{IG} \approx$ const. again we can evaluate the integral

$$\ln\frac{P_{SUB}}{P_{TP}} = \int_{P_{TP}}^{P_{SUB}} (1/P_{SUB})\, dP_{SUB} \approx \int_{T_{TP}}^{T} \frac{h_{IG}}{RT^2}\, dT \approx \frac{h_{IG}}{R}\left[\frac{1}{T_{TP}} - \frac{1}{T}\right]$$

or $P_{SUB} = \text{fn}(T)$

Property Relations

14.34

Use Gibbs relation du = Tds – Pdv and one of Maxwell's relations to find an expression for $(\partial u/\partial P)_T$ that only has properties P, v and T involved. What is the value of that partial derivative if you have an ideal gas?

du = Tds – Pdv divide this by dP so we get

$$\left(\frac{\partial u}{\partial P}\right)_T = T\left(\frac{\partial s}{\partial P}\right)_T - P\left(\frac{\partial v}{\partial P}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_P - P\left(\frac{\partial v}{\partial P}\right)_T$$

where we have used Maxwell Eq.14.19. Now for an ideal gas we get

Ideal gas: $Pv = RT \Rightarrow v = \frac{RT}{P}$

then the derivatives are

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} \quad \text{and} \quad \left(\frac{\partial v}{\partial P}\right)_T = -RTP^{-2}$$

and the derivative of u is

$$\left(\frac{\partial u}{\partial P}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_P - P\left(\frac{\partial v}{\partial P}\right)_T = -T\frac{R}{P} - P(-RTP^{-2}) = 0$$

This confirms that u is not sensitive to P and only a function of T.

14.35

The Joule-Thomson coefficient μ_J is a measure of the direction and magnitude of the temperature change with pressure in a throttling process. For any three properties x,y,z use the mathematical relation

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

to show the following relations for the Joule-Thomson coefficient:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

Let x = T, y = P and z = h and substitute into the relations as:

$$\left(\frac{\partial T}{\partial P}\right)_h \left(\frac{\partial P}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_P = -1$$

Then we have the definition of specific heat as $C_P = \left(\frac{\partial h}{\partial T}\right)_P$ so solve for the first term

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = -\frac{1}{C_P} / \left(\frac{\partial P}{\partial h}\right)_T = -\frac{1}{C_P}\left(\frac{\partial h}{\partial P}\right)_T$$

The last derivative is substituted with Eq.14.25 so we get

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P}$$

If we use the compressibility factor then we get

$$Pv = ZRT \quad \Rightarrow \quad \left(\frac{\partial v}{\partial T}\right)_P = \frac{ZR}{P} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P = \frac{v}{T} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P$$

so then

$$T\left(\frac{\partial v}{\partial T}\right)_P - v = v + \frac{RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P - v = \frac{RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P$$

and we have shown the last expression also.

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

14.36

Find the Joule-Thomson coefficient for an ideal gas from the expression given in Problem 14.35

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

For an ideal gas: v = RT/P so then the partial derivative

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} \quad \Rightarrow \quad T\left(\frac{\partial v}{\partial T}\right)_P - v = \frac{RT}{P} - v = v - v = 0$$

For an ideal gas Z = 1 so the very last derivative of Z is also zero.

14.37

Start from Gibbs relation dh = Tds + vdP and use one of Maxwell's equation to get $(\partial h/\partial v)_T$ in terms of properties P, v and T. Then use Eq.14.24 to also find an expression for $(\partial h/\partial T)_v$.

$$\text{Find } \left(\frac{\partial h}{\partial v}\right)_T \text{ and } \left(\frac{\partial h}{\partial T}\right)_v$$

dh = Tds + vdP and use Eq.14.18

$$\Rightarrow \quad \left(\frac{\partial h}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T + v\left(\frac{\partial P}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v + v\left(\frac{\partial P}{\partial v}\right)_T$$

Also for the second first derivative use Eq.14.24

$$\left(\frac{\partial h}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v + v\left(\frac{\partial P}{\partial T}\right)_v = C_v + v\left(\frac{\partial P}{\partial T}\right)_v$$

14.38

From Eqs. 14.23 and 14.24 and the knowledge that $C_p > C_v$ what can you conclude about the slopes of constant v and constant P curves in a T-s diagram? Notice that we are looking at functions T(s, P or v given).

Solution:

The functions and their slopes are:

Constant v: T(s) at that v with slope $\left(\frac{\partial T}{\partial s}\right)_v$

Constant P: T(s) at that P with slope $\left(\frac{\partial T}{\partial s}\right)_P$

Slopes of these functions are now evaluated using Eq.14.23 and Eq.14.24 as

$$\left(\frac{\partial T}{\partial s}\right)_P = \left(\left(\frac{\partial s}{\partial T}\right)_P\right)^{-1} = \frac{T}{C_p}$$

$$\left(\frac{\partial T}{\partial s}\right)_v = \left(\left(\frac{\partial s}{\partial T}\right)_v\right)^{-1} = \frac{T}{C_v}$$

Since we know $C_p > C_v$ then it follows that $T/C_v > T/C_p$ and therefore

$$\left(\frac{\partial T}{\partial s}\right)_v > \left(\frac{\partial T}{\partial s}\right)_P$$

which means that constant v-lines are steeper than constant P lines in a T-s diagram.

14.39

Derive expressions for $(\partial T/\partial v)_u$ and for $(\partial h/\partial s)_v$ that do not contain the properties h, u, or s. Use Eq. 14.30 with du = 0.

$$\left(\frac{\partial T}{\partial v}\right)_u = -\left(\frac{\partial u}{\partial v}\right)_T \Big/ \left(\frac{\partial u}{\partial T}\right)_v = \frac{P - T\left(\frac{\partial P}{\partial T}\right)_v}{C_v} \qquad \text{(see Eqs. 14.33 and 14.34)}$$

As dh = Tds + vdP => $\left(\frac{\partial h}{\partial s}\right)_v = T + v\left(\frac{\partial P}{\partial s}\right)_v = T - v\left(\frac{\partial T}{\partial v}\right)_s$ (Eq.14.20)

But $\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial s}{\partial v}\right)_T \Big/ \left(\frac{\partial s}{\partial T}\right)_v = -\frac{T\left(\frac{\partial P}{\partial T}\right)_v}{C_v}$ (Eq.14.22)

$$\Rightarrow \qquad \left(\frac{\partial h}{\partial s}\right)_v = T + \frac{vT}{C_v}\left(\frac{\partial P}{\partial T}\right)_v$$

14.40

Evaluate the isothermal changes in the internal energy, the enthalpy and the entropy for an ideal gas. Confirm the results in Chapters 5 and 8.

We need to evaluate du_T, dh_T and ds_T for an ideal gas: $P = RT/v$.

From Eq.14.31 we get

$$du_T = [\ T\ \left(\frac{\partial P}{\partial T}\right)_v - P\ ]\ dv_T = [\ T\ (\frac{R}{v}) - P\ ]\ dv_T = [\ P - P]\ dv_T = 0$$

From Eq.14.27 we get using $v = RT/P$

$$dh_T = [\ v - T\ \left(\frac{\partial v}{\partial T}\right)_P\ ]\ dP_T = [\ v - T\ (\frac{R}{P})\ ]\ dP_T = [\ v - v\ ]\ dP_T = 0$$

These two equations confirms the statements in chapter 5 that u and h are functions of T only for an ideal gas.

From Eq.14.32 or Eq.14.34 we get

$$ds_T = -\left(\frac{\partial v}{\partial T}\right)_P dP_T = \left(\frac{\partial P}{\partial T}\right)_v dv_T$$

$$= -\frac{R}{P}\, dP_T = \frac{R}{v}\, dv_T$$

so the change in s can be integrated to find

$$s_2 - s_1 = -R \ln \frac{P_2}{P_1} = R \ln \frac{v_2}{v_1} \qquad \text{when } T_2 = T_1$$

14.41

Develop an expression for the variation in temperature with pressure in a constant entropy process, $(\partial T/\partial P)_s$, that only includes the properties P–v–T and the specific heat, C_p. Follow the development for Eq.14.32.

$$\left(\frac{\partial T}{\partial P}\right)_s = -\frac{\left(\frac{\partial s}{\partial P}\right)_T}{\left(\frac{\partial s}{\partial T}\right)_P} = -\frac{-\left(\frac{\partial v}{\partial T}\right)_P}{(C_P/T)} = \frac{T}{C_P}\left(\frac{\partial v}{\partial T}\right)_P$$

$\left\{\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P\right.$, Maxwell relation Eq. 14.23 and the other is Eq.14.27$\left.\right\}$

14.42

Use Eq. 13.34 to get an expression for the derivative $(\partial T/\partial v)_s$. What is the general shape of a constant s process curve in a T-v diagram? For an ideal gas can you say a little more about the shape?

Equation 14.34 says

$$ds = C_v \frac{dT}{T} + \left(\frac{\partial P}{\partial T}\right)_v dv$$

so then in a constant s process we have ds = 0 and we find

$$\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T}{C_v}\left(\frac{\partial P}{\partial T}\right)_v$$

As T is higher the slope is steeper (but negative) unless the last term $(\partial P/\partial T)_v$ counteracts. If we have an ideal gas this last term can be determined

$$P = RT/v \qquad \Rightarrow \qquad \left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v}$$

$$\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T}{C_v}\frac{R}{v} = -\frac{P}{C_v}$$

and we see the slope is steeper for higher P and a little lower for higher T as C_v is an increasing function of T.

14.43

Show that the P-v-T relation as P(v – b) = RT satisfies the mathematical relation in Problem 14.35.

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

Let (x, y, z) be (P, v, T) so we have

$$\left(\frac{\partial P}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_v = -1$$

The first derivative becomes, P = RT/(v – b)

$$\left(\frac{\partial P}{\partial v}\right)_T = -RT\,(v - b)^{-2} = -P/(v - b)$$

The second derivative, v = b + RT/P

$$\left(\frac{\partial v}{\partial T}\right)_P = R/P$$

The third derivative, T = (P/R)(v – b)

$$\left(\frac{\partial T}{\partial P}\right)_v = (v - b)/R$$

Substitute all three derivatives into the relation

$$\left(\frac{\partial P}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_v = -\frac{RT}{(v-b)^2} \times \frac{R}{P} \times \frac{v-b}{R} = -\frac{RT}{(v-b)} \times \frac{1}{P} = -1$$

with the last one recognized as a rewrite of the original EOS.

Volume Expansivity and Compressibility

14.44

What are the volume expansivity α_p, the isothermal compressibility β_T, and the adiabatic compressibility β_s for an ideal gas?

The volume expansivity from Eq.14.37 and ideal gas v = RT/P gives

$$\alpha_p = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{v}\left(\frac{R}{P}\right) = \frac{1}{T}$$

The isothermal compressibility from Eq.14.38 and ideal gas gives

$$\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = -\frac{1}{v}\left(-RT\,P^{-2}\right) = \frac{1}{P}$$

The adiabatic compressibility β_s from Eq.14.40 and ideal gas

$$\beta_s = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s$$

From Eq.14.32 we get for constant s (ds = 0)

$$\left(\frac{\partial T}{\partial P}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P = \frac{T}{C_p}\frac{R}{P} = \frac{v}{C_p}$$

and from Eq.14.34 we get

$$\left(\frac{\partial v}{\partial T}\right)_s = -\frac{C_v}{T}\left(\frac{\partial P}{\partial T}\right)_v = -\frac{C_v}{T}\frac{v}{R} = -\frac{C_v}{P}$$

Finally we can form the desired derivative

$$\left(\frac{\partial v}{\partial P}\right)_s = \left(\frac{\partial v}{\partial T}\right)_s\left(\frac{\partial T}{\partial P}\right)_s = -\frac{C_v}{P}\frac{v}{C_p} = -\frac{v}{kP}$$

$$\beta_s = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s = \left(-\frac{1}{v}\right)\left(-\frac{v}{kP}\right) = \frac{1}{kP} = \frac{1}{k}\,\beta_T$$

14.45

Assume a substance has uniform properties in all directions with $V = L_xL_yL_z$ and show that volume expansivity $\alpha_p = 3\delta_T$. Hint: differentiate with respect to T and divide by V.

$$V = L_xL_yL_z$$

From Eq.14.37

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{L_xL_yL_z}\left(\frac{\partial\, L_xL_yL_z}{\partial T}\right)_P$$

$$= \frac{L_yL_z}{L_xL_yL_z}\left(\frac{\partial\, L_x}{\partial T}\right)_P + \frac{L_xL_z}{L_xL_yL_z}\left(\frac{\partial\, L_y}{\partial T}\right)_P + \frac{L_xL_y}{L_xL_yL_z}\left(\frac{\partial\, L_z}{\partial T}\right)_P$$

$$= \frac{1}{L_x}\left(\frac{\partial\, L_x}{\partial T}\right)_P + \frac{1}{L_y}\left(\frac{\partial\, L_y}{\partial T}\right)_P + \frac{1}{L_z}\left(\frac{\partial\, L_z}{\partial T}\right)_P$$

$$= 3\ \delta_T$$

This of course assumes isotropic properties (the same in all directions).

14.46

Determine the volume expansivity, α_P, and the isothermal compressibility, β_T, for water at 20°C, 5 MPa and at 300°C, and 15 MPa using the steam tables.

Water at 20^{o}C, 5 MPa (compressed liquid)

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{1}{v}\left(\frac{\Delta v}{\Delta T}\right)_P \quad ; \qquad \beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \approx -\frac{1}{v}\left(\frac{\Delta v}{\Delta P}\right)_T$$

Estimate by finite difference using values at 0^{o}C, 20^{o}C and 40^{o}C,

$$\alpha_P \approx \frac{1}{0.000\,9995}\,\frac{0.001\,0056 - 0.000\,9977}{40 - 0} = \mathbf{0.000\,1976\ {}^oC^{-1}}$$

Using values at saturation, 5 MPa and 10 MPa,

$$\beta_T \approx -\frac{1}{0.000\,9995}\,\frac{0.000\,9972 - 0.001\,0022}{10 - 0.0023} = \mathbf{0.000\,50\ MPa^{-1}}$$

Water at 300^{o}C, 15 MPa (compressed liquid)

$$\alpha_P \approx \frac{1}{0.001\,377}\,\frac{0.001\,4724 - 0.001\,3084}{320 - 280} = \mathbf{0.002\,977\ {}^oC^{-1}}$$

$$\beta_T \approx -\frac{1}{0.001\,377}\,\frac{0.001\,3596 - 0.001\,3972}{20 - 10} = \mathbf{0.002\,731\ MPa^{-1}}$$

14.47

Use the CATT3 software to solve the previous problem.

The benefit of the software to solve for the partial derivatives is that we can narrow the interval over which we determine the slope.

Water at 20°C, 5 MPa (compressed liquid)

$$\alpha_P = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{1}{v}\left(\frac{\Delta v}{\Delta T}\right)_P \quad ; \qquad \beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T \approx -\frac{1}{v}\left(\frac{\Delta v}{\Delta P}\right)_T$$

Estimate by finite difference using values at 19°C, 20°C and 21°C,

$$\alpha_P \approx \frac{1}{0.000\,9995}\,\frac{0.000\,9997 - 0.000\,9993}{21 - 19} = \mathbf{0.000\,40^{o}C^{-1}}$$

Using values at saturation, 4.5 MPa and 5.5 MPa,

$$\beta_T \approx -\frac{1}{0.000\,9995}\,\frac{0.000\,9993 - 0.000\,9997}{5.5 - 4.5} = \mathbf{0.000\,40\ MPa^{-1}}$$

Water at 300°C, 15 MPa (compressed liquid)

$$\alpha_P \approx \frac{1}{0.001\,377}\,\frac{0.001\,385 - 0.001\,369}{302 - 298} = \mathbf{0.011\,619\ ^{o}C^{-1}}$$

$$\beta_T \approx -\frac{1}{0.001\,377}\,\frac{0.001\,373 - 0.001\,381}{16 - 14} = \mathbf{0.002\,905\ MPa^{-1}}$$

14.48

A cylinder fitted with a piston contains liquid methanol at 20°C, 100 kPa and volume 10 L. The piston is moved, compressing the methanol to 20 MPa at constant temperature. Calculate the work required for this process. The isothermal compressibility of liquid methanol at 20°C is 1.22×10^{-9} m^2/N.

$$ {}_1w_2 = \int_1^2 P dv = \int P\left(\frac{\partial v}{\partial P}\right)_T dP_T = -\int_1^2 v\beta_T \, P dP_T $$

For v ≈ constant & β_T ≈ constant the integral can be evaluated

$$ {}_1w_2 = -\frac{v\beta_T}{2}\left(P_2^2 - P_1^2\right) $$

For liquid methanol, from Table A.4: $\rho = 787\ m^3/kg$

$V_1 = 10$ L, $m = 0.01 \times 787 = 7.87$ kg

$$ {}_1W_2 = \frac{0.01 \times 1220}{2}\left[(20)^2 - (0.1)^2\right] = 2440 \text{ J} = \mathbf{2.44\ kJ} $$

14.49

For commercial copper at 25°C (see table A.3) the speed of sound is about 4800 m/s. What is the adiabatic compressibility β_s?

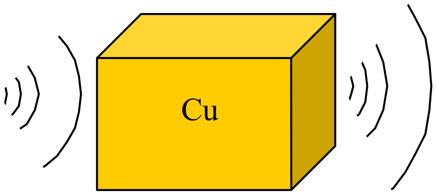
From Eq.14.43 and Eq.14.40

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho}$$

Then we get using density from Table A.3

$$\beta_s = \frac{1}{c^2\rho} = \frac{1}{4800^2 \times 8300}\ \frac{s^2\ m^3}{m^2\ kg} = \frac{1000}{4800^2 \times 8300}\ \frac{1}{kPa}$$

$$\mathbf{= 5.23 \times 10^{-9}\ \ kPa^{-1}}$$



14.50

Use Eq. 13.32 to solve for $(\partial T/\partial P)_s$ in terms of T, v, C_p and α_p. How large a temperature change does 25°C water ($\alpha_p = 2.1 \times 10^{-4}\ K^{-1}$) have, when compressed from 100 kPa to 1000 kPa in an isentropic process?

From Eq.14.32 we get for constant s (ds = 0) and Eq.14.37

$$\left(\frac{\partial T}{\partial P}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P = \frac{T}{C_p}\,\alpha_p\, v$$

Assuming the derivative is constant for the isentropic compression we estimate with heat capacity from Table A.3 and v from B.1.1

$$\Delta T_s = \left(\frac{\partial T}{\partial P}\right)_s \Delta P_s = \frac{T}{C_p}\,\alpha_p\, v\, \Delta P_s$$

$$= \frac{273.15 + 25}{4.18} \times 2.1 \times 10^{-4} \times 0.001003 \times (1000 - 100)$$

$= \mathbf{0.013\ K}$ barely measurable.

14.51

Sound waves propagate through a media as pressure waves that cause the media to go through isentropic compression and expansion processes. The speed of sound c is defined by $c^2 = (\partial P/\partial \rho)_s$ and it can be related to the adiabatic compressibility, which for liquid ethanol at 20°C is 9.4×10^{-10} m^2/N. Find the speed of sound at this temperature.

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho}$$

From Table A.4 for ethanol, $\rho = 783$ kg/m^3

$$\Rightarrow \qquad c = \left(\frac{1}{940\times10^{-12}\times783}\right)^{1/2} = \mathbf{1166\ m/s}$$

14.52

Use Table B.3 to find the speed of sound for carbon dioxide at 2500 kPa near 100^{o}C. Approximate the partial derivative numerically.

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$$

We will use the 2000 kPa and 3000 kPa table entries. We need to find the change in v between two states with the same s at those two pressures.

At 100^{o}C, 2500 kPa: s = (1.6843 + 1.5954)/2 = 1.63985 kJ/kg-K,
v = (0.03359 + 0.02182)/2 = 0.027705 m^3/kg

2000 kPa, s = 1.63985 kJ/kg-K: v = 0.031822 m^3/kg
3000 kPa, s = 1.63985 kJ/kg-K: v = 0.0230556 m^3/kg

$$c^2 \approx -v^2\left(\frac{\Delta P}{\Delta v}\right)_s = -\ 0.027705^2 \frac{3000 - 2000}{0.0230556-0.031822}\frac{\text{kJ}}{\text{kg}} = 87\ 557.8\ \frac{\text{J}}{\text{kg}}$$

$c = \sqrt{87\ 557.8}$ = **295.9 m/s**

14.53

Use the CATT3 software to solve the previous problem.

At $100^{o}C$, 2500 kPa: s = 1.636 kJ/kg-K, v = 0.02653 m^3/kg

$101^{o}C$, s = 1.636 kJ/kg-K: v = 0.02627 m^3/kg, P = 2.531 MPa
$99^{o}C$, s = 1.636 kJ/kg-K: v = 0.02679 m^3/kg, P = 2.469 MPa

$$c^2 \approx -v^2\left(\frac{\Delta P}{\Delta v}\right)_s = -\ 0.02653^2\ \frac{2531 - 2469}{0.02627 - 0.02679}\ \frac{kJ}{kg} = 83\ 919.5\ \frac{J}{kg}$$

$$c = \sqrt{83\ 919.5} = \mathbf{289.7\ m/s}$$

14.54

Consider the speed of sound as defined in Eq. 14.43. Calculate the speed of sound for liquid water at 20°C, 2.5 MPa, and for water vapor at 200°C, 300 kPa, using the steam tables.

From Eq. 14.43: $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$

Liquid water at 20^oC, 2.5 MPa, assume

$$\left(\frac{\partial P}{\partial v}\right)_s \approx \left(\frac{\Delta P}{\Delta v}\right)_T$$

Using saturated liquid at 20^oC and compressed liquid at 20^oC, 5 MPa,

$$c^2 = -\left(\frac{0.001\ 002+0.000\ 9995}{2}\right)^2\left(\frac{5-0.0023}{0.000\ 9995-0.001\ 002}\right)\frac{MJ}{kg}$$

$$= \ 2.002\times10^6\ \frac{J}{kg}$$

$$=> \qquad c \ = \ \mathbf{1415\ m/s}$$

Superheated vapor water at 200^oC, 300 kPa

$$v = 0.7163\ m^3/kg, \quad s = 7.3115\ kJ/kg\ K$$

At P = 200 kPa & $s = 7.3115$ kJ/kg K: $T = 157^oC$, $v = 0.9766\ m^3/kg$

At P = 400 kPa & $s = 7.3115$ kJ/kg K: $T = 233.8^oC$, $v = 0.5754\ m^3/kg$

$$c^2 = -(0.7163)^2\left(\frac{0.400-0.200}{0.5754-0.9766}\right)\frac{MJ}{kg} = 0.2558\times10^6\ m^2/s^2$$

$$=> \qquad c \ = \mathbf{506\ m/s}$$

14.55

Use the CATT3 software to solve the previous problem.

From Eq. 14.43: $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$

Liquid water at 20°C, 2.5 MPa, assume $\left(\frac{\partial P}{\partial v}\right)_s \approx \left(\frac{\Delta P}{\Delta v}\right)_s$

and CATT3: $v = 0.001001$ m^3/kg, $s = 0.2961$ kJ/kg-K

Using liquid at 3 MPa and 2 MPa at the same $s = 0.2961$ kJ/kg-K,

$$c^2 = -\ 0.001001^2 \left(\frac{3 - 2}{0.001 - 0.001\ 001}\right) \frac{\text{MJ}}{\text{kg}}$$

$$= 1.002\times 10^6 \frac{\text{J}}{\text{kg}}$$

=> $c =$ **1001 m/s**

Superheated vapor water at 200°C, 300 kPa

CATT3: $v = 0.7163$ m^3/kg, $s = 7.311$ kJ/kg K

At $P = 290$ kPa & $s = 7.311$ kJ/kg K: $T = 196.2$°C, $v = 0.7351$ m^3/kg

At $P = 310$ kPa & $s = 7.311$ kJ/kg K: $T = 203.7$°C, $v = 0.6986$ m^3/kg

$$c^2 = -(0.7163)^2 \left(\frac{0.310 - 0.290}{0.6986 - 0.7351}\right) \frac{\text{MJ}}{\text{kg}} = 0.28114 \times 10^6 \text{ m}^2/\text{s}^2$$

=> $c =$ **530 m/s**

14.56

Soft rubber is used as a part of a motor mounting. Its adiabatic bulk modulus is $B_s = 2.82 \times 10^6$ kPa, and the volume expansivity is $\alpha_p = 4.86 \times 10^{-4}\ \text{K}^{-1}$. What is the speed of sound vibrations through the rubber, and what is the relative volume change for a pressure change of 1 MPa?

From Eq.14.43 and Eq.14.40

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \rho} = \frac{1}{\beta_s \rho} = \frac{B_s}{\rho}$$

$$= 2.82 \times 10^6 \times 1000 \text{ Pa} / 1100 \text{ kg/m}^3 = 2.564 \times 10^6 \text{ m}^2/\text{s}^2$$

$$\mathbf{c = 1601\ m/s}$$

If the volume change is fast it is isentropic and if it is slow it is isothermal. We will assume it is isentropic

$$\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_s = -\beta_s = -\frac{1}{B_s}$$

then

$$\frac{\Delta V}{V} = -\frac{\Delta P}{B_s} = -\frac{1000}{2.82 \times 10^6} = \mathbf{-3.55 \times 10^{-4}}$$

14.57

Liquid methanol at 25°C has an adiabatic compressibility of 1.05×10^{-9} m²/N. What is the speed of sound? If it is compressed from 100 kPa to 10 MPa in an insulated piston/cylinder, what is the specific work?

From Eq.14.43 and Eq.14.40 and the density from table A.4

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = \frac{1}{\beta_s \rho} = \frac{1}{1.05 \times 10^{-9} \times 787}$$

$$= 1.210 \times 10^6 \ \mathrm{m^2/s^2}$$

$$c = \mathbf{1100\ m/s}$$

The specific work becomes

$$w = \int P\, dv = \int P\,(-\beta_s v)\, dP = -\int \beta_s v\, P\, dP = -\beta_s\, v \int_1^2 P\, dP$$

$$= -\beta_s\, v\, 0.5\, (P_2^2 - P_1^2)$$

$$= -1.05 \times 10^{-9}\ \mathrm{m^2/N} \times \frac{0.5}{787}\ \mathrm{m^3/kg} \times (10\,000^2 - 100^2) \times 1000^2\ \mathrm{Pa^2}$$

$$= \mathbf{-66.7\ J/kg}$$

14.58

Use Eq. 14.32 to solve for $(\partial T/\partial P)_s$ in terms of T, v, C_p and α_p. How much higher does the temperature become for the compression of the methanol in Problem 14.57? Use $\alpha_p = 2.4 \times 10^{-4}\ \mathrm{K}^{-1}$ for methanol at 25°C.

From Eq.14.32 we get for constant s (ds = 0) and Eq.14.37

$$\left(\frac{\partial T}{\partial P}\right)_s = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P = \frac{T}{C_p}\alpha_p v$$

Assuming the derivative is constant for the isentropic compression we estimate with heat capacity and density (v = 1/ρ) from Table A.4

$$\Delta T_s = \left(\frac{\partial T}{\partial P}\right)_s \Delta P_s = \frac{T}{C_p}\alpha_p v\, \Delta P_s$$

$$= \frac{298.15}{2.55}\frac{\mathrm{K\ kg\ K}}{\mathrm{kJ}} \times 2.4 \times 10^{-4}\ \mathrm{K}^{-1} \times \frac{1}{787}\frac{\mathrm{m}^3}{\mathrm{kg}} \times (10\,000 - 100)\ \mathrm{kPa}$$

= **0.353 K**

14.59

Find the speed of sound for air at 20°C, 100 kPa using the definition in Eq. 13.43 and relations for polytropic processes in ideal gases.

From problem 13.14 : $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$

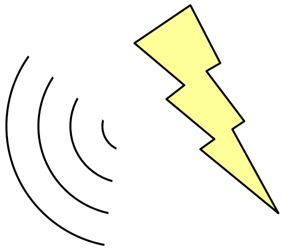
For ideal gas and isentropic process, Pv^k = constant

$P = Cv^{-k} \;\Rightarrow\; \frac{\partial P}{\partial v} = -kCv^{-k-1} = -kPv^{-1}$

$c^2 = -v^2(-kPv^{-1}) = kPv = kRT$

$c = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 293.15 \times 1000} = \mathbf{343.2\ m/s}$

For every 3 seconds after the lightning the sound travels about 1 km.



Equations of State

14.60

Use Table B3 and find the compressibility of carbon dioxide at the critical point.

$$Pv = Z\,RT$$

At the critical point from B.3:

$P = 7377.3$ kPa, $T = 31°C = 304.15$ K, $v = 0.002139\ m^3/kg$

from A.5: $R = 0.1889$ kJ/kg-K

$$Z = \frac{Pv}{RT} = \frac{7377.3 \times 0.002139}{0.1889 \times 304.15} = \mathbf{0.27}$$

14.61

Use the equation of state as shown in Example 14.3 where changes in enthalpy and entropy were found. Find the isothermal change in internal energy in a similar fashion; do not compute it from enthalpy.

The equation of state is

$$\frac{Pv}{RT} = 1 - C' \frac{P}{T^4}$$

and to integrate for changes in u from eq.14.31 we make it explicit in P as

$$P = T^4 \left(\frac{v}{R} T^3 + C'\right)^{-1}$$

Now perform the partial derivative of P

$$\left(\frac{\partial P}{\partial T}\right)_v = 4\, T^3 \left(\frac{v}{R} T^3 + C'\right)^{-1} - T^4 \left(\frac{v}{R} T^3 + C'\right)^{-2} 3 \frac{v}{R} T^2$$

$$= 4 \frac{P}{T} - \frac{P^2}{T^4} 3 \frac{v}{R} T^2 = 4 \frac{P}{T} - 3 \frac{P}{T} \times \frac{Pv}{RT} = \frac{P}{T} \left[4 - 3 \frac{Pv}{RT} \right]$$

Substitute into Eq.14.31

$$du_T = \left[T \left(\frac{\partial P}{\partial T}\right)_v - P \right] dv_T = \left[P\left(4 - 3 \frac{Pv}{RT}\right) - P \right] dv_T$$

$$= 3\, P \left(1 - \frac{Pv}{RT}\right) dv_T = 3\, P\, C' \frac{P}{T^4}\, dv_T$$

The P must be eliminated in terms of v or the opposite, we do the latter as from the equation of state

$$v = \frac{RT}{P} - C'\, R \frac{1}{T^3} \qquad \Rightarrow \qquad dv_T = -\frac{RT}{P^2}\, dP_T$$

so now

$$du_T = 3\, C' \frac{P^2}{T^4}\, dv_T = -\, 3\, C'\, R \frac{1}{T^3}\, dP_T$$

and the integration becomes

$$u_2 - u_1 = -\, 3\, C'\, R\, T^{-3} (P_2 - P_1)$$

14.62

Use Table B.4 to find the compressibility of R-410a at 60°C and a) saturated liquid b) saturated vapor and c) 3000 kPa.

Table A.2: $R = 8.31451 / 72.585 = 0.1145$ kJ/kg-K

a) Table B.4.1: $P = 3836.9$ kPa, $v = 0.001227\ \mathrm{m^3/kg}$

$$Z = \frac{Pv}{RT} = \frac{3836.9 \times 0.001227}{0.1145 \times 333.15} = \mathbf{0.1234}$$

b) Table B.4.1: $P = 3836.9$ kPa, $v = 0.00497\ \mathrm{m^3/kg}$

$$Z = \frac{Pv}{RT} = \frac{3836.9 \times 0.00497}{0.1145 \times 333.15} = \mathbf{0.5}$$

c) Table B.4.2: $P = 3000$ kPa, $v = 0.00858\ \mathrm{m^3/kg}$

$$Z = \frac{Pv}{RT} = \frac{3000 \times 0.00858}{0.1145 \times 333.15} = \mathbf{0.675}$$

The R-410a is not an ideal gas at any of these states.

14.63

Use a truncated virial EOS that includes the term with B for carbon dioxide at 20^o C, 1 MPa for which B = –0.128 m^3/kmol and T(dB/dT) = 0.266 m^3/kmol. Find the difference between the ideal-gas value and the real-gas value of the internal energy.

virial eq.: $$P = \frac{RT}{v} + \frac{BRT}{v^2} \;; \qquad \left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v} + \frac{BR}{v^2} + \frac{RT}{v^2}\left(\frac{dB}{dT}\right)$$

$$u\text{-}u^* = -\int_\infty^v \left[\left(\frac{\partial P}{\partial T}\right)_v - P \right] dv = -\int_\infty^v \left[\frac{RT^2}{v^2}\left(\frac{dB}{dT}\right)\right] dv = -\frac{RT}{v}\left[T\left(\frac{dB}{dT}\right)\right]$$

Solution of virial equation (quadratic formula):

$$\bar{v} = \frac{1}{2}\frac{\bar{R}T}{P}\left[1 + \sqrt{1 + 4BP/\bar{R}T}\,\right] \qquad \text{where: } \frac{\bar{R}T}{P} = \frac{8.3145\times 293.15}{1000} = 2.43737$$

$$\bar{v} = \frac{1}{2} \times 2.43737\; \left[1 + \sqrt{1 + 4(-0.128)/2.43737}\,\right] = 2.3018 \text{ m}^3\text{/kmol}$$

Using the minus-sign root of the quadratic formula results in a compressibility factor < 0.5, which is not consistent with such a truncated equation of state.

$$u - u^* = \frac{-8.3145 \times 293.15}{2.3018}\left[0.266\right]/\,44.01 = \mathbf{-\,6.4\ kJ/kg}$$

14.64

Solve the previous Problem with Table B.3 values and find the compressibility of the carbon dioxide at that state.

B.3: $v = 0.05236\ \text{m}^3/\text{kg}$, $u = 327.27$ kJ/kg, A5: $R = 0.1889$ kJ/kg-K

$$Z = \frac{Pv}{RT} = \frac{1000 \times 0.05236}{0.1889 \times 293.15} = 0.9455 \qquad \text{close to ideal gas}$$

To get u^* let us look at the lowest pressure 400 kPa, 20^oC: $v = 0.13551\ \text{m}^3/\text{kg}$ and $u = 331.57$ kJ/kg.

$$Z = Pv/RT = 400 \times 0.13551/(0.1889 \times 293.15) = 0.97883$$

It is not very close to ideal gas but this is the lowest P in the printed table.

$$u - u^* = 327.27 - 331.57 = \mathbf{-4.3\ kJ/kg}$$

14.65

A gas is represented by the virial EOS with the first two terms, B and C. Find an expression for the work in an isothermal expansion process in a piston-cylinder.

Virial EOS: $$P = \frac{RT}{v} + \frac{B(T)\,RT}{v^2} + \frac{C(T)\,RT}{v^3} + \dots$$

$$= RT[\, v^{-1} + B(T)\, v^{-2} + C(T)\, v^{-3} + ..\ ]$$

The work is $$w = \int P\, dv = RT \int [\, v^{-1} + B(T)\, v^{-2} + C(T)\, v^{-3} + ..\ ]\, dv$$

With just the first two terms we get

$$w = RT\, [\, \ln\frac{v_2}{v_1} - B(T)\, (v_2^{-1} - v_1^{-1}) - \frac{1}{2}\, C(T)\, (v_2^{-2} - v_1^{-2})\,]$$

14.66

Extend problem 14.63 to find the difference between the ideal-gas value and the real-gas value of the entropy and compare to table B.3.

Calculate the difference in entropy of the ideal-gas value and the real-gas value for carbon dioxide at the state 20°C, 1 MPa, as determined using the virial equation of state. Use numerical values given in Problem 14.63.

CO_2 at $T = 20^oC$, $P = 1$ MPa

$$s^*_{P*} - s_P = \int_{v(P)}^{RT/P*} \left(\frac{\partial P}{\partial T}\right)_v dv \;; \qquad \text{ID Gas:} \quad s^*_{P*} - s_P = \int_{v(P)}^{RT/P*} \frac{R}{v} dv = R \ln \frac{P}{P^*}$$

Therefore, at P: $s^*_P - s_P = -R \ln \frac{P}{P^*} + \int_{v(P)}^{RT/P*} \left(\frac{\partial P}{\partial T}\right)_v dv$

virial: $P = \frac{RT}{v} + \frac{BRT}{v^2}$ and $\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v} + \frac{BR}{v^2} + \frac{RT}{v^2}\left(\frac{dB}{dT}\right)$

Integrating,

$$s^*_P - s_P = -R \ln \frac{P}{P^*} + R \ln \frac{RT}{P^* v} + R\left[B + T\left(\frac{dB}{dT}\right)\right]\left(\frac{1}{v} - \frac{P^*}{RT}\right)$$

$$= R\left[\ln \frac{RT}{Pv} + \left(B + T\left(\frac{dB}{dT}\right)\right)\frac{1}{v}\right]$$

Using values for CO_2 from solution 14.63 and $R = 0.1889$ kJ/kgK

$$s^*_P - s_P = 0.1889\left[\ln \frac{2.437\ 37}{2.3018} + \left(-0.128 + 0.266\right)\frac{1}{2.3018}\right]$$

$= \mathbf{0.02214\ kJ/kg\ K}$

From Table B.3 take the ideal as the lowest P = 400 kPa:

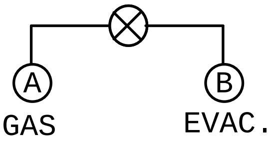
$s^*_P - s_P = 1.7904 - 1.6025 + 0.1889 \ln(400/1000) =$ **0.0148 kJ/kgK**

The lowest P = 400 kPa in B3 is not exactly ideal gas ($Z = Pv/RT = 0.9788$)

14.67

Two uninsulated tanks of equal volume are connected by a valve. One tank contains a gas at a moderate pressure P_1, and the other tank is evacuated. The valve is opened and remains open for a long time. Is the final pressure P_2 greater than, equal to, or less than $P_1/2$? Hint: Recall Fig. 14.5.

Assume the temperature stays constant then for an ideal gas the pressure will be reduced to half the original pressure. For the real gas the compressibility factor maybe different from 1 and then changes towards one as the pressure drops.

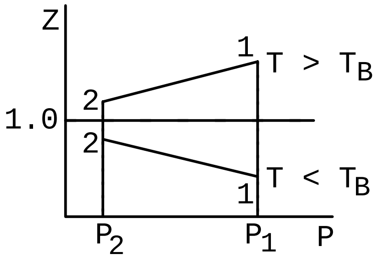


$$V_A = V_B \;\Rightarrow\; V_2 = 2V_1, \quad T_2 = T_1 = T$$

$$\frac{P_2}{P_1} = \frac{V_1}{V_2}\frac{Z_2}{Z_1}\frac{mRT}{mRT} = \frac{1}{2}\frac{Z_2}{Z_1}$$

$$\text{If } T < T_B, \quad Z_2 > Z_1 \;\Rightarrow\; \frac{P_2}{P_1} > \frac{1}{2}$$

$$\text{If } T > T_B, \quad Z_2 < Z_1 \;\Rightarrow\; \frac{P_2}{P_1} < \frac{1}{2}$$



14.68

Show how to get the constants in Eq.14.52 for van der Waals EOS.

van der Waals EOS: $P = \frac{RT}{v - b} - \frac{a}{v^2}$

The conditions at the critical point relate to these derivatives

$$\left(\frac{\partial P}{\partial v}\right)_T = -\frac{RT}{(v - b)^2} + \frac{2a}{v^3} \quad ; \quad \left(\frac{\partial^2 P}{\partial v^2}\right)_T = \frac{2RT}{(v - b)^3} - \frac{6a}{v^4}$$

Set both derivatives to zero at the critical point

$$-\frac{RT_c}{(v_c - b)^2} + \frac{2a}{v_c^3} = 0 \quad (1) \; ; \qquad \frac{2RT_c}{(v_c - b)^3} - \frac{6a}{v_c^4} = 0 \qquad (2)$$

we also have from the EOS

$$P_c = \frac{RT_c}{v_c - b} - \frac{a}{v_c^2} \qquad (3)$$

Now we need to solve theses three equations for v_c, a and b. Solve the first equation for a and substitute into the second equation to give

$$\frac{2a}{v_c^3} = \frac{RT_c}{(v_c - b)^2} \;\Rightarrow\; \frac{6a}{v_c^4} = \frac{3RT_c}{v_c\,(v_c - b)^2} \qquad \text{substitute into Eq.(2)}$$

$$\frac{3RT_c}{v_c\,(v_c - b)^2} = \frac{2RT_c}{(v_c - b)^3} \qquad \text{now solve to get} \qquad v_c = 3b$$

Substitute back into the first equation to get

$$2\,a = \frac{RT_c}{(v_c - b)^2}\,v_c^3 = RT_c\,\frac{27}{4}\,b$$

Now finally substitute a and v_c into the EOS Eq.(3) to get b.

$$P_c = \frac{RT_c}{v_c - b} - \frac{a}{v_c^2} = \frac{RT_c}{2b} - \frac{RT_c\,\frac{27}{8}\,b}{9\,b^2} \;=\; RT_c\,(0.5 - 3/8)\,/b$$

The result is as in Eq.14.52.

14.69

Show that the van der Waals equation can be written as a cubic equation in the compressibility factor involving the reduced pressure and reduced temperature as

$$Z^3 - \left(\frac{P_r}{8T_r} + 1\right) Z^2 + \left(\frac{27\,P_r}{64\,T_r^2}\right) Z - \frac{27\,P_r^2}{512\,T_r^3} = 0$$

van der Waals equation, Eq.14.55: $P = \frac{RT}{v-b} - \frac{a}{v^2}$

$$a = \frac{27}{64}\frac{R^2T_C^2}{P_C} \qquad\qquad b = \frac{RT_C}{8P_C}$$

multiply equation by $\frac{v^2(v-b)}{P}$

Get: $v^3 - (b + \frac{RT}{P})\, v^2 + (\frac{a}{P})\, v - \frac{ab}{P} = 0$

Multiply by $\frac{P^3}{R^3\,T^3}$ and substitute $Z = \frac{Pv}{RT}$

Get: $Z^3 - (\frac{bP}{RT} + 1)\, Z^2 + (\frac{aP}{R^2T^2})\, Z - (\frac{abP^2}{R^3\,T^3}) = 0$

Substitute for a and b, get:

$$Z^3 - \left(\frac{P_r}{8T_r} + 1\right) Z^2 + \left(\frac{27\,P_r}{64\,T_r^2}\right) Z - \frac{27\,P_r^2}{512\,T_r^3} = 0$$

Where $P_r = \frac{P}{P_c}$, $T_r = \frac{T}{T_c}$

14.70

Evaluate changes in an isothermal process for u, h and s for a gas with an equation of state as P (v – b) = RT.

From Eq.14.31 we get

$$du_T = [\ T\ \left(\frac{\partial P}{\partial T}\right)_v - P\ ]\ dv_T = [\ T\ (\frac{R}{v-b}) - P\ ]\ dv_T = [\ P - P]\ dv_T = 0$$

From Eq.14.27 we get using v = b + RT/P

$$dh_T = [\ v - T\ \left(\frac{\partial v}{\partial T}\right)_P\ ]\ dP_T = [\ v - T\ (\frac{R}{P})\ ]\ dP_T = b\ dP_T$$

From eq.14.32 or Eq.14.34 we get

$$ds_T = -\left(\frac{\partial v}{\partial T}\right)_P dP_T = \left(\frac{\partial P}{\partial T}\right)_v dv_T$$

$$= -\frac{R}{P}\,dP_T = \frac{R}{v-b}\,dv_T$$

Now the changes in u, h and s can be integrated to find

$$u_2 - u_1 = 0$$

$$h_2 - h_1 = \int b\ dP = b(P_2 - P_1)$$

$$s_2 - s_1 = -R \ln\frac{P_2}{P_1} = R \ln\frac{v_2 - b}{v_1 - b}$$

14.71

Develop expressions for isothermal changes in internal energy, enthalpy and entropy for a gas obeying the van der Waals equation of state.

van der Waals equation of state: $P = \frac{RT}{v-b} - \frac{a}{v^2}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b}$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v - P = \frac{RT}{v-b} - \frac{RT}{v-b} + \frac{a}{v^2}$$

$$(u_2-u_1)_T = \int_1^2 [T\left(\frac{\partial P}{\partial T}\right)_v - P]dv = \int_1^2 \frac{a}{v^2}dv = a(\frac{1}{v_1} - \frac{1}{v_2})$$

$$(h_2-h_1)_T = (u_2-u_1)_T + P_2v_2 - P_1v_1 = P_2v_2 - P_1v_1 + a(\frac{1}{v_1} - \frac{1}{v_2})$$

$$(s_2-s_1)_T = \int_1^2 \left(\frac{\partial P}{\partial T}\right)_v dv = \int_1^2 \frac{R}{v-b}dv = R\ln\left(\frac{v_2-b}{v_1-b}\right)$$

14.72

Consider the following equation of state, expressed in terms of reduced pressure and temperature: $Z = 1 + (P_r/14T_r)[1 - 6T_r^{-2}]$. What does this predict for the reduced Boyle temperature?

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\,T_r}\left(1 - \frac{6}{T_r^2}\right)$$

$$\left(\frac{\partial Z}{\partial P}\right)_T = \frac{1}{14P_cT_r}\left(1 - \frac{6}{T_r^2}\right) \qquad => \qquad \underset{P\to 0}{\text{Lim}}\left(\frac{\partial Z}{\partial P}\right)_T = 0 \text{ at } T_{boyle}$$

$$\left(1 - \frac{6}{T_r^2}\right) = 0 \quad \rightarrow \quad T_r = \sqrt{6} = \mathbf{2.45}$$

14.73

Use the result of Problem 14.35 to find the reduced temperature at which the Joule-Thomson coefficient is zero, for a gas that follows the EOS given in Problem 14.72

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\ T_r}\left(1 - \frac{6}{T_r^2}\right)$$

From Problem 14.35

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P}\left(\frac{\partial Z}{\partial T}\right)_P$$

$$\left(\frac{\partial Z}{\partial T}\right)_P = -\frac{P_r}{14T_c}\, T_r^{-2}\left(1 - 6\ T_r^{-2}\right) + \frac{P_r}{14\ T_r}\left(12\ T_r^{-3}\ /T_c\right)$$

$$= \frac{P_r}{14T_c}\, T_r^{-2}\ (18\ T_r^{-2} - 1)$$

So this is zero for $18\ T_r^{-2} = 1$ or

$$T_r = \sqrt{18}$$

14.74

What is the Boyle temperature for the following equation of state: $P = \frac{RT}{v\text{-}b} - \frac{a}{v^2T}$ where a and b are constants.

$$P = \frac{RT}{v\text{-}b} - \frac{a}{v^2T}$$

Multiplying by $\frac{v\text{-}b}{P}$ gives: $v - b = \frac{RT}{P} - \frac{a(1\text{-}b/v)}{PvT}$

Using solution from **13.56** for T_{Boyle}:

$$\lim_{P\to 0}\left(v - \frac{RT}{P}\right) = b - \frac{a(1\text{-}0)}{RT\times T} = b - \frac{a}{RT^2} = 0 \text{ at } T_{Boyle}$$

or $$T_{Boyle} = \sqrt{\frac{a}{Rb}} = \sqrt{\frac{27}{64}\frac{R^2T_C^3}{P_C}\frac{1}{R}\frac{8P_C}{RT_C}} = \sqrt{\frac{\mathbf{27}}{\mathbf{8}}}\,\mathbf{T_C}$$

14.75

Determine the reduced Boyle temperature as predicted by an equation of state (the experimentally observed value for most substances is about 2.5), using the van der Waals equation and the Redlich–Kwong equation. Note: It is helpful to use Eqs. 14.47 and 14.48 in addition to Eq. 14.46

The Boyle temp. is that T at which $\lim\limits_{P\to 0}\left(\frac{\partial Z}{\partial P}\right)_T = 0$

$$\text{But } \lim_{P\to 0}\left(\frac{\partial Z}{\partial P}\right)_T = \lim_{P\to 0}\frac{Z-1}{P-0} = \frac{1}{RT}\lim_{P\to 0}\left(v - \frac{RT}{P}\right)$$

van der Waals: $P = \frac{RT}{v-b} - \frac{a}{v^2}$

multiply by $\frac{v-b}{P}$, get

$$v-b = \frac{RT}{P} - \frac{a(v-b)}{Pv^2} \quad \text{or} \quad v - \frac{RT}{P} = b - \frac{a(1-b/v)}{Pv}$$

$$\& \quad RT \times \lim_{P\to 0}\left(\frac{\partial Z}{\partial P}\right)_T = b - \frac{a(1-0)}{RT} = 0 \quad \text{only at } T_{Boyle}$$

$$\text{or} \quad T_{Boyle} = \frac{a}{Rb} = \frac{27}{8}T_C = \mathbf{3.375\ T_C}$$

Redlich-Kwong: $P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}}$

as in the first part, get

$$v - \frac{RT}{P} = b - \frac{a(1-b/v)}{Pv(1+b/v)T^{1/2}}$$

$$\& \ RT \times \lim_{P\to 0}\left(\frac{\partial Z}{\partial P}\right)_T = b - \frac{a(1-0)}{Pv(1+0)T^{1/2}} = 0 \quad \text{only at } T_{Boyle}$$

$$\text{or} \quad T^{3/2}_{Boyle} = \frac{a}{Rb} = \frac{0.427\ 48\ R^2\ T_C^{5/2}}{RP_C} \times \frac{P_C}{0.08\ 664\ R\ T_C}$$

$$T_{Boyle} = \left(\frac{0.427\ 48}{0.086\ 64}\right)^{2/3} T_C = \mathbf{2.9\ T_C}$$

14.76

One early attempt to improve on the van der Waals equation of state was an expression of the form

$$P = \frac{RT}{v-b} - \frac{a}{v^2T}$$

Solve for the constants *a*, *b*, and v_C using the same procedure as for the van der Waals equation.

From the equation of state take the first two derivatives of P with v:

$$\left(\frac{\partial P}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3T} \quad \text{and} \quad \left(\frac{\partial^2 P}{\partial v^2}\right)_T = -\frac{2RT}{(v-b)^3} - \frac{6a}{v^4T}$$

Since both these derivatives are zero at the critical point:

$$-\frac{RT}{(v-b)^2} + \frac{2a}{v^3T} = 0 \qquad \text{and} \qquad -\frac{2RT}{(v-b)^3} - \frac{6a}{v^4T} = 0$$

Also, $$P_C = \frac{RT_C}{v_C-b} - \frac{a}{v_C^2\,T_C}$$

solving these three equations:

$$v_C = 3b, \quad a = \frac{27}{64}\frac{R^2T_C^3}{P_C}, \qquad b = \frac{RT_C}{8P_C}$$

14.77

Develop expressions for isothermal changes in internal energy, enthalpy and entropy for a gas obeying Redlich-Kwong equation of state.

Redlich-Kwong equation of state: $P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b} + \frac{a}{2v(v+b)T^{3/2}}$$

From Eq.14.31

$$(u_2 - u_1)_T = \int_1^2 \frac{3a}{2v(v+b)T^{1/2}}\,dv = \frac{-3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2+b}{v_2}\right)\left(\frac{v_1}{v_1+b}\right)\right]$$

We find change in h from change in u, so we do not do the derivative in Eq.14.27. This is due to the form of the EOS.

$$(h_2 - h_1)_T = P_2v_2 - P_1v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2+b}{v_2}\right)\left(\frac{v_1}{v_1+b}\right)\right]$$

Entropy follows from Eq.14.35

$$(s_2 - s_1)_T = \int_1^2 \left[\frac{R}{v-b} + \frac{a/2}{v(v+b)T^{3/2}}\right] dv$$

$$= R \ln\left(\frac{v_2-b}{v_1-b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_2+b}{v_2}\right)\left(\frac{v_1}{v_1+b}\right)\right]$$

14.78

Determine the second virial coefficient B(T) using the van der Waals equation of state. Also find its value at the critical temperature where the experimentally observed value is about $-0.34\ RT_c/P_c$.

From Eq.14.48: $B(T) = - \lim_{P\to 0} \alpha$ where Eq. 14.43: $\alpha = \frac{RT}{P} - v$

From Eq. 14.51:

van der Waals: $P = \frac{RT}{v-b} - \frac{a}{v^2}$ which we can multiply by $\frac{v-b}{P}$, get

$$v - b = \frac{RT}{P} - \frac{a(v-b)}{Pv^2} \quad \text{or} \quad v - \frac{RT}{P} = b - \frac{a(1-b/v)}{Pv}$$

Taking the limit for P -> 0 then (Pv -> RT and v -> ∞) we get :

$$B(T) = b - a/RT = \frac{RT_C}{P_C}\left(\frac{1}{8} - \frac{27\ T_C}{64\ T}\right)$$

where a,b are from Eq.14.52. At $T = T_C$ then we have

$$B(T_C) = \frac{RT_C}{P_C}\left(-\frac{19}{64}\right) = -0.297\,\frac{RT_C}{P_C}$$

14.79

Determine the second virial coefficient B(T) using the Redlich-Kwong equation of state. Also find its value at the critical temperature where the experimentally observed value is about –0.34 RT_c/P_c.

From Eq.14.48: $B(T) = - \lim_{P\to 0} \alpha$ where Eq.14.44: $\alpha = \frac{RT}{P} - v$

For Redlich Kwong the result becomes

$$v - \frac{RT}{P} = b - \frac{a(1- b/v)}{Pv(1 + b/v)\ T^{1/2}}$$

Taking the limit for P -> 0 then (Pv -> RT and v -> ∞) we get :

$$=> \quad B(T) = b - \frac{a}{RT^{3/2}}$$

Now substitute Eqs. 14.54 and 14.55 for a and b,

$$B(T) = \frac{RT_C}{P_C} \left[0.08664 - 0.42748 \left(\frac{T_C}{T}\right)^{3/2}\right]$$

and evaluated at T_C it becomes

$$B(T_C) = \frac{RT_C}{P_C} \left[0.08664 - 0.42748\right] = -0.341 \frac{RT_C}{P_C}$$

14.80

Oxygen in a rigid tank with 1 kg is at 160 K, 4 MPa. Find the volume of the tank by iterations using the Redlich-Kwong EOS. Compare the result with the ideal gas law.

For the ideal gas law: $Pv = RT$ so $v = RT/P$

$v = 0.2598 \times 160 / 4000 = 0.0104\ \text{m}^3\text{/kg}$; $V = mv = \mathbf{0.0104\ m^3}$

For Redlich-Kwong, Eq.14.53 and oxygen

$P_c = 5040$ kPa; $T_c = 154.6$ K; $R = 0.2598$ kJ/kg K

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.2598 \times 154.6}{5040} = 0.000\,690\,5\ \text{m}^3\text{/kg}$$

$$a = 0.427\,48 \frac{R^2 T_c^{5/2}}{P_c} = 0.427\,48 \times \frac{0.2598^2 \times 154.6^{5/2}}{5040} = 1.7013$$

$$P = \frac{RT}{v - b} - \frac{a}{v(v + b)T^{1/2}}$$ trial and error to get v due to nonlinearity

$v = 0.01\ \text{m}^3\text{/kg} \Rightarrow P = 4465.1 - 1279.9 = 3185.2$ kPa too low

$v = 0.008\ \text{m}^3\text{/kg} \Rightarrow P = 5686.85 - 1968.1 = 3718.8$ kPa too low

$v = 0.0075\ \text{m}^3\text{/kg} \Rightarrow P = 6104.41 - 2227.43 = 3876.98$ kPa

$v = 0.007\ \text{m}^3\text{/kg} \Rightarrow P = 6588.16 - 2541.70 = 4046.46$ kPa

Now we interpolate between the last two entries and check

$v = 0.00714\ \text{m}^3\text{/kg} \Rightarrow P = 6445.15 - 2447.3 = 3997.8$ kPa OK

$V = mv = \mathbf{0.00714\ m^3}$ (69% of the ideal gas value)

14.81

A flow of oxygen at 230 K, 5 MPa is throttled to 100 kPa in a steady flow process. Find the exit temperature and the specific entropy generation using Redlich-Kwong equation of state and ideal gas heat capacity. Notice this becomes iterative due to the non-linearity coupling *h, P, v* and *T.*

C.V. Throttle. Steady single flow, no heat transfer and no work.

Energy eq.: $h_1 + 0 = h_2 + 0$ so constant h

Entropy Eq.: $s_1 + s_{gen} = s_2$ so entropy generation

Find the change in h from Eq.14.26 assuming C_p is constant.

Redlich-Kwong equation of state: $P = \frac{RT}{v - b} - \frac{a}{v(v + b)T^{1/2}}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v - b} + \frac{a}{2v(v + b)T^{3/2}}$$

From Eq.14.31

$$(u_2 - u_1)_T = \int_1^2 \frac{3a}{2v(v + b)T^{1/2}} dv = \frac{-3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

We find change in h from change in u, so we do not do the derivative in Eq.14.27. This is due to the form of the EOS.

$$(h_2 - h_1)_T = P_2v_2 - P_1v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

Entropy follows from Eq.14.35

$$(s_2 - s_1)_T = \int_1^2 \left[\frac{R}{v - b} + \frac{a/2}{v(v + b)T^{3/2}}\right] dv$$

$$= R \ln\left(\frac{v_2 - b}{v_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

$P_c = 5040$ kPa; $T_c = 154.6$ K; $R = 0.2598$ kJ/kg K

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.2598 \times 154.6}{5040} = 0.000\ 690\ 5 \text{ m}^3/\text{kg}$$

$$a = 0.427\ 48\ \frac{R^2T_c^{5/2}}{P_c} = 0.427\ 48 \times \frac{0.2598^2 \times 154.6^{5/2}}{5040} = 1.7013$$

We need to find T_2 so the energy equation is satisfied

$$h_2 - h_1 = h_2 - h_x + h_x - h_1 = C_p(T_2 - T_1) + (h_2 - h_1)_T = 0$$

and we will evaluate it similar to Fig. 13.4, where the first term is done from state x to 2 and the second term is done from state 1 to state x (at T_1 = 230 K). We do this as we assume state 2 is close to ideal gas, but we do not know T_2.

We first need to find v_1 from the EOS, so guess v and find P

$v_1 = 0.011\ m^3/kg \quad \Rightarrow \quad P = 5796.0 - 872.35 = 4924$ too low

$v_1 = 0.01082\ m^3/kg \quad \Rightarrow \quad P = 5899.0 - 900.7 = 4998.3$ OK

Now evaluate the change in h along the 230 K from state 1 to state x, that requires a value for v_x. Guess ideal gas at T_x = 230 K,

$$v_x = RT_x/P_2 = 0.2598 \times 230/100 = 0.59754\ m^3/kg$$

From the EOS: $P_2 = 100.1157 - 0.3138 = 99.802$ kPa (close)

A few more guesses and adjustments gives

$v_x = 0.59635\ m^3/kg;\quad P_2 = 100.3157 - 0.3151 = 100.0006$ kPa OK

$$(h_x - h_1)_T = P_xv_x - P_1v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_x + b}{v_x}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

$$= 59.635 - 5000 \times 0.01082 - 243.694 \ln\left[\frac{0.59704}{0.59635} \times \frac{0.01082}{0.01151}\right]$$

$$= 59.635 - 54.1 + 14.78335 = 20.318\ kJ/kg$$

From energy eq.: $T_2 = T_1 - (h_x - h_1)_T/C_p = 230 - 20.318 / 0.922 =$ **208 K**

Now the change in s is done in a similar fashion,

$$s_{gen} = s_2 - s_1 = (s_x - s_1)_T + s_2 - s_x$$

$$= R \ln\left(\frac{v_x - b}{v_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_x + b}{v_x}\right)\left(\frac{v_1}{v_1 + b}\right)\right] + C_p \ln\frac{T_2}{T_x}$$

$$= 0.2598 \ln\left(\frac{0.59566}{0.0101295}\right) - 0.35318 \ln (0.94114) + 0.922 \ln\left(\frac{208}{230}\right)$$

$$= 1.05848 + 0.021425 - 0.092699$$

$= \mathbf{0.987\ kJ/kg\ K}$

Generalized Charts

14.82

A 200-L rigid tank contains propane at 9 MPa, 280°C. The propane is then allowed to cool to 50°C as heat is transferred with the surroundings. Determine the quality at the final state and the mass of liquid in the tank, using the generalized compressibility chart, Fig. D.1.

Propane C_3H_8: $V = 0.2\ m^3$, $P_1 = 9$ MPa, $T_1 = 280^oC = 553.2$ K

cool to $T_2 = 50\ ^oC = 323.2$ K

From Table A.2: $T_c = 369.8$ K, $P_c = 4.25$ MPa

$$P_{r1} = \frac{9}{4.25} = 2.118, \quad T_{r1} = \frac{553.2}{369.8} = 1.496 \qquad \text{From Fig. D.1:} \quad Z_1 = 0.825$$

$$v_2 = v_1 = \frac{Z_1RT_1}{P_1} = \frac{0.825 \times 0.188\ 55 \times 553.2}{9\ 000} = 0.00956\ m^3/kg$$

From Fig. D.1 at $T_{r2} = 0.874$,

$P_{G2} = 0.45 \times 4250 = 1912$ kPa

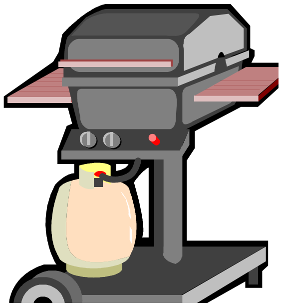
$v_{G2} = 0.71 \times 0.188\ 55 \times 323.2/1912 = 0.02263\ m^3/kg$

$v_{F2} = 0.075 \times 0.188\ 55 \times 323.2/1912 = 0.00239\ m^3/kg$

$0.00956 = 0.002\ 39 + x_2(0.02263 - 0.00239)$ => $x_2 = \mathbf{0.354}$

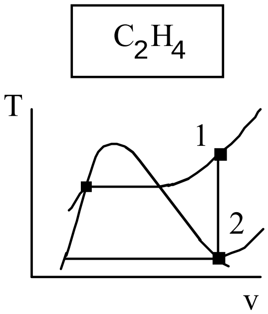
$m_{LIQ\ 2} = (1-0.354) \times 0.2/0.00956 = \mathbf{13.51\ kg}$

These tanks contain liquid propane.



14.83

A rigid tank contains 5 kg of ethylene at 3 MPa, 30°C. It is cooled until the ethylene reaches the saturated vapor curve. What is the final temperature?



V = const   m = 5 kg

$P_1 = 3$ MPa $T_1 = 30\ ^oC = 303.2$ K

cool to $x_2 = 1.0$

$$P_{r1} = \frac{3}{5.04} = 0.595, \quad T_{r1} = \frac{303.2}{282.4} = 1.074$$

Fig. D.1: $Z_1 = 0.82$

$$P_{r2} = P_{r1} \frac{Z_2 T_{r2}}{Z_1 T_{r1}} = 0.595 \frac{Z_{G2} T_{r2}}{0.82 \times 1.074} = 0.6756\ Z_{G2} T_{r2}$$

Trial & error, Table D.4 may be easier to use than Fig. D.1:

T_{r2}	Z_{G2}	P_{r2}	$P_{r2\ CALC}$		
0.866	0.72	0.42	0.421	~ OK	=> $T_2 =$ **244.6 K**

14.84

A 4-m^3 storage tank contains ethane gas at 10 MPa, 100°C. Using Lee-Kesler EOS and find the mass of the ethane.

The Lee-Kesler EOS is shown as the generalized charts.
Table A.2: $T_c = 305.4$ K, $P_c = 4.88$ MPa, Table A.5: $R = 0.2765$ kJ/kg-K
The reduced properties are:

$$P_{r1} = \frac{10}{4.88} = 2.05, \quad T_{r1} = \frac{373.15}{305.4} = 1.22 \quad ⇨ \quad \text{Fig. D.1:} \quad Z = 0.56$$

$$m = \frac{PV}{ZRT} = \frac{10\,000 \times 4}{0.56 \times 0.2765 \times 373.15} = \mathbf{692.3\ kg}$$

14.85

The ethane gas in the storage tank from the previous problem is cooled to 0^{o}C. Find the new pressure.

The new final state is given by: $(T_2, v_2 = v_1)$ $T_{r2} = \frac{273.15}{305.4} = 0.8944$

Since Z and P are unknown this becomes trial and error solution.

$$P_2 / Z_2 = mRT_2 /V = 692.3 \times 0.2765 \times 273.15 /4 = 13071.7 \text{ kPa}$$

Assume it is saturated $P_{r2} = 0.53$ (see Fig. D.1), $Z_g = 0.67$, $Z_f = 0.09$

$P_2 = 0.53 \times 4880 = 2586$ kPa and

$Z_2 = P_2 / 13071.7 = 0.198$ (two phase OK)

Ans.: $P_2 =$ **2586 kPa**

14.86

Use CATT3 to solve the previous two problems when the acentric factor is used to improve the accuracy.

Problem 14.84:
The Lee-Kesler EOS is shown as the generalized charts.
Table A.2: $T_c = 305.4$ K, $P_c = 4.88$ MPa, Table A.5: R = 0.2765 kJ/kg-K
Table D.4: $\omega = 0.099$
The reduced properties are:

$$P_{r1} = \frac{10}{4.88} = 2.05, \quad T_{r1} = \frac{373.15}{305.4} = 1.22 \quad \Rightarrow \quad \text{CATT3:} \quad Z = 0.605$$

$$m = \frac{PV}{ZRT} = \frac{10\,000 \times 4}{0.605 \times 0.2765 \times 373.15} = \mathbf{640.8\ kg}$$

Problem 14.85:

The new final state is given by: $(T_2, v_2 = v_1)$ $\qquad T_{r2} = \frac{273.15}{305.4} = 0.8944$

Since Z and P are unknown this becomes trial and error solution.

$$P_2 / Z_2 = mRT_2 /V = 640.8 \times 0.2765 \times 273.15 /4 = 12099.3 \text{ kPa}$$

Assume it is saturated vapor $P_{r2} = 0.483$ (CATT3), $Z_g = 0.69$, $Z_f = 0.078$

$P_2 = 0.483 \times 4880 = 2357$ kPa and

$Z_2 = P_2 / 12\,099.3 = 0.1948$ (two phase OK)

Ans.: $P_2 =$ **2357 kPa**

14.87

Consider the following equation of state, expressed in terms of reduced pressure and temperature:

$$Z = 1 + \frac{P_r}{14\, T_r}\left(1 - \frac{6}{T_r^2}\right)$$

What does this equation predict for enthalpy departure from the ideal gas value at the state $P_r = 0.4$, $T_r = 0.9$?

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\, T_r}\left(1 - \frac{6}{T_r^2}\right)$$

$$v = \frac{RT}{P} + \frac{RT_c}{14P_c}\left(1 - \frac{6T_c^2}{T^2}\right); \qquad \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{P} + \frac{12RT_c^3}{14P_cT^3}$$

$$v - T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT_c}{14P_c} - \frac{18RT_c^3}{14P_cT^2}$$

Now Eq.14.27 is integrated with limits similar to Eq.14.62

$$h - h^* = \int_0^P \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dP = \frac{RT_c}{14}\left(1 - \frac{18}{T_r^2}\right) P_r = \mathbf{0.606\ RT_c}$$

14.88

Consider the following equation of state, expressed in terms of reduced pressure and temperature:

$$Z = 1 + \frac{P_r}{14\ T_r}\left(1 - \frac{6}{T_r^2}\right)$$

What does this equation predict for entropy departure from the ideal gas value at the state $P_r = 0.4$, $T_r = 0.9$?

The entropy departure is the change in s for a real gas minus the change in s for an ideal gas, so from Eq.14.32 and eq.8.23 we get

$$d(s - s^*) = C_p\frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dP - \left[\ C_p\frac{dT}{T} - \frac{R}{P}\,dP\right] = \left[\frac{R}{P} - \left(\frac{\partial v}{\partial T}\right)_p\right] dP$$

Solve now for v from the compressibility factor (Z = Pv/RT) to get

$$Z = \frac{Pv}{RT} = 1 + \frac{P_r}{14\ T_r}\left(1 - \frac{6}{T_r^2}\right)$$

$$v = \frac{RT}{P} + \frac{RT_c}{14P_c}\left(1 - \frac{6T_c^2}{T^2}\right)\ ; \quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{P} + \frac{12RT_c^3}{14P_cT^3}$$

$$s - s^* = \int_0^P \left[\frac{R}{P} - \left(\frac{\partial v}{\partial T}\right)_p\right] dP = \int_0^P \left[- \frac{12RT_c^3}{14P_cT^3}\right] dP = -\frac{6}{7}R\frac{P_r}{T_r^3}$$

Evaluate at $P_r = 0.4$, $T_r = 0.9$ to get

$$s - s^* = \mathbf{-0.4703\ R}$$

14.89

The new refrigerant R-152a is used in a refrigerator with an evaporator temperature of $-20^{\circ}C$ and a condensing temperature of $30^{\circ}C$. What are the high and low pressures in this cycle?

Since we do not have the printed tables for R-152a we will use generalized charts. The critical properties are: $T_c = 386.4$ K, $P_c = 4.52$ MPa.

$T_{r1} = T/T_c = (273.15 - 20)/386.4 = 0.655$

Fig. D.1: $P_{G\,T1} = P_{r1\,sat}\,P_c = 0.06 \times 4.52 = \mathbf{0.271\ MPa}$

$T_{r2} = T/T_c = (273.15 + 30)/386.4 = 0.785$

Fig. D.1: $P_{G\,T2} = P_{r2\,sat}\,P_c = 0.22 \times 4.52 = \mathbf{0.994\ MPa}$

14.90

An ordinary lighter is nearly full of liquid propane with a small amount of vapor, the volume is 5 cm^3, and temperature is 23°C. The propane is now discharged slowly such that heat transfer keeps the propane and valve flow at 23°C. Find the initial pressure and mass of propane and the total heat transfer to empty the lighter.

Propane C_3H_8 $T_1 = 23^oC = 296.2\ K = \text{constant}$, $x_1 = 0.0$

$V_1 = 5\ cm^3 = 5\times10^{-6}\ m^3$, $T_{r1} = 296.2/369.8 = 0.804$

From Figs. D.1 and D.2,

$P_1 = P_{G\,T1} = 0.25\times4.25 =$ **1.063 MPa**, $Z_1 = 0.04$

$(h_1^*\text{-}h_1) = 0.188\ 55\times369.8\times4.51 = 314.5$

$$m_1 = \frac{P_1V_1}{Z_1RT_1} = \frac{1063\times5\times10^{-6}}{0.04\times0.188\ 55\times296.2} = \mathbf{0.00238\ kg}$$

State 2: Assume vapor at 100 kPa, 23^oC ideal gas so no corrections

Therefore, m_2 much smaller than m_1 (~ 9.0×10^{-6} kg)

$$Q_{CV} = m_2u_2 - m_1u_1 + m_eh_e$$

$$= m_2h_2 - m_1h_1 - (P_2-P_1)V + (m_1-m_2)h_e$$

$$= m_2(h_2-h_e) + m_1(h_e-h_1) - (P_2-P_1)V$$

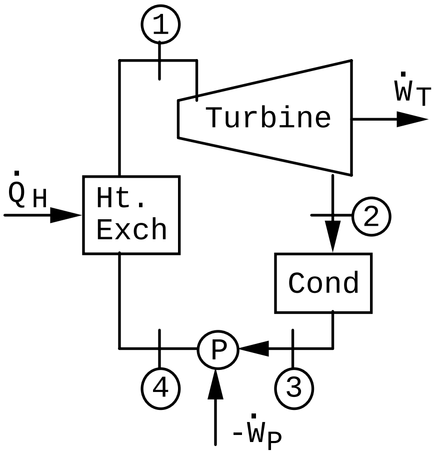
$(h_e - h_1) = 0 + 0 + 314.5$

$Q_{CV} = \approx 0 + 0.00238(314.5) - (100 - 1063)\times5\times10^{-6} =$ **0.753 kJ**

Actual lighters use butane and some propane.

14.91

A geothermal power plant uses butane as saturated vapor 80°C into the turbine and the condenser operates at 30°C. Find the reversible specific turbine work.



C_4H_{10} cycle

$T_1 = 80\ ^oC,\ x_1 = 1.0\ ;\ T_3 = 30^oC,\ x_3 = 0.0$

$$T_{r1} = \frac{353.2}{425.2} = 0.831$$

From D.1, D.2 and D.3:

$P_1 = 0.325 \times 3800 = 1235$ kPa

$(h^*_1 - h_1) = 0.143\ 04 \times 425.2 \times 0.56 = 34.1$

$(s^*_1 - s_1) = 0.143\ 04 \times 0.475 = 0.0680$

$$T_{r3} = \frac{303.2}{425.2} = 0.713$$

From D.1, D.2 and D.3: $P_3 = 0.113 \times 3800 = 429$ kPa

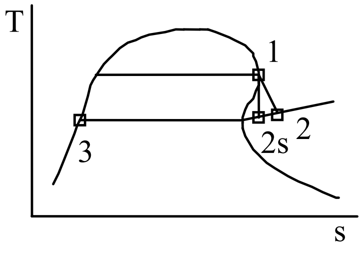
sat. liq.: $(h^* - h_f) = RT_c \times 4.81 = 292.5$; $(s^* - s_f) = R \times 6.64 = 0.950$

sat. vap.: $(h^* - h_g) = RT_c \times 0.235 = 14.3$; $(s^* - s_g) = R \times 0.22 = 0.031$

Because of the combination of properties of C_4H_{10} (particularly the large C_{P0}/R), s_1 is larger than s_g at T_3. To demonstrate,

$$(s^*_1 - s^*_{g3}) = 1.7164 \ln \frac{353.2}{303.2} - 0.143\ 04 \ln \frac{1235}{429} = 0.1107$$

$$(s_1 - s_{g3}) = -0.0680 + 0.1107 + 0.031 = +0.0737 \text{ kJ/kg K}$$



so that T_{2S} will be $> T_3$, as shown in the T-s diagram. A number of other heavy hydrocarbons also exhibit this behavior.

Assume $T_{2S} = 315$ K, $T_{r2S} = 0.741$

From D.2 and D.3:

$(h^*_{2S} - h_{2S}) = RT_c \times 0.21 = 12.8$ and $(s^*_{2S} - s_{2S}) = R \times 0.19 = 0.027$

$$(s_1^* - s_{2S}^*) = 1.7164 \ln\frac{353.2}{315} - 0.143\ 04 \ln\frac{1235}{429} = +0.0453$$

$$(s_1 - s_{2S}) = -0.0680 + 0.0453 + 0.027 \approx 0$$

$$\Rightarrow T_{2S} = 315\text{ K}$$

$$(h_1^* - h_{2S}^*) = 1.7164(353.2\text{-}315) = 65.6$$

$$w_{ST} = h_1 - h_{2S} = -34.1 + 65.6 + 12.8 = \mathbf{44.3\ kJ/kg}$$

14.92

A piston/cylinder contains 5 kg of butane gas at 500 K, 5 MPa. The butane expands in a reversible polytropic process to 3 MPa, 460 K. Determine the polytropic exponent n and the work done during the process.

C_4H_{10} $m = 5$ kg $T_1 = 500$ K $P_1 = 5$ MPa

Rev. polytropic process: $P_1V_1^n = P_2V_2^n$

$$T_{r1} = \frac{500}{425.2} = 1.176,\quad P_{r1} = \frac{5}{3.8} = 1.316 \qquad \text{From Fig. D.1:} \quad Z_1 = 0.68$$

$$T_{r2} = \frac{460}{425.2} = 1.082,\quad P_{r2} = \frac{3}{3.8} = 0.789 \qquad \text{From Fig. D.1:} \quad Z_2 = 0.74$$

$$V_1 = \frac{mZRT}{P} = \frac{5 \times 0.68 \times 0.1430 \times 500}{5000} = 0.0486 \text{ m}^3$$

$$V_2 = \frac{mZRT}{P} = \frac{5 \times 0.74 \times 0.1430 \times 460}{3000} = 0.0811 \text{ m}^3$$

Solve for the polytropic exponent, n, as

$$n = \ln(P_1/P_2) \,/\, \ln(V_2/V_1) = \ln\left(\frac{5}{3}\right) / \ln\left(\frac{0.0811}{0.0486}\right) = \mathbf{0.9976}$$

$$_1W_2 = \int_1^2 PdV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{3000\times0.0811 - 5000\times0.0486}{1 - 0.9976} = \mathbf{125\ kJ}$$

14.93

Calculate the heat transfer during the process described in Problem 14.92.

From solution **14.92**,

$V_1 = 0.0486 \text{ m}^3, \quad V_2 = 0.0811 \text{ m}^3, \quad {}_1W_2 = 125 \text{ kJ}$

$T_{r1} = \frac{500}{425.2} = 1.176, \quad P_{r1} = \frac{5}{3.8} = 1.316 \quad$ From Fig. D.1: $\quad Z_1 = 0.68$

$T_{r2} = 1.082, \quad P_{r2} = 0.789, \quad T_2 = 460 \text{ K}$

From Fig. D.2: $\quad (h^* - h)_1 = 1.30 \text{ RT}_C, \quad (h^* - h)_2 = 0.90 \text{ RT}_C$

$h^*_2 - h^*_1 = 1.716(460 - 500) = -83.1 \text{ kJ/kg}$

$h_2 - h_1 = -83.1 + \frac{8.3145 \times 425.2}{58.124}(-0.90 + 1.30) = -58.8 \text{ kJ/kg}$

$U_2 - U_1 = m(h_2 - h_1) - P_2V_2 + P_1V_1$

$= 5(-58.8) - 3000 \times 0.0811 + 5000 \times 0.0486 = -288.3 \text{ kJ}$

${}_1Q_2 = U_2 - U_1 + {}_1W_2 = \mathbf{-174.3 \text{ kJ}}$

14.94

A very low temperature refrigerator uses neon. From the compressor the neon at 1.5 MPa, 80 K goes through the condenser and comes out at saturated liquid 40 K. Find the specific heat transfer using generalized charts.

State 1: 80 K, 1.5 MPa : $T_{r1} = \frac{80}{44.4} = 1.802, \quad P_{r1} = \frac{1.5}{2.76} = 0.543$

State 2: 40 K, x = 0: $T_{r2} = 0.90, \quad P_{r2} = 0.532$

The enthalpy departure chart Fig. D.2:

$$(h^* - h)_1 = 0.22\ RT_C, \qquad (h^* - h)_2 = 4.10\ RT_C$$

$$h^*_2 - h^*_1 = 1.03\ (40 - 80) = -41.2\ \text{kJ/kg}$$

$$h_2 - h_1 = h^*_2 - h^*_1 - (h^* - h)_2 + (h^* - h)_1$$

$$= -41.2 + 0.412 \times 44.4\ (-4.10 + 0.22) = -112.2\ \text{kJ/kg}$$

$$q = h_2 - h_1 = \mathbf{-112.2\ kJ/kg}$$

14.95

Repeat the previous problem using CATT3 software for the neon properties.

From CATT3: $h_1 = 138.3$ kJ/kg, $h_2 = 30.24$ kJ/kg ($P = 1.46$ MPa)

$q = h_2 - h_1 = 30.24 – 138.3 =$ **-108.06 kJ/kg**

14.96

A cylinder contains ethylene, C_2H_4, at 1.536 MPa, −13°C. It is now compressed in a reversible isobaric (constant *P*) process to saturated liquid. Find the specific work and heat transfer.

Ethylene C_2H_4 ; $P_1 = 1.536$ MPa $= P_2$, $T_1 = -13^oC = 260.2$ K

State 2: saturated liquid, $x_2 = 0.0$

$$T_{r1} = \frac{260.2}{282.4} = 0.921 \quad P_{r1} = P_{r2} = \frac{1.536}{5.04} = 0.305$$

From Figs. D.1, D.2: $Z_1 = 0.85$, $(h^*_1\text{-}h_1)/RT_c = 0.40$

$$v_1 = \frac{Z_1RT_1}{P_1} = \frac{0.85 \times 0.29637 \times 260.2}{1536} = 0.042675$$

$$(h^*_1\text{-}h_1) = 0.296\ 37 \times 282.4 \times 0.40 = 33.5$$

From Figs. D.1, D.2: $T_2 = 0.824 \times 282.4 = 232.7$ K

$Z_2 = 0.05$, $(h^*_2\text{-}h_2)/RT_c = 4.42$

$$v_2 = \frac{Z_2RT_2}{P_2} = \frac{0.05 \times 0.29637 \times 232.7}{1536} = 0.002245$$

$$(h^*_2\text{-}h_2) = 0.296\ 37 \times 282.4 \times 4.42 = 369.9$$

$$(h^*_2\text{-}h^*_1) = C_{P0}(T_2\text{-}T_1) = 1.5482(232.7\text{-}260.2) = -42.6$$

$$w_{12} = \int P dv = P(v_2\text{-}v_1) = 1536(0.002\ 245\text{-}0.042\ 675) = \mathbf{-62.1\ kJ/kg}$$

$$q_{12} = (u_2\text{-}u_1) + w_{12} = (h_2\text{-}h_1) = -369.9 - 42.6 + 33.5 = \mathbf{-379\ kJ/kg}$$

14.97

A cylinder contains ethylene, C_2H_4, at 1.536 MPa, –13°C. It is now compressed isothermally in a reversible process to 5.12 MPa. Find the specific work and heat transfer.

Ethylene C_2H_4 $P_1 = 1.536$ MPa , $T_2 = T_1 = -13^{o}C = 260.2$ K

$T_{r2} = T_{r1} = 260.2 / 282.4 = 0.921$, $P_{r1} = 1.536 / 5.04 = 0.305$

From D.1, D.2 and D.3: $Z_1 = 0.85$

$(h^*_1 - h_1) = 0.2964 \times 282.4 \times 0.40 = 33.5$ and $(s^*_1 - s_1) = 0.2964 \times 0.30 = 0.0889$

From D.1, D.2 and D.3: $Z_2 = 0.17$, $P_{r2} = 5.12/5.04 = 1.016$ (comp. liquid)

$(h^*_2 - h_2) = 0.2964 \times 282.4 \times 4.0 = 334.8$ and $(s^*_2 - s_2) = 0.2964 \times 3.6 = 1.067$

Ideal gas: $(h^*_2 - h^*_1) = 0$ and $(s^*_2 - s^*_1) = 0 - 0.2964 \ln \frac{5.12}{1.536} = -0.3568$

${}_1q_2 = T(s_2 - s_1) = 260.2(-1.067 - 0.3568 + 0.0889) =$ **-347.3 kJ/kg**

$(h_2 - h_1) = -334.8 + 0 + 33.5 = -301.3$ kJ/kg

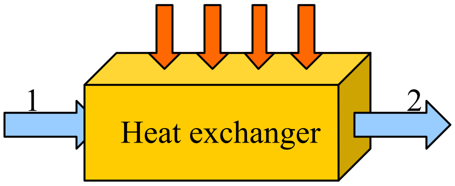
$(u_2 - u_1) = (h_2 - h_1) - RT(Z_2 - Z_1) = -301.3 - 0.2964 \times 260.2(0.17 - 0.85) = -248.9$

${}_1w_2 = {}_1q_2 - (u_2 - u_1) = -347.3 + 248.9 =$ **-98.4 kJ/kg**

14.98

Refrigerant-123, dichlorotrifluoroethane, which is currently under development as a potential replacement for environmentally hazardous refrigerants, undergoes an isothermal steady flow process in which the R-123 enters a heat exchanger as saturated liquid at 40°C and exits at 100 kPa. Calculate the heat transfer per kilogram of R-123, using the generalized charts, Fig. D.2

R-123: $M = 152.93$, $T_C = 456.9\text{ K}$, $P_C = 3.67\text{ MPa}$



$T_1 = T_2 = 40\ ^oC$, $x_1 = 0$
$P_2 = 100\text{ kPa}$

$T_{r1} = T_{r2} = 313.2/456.9 = 0.685$, $P_{r2} = 0.1/3.67 = 0.027$

From Fig. D.2: $P_{r1} = 0.084$, $(h^* - h)_1/RT_C = 4.9$

From D.1: saturated $P_1 = 0.084\times3670 = 308\text{ kPa}$

$P_2 < P_1$ with no work done, so process is irreversibel.

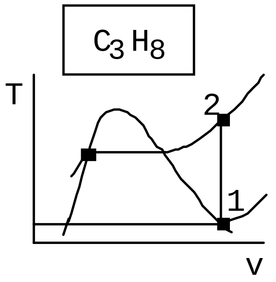
Energy Eq.: $q + h_1 = h_2$, Entropy Eq.: $s_1 + \int dq/T + s_{gen} = s_2$, $s_{gen} > 0$

From Fig. D.2: $(h^* - h)_2/RT_C = 0.056$

$q = h_2 - h_1 = 8.3145 \times 456.9 \left[-0.056 + 0 + 4.90\right]/152.93 = \mathbf{120.4\ kJ/kg}$

14.99

250-L tank contains propane at 30°C, 90% quality. The tank is heated to 300°C. Calculate the heat transfer during the process.



$V = 250\ \text{L} = 0.25\ \text{m}^3$

$T_1 = 30\ ^{o}\text{C} = 303.2\ \text{K},\ x_1 = 0.90$

Heat to $T_2 = 300\ ^{o}\text{C} = 573.2\ \text{K}$

$M = 44.094,\ T_C = 369.8\ \text{K},\ P_C = 4.25\ \text{MPa}$

$R = 0.188\ 55,\quad C_{P0} = 1.6794$

$T_{r1} = 0.82 \rightarrow$ Fig. D.1:

$Z_1 = (1- x_1)\, Z_{f1} + x_1\, Z_{g1} = 0.1 \times 0.05 + 0.9 \times 0.785 = 0.711$

Fig D.2: $\quad \dfrac{h^*_1\text{-}h_1}{RT_c} = 0.1 \times 4.43 + 0.9 \times 0.52 = 0.911$

$P_r^{SAT} = 0.30 \quad P_1^{SAT} = 1.275\ \text{MPa}$

$$m = \frac{1275\times0.25}{0.711\times0.188\ 55\times303.2} = 7.842\ \text{kg}$$

$$P_{r2} = \frac{7.842\times Z_2\times0.188\ 55\times573.2}{0.25\times4250} = \frac{Z_2}{1.254}$$

at $T_{r2} = 1.55$ Trial and error on P_{r2}

$P_{r2} = 0.743 \Rightarrow P_2 = 3.158\ \text{MPa},\quad Z_2 = 0.94\ ,\ (h^*\text{- } h)_2 = 0.35\ RT_C$

$(h^*_2\text{-}h^*_1) = 1.6794(300\text{-}30) \qquad = 453.4\ \text{kJ/kg}$

$(h^*_1\text{-}h_1) = 0.911\times0.188\ 55\times369.8 = \ 63.5\ \text{kJ/kg}$

$(h^*_2\text{-}h_2) = 0.35\times0.188\ 55\times369.8 = \ 24.4\ \text{kJ/kg}$

$Q_{12} = m(h_2\text{-}h_1) - (P_2\text{-}P_1)V = 7.842(-24.4+453.4+63.5) - (3158\text{-}1275)\times0.25$

$= +3862 - 471 =$ **3391 kJ**

14.100

Saturated vapor R-410a at 30°C is throttled to 200 kPa in a steady flow process. Calculate the exit temperature assuming no changes in the kinetic energy, using the generalized charts, Fig. D.2 and the R-410a tables, Table B.4.

R-410a throttling process

Energy Eq.: $h_2 - h_1 = 0 = (h_2 - h_2^*) + (h_2^* - h_1^*) + (h_1^* - h_1)$

Generalized Chart, Fig. D.2, $R = 8.31451/72.585 = 0.11455$ kJ/kg-K

$$T_{r1} = \frac{303.2}{344.5} = 0.88 \quad => \quad (h_1^* - h_1) = 0.11455 \times 344.5\ (0.85) = 33.54 \text{ kJ/kg}$$

For C_{P0}, use h values from Table B.4 at low pressure.

$$C_{P0} \approx (330.83 - 314.40) / (40 - 20) = 0.8215 \text{ kJ/kg K}$$

Substituting: $(h_2 - h_2^*) + 0.8215\ (T_2 - 30) + 33.54 = 0$

at $P_{r2} = 200/4900 = 0.041$

Assume $T_2 = -10^oC \;=>\; T_{r2} = 263.2/344.5 = 0.764$

$(h_2^* - h_2) = RT \times 0.1 = 0.11455 \times 344.5\ (0.1) = 3.95$

Substituting : $-3.95 + 0.8215\ (-10 - 30) + 33.54 = -3.27$

Assume $T_2 = -5^oC \;=>\; T_{r2} = 268.2/344.5 = 0.778$

$(h_2^* - h_2) = RT \times 0.1 = 0.11455 \times 344.5\ (0.1) = 3.95$

Substituting : $-3.95 + 0.8215\ (-5 - 30) + 33.54 = 0.84$

$\Rightarrow T_2 = \mathbf{-6.0\ ^oC}$

R-410a tables, B.4: at $T_1 = 30^oC$, $x_1 = 1.0$ => $h_1 = 284.16$ kJ/kg

$h_2 = h_1 = 284.16$, $P_2 = 0.2$ MPa => $T_2 = \mathbf{-13.4\ ^oC}$

14.101

Carbon dioxide collected from a fermentation process at 5°C, 100 kPa should be brought to 243 K, 4 MPa in a steady flow process. Find the minimum amount of work required and the heat transfer. What devices are needed to accomplish this change of state?

$$T_{ri} = \frac{278.2}{304.1} = 0.915, \quad P_{ri} = \frac{100}{7380} = 0.0136$$

From D.2 and D.3 : $(h^*\text{-}h)_{ri}/RT_C = 0.02, \quad (s^*\text{-}s)_{ri}/R = 0.01$

$$T_{re} = \frac{243}{304.1} = 0.80, \quad P_{re} = \frac{4}{7.38} = 0.542$$

From D.2 and D.3: $(h^*\text{-}h)_{re}/RT_C = 4.5\,, \quad (s^*\text{-}s)_{re}/R = 4.74$

$$(h_i\text{-}h_e) = -\,(h^*_i\text{-}h_i) + (h^*_i\text{-}h^*_e) + (h^*_e\text{-}h_e)$$

$$= -\,0.188\,92{\times}304.1{\times}0.01 + 0.8418(278.2\text{-}243)$$

$$+\,0.188\,92{\times}304.1{\times}4.5 \quad = 287.6 \text{ kJ/kg}$$

$$(s_i\text{-}s_e) = -\,(s^*_i\text{-}s_i) + (s^*_i\text{-}s^*_e) + (s^*_e\text{-}s_e)$$

$$= -\,0.188\,92{\times}0.01 + 0.8418 \ln(278.2/243)$$

$$-\,0.188\,92 \ln(0.1/4) + 0.188\,92{\times}4.74 \;= 1.7044 \text{ kJ/kg K}$$

$$w^{rev} = (h_i\text{-}h_e) - T_0(s_i\text{-}s_e) \;= 287.6 - 278.2(1.7044) = \mathbf{-186.6\ kJ/kg}$$

$$q^{rev} = (h_e\text{-}h_i) + w^{rev} \;= -287.6 - 186.6 = \mathbf{-474.2\ kJ/kg}$$

We need a compressor to bring the pressure up and a cooler to bring the temperature down. Cooling it before compression and intercooling between stages in the compressor lowers the compressor work. In an actual set-up we require more work than the above reversible limit.

14.102

A geothermal power plant on the Raft River uses isobutane as the working fluid. The fluid enters the reversible adiabatic turbine, as shown in Fig. P14.44, at 160°C, 5.475 MPa, and the condenser exit condition is saturated liquid at 33°C. Isobutane has the properties T_c= 408.14 K, P_c= 3.65 MPa, C_{P0}= 1.664 kJ/kg K and ratio of specific heats *k* = 1.094 with a molecular weight as 58.124. Find the specific turbine work and the specific pump work.

Turbine inlet: $T_1 = 160^oC$, $P_1 = 5.475$ MPa

Condenser exit: $T_3 = 33^oC$, $x_3 = 0.0$, $T_{r3} = 306.2 / 408.1 = 0.75$

From Fig. D.1:

$P_{r3} = 0.16$, $Z_3 = 0.03$ => $P_2 = P_3 = 0.16 \times 3.65 = 0.584$ MPa

$T_{r1} = 433.2 / 408.1 = 1.061$, $P_{r1} = 5.475 / 3.65 = 1.50$

From Fig. D.2 & D.3:

$$(h_1^* - h_1) = 0.143\ 05\times408.1\times2.84 = 165.8$$

$$(s_1^* - s_1) = 0.143\ 05\times2.15 = 0.3076$$

$$(s_2^* - s_1^*) = 1.664 \ln\frac{306.2}{433.2} - 0.143\ 05 \ln\frac{0.584}{5.475} = -0.2572$$

$$(s_2^* - s_2) = (s_2^* - s_{F2}) - x_2\, s_{FG2}$$

$$= 0.143\ 05\times6.12 - x_2\times0.143\ 05(6.12-0.29) = 0.8755 - x_2\times0.8340$$

$$(s_2 - s_1) = 0 = -0.8755 + x_2\times0.8340 - 0.2572 + 0.3076 \quad => \quad x_2 = 0.99$$

$$(h_2^* - h_1^*) = C_{P0}(T_2 - T_1) = 1.664(306.2 - 433.2) = -211.3$$

From Fig. D.2:,

$$(h_2^* - h_2) = (h_2^* - h_{F2}) - x_2h_{FG2} = 0.143\ 05\times408.1[4.69-0.99(4.69-0.32)]$$

$$= 273.8 - 0.99 \times 255.1 = 21.3$$

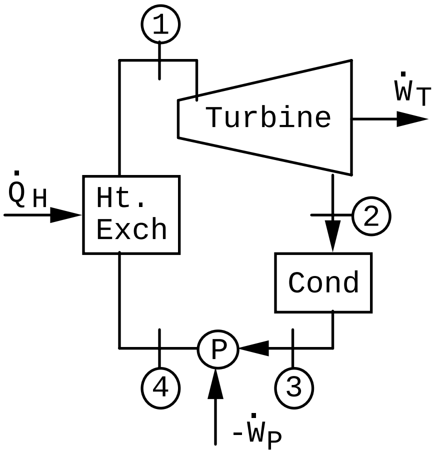
Turbine: $w_T = (h_1 - h_2) = -165.8 + 211.3 + 21.3 =$ **66.8 kJ/kg**

Pump: $$v_{F3} = \frac{Z_{F3}RT_3}{P_3} = \frac{0.03\times0.143\ 05\times306.2}{584} = 0.00225$$

$w_P = - \int v\, dP \approx v_{F3}(P_4 - P_3) = -0.00225\ (5475-584) =$ **-11.0 kJ/kg**

14.103

Repeat Problem 14.91 using CATT3 and include the acentric factor for butane to improve the accuracy.



C_4H_{10} cycle

$T_1 = 80\ ^oC$, $x_1 = 1.0$; $T_3 = 30^oC$, $x_3 = 0.0$

$$T_{r1} = \frac{353.2}{425.2} = 0.831$$

From CATT3 with ω = 0.199:

$P_1 = 0.2646 \times 3800 = 1005$ kPa

$(h_1^*-h_1) = 0.1430 \times 425.2 \times 0.5685 = 34.6$

$(s_1^*-s_1) = 0.1430 \times 0.4996 = 0.0714$

$$T_{r3} = \frac{303.2}{425.2} = 0.713$$

From CATT3 with ω = 0.199: $P_3 = 0.07443 \times 3800 = 282.8$ kPa

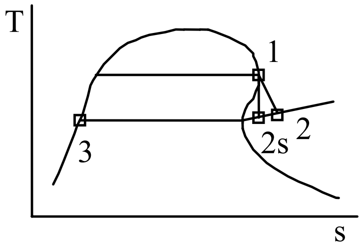
sat. liq.: $(h^*- h_f) = RT_c\times 6.048 = 367.74$; $(s^*- s_f) = R\times 8.399 = 1.201$

sat. vap.: $(h^*- h_g) = RT_c\times 0.202 = 12.28$; $(s^*- s_g) = R\times 0.201 = 0.0287$

Because of the combination of properties of C_4H_{10} (particularly the large C_{P0}/R), s_1 is larger than s_g at T_3. To demonstrate,

$$(s_1^*-s_{g3}^*) = 1.716 \ln\frac{353.2}{303.2} - 0.1430 \ln\frac{1005}{282.8} = 0.0806$$

$$(s_1-s_{g3}) = -0.0714 + 0.0806 + 0.0287 = +0.0379 \text{ kJ/kg K}$$



so that T_{2S} will be > T_3, as shown in the T-s diagram. A number of other heavy hydrocarbons also exhibit this behavior.

Assume $T_{2S} = 315$ K, $T_{r2S} = 0.741$

From CATT3:

$(h_{2S}^* - h_{2S}) = RT_c\times 0.183 = 11.13$ and $(s_{2S}^* - s_{2S}) = R\times 0.1746 = 0.025$

$$(s_1^* - s_{2S}^*) = 1.716 \ln\frac{353.2}{315} - 0.1430 \ln\frac{1005}{282.8} = +0.01509$$

$(s_1 - s_{2S}) = -0.0714 + 0.01509 + 0.025 = -0.031$

Repeat at $T_{2S} = 310$ K to get $T_{r2S} = 0.729$,

$(h^*_{2S} - h_{2S}) = RT_c\times0.1907 = 11.595$ and $(s^*_{2S} - s_{2S}) = R\times0.1853 = 0.0265$

$(s^*_1 - s^*_{2S}) = 1.716 \ln\frac{353.2}{310} - 0.1430 \ln\frac{1005}{282.8} = +0.04255$

$(s_1 - s_{2S}) = -0.0714 + 0.04255 + 0.0265 = -0.0023$ very close to 0, OK

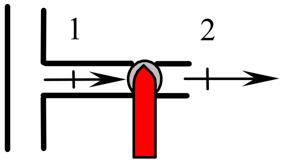
$\Rightarrow T_{2S} = 310$ K

$(h^*_1 - h^*_{2S}) = 1.716\ (353.2 - 310) = 74.13$

$w_{ST} = h_1 - h_{2S} = -34.6 + 74.13 + 11.6 =$ **51.1 kJ/kg**

14.104

A flow of oxygen at 230 K, 5 MPa is throttled to 100 kPa in a steady flow process. Find the exit temperature and the entropy generation.



Process: Throttling

Small surface area: $\dot{Q} = 0$;

No shaft: $\dot{W} = 0$

Irreversible: $\dot{S}_{gen} > 0$

We will solve the problem using generalized charts.

$$T_{ri} = \frac{230}{154.6} = 1.488, \quad P_{ri} = \frac{5}{5.04} = 0.992, \quad P_{re} = \frac{0.1}{5.04} = 0.02$$

From D.2: $(h^*_i - h_i) = RT_c\, \Delta h = 0.2598 \times 154.6 \times 0.50 = 20.1$ kJ/kg

Energy Eq.: $(h_e - h_i) = 0 = -(h^*_e - h_e) + (h^*_e - h^*_i) + (h^*_i - h_i)$

Assume $T_e = 208$ K , $T_{re} = 1.345$:

$(h^*_e - h^*_i) = C_p\,(T_e - T_i) = 0.922\,(208 - 230) = -20.3$ kJ/kg

From D.2: $(h^*_e - h_e) = RT_c\, \Delta h = 0.2598 \times 154.6 \times 0.01 = 0.4$

Check first law $(h_e - h_i) = -0.4 - 20.3 + 20.1 \approx 0$ OK => $T_e =$ **208 K**

From D.3,

$(s^*_i - s_i) = 0.2598 \times 0.25 = 0.0649$ and $(s^*_e - s_e) = 0.2598 \times 0.01 = 0.0026$

$$(s^*_e - s^*_i) = 0.9216 \ln \frac{208}{230} - 0.2598 \ln \frac{0.1}{5} = 0.9238 \text{ kJ/kg K}$$

$s_{gen} = (s_e - s_i) = -0.0026 + 0.9238 + 0.0649 =$ **0.9861 kJ/kg K**

14.105

A line with a steady supply of octane, C_8H_{18}, is at 400°C, 3 MPa. What is your best estimate for the availability in a steady flow setup where changes in potential and kinetic energies may be neglected?

Availability of Octane at $T_i = 400\ ^oC$, $P_i = 3$ MPa

From Table A.5: $R = 0.07279$ kJ/kgK, $C_P = 1.711$ kJ/kg

From Table A.2: $T_C = 568.8$ K, $P_C = 2.49$ MPa

$$P_{ri} = \frac{3}{2.49} = 1.205, \quad T_{ri} = \frac{673.2}{568.8} = 1.184$$

From D.2 and D.3,

$$(h^*_i - h_i) = RT_C \times \Delta\tilde{h} = 0.072\,79 \times 568.8 \times 1.13 = 46.8 \text{ kJ/kg};$$

$$(s^*_i - s_i) = R\,\Delta\tilde{s} = 0.072\,79 \times 0.69 = 0.05 \text{ kJ/kgK}$$

This is relative to the dead ambient state, assume $T_0 = 298.2$ K, $P_0 = 100$ kPa

$$T_{r0} = \frac{298.2}{568.8} = 0.524, \quad P_{r0} = \frac{0.1}{2.49} = 0.040$$

From D.2 and D.3, The s correction is outside chart (extrapolate or use CATT3)

$$(h^*_0 - h_0) = RT_C \times 5.4 = 223.6 \quad \text{and} \quad (s^*_0 - s_0) = R \times 9 = 0.655 \text{ kJ/kgK}$$

$$(h^*_i - h^*_0) = C_P(T_i - T_0) = 1.711\,(673.2 - 298.2) = 641.7 \text{ kJ/kg}$$

$$(s^*_i - s^*_0) = 1.711 \times \ln\frac{673.2}{298.2} - 0.072\,79 \times \ln\frac{3}{0.1} = 1.1459 \text{ kJ/kgK}$$

$$(h_i - h_0) = -46.8 + 641.7 + 223.6 = 818.5 \text{ kJ/kg}$$

$$(s_i - s_0) = -0.05 + 1.1459 + 0.655 = 1.7509 \text{ kJ/kgK}$$

$$\varphi_i = w^{rev} = (h_i - h_0) - T_0(s_i - s_0) = 818.5 - 298.2(1.7509) = \mathbf{296.5\ kJ/kg}$$

14.106

An alternative energy power plant has carbon dioxide at 6 MPa, 100^{o}C flowing into a turbine with an exit as saturated vapor at 1 MPa. Find the specific turbine work using generalized charts and repeat using Table B.3.

From Table A.5: $R = 0.1889$ kJ/kgK, $C_P = 0.842$ kJ/kg

From Table A.2: $T_C = 304.1$ K, $P_C = 7.38$ MPa

$$P_{ri} = \frac{6}{7.38} = 0.813, \quad T_{ri} = \frac{373.2}{304.1} = 1.227 \quad \Rightarrow \quad \Delta \tilde{h} = 0.70$$

From D.2 and D.3,

$$(h_i^* - h_i) = RT_C \times \Delta \tilde{h} = 0.1889 \times 304.1 \times 0.70 = 46.8 \text{ kJ/kg;}$$

$$P_{re} = \frac{1}{7.38} = 0.1355, \;\; x = 1 \;\text{ so }\; T_{re} = 0.73, \quad T_e = 0.73 \times 304.1 = 222 \text{ K}$$

From D.2 and D.3,

$$(h_e^* - h_e) = RT_C \times \Delta \tilde{h} = 0.1889 \times 304.1 \times 0.25 = 46.8 \text{ kJ/kg;}$$

$$w = h_i - h_e = h_i^* - h_e^* - (h_i^* - h_i) + (h_e^* - h_e)$$

$$= C_P(h_i - h_e) - RT_C (\Delta \tilde{h}_i - \Delta \tilde{h}_e)$$

$$= 0.842 (373.15 - 222) - 0.1889 \times 0304.1 (0.7 - 0.25)$$

$$= \mathbf{101.45 \text{ kJ/kg}}$$

From Table B.3

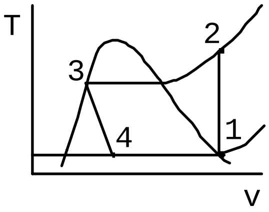
$$w = h_i - h_e = 421.69 - 322.39 = \mathbf{99.3 \text{ kJ/kg}}$$

14.107

The environmentally safe refrigerant R-152a is to be evaluated as the working fluid for a heat pump system that will heat a house. It uses an evaporator temperature of –20°C and a condensing temperature of 30°C. Assume all processes are ideal and R-152a has a heat capacity of $C_p = 0.996$ kJ/kg K. Determine the cycle coefficient of performance.

Ideal Heat Pump $T_H = 30\ ^oC$

From A.2: $M = 66.05$, $R = 0.125\ 88$, $T_C = 386.4$ K, $P_C = 4.52$ MPa



$$T_{r3} = \frac{303.2}{386.4} = 0.785$$

$P_{r3} = P_{r2} = 0.22 \quad => \quad P_3 = P_2 = 994$ kPa

Sat.liq.: $\overset{*}{h_3} - h_3 = 4.56 \times RT_C = 221.8$

$T_1 = -20\ ^oC = 253.2$ K, $T_{r1} = 0.655$, $P_{r1} = 0.058 \rightarrow P_1 = 262$ kPa

$\overset{*}{h_1} - h_1 = 0.14 \times RT_C = 6.8$ and $\overset{*}{s_1} - s_1 = 0.14 \times R = 0.0176$

Assume $T_2 = 307$ K, $T_{r2} = 0.795$ given $P_{r2} = 0.22$

From D.2, D.3: $\overset{*}{s_2} - s_2 = 0.34 \times R = 0.0428$; $\overset{*}{h_2} - h_2 = 0.40 \times RT_c = 19.5$

$$\overset{*}{s_2} - \overset{*}{s_1} = 0.996 \ln\frac{307}{253.2} - 0.125\ 88 \ln\frac{994}{262} = 0.0241$$

$$s_2 - s_1 = -0.0428 + 0.0241 + 0.0176 = -0.001 \approx 0 \ \text{ OK}$$

$$\Rightarrow h_2 - h_1 = -19.5 + 0.996(307-253.2) + 6.8 = 40.9$$

$$h_2 - h_3 = -19.5 + 0.996(307-303.2) + 221.8 = 206.1$$

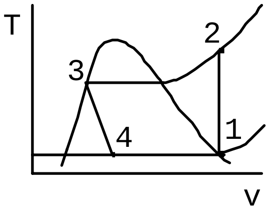
$$\beta = \frac{q_H}{w_{IN}} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{206.1}{40.9} = \mathbf{5.04}$$

14.108

Rework the previous problem using an evaporator temperature of 0°C.

Ideal Heat Pump $T_H = 30\ ^oC$

From A.2: $M = 66.05$, $R = 0.125\ 88$, $T_C = 386.4$ K, $P_C = 4.52$ MPa



$$T_{r3} = \frac{303.2}{386.4} = 0.785$$

$P_{r3} = P_{r2} = 0.22 \quad \Rightarrow \quad P_3 = P_2 = 994$ kPa

Sat.liq.: $h^*_3 - h_3 = 4.56 \times RT_C = 221.8$

$T_1 = 0\ ^oC = 273.2$ K, $T_{r1} = 0.707 \Rightarrow P_{r1} = 0.106$, $P_1 = 479$ kPa

$h^*_1 - h_1 = 0.22 \times RT_C = 10.7$ and $s^*_1 - s_1 = 0.21 \times R = 0.0264$

Assume $T_2 = 305$ K, $T_{r2} = 0.789$

$s^*_2 - s_2 = 0.35 \times R = 0.0441$ and $h^*_2 - h_2 = 0.38 \times RT_C = 18.5$

$$s^*_2 - s^*_1 = 0.996 \ln\frac{305.0}{273.2} - 0.125\ 88 \ln\frac{994}{479} = 0.0178$$

$s_2 - s_1 = -0.0441 + 0.0178 + 0.0264 = 0.0001 \approx 0$ OK

$h_2 - h_1 = -18.5 + 0.996(305.0-273.2) + 10.7 = 23.9$

$h_2 - h_3 = -18.5 + 0.996(305.0-303.2) + 221.8 = 205.1$

$$\beta = \frac{h_2 - h_3}{h_2 - h_1} = \frac{205.1}{23.9} = \mathbf{8.58}$$

14.109

The new refrigerant fluid R-123 (see Table A.2) is used in a refrigeration system that operates in the ideal refrigeration cycle, except the compressor is neither reversible nor adiabatic. Saturated vapor at -26.5°C enters the compressor and superheated vapor exits at 65°C. Heat is rejected from the compressor as 1 kW, and the R-123 flow rate is 0.1 kg/s. Saturated liquid exits the condenser at 37.5°C. Specific heat for R-123 is $C_P = 0.6$ kJ/kg. Find the coefficient of performance.

R-123: $T_c = 456.9$ K, $P_c = 3.67$ MPa, $M = 152.93$ kg/kmol, $R = 0.05438$ kJ/kg K

State 1: $T_1 = -26.5^oC = 246.7$ K, sat vap., $x_1 = 1.0$

$T_{r1} = 0.54$, Fig D.1, $P_{r1} = 0.01$, $P_1 = P_{r1}P_c = 37$ kPa

Fig. D.2, $h^*_1\text{-}h_1 = 0.03\ RT_C = 0.8$ kJ/kg

State 2: $T_2 = 65^oC = 338.2$ K

State 3: $T_3 = 37.5^oC = 310.7$ K, sat. liq., $x_3 = 0$

$T_{r3} = 0.68$, Fig. D.1: $P_{r3} = 0.08$, $P_3 = P_{r3}P_c = 294$ kPa

$P_2 = P_3 = 294$ kPa, $P_{r2} = 0.080$, $T_{r2} = 0.74$,

Fig. D.2: $h^*_2\text{-}h_2 = 0.25\ RT_C = 6.2$ kJ/kg

$h^*_3\text{-}h_3 = 4.92\ RT_C = 122.2$ kJ/kg

State 4: $T_4 = T_1 = 246.7$ K, $h_4 = h_3$

1st Law Evaporator: $q_L + h_4 = h_1 + w$; $w = 0$, $h_4 = h_3$

$$q_L = h_1 - h_3 = \left(h_1 - h^*_1\right) + \left(h^*_1 - h^*_3\right) + \left(h^*_3 - h_3\right)$$

$h^*_1 - h^*_3 = C_P(T_1 - T_3) = -38.4$ kJ/kg, $q_L = -0.8 - 38.4 + 122.2 = 83.0$ kJ/kg

1st Law Compressor: $q + h_1 = h_2 + w_c$; $\dot{Q} = -1.0$ kW, $\dot{m} = 0.1$ kg/s

$$w_c = h_1 - h_2 + q;\ h_1 - h_2 = \left(h_1 - h^*_1\right) + \left(h^*_1 - h^*_2\right) + \left(h^*_2 - h_2\right)$$

$h^*_1 - h^*_2 = C_P(T_1 - T_2) = -54.9$ kJ/kg,

$w_c = -0.8 - 54.9 + 6.2 - 10.0 = -59.5$ kJ/kg

$\beta = q_L/w_c = 83.0/59.5 = \mathbf{1.395}$

14.110

A distributor of bottled propane, C_3H_8, needs to bring propane from 350 K, 100 kPa to saturated liquid at 290 K in a steady flow process. If this should be accomplished in a reversible setup given the surroundings at 300 K, find the ratio of the volume flow rates $\dot{V}_{in}/\dot{V}_{out}$, the heat transfer and the work involved in the process.

From Table A.2: $T_{ri} = \frac{350}{369.8} = 0.946$, $P_{ri} = \frac{0.1}{4.25} = 0.024$

From D.1, D.2 and D.3,

$Z_i = 0.99$

$(h_i^* - h_i) = 0.1886 \times 369.8 \times 0.03 = 2.1$ kJ/kg

$(s_i^* - s_i) = 0.1886 \times 0.02 = 0.0038$ kJ/kg K

$T_{re} = \frac{290}{369.8} = 0.784$, and $x = 0$

From D.1, D.2 and D.3,

$P_{re} = 0.22$, $P_e = 0.22 \times 4.25 = 0.935$ MPa and $Z_e = 0.036$

$(h_e^* - h_e) = 0.1886 \times 369.8 \times 4.57 = 318.6$ kJ/kg

$(s_e^* - s_e) = 0.1886 \times 5.66 = 1.0672$ kJ/kg K

$(h_e^* - h_i^*) = 1.679(290 - 350) = -100.8$ kJ/kg

$(s_e^* - s_i^*) = 1.679 \ln \frac{290}{350} - 0.1886 \ln \frac{0.935}{0.1} = -0.7373$ kJ/kg K

$(h_e - h_i) = -318.6 - 100.8 + 2.1 = -417.3$ kJ/kg

$(s_e - s_i) = -1.0672 - 0.7373 + 0.0038 = -1.8007$ kJ/kg K

$$\frac{\dot{V}_{in}}{\dot{V}_{out}} = \frac{Z_i T_i / P_i}{Z_e T_e / P_e} = \frac{0.99}{0.036} \times \frac{350}{290} \times \frac{0.935}{0.1} = \mathbf{310.3}$$

$w^{rev} = (h_i - h_e) - T_0(s_i - s_e) = 417.3 - 300(1.8007) =$ **-122.9 kJ/kg**

$q^{rev} = (h_e - h_i) + w^{rev} = -417.3 - 122.9 =$ **-540.2 kJ/kg**

Mixtures

14.111

A 2 kg mixture of 50% argon and 50% nitrogen by mole is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) Kays rule with generalized compressibility charts.

a) Ideal gas mixture

Eq.13.5: $M_{mix} = \sum y_i M_i = 0.5 \times 39.948 + 0.5 \times 28.013 = 33.981$

$$V = \frac{m\bar{R}T}{M_{mix}P} = \frac{2 \times 8.3145 \times 180}{33.981 \times 2000} = \mathbf{0.044\ m^3}$$

b) Kay's rule Eq.14.86

$P_{c\ mix} = 0.5 \times 4.87 + 0.5 \times 3.39 = 4.13$ MPa

$T_{c\ mix} = 0.5 \times 150.8 + 0.5 \times 126.2 = 138.5$ K

Reduced properties: $P_r = \frac{2}{4.13} = 0.484, \quad T_r = \frac{180}{138.5} = 1.30$

Fig. D.1: $Z = 0.925$

$$V = Z \frac{m\bar{R}T}{M_{mix}P} = 0.925 \times 0.044 = \mathbf{0.0407\ m^3}$$

14.112

A 2 kg mixture of 50% argon and 50% nitrogen by mass is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) van der Waals equation of state with a, b for a mixture?

a) Ideal gas mixture

Eq.13.15: $R_{mix} = \sum c_i R_i = 0.5 \times 0.2081 + 0.5 \times 0.2968 = 0.25245$ kJ/kg K

$$V = \frac{mR_{mix}T}{P} = \frac{2 \times 0.25245 \times 180}{2000} = \mathbf{0.0454\ m^3}$$

b) van der Waals equation of state. before we can do the parameters a, b for the mixture we need the individual component parameters.

$$a_{Ar} = \frac{27}{64}\frac{R^2T_c^2}{P_c} = \frac{27}{64}\frac{(0.2081 \times 150.8)^2}{4870} = 0.08531$$

$$a_{N2} = \frac{27}{64}\frac{R^2T_c^2}{P_c} = \frac{27}{64}\frac{(0.2968 \times 126.2)^2}{3390} = 0.17459$$

$$b_{Ar} = \frac{RT_c}{8P_c} = \frac{0.2081 \times 150.8}{8 \times 4870} = 0.000\,805$$

$$b_{N2} = \frac{RT_c}{8P_c} = \frac{0.2968 \times 126.2}{8 \times 3390} = 0.001\,381$$

Now the mixture parameters are from Eq.14.87

$$a_{mix} = \left(\sum c_i\, a_i^{1/2}\right)^2 = (0.5 \times \sqrt{0.08531} + 0.5 \times \sqrt{0.17459})^2 = 0.126$$

$$b_{mix} = \sum c_i b_i = 0.5 \times 0.000\,805 + 0.5 \times 0.001\,381 = 0.001\,093$$

Using now Eq.14.52: $$P = \frac{RT}{v - b} - \frac{a}{v^2}$$

$$2000 = \frac{0.25245 \times 180}{v - 0.001\,093} - \frac{0.126}{v^2}$$

By trial and error we find the specific volume, $v = 0.02097$ m^3/kg

$$V = mv = \mathbf{0.04194\ m^3}$$

14.113

A 2 kg mixture of 50% argon and 50% nitrogen by mass is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) Redlich Kwong equation of state with a, b for a mixture.

a) Ideal gas mixture

Eq.13.15: $R_{mix} = \sum c_i R_i = 0.5 \times 0.2081 + 0.5 \times 0.2968 = 0.25245$ kJ/kg K

$$V = \frac{mR_{mix}T}{P} = \frac{2 \times 0.25245 \times 180}{2000} = \mathbf{0.0454\ m^3}$$

b) Redlich Kwong equation of state. Before we can do the parameters a, b for the mixture we need the individual component parameters, Eq.14.58, 13.59.

$$a_{Ar} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2081^2 \times 150.8^{2.5}}{4870} = 1.06154$$

$$a_{N2} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2968^2 \times 126.2^{2.5}}{3390} = 1.98743$$

$$b_{Ar} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.2081 \times 150.8}{4870} = 0.000\,558$$

$$b_{N2} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.2968 \times 126.2}{3390} = 0.000\,957$$

Now the mixture parameters are from Eq.14.87

$$a_{mix} = \left(\sum c_i a_i^{1/2}\right)^2 = (0.5 \times \sqrt{1.06154} + 0.5 \times \sqrt{1.98743})^2 = 1.4885$$

$$b_{mix} = \sum c_i b_i = 0.5 \times 0.000\,558 + 0.5 \times 0.000\,957 = 0.000\,758$$

Using now Eq.14.57: $$P = \frac{RT}{v - b} - \frac{a}{v(v + b)T^{1/2}}$$

$$2000 = \frac{0.25245 \times 180}{v - 0.000\,758} - \frac{1.4885}{v(v + 0.000\,758)\ 180^{1/2}}$$

By trial and error we find the specific volume, $v = 0.02102\ m^3/kg$

$$V = mv = \mathbf{0.04204\ m^3}$$

14.114

A modern jet engine operates so that the fuel is sprayed into air at a P, T higher than the fuel critical point. Assume we have a rich mixture of 50% n-octane and 50% air by mole at 500 K and 3.5 MPa near the nozzle exit. Do I need to treat this as a real gas mixture or is an ideal gas assumption reasonable? To answer find Z and the enthalpy departure for the mixture assuming Kay's rule and the generalized charts.

The mole fractions are:

$$y_{C8H18} = 0.5, \quad y_{N2} = 0.5 \times 0.79 = 0.395, \quad y_{O2} = 0.5 \times 0.21 = 0.105$$

Eq.13.5:

$$M_{mix} = \sum y_i M_i = 0.5 \times 114.232 + 0.395 \times 28.013 + 0.105 \times 31.999 = 71.541$$

Kay's rule Eq.14.86

$$P_{c\,mix} = 0.5 \times 2.49 + 0.395 \times 3.39 + 0.105 \times 5.04 = 3.113 \text{ MPa}$$

$$T_{c\,mix} = 0.5 \times 568.8 + 0.395 \times 126.2 + 0.105 \times 154.6 = 350.5 \text{ K}$$

Reduced properties: $P_r = \frac{3.5}{3.113} = 1.124, \quad T_r = \frac{500}{350.5} = 1.427$

Fig. D.1: **Z = 0.87** I must treat it as a real gas mixture.

Fig. D.2 $h^* - h = 0.70 \times RT_c = 0.70 \times \frac{8.3145}{71.541} \times 350.5 = \mathbf{28.51\ kJ/kg}$

14.115

R-410a is a 1:1 mass ratio mixture of R-32 and R-125. Find the specific volume at 20^{o}C, 1200 kPa using Kays rule and the generalized charts and compare to Table B.4

Kay's rule Eq.14.86

$$P_{c\ mix} = 0.5 \times 5.78 + 0.5 \times 3.62 = 4.70 \text{ MPa}$$

$$T_{c\ mix} = 0.5 \times 351.3 + 0.5 \times 339.2 = 345.25 \text{ K}$$

Reduced properties: $P_r = \frac{1.2}{4.70} = 0.255, \quad T_r = \frac{293.15}{345.25} = 0.849$

Table A.5: R = 0.1145 kJ/kg-K or compute from mix

Fig. D.1: Z = 0.85

$v = ZRT/P = 0.85 \times 0.1145 \times 293.15 / 1200 =$ **0.0238 m^3/kg**

Table B.4: v = **0.02260 m^3/kg**

14.116

A mixture of 60% ethylene and 40% acetylene by moles is at 6 MPa, 300 K. The mixture flows through a preheater where it is heated to 400 K at constant P. Using the Redlich Kwong equation of state with a, b for a mixture find the inlet specific volume. Repeat using Kays rule and the generalized charts.

To do the EOS we need the gas constant, so from Eq.13.5 we get

$$M_{mix} = \sum y_i\, M_i = 0.6 \times 28.054 + 0.4 \times 26.068 = 27.26$$

$$R_{mix} = 8.3145/27.26 = 0.305 \text{ kJ/kg K}$$

Redlich Kwong EOS the individual component parameters, Eq.14.58, 14.59.

$$a_{C2H4} = 0.42748 \frac{R^2T_c^{5/2}}{P_c} = 0.42748 \frac{0.2964^2 \times 282.4^{2.5}}{5040} = 9.9863$$

$$a_{C2H2} = 0.42748 \frac{R^2T_c^{5/2}}{P_c} = 0.42748 \frac{0.3193^2 \times 308.3^{2.5}}{6140} = 11.8462$$

$$b_{C2H4} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.2964 \times 282.4}{5040} = 0.001\,439$$

$$b_{C2H2} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.3193 \times 308.3}{6140} = 0.001\,389$$

Now the mixture parameters are from Eq.14.87 so we need the mass fractions

$$c_{C2H4} = \frac{y\,M}{M_{mix}} = \frac{0.6 \times 28.054}{27.26} = 0.6175, \qquad c_{C2H4} = 1 - c_{C2H4} = 0.3825$$

$$a_{mix} = \left(\sum c_i\, a_i^{1/2}\right)^2 = (0.6175 \times \sqrt{9.9863} + 0.3825 \times \sqrt{11.8462})^2 = 10.679$$

$$b_{mix} = \sum c_i\, b_i = 0.6175 \times 0.001\,439 + 0.3825 \times 0.001\,389 = 0.001\,42$$

Using now Eq.14.57: $P = \dfrac{RT}{v - b} - \dfrac{a}{v(v + b)T^{1/2}}$

$$6000 = \frac{0.305 \times 300}{v - 0.001\,42} - \frac{10.679}{v(v + 0.001\,42)\,300^{1/2}}$$

By trial and error we find the specific volume, v = **0.006683 m^3/kg**

Kay's rule Eq.14.86

$$P_{c\,mix} = 0.6 \times 5.04 + 0.4 \times 6.14 = 5.48 \text{ MPa}$$

$$T_{c\,mix} = 0.6 \times 282.4 + 0.4 \times 308.3 = 292.8 \text{ K}$$

Reduced properties: $P_r = \dfrac{6}{5.48} = 1.095, \quad T_r = \dfrac{300}{292.8} = 1.025$

Fig. D.1: Z = 0.4 (difficult to read)

v = ZRT/P = 0.4 × 0.305 × 300 / 6000 = **0.0061 m^3/kg**

14.117

For the previous problem, find the specific heat transfer using Kay's rule and the generalized charts.

To do the EOS we need the gas constant, so from Eq.12.5 we get

$$M_{mix} = \sum y_i M_i = 0.6 \times 28.054 + 0.4 \times 26.068 = 27.26$$

$$R_{mix} = 8.3145/27.26 = 0.305 \text{ kJ/kg K}$$

$$c_{C2H4} = \frac{y\,M}{M_{mix}} = \frac{0.6 \times 28.054}{27.26} = 0.6175, \qquad c_{C2H4} = 1 - c_{C2H4} = 0.3825$$

$$C_{P\,mix} = \sum c_i C_{P\,i} = 0.6175 \times 1.548 + 0.3825 \times 1.699 = 1.606 \text{ kJ/kg K}$$

Kay's rule Eq.14.86

$$P_{c\,mix} = 0.6 \times 5.04 + 0.4 \times 6.14 = 5.48 \text{ MPa}$$

$$T_{c\,mix} = 0.6 \times 282.4 + 0.4 \times 308.3 = 292.8 \text{ K}$$

Reduced properties 1: $P_{r1} = \frac{6}{5.48} = 1.095, \quad T_{r1} = \frac{300}{292.8} = 1.025$

Fig. D.1: $\left(h_1^* - h_1\right) = 2.1 \times RT_c = 2.1 \times 0.305 \times 292.8 = 187.5$ kJ/kg

Reduced properties 2: $P_{r2} = \frac{6}{5.48} = 1.095, \quad T_{r2} = \frac{400}{292.8} = 1.366$

Fig. D.1: $\left(h_2^* - h_2\right) = 0.7 \times RT_c = 0.7 \times 0.305 \times 292.8 = 62.5$ kJ/kg

The energy equation gives

$$_1q_2 = (h_2 - h_1) = \left(h_2 - h_2^*\right) + \left(h_2^* - h_1^*\right) + \left(h_1^* - h_1\right)$$

$$= -62.5 + 1.606\,(400 - 300) + 187.5$$

$$= \mathbf{285.6 \text{ kJ/kg mix}}$$

14.118

The R-410a in Problem 14.115 is flowing through a heat exchanger with an exit at 120°C, 1200 kPa. Find the specific heat transfer using Kays rule and the generalized charts and compare to solution using Table B.4

$R_{mix} = 0.5 \times 0.1598 + 0.5 \times 0.06927 = 0.1145$ kJ/kg-K,

$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.5 \times 0.822 + 0.5 \times 0.791 = 0.8065$ kJ/kg K

Kay's rule Eq.14.86

$P_{c\ mix} = 0.5 \times 5.78 + 0.5 \times 3.62 = 4.70$ MPa

$T_{c\ mix} = 0.5 \times 351.3 + 0.5 \times 339.2 = 345.25$ K

Reduced properties 1: $P_{r1} = \frac{1.2}{4.70} = 0.255, \quad T_{r1} = \frac{293.15}{345.25} = 0.849$

Fig. D.1: $\left(h_1^* - h_1\right) = 0.4 \times RT_c = 0.4 \times 0.1145 \times 345.25 = 15.81$ kJ/kg

Reduced properties 1: $P_{r2} = \frac{1.2}{4.70} = 0.255, \quad T_{r2} = \frac{393.15}{345.25} = 1.139$

Fig. D.1: $\left(h_2^* - h_2\right) = 0.2 \times RT_c = 0.2 \times 0.1145 \times 345.25 = 7.906$ kJ/kg

The energy equation gives

$$
\begin{aligned}
{}_1q_2 &= (h_2 - h_1) = \left(h_2 - h_2^*\right) + \left(h_2^* - h_1^*\right) + \left(h_1^* - h_1\right) \\
&= -7.906 + 0.8065\,(120 - 20) + 15.81 \\
&= \mathbf{88.55\ kJ/kg\ mix}
\end{aligned}
$$

Table B.4.2: $q = h_2 - h_1 = 393.13 - 290.51 =$ **102.62 kJ/kg**

The main difference is in the value of specific heat, about 1 kJ/kg-K at the avg. T, whereas it is 0.8 kJ/kg-K at 25°C.

14.119

Saturated-liquid ethane at T_1 = 14°C is throttled into a steady flow mixing chamber at the rate of 0.25 kmol/s. Argon gas at T_2 = 25°C, P_2 = 800 kPa, enters the chamber at the rate of 0.75 kmol/s. Heat is transferred to the chamber from a heat source at a constant temperature of 150°C at a rate such that a gas mixture exits the chamber at T_3 = 120°C, P_3 = 800 kPa. Find the rate of heat transfer and the rate of entropy generation.

Argon, $T_{a2} = 25^oC$, $P_2 = 800$ kPa, $\dot{n}_2 = 0.75$ kmol/s

$T_{ca} = 150$ K, $P_{ca} = 4.87$ MPa, $M_a = 39.948$ kg/kmol, $C_{pa} = 0.52$ kJ/kg K

$\bar{h}_{a3} - \bar{h}_{a2} = M_a C_{pa}(T_3 - T_{a2}) = 1973.4$ kJ/kmol

Inlet: Ethane, $T_{b1} = 14^oC$, sat. liq., $x_{b1} = 0$, $\dot{n}_1 = 0.25$ kmol/s

$T_{cb} = 305.4$ K, $P_{cb} = 4.88$ MPa, $M_b = 30.07$ kg/kmol, $C_{pb} = 1.766$ kJ/kg-K

$T_{r1} = 0.94$, $P_{b1} = P_{r1}P_{cb} = 0.69 \times 4880 = 3367$ kPa

$\bar{h}^*_{b1} - \bar{h}_{b1} = 3.81\ \bar{R}T_{cb} = 9674.5$ kJ/kmol, $\bar{s}^*_{b1} - \bar{s}_{b1} = 3.74\ \bar{R} = 31.1$

$\bar{h}^*_{b3} - \bar{h}^*_{b1} = M_b C_{pb}(T_3 - T_{b1}) = 5629.6$ kJ/kmol

Exit: Mix, $T_3 = 120^oC$, $P_3 = 800$ kPa consider this an ideal gas mixture.

Energy Eq.: $\dot{n}_1\bar{h}_{b1} + \dot{n}_2\bar{h}_{a2} + \dot{Q} = \dot{n}_3\bar{h}_3 = \dot{n}_1\bar{h}_{b3} + \dot{n}_2\bar{h}_{a3}$

$\dot{Q} = \dot{n}_1(\bar{h}_{b3} - \bar{h}_{b1}) + \dot{n}_2(\bar{h}_{a3} - \bar{h}_{a2}) = 0.25\ (5629.6 + 9674.5) + 0.75(1973.4)$

$= \mathbf{5306\ kW}$

Entropy Eq.: $\dot{S}_{gen} = \dot{n}_1(\bar{s}_{b3} - \bar{s}_{b1}) + \dot{n}_2(\bar{s}_{a3} - \bar{s}_{a2}) - \dot{Q}/T_H$; $T_H = 150^oC$

$y_a = \dot{n}_2/\dot{n}_{tot} = 0.75$; $y_b = \dot{n}_1/\dot{n}_{tot} = 0.25$

$$\bar{s}_{a3} - \bar{s}_{a2} = M_a C_{pa} \ln\frac{T_3}{T_{a2}} - \bar{R}\ln\frac{y_a P_3}{P_{a2}} = 8.14 \text{ kJ/kmol-K}$$

$$\bar{s}_{b3} - \bar{s}_{b1} = M_b C_{pb} \ln\frac{T_3}{T_{b1}} - \bar{R}\ln\frac{y_b P_3}{P_{b1}} + \bar{s}^*_{b1} - \bar{s}_{b1} =$$

$$= 40.172 + 31.1 = 71.27 \text{ kJ/kmol K}$$

$\dot{S}_{gen} = 0.25 \times 71.27 + 0.75 \times 8.14 - 5306 / 423 = \mathbf{11.38\ kW/K}$

14.120

One kmol/s of saturated liquid methane, CH_4, at 1 MPa and 2 kmol/s of ethane, C_2H_6, at 250°C, 1 MPa are fed to a mixing chamber with the resultant mixture exiting at 50°C, 1 MPa. Assume that Kay's rule applies to the mixture and determine the heat transfer in the process.

Control volume the mixing chamber, inlet CH_4 is 1, inlet C_2H_6 is 2 and the exit state is 3. Energy equation is

$$\dot{Q}_{CV} = \dot{n}_3 \bar{h}_3 - \dot{n}_1 \bar{h}_1 - \dot{n}_2 \bar{h}_2$$

Select the ideal gas reference temperature to be T_3 and use the generalized charts for all three states.

$P_{r1} = P_{rsat} = 1/4.60 = 0.2174 \Rightarrow \quad T_{rsat} = 0.783,$

$T_1 = 0.783 \times 190.4 = 149.1 \text{ K}, \quad \Delta h_1 = 4.57$

$P_{r2} = 1/4.88 = 0.205, \quad T_{r2} = 523/305.4 = 1.713, \quad \Delta h_2 = 0.08$

$\bar{h}_1 = \bar{C}_1(T_1 - T_3) - \Delta h_1 \bar{R} T_c = 36.15(149.1 - 323.2) - 4.57 \times 8.3145 \times 190.4$
$= -13528 \text{ kJ/kmol}$

$\bar{h}_2 = \bar{C}_2(T_2 - T_3) - \Delta h_2 \bar{R} T_c = 53.11(250 - 50) - 0.08 \times 8.3145 \times 305.4$
$= 10\,419 \text{ kJ/kmol}$

Kay's rule Eq.14.86

$T_{cmix} = (1 \times 190.4 + 2 \times 305.4)/3 = 267.1 \text{ K}$

$P_{cmix} = (1 \times 4.60 + 2 \times 4.88)/3 = 4.79 \text{ MPa}$

$T_{r3} = 323.2/267.1 = 1.21 \;, \quad P_{r3} = 1/4.79 = 0.21, \quad \Delta h_3 = 0.15$

$\bar{h}_3 = 0 - 0.15 \times 267.1 \times 8.3145 = -333 \text{ kJ/kmol}$

$\dot{Q}_{CV} = 3(-333) - 1(-13528) - 2(10\,419) =$ **- 8309 kW**

14.121

A cylinder/piston contains a gas mixture, 50% CO2 and 50% C2H6 (mole basis) at 700 kPa, 35°C, at which point the cylinder volume is 5 L. The mixture is now compressed to 5.5 MPa in a reversible isothermal process. Calculate the heat transfer and work for the process, using the following model for the gas mixture:

a. Ideal gas mixture.

b. Kay's rule and the generalized charts.

a) Ideal gas mixture

$U_2 - U_1 = mC_{v\,mix}(T_2 - T_1) = 0$

$Q_{12} = W_{12} = \int P\,dV = P_1V_1 \ln(V_2/V_1) = - P_1V_1 \ln(P_2/P_1)$

$= - 700 \times 0.005 \ln(5500/700) = \mathbf{-7.71\ kJ}$

b) Kay's rule

$T_{cmix} = 0.5 \times 304.1 + 0.5 \times 305.4 = 304.75\ \text{K}$

$P_{cmix} = 0.5 \times 7.38 + 0.5 \times 4.88 = 6.13\ \text{MPa}$

$T_{r1} = 308.15/304.75 = 1.011, \quad P_{r1} = 0.7/6.13 = 0.1142$

$Z_1 = 0.96, \quad \Delta h_1 = 0.12, \quad \Delta s_1 = 0.08$

$n = P_1V_1/Z_1\bar{R}\,T_1 = \dfrac{700*0.005}{0.962*8.3145*308.15} = 0.00142\ \text{kmol}$

$T_{r2} = T_{r1}\ , \quad P_{r2} = 5.5/6.13 = 0.897,$

$Z_2 = 0.58, \quad \Delta h_2 = 1.35, \quad \Delta s_2 = 1.0$

$\bar{h}_2 - \bar{h}_1 = (\bar{h}_2 - \bar{h}_1) - \bar{R}\,T_c(\Delta h_2 - \Delta h_1)$

$= 0 - 8.3145 \times 304.75(1.35 - 0.12) = - 3117$

$\bar{u}_2 - \bar{u}_1 = \bar{h}_2 - \bar{h}_1 + \bar{R}T(Z_1 - Z_2) = - 3117$

$+ 8.3145 \times 308.15(0.96 - 0.58) = -2143\ \text{kJ/kmol}$

$Q_{12} = nT(\bar{s}_2 - \bar{s}_1)_T = 0.00142 \times 308.15 \times 8.3145[\ 0 - \ln(5.5/0.7) -1.0$

$+ 0.08\ ] = \mathbf{-\ 10.85\ kJ}$

$W_{12} = Q_{12} - n(\bar{u}_2 - \bar{u}_1) = -10.85 - 0.00142(-2143) = \mathbf{-\ 7.81\ kJ}$

14.122

A cylinder/piston contains a gas mixture, 50% CO2 and 50% C2H6 (mole basis) at 700 kPa, 35°C, at which point the cylinder volume is 5 L. The mixture is now compressed to 5.5 MPa in a reversible isothermal process. Calculate the heat transfer and work for the process, using the following model for the gas mixture:

a. Ideal gas mixture.

b. The van der Waals equation of state.

a) Ideal gas mixture

$$U_2 - U_1 = mC_{v\,mix}(T_2 - T_1) = 0$$

$$Q_{12} = W_{12} = \int P\,dV = P_1V_1 \ln(V_2/V_1) = - P_1V_1 \ln(P_2/P_1)$$

$$= - 700 \times 0.005 \ln(5500/700) = \mathbf{-7.71\ kJ}$$

b) van der waal's equation

For CO_2 :

$$b = \bar{R}\,T_c/8P_c = 8.3145 \times 304.1/8 \times 7380 = 0.04282$$

$$a = 27\,P_c\,b^2 = 27 \times 7380 \times 0.04282^2 = 365.45$$

For C_2H_6 :

$$b = \bar{R}\,T_c/8P_c = 8.3145 \times 305.4/8 \times 4880 = 0.06504$$

$$a = 27\,P_c\,b^2 = 27 \times 4880 \times 0.06504^2 = 557.41$$

$$a_{mix} = (0.5\sqrt{365.45} + 0.5\sqrt{557.41})^2 = 456.384$$

$$b_{mix} = 0.5 \times 0.04282 + 0.5 \times 0.06504 = 0.05393$$

$$\frac{8.3145*308.2}{\bar{v}_1 - 0.05393} - \frac{456.384}{\bar{v}_1^2} - 700 = 0$$

By trial and error: $\bar{v}_1 = 3.5329\ m^3/kmol$

$$\frac{8.3145*308.2}{\bar{v}_2 - 0.05393} - \frac{456.384}{\bar{v}_2^2} - 5500 = 0$$

By trial and error: $\bar{v}_2 = 0.2815\ m^3/kmol$

$$n = V_1/\bar{v}_1 = 0.005/3.5329 = 0.00142$$

$$Q_{12} = nT(\bar{s}_2 - \bar{s}_1)_T = n\,\bar{R}\,T \ln\frac{\bar{v}_2 - b}{\bar{v}_1 - b}$$

$$= 0.00142 \times 8.3145 \times 308.2 \ln\frac{0.2815 - 0.05392}{3.5329 - 0.05392} = \mathbf{-\ 9.93\ kJ}$$

$$U_2\text{-}U_1 = 0.00142 \times 456.39(3.5329^{-1} - 0.2815^{-1}) = -2.12\ kJ$$

$$Q_{12} = U_2\text{-}U_1 + W_{12} \;\Rightarrow\; W_{12} = -9.93 -(-2.12) = \mathbf{-7.81\ kJ}$$

Helmholtz EOS

14.123

Verify that the ideal gas part of Helmholtz function substituted in Eq.14.86 does lead to the ideal gas law as in note after Eq.14.96.

The ideal gas Helmholtz function is from its definition, see eq.14.91,

$$a^* = u^* - Ts^* = h^* - RT - T\,s^*$$

We have

$$h^* = h_o^* + \int C_{Po}\, dT$$

$$s^* = s_o^* + \int \frac{C_{Po}}{T}\, dT - R \ln\left(\frac{\rho T}{\rho_o T_o}\right)$$

Now from eq.14.86 we need to look at $\left(\frac{\partial a^*}{\partial \rho}\right)_T$ so following 14.91

$$\left(\frac{\partial a^*}{\partial \rho}\right)_T = \left(\frac{\partial h^*}{\partial \rho}\right)_T - \left(\frac{\partial RT}{\partial \rho}\right)_T - T\left(\frac{\partial s^*}{\partial \rho}\right)_T$$

$$= 0 - 0 + TR\frac{\partial}{\partial \rho}\left(\ln \frac{\rho T}{\rho_o T_o}\right)_T$$

$$= RT / \rho$$

So then

$$\rho^2\left(\frac{\partial a^*}{\partial \rho}\right)_T = P = \rho RT \qquad \text{Ideal gas OK}$$

14.124

Gases like argon and neon have constant specific heats. Develop an expression for the ideal gas contribution to Helmholtz function in Eq.14.91 for these cases.

The ideal gas Helmholtz function is from its definition, see eq.14.91,

$$a^* = u^* - Ts^* = h^* - RT - T\,s^*$$

We have

$$h^* = h^*_o + \int C_{Po}\,dT = h^*_o + C_{Po}(T - T_o)$$

$$s^* = s^*_o + \int \frac{C_{Po}}{T}\,dT - R\ln\left(\frac{\rho T}{\rho_o T_o}\right)$$

$$= s^*_o + C_{Po}\ln(T/T_o) - R\ln\left(\frac{\rho T}{\rho_o T_o}\right)$$

So now we get

$$a^* = h^*_o + C_{Po}(T - T_o) - RT - T\,s^*$$

$$= h^*_o - T\,s^*_o + C_{Po}(T - T_o) - C_{Po}T\ln\left(\frac{T}{T_o}\right) - RT + RT\ln\left(\frac{\rho T}{\rho_o T_o}\right)$$

$$= C_o + C_1\,T - C_2\,T\ln\left(\frac{T}{T_o}\right) + RT\ln\left(\frac{\rho}{\rho_o}\right)$$

where

$$C_o = h^*_o - C_{Po}T_o\,; \qquad C_1 = C_{Po} - R - s^*_o\,; \qquad C_2 = C_{Po} - R$$

14.125

Use the equation of state in Example 14.3 and find an expression for isothermal changes in Helmholtz function between two states.

The EOS is

$$\frac{Pv}{RT} = 1 - C'\frac{P}{T^4} \qquad \text{or} \qquad v = \frac{RT}{P} - \frac{C}{T^3}$$

and we use the isothermal changes found in Ex.14.3 as

$$(h_2 - h_1)_T = -\frac{4C}{T^3}(P_2 - P_1)_T$$

$$(s_2 - s_1)_T = -R\ln\left(\frac{P_2}{P_1}\right)_T - \frac{3C}{T^4}(P_2 - P_1)_T$$

As Helmholtz function is: a = u – Ts; we get

$$(a_2 - a_1)_T = (u_2 - u_1)_T - T(s_2 - s_1)_T$$

$$= (h_2 - h_1)_T - (P_2v_2 - P_1v_1) - T(s_2 - s_1)_T$$

$$= -\frac{4C}{T^3}(P_2 - P_1)_T - [RT_2 - RT_1 - \frac{C}{T^3}(P_2 - P_1)_T\ ]$$

$$+ RT\ln\left(\frac{P_2}{P_1}\right)_T + \frac{3C}{T^3}(P_2 - P_1)_T$$

$$= -\frac{4C}{T^3}(P_2 - P_1)_T + \frac{C}{T^3}(P_2 - P_1)_T + RT\ln\left(\frac{P_2}{P_1}\right)_T + \frac{3C}{T^3}(P_2 - P_1)_T$$

This now reduces to the final answer

$$(a_2 - a_1)_T = RT\ln\left(\frac{P_2}{P_1}\right)_T$$

14.126

Find an expression for the change in Helmholtz function for a gas with an EOS as $P(v - b) = RT$.

From Eq.14.31 we get

$$du_T = [\, T \left(\frac{\partial P}{\partial T}\right)_v - P \,]\, dv_T = [\, T \left(\frac{R}{v - b}\right) - P \,]\, dv_T = [\, P - P]\, dv_T = 0$$

From eq.14.32 or Eq.14.34 we get

$$ds_T = -\left(\frac{\partial v}{\partial T}\right)_P dP_T = \left(\frac{\partial P}{\partial T}\right)_v dv_T$$

$$= -\frac{R}{P}\, dP_T = \frac{R}{v - b}\, dv_T$$

Now the changes in u and s can be integrated to find

$$(u_2 - u_1)_T = 0$$

$$(s_2 - s_1)_T = -R \ln \frac{P_2}{P_1} = R \ln \frac{v_2 - b}{v_1 - b}$$

to this we now need to add the variation due to T. For this we get

$$(u_2 - u_1)_v = \int_1^2 C_v \, dT \qquad \text{and} \qquad (s_2 - s_1)_v = \int_1^2 \frac{C_v}{T}\, dT$$

Finally since the Helmholtz function contains the product Ts we need the absolute value of the entropy so

$$s_1 = s_o + \int_o^1 \frac{C_v}{T}\, dT + R \ln \frac{v_1 - b}{v_o - b}$$

Then the change in Helmholtz function becomes

$$a_2 - a_1 = u_2 - u_1 - T_2 s_2 + T_1 s_1 = u_2 - u_1 - T_2(s_2 - s_1) + (T_1 - T_2)\, s_1$$

$$= \int_1^2 C_v \, dT - T_2[\, \int_1^2 \frac{C_v}{T}\, dT + R \ln \frac{v_2 - b}{v_1 - b}\,] + (T_1 - T_2)\, s_1$$

14.127

Assume a Helmholtz equation as

$$a^* = C_o + C_1\, T - C_2\, T \ln \left(\frac{T}{T_o}\right) + RT \ln \left(\frac{\rho}{\rho_o}\right)$$

where C_o, C_1, C_2 are constants and T_o and ρ_o reference values for temperature and density, see Eqs. 14.91-94. Find the properties P, u and s from this expression. Is anything assumed for this particular form.

Given Helmholtz function we can find the pressure and entropy from Eq. 14.21 and then u from the definition: $a = u - Ts$.

$$a^* = C_o + C_1\, T - C_2\, T \ln \left(\frac{T}{T_o}\right) - RT \ln \left(\frac{v}{v_o}\right)$$

$$\left(\frac{\partial a^*}{\partial v}\right)_T = - RT \frac{\partial}{\partial v} \ln \left(\frac{v}{v_o}\right) = - RT / v$$

$$P = -\left(\frac{\partial a^*}{\partial v}\right)_T = RT / v \qquad \text{i.e. Ideal gas.}$$

$$s = -\left(\frac{\partial a^*}{\partial T}\right)_v = - C_1 + C_2 \ln \left(\frac{T}{T_o}\right) + C_2 + R \ln \left(\frac{v}{v_o}\right)$$

Notice how it looks like Eq.8.17.

$$u = a + Ts = C_o + C_1\, T - C_2\, T \ln \left(\frac{T}{T_o}\right) - RT \ln \left(\frac{v}{v_o}\right)$$

$$- C_1T + C_2T \ln \left(\frac{T}{T_o}\right) + C_2T + RT \ln \left(\frac{v}{v_o}\right)$$

$$= C_o + C_2T$$

We find that u is linear in T.

Not only is it ideal gas but it also has constant specific heats.

Review Problems

14.128

An uninsulated piston/cylinder contains propene, C_3H_6, at ambient temperature, 19°C, with a quality of 50% and a volume of 10 L. The propene now expands very slowly until the pressure in the cylinder drops to 460 kPa. Calculate the mass of propene, the work, and heat transfer for this process.

Propene C_3H_6: $T_1 = 19^oC = 292.2$ K, $x_1 = 0.50$, $V_1 = 10$ L

From Fig. D.1: $T_{r1} = 292.2/364.9 = 0.80$,

$P_{r1} = P_{r\ sat} = 0.25$, $P_1 = 0.25 \times 4.6 = 1.15$ MPa

From D.1: $Z_1 = 0.5 \times 0.04 + 0.5 \times 0.805 = 0.4225$

$$m = \frac{P_1V_1}{Z_1RT_1} = \frac{1150\times0.010}{0.4225\times0.197\ 58\times292.2} = \mathbf{0.471\ kg}$$

Assume reversible and isothermal process (slow, no friction, not insulated)

$${}_1Q_2 = m(u_2-u_1) + {}_1W_2$$

$${}_1W_2 = \int_1^2 PdV \text{ (cannot integrate)}; \qquad {}_1Q_2 = \int_1^2 TdS = Tm(s_2-s_1)$$

From Figs. D.2 and D.3:

$$h^*_1 - h_1 = 0.19758 \times 364.9(0.5 \times 4.51 + 0.5 \times 0.46) = 179.2 \text{ kJ/kg}$$

$$(s^*_1 - s_1) = 0.197\ 58\ (0.5 \times 5.46 + 0.5 \times 0.39) = 0.5779 \text{ kJ/kg K}$$

The ideal gas change in h and s are

$$(h^*_2 - h^*_1) = 0 \text{ and } \quad (s^*_2 - s^*_1) = 0 - 0.197\ 58 \ln\frac{460}{1161} = +\ 0.1829 \text{ kJ/kg K}$$

At $T_{r2} = 0.80$, $P_{r2} = 0.10$, from D.1, D.2 and D.3, $Z_2 = 0.93$

$$(h^*_2 - h_2) = 0.197\ 58 \times 364.9 \times 0.16 = 11.5 \text{ kJ/kg}$$

$$(s^*_2 - s_2) = 0.197\ 58 \times 0.13 = 0.0257 \text{ kJ/kg K}$$

Now we can do the change in s and h from state 1 to state 2

$$(s_2 - s_1) = -(s^*_2 - s_2) + (s^*_2 - s^*_1) + (s^*_1 - s_1)$$
$$= -0.0257 + 0.1829 + 0.5779 = 0.7351 \text{ kJ/kg K}$$

$$(h_2 - h_1) = -(h^*_2 - h_2) + (h^*_2 - h^*_1) + h^*_1 - h_1$$
$$= -11.5 + 0 + 179.2 = 167.7 \text{ kJ/kg}$$

The heat transfer is found from the second law

$_1q_2 = 292.2 \times 0.7351 = 214.8$ kJ/kg => $_1Q_2 = m\ _1q_2 =$ **101.2 kJ**

We need the internal energy in the energy equation

$u_2 - u_1 = (h_2 - h_1) + RT(Z_1 - Z_2) = 167.7 + 0.197\ 58 \times 292.2\ (0.4225 - 0.93)$

$= 138.4$ kJ/kg

$_1w_2 = {_1q_2} - (u_2 - u_1) = 214.8 - 138.4 = 76.4$ kJ/kg

$_1W_2 = m\ _1w_2 =$ **36.0 kJ**

14.129

An insulated cylinder fitted with a frictionless piston contains saturated-vapor carbon dioxide at 0°C, at which point the cylinder volume is 20 L. The external force on the piston is now slowly decreased, allowing the carbon dioxide to expand until the temperature reaches - 30°C. Calculate the work done by the CO_2 during this process.

CO_2: $T_C = 304.1$ K, $P_c = 7.38$ MPa, $C_p = 0.842$ kJ/kg-K, $R = 0.1889$ kJ/kg K

State 1: $T_1 = 0^oC$, sat. vap., $x_1 = 1.0$, $V_1 = 20$ L

$T_{r1} = 0.9$, $P_1 = P_{r1}P_C = 0.53 \times 7380 = 3911$ kPa, $Z_1 = Z_g = 0.67$

$$\left(h_1^* - h_1\right)_g = 0.9\ RT_C,\ \left(s_1^* - s_1\right)_g/R = 0.72,\quad m = \frac{P_1V_1}{Z_1RT_1} = 2.262 \text{ kg}$$

State 2: $T_2 = -30^oC$

$T_{r2} = 0.8$, $P_2 = P_{r2}P_c = 0.25 \times 7380 = 1845$ kPa

2^{nd} Law: $\Delta S_{net} = m(s_2 - s_1) - {}_1Q_2/T$; ${}_1Q_2 = 0$, $\Delta S_{net} = 0$

$$s_2 - s_1 = \left(s_2 - s_2^*\right) + \left(s_2^* - s_1^*\right) + \left(s_1^* - s_1\right) = 0$$

$$s_2^* - s_1^* = C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 0.044 \text{ kJ/kg-K},\quad s_1^* - s_1 = 0.136 \text{ kJ/kg-K}$$

$$s_2^* - s_2 = 0.180 \text{ kJ/kg K},\ \left(s_2^* - s_2\right)_f = 5.46\ R,\ \left(s_2^* - s_2\right)_g = 0.39\ R$$

$$\left(s_2^* - s_2\right) = (1-x_2)\left(s_2^* - s_2\right)_f + x_2\left(s_2^* - s_2\right)_g \rightarrow x_2 = 0.889$$

1^{st} Law: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$; ${}_1Q_2 = 0$, $u = h - Pv$

$$Z_2 = (1 - x_2)Z_f + x_2Z_g = 0.111 \times 0.04 + 0.889 \times 0.81 = 0.725;$$

$$(h_2 - h_1) = \left(h_2 - h_2^*\right) + \left(h_2^* - h_1^*\right) + \left(h_1^* - h_1\right)$$

$$h_2^* - h_1^* = C_p(T_2 - T_1) = -25.3 \text{ kJ/kg},\ \left(h_1^* - h_1\right) = 51.7 \text{ kJ/kg}$$

$$\left(h_2^* - h_2\right)_f = 4.51\ RT_C,\ \left(h_2^* - h_2\right)_g = 0.46\ RT_C$$

$$\left(h_2^* - h_2\right) = (1 - x_2)\left(h_2^* - h_2\right)_f + x_2\left(h_2^* - h_2\right)_g = 52.2 \text{ kJ/kg}$$

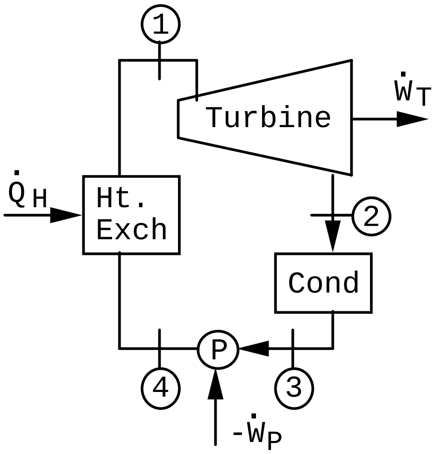
$$h_2 - h_1 = -52.2 - 25.3 + 51.7 = -25.8 \text{ kJ/kg}$$

$$u_2 - u_1 = (h_2 - h_1) - Z_2RT_2 + Z_1RT_1 = -25.8 - 0.725 \times 0.18892 \times 243.2 + 0.67 \times 0.18892 \times 273.2 = -24.5 \text{ kJ/kg}$$

${}_1W_2$ = **55.4 kJ**

14.130

A newly developed compound is being considered for use as the working fluid in a small Rankine-cycle power plant driven by a supply of waste heat. Assume the cycle is ideal, with saturated vapor at 200°C entering the turbine and saturated liquid at 20°C exiting the condenser. The only properties known for this compound are molecular weight of 80 kg/kmol, ideal gas heat capacity C_{PO}= 0.80 kJ/kg K and $T_C = 500$ K, $P_C = 5$ MPa. Calculate the work input, per kilogram, to the pump and the cycle thermal efficiency.



$T_1 = 200^oC = 473.2$ K, $x_1 = 1.0$

$T_3 = 20^oC = 293.2$ K, $x_3 = 0.0$

Properties known:

M = 80, $C_{PO} = 0.8$ kJ/kg K

$T_C = 500$ K, $P_C = 5.0$ MPa

$$T_{r1} = \frac{473.2}{500} = 0.946\ ,\quad T_{r3} = \frac{293.2}{500} = 0.586$$

R = R/M = 8.31451/80 = 0.10393 kJ/kg K

From Fig. D.1,

$P_{r1} = 0.72,\quad P_1 = 0.72 \times 5 = 3.6 \text{ MPa} = P_4$

$P_{r3} = 0.023, P_3 = 0.115 \text{ MPa} = P_2\ ,\quad Z_{F3} = 0.004$

$$v_{F3} = \frac{Z_{F3}RT_3}{P_3} = \frac{0.004 \times 0.10393 \times 293.2}{115} = 0.00106 \text{ m}^3\text{/kg}$$

$$w_P = - \int_3^4 vdP \approx v_{F3}(P_4 - P_3) = -0.00106(3600-115) = \mathbf{-3.7\ kJ/kg}$$

$q_H + h_4 = h_1$, but $h_3 = h_4 + w_P$ => $q_H = (h_1 - h_3) + w_P$

From Fig. D.2:

$(h^*_1 - h_1) = RT_C \times 1.25 = 0.103\,93 \times 500 \times 1.25 = 64.9$ kJ/kg

$(h^*_3 - h_3) = 0.103\,93 \times 500 \times 5.2 = 270.2$ kJ/kg

$(h^*_1 - h^*_3) = C_{P0}(T_1 - T_3) = 0.80(200-20) = 144.0$ kJ/kg

$(h_1\text{-}h_3) = -64.9 + 144.0 + 270.2 = 349.3$ kJ/kg

$q_H = 349.3 + (-3.7) = 345.6$ kJ/kg

Turbine, $(s_2 - s_1) = 0 = -(s_2^* - s_2)+(s_2^* - s_1^*) + (s_1^* - s_1)$

From Fig. D.3,

$(s_1^*\text{-}s_1) = 0.10393\times0.99 = 0.1029$ kJ/kg K

$$(s_2^*\text{-}s_1^*) = 0.80 \ln\frac{293.2}{473.2} - 0.103\,93 \ln\frac{115}{3600} = -0.0250$$

Substituting,

$s_2^*\text{-}s_2 = +0.1029 - 0.0250 = 0.0779 = (s_2^*\text{-}s_{F2}) - x_2s_{FG2}$

$0.0779 = 0.103\,93\times8.85 - x_2\times0.103\,93(8.85\text{-}0.06)$ => $x_2 = 0.922$

$(h_2^*\text{-}h_2) = (h_2^*\text{-}h_{F2}) - x_2h_{FG2}$

From Fig. D.2,

$h_{FG2} = 0.10393 \times 500\ (5.2\text{-}0.07) = 266.6$

$(h_2^*\text{-}h_2) = 270.2 -0.922 \times 266.6 = 25.0$

$w_T = (h_1\text{-}h_2) = -64.9 + 144.0 + 25.0 = 104.1$ kJ/kg

$$\eta_{TH} = \frac{w_{NET}}{q_H} = \frac{104.1\text{-}3.7}{345.6} = \mathbf{0.29}$$

14.131

An evacuated 100-L rigid tank is connected to a line flowing R-142b gas, chlorodifluoroethane, at 2 MPa, 100°C. The valve is opened, allowing the gas to flow into the tank for a period of time, and then it is closed. Eventually, the tank cools to ambient temperature, 20°C, at which point it contains 50% liquid, 50% vapor, by volume. Calculate the quality at the final state and the heat transfer for the process. The ideal-gas specific heat of R-142b is $C_p = 0.787$ kJ/kg K.

Rigid tank $V = 100$ L, $m_1 = 0$ Line: R-142b CH_3CClF_2

$M = 100.495$, $T_C = 410.3$ K, $P_C = 4.25$ MPa, $C_{P0} = 0.787$ kJ/kg K

$R = \bar{R}/M = 8.31451 / 100.495 = 0.082\,73$ kJ/kg K

Line $P_i = 2$ MPa, $T_i = 100$ °C, Flow in to $T_2 = T_0 = 20^oC$

$V_{LIQ\,2} = V_{VAP\,2} = 50$ L

Continuity: $m_i = m_2$; Energy: $Q_{CV} + m_i h_i = m_2 u_2 = m_2 h_2 - P_2 V$

From D.2 at i: $P_{ri} = 2 / 4.25 = 0.471$, $T_{ri} = 373.15 / 410.3 = 0.91$

$$(h_i^* - h_i) = 0.082\,73 \times 410.3 \times 0.72 = 24.4$$

$$(h_2^* - h_i^*) = C_{P0}(T_2 - T_i) = 0.787(20-100) = -63.0$$

From D.2: $T_{r2} = \dfrac{293.2}{410.3} = 0.715 \;\Rightarrow\; P_2 = 0.115 \times 4250 = 489$ kPa

sat. liq.: $Z_F = 0.02$, $(h^* - h_F) = RT_C \times 4.85 = 164.6$

sat. vap.: $Z_G = 0.88$, $(h^* - h_G) = RT_C \times 0.25 = 8.5$

$$m_{LIQ\,2} = \frac{P_2 V_{LIQ\,2}}{Z_F R T_2} = \frac{489 \times 0.050}{0.02 \times 0.082\,73 \times 293.2} = 50.4 \text{ kg}$$

$$m_{VAP\,2} = \frac{P_2 V_{VAP\,2}}{Z_G R T_2} = 1.15 \text{ kg}, \qquad m_2 = 51.55 \text{ kg}$$

$x_2 = m_{VAP\,2}/m_2 = 0.0223$

$$(h_2^* - h_2) = (1 - x_2)(h_2^* - h_{F2}) + x_2(h_2^* - h_{G2}) = 0.9777 \times 164.6 + 0.0223 \times 8.5 = 161.1$$

$$Q_{CV} = m_2(h_2 - h_i) - P_2 V = 51.55(-161.1 - 63.0 + 24.4) - 489 \times 0.10$$

$$= \mathbf{-10\,343 \text{ kJ}}$$

14.132

Saturated liquid ethane at 2.44 MPa enters a heat exchanger and is brought to 611 K at constant pressure, after which it enters a reversible adiabatic turbine where it expands to 100 kPa. Find the heat transfer in the heat exchanger, the turbine exit temperature and turbine work.

From D.2,

$P_{r1} = 2.44/4.88 = 0.50$, $T_{r1} = 0.89$, $T_1 = 0.89\times305.4 = 271.8$ K

$(h_1^*-h_1) = 0.2765\times305.4\times4.12 = 347.9$

$(h_2^*-h_1^*) = 1.766\ (611 - 271.8) = 599.0$

$P_{r2} = 0.50$, $T_{r2} = 611/305.4 = 2.00$

From D.2: $(h_2^*-h_2) = RT_c \times 0.14 = 0.2765\times305.4\times0.14 = 11.8$

$q = (h_2-h_1) = -11.8 + 599.0 + 347.9 =$ **935.1 kJ/kg**

From D.3,

$(s_2^*-s_2) = 0.2765\times0.05 = 0.0138$

$$(s_3^*-s_2^*) = 1.766 \ln \frac{T_3}{611} - 0.2765 \ln \frac{100}{2440}$$

Assume $T_3 = 368$ K , $T_{r3} = 1.205$

at $P_{r3} = 0.020$

$(s_3^*-s_2^*) = -0.8954 + 0.8833 = -0.0121$

From D.3,

$(s_3^*-s_3) = 0.2765\times0.01 = 0.0028$

$(s_3-s_2) = -0.0028 - 0.0121 + 0.0138 \approx 0$ OK

Therefore, $T_3 =$ **368 K**

From D.2,

$(h_3^*-h_3) = 0.2765\times305.4\times0.01 = 0.8$

$w = (h_2-h_3) = -11.8 + 1.766\ (611 - 368) + 0.8 =$ **418.1 kJ/kg**

14.133

A piston/cylinder initially contains propane at T = -7°C, quality 50%, and volume 10L. A valve connecting the cylinder to a line flowing nitrogen gas at T = 20°C, P = 1 MPa is opened and nitrogen flows in. When the valve is closed, the cylinder contains a gas mixture of 50% nitrogen, 50% propane on a mole basis at T = 20°C, P = 500 kPa. What is the cylinder volume at the final state, and how much heat transfer took place?

State 1: Propane, $T_1 = -7^oC$, $x_1 = 0.5$, $V_1 = 10$ L

$T_c = 369.8$ K, $P_c = 4.25$ kPa, $C_P = 1.679$ kJ/kg-K, $M = 44.097$ kg/kmol

Fig. D.1: $T_{r1} = 0.72$, $P_{r1} = 0.12$, $P_1 = P_{r1}P_c = 510$ kPa

Fig. D.1: $Z_{f1} = 0.020$, $Z_{g1} = 0.88$, $Z_1 = (1 - x_1)Z_{f1} + x_1Z_{g1} = 0.45$

$n_1 = P_1V_1/(Z_1\bar{R}T_1) = 510 \times 0.01/(0.45 \times 8.3145 \times 266.2) = 0.00512$ kmol

$\bar{h}_1 = \bar{h}^*_{1o} + \bar{C}_P(T_1 - T_o) + (\bar{h}_1 - \bar{h}^*_1)$; $\bar{h}^*_{1o} = 0$,

$(\bar{h}^*_1 - \bar{h}_1)_f /\bar{R}T_c = 4.79$, $(\bar{h}^*_1 - \bar{h}_1)_g /\bar{R}T_c = 0.25$

$\bar{h}^*_1 - \bar{h}_1 = (1 - x_1)(\bar{h}^*_1 - \bar{h}_1)_f + x_1(\bar{h}^*_1 - \bar{h}_1)_g = 7748$ kJ/kmol

$\bar{h}_1 = 0 + 1.679 \times 44.094(-7 - 20) - 7748 = -9747$ kJ/kmol

Inlet: Nitrogen, $T_i = 20^oC$, $P_i = 1.0$ MPa,

$T_c = 126.2$ K, $P_c = 3.39$ MPa, $C_{pn} = 1.042$ kJ/kg-K, $M = 28.013$ kg/kmol

$T_{ri} = 2.323$, $P_{ri} = 0.295$, $\bar{h}^*_i - \bar{h}_i = 0.06 \times 8.3145 \times 126.2 = 62.96$ kJ/kmol

$\bar{h}_i = \bar{h}^*_{io} + \bar{C}_{Pn}(T_i - T_o) + (\bar{h}_i - \bar{h}^*_i)$; $\bar{h}^*_{io} = 0$, $T_i - T_o = 0$

State 2: 50% Propane, 50% Nitrogen by mol, $T_2 = 20^oC$, $P_2 = 500$ kPa

$T_{cmix} = \Sigma y_iT_{ci} = 248$ K, $P_{cmix} = \Sigma y_iP_{ci} = 3.82$ MPa

$T_{r2} = 1.182$, $P_{r2} = 0.131$, $Z_2 = 0.97$, $(\bar{h}^*_2 - \bar{h}_2)/\bar{R}T_c = 0.06$

$\bar{h}_2 = \bar{h}^*_{2o} + \bar{C}_{Pmix}(T_2 - T_o) + (\bar{h}_2 - \bar{h}^*_2)$; $\bar{h}^*_{2o} = 0$, $T_2 - T_o = 0$

a) $n_i = n_1$ => $n_2 = n_1 + n_i = 0.1024$, $V_2 = n_2Z_2\bar{R}T_2/P_2 =$ **0.0484 m^3**

b) 1st Law: $Q_{cv} + n_i\bar{h}_i = n_2\bar{u}_2 - n_{21}\bar{u}_{21} + W_{cv}$; $\bar{u} = \bar{h} - P\bar{v}$

$W_{cv} = (P_1 + P_2)(V_2 - V_1)/2 = 19.88$ kJ

$Q_{cv} = n_2\bar{h}_2 - n_1\bar{h}_1 - n_i\bar{h}_i - P_2V_2 + P_1V_1 + W_{cv}$

$\bar{h}_i = -62.96$ kJ/kmol, $\bar{h}_2 = -123.7$ kJ/kmol, $Q_{cv} =$ **50.03 kJ**

14.134

A control mass of 10 kg butane gas initially at 80°C, 500 kPa, is compressed in a reversible isothermal process to one-fifth of its initial volume. What is the heat transfer in the process?

Butane C_4H_{10}: m = 10 kg, $T_1 = 80\ ^oC$, $P_1 = 500$ kPa

Compressed, reversible T = const, to $V_2 = V_1/5$

$$T_{r1} = \frac{353.2}{425.2} = 0.831, \; P_{r1} = \frac{500}{3800} = 0.132$$

From D.1 and D.3: $Z_1 = 0.92$, $(s^*_1 - s_1) = 0.143 \times 0.16 = 0.0230$

$$v_1 = \frac{Z_1RT_1}{P_1} = \frac{0.92 \times 0.143 \times 353.2}{500} = 0.09296\ m^3/kg$$

$v_2 = v_1/5 = 0.01859\ m^3/kg$

At $T_{r2} = T_{r1} = 0.831$

From D.1: $P_G = 0.325 \times 3800 = 1235$ kPa

sat. liq.: $Z_F = 0.05$, $(s^* - s_F) = R \times 5.08 = 0.7266$

sat. vap.: $Z_G = 0.775$, $(s^* - s_G) = R \times 0.475 = 0.0680$

Therefore

$$v_F = \frac{0.05 \times 0.143 \times 353.2}{1235} = 0.00205\ m^3/kg$$

$$v_G = \frac{0.775 \times 0.143 \times 353.2}{1235} = 0.0317\ m^3/kg$$

Since $v_F < v_2 < v_G \rightarrow x_2 = (v_2 - v_F)/(v_G - v_F) = 0.5578$

$$(s^*_2 - s_2) = (1 - x_2)(s^*_2 - s_{F2}) + x_2(s^*_2 - s_{G2})$$

$$= 0.4422 \times 0.7266 + 0.5578 \times 0.0680 = 0.3592\ kJ/kg\ K$$

$(s^*_2 - s^*_1) = C_{P0} \ln (T_2/T_1) - R \ln (P_2/P_1) = 0 - 0.143 \ln (1235/500) = -0.1293$

$(s_2 - s_1) = -0.3592 - 0.1293 + 0.0230 = -0.4655$ kJ/kg K

${}_1Q_2 = Tm(s_2 - s_1) = 353.2 \times 10\ (-0.4655) =$ **-1644 kJ**

14.135

An uninsulated compressor delivers ethylene, C_2H_4, to a pipe, $D = 10$ cm, at 10.24 MPa, 94°C and velocity 30 m/s. The ethylene enters the compressor at 6.4 MPa, 20.5°C and the work input required is 300 kJ/kg. Find the mass flow rate, the total heat transfer and entropy generation, assuming the surroundings are at 25°C.

$$T_{ri} = \frac{293.7}{282.4} = 1.040\ ,\ P_{ri} = \frac{6.4}{5.04} = 1.270$$

From D.2 and D.3,

$$(h^*_i - h_i) = 0.296\,37 \times 282.4 \times 2.65 = 221.8\ \text{kJ/kg}$$

$$(s^*_i - s_i) = 0.296\,37 \times 2.08 = 0.6164\ \text{kJ/kg K}$$

$$T_{re} = \frac{367.2}{282.4} = 1.30\ ,\ P_{re} = \frac{10.24}{5.04} = 2.032 \ \ \Rightarrow\ \ \text{From D.1:}\quad Z_e = 0.69$$

$$v_e = \frac{Z_e R T_e}{P_e} = \frac{0.69 \times 0.296\,37 \times 367.2}{10\,240} = 0.0073\ \text{m}^3/\text{kg}$$

$$A_e = \frac{\pi}{4} D_e^2 = 0.007\,85\ \text{m}^2 \quad \Rightarrow \quad \dot{m} = \frac{A_e \mathbf{V}_e}{v_e} = \frac{0.007\,85 \times 30}{0.0073} = \mathbf{32.26\ kg/s}$$

From D.2 and D.3,

$$(h^*_e - h_e) = 0.296\,37 \times 282.4 \times 1.6 = 133.9\ \text{kJ/kg}$$

$$(s^*_e - s_e) = 0.296\,37 \times 0.90 = 0.2667\ \text{kJ/kg K}$$

$$(h^*_e - h^*_i) = 1.5482(367.2 - 293.7) = 113.8$$

$$(s^*_e - s^*_i) = 1.5482 \ln \frac{367.2}{293.7} - 0.296\,37 \ln \frac{10.24}{6.4} = 0.2065$$

$$(h_e - h_i) = -133.9 + 113.8 + 221.8 = 201.7\ \text{kJ/kg}$$

$$(s_e - s_i) = -0.2667 + 0.2065 + 0.6164 = 0.5562\ \text{kJ/kg K}$$

First law:

$$q = (h_e - h_i) + KE_e + w = 201.7 + \frac{30^2}{2 \times 1000} - 300 = -97.9\ \text{kJ/kg}$$

$$\dot{Q}_{cv} = \dot{m}q = 32.26(-97.9) = \mathbf{-3158\ kW}$$

$$\dot{S}_{gen} = -\frac{\dot{Q}_{cv}}{T_o} + \dot{m}(s_e - s_i) = +\frac{3158}{298.2} + 32.26(0.5562) = \mathbf{28.53\ kW/K}$$

14.136

Consider the following reference state conditions: the entropy of real saturated liquid methane at −100°C is to be taken as 100 kJ/kmol K, and the entropy of hypothetical ideal gas ethane at −100°C is to be taken as 200 kJ/kmol K. Calculate the entropy per kmol of a real gas mixture of 50% methane, 50% ethane (mole basis) at 20°C, 4 MPa, in terms of the specified reference state values, and assuming Kay's rule for the real mixture behavior.

CH_4: $T_0 = -100\ ^oC$, $\quad \bar{s}_{LIQ\ 0} = 100$ kJ/kmol K

C_2H_6: $T_0 = -100\ ^oC$, $\quad P_0 = 1$ MPa, $\quad \bar{s}^*_0 = 200$ kJ/kmol K

Also for CH_4: $T_C = 190.4$ K, $\quad P_C = 4.60$ MPa

For a 50% mixture Kay's rule Eq.14.86:

$$T_{cmix} = 0.5 \times 190.4 + 0.5 \times 305.4 = 247.9\ \text{K}$$

$$P_{cmix} = 0.5 \times 4.60 + 0.5 \times 4.88 = 4.74\ \text{MPa}$$

IG MIX at T_0(=-100 ^oC), P_0(=1 MPa):

CH_4: $T_{r0} = 0.91$, $\quad P_G = 0.57 \times 4.60 = 2.622$ MPa

$$\bar{s}^*_{0\ CH4} = \bar{s}_{LIQ\ 0\ P_G} + (\bar{s}^*-\bar{s}_{LIQ})_{at\ P_G} - \bar{R}\ln(P_0/P_G)$$

$$= 100 + 4.01\times8.3145 - 8.3145\ln(1/2.622) = 141.36$$

$$\bar{s}^*_{0\ MIX} = 0.5\times141.36 + 0.5\times200 - 8.3145(0.5\ln 0.5 + 0.5\ln 0.5) = 176.44$$

$$\bar{C}_{P0\ MIX} = 0.5\times16.04\times2.254 + 0.5\times30.07\times1.766 = 44.629$$

$$\bar{s}^*_{TP\ MIX} = 176.44 + 44.629\ln\frac{293.2}{173.2} - 8.3145\ln\frac{4}{1} = 188.41\ \text{kJ/kmol K}$$

For the mixture at T, P: $T_r = 1.183$, $\quad P_r = 0.844$

Entropy departure $\quad \bar{s}^*_{TP\ MIX} - \bar{s}_{TP\ MIX} = 0.4363\times8.3145 = 3.63$ kJ/kmol K

Therefore,

$$\bar{s}_{TP\ MIX} = 188.41 - 3.63 = \mathbf{184.78\ kJ/kmol\ K}$$

An alternative is to form the ideal gas mixture at T, P instead of at T_0, P_0 :

$$\bar{s}^*_{TP\ CH4} = \underset{P_G,\ T_0}{\bar{s}_{LIQ\ 0}} + \underset{at\ P_G,\ T_0}{(\bar{s}^*-\bar{s}_{LIQ})} + \bar{C}_{P0\ CH4}\ln\frac{T}{T_0} - \bar{R}\ln\frac{P}{P_G}$$

$$= 100 + 33.34 + 16.04\times 2.254 \ln\frac{293.2}{173.2} - 8.3145 \ln\frac{4}{2.6}$$

$$= 100 + 33.34 + 19.03 - 3.53 = 148.84 \text{ kJ/kmol K}$$

$$\bar{s}^*_{TP\ C2H6} = 200 + 30.07\times 1.766 \ln\frac{293.2}{173.2} - 8.3145 \ln\frac{4}{1}$$

$$= 200 + 27.96 - 11.53 = 216.43 \text{ kJ/kmol K}$$

$$\bar{s}^*_{TP\ MIX} = 0.5\times 148.84 + 0.5\times 216.43$$

$$- 8.3145(0.5 \ln 0.5 + 0.5 \ln 0.5) = 188.41 \text{ kJ/kmol K}$$

$$\bar{s}_{TP\ MIX} = 188.41 - 3.63 = \mathbf{184.78\ kJ/kmol\ K}$$

14.137

A 200-L rigid tank contains propane at 400 K, 3.5 MPa. A valve is opened, and propane flows out until half the initial mass has escaped, at which point the valve is closed. During this process the mass remaining inside the tank expands according to the relation $Pv^{1.4}$ = constant. Calculate the heat transfer to the tank during the process.

C_3H_8: $V = 200\ \text{L},\ T_1 = 400\ \text{K},\ P_1 = 3.5\ \text{MPa}$

Flow out to $m_2 = m_1/2$; $Pv^{1.4} = \text{const}$ inside

$$T_{r1} = \frac{400}{369.8} = 1.082,\ P_{r1} = \frac{3.5}{4.25} = 0.824 \quad \text{Fig D.1: } Z_1 = 0.74$$

$$v_1 = \frac{0.74 \times 0.188\ 55 \times 400}{3500} = 0.01594,\ \ v_2 = 2v_1 = 0.03188$$

$$m_1 = \frac{0.2}{0.015\ 94} = 12.55\ \text{kg}, \quad m_2 = \frac{1}{2} m_1 = 6.275\ \text{kg},$$

$$P_2 = P_1 \left(\frac{v_1}{v_2}\right)^{1.4} = \frac{3500}{2^{1.4}} = 1326\ \text{kPa}$$

$$\left.\begin{array}{l} P_{r2} = \dfrac{1.326}{4.25} = 0.312 \\ P_2 v_2 = Z_2 R T_2 \end{array}\right\} \begin{array}{l} \text{Trial \& error: saturated with} \\ T_2 = 0.826 \times 369.8 = 305.5\ \text{K \&} \\ Z_2 = \dfrac{1326 \times 0.03188}{0.188\ 55 \times 305.5} = 0.734 \end{array}$$

$$Z_2 = Z_{F2} + x_2(Z_{G2} - Z_{F2}) = 0.734 = 0.05 + x_2(0.78-0.05) \quad => \quad x_2 = 0.937$$

$$(h^*_1 - h_1) = 0.188\ 55 \times 369.8(0.9) = 62.8$$

$$(h^*_2 - h^*_1) = 1.6794(305.5-400) = -158.7$$

$$(h^*_2 - h_2) = (h^*_2 - h_{F2}) - x_2 h_{FG2} = 0.188\ 55 \times 369.8 \Big[4.41 - 0.937(4.41-0.55)\Big]$$
$$= 55.3$$

1st law: $Q_{CV} = m_2 h_2 - m_1 h_1 + (P_1 - P_2)V + m_e h_{e\ AVE}$

Let $h^*_1 = 0$ then $\quad h_1 = 0 + (h_1 - h^*_1) = -62.8$

$$h_2 = h^*_1 + (h^*_2 - h^*_1) + (h_2 - h^*_2) = 0 - 158.7 - 55.3 = -214.0$$

$$h_{e\ AVE} = (h_1 + h_2)/2 = -138.4$$

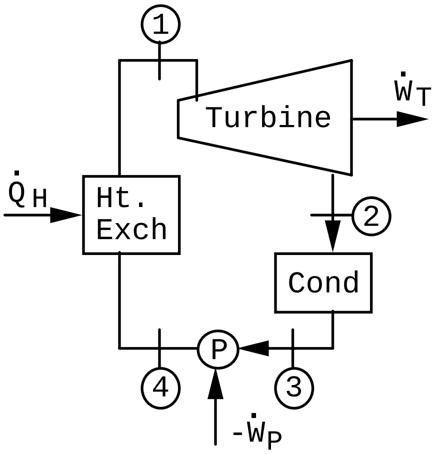
$$Q_{CV} = 6.275(-214.0) - 12.55(-62.8)$$
$$+ (3500-1326) \times 0.2 + 6.275(-138.4) = \mathbf{-981.4\ kJ}$$

14.138

One kilogram per second water enters a solar collector at 40°C and exits at 190°C, as shown in Fig. P14.138. The hot water is sprayed into a direct-contact heat exchanger (no mixing of the two fluids) used to boil the liquid butane. Pure saturated-vapor butane exits at the top at 80°C and is fed to the turbine. If the butane condenser temperature is 30°C and the turbine and pump isentropic efficiencies are each 80%, determine the net power output of the cycle.

H_2O cycle: solar energy input raises 1 kg/s of liquid H_2O from 40°C to 190°C. Therefore, corresponding heat input to the butane in the heat exchanger is

$$\dot{Q}_H = \dot{m}(h_{F\ 190\ C} - h_{F\ 40\ C})_{H2O} = 1(807.62 - 167.57) = 640.05 \text{ kW}$$



C_4H_{10} cycle

$T_1 = 80\ ^oC,\ x_1 = 1.0$; $T_3 = 30\ ^oC,\ x_3 = 0.0$

$\eta_{ST} = \eta_{SP} = 0.80$

$$T_{r1} = \frac{353.2}{425.2} = 0.831$$

From D.1, D.2 and D.3:

$P_1 = 0.325 \times 3800 = 1235$ kPa

$(h^*_1 - h_1) = 0.143\ 04 \times 425.2 \times 0.56 = 34.1$

$(s^*_1 - s_1) = 0.143\ 04 \times 0.475 = 0.0680$

$$T_{r3} = \frac{303.2}{425.2} = 0.713$$

From D.1, D.2 and D.3: $P_3 = 0.113 \times 3800 = 429$ kPa

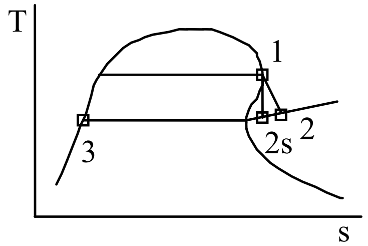
sat. liq.: $(h^* - h_F) = RT_C \times 4.81 = 292.5$; $(s^* - s_F) = R \times 6.64 = 0.950$

sat. vap.: $(h^* - h_G) = RT_C \times 0.235 = 14.3$; $(s^* - s_G) = R \times 0.22 = 0.031$

Because of the combination of properties of C_4H_{10} (particularly the large C_{P0} /R), s_1 is larger than s_G at T_3. To demonstrate,

$$(s^*_1 - s^*_{G3}) = 1.7164 \ln\frac{353.2}{303.2} - 0.143\ 04 \ln\frac{1235}{429} = 0.1107$$

$$(s_1 - s_{G3}) = -0.0680 + 0.1107 + 0.031 = +0.0737 \text{ kJ/kg K}$$



so that T_{2S} will be $> T_3$, as shown in the T-s diagram. A number of other heavy hydrocarbons also exhibit this behavior.
Assume $T_{2S} = 315$ K, $T_{r2S} = 0.741$

From D.2 and D.3:

$$(h^*_{2S}-h_{2S}) = RT_C\times0.21 = 12.8 \quad \text{and} \quad (s^*_{2S}-s_{2S}) = R\times0.19 = 0.027$$

$$(s^*_1-s^*_{2S}) = 1.7164 \ln\frac{353.2}{315} - 0.143\,04 \ln\frac{1235}{429} = +0.0453$$

$$(s_1-s_{2S}) = -0.0680 + 0.0453 + 0.027 \approx 0$$

$$\Rightarrow T_{2S} = 315 \text{ K}$$

$$(h^*_1-h^*_{2S}) = 1.7164(353.2-315) = 65.6$$

$$w_{ST} = h_1-h_{2S} = -34.1 + 65.6 + 12.8= 44.3 \text{ kJ/kg}$$

$$w_T = \eta_S\times w_{ST} = 0.80\times44.3 = 35.4 \text{ kJ/kg}$$

At state 3,

$$v_3 = \frac{0.019\times0.143\,04\times303.2}{429} = 0.001\,92 \text{ m}^3\text{/kg}$$

$$-w_{SP} \approx v_3(P_4-P_3) = 0.001\,92(1235-429) = 1.55 \text{ kJ/kg}$$

$$-w_P = \frac{-w_{SP}}{\eta_{SP}} = \frac{1.55}{0.8} = 1.94 \text{ kJ/kg}$$

$$w_{NET} = w_T + w_P = 35.4 - 1.94= 33.46 \text{ kJ/kg}$$

For the heat exchanger,

$$\dot{Q}_H = 640.05 = \dot{m}_{C4H10}(h_1-h_4)$$

But $h_1-h_4 = h_1-h_3+w_P$

$$h_1-h_3 = (h_1-h^*_1) + (h^*_1-h^*_3) + (h^*_3-h_3)$$

$$= -34.1 + 1.716(80 - 30) + 292.5 = 344.2 \text{ kJ/kg}$$

Therefore,

$$\dot{m}_{C4H10} = \frac{640.05}{344.2-1.94} = 1.87 \text{ kg/s}$$

$$\dot{W}_{NET} = \dot{m}_{C4H10}w_{NET} = 1.87 \times 33.46 = \mathbf{62.57\ kW}$$

14.139

A piston/cylinder contains ethane gas, initially at 500 kPa, 100 L, and at ambient temperature, 0°C. The piston is now moved, compressing the ethane until it is at 20°C, with a quality of 50%. The work required is 25% more than would have been required for a reversible polytropic process between the same initial and final states. Calculate the heat transfer and the net entropy change for the process.

Ethane: $T_c = 305.4$ K, $P_c = 4.88$ MPa,

$R = 0.2765$ kJ/kg-K, $C_p = 1.766$ kJ/kg K

State 1: $T_{r1} = 0.895$, $P_{r1} = 0.102$ → $Z_1 = 0.95$

$v_1 = Z_1RT_1/P_1 = 0.1435\ \text{m}^3/\text{kg}$, $\quad m_1 = V_1/v_1 = 0.697$ kg

$(h_1^* - h_1) = 0.13RT_c = 11.0$ kJ/kg, $\quad (s_1^* - s_1) = 0.09\ R = 0.025$ kJ/kg K

State 2: $T_2 = 20^oC$, $x_2 = 0.5$, $\quad {}_1W_2 = 1.25W_{rev}$

$T_{r2} = 0.96$, $\quad P_{r2} = 0.78$, $\quad P_2 = P_{r2}P_c = 3806$ kPa

$Z_{f2} = 0.14$, $\quad Z_{g2} = 0.54$, $\quad Z_2 = (1 - x_2)Z_f + x_2Z_g = 0.34$

$(h_2^* - h_2) = (1 - x_2)\ 3.65\ RT_c + x_2\ (1.39\ RT_c) = 212.8$ kJ/kg

$(s_2^* - s_2) = (1 - x_2)\ 3.45\ R + x_2 \times 1.10\ R = 0.629$ kJ/kg K

$v_2 = Z_2RT_2/P_2 = 0.0072\ \text{m}^3/\text{kg}$, $\quad V_2 = mv_2 = 0.005\ \text{m}^3$

$$P_1V_1^n = P_2V_2^n, \quad \ln\frac{P_2}{P_1} = n \ln\frac{V_1}{V_2} \rightarrow \quad n = 0.6783$$

$$W_{rev} = \int P\ dV = \frac{P_2V_2 - P_1V_1}{1 - n} = -96.3\ \text{kJ},\ {}_1W_2 = 1.25W_{rev} = -120.4\ \text{kJ}$$

a) 1^{st} Law: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$; $\quad u = h - Pv$

$h_2 - h_1 = (h_2 - h_2^*) + (h_2^* - h_1^*) + (h_1^* - h_1)$

$= -212.8 + 1.766(20 – 0) + 11.0 = -166.5$ kJ/kg

$u_2 - u_1 = (h_2 - h_1) - (P_2v_2 - P_1v_1) = -122.2$ kJ/kg

${}_1Q_2 = 0.697(-122.2) - 120.4 =$ **-205.6 kJ**

b) 2^{nd} Law: $\Delta S_{net} = m(s_2 - s_1) - {}_1Q_2 /T_o$; $\quad T_o = 0^oC$

$s_2 - s_1 = (s_2 - s_2^*) + (s_2^* - s_1^*) + (s_1^* - s_1)$

$(s_2^* - s_1^*) = C_p \ln(T_2 / T_1) – R \ln(P_2 / P_1) = -0.436$ kJ/kg K,

$$\Delta S_{net} = 0.697(-0.629 - 0.436 + 0.025) + \frac{205.6}{273.2} = \mathbf{0.028\ kJ/K}$$

14.140

Carbon dioxide gas enters a turbine at 5 MPa, 100°C, and exits at 1 MPa. If the isentropic efficiency of the turbine is 75%, determine the exit temperature and the second-law efficiency.

CO_2 turbine: $\eta_S = w/w_S = 0.75$

inlet: $T_1 = 100^oC$, $P_1 = 5$ MPa, exhaust: $P_2 = 1$ MPa

a) $P_{r1} = \frac{5}{7.38} = 0.678$, $T_{r1} = \frac{373.2}{304.1} = 1.227$, $P_{r2} = \frac{1}{7.38} = 0.136$

From D.2 and D.3,

$(h^*_1\text{-}h_1) = 0.188\ 92\times304.1\times0.52 = 29.9$

$(s^*_1\text{-}s_1) = 0.188\ 92\times0.30 = 0.0567$

Assume $T_{2S} = 253$ K, $T_{r2S} = 0.832$

From D.2 and D.3: $(h^*_{2S}\text{-}h_{2S}) = RT_C\times0.20 = 11.5$

$(s^*_{2S}\text{-}s_{2S}) = R\times0.17 = 0.0321$

$(s^*_{2S}\text{-}s^*_1) = 0.8418 \ln\frac{253}{373.2} - 0.188\ 92 \ln\frac{1}{5} = -0.0232$

$(s_{2S}\text{-}s_1) = -0.0321 - 0.0232 + 0.0567 \approx 0$

$\Rightarrow T_{2S} = 253$ K

$(h^*_{2S}\text{-}h^*_1) = 0.8418(253\text{-}373.2) = -101.2$

$w_S = (h_1\text{-}h_{2S}) = -29.9 + 101.2 + 11.5 = 82.8$ kJ/kg

$w = \eta_S\times w_S = 0.75\times82.8 = 62.1$ kJ/kg $= (h_1\text{-}h^*_1) + (h^*_1\text{-}h^*_2) + (h^*_2\text{-}h_2)$

Assume $T_2 = 275$ K, $T_{r2} = 0.904$

$(h^*_1\text{-}h^*_2) = 0.8418(373.2\text{-}275) = 82.7$

From D.2 and D.3,

$(h^*_2\text{-}h_2) = RT_C\times0.17 = 9.8$; $(s^*_2\text{-}s_2) = R\times0.13 = 0.0245$

Substituting,

$w = -29.9 + 82.7 + 9.8 = 62.7 \approx 62.1$ $\Rightarrow T_2 =$ **275 K**

b) $(s^*_2\text{-}s^*_1) = 0.8418 \ln\frac{275}{373.2} - 0.188\ 92 \ln\frac{1}{5} = +0.0470$

$(s_2 - s_1) = -0.0245 + 0.0470 + 0.0567 = +0.0792$

Assuming $T_0 = 25\ ^oC$,

$(\varphi_1 - \varphi_2) = (h_1 - h_2) - T_0(s_1 - s_2) = 62.1 + 298.2(0.0792) = 85.7$ kJ/kg

$$\eta_{2nd\ Law} = \frac{w}{\varphi_1 - \varphi_2} = \frac{62.1}{85.7} = \mathbf{0.725}$$

14.141

A 10- m^3 storage tank contains methane at low temperature. The pressure inside is 700 kPa, and the tank contains 25% liquid and 75% vapor, on a volume basis. The tank warms very slowly because heat is transferred from the ambient.

a. What is the temperature of the methane when the pressure reaches 10 MPa?

b. Calculate the heat transferred in the process, using the generalized charts.

c. Repeat parts (a) and (b), using the methane tables, Table B.7. Discuss the differences in the results.

CH_4: $V = 10\ m^3$, $P_1 = 700$ kPa

$V_{LIQ\,1} = 2.5\ m^3$, $V_{VAP\,1} = 7.5\ m^3$

a) $P_{r1} = \frac{0.70}{4.60} = 0.152,\ \ P_{r2} = \frac{10}{4.60} = 2.174$

From D.1: $Z_{F1} = 0.025$, $Z_{G1} = 0.87$ &

$T_1 = 0.74 \times 190.4 = 140.9$ K

$$v_{F1} = \frac{0.025 \times 0.518\ 35 \times 140.9}{700} = 0.00261$$

$$v_{G1} = \frac{0.87 \times 0.518\ 35 \times 140.9}{700} = 0.0908$$

$$m_{LIQ\,1} = \frac{2.5}{0.00261} = 957.9 \text{ kg},\ m_{VAP\,1} = \frac{7.5}{0.0908} = 82.6 \text{ kg}$$

Total m = 1040.3 kg

$$v_2 = v_1 = \frac{V}{m} = \frac{10}{1040.5} = 0.00961\ = \frac{Z_2 \times 0.518\ 35 \times 190.4 \times T_{r2}}{10\ 000}$$

or $Z_2 T_{r2} = 0.9737$ at $P_{r2} = 2.174$

By trial and error

$T_{r2} = 1.334$ & $Z_2 = 0.73$, $\quad T_2 = 1.334 \times 190.4 =$ **254.0 K**

b) 1st law:

$$Q_{12} = m(u_2 - u_1) = m(h_2 - h_1) - V(P_2 - P_1)$$

Using D.2 & $x_1 = \frac{82.6}{1040.5} = 0.0794$

$$(h_1^* - h_1) = (h_1^* - h_{F1}) - x_1 h_{FG1}$$

$= 0.518\ 35\times190.4\Big[4.72\text{-}0.0794(4.72\text{-}0.29)\Big] = 431.1$

$(h_2^*\text{-}h_1^*) = 2.2537(254.0\text{-}140.9) = 254.9$

$(h_2^*\text{-}h_2) = 0.518\ 35\times190.4(1.47) = 145.1$

$(h_2\text{-}h_1) = -145.1 + 254.9 + 431.1 = 540.9$ kJ/kg

$Q_{12} = 1040.5(540.9) - 10(10\ 000\text{-}700) =$ **469 806 kJ**

c) Using Table B.7 for CH_4

$T_1 = T_{SAT\ 1} = 141.7$ K, $v_{F1} = 0.002\ 675$, $u_{F1} = -178.47$

$v_{G1} = 0.090\ 45$, $u_{G1} = 199.84$

$m_{LIQ\ 1} = \frac{2.5}{0.002\ 675} = 934.6$, $m_{VAP\ 1} = \frac{7.5}{0.090\ 45} = 82.9$

Total mass m = 1017.5 kg and $v_2 = \frac{10}{1017.5} = 0.009\ 828$ m^3/kg

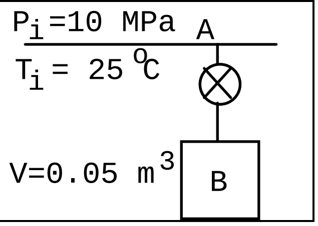
$$\text{At } v_2 \ \&\ P_2 = 10 \text{ MPa} \rightarrow \begin{cases} T_2 = 259.1 \text{ K} \\ u_2 = 296.11 \end{cases}$$

$Q_{12} = m(u_2\text{-}u_1) = 1017.5\times296.11 - 934.6(-178.47) - 82.9(199.84)$

= **451 523 kJ**

14.142

A gas mixture of a known composition is frequently required for different purposes, e.g., in the calibration of gas analyzers. It is desired to prepare a gas mixture of 80% ethylene and 20% carbon dioxide (mole basis) at 10 MPa, 25°C in an uninsulated, rigid 50-L tank. The tank is initially to contain CO2 at 25°C and some pressure *P*1. The valve to a line flowing C2H4 at 25°C, 10 MPa, is now opened slightly, and remains open until the tank reaches 10 MPa, at which point the temperature can be assumed to be 25°C. Assume that the gas mixture so prepared can be represented by Kay's rule and the generalized charts. Given the desired final state, what is the initial pressure of the carbon dioxide, *P*1?

	$A = C_2H_4$, $B = CO_2$ $T_1 = 25\ ^oC$ $P_2 = 10$ MPa, $T_2 = 25\ ^oC$ $y_{A2} = 0.8$, $y_{B2} = 0.2$	

Mixture at 2 :

$P_{C2} = 0.8 \times 5.04 + 0.2 \times 7.38 = 5.508$ MPa

$T_{C2} = 0.8 \times 282.4 + 0.2 \times 304.1 = 286.7$ K

$T_{r2} = 298.15/286.7 = 1.040$; $P_{r2} = 10/5.508 = 1.816$

D.1 : $Z_2 = 0.32$

$$n_2 = \frac{P_2V}{Z_2\bar{R}T_2} = \frac{10\,000\times0.05}{0.32\times8.3145\times298.2} = 0.6302 \text{ kmol}$$

$n_{A2} = n_i = 0.8\ n_2 = 0.5042$ kmol C_2H_4

$n_{B2} = n_1 = 0.2\ n_2 = 0.1260$ kmol CO_2

$$T_{r1} = \frac{298.2}{304.1} = 0.981$$

$$P_{r1} = \frac{n_1Z_{B1}\bar{R}T_1}{P_{CB}V} = \frac{0.126\ Z_{B1}\times 8.3145\times298.2}{7380\times0.05} = 0.8466\ Z_{B1}$$

By trial & error: $P_{r1} = 0.618$ & $Z_{B1} = 0.73$

$\Rightarrow P_1 = 0.618 \times 7.38 =$ **4.56 MPa**

14.143

Determine the heat transfer and the net entropy change in the previous problem. Use the initial pressure of the carbon dioxide to be 4.56 MPa before the ethylene is flowing into the tank.

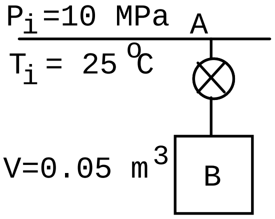
A gas mixture of a known composition is frequently required for different purposes, e.g., in the calibration of gas analyzers. It is desired to prepare a gas mixture of 80% ethylene and 20% carbon dioxide (mole basis) at 10 MPa, 25°C in an uninsulated, rigid 50-L tank. The tank is initially to contain CO2 at 25°C and some pressure *P*1. The valve to a line flowing C2H4 at 25°C, 10 MPa, is now opened slightly, and remains open until the tank reaches 10 MPa, at which point the temperature can be assumed to be 25°C. Assume that the gas mixture so prepared can be represented by Kay's rule and the generalized charts. Given the desired final state, what is the initial pressure of the carbon dioxide, *P*1?

$A = C_2H_4$, $B = CO_2$

$T_1 = 25\ ^oC$

$P_2 = 10$ MPa, $T_2 = 25\ ^oC$

$y_{A2} = 0.8$, $y_{B2} = 0.2$



Mixture at 2 :

$P_{C2} = 0.8 \times 5.04 + 0.2 \times 7.38 = 5.508$ MPa

$T_{C2} = 0.8 \times 282.4 + 0.2 \times 304.1 = 286.7$ K

$T_{r2} = 298.15/286.7 = 1.040$; $P_{r2} = 10/5.508 = 1.816$

D.1 : $Z_2 = 0.32$

$$n_2 = \frac{P_2V}{Z_2\bar{R}T_2} = \frac{10\,000\times0.05}{0.32\times8.3145\times298.2} = 0.6302 \text{ kmol}$$

$n_{A2} = n_i = 0.8\ n_2 = 0.5042$ kmol C_2H_4

$n_{B2} = n_1 = 0.2\ n_2 = 0.1260$ kmol CO_2

$$T_{r1} = \frac{298.2}{304.1} = 0.981 \text{ and } P_{r1} = \frac{4560}{7380} = 0.618$$

1st law: $Q_{CV} + n_i\bar{h}_i = n_2\bar{u}_2 - n_1\bar{u}_1 = n_2\bar{h}_2 - n_1\bar{h}_1 - (P_2\text{-}P_1)V$

or $Q_{CV} = n_2(\bar{h}_2\text{-}\bar{h}^*_2) - n_1(\bar{h}_1\text{-}\bar{h}^*_1) - n_i(\bar{h}_i\text{-}\bar{h}^*_i) - (P_2\text{-}P_1)V$

(since $T_i = T_1 = T_2$, $\bar{h}^*_i = \bar{h}^*_1 = \bar{h}^*_2$)

$(\bar{h}^*_1 - \bar{h}_1) = 0.83 \times 8.3145 \times 304.1 = 2099$ kJ/kmol

$(\bar{h}^*_2 - \bar{h}_2) = 3.40 \times 8.3145 \times 286.7 = 8105$ kJ/kmol

$T_{ri} = \frac{298.2}{282.4} = 1.056,\ P_{ri} = \frac{10}{5.04} = 1.984$

$(\bar{h}^*_i - \bar{h}_i) = 3.35 \times 8.3145 \times 282.4 = 7866$ kJ/kmol

$Q_{CV} = 0.6302(-8105) - 0.126(-2099) - 0.5042(-7866) - (10\,000-4560) \times 0.05$

$= \mathbf{-1149\ kJ}$

$\Delta S_{CV} = n_2\bar{s}_2 - n_1\bar{s}_1\ ,\quad \Delta S_{SURR} = -\,Q_{CV}/T_0 - n_i\bar{s}_i$

$\Delta S_{NET} = n_2\bar{s}_2 - n_1\bar{s}_1 - Q_{CV}/T_0 - n_i\bar{s}_i$

Let $\bar{s}^*_{A0} = \bar{s}^*_{B0} = 0$ at $T_0 = 25\ ^oC$, $P_0 = 0.1$ MPa

Then $\bar{s}^*_{MIX\ 0} = -8.3145\ (0.8 \ln 0.8 + 0.2 \ln 0.2) = 4.161$ kJ/kmol K

$\bar{s}_1 = \bar{s}^*_{B0} + (\bar{s}^*_{P1\ T1} - \bar{s}^*_{P0\ T0})_B + (\bar{s}_1 - \bar{s}^*_{P1\ T1})_B$

$= 0 + (0 - 8.3145 \ln \frac{4.56}{0.1}) - 0.60 \times 8.3145 = -36.75$ kJ/kmol K

$\bar{s}_i = \bar{s}^*_{A0} + (\bar{s}^*_{Pi\ Ti} - \bar{s}^*_{P0\ T0})_A + (\bar{s}_i - \bar{s}^*_{Pi\ Ti})_A$

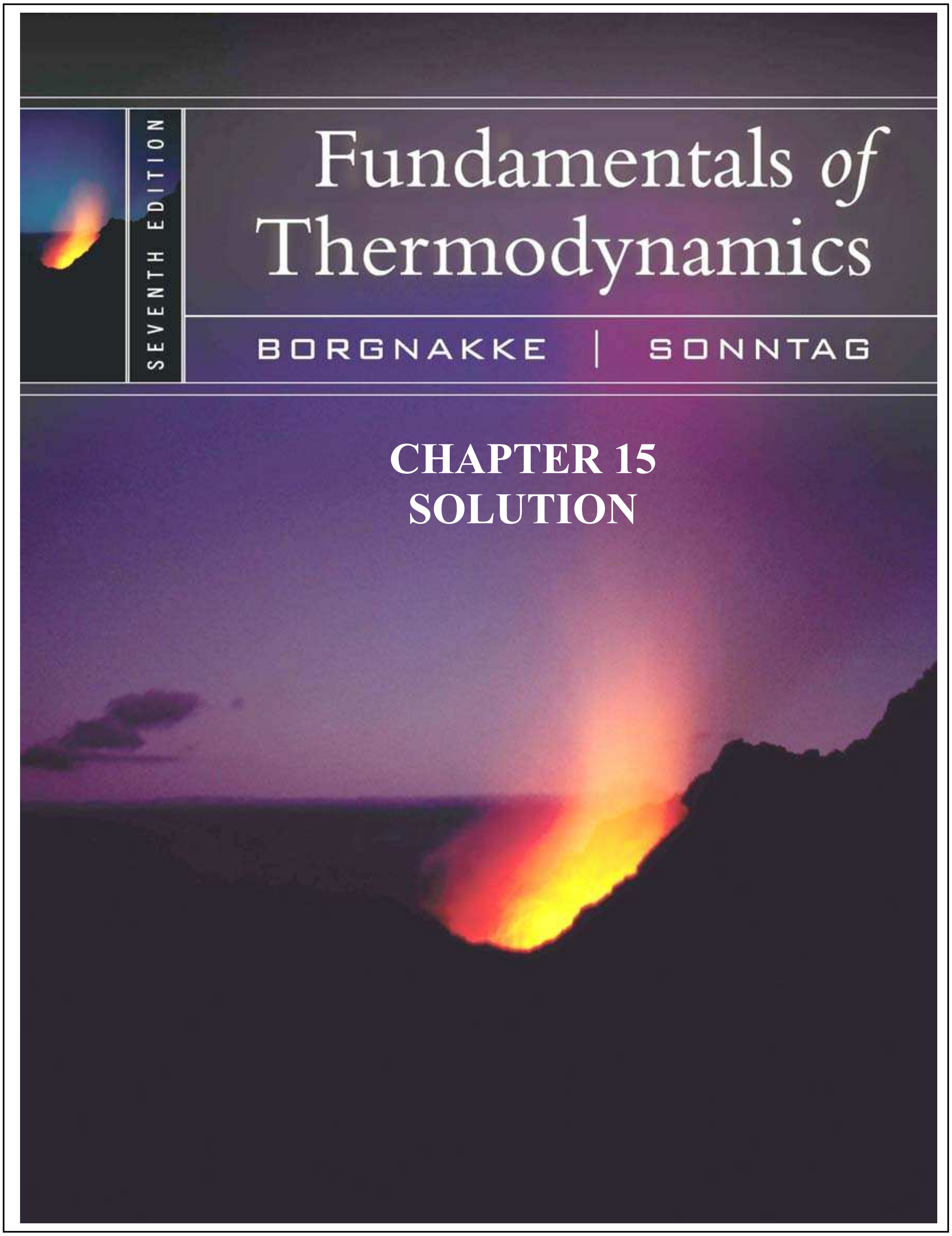
$= 0 + (0 - 8.3145 \ln \frac{10}{0.1}) - 2.44 \times 8.3145 = -58.58$ kJ/kmol K

$\bar{s}_2 = \bar{s}^*_{MIX\ 0} + (\bar{s}^*_{P2\ T2} - \bar{s}^*_{P0\ T0})_{MIX} + (\bar{s}_2 - \bar{s}^*_{P2\ T2})_{MIX}$

$= 4.161 + (0 - 8.3145 \ln \frac{10}{0.1}) - 2.551 \times 8.3145 = -55.34$ kJ/kmol K

$\Delta S_{NET} = 0.6302(-55.33) - 0.126(-36.75) - 0.5042(-58.58) + 1149/298.2$

$= \mathbf{+3.15\ kJ/K}$



SEVENTH EDITION

Fundamentals of Thermodynamics

BORGNAKKE | SONNTAG

CHAPTER 15
SOLUTION

CONTENT

In-Text Concept Questions

15.a

How many kmol of air are needed to burn 1 kmol of carbon?

Combustion Eq.: $C + O_2 \Rightarrow 1\ CO_2$

One kmol of O_2 is required to form one kmol CO_2. Since air is 21 % O_2, this means 4.76 kmol of air.

15.b

If I burn 1 kmol of hydrogen H_2 with 6 kmol of air what is the air-fuel ratio on a mole basis and what is the percent theoretical air?

Combustion Eq. stoichiometric:

$$H_2 + \nu_{O2}(O_2 + 3.76\ N_2) \Rightarrow 1\ H_2O + 3.76\ \nu_{O2}\ N_2$$

$$\nu_{O2} = 0.5\ ; \qquad (A/F)_S = \nu_{O2} \times (1 + 3.76) / 1 = 2.38$$

Six kmol of air is: $1.26\ O_2 + 4.74\ N_2$.

The A/F mole ratio is 6, so the percent theoretical air is

$$\%\text{Theoretical air} = \frac{(A/F)_{ac}}{(A/F)_S} \times 100 = \frac{6}{2.38} \times 100 = 252\ \%$$

15.c

For the 110% theoretical air in Eq.15.8 what is the equivalence ratio? Is that mixture rich or lean?

110% Theoretical air means also $AF = 1.1\ AF_S$

so from the definition in Eq.15.6

$$\Phi = \frac{AF_S}{AF} = \frac{1}{1.10} = 0.909 \qquad \text{a lean mixture}$$

15.d

In most cases combustion products are exhausted above the dew point. Why?

If any water in the products condenses, it will be acidic due to the other gases in the products. There are always minute amounts of unburned or partially burned fuel and intermediate species in the products that can combine with water and create a very corrosive mixture.

15.e

How is a fuel enthalpy of combustion connected to its enthalpy of formation?

The enthalpy of combustion of a fuel is the difference in enthalpy of the products and reactants for the combustion involving the fuel, these enthalpies include the various enthalpies of formation.

At reference condition: $H^{\circ}_{RP} = H^{\circ}_{P} - H^{\circ}_{R} = H^{\circ}_{P} - \bar{h}^{0}_{f\ \text{fuel vap or liq}}$

15.f

What is the higher and lower heating value HHV, LHV of n-butane?

The heating value is the negative of the enthalpy of combustion. From Table 15.3, the HHV of gaseous n-butane at 25°C is

$$HHV = 49\,500 \text{ kJ/kg},$$

and the corresponding LHV is

$$LHV = 45\,714 \text{ kJ/kg}.$$

Notice the table is on a mass basis (per kg fuel).

15.g

What is the value of h_{fg} for n-octane?

This can be obtained from two places. From Table A.10 we get

$$h_{fg} = (\bar{h}^0_{f\,vap} - \bar{h}^0_{f\,liq}) / M = [(-208\,600 - (-250\,105)] / 114.232 = \mathbf{363\ kJ/kg}$$

The h_{fg} of a fuel listed in Table 15.3 is the difference between the first two columns in the table (or the third and fourth). For n-octane, this value is

$$h_{fg} = -47\,893 - (-48\,256) = \mathbf{363\ kJ/kg}$$

To see this remember

$$H^\circ_{RP} = H^\circ_P - H^\circ_R = H^\circ_P - \bar{h}^0_{f\ fuel\ vap\ or\ liq}$$

so when we take the difference between fuel as gas and fuel as liquid all other terms will cancel out leaving h_{fg} for the fuel.

15.h

What happens to the adiabatic flame temperature if I burn rich or lean?

The higher the percent theoretical air used in combustion (the leaner), the larger the number of moles of products, especially nitrogen, which decreases the adiabatic flame temperature. Burning rich causes incomplete combustion, however, with a smaller release of energy.

Experimentally the highest temperature is reached for slightly rich.

Heavy molecules show up as yellow. Oxygen diffuses in from the air and the fuel evaporates from the wick. As air mixes in, the flame cools.

15.i

Is the irreversibility in a combustion process significant? Why is that?

A combustion process is highly irreversible with a large increase in entropy. It takes place at a rapid rate, due to large driving forces, and results in stable products of combustion that have little or no tendency to return to their former constituents and states.

15.j

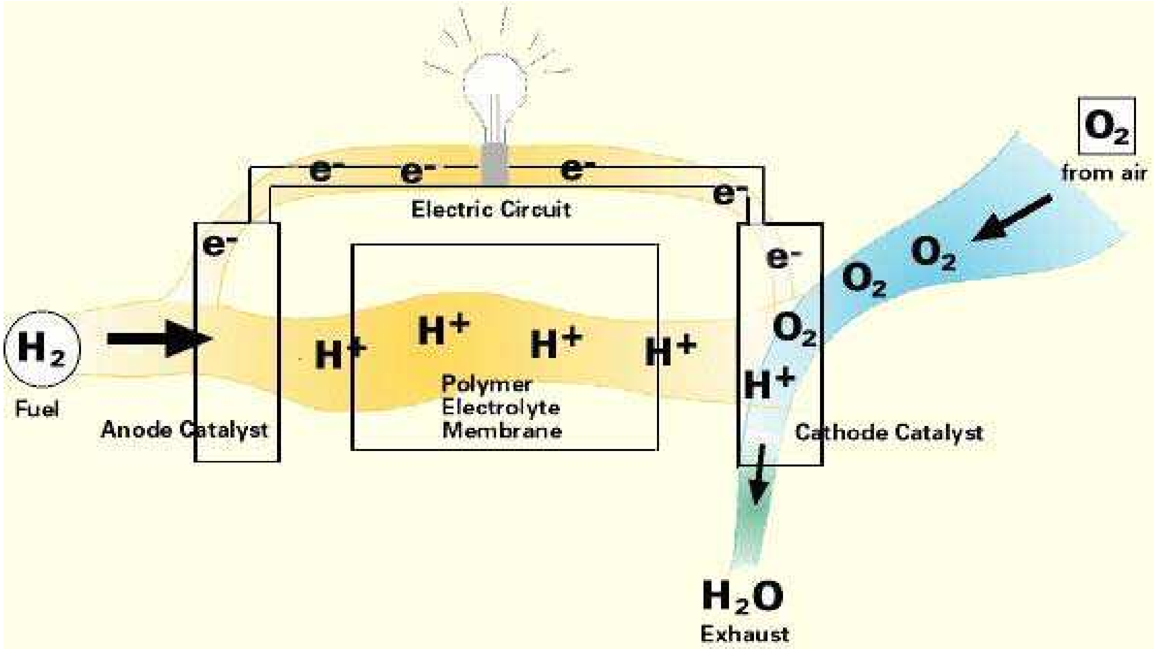
If the air-fuel ratio is larger than stoichiometric is it more or less reversible?

Less reversible more irreversible. The excess oxidizer (air) is being heated up, Q over a finite temperature difference is an irreversible process. The same is true for A/F smaller than one where the excess fuel is heated up.

15.k

What makes the fuel cell attractive from a power-generating point of view?

Fuel cells are attractive for power generation because their direct output is electrical energy. They also have a much higher power density as power per unit volume or power per unit mass and thus can be used in mobile applications.



Concept-Study Guide Problems

15.1

Is mass conserved in combustion? Is the number of moles constant?

Yes mass is conserved as we do not consider the quantum physic effect of mass and energy conversion.
Moles are not conserved, what is conserved are the atoms so each atom in the reactants must be found in the products.

15.2

Does all combustion take place with air?

Most combustion takes place with air as the oxidizer, special cases do occur where a different oxidizer is used. To reach very high temperatures a gas welder uses a bottle of pure oxygen (and one with the fuel) instead of air and then the flame can cut through steel.

15.3

Why would I sometimes need A/F on a mole basis? on a mass basis?

If you want to meter (measure) the fuel and air flows it can be done as a volume flowrate which is proportional to moles ($P\dot{V} = \dot{n}\bar{R}T$) in which case concentrations on a mole basis are needed.

The fuel and air flows can also be measured with a method that measures mass flow rate $\dot{m}$ or if you are filling up tanks to store the fuel and oxidizer as in a rocket in both cases the concentrations on a mass basis are needed.

15.4

Why is there no significant difference between the number of moles of reactants versus products in combustion of hydrocarbon fuels with air?

In most hydrocarbon fuels, there are approximately twice as many hydrogen atoms as carbon atoms, so the numbers of moles of CO_2 and H_2O in the products are roughly equal, the total of which is not too different from the O_2 required in the reactants. The number of excess O_2 is the same in reactants and products. The total number of moles is dominated by the N_2 in each, especially with excess air.

15.5

Why are products measured on a dry basis?

Combustion products have traditionally been measured by passing the gas mixture through a series of solutions that selectively absorb the components one-by-one and measuring the resulting gas volume decreases. The water component is condensed out in these processes, leaving the others – that is, a dry basis. Other and newer instruments measure the concentrations by optical means and these are sensitive to moisture content, which can corrode the surfaces and destroy the sensors. If the water stays in the mixture it typically has to stay hot to prevent condensation at undesirable locations where that would alter the concentrations of the remaining gas mixture components.

15.6

What is the dew point of hydrogen burned with stoichiometric pure oxygen? air?

For H_2 burned with pure oxygen

$$H_2 + 0.5\ O_2 \quad \Rightarrow \quad 1\ H_2O$$

with the only product as water, so the dew-point at 100 kPa pressure is **99.6°C**.
For H_2 burned with stoichiometric air

$$H_2 + 0.5\ (O_2 + 3.76\ N_2) \quad \Rightarrow \quad 1\ H_2O\ + 1.88\ N_2$$

the product mixture is water and nitrogen. The partial pressure of the water at a total pressure of 100 kPa is

$$P_v = P_{tot}\ y_v = 100 \times \frac{1}{1 + 1.88} = 34.7\ \text{kPa},$$

corresponding to a dew-point of **72.3°C** .

15.7

How does the dew point change as equivalence ratio goes from 0.9 to 1 to 1.1?

For a given amount of water in the products, the smaller the total number of moles of other gases is (as Φ increases), the higher the partial pressure of the water and therefore the dew-point temperature. As Φ becomes greater than 1.0, there will be incomplete combustion, and the resulting composition will be affected to have some unburned fuel and therefore relative less water. The relative maximum amount of water is then at a stoichiometric mixture $\Phi = 1$ and this is also maximum dew point temperature.

15.8

Why does combustion contribute to global warming?

Any combustion of a hydrocarbon fuel produces carbon dioxide, which in the atmosphere is a contributing factor to global warming. Carbon dioxide absorbs radiation over a wide spectrum and thus heats the atmosphere. This is not just man-made, but nature has forest fires and volcanic action that liberate gases into the atmosphere.

15.9

What is the enthalpy of formation for oxygen as O_2? If O? For CO_2?

The enthalpy of formation of O_2 is zero, by choice of the reference base. Relative to this base, the value for the monatomic form O is

$$\bar{h}^{\circ}_{f\,O} = +249\ 170\ \text{kJ/kmol (Table A.9)},$$

and the value for CO_2 is

$$\bar{h}^{\circ}_{f\,CO2} = -393\ 522\ \text{kJ/kmol (Table A.9 or A.10)}.$$

15.10

If the nitrogen content of air could be lowered will the adiabatic flame temperature increase or decrease?

Since that would reduce the number of nitrogen molecules in the products it will increase the adiabatic flame temperature, more energy per molecule.

Energy Eq.: $\Delta H_P = HV + \Delta H_R = \text{given}$

$$\Delta H_P = \nu_{CO_2}\, \Delta\bar{h}_{CO_2} + \nu_{H_2O}\, \Delta\bar{h}_{H_2O} + \nu_{N_2}\, \Delta\bar{h}_{N_2} \quad \text{at } T_{ad}$$

15.11

Does the enthalpy of combustion depend on AF?

NO.

$H_{RP} = H_P - H_R$ depends on T, P in general.

Assuming ideal gas it does not depend on P.

It is assumed the number is scaled to 1 kmol or 1 kg of fuel burned.

15.12

Why do some fuels not have entries for liquid fuel in Table 15.3?

Those fuels cannot exist as liquids at 25°C (above their critical temperature).

15.13

Is a heating value a fixed number for a fuel?

Yes.

The heating value is the negative of the enthalpy of combustion and is only useful if it is a well defined number. It is fixed, but there are normally 4 values quoted as the **higher** (water liquid) and **lower** (water vapor) **heating value** for the fuel as a liquid or vapor if applicable. It should also be clear whether it is the enthalpies or the internal energies that are used, i.e. do you have a flow or a non flow situation. So these qualifiers must accompany the number for it to be unique.

15.14

Is an adiabatic flame temperature a fixed number for a fuel?

Qualified **Yes.**
It is a single number if we assume reactants are supplied at the reference conditions in a stoichiometric ratio and we know the phase of the fuel (liquid or gas) and the type of oxidizer (air or pure oxygen). Also you need to know if the combustion takes place at constant pressure (the most common) or it may take place at constant volume (unusual). If anyone of these conditions is changed the adiabatic flame temperature is changed. This is less fixed than the heating value, more variables can change the adiabatic flame temperature, say, adding an inert gas will lower the flame temperature, but keep the heating value the same. Chemical equilibrium reactions and incomplete combustion makes the real flame temperature lower than the theoretical adiabatic flame temperature.

15.15

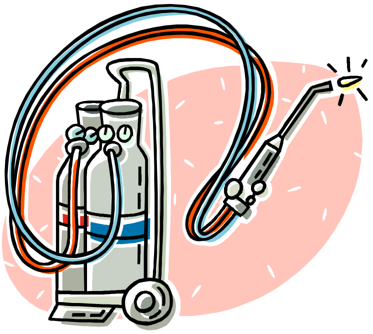
Does it make a difference for the enthalpy of combustion whether I burn with pure oxygen or air? What about the adiabatic flame temperature?

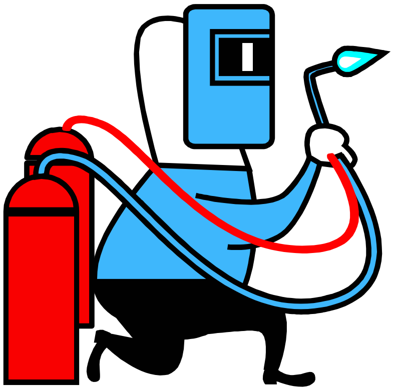
No difference in the enthalpy of combustion – the nitrogen in the air is the same in the reactants and products, and its enthalpy cancels out.

The adiabatic flame temperature is much lower for combustion with air, because a significant part of the energy release from the combustion goes into heating the nitrogen (as well as the other products) to the flame temperature.

15.16

A welder uses a bottle with acetylene and a bottle with oxygen. Why does he use oxygen instead of air?





The oxygen in the air comes with nitrogen that is also being heated up to the flame temperature. The heating value is being shared among all the product components so when they increase (having the nitrogen also) the adiabatic flame temperature drops correspondingly.

15.17

Some gas-welding is done using bottles of fuel, oxygen and argon. Why do you think argon is used?

Argon is used to shield the welding spot from the oxygen in the surrounding air. If oxygen is on the surface of the liquid metal it may form metal-oxides creating a weak location in the weld.

15.18

Is combustion a reversible process?

NO. It is a highly irreversible process.

Fuels and the Combustion Process

15.19

In a picnic grill gaseous propane is fed to a burner together with stoichiometric air. Find the air-fuel ratio on a mass basis and the total amount of reactant mass for 1 kg propane burned.

The reaction equation for stoichiometric mixture is:

$$C_3H_8 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \ \rightarrow\ a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $3 = b$;

H balance: $8 = 2a \quad => \quad a = 4$

O balance: $2\ \nu_{O2} = a + 2b = 4 + 2 \times 3 \quad => \quad \nu_{O2} = 5$

N_2 balance: $3.76\ \nu_{O2} = c = 18.8$

The reaction is

$$C_3H_8 + 5\left(O_2 + 3.76\ N_2\right) \ \rightarrow\ 4\,H_2O + 3\,CO_2 + 18.8\,N_2$$

$$AF_{mol} = \frac{5 + 18.8}{1} = 23.8$$

$$AF_{mass} = AF_{mol}\,\frac{M_{air}}{M_{fuel}} = 23.8 \times \frac{28.97}{44.097} = \mathbf{15.64}$$

Total mass of reactants = 1 + 15.64 = **16.64 kg**

15.20

Calculate the theoretical air-fuel ratio on a mass and mole basis for the combustion of ethanol, C_2H_5OH.

Reaction Eq.: $C_2H_5OH + \nu_{O2}(O_2 + 3.76N_2) \Rightarrow aCO_2 + bH_2O + cN_2$

Do the atom balance

Balance C: $2 = a$

Balance H: $6 = 2b \quad \Rightarrow \quad b = 3$

Balance O: $1 + 2\nu_{O2} = 2a + b = 4 + 3 = 7 \qquad \Rightarrow \quad \nu_{O2} = 3$

$(A/F)_{mol} = \nu_{O2}(1 + 3.76)/1 = 3 \times 4.76 = \mathbf{14.28}$

$(A/F)_{mass} = (\nu_{O2}M_{O2} + \nu_{N2}\, M_{N2})/M_{Fuel}$

$= (3\times31.999 + 11.28\times28.013) / 46.069 = \mathbf{8.943}$

15.21

A certain fuel oil has the composition $C_{10}H_{22}$. If this fuel is burned with 150% theoretical air, what is the composition of the products of combustion?

$$C_{10}H_{22} + (1/\phi)\,\nu_{O2}\,(O_2 + 3.76\,N_2) \rightarrow a\,H_2O + b\,CO_2 + c\,N_2 + d\,O_2$$

Stoichiometric combustion: $\phi = 1$, $d = 0$,

C balance: $b = 10$

H balance: $a = 22/2 = 11$,

O balance: $2\,\nu_{O_2} = a + 2b = 11 + 20 = 31 \quad => \quad \nu_{O2} = 15.5$

Actual case: $1/\phi = 1.5 \quad => \quad \nu_{O2} = 1.5 \times 15.5 = 23.25$

H balance: $a = 11$, C balance: $b = 10$,

N balance: $c = 23.25 \times 3.76 = 87.42$

O_2 balance: $d = 23.25 - 10 - 11/2 = 7.75$ (excess oxygen)

The combustion process is

$$C_{10}H_{22} + 23.25\,(O_2 + 3.76\,N_2) \rightarrow 11\,H_2O + 10\,CO_2 + 87.42\,N_2 + 7.75\,O_2$$

15.22

Methane is burned with 200% theoretical air. Find the composition and the dew point of the products.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\,N_2\right) \rightarrow a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $1 = b$; H balance: $4 = 2a$

O balance: $2\,\nu_{O2} = a + 2b = 2 + 2 \times 1 \;\;=>\;\; \nu_{O2} = 2$

N_2 balance: $3.76\,\nu_{O2} = c = 7.52$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\,N_2\right) \rightarrow a\,H_2O + b\,CO_2 + c\,N_2 + d\,O_2$$

N_2 balance: $3.76\,\nu_{O2} = c = 15.04$

Extra oxygen: $d = 4 – 1 - 1 = 2$

Products: $2\,H_2O + 1\,CO_2 + 15.04\,N_2 + 2\,O_2$

Water vapor mole fraction: $y_v = \dfrac{2}{1 + 2 + 2 + 15.04} = 0.0998$

Partial water vapor pressure: $P_v = y_v\,P_o = 0.0998 \times 101 = 9.98$ kPa

$$P_g(T_{dew}) = P_v = 9.98\text{ kPa} \;\;\Rightarrow\;\; T_{dew} = \mathbf{45.8^oC}$$

15.23

Natural gas B from Table 15.2 is burned with 20% excess air. Determine the composition of the products.

The reaction equation (stoichiometric and complete combustion) with the fuel composition is:

$$60.1\ CH_4 + 14.8\ C_2H_6 + 13.4\ C_3H_8 + 4.2\ C_4H_{10} + 7.5\ N_2$$

$$+ \nu_{O2}\left(O_2 + 3.76\ N_2\right) \ \rightarrow\ a\,H_2O + b\,CO_2 + c\ N_2$$

C balance: $60.1 + 2\times14.8 + 3\times13.4 + 4\times4.2 = b = 146.7$

H balance: $4\times60.1 + 6\times14.8 + 8\times13.4 + 10\times4.2 = 2a = 478.4 \ \Rightarrow\ a = 239.2$

O balance: $2\ \nu_{O2} = a + 2b = 239.2 + 2\times146.7 \ \Rightarrow\ \nu_{O2} = 266.3$

N_2 balance: $7.5 + 3.76\ \nu_{O2} = c = 1008.8$

20% excess air: $\nu_{O2} = 1.2\times266.3 = 319.56$ so now more O_2 and N_2

Extra oxygen: $d = 319.56 - 266.3 = 53.26$, $c = 7.5 + 3.76\times319.56 = 1209$

Products: $239.2\ H_2O + 146.7\ CO_2 + 1209\ N_2 + 53.26\ O_2$

To the expert the color of the flame can tell about the composition. It can also tell about other gases present if they have distinct color emission.

15.24

For complete stoichiometric combustion of gasoline, C_7H_{17}, determine the fuel molecular weight, the combustion products, and the mass of carbon dioxide produced per kg of fuel burned.

Stoichiometric combustion:

$$C_7H_{17} + \nu_{O2} \left(O_2 + 3.76\ N_2\right) \rightarrow \ a\ H_2O + b\ CO_2 + c\ N_2$$

C balance: $7 = b$

H balance: $17 = 2a, \quad \Rightarrow \quad a = 8.5$

O balance: $2\ \nu_{O2} = a + 2b = 8.5 + 14 = 22.5 \quad \Rightarrow \quad \nu_{O2} = 11.25$

N balance: $c = 3.76\ \ \nu_{O2} = 3.76 \times 11.25 = 42.3$

$$M_{FUEL} = 7\ M_C + 17\ M_H = 7 \times 12.011 + 17 \times 1.008 = \mathbf{101.213}$$

$$\frac{m_{CO2}}{m_{FUEL}} = \frac{7\ M_{CO2}}{M_{FUEL}} = \frac{7 \times 44.01}{101.213} = \mathbf{3.044\ kg\ CO_2\ per\ kg\ fuel}$$

15.25

A Pennsylvania coal contains 74.2% C, 5.1% H, 6.7% O, (dry basis, mass percent) plus ash and small percentages of N and S. This coal is fed into a gasifier along with oxygen and steam, as shown in Fig. P14.26. The exiting product gas composition is measured on a mole basis to: 39.9% CO, 30.8% H_2, 11.4% CO_2, 16.4% H_2O plus small percentages of CH_4, N_2, and H_2S. How many kilograms of coal are required to produce 100 kmol of product gas? How much oxygen and steam are required?

Convert the mass concentrations to number of kmol per 100 kg coal:

C : n = 74.2/12.01 = 6.178 H_2: n = 5.1/2.016 = 2.530

O_2: n = 6.7/31.999 = 0.209

Now the combustion equation reads

$$x(6.178\,C + 2.53\,H_2 + 0.209\,O_2) + y\,H_2O + z\,O_2 \;\text{ in and}$$

$$39.9\,CO + 30.8\,H_2 + 11.4\,CO_2 + 16.4\,H_2O \;\text{ out}$$

in 100 kmol of mix out

Now we can do the atom balance to find (x, y, z)

C balance: $6.178\,x = 39.9 + 11.4 \quad\rightarrow\quad x = 8.304$

H_2 balance: $2.53 \times 8.304 + y = 30.8 + 16.4 \quad\rightarrow\quad y = 26.191$

O_2 balance: $0.209 \times 8.304 + \dfrac{26.191}{2} + z = \dfrac{39.9}{2} + 11.4 + \dfrac{16.4}{2} \quad\rightarrow z = 24.719$

Therefore, for 100 kmol of mixture out

require: **830.4 kg** of coal

26.191 kmol of steam

24.719 kmol of oxygen

15.26

Liquid propane is burned with dry air. A volumetric analysis of the products of combustion yields the following volume percent composition on a dry basis: 8.6% CO_2, 0.6% CO, 7.2% O_2 and 83.6% N_2. Determine the percent of theoretical air used in this combustion process.

$$a\,C_3H_8 + b\,O_2 + c\,N_2 \rightarrow 8.6\,CO_2 + 0.6\,CO + d\,H_2O + 7.2\,O_2 + 83.6\,N_2$$

C balance: $3a = 8.6 + 0.6 = 9.2 \Rightarrow a = 3.067$

H_2 balance: $4a = d \Rightarrow d = 12.267$

N_2 balance: $c = 83.6$

O_2 balance: $b = 8.6 + \frac{0.6}{2} + \frac{12.267}{2} + 7.2 = 22.234$

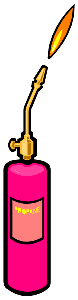
$$\text{Air-Fuel ratio} = \frac{22.234 + 83.6}{3.067} = 34.51$$

Theoretical:

$$C_3H_8 + 5\,O_2 + 18.8\,N_2 \rightarrow 3\,CO_2 + 4\,H_2O + 18.8\,N_2$$

$$\Rightarrow \text{theo. A-F ratio} = \frac{5 + 18.8}{1} = 23.8$$

$$\%\text{ theoretical air} = \frac{34.51}{23.8} \times 100\,\% = \mathbf{145\ \%}$$



15.27

In a combustion process with decane, $C_{10}H_{22}$, and air, the dry product mole fractions are 83.61% N_2, 4.91% O_2, 10.56% CO_2 and 0.92% CO. Find the equivalence ratio and the percent theoretical air of the reactants.

$$x\, C_{10}H_{22} + (1/\phi)\, \nu_{O2} \left(O_2 + 3.76\, N_2\right) \rightarrow a\, H_2O + b\, CO_2 + c\, CO + d\, N_2 + e\, O_2$$

Stoichiometric combustion: $\phi = 1, \quad c = 0, \quad e = 0,$

C balance: $b = 10x$

H balance: $a = 22x/2 = 11x,$

O balance: $2\, \nu_{O2} = a + 2b = 11x + 20x = 31x$

$\nu_{O2} = 15.5x, \quad \nu_{N2} = 58.28x \qquad \Rightarrow \; (A/F)_s = (\nu_{O2} + \nu_{N2})/x = \mathbf{73.78}$

Actual combustion: $d = 83.61 \quad \rightarrow$

N balance: $(1/\phi)\, \nu_{O2} \times 3.76 = 83.61 \quad \rightarrow (1/\phi)\, \nu_{O2} = 22.24$

C balance: $10x = 10.56 + 0.92 = 11.48 \quad \Rightarrow \; x = 1.148$

$(A/F)_{ac} = (1/\phi)\, \nu_{O2} \times 4.76/1.148 = \mathbf{92.215}$

$\phi = (F/A)_{ac} / (F/A)_s = (A/F)_s / (A/F)_{ac} = 73.78 / 92.215 = \mathbf{0.80}$ **or** $\boldsymbol{\phi = 0.8}$

Percent theoretical air $= 100\, (1/\phi) = \mathbf{125\%}$

15.28

A sample of pine bark has the following ultimate analysis on a dry basis, percent by mass: 5.6% H, 53.4% C, 0.1% S, 0.1% N, 37.9% O and 2.9% ash. This bark will be used as a fuel by burning it with 100% theoretical air in a furnace. Determine the air–fuel ratio on a mass basis.

Converting the Bark Analysis from a mass basis:

Substance	S	H_2	C	O_2	N_2
c/M =	0.1/32	5.6/2	53.4/12	37.9/32	0.1/28
kmol / 100 kg coal	0.003	2.80	4.45	1.184	0.004
Product	SO_2	H_2O	CO_2		
oxygen required	0.003	1.40	4.45	--	--

Combustion requires: 0.003 + 1.40 + 4.45 = 5.853 kmol O_2 there is in the bark 1.184 kmol O_2 so the net from air is 4.669 kmol O_2

$$AF = (4.669 + 4.669 \times 3.76) \times \frac{28.97}{100} = \mathbf{6.44} \, \frac{\textbf{kg air}}{\textbf{kg bark}}$$

15.29

Methanol, CH_3OH, is burned with 200% theoretical air in an engine and the products are brought to 100 kPa, 30°C. How much water is condensed per kilogram of fuel?

$$CH_3OH + \nu_{O2}\{O_2 + 3.76\ N_2\} \rightarrow CO_2 + 2\ H_2O + 3.76\ \nu_{O2}N_2$$

$$\text{Stoichiometric} \quad \nu_{O2\ S} = 1.5 \quad \Rightarrow \quad \nu_{O2\ AC} = 3$$

Actual products: $CO_2 + 2\ H_2O + 1.5\ O_2 + 11.28\ N_2$

$P_{sat}(30°C) = 4.246$ kPa

$$\Rightarrow y_{H2O} = 0.04246 = \frac{\nu_{H2O}}{1 + \nu_{H2O} + 1.5 + 11.28}$$

$$\Rightarrow \nu_{H2O} = 0.611 \quad \Rightarrow \quad \Delta\nu_{H2O\ cond} = 2 - 0.611 = 1.389$$

$$M_{Fu} = 32.042 \qquad \frac{\Delta M_{H2O}}{M_{Fu}} = \frac{1.389 \times 18}{32.042} = \mathbf{0.781}\ \frac{\mathbf{kg\ H_2O}}{\mathbf{kg\ fuel}}$$

15.30

The coal gasifier in an integrated gasification combined cycle (IGCC) power plant produces a gas mixture with the following volumetric percent composition:

Product	CH_4	H_2	CO	CO_2	N_2	H_2O	H_2S	NH_3
% vol.	0.3	29.6	41.0	10.0	0.8	17.0	1.1	0.2

This gas is cooled to 40°C, 3 MPa, and the H_2S and NH_3 are removed in water scrubbers. Assuming that the resulting mixture, which is sent to the combustors, is saturated with water, determine the mixture composition and the theoretical air–fuel ratio in the combustors.

CH_4	H_2	CO	CO_2	N_2	n
0.3	29.6	41.0	10.0	0.8	81.7

$y_{H2O} = \frac{n_V}{n_V + 81.7}$, where n_V = number of moles of water vapor

Cool to 40°C $P_G = 7.384$, $P = 3000$ kPa

$$y_{H2O\ MAX} = \frac{7.384}{3000} = \frac{n_V}{n_V + 81.7} \quad \rightarrow \quad n_V = 0.2016$$

a) Mixture composition:

CH_4	H_2	CO	CO_2	N_2	$H_2O(v)$
0.3 kmol	29.6	41.0	10.0	0.8	0.2016

81.9016 kmol (from 100 kmol of the original gas mixture)

$$0.3\ CH_4 + 0.6\ O_2 \rightarrow 0.3\ CO_2 + 0.6\ H_2O$$

$$29.6\ H_2 + 14.8\ O_2 \rightarrow 29.6\ H_2O$$

$$41\ CO + 20.5\ O_2 \rightarrow 41\ CO_2$$

$\Rightarrow$ Number of moles of $O_2 = 0.6 + 14.8 + 20.5 = 35.9$

Number of moles of air $= 35.9 + 3.76 \times 35.9$ (N_2)

$$A/F = \frac{28.97(35.9 + 3.76(35.9))}{0.3(16) + 29.6(2) + 41(28) + 10(44) + 0.8(28) + 0.2016(18)}$$

$= \mathbf{2.95\ kg\ air/kg\ fuel}$

15.31

Butane is burned with dry air at 40°C, 100 kPa with AF = 26 on a mass basis. For complete combustion find the equivalence ratio, % theoretical air and the dew point of the products. How much water (kg/kg fuel) is condensed out, if any, when the products are cooled down to ambient temperature?

Solution:

$$C_4H_{10} + \nu_{O_2}\{O_2 + 3.76\ N_2\} \rightarrow 4\ CO_2 + 5\ H_2O + 3.76\ \nu_{O_2}N_2$$

Stoichiometric $\nu_{O_2\ S} = 4 + 5/2 = 6.5; \qquad 3.76\ \nu_{O_2} = 24.44$

$$(A/F)_S = 6.5(31.999 + 3.76 \times 28.013)/58.124 = 15.3574$$

Actual: $\nu_{O_2ac} = \frac{(A/F)_{ac}}{(A/F)_s} \nu_{O_2\ S} = \frac{26}{15.3574} 6.5 = 11$

$$\%\ \text{Theoretical air} = \frac{26}{15.3574} 100 = \mathbf{169.3\%}$$

Equivalence ratio $\Phi = 1/1.693 = \mathbf{0.59}$

Actual products: $4\ CO_2 + 5\ H_2O + 4.5\ O_2 + 41.36\ N_2$

The water partial pressure becomes

$$P_v = y_v\ P_{tot} = \frac{5}{4 + 5 + 4.5 + 41.36} 100 = 9.114\ \text{kPa}$$

$$T_{dew} = \mathbf{43.85^oC}$$

$P_{g\ 40} = 7.348\ \text{kPa} \quad \Rightarrow \quad y_{v\ max} = \frac{7.384}{100} = \frac{\nu_{H2O}}{4 + \nu_{H2O} + 4.5 + 41.36}$

Solve for $\nu_{H2O\ vap}$:

$\nu_{H2O\ vap} = 3.975$ still vapor,

$\nu_{H2O\ LIQ} = 5 - 3.975 = 1.025$ is liquid

$$\frac{m_{H2O\ LIQ}}{m_{Fuel}} = \frac{1.025 \times 18.015}{58.124} = \mathbf{0.318}$$

15.32

The output gas mixture of a certain air–blown coal gasifier has the composition of producer gas as listed in Table 14.2. Consider the combustion of this gas with 120% theoretical air at 100 kPa pressure. Determine the dew point of the products and find how many kilograms of water will be condensed per kilogram of fuel if the products are cooled 10°C below the dew-point temperature.

$$\{3\ CH_4 + 14\ H_2 + 50.9\ N_2 + 0.6\ O_2 + 27\ CO + 4.5\ CO_2\}$$

$$+ 31.1\ O_2 + 116.9\ N_2 \rightarrow 34.5\ CO_2 + 20\ H_2O + 5.2\ O_2 + 167.8\ N_2$$

Products:

$$y_{H2O} = y_{H2O\ MAX} = P_G/100 = \frac{20}{34.5 + 20 + 5.2 + 167.8}$$

$$\Rightarrow P_G = 8.79\text{ kPa} \quad \rightarrow \quad T_{DEW\ PT} = \mathbf{43.2°C}$$

At T = 33.2°C, $P_G = 5.13$ kPa

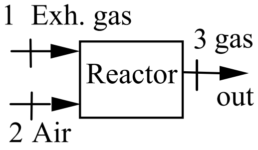
$$y_{H2O} = \frac{5.13}{100} = \frac{n_{H2O}}{n_{H2O}+34.5+5.2+167.8} \quad \rightarrow \quad n_{H2O} = 11.22$$

$$m_{H2O\ LIQ} = \frac{8.78(18)}{3(16) + 14(2) + 50.9(28) + 0.6(32) + 27(28) + 4.5(44)} = \mathbf{0.0639\ kg/kg\ fuel}$$

15.33

The hot exhaust gas from an internal combustion engine is analyzed and found to have the following percent composition on a volumetric basis at the engine exhaust manifold. 10% CO_2, 2% CO, 13% H_2O, 3% O_2 and 72% N_2. This gas is fed to an exhaust gas reactor and mixed with a certain amount of air to eliminate the carbon monoxide, as shown in Fig. P15.33. It has been determined that a mole fraction of 10% oxygen in the mixture at state 3 will ensure that no CO remains. What must the ratio of flows be entering the reactor?

Exhaust gas at state 1: CO_2 10 %, H_2O 13%,
CO 2%, O_2 3%, N_2 72%



Exhaust gas at state 3: CO = 0 %, O_2 = 10 %

Reaction equation for the carbon monoxide

$$\Rightarrow 0.02\,CO + x\,O_2 + 3.76x\,N_2 \rightarrow 0.02\,CO_2 + (x-0.01)\,O_2 + 3.76x\,N_2$$

At 3: $\nu_{CO_2} = 0.10 + 0.02 = 0.12,\qquad \nu_{H_2O} = 0.13$

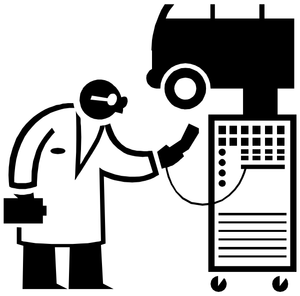
$\nu_{O_2} = (x-0.01) + 0.03 = x + 0.02\qquad \nu_{N_2} = 0.72 + 3.76x$

or

$$n_{TOT} = 0.12 + 0.13 + x + 0.02 + 0.72 + 3.76x = 0.99 + 4.76x$$

$$y_{O2} = 0.10 = \frac{x + 0.02}{0.99 + 4.76x} \quad\rightarrow\quad x = 0.151$$

or $$\frac{\text{air}_{\ 2}}{\text{Exh. Gas}_{\ 1}} = \frac{4.76x}{1} = \mathbf{0.718}\ \frac{\textbf{kmol air}}{\textbf{kmol Exh. gas}}$$



Energy Equation, Enthalpy of Formation

15.34

Hydrogen is burned with stoichiometric air in a steady flow process where the reactants are supplied at 100 kPa, 298 K. The products are cooled to 800 K in a heat exchanger. Find the heat transfer per kmol hydrogen.

The combustion equation is

$$H_2 + 0.5 \times [\,O_2 + 3.76\,N_2\,] \rightarrow 1\,H_2O + 1.88\,N_2$$

The reactants are at 298 K

$$H_R = 0 + 0 + 0 = 0$$

The products are at 800 K so water is gaseous and from A.9

$$H_P = 1[-241\,826 + 18\,002)] + 1.88\,(0 + 15\,046) = -195\,538 \text{ kJ/kmol fuel}$$

$$Q_{CV} = H_P - H_R = \mathbf{-195\,538\ kJ/kmol\ fuel}$$

15.35

Butane gas and 200% theoretical air, both at 25°C, enter a steady flow combustor. The products of combustion exits at 1000 K. Calculate the heat transfer from the combustor per kmol of butane burned.

$C_4H_{10} + (1/\phi)\,\nu_{O2}(O_2 + 3.76\,N_2) \;\rightarrow\; a\,CO_2 + b\,H_2O + c\,N_2 + d\,O_2$

First we need to find the stoichiometric air ($\phi = 1$, $d = 0$)

C balance: $4 = a$, H balance: $10 = 2b \;\Rightarrow\; b = 5$

O balance: $2\nu_{O2} = 2a + b = 8 + 5 = 13 \;\Rightarrow\; \nu_{O2} = 6.5$

Now we can do the actual air: $(1/\phi) = 2 \;\Rightarrow\; \nu_{O2} = 2 \times 6.5 = 13$

N balance: $c = 3.76\,\nu_{O2} = 48.88$, O balance: $d = 13 - 6.5 = 6.5$

Energy Eq.: $q = H_R - H_P = H^o_R - H^o_P - \Delta H_P$

Table A.10: $H^o_R = -126\,200 + 0 + 0 = -126\,200$ kJ/kmol fuel

$H^o_P = 4\,(-393\,522) + 5(-241\,826) + 0 + 0 = -2\,783\,218$ kJ/kmol fuel

The rest of the values are from Table A.9 at 1000 K

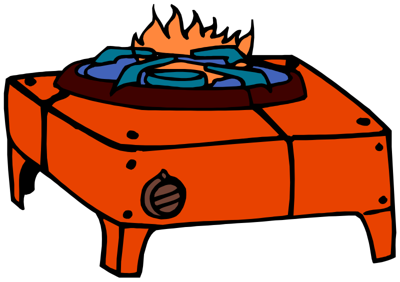
$\Delta\bar{h}_{CO2} = 33397$, $\Delta\bar{h}_{N2} = 21463$, $\Delta\bar{h}_{O2} = 22703$, $\Delta\bar{h}_{H2O} = 26000$ kJ/kmol

$\Delta H_P = 4 \times 33\,397 + 5 \times 26\,000 + 48.88 \times 21\,463 + 6.5 \times 22\,703$

$= 1\,460\,269$ kJ/kmol fuel

From the energy equation we get

$q = -126\,200 - (-2\,783\,218) - 1\,460\,269 =$ **1 196 749 kJ/kmol butane**



15.36

One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 120% theoretical air in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products exit at 60°C, 100 kPa. Calculate the heat transfer per kilomole of ethanol.

$$C_2H_5OH + 1.2 \times 3 \left(O_2 + 3.76\ N_2\right) \ \rightarrow\ 2CO_2 + 3H_2O + 0.6O_2 + 13.54N_2$$

Fuel: $\bar{h}_f^0 = -277\ 380$ kJ/kmol for liquid from Table A.10,

Products at 60°C, 100 kPa, check for condensation of water

$$y_{H2O\ MIX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX}+2+0.6+13.54} \quad => \quad n_{V\ MAX} = 4.0 > 3 \ \Rightarrow\ \text{No liq.}$$

$H_R = 1(-277\ 380) + 0 + 0 = -277\ 380$ kJ/kmol fuel

$H_P = 2(-393\ 522 + 1327) + 3(-241\ 826 + 1178)$

$\qquad + 0.6(0 + 1032) + 13.54(0 + 1020) = -1\ 491\ 904$ kJ/kmol fuel

$Q_{CV} = H_P - H_R =$ **-1 214 524 kJ/kmol fuel**

15.37

Do the previous problem with the ethanol fuel delivered as a vapor.

One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 120% theoretical air in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products exit at 60°C, 100 kPa. Calculate the heat transfer per kilomole of ethanol.

$$C_2H_5OH + 1.2 \times 3 \left(O_2 + 3.76\ N_2\right) \rightarrow 2CO_2 + 3H_2O + 0.6O_2 + 13.54N_2$$

Fuel: $\bar{h}_f^0 = -235\ 000$ kJ/kmol for IG from Table A.10

Products at 60°C, 100 kPa, check for condensation of water

$$y_{H2O\ MIX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX} + 2 + 0.6 + 13.54} \quad => \quad n_{V\ MAX} = 4.0$$

The mixture can then hold 4 moles of vapor > 3 present ⇒ No liquid.

$H_R = 1(-235\ 000) + 0 + 0 = -235\ 000$ kJ/kmol fuel

$H_P = 2(-393\ 522 + 1327) + 3(-241\ 826 + 1178)$

$+ 0.6(0 + 1032) + 13.54(0 + 1020) = -1\ 491\ 904$ kJ/kmol fuel

$Q_{CV} = H_P - H_R =$ **-1 256 904 kJ/kmol fuel**

15.38

As an alternative fuel consider liquid methanol burned with stoichiometric air both supplied at P_o, T_o in a constant pressure process exhausting the products at 900 K. What is the heat transfer per kmol of fuel.

$$CH_3OH + 1.5\,(O_2 + 3.76\ N_2) \rightarrow 1\ CO_2 + 2\ H_2O + 5.64\ N_2$$

Reactants at 25°C, products are at 900 K, 100 kPa.

CH_3OH: $\bar{h}_f^o = -239\ 220$ kJ/kmol from table A.10 for the liquid state

$H_R = 1\ \bar{h}_{LIQ} = -239\ 220$ kJ/kmol fuel

$H_P = 1(-393\ 522 + 28\ 030) + 2(-241\ 826 + 21\ 937) + 5.64(18\ 223)$

$= -702\ 492$ kJ/kmol fuel

$Q = H_P - H_R = -702\ 492 - (-239\ 220) = \mathbf{-463\ 272\ kJ/kmol\ fuel}$

15.39

Another alternative fuel to be seriously considered is hydrogen. It can be produced from water by various techniques that are under extensive study. Its biggest problem at the present time are cost, storage, and safety. Repeat Problem 15.36 using hydrogen gas as the fuel instead of ethanol.

$$H_2 + 1.2 \times 0.5\ O_2 + 1.2 \times 3.76 \times 0.5\ N_2 \ \rightarrow 1\ H_2O + 0.1\ O_2 + 2.256\ N_2$$

Products at 60°C, 100 kPa, check for condensation of water

$$y_{H2O\ MAX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX} + 0.1 + 2.256}$$

Solving, $n_{V\ MAX} = 0.587 < 1 \quad => \ n_V = 0.587, \quad n_{LIQ} = 0.413$

$H_R = 0 + 0 + 0 = 0$

Notice the products are at 60°C so add for water liquid from steam tables

$$H_P = 0.413[-285\ 830 + 18.015(251.1 - 104)] + 0.587(-241\ 826 + 1178)$$
$$+ 0.1(0 + 1032) + 2.256(0 + 1020) \ = -255\ 816 \text{ kJ}$$

$Q_{CV} = H_P - H_R =$ **$-255\ 816$ kJ/kmol fuel**

15.40

The combustion of heptane C_7H_{16} takes place in a steady flow burner where fuel and air are added as gases at P_o, T_o. The mixture has 125% theoretical air and the products are going through a heat exchanger where they are cooled to 600 K.Find the heat transfer from the heat exchanger per kmol of heptane burned.

The reaction equation for stoichiometric ratio is:

$$C_7H_{16} + v_{O2} (O_2 + 3.76\ N_2) \quad => \ 7CO_2 + 8\ H_2O + v_{O2} \times 3.76\ N_2$$

So the balance (C and H was done in equation) of oxygen gives $v_{O2} = 7 + 4 = 11$, and actual one is $11 \times 1.25 = 13.75$. Now the actual reaction equation is:

$$C_7H_{16} + 13.75\ O_2 + 51.7\ N_2 \quad => \ 7CO_2 + 8\ H_2O + 51.7\ N_2 + 2.75\ O_2$$

To find the heat transfer take control volume as combustion chamber and heat exchanger

$$H_R + Q = H_P \quad => \quad Q = H^o_P + \Delta H_P - H^o_R$$

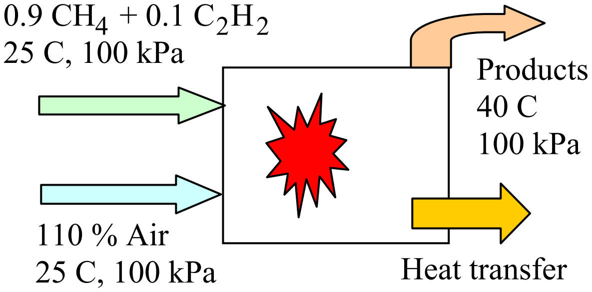
Take the enthalpies from Tables A.9 for the products and A.10 for the fuel

$$Q = 7(-393\ 522 + 12\ 906) + 8\ (-241\ 826 + 10\ 499) + 51.7(8894) + 2.75(9245) - (-187\ 900)$$

$$= \mathbf{-\ 3\ 841\ 784\ kJ/kmol\ fuel}$$

15.41

In a new high-efficiency furnace, natural gas, assumed to be 90% methane and 10% ethane (by volume) and 110% theoretical air each enter at 25°C, 100 kPa, and the products (assumed to be 100% gaseous) exit the furnace at 40°C, 100 kPa. What is the heat transfer for this process? Compare this to an older furnace where the products exit at 250°C, 100 kPa.



$$0.9\ CH_4 + 0.1\ C_2H_6 + 1.1 \times 2.15\ O_2 + 3.76 \times 2.365\ N_2$$

$$\rightarrow 1.1\ CO_2 + 2.1\ H_2O + 0.215\ O_2 + 8.892\ N_2$$

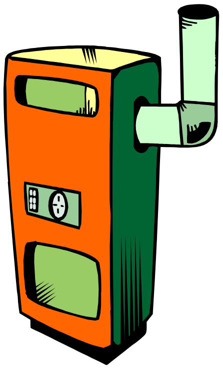
Fuel values from table A.10 and the rest from Table A.9

$H_R = 0.9(-74\ 873) + 0.1(-84\ 740) = -75860$ kJ/kmol fuel

$H_P = 1.1(-393\ 522 + 562) + 2.1(-241\ 826 + 504) + 0.215(441) + 8.892(437)$

$= -935\ 052$ kJ/kmol fuel   assuming all gas

$Q_{CV} = H_P - H_R =$ **-859 192 kJ/kmol fuel**



b) $T_P = 250\ ^oC$

$H_P = 1.1(-393\ 522 + 9346) + 2.1(-241\ 826 + 7740)$

$+ 0.215(6808) + 8.892(6597) = -854\ 050$ kJ

$Q_{CV} = H_P - H_R =$ **-778 190 kJ/kmol fuel**

15.42

Repeat the previous problem, but take into account the actual phase behavior of the products exiting the furnace.

$$0.9\,CH_4 + 0.1\,C_2H_6 + 1.1 \times 2.15\,O_2 + 3.76 \times 2.365\,N_2$$
$$\rightarrow 1.1\,CO_2 + 2.1\,H_2O + 0.215\,O_2 + 8.892\,N_2$$

Same as 14.49, except check products for saturation at 40°C, 100 kPa

$$y_{V\,MAX} = \frac{7.384}{100} = \frac{n_{V\,MAX}}{n_{V\,MAX}+10.207} \quad => \quad \text{Solving, } n_{V\,MAX} = 0.814$$

$$n_V = 0.814, \quad n_{LIQ} = 2.1 - 0.814 = 1.286$$

Fuel values from table A.10 and the rest from Table A.9

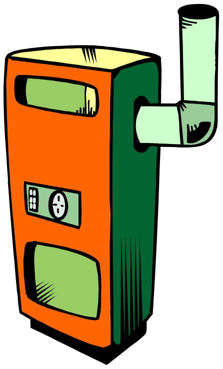
$$H_R = 0.9(-74\,873) + 0.1(-84\,740) = -75\,860 \text{ kJ/kmol fuel}$$

For the liquid water add difference (40°C – 25°C) from steam tables

$$H_{LIQ} = 1.286\big[-285\,830 + 18.015(167.6 - 104.9)\big] = -366\,125 \text{ kJ/kmol fuel}$$

$$H_{GAS} = 1.1(-393\,522 + 562) + 0.814(-241\,826 + 504\,)$$
$$+ 0.215(441) + 8.892(437) = -624\,711 \text{ kJ/kmol fuel}$$

$$Q_{CV} = H_P - H_R = (-366\,125 - 624\,711) + 75\,860$$
$$= \mathbf{-914\,976 \text{ kJ/kmol fuel}}$$



b) $T_P = 250\ ^oC$

$$H_P = 1.1(-393\,522 + 9346) + 2.1(-241\,826 + 7740)$$
$$+ 0.215(6808) + 8.892(6597)$$
$$= -854\,050 \text{ kJ/kmol}$$

$$Q_{CV} = H_P - H_R = \mathbf{-778\,190 \text{ kJ/kmol fuel}}$$

15.43

Pentene, C_5H_{10} is burned with stoichiometric pure oxygen in a steady flow process. After giving some heat out the products are at 700 K and used in a heat exchanger, where they are cooled to 25°C. Find the specific heat transfer in the heat exchanger.

$$C_5H_{10} + \nu_{O_2}O_2 \rightarrow 5\ CO_2 + 5\ H_2O \qquad \Rightarrow \qquad \nu_{O_2} = \mathbf{7.5}$$

The heat exchanger cools the products so energy equation is

$$5\ \dot{n}_F\ \bar{h}_{CO_2} + 5\ \dot{n}_F\ \bar{h}_{H_2O} + \dot{Q} = 5\ \dot{n}_F\ \bar{h}^{\circ}_{f\ CO_2}$$

$$+ (5 - x)\ \dot{n}_F\ \bar{h}^{\circ}_{liq\ H_2O} + (x)\ \dot{n}_F\ \bar{h}^{\circ}_{vap\ H_2O}$$

Check for condensation amount

$$\text{Find x:} \quad y_{H_2O\ max} = \frac{P_g(25°)}{P_{tot}} = 0.0313 = \frac{x}{5 + x} \qquad \Rightarrow \qquad x = \mathbf{0.1614}$$

Out of the 5 H_2O only 0.1614 still vapor.

Find the enthalpy at 700 K in table A.9 and the formation enthalpy in table A.10

$$\frac{\dot{Q}}{\dot{n}_F} = -5\ \Delta\bar{h}_{CO_2,700} + (5\text{-}x)(\bar{h}^{\circ}_{f\ liq} - \bar{h}^{\circ}_{f\ vap} - \Delta\bar{h}_{700}) + x(\bar{h}^{\circ}_{f\ vap} - \bar{h}^{\circ}_{f\ vap} - \Delta\bar{h}_{700})$$

$$= -5(17\ 761) + 4.84(-44\ 011 - 14\ 184) - 0.16(14\ 184)$$

$$= \mathbf{-372\ 738\ kJ/kmol\ Fu}$$

15.44

A rigid container has a 1:1 mole ratio of propane and butane gas together with a stoichiometric ratio of air at P_o, T_o. The charge burns and there is heat transfer to a final temperature of 1000 K. Find the final pressure and the heat transfer per kmol of fuel mixture.

$$C_3H_8 + C_4H_{10} + \nu_{O2}(O_2 + 3.76\ N_2) \ \rightarrow 7\ CO_2 + 9\ H_2O + 3.76\ \nu_{O2}\ N_2$$

The carbon and hydrogen balance has been used so oxygen gives

$$\nu_{O2} = 7 + 9/2 = 11.5\ ; \quad \nu_{N_2} = 43.24$$

The energy equation is

$$U_P - U_R = Q - 0 = (H_P - H_R) - n_P\bar{R}T_P + n_R\bar{R}T_R$$

$$= H^o_P + \Delta H_P - H^o_R - n_P\bar{R}T_P + n_R\bar{R}T_R$$

$$n_R = 1 + 1 + 11.5 \times 4.76 = 56.74, \qquad n_P = 7 + 9 + 43.24 = 59.24$$

From Table A.9

$$\Delta H_P = 7\ (33\ 397) + 9\ (26\ 000) + 43.24\ (21463) = 1\ 395\ 839\ \text{kJ}$$

From Table 15.3 (convert kJ/kg to kJ/kmol with M)

$$H^o_P - H^o_R = 44.094\ (-46\ 352) + 58.124\ (-45\ 714) = -\ 4\ 700\ 926\ \text{kJ}$$

Substitute into energy equation

$$Q = -\ 4\ 700\ 926 + 1\ 395\ 839 - 8.3145(59.24 \times 1000 - 56.74 \times 298.15)$$

$= -\ 3\ 656\ 981$ kJ/2 kmol fuel

= - 1 828 490 kJ/kmol fuel

15.45

A rigid vessel initially contains 2 kmol of carbon and 2 kmol of oxygen at 25°C, 200 kPa. Combustion occurs, and the resulting products consist of 1 kmol of carbon dioxide, 1 kmol of carbon monoxide, and excess oxygen at a temperature of 1000 K. Determine the final pressure in the vessel and the heat transfer from the vessel during the process.

$$2\,C + 2\,O_2 \;\rightarrow 1\,CO_2 + 1\,CO + \tfrac{1}{2}O_2$$

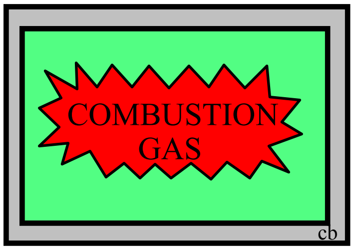
Process $V = \text{constant}$, C: solid, $n_{1(GAS)} = 2$, $n_{2(GAS)} = 2.5$

$$P_2 = P_1 \times \frac{n_2T_2}{n_1T_1} = 200 \times \frac{2.5 \times 1000}{2 \times 298.2} = \mathbf{838.4\ kPa}$$

$$H_1 = 0$$

$$H_2 = 1(-393\,522 + 33\,397) + 1(-110\,527 + 21\,686) + (1/2)(0 + 22\,703) = \mathbf{-437\,615\ kJ}$$

$${}_1Q_2 = (U_2 - U_1) = (H_2 - H_1) - n_2\bar{R}T_2 + n_1\bar{R}T_1$$
$$= (-437\,615 - 0) - 8.3145(2.5 \times 1000 - 2 \times 298.2) = \mathbf{-453\,442\ kJ}$$



15.46

A closed, insulated container is charged with a stoichiometric ratio of oxygen and hydrogen at 25°C and 150 kPa. After combustion, liquid water at 25°C is sprayed in such that the final temperature is 1200 K. What is the final pressure?

Combustion reaction: $H_2 + \frac{1}{2}\left(O_2\right) \rightarrow H_2O$

Products: $1\ H_2O + x_iH_2O$

$$U_2 - U_1 = x_i\bar{h}_i = x_i\overset{o}{h}_{f\,liq} = \left(1 + x_i\right)H_P - H_R - \left(1 + x_i\right)\bar{R}T_P + \tfrac{3}{2}\bar{R}T_R$$

From Table A.9: $H_R = 0$, $H_P = -241\ 826 + 34\ 506 = -207\ 320$ kJ/kmol

From Table A.10: $\overset{o}{h}_{f\,liq} = -285\ 830$ kJ/kmol

Substitute

$$x_i\left(-285830 + 207320 + 8.3145 \times 1200\right) =$$

$$-207\ 320 - 8.3145\left(1200 - \tfrac{3}{2}\times 298.15\right) = -213\ 579$$

$$x_i = 3.116$$

$$P_1V_1 = n_R\bar{R}T_1, \quad P_2V_1 = n_p\bar{R}T_p$$

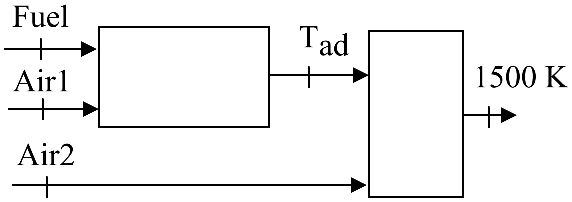
$$P_2 = \frac{P_1\left(1 + x_i\right)T_P}{\frac{3}{2}\left(T_1\right)} = \frac{150 \times 4.116 \times 1200}{\frac{3}{2} \times 298.15} = \mathbf{1657\ kPa}$$

15.47

In a gas turbine natural gas (methane) and stoichiometric air flows into the combustion chamber at 1000 kPa, 500 K. Secondary air, see Fig.P15.84, also at 1000 kPa, 500 K is added right after the combustion to result in a product mixture temperature of 1500 K. Find the A/F ratio mass basis for the primary reactant flow and the ratio of the secondary air to the primary air (mass flow rates ratio).

$$CH_4 + \nu(O_2 + 3.76\ N_2) \rightarrow$$
$$CO_2 + 2H_2O + 3.76\ \nu\ N_2$$

O balance: $\nu = 2, \quad \nu_{N2} = 7.52$



For the primary flow: $\quad (A/F)_{mass} = \dfrac{2 \times 32 + 7.52 \times 28.013}{16.043} = \mathbf{17.12}$

Do a C.V. around the entire set-up, then the energy equation becomes

Energy Eq.: $\quad H_{R1} + H_{R2} = H_{P\ 1500} = H^{\circ}_P + \Delta H_{P\ 1500}$

$$H_{R1} = H^{\circ}_R + \Delta H_{R1} = \bar{h}^{o}_{f\ fuel} + \Delta H_{Fuel} + \Delta H_{air1}$$

$$\Delta H_{air1} = 2(\Delta H_{O2} + 3.76\ \Delta H_{N2}) = 2(6086 + 3.76 \times 5911) = 56\ 623\ \text{kJ/kmol}$$

$$H_{R2} = 0 + \Delta H_{air2} = \nu_{O2\ add}\ (\Delta H_{O2} + 3.76\ \Delta H_{N2}) = \ \nu_{O2\ add}\ 28\ 311\ \text{kJ/kmol}$$

$$\Delta H_{Fuel} \approx M\ C_P\ \Delta T = 16.043 \times 2.254\ (500 - 298) = 7304.5\ \text{kJ/kmol}$$

$$\bar{h}^{o}_{f\ fuel} - H^{\circ}_P = HV = 16.043 \times 50\ 010 = 802\ 310\ \text{kJ/kmol}$$

$$\Delta H_{P\ 1500} = \Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + 7.52\ \Delta\bar{h}_{N_2} + \nu_{O2\ add}\ (\Delta\bar{h}_{O_2} + 3.76\ \Delta\bar{h}_{N_2})$$
$$= 61\ 705 + 2 \times 48\ 149 + 7.52 \times 38\ 405 + \nu_{O2\ add}\ (40\ 600$$
$$+ 3.76 \times 38\ 405) = 446\ 809 + \nu_{O2\ add}\ 185\ 003\ \text{kJ/kmol}$$

Now substitute everything into energy equation

$$802\ 310 + 7304.5 + 56\ 623 + \nu_{O2\ add}\ 28\ 311 = 446\ 809 + \nu_{O2\ add}\ 185\ 003$$

$$\nu_{O2\ add} = 2.677$$
$$m_{air2}/m_{air1} = n_{air2}/n_{air1} = \nu_{O2\ add}/\ \nu = 2.677/2 = 1.34$$

15.48

Methane, CH_4, is burned in a steady adiabatic flow process with two different oxidizers: Case **A**: Pure oxygen, O_2 and case **B**: A mixture of $O_2 + x$ Ar. The reactants are supplied at T_0, P_0 and the products for both cases should be at 1800 K. Find the required equivalence ratio in case (A) and the amount of Argon, x, for a stoichiometric ratio in case (B).

a) Stoichiometric has $\nu = 2$, actual has:

$$CH_4 + \nu O_2 \rightarrow CO_2 + 2H_2O + (\nu - 2)O_2$$

Energy eq.: $H^\circ_R = H^\circ_P + \Delta H_{P\ 1800}$

$$\Delta H_{P\ 1800} = H^\circ_R - H^\circ_P = \bar{h}^o_{f\ fuel} + 0 - \bar{h}^o_{f\ CO_2} - 2\bar{h}^o_{f\ H_2O} - 0$$

$$\text{RHS} = -74\ 873 - (-393\ 522) - 2(-241\ 826) = 802\ 301 \text{ kJ/kmol}$$

$\Delta\bar{h}_{CO_2} = 79\ 432$, $\Delta\bar{h}_{H_2O} = 62\ 693$, $\Delta\bar{h}_{O_2} = 51\ 674$ all in kJ/kmol

$$\Delta H_{P\ 1800} = \Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + (\nu - 2)\,\Delta\bar{h}_{O_2}$$

$$= 79\ 432 + 2 \times 62\ 693 + (\nu - 2) \times 51\ 674$$

$$= 101\ 470 + \nu\ 51\ 674 = 802\ 301 \text{ kJ/kmol fuel}$$

$$\Rightarrow \quad \boldsymbol{\nu = 13.56}, \quad \Phi = \frac{AF_S}{AF} = \frac{2}{13.56} = \mathbf{0.1475}$$

b) $$CH_4 + 2\ O_2 + 2x\ Ar \rightarrow CO_2 + 2H_2O + 2x\ Ar$$

$\Delta\bar{h}_{Ar} = \bar{C}_{P\ Ar}\,\Delta T = C_{P\ Ar}\,M_{Ar}\,\Delta T$ from table A.5

$$\Delta H_{P\ 1800} = 79\ 432 + 2 \times 62\ 693$$

$$+ 2x \times 0.52 \times 39.948(1800 - 298) = 204\ 818 + x\ 62\ 402$$

Now the energy equation becomes

$$802\ 301 = 204\ 818 + x\ 62\ 402 \quad \Rightarrow \quad x = \mathbf{9.575}$$

15.49

Gaseous propane mixes with air, both supplied at 500 K, 0.1 MPa. The mixture goes into a combustion chamber and products of combustion exit at 1300 K, 0.1 MPa. The products analyzed on a dry basis are 11.42% CO_2, 0.79% CO, 2.68% O_2, and 85.11% N_2 on a volume basis. Find the equivalence ratio and the heat transfer per kmol of fuel.

$$C_3H_8 + \alpha\, O_2 + 3.76\,\alpha\, N_2 \rightarrow \beta\, CO_2 + \gamma\, H_2O + 3.76\,\alpha\, N_2$$

$$\beta = 3, \quad \gamma = 4, \quad \alpha = \beta + \gamma/2 = 5, \qquad (A/F)_S = 4.76\alpha = 23.8$$

The actual combustion reaction is

$$xC_3H_8 + \alpha\, O_2 + 3.76\,\alpha\, N_2 \rightarrow$$

$$11.42\, CO_2 + y\, H_2O + 85.11\, N_2 + 0.79\, CO + 2.68\, O_2$$

C balance: $3x = 11.42 + 0.79 \quad => \quad x = 4.07$

H balance: $8x = 2y \quad => \quad y = 4x = 16.28$

O balance: $2\alpha = 2 \times 11.42 + y + 0.79 + 2 \times 2.68 = 45.27 \quad => \quad \alpha = 22.635$

N balance: $3.76\,\alpha = 85.11 \quad => \quad \alpha = 22.6356$ checks close enough

Rescale the equation by dividing with x to give

$$C_3H_8 + 5.5614\,(O_2 + 3.76\,N_2) \rightarrow$$

$$2.806\, CO_2 + 4\, H_2O + 20.91\, N_2 + 0.194\, CO + 0.6584\, O_2$$

$$A/F = 5.5614\,(1 + 3.76)\,/\,1 = 26.472$$

$\phi = (A/F)_S/(A/F) = 23.8\,/\,26.472 = \mathbf{0.899}$, %Theo. air = $1/\phi$ = **111%**

$$q = h_P - h_R = h_P^o + \sum \nu_i \Delta h(1300\text{ K}) - h_R$$

$$h_R = h^o_{f\,fuel} + \Delta h_{fuel} + 5.5614\,\Delta h_{O2} + 20.91\,\Delta h_{N2}$$

$$= -103\,900 + 1.679 \times 44.094\,(500 - 298) + 5.5614\,(6086) + 20.91\,(5911) = 68\,500 \text{ kJ/kmol fuel}$$

$$h_P = h_P^o + \sum \nu_i \Delta h(1300\text{ K})$$

$$= 2.806\,(-393\,522 + 50\,148) + 4(-241\,826 + 38\,941) + 20.91\,(0 + 31\,503) + 0.194\,(-110\,527 + 31\,867) + 0.6584\,(0 + 33\,345)$$

$$= -1\,109\,625 \text{ kJ/kmol fuel}$$

$q = -1\,109\,625 - 68\,500 =$ **$-$1 178 125 kJ/kmol fuel**

Enthalpy of Combustion and Heating Value

15.50

Find the enthalpy of combustion and the heating value for pure carbon.

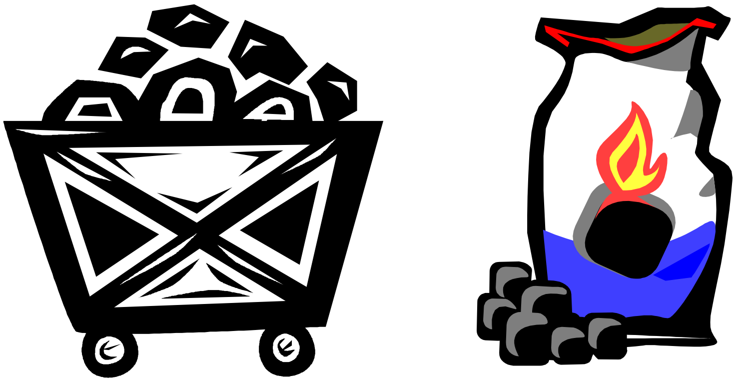
Reaction: $C + \nu_{O_2}[\,O_2 + 3.76\,N_2\,] \rightarrow\ CO_2 + \nu_{O_2}3.76\,N_2$

oxygen balance: $\nu_{O_2} = 1$

$H^{\circ}_{RP} = H^{o}_{P} - H^{o}_{R} = \bar{h}^{\circ}_{f\,CO_2} - \bar{h}^{\circ}_{f\,C} = -393\,522 - 0$

$= $ **$-393\,522$ kJ/kmol** $=$ **$-32\,791$ kJ/kg** (M = 12.011)

HV = 393 522 kJ/kmol = **32 791 kJ/kg**



15.51

Phenol has an entry in Table 15.3, but it does not have a corresponding value of the enthalpy of formation in Table A.10. Can you calculate it?

$$C_6H_5OH + \nu_{O2}[\ O_2 + 3.76\ N_2\ ] \rightarrow\ 3\ H_2O + 6\ CO_2 + 3.76\nu_{O2}\ N_2$$

The C and H balance was introduced (6 C's and 6 H's). At the reference condition the oxygen and nitrogen have zero enthalpy of formation.

Energy Eq.: $H_P = H_R = H^o_P = H^o_R$ since ref. T is assumed.

$$H^\circ_{RP} = H_P - H_R = H^o_P - H^o_R = 3\ \bar{h}^\circ_{f\,H2O} + 6\ \bar{h}^\circ_{f\,CO2} - \bar{h}^\circ_{f\,fuel}$$

Table 14.3 is on a mass basis and let us chose liquid fuel, so we get the molecular weight from the composition

$$M = 6 \times 12.011 + 3 \times 2.016\ + 16 = 94.114$$

$$H^\circ_{RP} = 94.114\ (-31\ 117) = -2\ 928\ 545\ \text{kJ/kmol}$$

Solve the energy equation for fuel formation enthalpy

$$\bar{h}^\circ_{f\,fuel} = 3\ \bar{h}^\circ_{f\,H2O} + 6\ \bar{h}^\circ_{f\,CO2} - H^\circ_{RP}$$

$$= 3\ (-241\ 826) + 6(-393\ 522) - (-2\ 928\ 545)$$

$$= \mathbf{-158\ 065\ kJ/kmol}$$

For fuel as vapor we get

$$H^\circ_{RP} = 94.114\ (-31\ 774) = -2\ 990\ 378\ \text{kJ/kmol}$$

$$\bar{h}^\circ_{f\,fuel} = 3\ \bar{h}^\circ_{f\,H2O} + 6\ \bar{h}^\circ_{f\,CO2} - H^\circ_{RP}$$

$$= 3\ (-241\ 826) + 6(-393\ 522) - (-2\ 990\ 378)$$

$$= \mathbf{-96\ 232\ kJ/kmol}$$

Notice if I took liquid water in products to do H°_{RP} then I must use liquid value for $\bar{h}^\circ_{f\,H2O} = -\ 285\ 830$ kJ/kmol and the final result is the same.

15.52

Some type of wood can be characterized as $C_1H_{1.5}O_{0.7}$ with a lower heating value of 19 500 kJ/kg. Find its formation enthalpy.

$$C_1H_{1.5}O_{0.7} + \nu_{O_2}[\,O_2 + 3.76\,N_2\,] \rightarrow \quad CO_2 + 0.75\,H_2O + 3.76\nu_{O_2}\,N_2$$

O balance: $0.7 + 2\nu_{O_2} = 2 + 0.75 \quad ⇨ \quad \nu_{O_2} = 1.025,\ \nu_{N2} = 3.854$

$$M = 1 \times 12.0111 + 1.5 \times 1.008 + 0.7 \times 16 = 24.712$$

$$H^{\circ}_{RP} = H^{o}_{P} - H^{o}_{R} = -M \times HV = -24.712 \times 19\,500 = -481\,884 \text{ kJ/kmol}$$

$$= 0.75\,\bar{h}^{\circ}_{f\,H2O} + 1\,\bar{h}^{\circ}_{f\,CO_2} - \bar{h}^{\circ}_{f\text{ fuel}}$$

$$\bar{h}^{\circ}_{f\text{ fuel}} = 0.75\,\bar{h}^{\circ}_{f\,H2O} + 1\,\bar{h}^{\circ}_{f\,CO_2} + M \times HV$$

$$= 0.75\,(-241\,826) + (-393\,522) + 481\,884$$

$$= \mathbf{-93\,008\ kJ/kmol}$$

15.53

Do problem 15.36 using table 15.3 instead of Table A.10 for the solution.
One alternative to using petroleum or natural gas as fuels is ethanol (C_2H_5OH), which is commonly produced from grain by fermentation. Consider a combustion process in which liquid ethanol is burned with 120% theoretical air in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products exit at 60°C, 100 kPa. Calculate the heat transfer per kilomole of ethanol.

$$C_2H_5OH + 1.2 \times 3\left(O_2 + 3.76\ N_2\right) \rightarrow 2\ CO_2 + 3\ H_2O + 0.6\ O_2 + 13.54\ N_2$$

Products at 60°C, 100 kPa, so check for condensation of water

$$y_{H2O\ MIX} = \frac{19.94}{100} = \frac{n_{V\ MAX}}{n_{V\ MAX}+2+0.6+13.54} \quad => \quad n_{V\ MAX} = 4.0 > 3 \ \Rightarrow \ \text{No liq.}$$

Fuel: table 15.3 select (liquid fuel, water vapor) and convert to mole basis

$$H^{\circ}_{RP} = 46.069\ (-26\ 811) = -1\ 235\ 156\ \text{kJ/kmol}$$

Since the reactants enter at the reference state the energy equation becomes

$$Q_{CV} = H_P - H_R = H^{o}_P + \Delta H_P - H^{o}_R = H^{\circ}_{RP} + \Delta H_P$$

$$\Delta H_P = 2\ \Delta\bar{h}_{CO_2} + 3\ \Delta\bar{h}_{H_2O} + 0.6\ \Delta\bar{h}_{O_2} + 13.54\ \Delta\bar{h}_{N_2}$$

$$= 2(1327) + 3(1178) + 0.6(1032) + 13.54(1020) = 20\ 618\ \text{kJ/kmol}$$

$$Q_{CV} = -1\ 235\ 156 + 20\ 618 = \mathbf{-1\ 214\ 538\ kJ/kmol\ fuel}$$

15.54

Liquid pentane is burned with dry air and the products are measured on a dry basis as: 10.1% CO_2, 0.2% CO, 5.9% O_2 remainder N_2. Find the enthalpy of formation for the fuel and the actual equivalence ratio.

$$\nu_{Fu} C_5H_{12} + \nu_{O2} O_2 + 3.76\ \nu_{O2} N_2 \rightarrow$$

$$x\ H_2O + 10.1\ CO_2 + 0.2\ CO + 5.9\ O_2 + 83.8\ N_2$$

Balance of C: $5\ \nu_{Fu} = 10.1 + 0.2 \ \Rightarrow\ \nu_{Fu} = \mathbf{2.06}$

Balance of H: $12\ \nu_{Fu} = 2\ x \ \Rightarrow\ x = 6\ \nu_{Fu} = \mathbf{12.36}$

Balance of O: $2\ \nu_{O2} = x + 20.2 + 0.2 + 2 \times 5.9 \ \Rightarrow\ \nu_{O2} = \mathbf{22.28}$

Balance of N: $2 \times 3.76\ \nu_{O2} = 83.8 \times 2 \ \Rightarrow\ \nu_{O2} = \mathbf{22.287 \Rightarrow OK}$

ν_{O2} for 1 kmol fuel = 10.816

$\phi = 1,\ C_5H_{12} + 8\ O_2 + 8 \times 3.76\ N_2 \rightarrow 6\ H_2O + 5\ CO_2 + 30.08\ N_2$

$$H^\circ_{RP} = H^\circ_P - H^\circ_R = 6\ \bar{h}^\circ_{f\,H2O} + 5\ \bar{h}^\circ_{f\,CO2} - \bar{h}^\circ_{f\,fuel}$$

14.3: $H^\circ_{RP} = 44\ 983 \times 72.151 \qquad \Rightarrow \qquad \bar{h}^\circ_{f\,fuel} = \mathbf{-172\ 998\ kJ/kmol}$

$$\phi = AFs / AF = \nu_{O2\ stoich}/\nu_{O2\ AC} = 8/10.816 = \mathbf{0.74}$$

15.55

Agriculturally derived butanol, $C_4H_{10}O$, molecular mass 74.12, also called bio-butanol has a lower heating value LHV = 33 075 kJ/kg for liquid fuel. Find its formation enthalpy.

The reaction equation for stiochiometric ratio is:

$$C_4H_{10}O \;+\; \nu_{O2}\,(O_2 + 3.76\ N_2) \quad => \; 4CO_2 + 5\ H_2O + \nu_{O2} \times 3.76\ N_2$$

where the carbon and hydrogen balance have been done. Now the oxygen

$$O: \quad 1 + 2\nu_{O2} = 4 \times 2 + 5 \quad => \quad \nu_{O2} = 6; \quad \nu_{N2} = 22.56$$

The lower heating value per kmol fuel is

$$LHV = -\,H^{\circ}_{RP} = H^{o}_{R} - H^{o}_{P} = \bar{h}^{\circ}_{f\ fuel} - 4\ \bar{h}^{\circ}_{f\ CO_2} - 5\ \bar{h}^{\circ}_{f\ H_2O}$$

so the fuel formation enthalpy becomes

$$\bar{h}^{\circ}_{f\ fuel} = LHV + 4\ \bar{h}^{\circ}_{f\ CO_2} + 5\ \bar{h}^{\circ}_{f\ H_2O}$$

$$= 74.12 \times 33\ 075 + 4\ (-393\ 522\ ) + 5(-241\ 826)$$

$$= 2\ 451\ 519 - 1\ 574\ 088 - 1\ 209\ 130$$

$$= \mathbf{-331\ 699\ kJ/kmol}$$

15.56

Do Problem 15.38 using Table 15.3 instead of Table A.10 for the solution.

As an alternative fuel consider liquid methanol burned with stoichiometric air both supplied at P_o, T_o in a constant pressure process exhausting the products at 900 K. What is the heat transfer per kmol of fuel.

$$CH_3OH + 1.5\,(O_2 + 3.76\,N_2) \rightarrow 1\,CO_2 + 2\,H_2O + 5.64\,N_2$$

Reactants at 25°C, products are at 900 K, 100 kPa.

CH_3OH: From table 15.3 for liquid fuel, water vapor

$$\overset{o}{H}_{RP} = –19\,910 \text{ kJ/kg } = –19\,910 \times 32.042 = –\,637\,956 \text{ kJ/kmol fuel}$$

A9: $\Delta H_P = 1 \times 28\,030 + 2 \times 21\,937 + 5.64 \times 18\,223 = 174\,682$ kJ/kmol fuel

$$Q = H_{Po} + \Delta H_P - H_{Ro} = \overset{o}{H}_{RP} + \Delta H_P = –\,637\,956 + 174\,682$$

$$= \mathbf{–463\,274 \text{ kJ/kmol fuel}}$$

15.57

Wet biomass waste from a food-processing plant is fed to a catalytic reactor, where in a steady flow process it is converted into a low-energy fuel gas suitable for firing the processing plant boilers. The fuel gas has a composition of 50% methane, 45% carbon dioxide, and 5% hydrogen on a volumetric basis. Determine the lower heating value of this fuel gas mixture per unit volume.

For 1 kmol fuel gas,

$$0.5\ CH_4 + 0.45\ CO_2 + 0.05\ H_2 + 1.025\ O_2$$

$$\rightarrow (0.5 + 0.45)\ CO_2 + 1.05\ H_2O$$

The lower heating value is with water vapor in the products. Since the $0.45\ CO_2$ cancels,

$$\bar{h}_{RP} = 0.5(-393\ 522) + 1.05(-241\ 826) - 0.5(-74\ 873) - 0.05(0)$$

$$= -413242 \text{ kJ/kmol fuel gas}$$

With $\frac{n}{V} = P/\bar{R}T = \frac{100}{8.3145 \times 298.2} = 0.04033 \text{ kmol/m}^3$

LHV = +413 242 × 0.04033 = **16 666 kJ/m^3**

15.58

Determine the lower heating value of the gas generated from coal as described in Problem 15.30. Do not include the components removed by the water scrubbers.

The gas from problem 15.30 is saturated with water vapor. Lower heating value LHV has water as vapor.

$$\mathrm{LHV} = -H^{\circ}_{RP} = H^{\circ}_{P} - H^{\circ}_{R}$$

Only CH_4, H_2 and CO contributes. From 15.12 the gas mixture after the scrubbers has $\sum \nu_i = 81.9$ of composition:

$$0.3\ CH_4 + 29.6\ H_2 + 41\ CO + 10\ CO_2 + 0.8\ N_2 + 0.2016\ H_2O$$

$$\mathrm{LHV} = -[0.3\bar{H}^{\circ}_{RP_{CH4}} + 29.6\bar{H}^{\circ}_{RP_{H2}} + 41\bar{H}^{\circ}_{RP_{CO}}]/81.9$$

$$= -[0.3(-50\ 010 \times 16.043) + 29.6(-241\ 826)$$
$$+ 41(-393\ 522 + 110\ 527)]/81.9$$

$$= \mathbf{232\ 009}\ \frac{\mathbf{kJ}}{\mathbf{kmol\ gas}}$$

15.59

In a picnic grill, gaseous propane and stoichiometric air are mixed and fed to the burner both at ambient P_o and T_o. After combustion the products cool down and at some point exit at 500 K. How much heat transfer was given out for 1 kg of propane?

The reaction equation for stoichiometric mixture is:

$$C_3H_8 + \nu_{O2}\,(O_2 + 3.76\ N_2) \ \rightarrow\ a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $3 = b$;

H balance: $8 = 2a \quad => \quad a = 4$

O balance: $2\,\nu_{O2} = a + 2b = 4 + 2 \times 3$

$=> \quad \nu_{O2} = 5$

N_2 balance: $3.76\,\nu_{O2} = c = 18.8$

$A/F = 5 \times 4.76 \times 28.97 / 44.094 = 15.637$

The reaction is

$$C_3H_8 + 5\,(O_2 + 3.76\ N_2) \ \rightarrow\ 4\,H_2O + 3\,CO_2 + 18.8\,N_2$$

All the water is in the vapor phase (500 K products).

Energy Eq.: $H_R = Q_{out} + H_P \quad => \quad H^o_R = Q_{out} + H^o_P + \Delta H_P$

A9: $\Delta H_P = 4 \times 6922 + 3 \times 8305 + 18.8 \times 5911 = 174\ 682$ kJ/kmol fuel

$= (174\ 682 / 44.094)$ kJ/kg $= 3961.6$ kJ/kg

Table 15.3: $-H^o_{RP} = HV = 46\ 352$ kJ/kg

$$Q_{out} = H^o_R - H^o_P - \Delta H_P = -H^o_{RP} - \Delta H_P = HV - \Delta H_P$$

$= (46\ 352 - 3961.6)$ kJ/kg fuel

$=$ **42390 kJ/kg fuel**

15.60

Do problem 15.40 using table 15.3 instead of Table A.10 for the solution.

The reaction equation for stiochiometric ratio is:

$$C_7H_{16} + \nu_{O2} (O_2 + 3.76\ N_2) \quad => \ 7CO_2 + 8\ H_2O + \nu_{O2} \times 3.76\ N_2$$

So the balance (C and H was done in equation) of oxygen gives $\nu_{O2} = 7 + 4 = 11$, and actual one is $11 \times 1.25 = 13.75$. Now the actual reaction equation is:

$$C_7H_{16} + 13.75\ O_2 + 51.7\ N_2 \quad => \ 7CO_2 + 8\ H_2O + 51.7\ N_2 + 2.75\ O_2$$

To find the heat transfer take a control volume as combustion chamber and heat exchanger

$$H_R + Q = H_P \quad => \quad Q = H_P^o + \Delta H_P - H_R^o = H_{RP}^{\circ} + \Delta H_P$$

Now we get the enthalpy of combustion from table 14.3, which is per kg, so scale it with the molecular weight for the fuel. Add all the ΔH_P from A.9

$$\Delta H_P = 7\ \Delta\bar{h}_{CO_2} + 8\ \Delta\bar{h}_{H_2O} + 51.7\ \Delta\bar{h}_{N_2} + 2.75\ \Delta\bar{h}_{O_2}$$

$$= 7(12\ 906) + 8(10\ 499) + 51.7(8894) + 2.75(9245) = 659\ 578\ \frac{kJ}{kmol}$$

$$Q = M\ H_{RP}^{\circ} + \Delta H_P = 100.205(-44\ 922) + 659\ 578$$

$= \mathbf{-\ 3\ 841\ 831\ kJ/kmol\ fuel}$

15.61

Propylbenzene, C_9H_{12}, is listed in Table 15.3, but not in table A.10. No molecular weight is listed in the book. Find the molecular weight, the enthalpy of formation for the liquid fuel and the enthalpy of evaporation.

$$C_9H_{12} + 12\ O_2 \rightarrow\ 9\ CO_2 + 6\ H_2O$$

$$\hat{M} = 9 \times 12.011 + 6 \times 2.016 = \mathbf{120.195}$$

$$\bar{h}^{\circ}_{RP} = = H^{o}_{P} - H^{o}_{R} = \sum_{P} \nu_i \bar{h}^{\circ}_{f_i} - \bar{h}^{\circ}_{fFu} \quad \Rightarrow \quad \bar{h}^{\circ}_{fFu} = \sum_{P} \nu_i \bar{h}^{\circ}_{f_i} - \bar{h}^{\circ}_{RP}$$

Formation enthalpies from Table A.10 and enthalpy of combustion from Table 15.3

$$\bar{h}^{\circ}_{fFu} = 9\bar{h}^{\circ}_{fCO_2} + 6\bar{h}^{\circ}_{fH_2O\ g} - \hat{M}(-41\ 219)_{liq\ Fu\ H_2O\ vap}$$

$$= 9(-393\ 522) + 6(-241\ 826) - 120.195(-41\ 219)$$

$$= \mathbf{-38\ 336\ kJ/kmol}$$

Take the enthalpy of combustion from Table 15.3 for fuel as a gas and as a vapor, the difference is the enthalpy of evaporation

$$h_{fg} = -(h^{\circ}_{RP\ gas} - h^{\circ}_{RP\ liq}) = 41\ 603 - 41\ 219 = \mathbf{384\ kJ/kg}$$

15.62

Consider natural gas A which are listed in Table 15.2. Calculate the enthalpy of combustion of each gas at 25°C, assuming that the products include vapor water. Repeat the answer for liquid water in the products.

Natural Gas A

$$0.939\ CH_4 + 0.036\ C_2H_6 + 0.012\ C_3H_8 + 0.013\ C_4H_{10}$$

$$+ 2.1485\ O_2 + 3.76 \times 2.1485\ N_2 \rightarrow 1.099\ CO_2 + 2.099\ H_2O + 8.0784\ N_2$$

$$H_R = 0.939(-74\,878) + 0.036(-84\,740) + 0.012(-103\,900)$$

$$+ 0.013(-126\,200) = -76244\ \text{kJ}$$

a) vapor H_2O

$$H_P = 1.099(-393\,522) + 2.099(-241\,826) = -940\,074\ \text{kJ/kmol}$$

$$\bar{h}_{RP} = H_P - H_R = \mathbf{-863\,830\ kJ/kmol}$$

b) Liq. H_2O

$$H_P = 1.099(-393\,522) + 2.099(-285\,830) = -1\,032\,438$$

$$\bar{h}_{RP} = \mathbf{-956\,194\ kJ/kmol}$$

15.63

Consider natural gas D which are listed in Table 15.2. Calculate the enthalpy of combustion of each gas at 25°C, assuming that the products include vapor water. Repeat the answer for liquid water in the products.

Natural Gas D:

$$0.543\ CH_4 + 0.163\ C_2H_6 + 0.162\ C_3H_8 + 0.074\ C_4H_{10}$$

$$+ 0.058\ N_2 + O_2 + N_2 \quad \rightarrow \quad 1.651\ CO_2 + 2.593\ H_2O + N_2$$

$$H_R = 0.543(-74\ 873) + 0.163(-84\ 740) + 0.162(-130\ 900)$$

$$+ 0.074(-126\ 200) = -80\ 639 \text{ kJ}$$

a) vapor H_2O

$$H_P = 1.651(-393\ 522) + 2.593(-241\ 826) = -1\ 276\ 760 \text{ kJ}$$

$\bar{h}_{RP}$ = **-1 196 121 kJ/kmol**

b) Liq. H_2O

$$H_P = 1.651(-393\ 522) + 2.593(-285\ 830) = -1\ 390\ 862 \text{ kJ}$$

$\bar{h}_{RP}$ = **-1 310 223 kJ/kmol**

15.64

Gaseous propane and stoichiometric air are mixed and fed to a burner both at ambient P_o and T_o. After combustion the products cool down and eventually reach ambient T_o. How much heat transfer was given out for 1 kg of propane?

The reaction equation for stoichiometric mixture is:

$$C_3H_8 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow a\,H_2O + b\,CO_2 + c\,N_2$$

C balance: $3 = b$;

H balance: $8 = 2a \quad => \quad a = 4$

O balance: $2\ \nu_{O2} = a + 2b = 4 + 2 \times 3$

$=> \quad \nu_{O2} = 5$

N_2 balance: $3.76\ \nu_{O2} = c = 18.8$

$A/F = 5 \times 4.76 \times 28.97 / 44.094 = 15.637$

The reaction is

$$C_3H_8 + 5\left(O_2 + 3.76\ N_2\right) \rightarrow 4\,H_2O + 3\,CO_2 + 18.8\,N_2$$

First we solve the problem assuming all the water is in the vapor phase and then we can solve with the proper split of water into liquid and vapor masses.

Energy Eq.: $H_R = Q_{out} + H_P \quad => \quad H^o_R = Q_{out} + H^o_P$

$$Q_{out} = H^o_R - H^o_P = -H^o_{RP} = HV = 46\ 352 \text{ kJ/kg fuel}$$

At T_o we need to check for condensation of water partial P limited to P_g.

$$y_{v\ max} = \frac{P_g}{P} = \frac{3.169}{100} = \frac{n_{v\ max}}{n_{v\ max} + 3 + 18.8}$$

$$\Rightarrow \quad n_{v\ max} = 0.7135 \quad \text{and then} \quad n_{liq} = 4 - n_{v\ max} = 3.2865$$

So now we know the split of the 4 moles of water into vapor and liquid. The change in the heating value becomes ($HV_{liq\ water} = 50\ 343$ kJ/kg fuel)

$$Q_{out} = HV = \frac{0.7135}{4}\, 46\ 352 + \frac{3.2865}{4}\, 50\ 343 = \mathbf{49\ 631\ kJ/kg\ fuel}$$

15.65

Blast furnace gas in a steel mill is available at 250°C to be burned for the generation of steam. The composition of this gas is, on a volumetric basis,

Component	CH_4	H_2	CO	CO_2	N_2	H_2O
Percent by volume	0.1	2.4	23.3	14.4	56.4	3.4

Find the lower heating value (kJ/m^3) of this gas at 250°C and ambient pressure.

Of the six components in the gas mixture, only the first 3 contribute to the heating value. These are, per kmol of mixture:

$$0.024\ H_2, \quad 0.001\ CH_4, \quad 0.233\ CO$$

For these components,

$$0.024\ H_2 + 0.001\ CH_4 + 0.233\ CO + 0.1305\ O_2$$
$$\rightarrow 0.026\ H_2O + 0.234\ CO_2$$

The remainder need not be included in the calculation, as the contributions to reactants and products cancel. For the lower HV(water as vapor) at 250°C

$$\bar{h}_{RP} = 0.026(-241\,826 + 7742) + 0.234(-393\,522 + 9348)$$
$$- 0.024(0 + 6558) - 0.001(-74\,873 + 2.254 \times 16.04(250-25))$$
$$- 0.233(-110\,527 + 6625) - 0.1305(0 + 6810)$$
$$= -72\,573 \frac{\text{kJ}}{\text{kmol fuel}}$$

$$\bar{v}_0 = \bar{R}\ T_o/P_o = 8.3145 \times 523.2/100 = 43.5015\ \text{m}^3/\text{kmol}$$

LHV = +72 573 / 43.5015 = **1668 kJ/m^3**

15.66

A burner receives a mixture of two fuels with mass fraction 40% n-butane and 60% methanol, both vapor. The fuel is burned with stoichiometric air. Find the product composition and the lower heating value of this fuel mixture (kJ/kg fuel mix).

Since the fuel mixture is specified on a mass basis we need to find the mole fractions for the combustion equation. From Eq.13.4 we get

$$y_{butane} = (0.4/58.124) / [0.4/58.124 + 0.6/32.042] = 0.26875$$

$$y_{methanol} = 1 - y_{butane} = 0.73125$$

The reaction equation is

$$0.73125\ CH_3OH + 0.26875\ C_4H_{10} + \nu_{O2}\ (O_2 + 3.76\ N_2)$$

$$\rightarrow\ \nu_{CO2}\ CO_2 + \nu_{H2O}H_2O + 3.76\ \nu_{O2}\ N_2$$

C balance: $0.73125 + 4 \times 0.26875 = \nu_{CO2} = 1.80625$

H_2 balance: $2 \times 0.73125 + 5 \times 0.26875 = \nu_{H2O} = 2.80625$

O balance: $0.73125 + 2\ \nu_{O2} = 2\ \nu_{CO2} + \nu_{H2O} = 6.41875 \quad => \nu_{O2} = 2.84375$

Now the products are:

$$1.80625\ CO_2 + 2.80625\ H_2O + 10.6925\ N_2$$

Since the enthalpy of combustion is on a mass basis in table 15.3 (this is also the negative of the heating value) we get

LHV = 0.4 × 45 714 + 0.6 × 21 093 = **30 941 kJ/kg fuel mixture**

Notice we took fuel vapor and water as vapor (lower heating value).

15.67

In an experiment a 1:1 mole ratio propane and butane is burned in a steady flow with stoichiometric air. Both fuels and air are supplied as gases at 298 K and 100 kPa. The products are cooled to 1000 K as they give heat to some application. Find the lower heating value (per kg fuel mixture) and the total heat transfer for 1 kmol of fuel mixture used.

Combustion: $C_3H_8 + C_4H_{10} + 11.5(O_2 + 3.76\ N) \rightarrow 7\ CO_2 + 9\ H_2O + 43.24\ N_2$

$$\nu_{O2} = 7 + 9/2 = 11.5 \qquad => \qquad \nu_{N2} = 3.76 \times 11.5 = 43.24$$

The enthalpy of combustion for the 2 kmol of fuel becomes from Tbl 15.3

$$H^o_P - H^o_R = (-46\ 352)\ 44.094 + (-45\ 714)\ 58.124 = -\ 4\ 700\ 925.6 \text{ kJ/2 kmol fuel}$$

$$HV = -H^\circ_{RP} = \frac{4\ 700\ 925.6}{44.094 + 58.124} = \mathbf{45\ 989\ kJ/kg\ fuel}$$ Convert to mass basis

$$\Delta H_P = 7 \times 33\ 397 + 9 \times 26\ 000 + 43.24 \times 21\ 463 = 1\ 395\ 839 \text{ kJ/2 kmol fuel}$$

The heat transfer for the two kmol of fuel is

$$Q = H_R - H_P = -H^\circ_{RP} - \Delta H_P = 4\ 700\ 925.6 - 1\ 395\ 839$$
$$= 3\ 305\ 087 \text{ kJ/2 kmol fuel}$$
$$= \mathbf{1\ 652\ 544\ kJ/kmol\ fuel}$$

15.68

Liquid nitromethane is added to the air in a carburetor to make a stoichiometric mixture where both fuel and air are added at 298 K, 100 kPa. After combustion a constant pressure heat exchanger brings the products to 600 K before being exhausted. Assume the nitrogen in the fuel becomes N_2 gas. Find the total heat transfer per kmole fuel in the whole process.

$$CH_3NO_2 + \ \nu_{O2}\,(O_2 + 3.76\,N_2) \ \rightarrow \ 1.5\,H_2O + 1\,CO_2 + a\,N_2$$

C and H balances done in equation. The remaining

O balance: $2 + 2\,\nu_{O2} = 1.5 + 2 \quad => \quad \nu_{O2} = 0.75$

N balance: $1 + 3.76\,\nu_{O2} \times 2 = 2a \quad => \quad a = 3.32$

Energy eq.: $H_R + Q = H_P \quad => \quad Q = H_P - H_R = H^{\circ}_P - H^{\circ}_R + \Delta H_P - \Delta H_R$

The reactants enter at the reference state, $\Delta H_R = 0$, and the products at 600 K from table A.9

$$\Delta H_P = 1.5\,\Delta\bar{h}_{H2O} + \Delta\bar{h}_{CO_2} + 3.32\,\Delta\bar{h}_{N_2}$$

$$= 1.5\,(10\,499) + 1\,(12\,906) + 3.32\,(8894) = 58\,183 \text{ kJ/kmol fuel}$$

$$H^{\circ}_P - H^{\circ}_R = H^{\circ}_{RP} = 61.04\,(-10\,537) = -643\,178 \text{ kJ/kmol}$$

$Q = -643\,178 + 58\,183 =$ **$-584\,995$ kJ/kmol fuel**

15.69

Natural gas, we assume methane, is burned with 200% theoretical air and the reactants are supplied as gases at the reference temperature and pressure. The products are flowing through a heat exchanger where they give off energy to some water flowing in at 20°C, 500 kPa and out at 700°C, 500 kPa. The products exit at 400 K to the chimney. How much energy per kmole fuel can the products deliver and how many kg water per kg fuel can they heat?

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \;\rightarrow\; 2\ H_2O + 1\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\left(O_2 + 3.76\ N_2\right) \;\rightarrow\; 2\ H_2O + 1\ CO_2 + 15.04\ N_2 + 2\ O_2$$

The products are cooled to 400 K (so we do not consider condensation) and the energy equation is

Energy Eq.: $H_R + Q = H_P = \overset{\circ}{H}_P + \Delta H_P = \overset{\circ}{H}_R + Q$

$$Q = \overset{\circ}{H}_P - \overset{\circ}{H}_R + \Delta H_P = \overset{\circ}{H}_{RP} + \Delta H_P$$

From Table 15.3: $\overset{\circ}{H}_{RP} = 16.04\ (-50\ 010) = -802\ 160$ kJ/kmol

$$\Delta H_P = \Delta\bar{h}^*_{CO2} + 2\ \Delta\bar{h}^*_{H2O} + 2\ \Delta\bar{h}^*_{O2} + 15.04\ \Delta\bar{h}^*_{N2}$$

From Table A.9

$$\Delta H_{P\ 400} = 4003 + 2 \times 3450 + 2 \times 3027 + 15.04 \times 2971 = 61\ 641 \text{ kJ/kmol}$$

$$Q = \overset{\circ}{H}_{RP} + \Delta H_P = -802\ 160 + 61\ 641 = \mathbf{-740\ 519\ kJ/kmol}$$

$$q_{prod} = -Q / M = 740\ 519 / 16.04 = 46\ 167 \text{ kJ/kg fuel}$$

The water flow has a required heat transfer, using B.1.3 and B.1.4 as

$$q_{H2O} = h_{out} - h_{in} = 3925.97 - 83.81 = 3842.2 \text{ kJ/kg water}$$

The mass of water becomes

$$m_{H2O} / m_{fuel} = q_{prod} / q_{H2O} = \mathbf{12.0\ kg\ water\ /\ kg\ fuel}$$

15.70

An isobaric combustion process receives gaseous benzene C_6H_6 and air in a stoichiometric ratio at P_o, T_o. To limit the product temperature to 2000 K, liquid water is sprayed in after the combustion. Find the kmol of liquid water added per kmol of fuel and the dew point of the combined products.

The reaction equation for stoichiometric mixture with C and H balance done is:

$$C_6H_6 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow 3\ H_2O + 6\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O_2} = 3 + 6 \times 2 = 15 \quad \Rightarrow \quad \nu_{O_2} = 7.5$

N balance: $c = 3.76\ \nu_{O_2} = 3.76 \times 7.5 = 28.2$

With x kmol of water added per kmol fuel the products are
Products: $(3 + x)\ H_2O + 6\ CO_2 + 28.2\ N_2$

Energy Eq.: $H_R = H^\circ_R + x\ \bar{h}^\circ_{f\ H_2O\ liq} = H^\circ_P + \Delta H_P$

$$= H^\circ_P + x\ \bar{h}^\circ_{f\ H_2O\ vap} + (3 + x)\ \Delta\bar{h}_{H_2O} + 6\ \Delta\bar{h}_{CO_2} + 28.2\ \Delta\bar{h}_{N_2}$$

Where the extra water is shown explicitly. Rearrange to get

$$H^\circ_R - H^\circ_P - 6\Delta\bar{h}_{CO_2} - 28.2\Delta\bar{h}_{N_2} - 3\Delta\bar{h}_{H_2O}$$

$$= x\ (\bar{h}^\circ_{f\ H_2O\ vap} - \bar{h}^\circ_{f\ H_2O\ liq} + \Delta\bar{h}_{H_2O})$$

$$40\ 576 \times 78.114 - 6 \times 91\ 439 - 28.2 \times 56\ 137 - 3 \times 72\ 788$$

$$= x\ [-241\ 826 - (-285\ 830) + 72\ 788\ ]$$

$819\ 493 = x\ (116\ 792) \Rightarrow \qquad x =$ **7.017 kmol/kmol fuel**

Dew point: $y_v = \dfrac{3 + x}{6 + 28.2 + 3 + x} = 0.2265$

$\Rightarrow P_v = y_vP = 0.2265 \times 101.325 = 22.95$ kPa

B.1.2: $T_{dew} =$ **63°C**

15.71

Gasoline, C_7H_{17}, is burned in a steady state burner with stoichiometric air at P_o, T_o. The gasoline is flowing as a liquid at T_o to a carburetor where it is mixed with air to produce a fuel air gas mixture at T_o. The carburetor takes some heat transfer from the hot products to do the heating. After the combustion the products go through a heat exchanger, which they leave at 600 K. The gasoline consumption is 10 kg per hour. How much power is given out in the heat exchanger and how much power does the carburetor need?

Stoichiometric combustion:

$$C_7H_{17} + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow\ 8.5\ H_2O + 7\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O2} = 8.5 + 14 = 22.5 \quad \Rightarrow \quad \nu_{O2} = 11.25$

N balance: $c = 3.76\ \ \nu_{O2} = 3.76 \times 11.25 = 42.3$

$$M_{FUEL} = 7\ M_C + 17\ M_H = 7 \times 12.011 + 8.5 \times 2.016 = 101.213$$

C.V. Total, heat exchanger and carburetor included, Q out.

Energy Eq.: $H_R = \overset{o}{H}_R = \overset{o}{H}_P + \Delta H_P + Q_{out}$

From Table A.9

$$\Delta H_P = 8.5 \times 10\ 499 + 7 \times 12\ 906 + 42.3 \times 8894 = 555\ 800\ \text{kJ/kmol}$$

From energy equation and Table 15.3

$$Q_{out} = \overset{o}{H}_R - \overset{o}{H}_P - \Delta H_P = -\overset{o}{H}_{RP} - \Delta H_P$$

$$= 101.213\ (44\ 506) - 555\ 800 = 3\ 948\ 786\ \text{kJ/kmol}$$

Now the power output is

$$\dot{Q} = \dot{n}\ Q_{out} = Q_{out}\ \dot{m}/M = 3\ 948\ 786 \times \frac{10}{3600} / \ 101.213 = \mathbf{108.4\ kW}$$

The carburetor air comes in and leaves at the same T so no change in energy, all we need is to evaporate the fuel, h_{fg} so

$$\dot{Q} = \dot{m}\ h_{fg} = \frac{10}{3600}\,(44\ 886 - 44\ 506) = \frac{1}{360} \times 380 = \mathbf{1.06\ kW}$$

Here we used Table 15.3 for fuel liquid and fuel vapor to get h_{fg} and any phase of the water as long as it is the same for the two.

Adiabatic Flame Temperature

15.72

In a rocket, hydrogen is burned with air, both reactants supplied as gases at P_o, T_o. The combustion is adiabatic and the mixture is stoichiometeric (100% theoretical air). Find the products dew point and the adiabatic flame temperature (~2500 K).

The reaction equation is:

$$H_2 + \nu_{O2} (O_2 + 3.76 N_2) \Rightarrow H_2O + 3.76 \nu_{O2} N_2$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$ and thus we have 1.88 for nitrogen.

$y_v = 1/(1+1.88) = 0.3472 \Rightarrow P_v = 101.325 \times 0.3472 = 35.18$ kPa $= P_g$

Table B.1.2: **$T_{dew} = 72.6$ C.**

$H_R = H_P \Rightarrow 0 = -241826 + \Delta h_{water} + 1.88\ \Delta h_{nitrogen}$

Find now from table A.9 the two enthalpy terms

At 2400 K : $\Delta H_P = 93741 + 1.88 \times 70640 = 226544$ kJ/kmol fuel

At 2600 K : $\Delta H_P = 104520 + 1.88 \times 77963 = 251090$ kJ/kmol fuel

Then interpolate to hit 241 826 to give **T = 2525 K**

15.73

Hydrogen gas is burned with pure oxygen in a steady flow burner where both reactants are supplied in a stoichiometric ratio at the reference pressure and temperature. What is the adiabatic flame temperature?

The reaction equation is:

$$H_2 + \nu_{O2}\, O_2 \;\;\Rightarrow\;\; H_2O$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$.

Energy Eq.: $H_R = H_P \;\Rightarrow\; 0 = -241\,826 + \Delta\bar{h}_{H2O}$

$$\Rightarrow \quad \Delta\bar{h}_{H2O} = 241\,826 \text{ kJ/kmol}$$

Interpolate now in table A.9 for the temperature to give this enthalpy

T = 4991 K

15.74

Some type of wood can be characterized as $C_1H_{1.5}O_{0.7}$ with a lower heating value of 19 500 kJ/kg. Find its adiabatic flame temperature when burned with stoichiometric air at 100 kPa, 298 K.

$$C_1H_{1.5}O_{0.7} + \nu_{O2}[\,O_2 + 3.76\ N_2\,] \rightarrow \quad CO_2 + 0.75\ H_2O + 3.76\nu_{O2}\ N_2$$

$$\text{O balance:}\quad 0.7 + 2\nu_{O2} = 2 + 0.75 \quad \Rightarrow \quad \nu_{O2} = 1.025,\ \ \nu_{N2} = 3.854$$

$$M = 1 \times 12.0111 + 1.5 \times 1.008 + 0.7 \times 16 = 24.712$$

$$\overset{o}{H}_{RP} = H^o_P - H^o_R = -M \times HV = -24.712 \times 19\,500 = -481\,884\ \text{kJ/kmol}$$

$$\text{Energy Eq.:}\quad H_P = \overset{o}{H}_P + \Delta H_P = H_R = \overset{o}{H}_R \quad \Rightarrow \quad \Delta H_P = \overset{o}{H}_R - \overset{o}{H}_P = -\overset{o}{H}_{RP}$$

$$\Delta H_P = \Delta\bar{h}_{CO2} + 0.75\ \Delta\bar{h}_{H2O} + 3.854\ \Delta\bar{h}_{N2} = 481\,884\ \text{kJ/kmol}$$

at 2400 K: $\quad \Delta H_P = 115\,779 + 0.75 \times 93\,741 + 3.854 \times 70\,640 = 458\,331$ kJ

at 2600 K: $\quad \Delta H_P = 128\,074 + 0.75 \times 104\,520 + 3.854 \times 77\,963 = 506\,933$ kJ

$$T = 2400 + 200\,\frac{481\,884 - 458\,331}{506\,933 - 458\,331} = \mathbf{2497\ K}$$

Comment: Most wood has some water and some non-combustible solids material so the actual flame temperature will be much lower.

15.75

Carbon is burned with air in a furnace with 150% theoretical air and both reactants are supplied at the reference pressure and temperature. What is the adiabatic flame temperature?

$$C + \nu_{O_2} O_2 + 3.76\, \nu_{O_2} N_2 \rightarrow 1\, CO_2 + 3.76\, \nu_{O_2} N_2$$

From this we find $\nu_{O_2} = 1$ and the actual combustion reaction is

$$C + 1.5\, O_2 + 5.64\, N_2 \rightarrow 1\, CO_2 + 5.64\, N_2 + 0.5\, O_2$$

$$H_P = H^{\circ}_P + \Delta H_P = H_R = H^{\circ}_R \quad \Rightarrow$$

$$\Delta H_P = H^{\circ}_R - H^{\circ}_P = 0 - (-393\,522) = 393\,522 \text{ kJ/kmol}$$

$$\Delta H_P = \Delta\bar{h}_{CO2} + 5.64\, \Delta\bar{h}_{N2} + 0.5\, \Delta\bar{h}_{O2}$$

Find T so ΔH_P takes on the required value. To start guessing assume all products are nitrogen (1 + 5.64 + 0.5 = 7.14) that gives 1900 < T < 2000 K from Table A.9.

$$\Delta H_{P\,1900} = 85\,420 + 5.64 \times 52\,549 + 0.5 \times 55\,414 = 409\,503 \quad \text{too high}$$

$$\Delta H_{P\,1800} = 79\,432 + 5.64 \times 48\,979 + 0.5 \times 51\,674 = 381\,511$$

Linear interpolation to find

$$T = 1800 + 100\, \frac{393\,522 - 381\,511}{409\,503 - 381\,511} = \mathbf{1843\ K}$$

15.76

Hydrogen gas is burned with 200% theoretical air in a steady flow burner where both reactants are supplied at the reference pressure and temperature. What is the adiabatic flame temperature?

The stoichiometric reaction equation is:

$$H_2 + \nu_{O2} (O_2 + 3.76\ N_2) \;=>\; H_2O + 3.76\ \nu_{O2}\ N_2$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$ and thus we have for the actual mixture $\nu_{O2} = 1$. The actual reaction is

$$H_2 + 1\ (O_2 + 3.76\ N_2) \;=>\; 1\ H_2O + 3.76\ N_2 + 0.5\ O_2$$

The energy equation with formation enthalpy from A.9 or A.10 for water is

$$H_R = H_P \;=>\; 0 = -241\ 826 + \Delta h_{H2O} + 3.76\ \Delta h_{N2} + 0.5\ \Delta h_{O2}$$

Find now from table A.9 the two enthalpy terms

At 2000 K : $\Delta H_P = 72\ 788 + 3.76 \times 56\ 137 + 0.5 \times 59\ 176 = 313\ 451$

At 1800 K : $\Delta H_P = 62\ 693 + 3.76 \times 48\ 979 + 0.5 \times 51\ 674 = 272\ 691$

At 1600 K : $\Delta H_P = 52\ 907 + 3.76 \times 41\ 904 + 0.5 \times 44\ 267 = 232\ 600$

At 1700 K : $\Delta H_P = 57\ 757 + 3.76 \times 45\ 430 + 0.5 \times 47\ 959 = 252\ 553$

Then interpolate to hit 241 826 to give

$$T = 1600 + 100\ \frac{241\ 826 - 232\ 600}{252\ 553 - 232\ 600} = \mathbf{1646\ K}$$

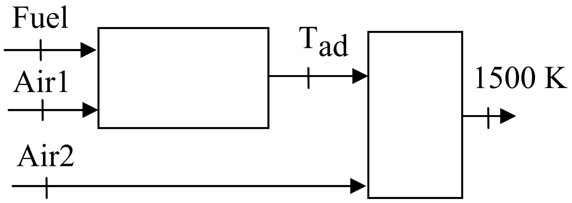
15.77

What is the adiabatic flame temperature before the secondary air is added in Problem 15.47

$CH_4 + \nu(O_2 + 3.76\ N_2) \rightarrow$

$CO_2 + 2H_2O + 3.76\ \nu\ N_2$

O balance: $\nu = 2$, $\nu_{N2} = 7.52$



Do a C.V. around the combustion chamber, then the energy equation becomes

Energy Eq.: $H_{R1} = H_{P\ 1} = H^{\circ}_P + \Delta H_{P\ 1}$

$$H_{R1} = H^{\circ}_R + \Delta H_{R1} = \bar{h}^{o}_{f\ fuel} + \Delta H_{Fuel} + \Delta H_{air1}$$

$$\Delta H_{air1} = 2(\Delta H_{O2} + 3.76\ \Delta H_{N2}) = 2(6086 + 3.76 \times 5911) = 56\ 623\ \text{kJ/kmol}$$

$$\Delta H_{Fuel} \approx M\ C_P\ \Delta T = 16.043 \times 2.254\ (500 - 298) = 7304.5\ \text{kJ/kmol}$$

$$\bar{h}^{o}_{f\ fuel} - H^{\circ}_P = HV = 16.043 \times 50\ 010 = 802\ 310\ \text{kJ/kmol}$$

$$\Delta H_{P\ 1} = \Delta\bar{h}_{CO2} + 0.75\ \Delta\bar{h}_{H2O} + 3.854\ \Delta\bar{h}_{N2}$$
$$= \bar{h}^{o}_{f\ fuel} + \Delta H_{Fuel} + \Delta H_{air1} - H^{\circ}_P \quad \text{(from energy Eq.)}$$
$$= 802\ 310 + 7304.5 + 56\ 623 = 866\ 237\ \text{kJ/kmol fuel}$$

at 2400 K: $\Delta H_{P\ 1} = 115\ 779 + 2 \times 93\ 741 + 7.52 \times 70\ 640 = 834\ 474$ kJ

at 2600 K: $\Delta H_{P\ 1} = 128\ 074 + 2 \times 104\ 520 + 7.52 \times 77\ 963 = 923\ 396$ kJ

$$T = 2400 + 200\ \frac{866\ 237\ - 834\ 474}{923\ 396\ - 834\ 474} = \mathbf{2471\ K}$$

15.78

Butane gas at 25°C is mixed with 150% theoretical air at 600 K and is burned in an adiabatic steady flow combustor. What is the temperature of the products exiting the combustor?

$$C_4H_{10} + 1.5{\times}6.5\left(O_2 + 3.76\ N_2\right) \rightarrow 4\ CO_2 + 5\ H_2O + 3.25\ O_2 + 36.66\ N_2$$

Energy Eq.: $H_P - H_R = 0 \quad \Rightarrow \quad \Delta H_P = H^\circ_R + \Delta H_R - H^\circ_P$

Reactants: $\Delta H_R = 9.75(9245) + 36.66(8894) = 416\ 193$ kJ ;

$$H^\circ_R = \bar{h}_{C4H10} = \bar{h}^{o}_{f\,IG} = -126\ 200 \text{ kJ} \quad \Rightarrow \quad H_R = +289\ 993 \text{ kJ}$$

$$H^\circ_P = 4(-393522) + 5(-241826) = -2\ 783\ 218 \text{ kJ/kmol}$$

$$\Delta H_P = 4\ \Delta\bar{h}^*_{CO2} + 5\ \Delta\bar{h}^*_{H2O} + 3.25\ \Delta\bar{h}^*_{O2} + 36.66\ \Delta\bar{h}^*_{N2}$$

From the energy equation we then get

$$\Delta H_P = -126\ 200 + 416\ 193 - (-2\ 783\ 218) = 3\ 073\ 211 \text{ kJ/kmol}$$

Trial and Error: $LHS_{2000\ K} = 2\ 980\ 000, \quad LHS_{2200\ K} = 3\ 369\ 866$

Linear interpolation to match RHS => $T_P =$ **2048 K**

15.79

A gas turbine burns methane with 200% theoretical air. The air and fuel comes in through two separate compressors bringing them from 100 kPa, 298 K to 1400 kPa and after mixing enters the combustion chamber at 600 K. Find the adiabatic flame temperature using constant specific heat for the ΔH_P terms.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2} \left(O_2 + 3.76\ N_2\right) \ \rightarrow\ 2\ H_2O + 1\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4 \left(O_2 + 3.76\ N_2\right) \ \rightarrow\ 2\ H_2O + 1\ CO_2 + 15.04\ N_2 + 2\ O_2$$

The energy equation around the combustion chamber becomes

Energy Eq.: $H_P - H_R = 0 \quad => \quad \Delta H_P = H^{\circ}_R + \Delta H_R - H^{\circ}_P = -H^{\circ}_{RP} + \Delta H_R$

$\Delta H_R = \Delta H_{Fuel} + \Delta H_{air} = M\ C_P\ \Delta T + 4(\Delta\bar{h}_{O2} + 3.76\ \Delta\bar{h}_{N2})$

$= 16.043 \times 2.254\ (600–298) + 4(9245 + 3.76 \times 8894) = 181\ 666$ kJ/kmol

$-H^{\circ}_{RP} = 16.043 \times 50\ 010 = 802\ 310$ kJ/kmol

$\Delta H_P = \Delta\bar{h}_{CO2} + 2\ \Delta\bar{h}_{H2O} + 15.04\ \Delta\bar{h}_{N2} + 2\ \Delta\bar{h}_{O2} \approx\ \Delta T \sum \nu_i \bar{C}_{Pi}$

$= 802\ 310 + 181\ 666 = 983\ 976$ kJ/kmol (from energy Eq.)

$\sum \nu_i \bar{C}_{Pi} = 0.842 \times 44.01 + 2 \times 1.872 \times 18.015 + 15.04 \times 1.042 \times 28.013$

$+ 2 \times 0.922 \times 31.999 = 602.52$ kJ/kmol-K

$\Delta T = \Delta H_P / \sum \nu_i \bar{C}_{Pi} = 983\ 976 / 602.52 = 1633.1$ K

T = 298 + 1633 = **1931 K**

15.80

Extend the solution to the previous problem by using Table A.9 for the ΔH_P terms.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow 2\ H_2O + 1\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\left(O_2 + 3.76\ N_2\right) \rightarrow 2\ H_2O + 1\ CO_2 + 15.04\ N_2 + 2\ O_2$$

The energy equation around the combustion chamber becomes

$$\text{Energy Eq.:} \qquad H_P - H_R = 0 \quad => \quad \Delta H_P = H^\circ_R + \Delta H_R - H^\circ_P = -H^\circ_{RP} + \Delta H_R$$

$$\Delta H_R = \Delta H_{Fuel} + \Delta H_{air} = M\ C_P\ \Delta T + 4(\Delta\bar{h}_{O2} + 3.76\ \Delta\bar{h}_{N2})$$

$$= 16.043 \times 2.254\ (600–298) + 4(9245 + 3.76 \times 8894) = 181\ 666\ \text{kJ/kmol}$$

$$-H^\circ_{RP} = 16.043 \times 50\ 010 = 802\ 310\ \text{kJ/kmol}$$

$$\Delta H_P = \Delta\bar{h}_{CO2} + 2\ \Delta\bar{h}_{H2O} + 15.04\ \Delta\bar{h}_{N2} + 2\ \Delta\bar{h}_{O2}$$

$$= 802\ 310 + 181\ 666 = 983\ 976\ \text{kJ/kmol} \quad \text{(from energy Eq.)}$$

Trial and error with $\Delta\bar{h}$ from Table A.9

At 1800 K

$\Delta H_P = 79\ 432 + 2 \times 62\ 693 + 15.04 \times 48\ 979 + 2 \times 51\ 674 = 1\ 044\ 810$ kJ/kmol

At 1700 K

$\Delta H_P = 73\ 480 + 2 \times 57\ 757 + 15.04 \times 45\ 430 + 2 \times 47\ 959 = 968\ 179$ kJ/kmol

Linear interpolation: $T = 1700 + 100\ \dfrac{983\ 976\ - 968\ 179}{1\ 044\ 810 - 968\ 179} = \mathbf{1721\ K}$

15.81

A stoichiometric mixture of benzene, C_6H_6, and air is mixed from the reactants flowing at 25°C, 100 kPa. Find the adiabatic flame temperature. What is the error if constant specific heat at T_0 for the products from Table A.5 are used?

$$C_6H_6 + \nu_{O2}O_2 + 3.76\ \nu_{O2}N_2 \rightarrow 6CO_2 + 3H_2O + 3.76\ \nu_{O2}N_2$$

$$\nu_{O2} = 6 + 3/2 = 7.5 \Rightarrow \nu_{N2} = 28.2$$

$$H_P = H^\circ_P + \Delta H_P = H_R = H^\circ_R \quad \Rightarrow$$

$$\Delta H_P = -H^\circ_{RP} = 40576 \times 78.114 = 3\ 169\ 554 \text{ kJ/kmol}$$

$$\Delta H_P = 6\ \Delta\bar{h}_{CO2} + 3\ \Delta\bar{h}_{H2O} + 28.2\ \Delta\bar{h}_{N2}$$

$$\Delta H_{P\ 2600K} = 6(128074) + 3(104\ 520) + 28.2(77\ 963) = 3\ 280\ 600,$$

$$\Delta H_{P\ 2400K} = 6(115\ 779) + 3(93\ 741) + 28.2(70\ 640) = 2\ 968\ 000$$

Linear interpolation $\Rightarrow$ T_{AD}= **2529 K**

$$\sum\nu_i\bar{C}_{Pi} = 6 \times 0.842 \times 44.01 + 3 \times 1.872 \times 18.015 + 28.2 \times 1.042 \times 28.013$$

$$= 1146.66 \text{ kJ/kmol K}$$

$$\Delta T = \Delta H_P/\sum\nu_i\bar{C}_{Pi} = 3\ 169\ 554 / 1146.66 = 2764$$

$\Rightarrow$ T_{AD}= 3062 K, **21% high**

15.82

A gas-turbine burns natural gas (assume methane) where the air is supplied to the combustor at 1000 kPa, 500 K and the fuel is at 298 K, 1000 kPa. What is the equivalence ratio and the percent theoretical air if the adiabatic flame temperature should be limited to 1800 K?

The reaction equation for a mixture with excess air is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow\ 2\ H_2O + 1\ CO_2 + 3.76\nu_{O2}\ N_2 + (\nu_{O2} - 2)O_2$$

$$H_P = H^{\circ}_P + \Delta H_P = H_R = H^{\circ}_R + \Delta H_R$$

From table A.9 at 500 K (notice fuel is at 298 K)

$$\Delta H_R = 0 + \nu_{O2}(\Delta h_{O2} + 3.76\ \Delta h_{N2}) = \nu_{O2}(6086 + 3.76 \times 5911) = 28\ 311.4\ \nu_{O2}$$

From table A.9 at 1800 K:

$$\Delta H_P = 2\ \Delta h_{H2O} + \Delta h_{CO2} + 3.76\ \nu_{O2}\ \Delta h_{N2} + (\nu_{O2} - 2)\ \Delta h_{O2}$$

$$= 2 \times 62\ 693 + 79432 + 3.76\ \nu_{O2} \times 48\ 979 + (\nu_{O2} - 2)\ 51\ 674$$

$$= 101\ 470 + 235\ 835\ \nu_{O2}$$

From table 15.3: $H^{\circ}_P - H^{\circ}_R = H^{\circ}_{RP} = 16.04(-50\ 010) = -802\ 160$ kJ/kmol

Now substitute all terms into the energy equation

$$-802\ 160 + 101\ 470 + 235\ 835\ \nu_{O2} = 28\ 311.4\ \nu_{O2}$$

Solve for ν_{O2}

$$\nu_{O2} = \frac{802\ 160 - 101\ 470}{235\ 835 - 28\ 311.4} = 3.376$$

%Theoretical air = 100 (3.376 / 2) = **168.8 %**

Φ = AFs / AF = 2 / 3.376 = **0.592**

15.83

Acetylene gas at 25°C, 100 kPa is fed to the head of a cutting torch. Calculate the adiabatic flame temperature if the acetylene is burned with

a. 100% theoretical air at 25°C.

b. 100% theoretical oxygen at 25°C.

a) $C_2H_2 + 2.5\ O_2 + 2.5 \times 3.76\ N_2 \rightarrow 2\ CO_2 + 1\ H_2O + 9.4\ N_2$

$H_R = \bar{h}^o_{f\ C2H2} = +226\ 731$ kJ/kmol from table A.10

$H_P = 2(-393\ 522 + \Delta\bar{h}^*_{CO2}) + 1(-241\ 826 + \Delta\bar{h}^*_{H2O}) + 9.4\ \Delta\bar{h}^*_{N2}$

$Q_{CV} = H_P - H_R = 0 \;\;\Rightarrow\;\; 2\ \Delta\bar{h}^*_{CO2} + 1\ \Delta\bar{h}^*_{H2O} + 9.4\ \Delta\bar{h}^*_{N2} = 1\ 255\ 601$ kJ

Trial and Error A.9: $LHS_{2800} = 1\ 198\ 369$, $LHS_{3000} = 1\ 303\ 775$

Linear interpolation: $T_{PROD} =$ **2909 K**

b) $C_2H_2 + 2.5\ O_2 \rightarrow 2\ CO_2 + H_2O$

$H_R = +226\ 731$ kJ ; $H_P = 2(-393\ 522 + \Delta\bar{h}^*_{CO2}) + 1(-241\ 826 + \Delta\bar{h}^*_{H2O})$

$\Rightarrow\;\; 2\ \Delta\bar{h}^*_{CO2} + 1\ \Delta\bar{h}^*_{H2O} = 1\ 255\ 601$ kJ/kmol fuel

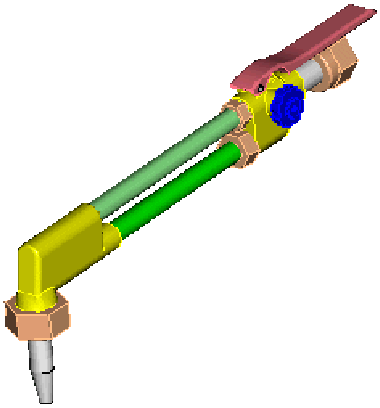
At 6000 K (limit of A.9) $2 \times 343\ 782 + 302\ 295 = 989\ 859$

At 5600 K $2 \times 317\ 870 + 278\ 161 = 913\ 901$

Slope 75 958/400 K change

Extrapolate to cover the difference above 989 859 kJ/kmol fuel

$T_{PROD} \approx 6000 + 400(265\ 742/75\ 958) \approx$ **7400 K**

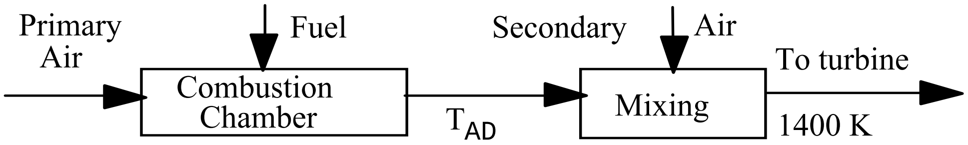


15.84

Liquid *n*-butane at T_0, is sprayed into a gas turbine with primary air flowing at 1.0 MPa, 400 K in a stoichiometric ratio. After complete combustion, the products are at the adiabatic flame temperature, which is too high, so secondary air at 1.0 MPa, 400 K is added, with the resulting mixture being at 1400 K. Show that $T_{ad} > 1400$ K and find the ratio of secondary to primary air flow.

C.V. Combustion Chamber.

$$C_4H_{10} + 6.5\ O_2 + 6.5 \times 3.76\ N_2 \rightarrow 5\ H_2O + 4\ CO_2 + 24.44\ N_2$$



Energy Eq.: $H_{air} + H_{fuel} = H_R = H_P$

$$\overset{o}{H}_P + \Delta H_P = \overset{o}{H}_R + \Delta H_R \quad \Rightarrow \quad \Delta H_P = \overset{o}{H}_R + \Delta H_R - \overset{o}{H}_P = -\overset{o}{H}_{RP} + \Delta H_R$$

$$\Delta H_P = 45344 \times 58.124 + 6.5(3.76 \times 2971 + 3027) = \mathbf{2\ 727\ 861\ kJ/kmol}$$

$$\Delta H_{P\ 1400} = 5 \times 43491 + 4 \times 55895 + 24.44 \times 34936 = \mathbf{1\ 294\ 871 < \Delta H_P}$$

Remark: Try $T_{AD} > 1400$:

$$\Delta H_P = 2\ 658\ 263\ @2400\ K, \quad \Delta H_P = 2\ 940\ 312\ @2600\ K$$

C.V. Mixing Chamber. Air Second: $\nu_{O2\ s}O_2 + 3.76\ N_2$

$$\Delta H_P + \nu_{O2\ second}\,\Delta H_{air} = \Delta H_{P\ 1400} + \nu_{O2\ second}\,\Delta H_{air\ 1400}$$

$$\Rightarrow \nu_{O2\ second} = \frac{\Delta H_P - \Delta H_{P\ 1400}}{\Delta H_{air\ 1400} - \Delta H_{air\ 400}} = \frac{1432990}{168317 - 14198} = \mathbf{9.3}$$

$$\text{ratio} = \nu_{O2\ sec}/\nu_{O2\ prim} = 9.3/6.5 = \mathbf{1.43}$$

15.85

Ethene, C_2H_4, burns with 150% theoretical air in a steady flow constant-pressure process with reactants entering at P_0, T_0. Find the adiabatic flame temperature.

Stoichiometric

$$C_2H_4 + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 11.28N_2$$

Actual

$$C_2H_4 + 4.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 1.5\ O_2 + 16.92N_2$$

$$H_P = H^\circ_P + 2\Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + 1.5\Delta\bar{h}_{O_2} + 16.92\Delta\bar{h}_{N_2}$$

$$H^\circ_R = \bar{h}^\circ_{f\,Fu} \qquad\qquad \Delta H_P + H^\circ_P = H^\circ_R$$

$$\Rightarrow \Delta H_P = -H^\circ_{RP} = 28.054 \times 47158 = 1\ 322\ 970.5\ \frac{kJ}{kmol\ Fu}$$

$$\Delta H_P = 2\Delta\bar{h}_{CO_2} + 2\Delta\bar{h}_{H_2O} + 1.5\Delta\bar{h}_{O_2} + 16.92\Delta\bar{h}_{N_2}$$

Initial guess based on (2+2+1.5+16.92) N_2 from A.9: $T_1 = 2100$ K

$$\Delta H_P(2000) = 1\ 366\ 982, \quad \Delta H_P(1900) = 1\ 278\ 398$$

=> $\mathbf{T_{AD} \cong 1950\ K}$

15.86

Natural gas, we assume methane, is burned with 200% theoretical air and the reactants are supplied as gases at the reference temperature and pressure. The products are flowing through a heat exchanger and then out the exhaust, as in Fig. P15.86. What is the adiabatic flame temperature right after combustion before the heat exchanger?

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow 1\ CO_2 + 2\ H_2O + c\ N_2$$

O balance: $2\ \nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\left(O_2 + 3.76\ N_2\right) \rightarrow 1\ CO_2 + 2\ H_2O + 15.04\ N_2 + 2\ O_2$$

Energy Eq.: $H_{air} + H_{fuel} = H_R = H_P$

$$\overset{o}{H}_P + \Delta H_P = \overset{o}{H}_R + \Delta H_R \quad \Rightarrow \quad \Delta H_P = \overset{o}{H}_R + \Delta H_R - \overset{o}{H}_P = -\overset{o}{H}_{RP} + 0$$

From Table 14.3: $-\overset{o}{H}_{RP} = -16.04\ (-50\ 010) = 802\ 160$ kJ/kmol

$$\Delta H_P = \Delta\bar{h}^*_{CO2} + 2\ \Delta\bar{h}^*_{H2O} + 2\ \Delta\bar{h}^*_{O2} + 15.04\ \Delta\bar{h}^*_{N2}$$

From Table A.9

$$\Delta H_{P\ 1600} = 67\ 659 + 2 \times 52\ 907 + 2 \times 44\ 267 + 15.04 \times 41\ 904 = 892\ 243$$

$$\Delta H_{P\ 1500} = 61\ 705 + 2 \times 48\ 149 + 2 \times 40\ 600 + 15.04 \times 38\ 405 = 816\ 814$$

$$\Delta H_{P\ 1400} = 55\ 895 + 2 \times 43\ 491 + 2 \times 36\ 958 + 15.04 \times 34\ 936 = 742\ 230$$

Linear interpolation to get 802 160

$$T = 1400 + 100\ \frac{802\ 160 - 742\ 230}{816\ 814 - 742\ 230} = \mathbf{1480\ K}$$

15.87

Solid carbon is burned with stoichiometric air in a steady flow process. The reactants at T_0, P_0 are heated in a preheater to T_2 = 500 K as shown in Fig. P15.87, with the energy given by the product gases before flowing to a second heat exchanger, which they leave at T_0. Find the temperature of the products T_4, and the heat transfer per kmol of fuel (4 to 5) in the second heat exchanger.

Combustion of carbon:

$$C + O_2 + 3.76\ N_2 \rightarrow CO_2 + 3.76\ N_2$$

C.V. Combustion chamber and preheater from 1 to 4, no external Q. For this CV states 2 and 3 are internal and do not appear in equations.

Energy Eq.:

$$H_R = H^{\circ}_R = H_{P_4} = H^{\circ}_P + \Delta H_{P_4} = \bar{h}_{f\,CO_2} + \Delta\bar{h}_{CO_2} + 3.76\Delta\bar{h}_{N_2}$$

Table A.9 or A.10: $\bar{h}_{f\,CO_2}$= -393 522 kJ/kmol, reference $H^{\circ}_R = 0$

$$\Delta H_{P_4} = H^{\circ}_R - H^{\circ}_P = 0 - \bar{h}_{f\,CO_2} = 393\ 522 \text{ kJ/kmol fuel}$$

$$\Delta H_{P4\ 2400} = 115\ 779 + 3.76 \times 70\ 640 = 381\ 385 \text{ kJ/kmol fuel,}$$

$$\Delta H_{P4\ 2600} = 128\ 074 + 3.76 \times 77\ 963 = 421\ 215 \text{ kJ/kmol fuel}$$

interpolate $\Rightarrow$ $T_4 = T_{ad.flame} = \mathbf{2461\ K}$

Control volume: Total. Then energy equation:

$$H^{\circ}_R + \bar{Q} = H^{\circ}_P$$

$$\bar{Q} = \bar{H}^{\circ}_{RP} = \bar{h}^{\circ}_{f\,CO_2} - 0 = \mathbf{-393\ 522\ \frac{kJ}{kmol\ fuel}}$$

15.88

Liquid butane at 25°C is mixed with 150% theoretical air at 600 K and is burned in a steady flow burner. Use the enthalpy of combustion from Table 15.3 to find the adiabatic flame temperature out of the burner.

$$C_4H_{10} + 1.5\times 6.5\,(O_2 + 3.76\ N_2) \rightarrow 4\ CO_2 + 5\ H_2O + 3.25\ O_2 + 36.66\ N_2$$

Energy Eq.: $H_P - H_R = 0 \quad => \quad \Delta H_P = H^{\circ}_R + \Delta H_R - H^{\circ}_P = -H^{\circ}_{RP} + \Delta H_R$

Reactants: $\Delta H_R = 9.75(9245) + 36.66(8894) = 416\ 193$ kJ/kmol;

$$H^{\circ}_{RP} = M \times HV = 58.124 \times (-45\ 344) = -2\ 635\ 575 \text{ kJ/kmol}$$

$$\Delta H_P = 4\Delta\bar{h}^*_{CO2} + 5\Delta\bar{h}^*_{H2O} + 3.25\ \Delta\bar{h}^*_{O2} + 36.66\ \Delta\bar{h}^*_{N2}$$

So the energy equation becomes

$$\Delta H_P = 2\ 635\ 575 + 416\ 193 = 3\ 051\ 768 \text{ kJ/kmol}$$

Trial and Error: $LHS_{2000\ K} = 2\ 980\ 000, \qquad LHS_{2200\ K} = 3\ 369\ 866$

Linear interpolation to match RHS $\quad => \quad T_P =$ **2037 K**

15.89

Gaseous ethanol, C_2H_5OH, is burned with pure oxygen in a constant volume combustion bomb. The reactants are charged in a stoichiometric ratio at the reference condition. Assume no heat transfer and find the final temperature (> 5000 K).

$$C_2H_5OH + 3\ O_2 \ \rightarrow\ 2\ CO_2 + 3\ H_2O$$

Energy Eq.:

$$U_P = U_R = \overset{\circ}{H}_R + \Delta H_R - n_R\bar{R}T_R = \overset{\circ}{H}_P + \Delta H_P - n_P\bar{R}T_P$$

Solve for the properties that depends on T_P and recall $\Delta H_R = 0$

$$\Delta H_P - n_P\bar{R}T_P = \overset{\circ}{H}_R - \overset{\circ}{H}_P - n_R\bar{R}T_R = \bar{h}^0_{f\,fuel} - 2\ \bar{h}^0_{f\,CO2} - 3\ \bar{h}^0_{f\,H2O} - 4\bar{R}T_R$$

Fuel: $\bar{h}^0_{f\,fuel} = -235\,000$ kJ/kmol for IG from Table A.10 so

$$\Delta H_P - n_P\bar{R}T_P = -235\,000 - 2(-393\,522) - 3(-241\,826)$$

$$- 4 \times 8.31451 \times 298.15 = 1\,267\,606 \text{ kJ/kmol}$$

$$\text{LHS} = \Delta H_P - n_P\bar{R}T_P = 2\ \Delta\bar{h}_{CO_2} + 3\ \Delta\bar{h}_{H_2O} - 5 \times 8.31451 \times T_P$$

From Table A.9 we find

$$\text{LHS}_{5600} = 2 \times 317\,870 + 3 \times 278\,161 - 41.5726 \times 5600 = 1\,237\,417$$

$$\text{LHS}_{6000} = 2 \times 343\,782 + 3 \times 302\,295 - 41.5726 \times 6000 = 1\,345\,014$$

$$T_{ad.flame} = \mathbf{5712\ K}$$

15.90

The enthalpy of formation of magnesium oxide, MgO(s), is −601 827 kJ/kmol at 25°C. The melting point of magnesium oxide is approximately 3000 K, and the increase in enthalpy between 298 and 3000 K is 128 449 kJ/kmol. The enthalpy of sublimation at 3000 K is estimated at 418 000 kJ/kmol, and the specific heat of magnesium oxide vapor above 3000 K is estimated at 37.24 kJ/kmol K.

a. Determine the enthalpy of combustion per kilogram of magnesium.

b. Estimate the adiabatic flame temperature when magnesium is burned with theoretical oxygen.

a) $Mg + \frac{1}{2}O_2 \rightarrow MgO(s)$

$$\Delta h_{COMB} = \frac{\Delta\bar{h}_{COMB}}{M} = \frac{\bar{h}^\circ_f}{M} = \frac{-601\ 827}{24.32} = \mathbf{-24\ 746\ kJ/kg}$$

b) assume T_R = 25°C and also that T_P > 3000 K, (MgO = vapor phase)

1st law: $Q_{CV} = H_P - H_R = 0$, but $H_R = 0$

$$\Rightarrow H_P = \bar{h}^\circ_f + (\bar{h}_{3000} - \bar{h}_{298})_{SOL} + \Delta\bar{h}_{SUB} + \bar{C}_{P\ VAP}(T_P - 3000)$$

$$= -601\ 827 + 128\ 449 + 418\ 000 + 37.24(T_P - 3000) = 0$$

Solving, $T_P = \mathbf{4487\ K}$

Second Law for the Combustion Process

15.91

Consider the combustion of hydrogen with pure oxygen in a stoichiometric ratio under steady flow adiabatic conditions. The reactants enter separately at 298 K, 100 kPa and the product(s) exit at a pressure of 100 kPa. What is the exit temperature and what is the irreversibility?

The reaction equation is:

$$H_2 + \nu_{O2}\, O_2 \;\; => \;\; H_2O$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$.

Energy Eq.: $H_R = H_P \;\; => \;\; 0 = -241\,826 + \Delta\bar{h}_{H2O}$

$$=> \;\; \Delta\bar{h}_{H2O} = 241\,826 \text{ kJ/kmol}$$

Interpolate now in table A.9 for the temperature to give this enthalpy

$$\mathbf{T = 4991\ K}$$

For this temperature we find from Table A.9, $P = P_o$, so we do not need any pressure correction for the entropy

$$S_P = \overset{o}{S}_P = \overset{o}{\bar{s}}_{H2O} = 315.848 \text{ kJ/kmol K}$$

For the reactants we have (again no pressure correction)

$$S_R = \overset{o}{\bar{s}}_{H2} + 0.5\, \overset{o}{\bar{s}}_{O2} = 130.678 + 0.5 \times 205.148 = 233.252 \text{ kJ/kmol K}$$

$$S_{gen} = S_P - S_R = 315.848 - 233.252 = 82.596 \text{ kJ/kmol } H_2 \text{ K}$$

$$I = T_o\, S_{gen} = 298.15 \times 82.596 = \mathbf{24\,626\ kJ/kmol\ H_2}$$

15.92

Consider the combustion of methanol, CH_3OH, with 25% excess air. The combustion products are passed through a heat exchanger and exit at 200 kPa, 400 K. Calculate the absolute entropy of the products exiting the heat exchanger assuming all the water is vapor.

$$CH_3OH + 1.25 \times 1.5 \left(O_2 + 3.76\, N_2\right) \rightarrow CO_2 + 2\, H_2O + 0.375\, O_2 + 7.05\, N_2$$

We need to find the mole fractions to do the partial pressures,

$$n = 1 + 2 + 0.375 + 7.05 = 10.425 \quad => \quad y_i = n_i / n$$

Gas mixture:

	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln \frac{y_i P}{P_0}$	$\bar{S}_i$
CO_2	1.0	0.0959	225.314	+13.730	239.044
H_2O	2	0.1918	198.787	+7.967	206.754
O_2	0.375	0.0360	213.873	+20.876	234.749
N_2	7.05	0.6763	200.181	-2.511	197.670

$S_{GAS\ MIX} = \sum n_i \bar{S}_i =$ **2134.5 kJ/K kmol fuel**

15.93

Two kilomoles of ammonia are burned in a steady flow process with *x* kmol of oxygen. The products, consisting of H_2O, N_2, and the excess O_2, exit at 200°C, 7 MPa.

a. Calculate *x* if half the water in the products is condensed.
b. Calculate the absolute entropy of the products at the exit conditions.

$$2NH_3 + xO_2 \rightarrow 3H_2O + N_2 + (x - 1.5)O_2$$

Products at 200 ^oC, 7 MPa with $n_{H2O\ LIQ} = n_{H2O\ VAP} = 1.5$

a) $y_{H2O\ VAP} = P_G/P = \frac{1.5538}{7} = \frac{1.5}{1.5 + 1 + x - 1.5}$ => x = **5.757**

b) $S_{PROD} = S_{GAS\ MIX} + S_{H2O\ LIQ}$

Gas mixture:	n_i	y_i	$\bar{s}_i^o$	$-\bar{R}\ln(y_iP/P_0)$	S_i
H_2O	1.5	0.222	204.595	-22.810	181.785
O_2	4.257	0.630	218.985	-31.482	187.503
N_2	1.0	0.148	205.110	-19.439	185.671

$S_{GAS\ MIX} = 1.5(181.785) + 4.257(187.503) + 1.0(185.67) = 1256.55$ kJ/K

$S_{H2O\ LIQ} = 1.5[69.950 + 18.015(2.3223 - 0.3674)] = 157.75$ kJ/K

$S_{PROD} = 1256.55 + 157.75 =$ **1414.3 kJ/K**

15.94

Propene, C_3H_6, is burned with air in a steady flow burner with reactants at P_o, T_o. The mixture is lean so the adiabatic flame temperature is 1800 K. Find the entropy generation per kmol fuel neglecting all the partial pressure corrections.

The reaction equation for a mixture with excess air is:

$$C_3H_6 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow\ 3\ H_2O + 3\ CO_2 + 3.76\nu_{O2}\ N_2 + (\nu_{O2} - 4.5)O_2$$

Energy Eq.: $H_R = H^\circ_R + \Delta H_R = H^\circ_R = H_P = H^\circ_P + \Delta H_P$

The entropy equation: $S_R + S_{gen} = S_P \quad => \quad S_{gen} = S_P - S_R = S_P - S^\circ_R$

From table A.9 at reference T

$$\Delta H_R = \Delta h_{Fu} + \nu_{O2}(\Delta h_{O2} + 3.76\ \Delta h_{N2}) = 0$$

From table A.9 at 1800 K:

$$\Delta H_P = 3\ \Delta h_{H2O} + 3\ \Delta h_{CO2} + 3.76\ \nu_{O2}\ \Delta h_{N2} + (\nu_{O2} - 4.5)\ \Delta h_{O2}$$

$$= 3 \times 62\ 693 + 3 \times 79432 + 3.76\ \nu_{O2} \times 48\ 979 + (\nu_{O2} - 4.5)\ 51\ 674$$

$$= 193\ 842 + 235\ 835\ \nu_{O2}$$

From table 15.3: $H^\circ_P - H^\circ_R = H^\circ_{RP} = 42.081(-45\ 780) = -1\ 926\ 468$ kJ/kmol

Now substitute all terms into the energy equation

$$-1\ 926\ 468 + 193\ 842 + 235\ 835\ \nu_{O2} = 0$$

Solve for ν_{O2}: $\quad \nu_{O2} = \dfrac{1\ 926\ 468 - 193\ 842}{235\ 835} = 7.3468, \quad \nu_{N2} = 27.624$

Table A.9-10 contains the entropies at 100 kPa so we get:

$$S_P = 3 \times 259.452 + 3 \times 302.969 + (7.3468 - 4.5)\ 264.797 + 27.624 \times 248.304$$

$$= 9300.24 \text{ kJ/kmol-K}$$

$$S_R = 267.066 + 7.3468 \times 205.148 + 27.624 \times 191.609 = 7067.25 \text{ kJ/kmol K}$$

$S_{gen} = 9300.24 - 7067.25 =$ **2233 kJ/kmol-K**

15.95

A flow of hydrogen gas is mixed with a flow of oxygen in a stoichiometric ratio, both at 298 K and 50 kPa. The mixture burns without any heat transfer in complete combustion. Find the adiabatic flame temperature and the amount of entropy generated per kmole hydrogen in the process.

The reaction equation is:

$$H_2 + \nu_{O2}\, O_2 \;\Rightarrow\; H_2O$$

The balance of hydrogen is done, now for oxygen we need $\nu_{O2} = 0.5$.

Energy Eq.: $H_R = H_P \;\Rightarrow\; 0 = -241\,826 + \Delta\bar{h}_{H2O}$

$\Rightarrow\; \Delta\bar{h}_{H2O} = 241\,826$ kJ/kmol

Interpolate now in table A.9 for the temperature to give this enthalpy

T = 4991 K

For this temperature we find from Table A.9

$$S_P = \bar{s}^{\circ}_{H2O} - \bar{R}\ln(P/P_o) = 315.848 - 8.31451 \ln(0.5) = 321.611 \text{ kJ/kmol K}$$

For the reactants we have

$$S_R = \bar{s}^{\circ}_{H2} - \bar{R}\ln(P/P_o) + 0.5\,[\bar{s}^{\circ}_{O2} - \bar{R}\ln(P/P_o)\,]$$

$$= 130.678 + 0.5 \times 205.148 - 1.5 \times 8.31451 \ln(0.5)$$

$$= 241.897 \text{ kJ/kmol K}$$

$S_{gen} = S_P - S_R = 321.611 - 241.897 =$ **79.714 kJ/kmol H_2 K**

Recall that this includes the mixing process.

15.96

Calculate the irreversibility for the process described in Problem 15.45.

$$2\,C + 2\,O_2 \;\rightarrow 1\,CO_2 + 1\,CO + \tfrac{1}{2}O_2$$

Process V = constant, C: solid, $n_{1(GAS)} = 2$, $n_{2(GAS)} = 2.5$

$$P_2 = P_1 \times \frac{n_2T_2}{n_1T_1} = 200 \times \frac{2.5 \times 1000}{2 \times 298.2} = 838.4 \text{ kPa}$$

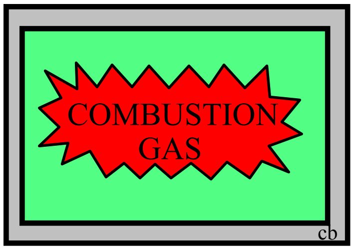
$$H_1 = H_R = 0$$

$$H_2 = H_P = 1(-393522 + 33397) + 1(-110527 + 21686)$$

$$+ (1/2)(0 + 22703) = -437\,615 \text{ kJ}$$

$${}_1Q_2 = (U_2 - U_1) = (H_2 - H_1) - n_2\bar{R}T_2 + n_1\bar{R}T_1$$

$$= (-437\,615 - 0) - 8.3145(2.5 \times 1000 - 2 \times 298.2) = -453\,442 \text{ kJ}$$



Reactants:

$$S_R = 2(5.740) + 2(205.148 - 8.31451 \ln\frac{200}{100}) = 410.250 \text{ kJ/K}$$

Products:

	n_i	y_i	$\bar{s}_i^\circ$	$-\bar{R}\ln\frac{y_iP}{P_0}$	S_i
CO_2	1.0	0.40	269.299	-10.061	259.238
CO	1.0	0.40	234.538	-10.061	224.477
O_2	0.5	0.20	243.579	-4.298	239.281

$$S_P = 1.0(259.238) + 1.0(224.477) + 0.5(239.281) = 603.355 \text{ kJ/K}$$

$$I = T_0(S_P - S_R) - {}_1Q_2$$

$$= 298.15(603.355 - 410.250) - (-453\,442) = \mathbf{+511\,016\ kJ}$$

15.97

Consider the combustion of methanol, CH_3OH, with 25% excess air. The combustion products are passed through a heat exchanger and exit at 200 kPa, 40°C. Calculate the absolute entropy of the products exiting the heat exchanger per kilomole of methanol burned, using the proper amounts of liquid and vapor for the water.

$$CH_3OH + 1.25 \times 1.5\left(O_2 + 3.76\,N_2\right) \rightarrow CO_2 + 2\,H_2O + 0.375\,O_2 + 7.05\,N_2$$

Products exit at 40 ^oC, 200 kPa, check for saturation:

$$y_{V\,MAX} = \frac{P_G}{P} = \frac{7.384}{200} = \frac{n_{V\,MAX}}{n_{V\,MAX} + 1 + 0.375 + 7.05}$$

$$n_V = n_{V\,MAX} = 0.323 \quad n_{LIQ} = 1.677$$

Gas mixture:

	n_i	y_i	$\bar{s}_i^o$	$-\bar{R}\ln\frac{y_iP}{P_0}$	$\bar{S}_i$
CO_2	1.0	0.1143	215.633	+12.270	227.903
H_2O	0.323	0.0369	190.485	+21.671	212.156
O_2	0.375	0.0429	206.592	+20.418	227.01
N_2	7.05	0.8059	193.039	-3.969	189.07

$S_{GAS\,MIX} = \sum n_i\bar{S}_i = 1714.50$ kJ/K kmol fuel

$\bar{s}_{LIQ} = 69.950 + 18.015(0.5725 - 0.3674) = 73.645$ kJ/kmol

$S_{LIQ} = 1.677 \times 73.645 = 123.50$ kJ/K kmol fuel

$S_{PROD} = 1714.50 + 123.50 =$ **1838 kJ/K kmol fuel**

15.98

Graphite, C, at P_0, T_0 is burned with air coming in at P_0, 500 K in a ratio so the products exit at P_0, 1200 K. Find the equivalence ratio, the percent theoretical air, and the total irreversibility.

$$C + (1/\phi)b(O_2 + 3.76\ N_2) \rightarrow CO_2 + \big((1/\phi) - 1\big)O_2 + 3.76\ (1/\phi)\ N_2$$

Stoichiometric: b = 1

Energy Eq.: $H_P = H_R \Rightarrow \Delta H_{P\ 1200} - \Delta H_R = H^{\circ}_R - H^{\circ}_P$

$$44\ 473 + \big((1/\phi) - 1\big)29\ 761 + 3.76(1/\phi)28\ 109$$
$$- (1/\phi)\big(6086 + 3.76\times 5911\big) = 0 - (-393\ 522) \ \Rightarrow\ (1/\phi) = 3.536$$

$$S_{gen} = S_P - S_R = \sum_{P-R} \nu\big(\bar{s}^{\circ} - \bar{R}\ \ln(y)\big)$$

Reactants: $y_{O_2} = 0.21$, $y_{N_2} = 0.79$ (carbon is solid)

Products: $y_{O_2} = 0.1507$, $y_{N_2} = 0.79$, $y_{CO_2} = 0.0593$

$$S^{\circ}_P = 279.39 + 2.536 \times 250.011 + 13.295 \times 234.227 = 4027.5$$

$$S^{\circ}_R = 5.74 + 3.536(220.693 + 3.76 \times 206.74) = 3534.8$$

For the pressure correction the term with the nitrogen drops out (same y).

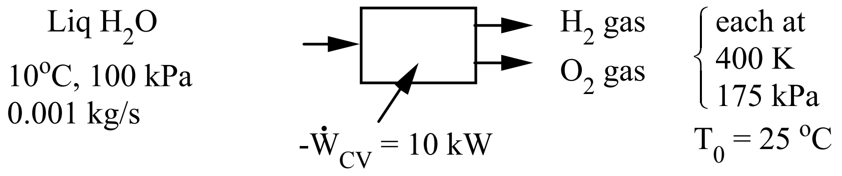
$$\bar{R} \sum_{P-R} -\nu\ \ln(y) = \bar{R}\ [-\ln(y_{CO_2}) - 2.536\ \ln(y_{O_2\ prod}) + 3.536\ \ln(y_{O_2\ reac})]$$
$$= \bar{R}\ [\ -\ln(0.0593) - 2.536\ \ln(0.1507) + 3.536\ \ln(0.21)\ ]$$
$$= 8.3145(2.8235 + 2.536 \times 1.8927 - 3.536 \times 1.5606) = 17.503$$

$S_{gen} = 4027.5 - 3534.8 + 17.503 = 510.2$ kJ/kmol carbon-K

$$I = T_0\ S_{gen} = \mathbf{152\ 117}\ \frac{\mathbf{kJ}}{\mathbf{kmol\ C}}$$

15.99

An inventor claims to have built a device that will take 0.001 kg/s of water from the faucet at 10°C, 100 kPa, and produce separate streams of hydrogen and oxygen gas, each at 400 K, 175 kPa. It is stated that this device operates in a 25°C room on 10-kW electrical power input. How do you evaluate this claim?



$$H_2O \rightarrow H_2 + \tfrac{1}{2}O_2$$

$$H_i - H_e = [-285830 + 18.015(42.01 - 104.89)] - 2961 - \tfrac{1}{2}(3027)$$

$$= -291\ 437 \text{ kJ/kmol}$$

$$(S_i - S_e) = [69.950 + 18.015(0.151 - 0.3674)] - (139.219 - 8.3145 \ln 1.75)$$

$$- \tfrac{1}{2}(213.873 - 8.3145 \ln 1.75) = -173.124 \text{ kJ/kmol K}$$

$$W_{REV} = (H_i - H_e) - T_0(S_i - S_e) = -291\ 437 - 298.15(-173.124)$$

$$= -239820 \text{ kJ/kmol}$$

$$\dot{W}_{REV} = (0.001/18.015)(-239\ 820) = -13.31 \text{ kW}$$

$$\dot{I} = \dot{W}_{REV} - \dot{W}_{CV} = -13.31 - (-10) < 0$$ **Impossible**

15.100

Hydrogen peroxide, H_2O_2, enters a gas generator at 25°C, 500 kPa at the rate of 0.1 kg/s and is decomposed to steam and oxygen exiting at 800 K, 500 kPa. The resulting mixture is expanded through a turbine to atmospheric pressure, 100 kPa, as shown in Fig. P15.100. Determine the power output of the turbine, and the heat transfer rate in the gas generator. The enthalpy of formation of liquid H_2O_2 is −187 583 kJ/kmol.

$$H_2O_2 \rightarrow H_2O + \frac{1}{2} O_2 \qquad \dot{n}_{H2O2} = \frac{\dot{m}_{H2O2}}{M} = \frac{0.1}{34.015} = 0.00294 \text{ kmol/s}$$

$$\dot{n}_{MIX} = \dot{n}_{H2O2} \times 1.5 = 0.00441 \text{ kmol/s}$$

$$\bar{C}_{P0\ MIX} = \frac{2}{3} \times 1.872 \times 18.015 + \frac{1}{3} \times 0.922 \times 31.999 = 32.317$$

$$\bar{C}_{V0\ MIX} = 32.317 - 8.3145 = 24.0 \quad => \quad k_{MIX} = 32.317/24.0 = 1.3464$$

CV: turbine. Assume reversible → $s_3 = s_2$

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} = 800\left(\frac{100}{500}\right)^{0.2573} = 528.8 \text{ K}$$

$$w = \bar{C}_{P0}(T_2 - T_3) = 32.317(800 - 528.8) = 8765 \text{ kJ/kmol}$$

$$\dot{W}_{CV} = 0.00441 \times 8765 = \mathbf{38.66\ kW}$$

CV: Gas Generator

$$\dot{H}_1 = 0.00294(-187\,583 + 0) = -551.49$$

$$\dot{H}_2 = 0.00294(-241\,826 + 18002) + 0.00147(0 + 15836) = -634.76$$

$$\dot{Q}_{CV} = \dot{H}_2 - \dot{H}_1 = -634.76 + 551.49 = \mathbf{-83.27\ kW}$$

15.101

Methane is burned with air, both of which are supplied at the reference conditions. There is enough excess air to give a flame temperature of 1800 K. What are the percent theoretical air and the irreversibility in the process?

The combustion equation with X times theoretical air is

$$CH_4 + 2X(O_2 + 3.76\ N_2) \rightarrow CO_2 + 2H_2O + 2(X\text{-}1)O_2 + 7.52X\ N_2$$

Energy Eq.: $\quad H_{air} + H_{fuel} = H_R = H_P = H^\circ_P + \Delta H_P = H^\circ_R + \Delta H_R$

$$\Rightarrow \quad \Delta H_P = H^\circ_R + \Delta H_R - H^\circ_P = -H^\circ_{RP} + 0$$

From Table 15.3: $\quad -H^\circ_{RP} = -16.04\ (-50\ 010) = 802\ 160$ kJ/kmol

$$\Delta H_P = \Delta\bar{h}^*_{CO2} + 2\ \Delta\bar{h}^*_{H2O} + 2(X\text{-}1)\ \Delta\bar{h}^*_{O2} + 7.52X\ \Delta\bar{h}^*_{N2}$$

From Table A.9 and the energy equation

$$\Delta H_{P\ 1800} = 79\ 432 + 2 \times 62\ 693 + 2(X\text{-}1)\ 51\ 674 + 7.52X \times 48\ 979 = 802\ 160$$

so

$$101\ 470 + 471\ 670\ X = 802\ 160 \quad => \qquad X = 1.4856$$

%Theoretical air = **148.6%**

The products are

Products: $\quad CO_2 + 2H_2O + 0.9712\ O_2 + 11.172\ N_2$

The second law

$$S_{gen} = S_P - S_R \quad \text{and} \qquad I = T_o\ S_{gen}$$

Reactants: $P_i = 100$ kPa, $P_o = 100$ kPa, $\bar{s}^o_f$ from Table A.9

	n_i	y_i	$\bar{s}^o_f$	$-\bar{R}\ln\frac{y_iP_i}{P_o}$	$\bar{S}_i\ \frac{kJ}{kmol\ K}$
CH_4	1	1	186.251	0	186.251
O_2	2X	0.21	205.148	12.976	218.124
N_2	7.52 X	0.79	191.609	1.96	193.569

$$S_R = \sum n_i\bar{S}_i = 2996.84 \text{ kJ/K kmol fuel}$$

Products: $P_e = 100$ kPa, $P_o = 100$ kPa

	n_i	y_i	$\bar{s}^o_{1800}$	$-\bar{R}\ln\frac{y_i P_e}{P_o}$	$\bar{S}_i$ $\frac{\text{kJ}}{\text{kmol K}}$
CO_2	1	0.06604	302.969	22.595	325.564
H_2O	2	0.13208	259.452	16.831	276.283
O_2	0.9712	0.06413	264.797	22.838	287.635
N_2	11.172	0.73775	248.304	2.529	250.833

$S_P = \sum n_i \bar{S}_i = 3959.72$ kJ/K kmol fuel;

$I = T_o(S_P - S_R) = 298.15(3959.72 - 2996.84) =$ **287 MJ/kmol fuel**

15.102

Pentane gas at 25°C, 150 kPa enters an insulated steady flow combustion chamber. Sufficient excess air to hold the combustion products temperature to 1800 K enters separately at 500 K, 150 kPa. Calculate the percent theoretical air required and the irreversibility of the process per kmol of pentane burned.

$$C_5H_{12} + 8X(\ O_2 + 3.76\ N_2) \rightarrow 5\ CO_2 + 6\ H_2O + 8(X\text{-}1)\ O_2 + 30.08X\ N_2$$

Energy Eq.: $Q_{cv} + H_R = H_P + W_{CV}$; $W_{CV} = 0$, $Q_{cv} = 0$

Reactants: C_5H_{12} : $\bar{h}_f^o$ from A.9 and $\Delta\bar{h}_{500}$ for O_2 and N_2 from A.9

$$H_R = \left(\bar{h}_f^o\right)_{C5H12} + 8X\ \Delta\bar{h}_{O2} + 30.08X\ \Delta\bar{h}_{N2}$$

$$= -146\ 500 + 8X\ 6086 + 30.08\ X\ 5911 = 226\ 491\ X - 146\ 500$$

$$H_P = 5\left(\bar{h}_f^o + \Delta\bar{h}\right)_{CO2} + 6\left(\bar{h}_f^o + \Delta\bar{h}\right)_{H2O} + 8(X\text{-}1)\ \Delta\bar{h}_{O2} + 30.08\ X\ \Delta\bar{h}_{N2}$$

$$= 5(-393\ 522 + 79\ 432) + 6(-241\ 826 + 62\ 693) + 8(X\text{-}1)\ 51\ 674$$

$$+ 30.08\ X\ 48\ 979 = 1\ 886\ 680\ X - 3\ 058\ 640$$

Energy Eq. solve for X;

$$H_R = H_P = 226\ 491\ X - 146\ 500 = 1\ 886\ 680\ X - 3\ 058\ 640$$

$$\Rightarrow \quad X = \mathbf{1.754}$$

b) Reactants: $P_{in} = 150$ kPa, $P_o = 100$ kPa, $\bar{s}_f^o$

	n_i	y_i	$\bar{s}_f^o$, $\bar{s}_{500}^o$	$-\bar{R}\ln\frac{y_iP_{in}}{P_o}$	$\bar{S}_i$ $\frac{kJ}{kmol\ K}$
C_5H_{12}	1	1	348.945	-3.371	345.574
O_2	8X	0.21	220.693	9.605	230.298
N_2	30.08 X	0.79	206.74	-1.411	205.329

$$S_R = \sum n_i\bar{S}_i = 14410.34\ \text{kJ/K kmol fuel}$$

Products: $P_e = 150$ kPa, $P_o = 100$ kPa

	n_i	y_i	$\bar{s}^o_{1800}$	$-\bar{R}\ln\frac{y_iP_e}{P_o}$	$\bar{S}_i \ \frac{kJ}{kmol\ K}$
CO_2	5	0.0716	302.969	18.550	321.519
H_2O	6	0.086	259.452	17.027	276.479
O_2	8(X-1)	0.0864	264.797	16.988	281.785
N_2	30.08X	0.756	248.304	-1.045	247.259

$$S_P = \sum n_i\bar{S}_i = 17\ 732.073 \ \text{ kJ/K kmol fuel;}$$

$$I = T_o(S_P - S_R) = 298.15(17\ 732.07 - 14\ 410.34)$$

= **990 MJ/kmol fuel**

15.103

A closed rigid container is charged with propene, C_3H_6, and 150% theoretical air at 100 kPa, 298 K. The mixture is ignited and burns with complete combustion. Heat is transferred to a reservoir at 500 K so the final temperature of the products is 700 K. Find the final pressure, the heat transfer per kmole fuel and the total entropy generated per kmol fuel in the process.

$$C_3H_6 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow 3\ CO_2 + 3\ H_2O + x\ N_2$$

Oxygen, O_2, balance: $\quad 2\ \nu_{O2} = 6 + 3 = 9 \quad \Rightarrow \quad \nu_{O2} = 4.5$

Actual Combustion: $\quad \phi = 1.5 \ \Rightarrow \ \nu_{O_2\ ac} = 1.5 \times 4.5 = 6.75$

$$C_3H_6 + 6.75\ O_2 + 25.38\ N_2 \ \rightarrow \ 3\ CO_2 + 3\ H_2O + 25.38\ N_2 + 2.25\ O_2$$

$$P_2 = P_1 \frac{n_P T_2}{n_R T_1} = 100 \times \frac{33.63 \times 700}{33.13 \times 298.15} = \mathbf{238.3\ kPa}$$

Enthalpies from Table A.9

$$\Delta H_{P\ 700} = 3{\times}17\ 754 + 3{\times}14\ 190 + 25.38{\times}11\ 937 + 2.25{\times}12\ 499$$

$$= 426\ 916\ \frac{\text{kJ}}{\text{kmol fuel}}$$

Enthalpy of combustion from table 15.3 converted to mole basis

$$H^{\circ}_{RP} = -45\ 780 \times 42.081 = -1\ 926\ 468 \text{ kJ/kmol fuel}$$

$$U_2 - U_1 = {}_1Q_2 - 0 = H_2 - H_1 - n_2\bar{R}T_2 + n_1\bar{R}T_1$$

$${}_1Q_2 = H^{\circ}_{RP} + \Delta H_{P\ 700} - n_P\bar{R}T_2 + n_1\bar{R}T_1$$

$$= -1\ 926\ 468 + 426\ 916 - 33.63 \times 8.3145 \times 700$$

$$+ 33.13 \times 8.3145 \times 298.15 = \mathbf{-1.613{\times}10^6\ \frac{kJ}{kmol\ fuel}}$$

Entropies from Table A.9 and pressure correction

Reactants:	n_i	y_i	$\bar{s}^{\circ}_i$	$-\bar{R}\ln(y_iP/P_0)$	S_i
C_3H_8	1.0	0.0302	267.066	29.104	296.17
O_2	6.75	0.2037	205.143	13.228	218.376
N_2	25.38	0.7661	191.609	2.216	189.393

$$S_1 = 296.17 + 6.75 \times 218.376 + 25.38 \times 189.393 = 6577 \ \frac{\text{kJ}}{\text{kmol fuel K}}$$

Products:	n_i	y_i	$\bar{s}^\circ_i$	$-\bar{R}\ln(y_iP/P_0)$	S_i
CO_2	3	0.0892	250.752	+12.875	263.627
H_2O	3	0.0892	218.739	+12.875	231.614
O_2	2.25	0.0669	231.465	+15.266	246.731
N_2	25.38	0.7547	216.865	- 4.88	211.985

$$S_2 = 3(263.627 + 231.614) + 2.25 \times 246.731 + 25.38 \times 211.985$$
$$= 7421 \text{ kJ/kmol fuel K}$$

$$_1S_{2\,gen} = S_2 - S_1 - {}_1Q_2/T_{res} = 7421 - 6577 + \frac{1.613\times10^6}{500} = \mathbf{4070} \ \frac{\textbf{kJ}}{\textbf{kmol fuel K}}$$

Problems Involving Generalized Charts or Real Mixtures

15.104

A gas mixture of 50% ethane and 50% propane by volume enters a combustion chamber at 350 K, 10 MPa. Determine the enthalpy per kilomole of this mixture relative to the thermochemical base of enthalpy using Kay's rule.

$\bar{h}^*_{MIX\ O} = 0.5(-84740) + 0.5(-103900) = -94320$ kJ/kmol

$\bar{C}_{P0\ MIX} = 0.5 \times 30.07 \times 1.7662 + 0.5 \times 44.097 \times 1.67 = 63.583$

$\bar{h}^*_{350} - \bar{h}^*_{298} = 63.583(350 - 298.2) = 3294$ kJ/kmol

Kay's rule: $T_{C\ MIX} = 0.5 \times 305.4 + 0.5 \times 369.8 = 337.6$ K

$P_{C\ MIX} = 0.5 \times 4.88 + 0.5 \times 4.25 = 4.565$ MPa

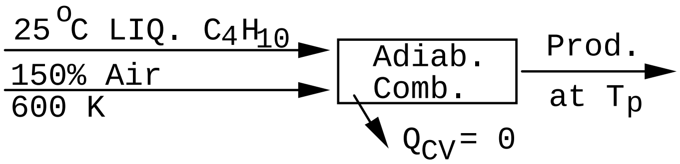
$T_r = 350/337.6 = 1.037, \quad P_r = 10/4.565 = 2.19$

From Fig. D.2: $\bar{h}^* - \bar{h} = 8.3145 \times 337.6 \times 3.53 = 9909$ kJ/kmol

$\bar{h}_{MIX\ 350K,10MPa} = -94320 + 3294 - 9909 =$ **-100 935 kJ/kmol**

15.105

Liquid butane at 25°C is mixed with 150% theoretical air at 600 K and is burned in an adiabatic steady state combustor. Use the generalized charts for the liquid fuel and find the temperature of the products exiting the combustor.



$T_C = 425.2$ K

$T_R = 0.701$

$\bar{h}_{C4H10} = \bar{h}^o_{f\,IG} + (\bar{h}_{LIQ} - \bar{h}^*)$ see Fig. D.2

$= -126200 + (-4.85 \times 8.3145 \times 425.2) = -143\,346$ kJ

$C_4H_{10} + 1.5 \times 6.5\ O_2 + 3.76 \times 9.75\ N_2$

$\rightarrow 4\ CO_2 + 5\ H_2O + 3.25\ O_2 + 36.66\ N_2$

$\bar{h}_{AIR} = 9.75(9245) + 36.66(8894) = 416\,193$ kJ

$\Rightarrow H_R = 416\,193 - 143\,346 = +272\,847$ kJ

$H_P = 4(-393522 + \Delta\bar{h}^*_{CO2}) + 5(-241826 + \Delta\bar{h}^*_{H2O}) + 3.25\ \Delta\bar{h}^*_{O2} + 36.66\ \Delta\bar{h}^*_{N2}$

Energy Eq.: $H_P - H_R = 0$

$4\ \Delta\bar{h}^*_{CO2} + 5\ \Delta\bar{h}^*_{H2O} + 3.25\ \Delta\bar{h}^*_{O2} + 36.66\ \Delta\bar{h}^*_{N2} = 3\,056\,065$

Trial and Error: $LHS_{2000\,K} = 2\,980\,000$, $LHS_{2200\,K} = 3\,369\,866$

Linear interpolation to match RHS => $T_P =$ **2039 K**

15.106

Repeat Problem 15.135, but assume that saturated-liquid oxygen at 90 K is used instead of 25°C oxygen gas in the combustion process. Use the generalized charts to determine the properties of liquid oxygen.

Problem same as 15.135, except oxygen enters at 2 as saturated liquid at 90 K.

$\dot{m}_{O2}/\dot{m}_{N2H4} = 0.5 = 32\dot{n}_{O2}/32\dot{n}_{N2H4}$ and $\dot{Q}/\dot{m}_{N2H4} = -100$ kJ/kg

Energy Eq.: $Q_{CV} = H_P - H_R = -100 \times 32.045 = -3205$ kJ/kmol fuel

Reaction equation: $1\, N_2H_4 + \frac{1}{2} O_2 \rightarrow H_2O + H_2 + N_2$

At 90 K, $T_{r2} = 90/154.6 = 0.582 \Rightarrow \Delta\tilde{h}_f = 5.2$

Figure D.2, $(\bar{h}^* - \bar{h}) = 8.3145 \times 154.6 \times 5.2 = 6684$ kJ/kmol

$\Delta\bar{h}_{AT\,2} = -6684 + 0.922 \times 32(90 - 298.15) = -12825$ kJ/kmol

$H_R = 50417 + \frac{1}{2}(0 - 12825) = 44005$ kJ, $H_P^\circ = -241826$

1st law: $\Delta\bar{h}_P = \Delta\bar{h}_{H2O} + \Delta\bar{h}_{H_2} + \Delta\bar{h}_{N_2} = Q_{cv} + H_R - H_P^\circ = 282626$

From Table A.9, $\Delta H_{P\ 2800K} = 282141$, $\Delta H_{P\ 3000K} = 307988$

Therefore, $T_P =$ **2804 K**

15.107

A mixture of 80% ethane and 20% methane on a mole basis is throttled from 10 MPa, 65°C, to 100 kPa and is fed to a combustion chamber where it undergoes complete combustion with air, which enters at 100 kPa, 600 K. The amount of air is such that the products of combustion exit at 100 kPa, 1200 K. Assume that the combustion process is adiabatic and that all components behave as ideal gases except the fuel mixture, which behaves according to the generalized charts, with Kay's rule for the pseudocritical constants. Determine the percentage of theoretical air used in the process and the dew-point temperature of the products.

Reaction equation:

Fuel mix: $\bar{h}^0_{f\,FUEL} = 0.2(-74873) + 0.8(-84740) = -82767$ kJ/kmol

$\bar{C}_{P0\,FUEL} = 0.2 \times 2.2537 \times 16.04 + 0.8 \times 1.7662 \times 30.07 = 49.718$

$\Delta\bar{h}^*_{FUEL} = 49.718(65 - 25) = 1989$ kJ/kmol

$T_{CA} = 305.4$ K, $T_{CB} = 190.4$ K $\Rightarrow$ $T_{c\,mix}$=282.4 K

$P_{CA} = 4.88$, P_{CB}=4.60 $\Rightarrow$ $P_{c\,mix}$= 4.824 MPa

$T_r = 338.2/282.4 = 1.198$, $P_r = 10/4.824 = 2.073$

$(\bar{h}^* - \bar{h})_{FUEL\,IN} = 8.31451 \times 282.4 \times 2.18 = 5119$

$\Rightarrow \bar{h}_{FUEL\,IN} = -82767 + 1989 - 5119 = -85897 \frac{kJ}{kmol}$

Energy Eq.:

$$1.8(-393522 + 44473) + 2.8(-241826 + 34506)$$
$$+ 3.2(x - 1)(29761) + (12.03x)(28109)$$
$$+ 85897 - (3.2x)(9245) - (12.03x)(8894) = 0$$

a) x = 4.104 or **410.4 %**

b) $n_P = 1.8 + 2.8 + 3.2(4.104 - 1) + 12.03 \times 4.104 = 63.904$

$y_{H2O} = 2.8/63.904 = P_V/100$; $P_V = 4.38$ kPa, T = **30.5°C**

15.108

Saturated liquid butane enters an insulated constant pressure combustion chamber at 25°C, and *x* times theoretical oxygen gas enters at the same *P* and *T*. The combustion products exit at 3400 K. With complete combustion find *x*. What is the pressure at the chamber exit? and what is the irreversibility of the process?

Butane: $T_1 = T_o = 25^oC$, sat liq., $x_1 = 0$, $T_c = 425.2$ K, $P_c = 3.8$ MPa

Do the properties from the generalized charts

Fig. D.1: $T_{r1} = 0.7$, $P_{r1} = 0.1$, $P_1 = P_{r1}P_c = 380$ kPa

Figs. D.2 and D.3: $\left(\bar{h}_1^* - \bar{h}_1\right)_f = 4.85\ \bar{R}T_c$, $(\bar{s}_1^* - \bar{s}_1)_f = 6.8\ \bar{R}$

Oxygen: $T_2 = T_o = 25^oC$, X - Theoretical O_2

Products: $T_3 = 3400$ K, Assumes complete combustion

$$C_4H_{10} + 6.5X\ O_2 \rightarrow 4\ CO_2 + 5\ H_2O + 6.5(X-1)\ O_2$$

Energy Eq.: $Q_{CV} + H_R = H_P + W_{CV}$; $Q_{CV} = 0$, $W_{CV} = 0$

$$H_R = n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{C4H10} = 1(-126\ 200 + -17\ 146) = -143\ 346 \text{ kJ}$$

Products: CO_2 $n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{CO2} = 4(-393\ 522 + 177\ 836) = -862\ 744$ kJ

H_2O $n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{H2O} = 5(-241\ 826 + 149\ 073) = -463\ 765$ kJ

O_2 $n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{O2} = 6.5(X-1)(0 + 114\ 101) = (X-1)741\ 657$ kJ

$$H_P = \sum n_i \left(\bar{h}_f^o + \Delta\bar{h}\right)_i = 741\ 657X - 2\ 068\ 166$$

Energy Eq.: $H_P = H_R$ solve for X; $\mathbf{X = 2.594}$

Assume that the exit pressure equals the inlet pressure: $P_e = P_i = 380$ kPa

$$\bar{s}_{C_4H_{10}} = \left[\bar{s}_f^o - \bar{R}\ln\frac{P_1}{P_o} - (\bar{s}_1^* - \bar{s}_1)_f\right] \ ; \ \bar{s}_{O_2} = \left[\bar{s}^o - \bar{R}\ln\frac{P_1}{P_o}\right]$$

$$S_R = S_{C4H10} + S_{O2} = [306.647 - 11.10 - 56.539]$$
$$+ [205.48 - 11.10] \times 6.5 \times 2.594 = 3516.45 \text{ kJ/K}$$

Products:

	n_i	y_i	$\bar{s}_i^o$	$-\bar{R}\ln\frac{y_i P_e}{P_o}$	$\bar{S}_i \ \frac{kJ}{kmol\ K}$
CO_2	4	0.2065	341.988	2.016	344.004
H_2O	5	0.2582	293.550	0.158	293.708
O_2	10.368	0.5353	289.499	-5.904	283.595

$S_P = \sum n_i \bar{S}_i = 5784.87$ kJ/K;

$I = T_o(S_P - S_R) = 298.15\ (5784.87 - 3516.45) =$ **676 329 kJ**

15.109

Liquid hexane enters a combustion chamber at 31°C, 200 kPa, at the rate 1 kmol/s 200% theoretical air enters separately at 500 K, 200 kPa, and the combustion products exit at 1000 K, 200 kPa. The specific heat of ideal gas hexane is C_p = 143 kJ/kmol K. Calculate the rate of irreversibility of the process.

Hexane: $T_c = 507.5$ K, $P_c = 3010$ kPa

$T_{r1} = 0.6$, Fig. D.1: $P_{rg} = 0.028$, $P_{g1} = P_{r1}P_c = 84.47$ kPa

Figs D.2 and D.3: $(\bar{h}^*_1 - \bar{h}_1)_f = 5.16\,\bar{R}T_c$, $(\bar{s}^*_1 - \bar{s}_1)_f = 8.56\,\bar{R}$

Air: $T_2 = 500$ K, $P_2 = 200$ kPa, 200% theoretical air

Products: $T_3 = 1000$ K, $P_3 = 200$ kPa

a) $\bar{h}_{C6H14} = \bar{h}^o_f - (\bar{h}^*_1 - \bar{h}_1)_f + (\bar{h}^*_1 - \bar{h}^*_0) + (\bar{h}^*_0 - \bar{h}_0)$

$\bar{h}^*_0 - \bar{h}_0 = 0$, $\bar{h}^*_1 - \bar{h}^*_0 = \bar{C}_P (T_1 - T_o) = 858$ kJ/kmol, $\bar{h}^o_f = -167300$ kJ/kmol

$\bar{h}^*_1 - \bar{h}_1 = 5.16 \times 8.3145 \times 507.5 = 21773$ kJ/kmol, $\bar{h}_{C6H14} =$ **-188215 kJ/kmol**

$$\bar{s}_{C6H14} = \bar{s}^o_{T_o} + \bar{C}_P \ln\frac{T_1}{T_o} - \bar{R}\ln\frac{P_1}{P_o} + (\bar{s}_1 - \bar{s}^*_1)$$

$$\bar{s}^o_{T_o} + \bar{C}_P \ln\frac{T_1}{T_o} - \bar{R}\ln\frac{P_1}{P_o} = 387.979 + 2.85 - 5.763 = 385.066 \text{ kJ/kmol-K}$$

$\bar{s}^*_1 - \bar{s}_1 = 8.56 \times 8.3145 = 71.172$ kJ/kmol-K, $\bar{s}_{C6H14} =$ **313.894 kJ/kmol-K**

b) $C_6H_{14} + 19O_2 + 71.44N_2 \rightarrow 6CO_2 + 7H_2O + 9.5O_2 + 71.44N_2$

$T_{c\ prod} = \sum y_i T_{ci} = 179.3$ K, $T_{r3} = \dfrac{T_3}{T_{c\ prod}} =$ **5.58 → Ideal Gas**

c) 1st Law: $Q + H_R = H_P + W$; $W = 0$ => $Q = H_P - H_R$

$H_R = \left(\bar{h}\right)_{C6H14} + 19\Delta\bar{h}_{O2} + 71.44\,\Delta\bar{h}_{N2}$

$= -188\,215 + 19\ \ 6086 + 71.44\ \ 5911 = 349701$ kJ/kmol fuel

$$H_P = 6\left(\bar{h}^o_f + \Delta\bar{h}\right)_{CO2} = 7\left(\bar{h}^o_f + \Delta\bar{h}\right)_{H2O} + 9.5\left(\bar{h}^o_f + \Delta\bar{h}\right)_{O2} + 71.44\left(\bar{h}^o_f + \Delta\bar{h}\right)_{N2}$$

CO_2 - $\left(\bar{h}^o_f + \Delta\bar{h}\right) = (-393522 + 33397) = -360125$ kJ/kmol

H_2O - $\left(\bar{h}^o_f + \Delta\bar{h}\right) = (-241826 + 26000) = -215826$ kJ/kmol

O_2 - $\left(\bar{h}^o_f + \Delta\bar{h}\right) = (0 + 22703) = 22703$ kJ/kmol

$N_2 \quad - \quad \left(\bar{h}_f^o + \Delta\bar{h}\right) = (0 + 21463) = 21463$ kJ/kmol

$H_P = -1922537$ kJ; $\dot{Q} = \mathbf{-2272238\ kW}$

d) $\dot{I} = T_o\, \dot{n}\, (S_P - S_R) - \dot{Q}$; $T_o = 25^oC$

$$S_R = \left(\bar{s}\right)_{C6H14} + 19\,\left(\bar{s}^o_{500} - \bar{R}\ln\frac{y_{O2}P_2}{P_o}\right)_{O2} + 71.44\left(\bar{s}^o_{500} - \bar{R}\ln\frac{y_{N2}P_2}{P_o}\right)_{N2}$$

$\left(\bar{s}\right)_{C6H14} = 313.894$ kJ/kmol K, $\left(\bar{s}^o_{500}\right)_{O2} = 220.693$ kJ/kmol K

$\left(\bar{s}^o_{500}\right)_{N2} = 206.740$ kJ/kmol K, $y_{O2} = 0.21$, $y_{N2} = 0.79$

$\dot{S}_R = 19141.9$ kW/K

Products:

	n_i	y_i	$\bar{s}_i^o$	$-\bar{R}\ln\frac{y_iP_e}{P_o}$	$\bar{S}_i$ (kJ/kmol-K)
CO_2	6	0.0639	269.299	17.105	286.404
H_2O	7	0.0745	232.739	15.829	248.568
O_2	9.5	0.1011	243.579	13.291	256.87
N_2	71.44	0.7605	228.171	-3.487	224.684

$S_P = \sum n_i\bar{s}_i = 21950.1$ kJ/K;

$\dot{I} = T_o\, \dot{n}\, (S_P - S_R) - \dot{Q} = \mathbf{3\ 109\ 628\ kW}$

Fuel Cells

15.110

In Example 15.16, a basic hydrogen–oxygen fuel cell reaction was analyzed at 25°C, 100 kPa. Repeat this calculation, assuming that the fuel cell operates on air at 25°C, 100 kPa, instead of on pure oxygen at this state.

Anode: $2\ H_2 \rightarrow 4\ e^- + 4\ H^+$

Cathode: $4\ H^+ + 4\ e^- \rightarrow 1\ O_2 + 2\ H_2O$

Overall: $2\ H_2 + 1\ O_2 \rightarrow 2\ H_2O$

Example 14.16: $\Delta G_{25°C} = -474\ 283$ kJ (for pure O_2)

For $P_{O2} = 0.21 \times 0.1$ MPa:

$\bar{S}_{O_2} = 205.148 - 8.3145 \ln 0.21 = 218.124$ kJ/kmol

$\Delta S = 2(69.950) - 2(130.678) - 1(218.124) = -339.58$ kJ/kmol K

$\Delta G_{25°C} = -571\ 660 - 298.15(-339.58) = -470\ 414$ kJ/kmol

$$E° = \frac{470414}{96487 \times 4} = \mathbf{1.219\ V}$$

15.111

Assume that the basic hydrogen-oxygen fuel cell operates at 600 K instead of 298 K as in example 15.16. Find the change in the Gibbs function and the reversible EMF it can generate.

Reaction: $2\ H_2 + O_2 \ \Rightarrow\ 2\ H_2O$

At a temperature of 600 K the water formed is in a vapor state. We can thus find the change in the enthalpy as

$$\Delta H^0_{600\text{ K}} = 2\left(\bar{h}^o_f + \Delta\bar{h}\right)_{H2O\ g} - 2\left(\bar{h}^o_f + \Delta\bar{h}\right)_{H2} - \left(\bar{h}^o_f + \Delta\bar{h}\right)_{O2}$$

$$= 2(-241\ 826 + 10\ 499) - 2(0 + 8799) - 0 - 9245$$

$$= -489\ 497\ \text{kJ/4 kmol e-}$$

$$\Delta S^0_{600\text{ K}} = 2\ \bar{s}^o_{f\,H2O\ g} - 2\ \bar{s}^o_{f\,H2} - \bar{s}^o_{f\,O2}$$

$$= 2 \times 213.051 - 2 \times 151.078 - 226.45$$

$$= -102.504\ \text{kJ/4 kmol e- K}$$

$$\Delta G^0_{600\text{ K}} = \Delta H^0_{600\text{ K}} - T\Delta S^0_{600\text{ K}} = -489\ 497 - 600(-102.504)$$

$$= -\ 427\ 995\ \text{kJ/4 kmol e-}$$

$$W^{rev} = -\Delta\bar{G}^0 = 427\ 995\ \text{kJ/4 kmol e-}$$

$$E^0 = \frac{-\Delta\bar{G}^0}{96485 \times 8} = \frac{427\ 995}{96\ 485 \times 4} = \mathbf{1.109\ V}$$

15.112

For a PEC fuel cell operating at 350 K the constants in Eq.15.29 are: i_{leak} = 0.01, $i_L = 2$, $i_o = 0.013$ all A/cm^2, b = 0.08 V, c = 0.1 V, ASR = 0.01 Ω cm^2 and EMF = 1.22 V. Find the voltage and the power density for the current density i = 0.25, 0.75 and 1.0 A/cm^2.

The fuel cell output voltage is from Eq.15.29:

$$V = \text{EMF} - b \ln\left(\frac{i + i_{leak}}{i_o}\right) - i\,\text{ASR}_{ohmic} - c \ln\left(\frac{i_L}{i_L - (i + i_{leak})}\right)$$

i = 0.25: $V = 1.22 - 0.23966 - 0.0025 - 0.01393 = 0.9639$ V,
$p = Vi = 0.241$ W/cm^2

i = 0.75: $V = 1.22 - 0.32547 - 0.0075 - 0.0478 = 0.8392$ V,
$p = Vi = 0.6294$ W/cm^2

i = 1.0: $V = 1.22 - 0.34822 - 0.01 - 0.0703 = 0.7915$ V,
$p = Vi = 0.7915$ W/cm^2

15.113

Assume the PEC fuel cell in the previous problem. How large an area does the fuel cell have to deliver 1 kW with a current density of 1 A/ cm^2?

The fuel cell output voltage is from Eq.15.29:

$$V = \text{EMF} - b \ln \left(\frac{i + i_{\text{leak}}}{i_o} \right) - i\, \text{ASR}_{\text{ohmic}} - c \ln \left(\frac{i_L}{i_L - (i + i_{\text{leak}})} \right)$$

i = 1.0: $V = 1.22 - 0.34822 - 0.01 - 0.0703 = 0.7915$ V,

$p = Vi = 0.7915$ W/cm^2

Total power $P = p\,A = 0.7915 \text{ W/cm}^2 \times A = 1000 \text{ W}$

$A = (1000 / 0.7915) \text{ cm}^2 = \mathbf{1263\ cm^2}$

15.114

Consider a methane-oxygen fuel cell in which the reaction at the anode is

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

The electrons produced by the reaction flow through the external load, and the positive ions migrate through the electrolyte to the cathode, where the reaction is

$$8\ e^- + 8\ H^+ + 2\ O_2 \rightarrow 4\ H_2O$$

Calculate the reversible work and the reversible EMF for the fuel cell operating at 25°C, 100 kPa.

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

$$\text{and}\quad 8e^- + 8H^+ + 2CO_2 \rightarrow 4H_2O$$

$$\text{Overall}\quad CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

a) 25 ^oC assume all liquid H_2O and all comp. at 100 kPa

$$\Delta H^0_{25\ C} = -393\ 522 + 2(-285\ 830) - (-74\ 873) - 0 = -890\ 309\ \text{kJ}$$

$$\Delta S^0_{25\ C} = 213.795 + 2(69.950) - 186.251 - 2(205.148) = -\ 242.852\ \text{kJ/K}$$

$$\Delta G^0_{25\ C} = -890\ 309 - 298.15(-242.852) = -\ 817\ 903\ \text{kJ}$$

$$W^{rev} = -\Delta\bar{G}^0 = +817903\ \text{kJ}$$

$$E^0 = \frac{-\Delta\bar{G}^0}{96485 \times 8} = \frac{+817903}{96485 \times 8} = \mathbf{1.06\ V}$$

15.115

Consider a methane-oxygen fuel cell in which the reaction at the anode is

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

The electrons produced by the reaction flow through the external load, and the positive ions migrate through the electrolyte to the cathode, where the reaction is

$$8\ e^- + 8\ H^+ + 2\ O_2 \rightarrow 4\ H_2O$$

Assume that the fuel cell operates at 1200 K instead of at room temperature.

$$CH_4 + 2H_2O \rightarrow CO_2 + 8e^- + 8H^+$$

$$\text{and}\quad 8e^- + 8H^+ + 2CO_2 \rightarrow 4H_2O$$

$$\text{Overall}\qquad CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

At 1200 K assume all gas H_2O and all comp. at 100 kPa

$$\Delta H^0_{1200\ K} = 1(-393522 + 44473) + 2(-241826 + 34506) - 2(0 + 29761)$$
$$- 1[-74873 + 16.043 \times 2.254(1200 - 298.2)] = -780\ 948\ kJ$$

$$\Delta S^0_{1200\ K} = 1(279.390) + 2(240.485)$$
$$- 1(186.251 + 16.043 \times 2.254 \ln \frac{1200}{298.2}) - 2(250.011)$$
$$= 23.7397\ kJ/K$$

$$\Delta G^0_{1200\ K} = \Delta H^0_{1200\ K} - T\Delta S^0_{1200\ K} = -780\ 948 - 1200(23.7397)$$
$$= -809\ 436\ kJ$$

W^{rev} = **+809 436 kJ** $\qquad E^0 = \frac{+809\ 436}{96\ 485 \times 8} =$ **1.049 V**

15.116

A SOC fuel cell at 900 K can be described by an EMF = 1.06 V and the constants in Eq.15.29 as : b = 0 V, c = 0.1 V, ASR = 0.04 Ω cm^2, i_{leak} = 0.01, i_L = 2, i_o = 0.13 all A/cm^2. Find the voltage and the power density for the current density i = 0.25, 0.75 and 1.0 A/cm^2.

The fuel cell output voltage is from Eq.15.29:

$$V = \text{EMF} - b \ln \left(\frac{i + i_{leak}}{i_o} \right) - i \, \text{ASR}_{ohmic} - c \ln \left(\frac{i_L}{i_L - (i + i_{leak})} \right)$$

i = 0.25: $V = 1.06 - 0.0 - 0.01 - 0.01393 = 1.036$ V,
$p = Vi = 0.259$ W/cm^2

i = 0.75: $V = 1.06 - 0.0 - 0.03 - 0.0478 = 0.9822$ V,
$p = Vi = 0.737$ W/cm^2

i = 1.0: $V = 1.06 - 0.0 - 0.04 - 0.0703 = 0.95$ V,
$p = Vi = 0.95$ W/cm^2

15.117

Assume the SOC fuel cell in the previous problem. How large an area does the fuel cell have to deliver 1 kW with a current density of 1 A/ cm^2?

The fuel cell output voltage is from Eq.15.29:

$$V = \text{EMF} - b \ln \left(\frac{i + i_{\text{leak}}}{i_o} \right) - i \, \text{ASR}_{\text{ohmic}} - c \ln \left(\frac{i_L}{i_L - (i + i_{\text{leak}})} \right)$$

$i = 1.0$: $V = 1.06 - 0.0 - 0.04 - 0.0703 = 0.95$ V,

$p = Vi = 0.95 \text{ W/cm}^2$

Total power $P = p\,A = 0.95 \text{ W/cm}^2 \times A = 1000$ W

$A = (1000 / 0.95) \text{ cm}^2 = \mathbf{1053\ cm^2}$

15.118

A PEC fuel cell operating at 25°C, generates 1.0 V that also account for losses. For a total power of 1 kW what is the hydrogen mass flow rate?

The work term for the fuel cell is

$$W = -\Delta G^0_{298\text{ K}} \qquad \text{so} \quad \dot{W} = -\dot{n}\,\Delta G^0_{298\text{ K}}$$

$$\dot{m} = M\dot{n} = \frac{M\dot{W}}{-\Delta G^0_{298\text{ K}}} = \frac{2.016\text{ kg/kmol} \times 1\text{ kW}}{0.5 \times 474\,283\text{ kJ/kmol}} = 8.5 \times 10^{-6}\text{ kg/s}$$

Since the actual output was 1 V rather than the ideal 1.229 V we then infer an efficiency of (1/1.229 = 0.814) so

$$\dot{m}_{ac} = \dot{m} / 0.814 = \mathbf{10.45 \times 10^{-6}\ kg/s}$$

15.119

A basic hydrogen-oxygen fuel cell operates at 600 K instead of 298 K as in example 15.16. For a total power of 5 kW, find the hydrogen mass flow rate and the exergy in the exhaust flow.

Reaction: $2\ H_2 + O_2 \ \Rightarrow\ 2\ H_2O$

At a temperature of 600 K the water formed is in a vapor state. We can thus find the change in the enthalpy as

$$\Delta H^0_{600\ K} = 2\left(\bar{h}^o_f + \Delta\bar{h}\right)_{H2O\ g} - 2\left(\bar{h}^o_f + \Delta\bar{h}\right)_{H2} - \left(\bar{h}^o_f + \Delta\bar{h}\right)_{O2}$$

$$= 2(-241\ 826 + 10\ 499) - 2(0 + 8799) - 0 - 9245$$

$$= -489\ 497 \text{ kJ/4 kmol e-}$$

$$\Delta S^0_{600\ K} = 2\ \bar{s}^o_{f\,H2O\ g} - 2\ \bar{s}^o_{f\,H2} - \bar{s}^o_{f\,O2}$$

$$= 2 \times 213.051 - 2 \times 151.078 - 226.45$$

$$= -102.504 \text{ kJ/4 kmol e- K}$$

$$\Delta G^0_{600\ K} = \Delta H^0_{600\ K} - T\Delta S^0_{600\ K} = -489\ 497 - 600(-102.504)$$

$$= -\ 427\ 995 \text{ kJ/4 kmol e-}$$

$$W^{rev} = -\Delta\bar{G}^0 = 427\ 995 \text{ kJ/4 kmol e-}$$

This was for 2 kmol hydrogen

$$\dot{m} = M\dot{n} = \frac{M\dot{W}}{-\Delta G^0_{600\ K}} = \frac{2.016 \text{ kg/kmol} \times 5 \text{ kW}}{0.5 \times 427\ 995 \text{ kJ/kmol}} = \mathbf{47.1 \times 10^{-6}\ kg/s}$$

The exhaust flow is water vapor at 600 K, 100 kPa so use Table B.1

$$\dot{m}\psi = \dot{m}\ [(h - h_o - T_o(s - s_o)]$$

$$= 47.1 \times 10^{-6}\ [3129 - 104.87 - 298.15\ (8.3037 - 0.3673)]$$

$$= \mathbf{0.031\ kW}$$

Combustion applications and efficiency

15.120

For the combustion of methane 150% theoretical air is used at 25°C, 100 kPa and relative humidity of 70%. Find the composition and dew point of the products.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \;\rightarrow\; 2\ H_2O + 1\ CO_2 + 3.76\ \nu_{O2}\ N_2$$

C balance: $\nu_{CO2} = 1$, H balance: $2\nu_{H2O} = 4$, N_2 balance: $\nu_{N2} = 3.76\ \nu_{O2}$

O balance: $2\ \nu_{O2} = \nu_{H2O} + 2\nu_{CO2} = 2 + 2 \times 1 \quad \Rightarrow \quad \nu_{O2} = 2$

150% theoretical air: $\nu_{O2} = 1.5 \times 2 = 3$ so now more O_2 and N_2

$$CH_4 + 3\left(O_2 + 3.76\ N_2\right) \;\rightarrow\; 2\ H_2O + 1\ CO_2 + 11.28\ N_2 + 1\ O_2$$

Add water to the dry air from Eq.12.28

$$w = 0.622\,\frac{\phi P_g}{P_{tot} - \phi P_g} = 0.622\,\frac{0.7 \times 3.169}{100 - 0.7\ \times 3.169} = 0.0141$$

So the number of moles to add is from Eq.14.9

$$x = 7.655\ w = 7.655 \times 0.0141 = 0.108$$

and the added number of moles is $\nu_{O2}\ x = 0.324$, the products are then

Products: $2.324\ H_2O + 1\ CO_2 + 11.28\ N_2 + 1\ O_2$

The water partial pressure becomes

$$P_v = y_v\ P_{tot} = \frac{2.324}{2.324 + 1 + 11.28 + 1}\ 100 = 14.894\ \text{kPa}$$

T_{dew} = **53.8°C**

15.121

Pentane is burned with 120% theoretical air in a constant pressure process at 100 kPa. The products are cooled to ambient temperature, 20°C. How much mass of water is condensed per kilogram of fuel? Repeat the answer, assuming that the air used in the combustion has a relative humidity of 90%.

$$C_5H_{12} + 1.2 \times 8\,(O_2 + 3.76\,N_2) \rightarrow 5\,CO_2 + 6\,H_2O + 0.96\,O_2 + 36.1\,N_2$$

Products cooled to 20°C, 100 kPa, so for H_2O at 20°C: $P_g = 2.339$ kPa

$$y_{H2O\,MAX} = P_g/P = \frac{2.339}{100} = \frac{n_{H2O\,MAX}}{n_{H2O\,MAX} + 42.06} \;\Rightarrow\; n_{H2O\,MAX} = 1.007 < n_{H2O}$$

Therefore, $n_{H2O\,VAP} = 1.007$, $n_{H2O\,LIQ} = 6 - 1.007 = 4.993$

$$m_{H2O\,LIQ} = \frac{4.993 \times 18.015}{72.151} = \mathbf{1.247\ kg/kg\ fuel}$$

$$P_{v1} = 0.9 \times 2.339 = 2.105\text{ kPa} \;\Rightarrow\; w_1 = 0.622 \times \frac{2.105}{97.895} = 0.013375$$

$$n_{H2O\,IN} = 0.013375 \times \frac{28.97}{18.015} \times (9.6 + 36.1) = 0.983\text{ kmol}$$

$$n_{H2O\,OUT} = 0.983 + 6 = 6.983 \;\Rightarrow\; n_{H2O\,LIQ} = 6.983 - 1.007 = 5.976\text{ kmol}$$

$$m_{H2O\,LIQ} = \frac{5.976 \times 18.015}{72.151} = \mathbf{1.492\ kg/kg\ fuel}$$

15.122

A gas turbine burns methane with 150% theoretical air. Assume the air is 25°C, 100 kPa and relative humidity of 80%. How large a fraction of the product mixture water comes from the moist inlet air?

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2} \left(O_2 + 3.76\ N_2\right) \ \rightarrow\ 2\ H_2O + 1\ CO_2 + 3.76\ \nu_{O2}\ N_2$$

C balance: $\nu_{CO2} = 1$, H balance: $2\nu_{H2O} = 4$, N_2 balance: $\nu_{N2} = 3.76\ \nu_{O2}$

O balance: $2\ \nu_{O2} = \nu_{H2O} + 2\nu_{CO2} = 2 + 2 \times 1 \quad => \quad \nu_{O2} = 2$

150% theoretical air: $\nu_{O2} = 1.5 \times 2 = 3$ so now more O_2 and N_2

$$CH_4 + 3\left(O_2 + 3.76\ N_2\right) \ \rightarrow\ 2\ H_2O + 1\ CO_2 + 11.28\ N_2 + 1\ O_2$$

Add water to the dry air from Eq.12.28

$$w = 0.622\,\frac{\phi P_g}{P_{tot} - \phi P_g} = 0.622\,\frac{0.7 \times 3.169}{100 - 0.7\ \times 3.169} = 0.0141$$

So the number of moles to add is from Eq.15.??9

$$x = 7.655\ w = 7.655 \times 0.0141 = 0.108$$

and the added number of moles is $\nu_{O2}\ x = 0.324$, the products are then

Products: $2.324\ H_2O + 1\ CO_2 + 11.28\ N_2 + 1\ O_2$

Fraction of water is 0.324/2.324 = 0.1394 so **13.9%**

15.123

In an engine a mixture of liquid octane and ethanol, mole ratio 9:1, and stoichiometric air are taken in at T_0, P_0. In the engine the enthalpy of combustion is used so that 30% goes out as work, 30% goes out as heat loss and the rest goes out the exhaust. Find the work and heat transfer per kilogram of fuel mixture and also the exhaust temperature.

$$0.9\ C_8H_{18} + 0.1\ C_2H_5OH + 11.55\ O_2 + 43.428\ N_2$$
$$\rightarrow 8.4\ H_2O + 7.4\ CO_2 + 43.428\ N_2$$

For 0.9 octane + 0.1 ethanol, convert to mole basis

$$\bar{H}^\circ_{RP\ mix} = 0.9\ H^\circ_{RP\ C_8H_{18}} + 0.1\ H^\circ_{RP\ C_2H_5OH}$$
$$= 0.9\ (-44\ 425) \times 114.232 + 0.1\ (-26\ 811) \times 46.069$$
$$= -4\ 690\ 797\ \frac{kJ}{kmol}$$

$$\hat{M}_{mix} = 0.9\ \hat{M}_{oct} + 0.1\ \hat{M}_{alc} = 107.414$$

Energy: $\bar{h}^\circ_{in} + q_{in} = \bar{h}_{ex} + \omega_{ex} = \bar{h}^\circ_{ex} + \Delta\bar{h}_{ex} + \omega_{ex}$

$$\bar{h}^\circ_{ex} - \bar{h}^\circ_{in} = \bar{H}^\circ_{RP\ mix} \quad \Rightarrow \quad \omega_{ex} + \Delta\bar{h}_{ex} - q_{in} = -\bar{H}^\circ_{RP\ mix}$$

$$\omega_{ex} = -q_{in} = 0.3\left(-\bar{H}^\circ_{RP}\right) = \mathbf{1\ 407\ 239}\ \frac{\mathbf{kJ}}{\mathbf{kmol}} = \mathbf{13\ 101}\ \frac{\mathbf{kJ}}{\mathbf{kg\ Fu}}$$

$$\Delta\bar{h}_{prod} = \Delta\bar{h}_{ex} = 0.4\left(-\bar{H}^\circ_{RP}\right) = 1\ 876\ 319\ \frac{kJ}{kmol\ Fu}$$

$$\Delta\bar{h}_{prod} = 8.4\ \Delta\bar{h}_{H_2O} + 7.4\Delta\bar{h}_{CO_2} + 43.428\ \Delta\bar{h}_{N_2}$$

$$\Delta\bar{h}_{prod\ 1300} = 8.4\times 38\ 941 + 7.4\times 50\ 148 + 43.428\times 31\ 503 = 2\ 066\ 312$$

$$\Delta\bar{h}_{prod\ 1200} = 8.4\times 34\ 506 + 7.4\times 44\ 473 + 43.428\times 28\ 109 = 1\ 839\ 668$$

Linear interpolation to get the value of $\Delta\bar{h}_{prod} = 1\ 876\ 319$

$\Rightarrow$ satisfied for T = **1216 K**

15.124

The gas turbine cycle in problem 12.21 has a q_H = 960 kJ/kg air added by combustion. Assume the fuel is methane gas and the q_H equals the heating value at T_o. Find the A/F ratio on a mass basis.

The heating value and the q_H are scaled with the fuel mass and total mixture mass respectively so the relation is

$$q_H = \frac{HV}{m_{mix}/m_{fuel}} = \frac{HV}{A/F + 1} = 960 \text{ kJ/kg-mix}$$

$$A/F = \frac{HV}{q_H} - 1 = \frac{50\,010}{960} - 1 = \mathbf{51.09}$$

Comment: This is about 300% theoretical air.

15.125

A gas turbine burns methane with 200% theoretical air. The air and fuel comes in through two separate compressors bringing them from 100 kPa, 298 K to 1400 kPa and enters a mixing chamber and combustion chamber. What are the specific compressor work and q_H to be used in Brayton cycle calculation? Use constant specific heat to solve.

The reaction equation for stoichiometric mixture is:

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow \ 2\ H_2O + 1\ CO_2 + c\ N_2$$

O balance: $2\ \nu_{O2} = 2 + 2 \quad => \quad \nu_{O2} = 2$

200% theoretical air: $\nu_{O2} = 2 \times 2 = 4$ so now more O_2 and N_2

$$CH_4 + 4\left(O_2 + 3.76\ N_2\right) \rightarrow \ 2\ H_2O + 1\ CO_2 + 15.04\ N_2 + 2\ O_2$$

For 1 kmol fuel: $m = 16.043 + 4 \times 4.76 \times 28.97$

$= 16.043 + 551.59 = 567.63$ kg reactants

The temperatures after compression are:

$$T_{2\ air} = T_1\left(P_2/P_1\right)^{(k-1)/k} = 298.15 \times 14^{0.2857} = 633.7\ K \qquad (k = 1.4)$$

$$T_{2\ fuel} = T_1\left(P_2/P_1\right)^{(k-1)/k} = 298.15 \times 14^{0.230177} = 547.3\ K \quad (k = 1.299)$$

$$w_{c\ air} = C_p\ (T_2 - T_1) = 1.004(633.7 - 298.15) = 336.89\ kJ/kg\ air$$

$$w_{c\ fuel} = C_p\ (T_2 - T_1) = 2.254(547.3 - 298.15) = 561.68\ kJ/kg\ fuel$$

$$w_{c\ total} = \frac{m_{fuel}}{m} w_{c\ fuel} + \frac{m_{air}}{m} w_{c\ air} = \frac{16.04}{567.63} 561.58 + \frac{551.59}{567.63} 336.89$$

= **343.24 kJ/kg mix**

$q = -\overset{o}{H}_{RP} = HV$ but scaled to be per kg reactants

$$q = \frac{50\ 010 \times 16.043}{567.63} = \mathbf{1413.4\ kJ/kg} \quad \left(= \frac{HV}{A/F + 1}\right)$$

15.126

Find the equivalent heat transfer q_H to be used in a cycle calculation for constant pressure combustion when the fuel is (a) methane and (b) gaseous octane in both cases use water vapor in the products and a stoichiometric mixture.

Methane combustion becomes

$$CH_4 + \nu_{O2} (O_2 + 3.76\ N_2) \rightarrow 1\ CO_2 + 2\ H_2O + 7.52\ N_2$$

Oxygen balance: $2\nu_{O2} = 2 + 2 = 4 \quad \Rightarrow \quad \nu_{O2} = 2, \quad \nu_{N2} = 7.52$

$$A/F = \frac{2 \times 31.999 + 7.52 \times 28.013}{16.043} = 17.12 = m_{air}/m_{fuel}$$

$$q = \frac{HV}{m_{mix}/m_{fuel}} = \frac{HV}{A/F + 1} = \frac{50\ 010}{17.12 + 1} = \mathbf{2760\ kJ/kg}$$

Octane combustion, see reaction and A/F in Example 15.1, HV from Table 15.3

$$q = \frac{HV}{m_{mix}/m_{fuel}} = \frac{HV}{A/F + 1} = \frac{44\ 788}{15 + 1} = \mathbf{2799\ kJ/kg}$$

15.127

Consider the steady combustion of propane at 25°C with air at 400 K. The products exit the combustion chamber at 1200 K. It may be assumed that the combustion efficiency is 90%, and that 95% of the carbon in the propane burns to form carbon dioxide; the remaining 5% forms carbon monoxide. Determine the ideal fuel–air ratio and the heat transfer from the combustion chamber.

Ideal combustion process, assumed adiabatic, excess air to keep 1200 K out.

$C_3H_8 + 5x\ O_2 + 18.8x\ N_2 \rightarrow 3\ CO_2 + 4\ H_2O + 5(x - 1)\ O_2 + 18.8x\ N_2$

$H_R = -103900 + 5x(0 + 3027) + 18.8x(0 + 2971) = -103900 + 70990x$

$H_P = 3(-393522 + 44473) + 4(-241826 + 34506)$

$+ 5(x - 1)(0 + 29761) + 18.8x(0 + 28109) = -2025232 + 677254x$

1st law: $H_P - H_R = 0$ Solving, $x = 3.169$

$FA_{IDEAL} = 1/(23.8 \times 3.169) = \mathbf{0.01326}$

b) $FA_{ACTUAL} = 0.01326/0.90 = 0.01473$

$C_3H_8 + 14.26\ O_2 + 53.62\ N_2$

$\rightarrow 2.85\ CO_2 + 0.15\ CO + 4\ H_2O + 9.335\ O_2 + 53.62\ N_2$

$H_R = -103900 + 14.26(0 + 3027) + 53.62(0 + 2971) = +98\ 570$ kJ

$H_P = 2.85(-393522 + 44473) + 0.15(-110527 + 28427) + 4(-241826 + 34506)$

$+ 9.335(0 + 29761) + 53.62(0 + 28109) = -51\ 361$ kJ

$Q_{CV} = H_P - H_R =$ **-149 931 kJ**

15.128

A gasoline engine is converted to run on propane. Assume the propane enters the engine at 25°C, at the rate 40 kg/h. Only 90% theoretical air enters at 25°C such that 90% of the C burns to form CO_2, and 10% of the C burns to form CO. The combustion products also include H_2O, H_2 and N_2, exit the exhaust at 1000 K. Heat loss from the engine (primarily to the cooling water) is 120 kW. What is the power output of the engine? What is the thermal efficiency?

Propane: $T_1 = 25^oC$, $\dot{m} = 40$ kg/hr, $M = 44.094$ kg/kmol

Air: $T_2 = 25^oC$, 90% theoretical Air produces 90% CO_2, 10% CO

Products: $T_3 = 1000$ K, CO_2, CO, H_2O, H_2, N_2

$$C_3H_8 + 4.5O_2 + 16.92N_2 \rightarrow 2.7\,CO_2 + 0.3CO + 3.3H_2O + 0.7H_2 + 16.92N_2$$

$$\dot{n}_{C3H8} = \dot{m}/(M\times3600) = 0.000252 \text{ kmol/s}$$

1st Law: $\dot{Q} + H_R = H_P + \dot{W}$; $\dot{Q} = -120$ kW

$$H_R = n_{C3H8}\,\bar{h}_f^o = -103\ 900 \text{ kJ}$$

Products:

CO_2 - $n_{CO2}\left(\bar{h}_f^o + \Delta\bar{h}\right) = 2.7(-393522 + 33397) = -972337.5$ kJ

CO - $n_{CO}\left(\bar{h}_f^o + \Delta\bar{h}\right) = 0.3(-110527 + 21686) = -26652$ kJ

H_2O - $n_{H2O}\left(\bar{h}_f^o + \Delta\bar{h}\right) = 3.3(-241826 + 26000) = -712226$ kJ

H_2 - $n_{H2}\left(\bar{h}_f^o + \Delta\bar{h}\right) = 0.7(0 + 20663) = 14464.1$ kJ

N_2 - $n_{N2}\left(\bar{h}_f^o + \Delta\bar{h}\right) = 16.92(0 + 21463) = 363154$ kJ

$$H_P = \sum n_i\left(\bar{h}_f^o + \Delta\bar{h}\right)_i = -1\ 333\ 598 \text{ kJ}$$

$$\dot{W} = \dot{Q} + \dot{n}(H_R - H_P) = \mathbf{189.9\ kW}$$

C_3H_8: Table 14.3 $\quad H_{RPo} = -50343$ kJ/kg

$$\dot{HHV} = \dot{n}_{C3H8}\,M(-H_{RPo}) = 559.4 \text{ kW}$$

$$\eta_{th} = \dot{W}/\dot{HHV} = \mathbf{0.339}$$

15.129

A small air-cooled gasoline engine is tested, and the output is found to be 1.0 kW. The temperature of the products is measured to 600 K. The products are analyzed on a dry volumetric basis, with the result: 11.4% CO_2, 2.9% CO, 1.6% O_2 and 84.1% N_2. The fuel may be considered to be liquid octane. The fuel and air enter the engine at 25°C, and the flow rate of fuel to the engine is 1.5×10^{-4} kg/s. Determine the rate of heat transfer from the engine and its thermal efficiency.

$$a\,C_8H_{18} + b\,O_2 + 3.76b\,N_2$$

$$\rightarrow 11.4\,CO_2 + 2.9\,CO + c\,H_2O + 1.6\,O_2 + 84.1\,N_2$$

$$b = \frac{84.1}{3.76} = 22.37, \qquad a = \frac{1}{8}(11.4 + 2.9) = 1.788$$

$$c = 9a = 16.088$$

$$C_8H_{18} + 12.5\,O_2 + 47.1\,N_2$$

$$\rightarrow 6.38\,CO_2 + 1.62\,CO + 9\,H_2O + 0.89\,O_2 + 47.1\,N_2$$

$$H_R = \bar{h}^0_{f\,C_8H_{18}} = -250\ 105 \text{ kJ/kmol}$$

$$H_P = 6.38(-393\ 522 + 15\ 788) + 1.62(-110527 + 10\ 781)$$

$$+ 9(-241\ 826 + 12\ 700) + 0.89(0 + 11187)$$

$$+ 47.1(0 + 10712) = -4\ 119\ 174 \text{ kJ/kmol}$$

$$H_P - H_R = -4\ 119\ 174 - (-250\ 105) = -3\ 869\ 069 \text{ kJ/kmol}$$

$$\dot{H}_P - \dot{H}_R = (0.00015/114.23)(-3\ 869\ 069) = -5.081 \text{ kW}$$

$$\dot{Q}_{CV} = -5.081 + 1.0 = \mathbf{-4.081\ kW}$$

Fuel heating value from table 15.3

$$\dot{Q}_H = \dot{m}\,HV = 0.00015\,(47\ 893) = 7.184 \text{ kW}$$

$$\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = 1.0/7.184 = \mathbf{0.139}$$

15.130

A gasoline engine uses liquid octane and air, both supplied at P_0, T_0, in a stoichiometric ratio. The products (complete combustion) flow out of the exhaust valve at 1100 K. Assume that the heat loss carried away by the cooling water, at 100°C, is equal to the work output. Find the efficiency of the engine expressed as (work/lower heating value) and the second law efficiency.

$$C_8H_{18} + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow 8\ CO_2 + 9\ H_2O + 47\ N_2$$

$$2\ \nu_{O_2} = 16 + 9 \Rightarrow \nu_{O_2} = 12.5$$

$$\text{LHV} = 44425\ \frac{\text{kJ}}{\text{kg fuel}} \ \Rightarrow\ \text{LHV} = 5.07476\times10^6\ \frac{\text{kJ}}{\text{kmol fuel}}$$

$$\Delta H_{P\ 1100} = 8 \times 38885 + 9 \times 30190 + 47 \times 24760 = 1746510$$

C.V. Total engine

$$H_{in} = H_{ex} + W + Q_{loss} = H_{ex} + 2\ W$$

$$\Rightarrow 2\ W = H_{in} - H_{ex} = H_R - H_v = -H^\circ_{RP} + \Delta H_R - \Delta H_{P\ 1100}$$

$$= 5.07476\times10^6 + 0 - 1746510 = 3328250$$

$$W = 1.664\times10^6\ \frac{\text{kJ}}{\text{kmol fuel}}$$

$$\eta_{th} = \frac{W}{\text{LHV}} = \frac{1.664\times10^6}{5.07476\times10^6} = \mathbf{0.328}$$

Find entropies in and out:

inlet:

$$\bar{S}^\circ_{Fu} = 360.575$$

$$\bar{S}^\circ_{O_2} = 205.148 - 8.3145\ \ln\frac{1}{4.76} = 218.12$$

$$\bar{S}^\circ_{N_2} = 191.609 - 8.3145\ \ln\frac{3.76}{4.76} = 193.57$$

$$\bar{S}^\circ_{in} = 360.575 + 12.5 \times 218.12 + 47 \times 193.57 = 12185$$

exit: $\bar{S}^\circ_{CO_2} = 275.528 - 8.3145\ \ln\frac{8}{64} = 292.82$

$$\bar{S}^\circ_{H_2O} = 236.732 - 8.3145\ \ln\frac{9}{64} = 253.04$$

$$\bar{S}^{\circ}_{N_2} = 231.314 - 8.1345 \ln \frac{47}{64} = 233.88$$

$$\bar{S}^{\circ}_{ex} = 8 \times 292.82 + 9 \times 253.04 + 47 \times 233.88 = 15612$$

Assume the same Q_{loss} out to 100°C reservoir in the reversible case and compute Q_0^{rev}:

$$\bar{S}_{in} + Q_0^{rev}/T_0 = \bar{S}_{ex} + Q_{loss}/T_{res}$$

$$\begin{aligned} Q_0^{rev} &= T_0\left(\bar{S}_{ex} - \bar{S}_{in}\right) + Q_{loss}\, T_0/T_{res} \\ &= 298.15(15612 - 12185) + 1.664\times10^6 \times 298.15/373.15 \\ &= 2.351\times10^6 \ \frac{\text{kJ}}{\text{kmol fuel}} \end{aligned}$$

$$H_{in} + Q_0^{rev} = H_{ex} + W^{rev} + Q_{loss}$$

$$\Rightarrow W^{rev} = H_{in} - H_{ex} - Q_{loss} + Q_0^{rev} = W_{ac} + Q_0^{rev} = 4.015\times10^6 \ \frac{\text{kJ}}{\text{kmol fuel}}$$

$$\eta_{II} = W_{ac}/\, W^{rev} = 1.664\times10^6/4.015\times10^6 = \mathbf{0.414}$$

Review Problems

15.131

Repeat Problem 15.25 for a certain Utah coal that contains, according to the coal analysis, 68.2% C, 4.8% H, 15.7% O on a mass basis. The exiting product gas contains 30.9% CO, 26.7% H_2, 15.9% CO_2 and 25.7% H_2O on a mole basis.

Convert the mass concentrations to number of kmol per 100 kg coal:

C : 68.2/12.01 = 5.679 $\qquad$ H_2: 4.8/2.016 = 2.381

O_2: 15.7/32.00 = 0.491

Now the combustion equation reads

$$x(5.679\,C + 2.381\,H_2 + 0.491\,O_2) + y\,H_2O + z\,O_2 \;\text{ in}$$

$$30.9\,CO + 26.7\,H_2 + 15.9\,CO_2 + 25.7\,H_2O \;\text{ out}$$

in 100 kmol of mix out

Now we can do the atom balance to find (x, y, z)

$$C: \quad 5.679x = 30.9 + 15.9 \quad \rightarrow \quad x = 8.241$$

$$H_2: \quad 2.381 \times 8.241 + y = 26.7 + 25.7 \quad \rightarrow \quad y = 32.778$$

$$O_2: \quad 0.491 \times 8.241 + \frac{32.778}{2} + z = \frac{30.9}{2} + 15.9 + \frac{25.7}{2}$$

$$\rightarrow \quad z = 23.765$$

Therefore, for 100 kmol of mixture out,

require: **824.1 kg** of coal

32.778 kmol of steam

23.765 kmol of oxygen

15.132

Many coals from the western United States have a high moisture content. Consider the following sample of Wyoming coal, for which the ultimate analysis on an as-received basis is, by mass:

Component	Moisture	H	C	S	N	O	Ash
% mass	28.9	3.5	48.6	0.5	0.7	12.0	5.8

This coal is burned in the steam generator of a large power plant with 150% theoretical air. Determine the air–fuel ratio on a mass basis.

Converting from mass analysis:

Substance	S	H_2	C	O_2	N_2
c/M =	0.5/32	3.5/2	4.86/12	12/32	0.7/28
kmol / 100 kg coal	0.0156	1.75	4.05	0.375	0.025
Product	SO_2	H_2O	CO_2		
oxygen required	0.0156	0.875	4.05	--	--

Combustion requires then oxygen as: 0.0156 + 0.875 + 4.05 = 4.9406

The coal does include 0.375 O_2 so only 4.5656 O_2 from air/100 kg coal

AF = 1.5 × (4.5656 + 4.5656 × 3.76) × 28.97/100 = **9.444 kg air/kg coal**

15.133

A fuel, C_xH_y, is burned with dry air and the product composition is measured on a dry mole basis to be: 9.6% CO_2, 7.3% O_2 and 83.1% N_2. Find the fuel composition (*x*/*y*) and the percent theoretical air used.

$$\nu_{Fu}C_xH_y + \nu_{O2}O_2 + 3.76\nu_{O2}N_2 \rightarrow 9.6\ CO_2 + 7.3\ O_2 + 83.1\ N_2 + \nu_{H2O}H_2O$$

N_2 balance: $3.76\nu_{O2} = 83.1 \Rightarrow \nu_{O2} = \mathbf{22.101}$

O_2 balance: $\nu_{O2} = 9.6 + 7.3 + \frac{1}{2}\nu_{H2O} \Rightarrow \nu_{H2O} = \mathbf{10.402}$

H balance: $\nu_{Fu}\, y = 2\ \nu_{H2O} = \mathbf{20.804}$

C balance: $\nu_{Fu}\, x = \mathbf{9.6}$

Fuel composition ratio = x/y = 9.6/20.804 = **0.461**

$$\text{Theoretical air} = \frac{\nu_{O2AC}}{\nu_{O2stoich}}\ 100 = \frac{22.101}{9.6 + \frac{1}{4} \times 29.804}\ 100 = \mathbf{149.3\%}$$

15.134

In an engine liquid octane and ethanol, mole ration 9:1, and stoichiometric air are taken in at 298 K, 100 kPa. After complete combustion, the products run out of the exhaust system where they are cooled to 10^oC. Find the dew point of the products and the mass of water condensed per kilogram of fuel mixture.

Reaction equation with 0.9 octane and 0.1 ethanol is

$$0.9\ C_8H_{18} + 0.1\ C_2H_5OH + 11.55\ O_2 + 43.428\ N_2$$

$$\rightarrow 8.4\ H_2O + 7.4\ CO_2 + 43.428\ N_2$$

$$y_{H_2O} = \frac{8.4}{8.4 + 7.4 + 43.428} = 0.1418$$

$$P_{H_2O} = y_{H_2O}P_{tot} = \mathbf{14.3\ kPa} \qquad \Rightarrow\ T_{dew} = \mathbf{52.9\ °C}$$

$$10\ °C \Rightarrow P_{H_2O} = 1.2276 \quad \Rightarrow \quad y_{H_2O} = 0.012276 = \frac{x}{x + 7.4 + 43.428}$$

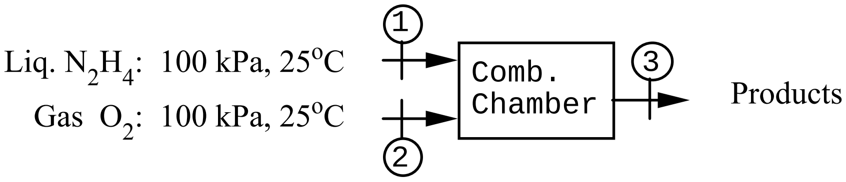
$$\Rightarrow x = \mathbf{0.6317} \quad \Rightarrow \quad \Delta\nu_{H_2O} = -7.77\ \frac{\text{kmol}}{\text{kmol Fu mix}}$$

$$M_{mix} = \sum y_iM_i = 0.9 \times 114.23 + 0.1 \times 46.069 = 107.414$$

$$m_{H_2O\ cond} = \frac{-\Delta\nu_{H_2O} \times 18.015}{107.414} = \mathbf{1.303\ \frac{kg}{kg\ Fu\ mix}}$$

15.135

In a test of rocket propellant performance, liquid hydrazine (N_2H_4) at 100 kPa, 25°C, and oxygen gas at 100 kPa, 25°C, are fed to a combustion chamber in the ratio of 0.5 kg O_2/kg N_2H_4. The heat transfer from the chamber to the surroundings is estimated to be 100 kJ/kg N_2H_4. Determine the temperature of the products exiting the chamber. Assume that only H_2O, H_2, and N_2 are present. The enthalpy of formation of liquid hydrazine is +50 417 kJ/kmol.



$$\dot{m}_{O2}/\dot{m}_{N2H4} = 0.5 = 32\dot{n}_{O2}/32\dot{n}_{N2H4} \quad \text{and} \quad \dot{Q}/\dot{m}_{N2H4} = -100 \text{ kJ/kg}$$

Energy Eq.: $Q_{CV} = H_P - H_R = -100 \times 32.045 = -3205$ kJ/kmol fuel

Combustion eq.: $1\, N_2H_4 + \frac{1}{2} O_2 \rightarrow H_2O + H_2 + N_2$

$$H_R = 1(50417) + \tfrac{1}{2}(0) = 50417 \text{ kJ}$$

$$H_P = -241\,826 + \Delta\bar{h}_{H2O} + \Delta\bar{h}_{H2} + \Delta\bar{h}_{N2}$$

Energy Eq. now reads

$$H_P = H_R + Q_{CV} = H_P^o + \Delta H_P$$

$$\Delta H_P = \Delta\bar{h}_{H2O} + \Delta\bar{h}_{H2} + \Delta\bar{h}_{N2} = -H_P^o + H_R + Q_{CV}$$

$$= 241\,826 + 50\,417 - 3205 = 289\,038 \text{ kJ/kmol fuel}$$

Table A.9 : Guess T and read for water, hydrogen and nitrogen

2800 K: $\Delta H_P = 115\,463 + 81\,355 + 85\,323 = 282\,141$ too low

3000 K: $\Delta H_P = 126\,548 + 88\,725 + 92\,715 = 307\,988$ too high

Interpolate to get $T_P =$ **2854 K**

15.136

Find the lower heating value for the fuel blend in problem 15.134 with a scaling as in Table 15.3

Reaction equation with 0.9 octane and 0.1 ethanol is

$$0.9\ C_8H_{18} + 0.1\ C_2H_5OH + 11.55\ O_2 + 43.428\ N_2$$

$$\rightarrow 8.4\ H_2O + 7.4\ CO_2 + 43.428\ N_2$$

We can solve several ways, use Table A.10 to find $\overset{\circ}{H}_{RP}$ and do it on a mole basis and then divide with M for mixture or we can convert the mole fractions to mass fractions and use Table 15.3 values. Let us use Table 15.3.

$$M_{mix} = \sum y_i M_i = 0.9 \times 114.23 + 0.1 \times 46.069 = 107.414$$

$$c_{ethanol} = yM/M_{mix} = \frac{0.1 \times 46.069}{107.414} = 0.042889, \quad c_{octane} = 0.957111$$

$$LHV = c_{ethanol}\ LHV_{ethanol} + c_{octane}\ LHV_{octane}$$

$$= 0.042889\ (26\ 811) + 0.957111\ (44\ 425)\quad kJ/kg$$

$$= \mathbf{43\ 670\ kJ/kg}$$

15.137

E85 is a liquid mixture of 85% ethanol and 15% gasoline (assume octane) by mass. Find the lower heating value for this blend.

The heating value of the blend becomes

$$\begin{aligned} \text{LHV} &= 0.85\ \text{LHV}_{\text{ethanol}} + 0.15\ \text{LHV}_{\text{octane}} \\ &= 0.85\ (26\ 811) + 0.15\ (44\ 425)\quad \text{kJ/kg} \\ &= \mathbf{29\ 453\ kJ/kg} \end{aligned}$$

As this is lower than gasoline a larger amount of fuel is being used for the same energy release in the engine. If the mixture was given on a mole basis we would have used LHV on a mole basis instead (multiplying Table 15.3 entries by M).

15.138

Determine the higher heating value of the sample Wyoming coal as specified in Problem 15.132.

The higher heating value is for liquid water in the products. We need the composition of the products.

Converting from mass analysis:

Substance	S	H_2	C	O_2	N_2
c/M =	0.5/32	3.5/2	4.86/12	12/32	0.7/28
kmol / 100 kg coal	0.0156	1.75	4.05	0.375	0.025
Product	SO_2	H_2O	CO_2		

So the combustion equation becomes (for 100 kg coal)

$$\text{Fuel} + \text{Air} \rightarrow 1.75\ H_2O + 4.05\ CO_2 + 0.0156\ SO_2$$

The formation enthalpies are from Table A.10. Therefore,

$$\bar{h}_{RP0} = H_P^o - H_R^o = 4.05(-393\ 522) + 1.75(-285\ 830) + 0.0156(-296\ 842)$$

$$= -2\ 098\ 597 \text{ kJ/100 kg coal}$$

So that HHV = **+20 986 kJ/kg coal**

15.139

Ethene, C_2H_4, and propane, C_3H_8, in a 1:1 mole ratio as gases are burned with 120% theoretical air in a gas turbine. Fuel is added at 25°C, 1 MPa and the air comes from the atmosphere, 25°C, 100 kPa through a compressor to 1 MPa and mixed with the fuel. The turbine work is such that the exit temperature is 800 K with an exit pressure of 100 kPa. Find the mixture temperature before combustion, and also the work, assuming an adiabatic turbine.

$\phi = 1$: $C_2H_4 + C_3H_8 + 8\ O_2 + 30.08\ N_2 \rightarrow 5\ CO_2 + 6\ H_2O + 30.08\ N_2$

$\phi = 1.2$: $C_2H_4 + C_3H_8 + 9.6\ O_2 + 36.096\ N_2$

$\rightarrow 5\ CO_2 + 6\ H_2O + 1.6\ O_2 + 36.096\ N_2$

45.696 kmol air per 2 kmol fuel

C.V. Compressor (air flow)

Energy Eq.: $w_c = h_2 - h_1$ Entropy Eq.: $s_2 = s_1 \Rightarrow$

$P_{r2} = P_{r1} \times P_2/P_1 = 13.573 \Rightarrow T_{2\ air} = \mathbf{570.8\ K}$

$w_c = 576.44 - 298.34 = \mathbf{278.1\ kJ/kg = 8056.6\ kJ/kmol\ air}$

C.V. Mixing Chanber (no change in composition)

$$\dot{n}_{air}\bar{h}_{air\ in} + \dot{n}_{Fu1}\bar{h}_{1\ in} + \dot{n}_{Fu2}\bar{h}_{2\ in} = (\text{SAME})_{exit}$$

$$\left(\bar{C}_{P\ F1} + \bar{C}_{P\ F2}\right)\left(T_{exit} - T_0\right) = 45.696\ \bar{C}_{P\ air}\left(T_{2\ air} - T_{exit}\right)$$

C_2H_4: $\bar{C}_{P\ F1} = 43.43$, C_3H_8: $\bar{C}_{P\ F2} = 74.06$, $\bar{C}_{P\ air} = 29.07$

$$T_{exit} = \frac{\left(45.696\bar{C}_{P\ air}T_2 + \left(\bar{C}_{P\ F1} + \bar{C}_{P\ F2}\right)T_0\right)}{\bar{C}_{P\ F1} + \bar{C}_{P\ F2} + 45.696\ \bar{C}_{P\ air}} = \mathbf{548.7\ K}$$

Dew Point Products: $y_{H2O} = \frac{6}{5 + 6 + 1.6 + 36.096} = 0.1232$

$P_{H2O} = y_{H2O}P_{tot} = 123.2\ \text{kPa} \Rightarrow T_{dew} = \mathbf{105.5°C}$

C.V. Turb. + combustor + mixer + compressor (no Q)

$$w_{net} = H_{in} - H_{out} = H_R - H_{P\ 800} \quad (800°\text{K out so no liquid } H_2O)$$

$$= \bar{h}^\circ_{fC2H4} + \bar{h}^\circ_{fC3H8} - 5\ \bar{h}_{CO_2} - 6\ \bar{h}_{H_2O} - 1.6\ \bar{h}_{O_2} - 36.096\ \bar{h}_{N_2}$$

$$= \mathbf{2\ 576\ 541}\ \frac{\mathbf{kJ}}{\mathbf{2\ kmol\ Fu}}$$

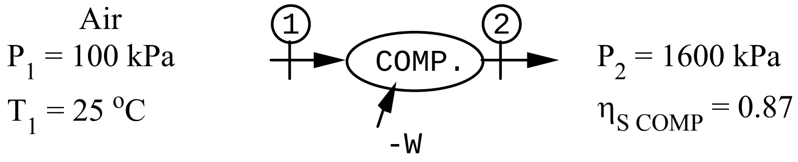
$$w_T = w_{net} + w_{comp} = \mathbf{2\ 944\ 695}\ \frac{\mathbf{kJ}}{\mathbf{2\ kmol\ Fu}}$$

15.140

A study is to be made using liquid ammonia as the fuel in a gas-turbine engine. Consider the compression and combustion processes of this engine.

a. Air enters the compressor at 100 kPa, 25°C, and is compressed to 1600 kPa, where the isentropic compressor efficiency is 87%. Determine the exit temperature and the work input per kilomole.

b. Two kilomoles of liquid ammonia at 25°C and *x* times theoretical air from the compressor enter the combustion chamber. What is *x* if the adiabatic flame temperature is to be fixed at 1600 K?



a) ideal compressor process (adiabatic reversible):

$$s_{2S} = s_1 \Rightarrow T_{2S} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 298.2\left(\frac{1600}{100}\right)^{0.286} = 659\text{ K}$$

$$-w_S = C_{P0}(T_{2S} - T_1) = 1.004(659 - 298.2) = 362.2$$

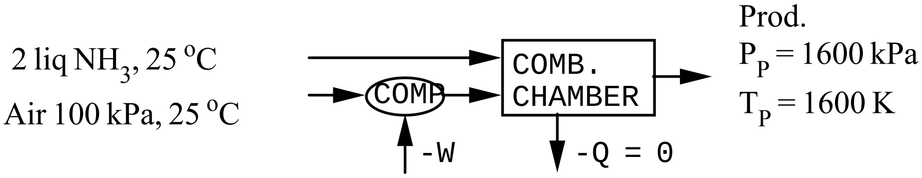
Real process:

$$-w = -w_S/\eta_S \ = 362.2/0.87 = 416.3\text{ kJ/kg}$$

$$T_2 = T_1 - w/C_{P0} = 298.2 + 416.3/1.004 \ = \mathbf{713\ K}$$

Also $-w = 416.3 \times 28.97 =$ **12060 kJ/kmol**

b)



$$2\,NH_3 + 1.5x\,O_2 + 5.64x\,N_2 \rightarrow 3\,H_2O + 1.5(x - 1)\,O_2 + (5.64x + 1)\,N_2$$

Using Tables 15.3, A.10 and A.2,

$$\bar{h}_{NH3} = -45\ 720 + 17.031(298.36 - 1530.04) \ = -66\ 697\text{ kJ/kmol}$$

$$H_R = 2(-66\ 697) + 0 = -133\ 394\text{ kJ}$$

$$-W = 12\ 060 \times 7.14x = 86\ 108\ x\text{ kJ}$$

$$H_P = 3(-241\ 826 + 52\ 907) + 1.5(x - 1)(0 + 44267) + (5.64x + 1)(0 + 41904)$$
$$= 302\ 739x - 591\ 254$$

Energy Eq.: $H_R = H_P + W$

$$-133\ 394 = 302\ 739\ x - 591\ 254 - 86\ 108\ x$$
$$\Rightarrow \quad x = \mathbf{2.11}$$

15.141

Consider the gas mixture fed to the combustors in the integrated gasification combined cycle power plant, as described in Problem 15.30. If the adiabatic flame temperature should be limited to 1500 K, what percent theoretical air should be used in the combustors?

Product	CH_4	H_2	CO	CO_2	N_2	H_2O	H_2S	NH_3
% vol.	0.3	29.6	41.0	10.0	0.8	17.0	1.1	0.2

Mixture may be saturated with water so the gases are (H_2S and NH_3 out)

CH_4	H_2	CO	CO_2	N_2	n
0.3	29.6	41.0	10.0	0.8	81.7

$y_{V\,MAX} = 7.384/3000 = n_V/(n_V + 81.7)$

Solving, $n_V = 0.2$ kmol, the rest is condensed

$$\{0.3\,CH_4 + 29.6\,H_2 + 41.0\,CO + 10.0\,CO_2 + 0.8\,N_2$$
$$+ 0.2\,H_2O + 35.9x\,O_2 + 3.76 \times 35.9x\,N_2\} \rightarrow$$
$$51.3\,CO_2 + 30.4\,H_2O + 35.9(x - 1)\,O_2 + (135.0x + 0.8)\,N_2$$

For the fuel gas mixture,

$$n\bar{C}_{P0\,MIX} = 0.3 \times 16.04 \times 2.2537 + 29.6 \times 2.016 \times 14.2091$$
$$+ 41.0 \times 28.01 \times 1.0413 + 10.0 \times 44.01 \times 0.8418$$
$$+ 0.8 \times 28.013 \times 1.0416 + 0.2 \times 18.015 \times 1.8723 \quad = 2455.157$$

$$n\bar{h}^0_{f\,MIX} = 0.3(-74873) + 29.6(0) + 41.0(-110527)$$
$$+ 10.0(-393522) + 0.8(0) + 0.2(-241826) \quad = -8537654 \text{ kJ}$$

At 40°C, for the fuel mixture:

$$H_{MIX} = -8\,537\,654 + 2455.157(40 - 25) = -8\,500\,827 \text{ kJ}$$

Assume air enters at 25°C: $\bar{h}_{AIR} = 0$

Products at 1500 K:

$$H_P = 51.3(-393522 + 61705) + 30.4(-241826 + 48149)$$
$$+ 35.9(x - 1)(0 + 40600) + (135x + 0.8)(0 + 38405)$$
$$= -24\,336\,806 + 6\,642\,215x$$

Energy Eq.: $H_P = H_R = H_{MIX}$

$$x = \frac{+24336809 - 8500827}{6642215} = 2.384 \quad \text{or} \quad \mathbf{238\ \%\ theo.\ air}$$

15.142

Carbon monoxide, CO, is burned with 150% theoretical air and both gases are supplied at 150 kPa and 600 K. Find the reference enthalpy of reaction and the adiabatic flame temperature.

$$\mathrm{CO} + \nu_{O2}\left(O_2 + 3.76\, N_2\right) \rightarrow CO_2 + \nu_{N2}\, N_2$$

$$\text{O balance:} \quad 1 + 2\nu_{O_2} = 1 \quad \Rightarrow \quad \nu_{O_2} = 0.5 \quad \Rightarrow \nu_{O2\ \text{actual}} = 0.75$$

Now the actual reaction equation has excess oxygen as

$$\mathrm{CO} + 0.75\left(O_2 + 3.76\, N_2\right) \rightarrow CO_2 + 2.82\, N_2 + 0.25\, O_2$$

From the definition of enthalpy of combustion, Eq.14.14 or 14.15

$$\overset{o}{H}_{RP} = \overset{o}{H}_P - \overset{o}{H}_R = \bar{h}^o_{f\,CO_2} + 0 - \bar{h}^o_{f\,CO}$$

$$= -393\,522 - (-110\,527) = \mathbf{-282\,995\ kJ/kmol\ CO}$$

$$= \mathbf{-\,10\,103\ kJ/kg\ CO} \qquad \text{(as for Table 14.3)}$$

$$\text{Actual energy Eq.:} \quad H_R = H_P = \overset{o}{H}_P + \Delta H_P = \overset{o}{H}_R + \Delta H_R$$

$$\Delta H_P = \overset{o}{H}_R + \Delta H_R - \overset{o}{H}_P = -\overset{o}{H}_{RP} + \Delta\bar{h}_{CO} + 0.75\, \Delta\bar{h}_{O_2} + 2.82\, \Delta\bar{h}_{N_2}$$

$$= 282\,995 + 8942 + 0.75 \times 9245 + 2.82 \times 8894$$

$$= 323\,952\ \text{kJ/kmol}$$

The left hand side is

$$\Delta H_P = \Delta\bar{h}_{CO_2} + 0.25\, \Delta\bar{h}_{O_2} + 2.82\, \Delta\bar{h}_{N_2}$$

$$\Delta H_{P\,2600} = 128\,074 + 0.25 \times 82\,225 + 2.82 \times 77\,963 = 368\,486$$

$$\Delta H_{P\,2400} = 115\,779 + 0.25 \times 74\,453 + 2.82 \times 70\,640 = 333\,597$$

$$\Delta H_{P\,2200} = 103\,562 + 0.25 \times 66\,770 + 2.82 \times 63\,362 = 298\,935$$

Now we can do a linear interpolation for the adiabatic flame temperature

$$T = 2200 + 200\, \frac{323\,952 - 298\,935}{333\,597 - 298\,935} = \mathbf{2344\ K}$$

15.143

A rigid container is charged with butene, C_4H_8, and air in a stoichiometric ratio at P_0, T_0. The charge burns in a short time with no heat transfer to state 2. The products then cool with time to 1200 K, state 3. Find the final pressure, P_3, the total heat transfer, ${}_1Q_3$, and the temperature immediately after combustion, T_2.

The reaction equation is, having used C and H atom balances:

$$C_4H_8 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow 4\ CO_2 + 4\ H_2O + 3.76\ \nu_{O2}N_2$$

Counting now the oxygen atoms we require $\nu_{O2} = 6$.

C.V. analysis gives:

$$U_3 - U_1 = Q - W = Q = H_3 - H_1 - P_2V_2 + P_1V_1 = H_3 - H_1 - \bar{R}(n_2T_3 - n_1T_1)$$

$$H_3 - H_1 = H_{P\ 1200} - \overset{o}{H_R} = \overset{o}{H_P} - \overset{o}{H_R} + \Delta H_P = \hat{M}\ \overset{o}{H_{RP}} + \Delta H_P$$

$$= 56.108 \times (-45\ 316) + 950055 = -1\ 592\ 535\ \text{kJ/kmol}$$

Where $\hat{M} = 56.108$ and $n_1 = 1 + 6 \times 4.76 = 29.56$,

$$n_2 = 4 + 4 + 6 \times 3.76 = 30.56,$$

Table A.9 at 1200 K: $\Delta h_{CO2} = 44473$, $\Delta h_{H2O} = 34506$, $\Delta h_{N2} = 28109$.

Now solving for the heat transfer:

$$Q = -1592535 - 8.3145(30.56 \times 1200 - 29.56 \times 298.15) = -1824164\ \frac{\text{kJ}}{\text{kmol fuel}}$$

To get the pressure, assume ideal gases:

$$P_3 = \frac{n_2\bar{R}T_3}{V_2} = P_1\frac{n_2T_3}{n_1T_1} = 421.6\ \text{kPa}$$

Before heat transfer takes place we have constant U so:

$$U_2 - U_1 = 0 = H_2 - H_1 - n_2\bar{R}T_2 + n_1\bar{R}T_1$$

Now split the enthalpy $H_2 = \overset{o}{H_P} + \Delta H_P(T_2)$ and arrange things with the unknowns on LHS and knowns on RHS:

$$\Delta H_P - n_2\bar{R}T_2 = H_R - \overset{o}{H_P} - n_1\bar{R}T_1 = 2\ 542\ 590 - 73278 = 2\ 469\ 312$$

Trial and error leads to:

$$\text{LHS}\ (3000\ \text{K}) = 3\ 209\ 254 - 30.56 \times 8.31451 \times 3000 = 2\ 446\ 980$$

$$\text{LHS}\ (3200\ \text{K}) = 3\ 471\ 331 - 30.56 \times 8.31451 \times 3200 = 2\ 658\ 238$$

linear interpolation T = **3021 K**

15.144

Natural gas (approximate it as methane) at a ratio of 0.3 kg/s is burned with 250% theoretical air in a combustor at 1 MPa where the reactants are supplied at T_0. Steam at 1 MPa, 450°C at a rate of 2.5 kg/s is added to the products before they enter an adiabatic turbine with an exhaust pressure of 150 kPa. Determine the turbine inlet temperature and the turbine work assuming the turbine is reversible.

$$CH_4 + \nu_{O2}\left(O_2 + 3.76\ N_2\right) \rightarrow CO_2 + 2\ H_2O + 7.52\ N_2$$

$$2\ \nu_{O2} = 2 + 2\ \Rightarrow\ \nu_{O2} = 2 \quad => \quad \text{Actual } \nu_{O2} = 2 \times 2.5 = 5$$

$$CH_4 + 5\ O_2 + 18.8\ N_2 \rightarrow CO_2 + 2\ H_2O + 3\ O_2 + 18.8\ N_2$$

C.V. combustor and mixing chamber

$$H_R + n_{H2O}\bar{h}_{H2O\ in} = H_{P\ ex}$$

$$n_{H2O} = \frac{\dot{n}_{H2O}}{\dot{n}_{Fu}} = \frac{\dot{m}_{H2O}M_{Fu}}{\dot{m}_{Fu}M_{H2O}} = \frac{2.5 \times 16.043}{0.3 \times 18.015} = 7.421\ \frac{\text{kmol steam}}{\text{kmol fuel}}$$

Energy equation becomes

$$n_{H2O}\left(\bar{h}_{ex} - \bar{h}_{in}\right)_{H2O} + \left(\Delta\bar{h}_{CO2} + 2\Delta\bar{h}_{H2O} + 3\Delta\bar{h}_{O2} + 18.8\Delta\bar{h}_{N2}\right)_{ex}$$

$$= -\overset{o}{H}_{RP} = 50\ 010 \times 16.043 = 802\ 310$$

$$\left(\bar{h}_{ex} - \bar{h}_{in}\right)_{H2O} = \Delta\bar{h}_{H2O\ ex} - 15072.5, \text{ so then:}$$

$$\left(\Delta\bar{h}_{CO2} + 9.421\Delta\bar{h}_{H2O} + 3\Delta\bar{h}_{O2} + 18.8\Delta\bar{h}_{N2}\right)_{ex} = 914\ 163\ \frac{\text{kJ}}{\text{kmol fuel}}$$

Trial and error on T_{ex}

$T_{ex} = 1000$ K $\Rightarrow$ LHS = 749 956 ; $T_{ex} = 1100$ K $\Rightarrow$ LHS = 867429

$T_{ex} = 1200$ K $\Rightarrow$ LHS = 987 286 $\Rightarrow$ $T_{ex} \cong$ **1139 K =** $\mathbf{T_{in\ turbine}}$

If air then $T_{ex\ turbine} \approx 700$ K and $T_{avg} \approx 920$ K. Find $\bar{C}_{P\ mix}$ between 900 and 1000 K. From Table A.9:

$$\bar{C}_{P\ mix} = \sum n_i\bar{C}_{P_i}/\sum n_i = \frac{53.67 + 9.421(40.63) + 3(34.62) + 18.8(32.4)}{32.221}$$

$$= 35.673 \text{ kJ/kmol K}$$

$$\bar{C}_{V\ mix} = \bar{C}_{P\ mix} - \bar{R} = 27.3587 \text{ kJ/kmol}, \quad k_{mix} = 1.304$$

$$T_{ex\ turbine} = 1139\ (150 / 1000)^{0.2331} = 732 \text{ K}$$

$\Delta H_{732} = 19370.6 + 9.421(15410) + 3(13567) + 18.8(12932)$

$= 448\ 371\ \text{kJ/kmol}$

$w_T = H_{in} - H_{ex} = \Delta H_{in} - \Delta H_{ex} = 914\ 163 - 448\ 371 = 465\ 792\ \frac{\text{kJ}}{\text{kmol fuel}}$

$\dot{W}_T = \dot{n}_{Fu} w_T = \dot{m}_{Fu} w_T / \hat{M}_{Fu} = (0.3 \times 465\ 792)16.043 =$ **8710 kW**

15.145

The turbine in Problem 15.139 is adiabatic. Is it reversible, irreversible, or impossible?

Inlet to the turbine is the exit from the mixing of air and fuel at 1 MPa. From solution to 15.139, we have:

$\bar{C}_{P\,C_2H_2} = 43.43$, $\bar{C}_{P\,C_3H_8} = 74.06$, $T_{turbine,in} = 548.7$ K

$$C_2H_4 + C_3H_8 + 9.6\ O_2 + 36.096\ N_2$$
$$\rightarrow 5\ CO_2 + 6\ H_2O + 1.6\ O_2 + 36.096\ N_2$$

$$S_{ex} - S_{in} = \int dQ/T + S_{gen} = S_{gen} \geq \phi$$

Inlet: 1 MPa, 548.7 K $\qquad \bar{S}_{Fu} = \bar{s}^{o}_{i} + \bar{C}_{P\,Fu} \ln\left(T/T_0\right)$

	n_i	y_i	$\bar{s}^{o}_{i}$	$-\bar{R}\ln\frac{y_iP}{P_0}$	$\bar{S}_i$
C_2H_4	1	0.02097	245.82	12.989	258.809
C_3H_8	1	0.02097	315.09	12.989	328.079
O_2	9.6	0.2013	223.497	-5.816	217.681
N_2	36.096	0.7568	209.388	-16.828	192.56

$$S_{in} = 258.809 + 328.079 + 9.6 \times 217.681 + 36.096 \times 192.56 = 9627.3$$

	n_i	y_i	$\bar{s}^{o}_{i}$	$-\bar{R}\ln\frac{y_iP}{P_0}$	S_i
CO_2	5	0.1027	257.496	18.925	276.421
H_2O	6	0.1232	223.826	17.409	241.235
O_2	1.6	0.0329	235.92	28.399	264.319
N_2	36.096	0.7413	221.016	2.489	223.505

$$S_{ex} = 5 \times 276.421 + 6 \times 241.235\ 1.6 \times 264.319$$
$$+ 36.096 \times 223.505 = 11320 \frac{\text{kJ}}{\text{2kmol Fu K}}$$

$$S_{gen} = S_{ex} - S_{in} = \mathbf{1693} \frac{\textbf{kJ}}{\textbf{2kmol Fu K}} \mathbf{> 0}$$

Possible, but one should check the state after combustion to account for generation by combustion alone and then the turbine expansion separately.

15.146

Consider the combustion process described in Problem 15.107.

a. Calculate the absolute entropy of the fuel mixture before it is throttled into the combustion chamber.

b. Calculate the irreversibility for the overall process.

From solution to 15.107, fuel mixture 0.8 C_2H_6 + 0.2 CH_4 at 65°C, 10 MPa

$\bar{C}_{P0\ FUEL}$ = 49.718 kJ/kmol K. Using Kay's rule: T_{r1} = 1.198, P_{r1} = 2.073

and x = 410.4 % theoretical air

or 13.13 O_2 + 49.36 N_2 in at 600 K, 100 kPa

and $1.8CO_2 + 2.8H_2O + 9.93O_2 + 49.36N_2$ out at 100 kPa, 1200 K

$\bar{s}^*_{0\ FUEL} = 0.2(186.251) + 0.8(229.597) - 8.3145(0.2 \ln 0.2 + 0.8 \ln 0.8) =$ 225.088

$$\Delta s^*_{TP} = 49.718 \ln \frac{338.2}{298.2} - 8.3145 \ln \frac{10}{0.1} = -32.031$$

From Fig. D.3: $(\bar{s}^* - \bar{s})_{FUEL} = 1.37 \times 8.3145 = 11.391$

$\bar{s}_{FUEL}$ = 225.088 - 32.031 - 11.391 = **181.66 kJ/kmol K**

Air at 600 K, 100 kPa

	n_i	y_i	$\bar{s}^\circ_i$	$-\bar{R}\ln(y_iP/P_0)$	S_i
O_2	13.13	0.21	226.45	+12.976	239.426
N_2	49.36	0.79	212.177	+1.96	214.137

$S_{AIR} = \sum n_i\bar{S}_i$ = 13713.47 kJ/K

S_R = 181.66 + 13713.47 = 13895.1 kJ/K

Products at 1200 K, 100 kPa

PROD	n_i	y_i	$\bar{s}^o_i$	$-\bar{R}\ln(y_iP/P_0)$	S_i
CO_2	1.8	0.0282	279.390	+29.669	309.059
H_2O	2.8	0.0438	240.485	+26.008	266.493
O_2	9.93	0.1554	250.011	+15.479	265.490
N_2	49.36	0.7726	234.227	+2.145	236.372

$S_P = \sum n_i\bar{S}_i$ = 15606.1 kJ/K

$$I = T_0(S_P - S_R) - Q_{CV} = 298.15(15\ 606.1 - 13\ 895.1) + 0 = \mathbf{510\ 132\ kJ}$$

15.147

Consider one cylinder of a spark-ignition, internal-combustion engine. Before the compression stroke, the cylinder is filled with a mixture of air and methane. Assume that 110% theoretical air has been used, that the state before compression is 100 kPa, 25°C. The compression ratio of the engine is 9 to 1.

a. Determine the pressure and temperature after compression, assuming a reversible adiabatic process.

b. Assume that complete combustion takes place while the piston is at top dead center (at minimum volume) in an adiabatic process. Determine the temperature and pressure after combustion, and the increase in entropy during the combustion process.

c. What is the irreversibility for this process?

$$1\,CH_4 + 1.1 \times 2\,O_2 + 3.76 \times 2.2\,N_2 \rightarrow 1\,CO_2 + 2\,H_2O + 0.2\,O_2 + 2\,H_2O$$

$P_1 = 100$ kPa, $T_1 = 298.2$ K, $V_2/V_1 = 1/8$, Rev. Ad. $s_2 = s_1$

Assume $T_2 \sim 650\text{ K} \rightarrow T_{AVE} \sim 475$ K

Table A.6: $\bar{C}_{P0\,CH4} = 44.887$, $\bar{C}_{P0\,O2} = 30.890$, $\bar{C}_{P0\,N2} = 29.415$

$\bar{C}_{P0\,MIX} = (1 \times 44.887 + 2.2 \times 30.890 + 8.27 \times 29.415)/11.47 = 31.047$

$\bar{C}_{V0\,MIX} = \bar{C}_{P0} - \bar{R} = 22.732$, $k = \bar{C}_{P0}/\bar{C}_{V0} = 1.366$

a) $T_2 = T_1(V_1/V_2)^{k-1} = 298.2\,(9)^{0.366} = 666.4$ K (avg OK)

$P_2 = P_1(V_1/V_2)^{k} = 100\,(9)^{1.366} =$ **2011 kPa**

b) comb. 2-3 const. vol., Q = 0

$${}_2Q_3 = 0 = (H_3 - H_2) - \bar{R}(n_3T_3 - n_2T_2)$$

$$H_2 = 1\,\bar{h}^0_{f\,CH4} + n_2\,\bar{C}_{P0\,MIX}(T_2 - T_1)$$

$$H_2 = -74873 + 11.47 \times 31.047(666.4 - 298.2) = +56246\text{ kJ}$$

$$H_3 = 1(-393522 + \Delta\bar{h}^*_{CO2}) + 2(-241826 + \Delta\bar{h}^*_{H2O}) + 0.2\,\Delta\bar{h}^*_{O2} + 8.27\,\Delta\bar{h}^*_{N2}$$

Substituting,

$$1\,\Delta\bar{h}^*_{CO2} + 2\,\Delta\bar{h}^*_{H2O} + 0.2\,\Delta\bar{h}^*_{O2} + 8.27\,\Delta\bar{h}^*_{N2} - 95.366\,T_3 - 869868 = 0$$

Trial & error: $T_3 =$ **2907 K**

$1 \times 147072 + 2 \times 121377 + 0.2 \times 94315 + 8.27 \times 89274$

$- 95.366 \times 2907 - 869868 \approx 0$ **OK**

$$P_3 = P_2 \frac{n_3T_3}{n_2T_2} = P_2 \frac{T_3}{T_2} = 2011 \times \frac{2907}{666.4} = \mathbf{8772\ kPa}$$

c) state 1

REAC	n_i	y_i	$\bar{s}_i^\circ$	$- \bar{R} \ln(y_iP/P_0)$	S_i
CH_4	1	0.0872	186.251	+20.283	206.534
O_2	2.2	0.1918	205.148	+13.730	218.878
N_2	8.27	0.7210	191.609	+2.720	194.329
	11.47				

$S_2 = S_1 = \sum n_i\bar{S}_i = 2295.17$ kJ/K

state 3

PROD	n_i	y_i	$\bar{s}_i^\circ$	$- \bar{R} \ln(y_iP/P_0)$	S_i
CO_2	1	0.0872	332.213	-16.916	315.297
H_2O	2	0.1744	284.753	-22.680	262.073
O_2	0.2	0.0174	283.213	-3.516	279.697
N_2	8.27	0.7210	265.726	-34.480	231.246
	11.47				

$S_3 = \sum n_i\bar{S}_i = 2807.79$ kJ/K

$S_3 - S_2 = 2807.79 – 2295.17 = 512.62$ kJ/K

$I = T_0(S_3 - S_2) = 298.2 \times 512.62 =$ **152 860 kJ**

15.148

Liquid acetylene, C_2H_2, is stored in a high-pressure storage tank at ambient temperature, 25°C. The liquid is fed to an insulated combustor/steam boiler at the steady rate of 1 kg/s, along with 140% theoretical oxygen, O_2, which enters at 500 K, as shown in Fig. P15.148. The combustion products exit the unit at 500 kPa, 350 K. Liquid water enters the boiler at 10°C, at the rate of 15 kg/s, and superheated steam exits at 200 kPa.

a. Calculate the absolute entropy, per kmol, of liquid acetylene at the storage tank state.

b. Determine the phase(s) of the combustion products exiting the combustor boiler unit, and the amount of each, if more than one.

c. Determine the temperature of the steam at the boiler exit.

a) C_2H_2: $\bar{s}^{\circ}_{IG\ 25°C} = 200.958$

$T_{R_1} = 298.2/308.3 = 0.967 \;\Rightarrow\;$ From Fig. D.1: $\quad P_{R_1} = 0.82$

$P_1 = 0.82 \times 6.14 = 5.03$ MPa, $\quad (\bar{s}^* - \bar{s})_1 = 3.33\bar{R} = 27.687$

$$\bar{s}_{liq\,T_1\,P_1} = \bar{s}^{\circ}_{T_0\,P_0} + \Delta T - \bar{R}\ln(P_1/P^{\circ}) + (\bar{s} - \bar{s}^*)_{P_1\,T_1} = \mathbf{140.695}\ \mathbf{\frac{kJ}{kmol\ K}}$$

b) $1\ C_2H_2 + 1.4 \times 2.5\ O_2 \rightarrow 2\ CO_2 + 1\ H_2O + 1\ O_2$

$H_1 = 226731 + (-3.56 \times \bar{R} \times 308.3) = 217605$ kJ

$H_2 = 3.5(0 + 6086) = 21301$ kJ

Products $T_3 = 350\ K = 76.8°C \;\Rightarrow\; P_G = 41.8$ kPa

$$y_{V\,max} = \frac{P_G}{P} = \frac{41.8}{500} = 0.0836 = \frac{n_{V\,max}}{n_{V\,max} + 2 + 1} \;\Rightarrow\; n_{V\,max} = 0.2737 = n_{V\ gas\ mix}$$

$\Rightarrow n_{liq} = 1 - 0.2737 = \mathbf{0.7263}$

Gas Mix = $\mathbf{2\ CO_2 + 0.2737\ H_2O + 1\ O_2}$

c) $H_{liq3} = 0.7263(-285830 + 18.015(321.5 - 104.9)) = -204764$ kJ

$H_{gas\ mix3} = 2(-393522 + 2036) + 0.2737(-241826 + 1756) + 1541$

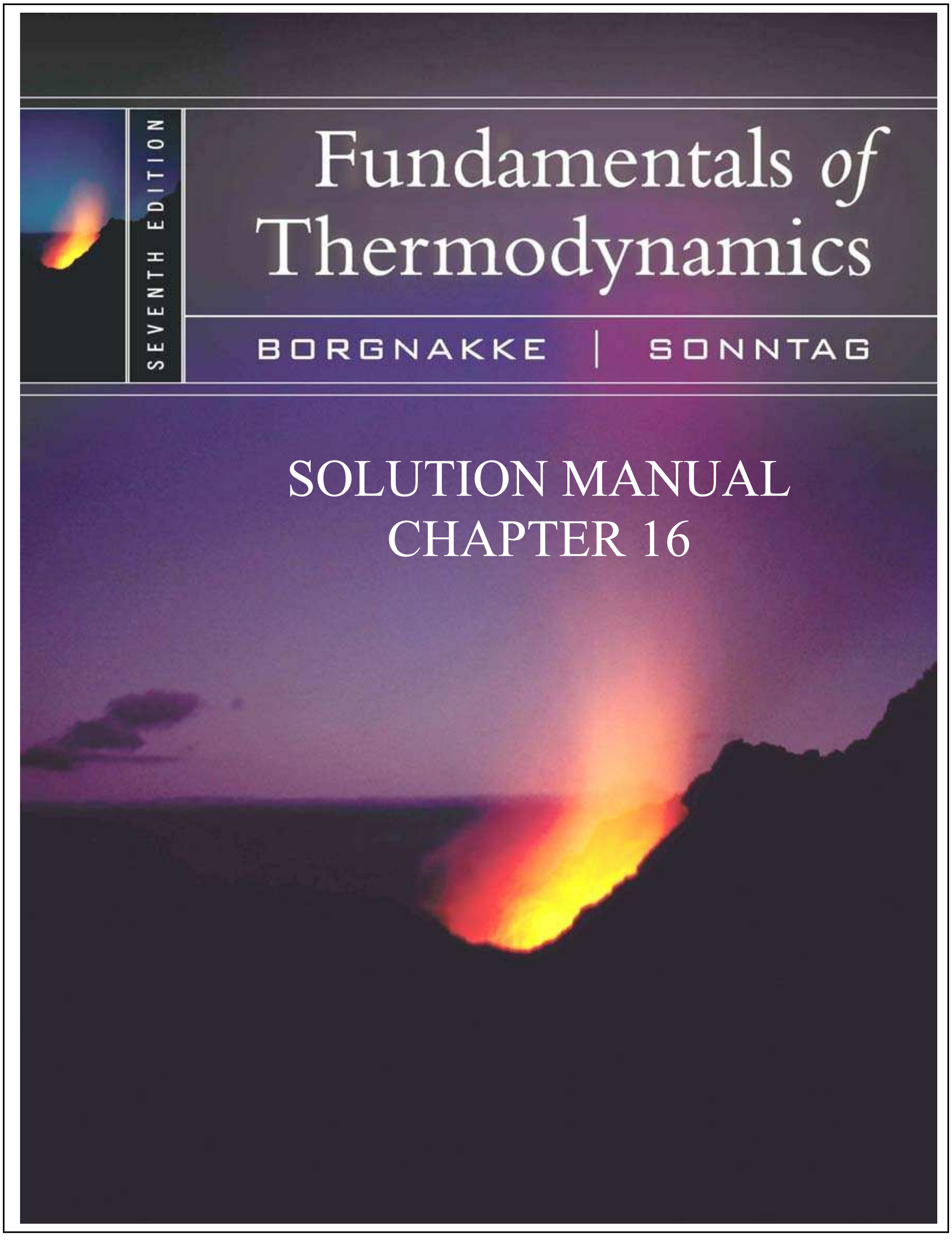
$= -847138$ kJ

$H_3 = H_{liq3} + H_{gas\ mix3} = -204\,764 - 847\,138 = -1\,051\,902$ kJ

$H_3 - H_1 - H_2 = -1\,290\,808$ kJ

or $\dot{H}_3 - \dot{H}_1 - \dot{H}_2 = -1\,290\,808/26.038 = -49\,574\ kW = \dot{m}_{H_2O}(h_4 - h_5)$

$$h_5 = 42.01 + \frac{49574}{15} = 3346.9 \quad \Rightarrow \quad T_5 = \mathbf{433.4°C}$$



SEVENTH EDITION

Fundamentals *of* Thermodynamics

BORGNAKKE | SONNTAG

SOLUTION MANUAL
CHAPTER 16

CONTENT

In-Text Concept Questions

16.a

For a mixture of O_2 and O the pressure is increased at constant T; what happens to the composition?

An increase in pressure causes the reaction to go toward the side of smaller total number of moles, in this case toward the O_2 .

16.b

For a mixture of O_2 and O the temperature is increased at constant P; what happens to the composition?

A temperature increase causes more O_2 to dissociate to O.

16.c

For a mixture of O_2 and O I add some argon keeping constant T, P; what happens to the moles of O?

Diluting the mixture with a non-reacting gas has the same effect as decreasing the pressure, causing the reaction to shift toward the side of larger total number of moles, in this case the O.

16.d

When dissociations occur after combustion, does T go up or down?

Dissociation reactions of combustion products lower the temperature.

16.e

For nearly all the dissociations and ionization reactions what happens with the composition when the pressure is raised?

The reactions move towards the side with fewer moles of particles, that is ions and electrons towards the monatomic side and monatomic species combine to form the molecules (diatomic or more)

16.f

How does the time scale for NO formation change when P is lower at same T?

Look at the expression for the time scale in Eq.16.40. As P is lowered the time scale becomes larger. The formation rates drops, and the effect is explained by the larger distance between the molecules/atoms (density lower), the same T essentially means that they have the same characteristic velocity.

16.g

Which atom in air ionizes first as T increases? What is the explanation?

Using Fig. 16.11, we note that as temperature increases, atomic N ionizes to N^+, becoming significant at about 6-8000 K. N has a lower ionization potential compared to O or Ar.

Concept-Study Guide Problems

16.1

Is the concept of equilibrium limited to thermodynamics?

Equilibrium is a condition in which the driving forces present are balanced, with no tendency for a change to occur spontaneously. This concept applies to many diverse fields of study – one no doubt familiar to the student being that of mechanical equilibrium in statics, or engineering mechanics.

16.2

How does Gibbs function vary with quality as you move from liquid to vapor?

There is no change in Gibbs function between liquid and vapor. For equilibrium we have $g_g = g_f$.

16.3

How is a chemical equilibrium process different from a combustion process?

Chemical equilibrium occurs at a given state, T and P, following a chemical reaction process, possibly a combustion followed by one or more dissociation reactions within the combustion products. Whereas the combustion is a one-way process (irreversible) the chemical equilibrium is a reversible process that can proceed in both directions.

16.4

Must P and T be held fixed to obtain chemical equilibrium?

No, but we commonly evaluate the condition of chemical equilibrium at a state corresponding to a given temperature and pressure. If T and P changes in a process it means that the chemical composition adjusts towards equilibrium and the composition changes along with the process.

16.5

The change in Gibbs function ΔG^o for a reaction is a function of which property?

The change in Gibbs function for a reaction ΔG is a function of T and P. The change in standard-state Gibbs function ΔG^o is a function only of T.

16.6

In a steady flow burner T is not controlled, which properties are?

The pressure tends to be constant, only minor pressure changes due to acceleration of the products as density decreases velocity must increase to have the same mass flow rate. The product temperature depends on heat losses (radiation etc.) and any chemical reactions that may take place generally lowering the temperature below the standard adiabatic flame temperature.

16.7

In a closed rigid combustion bomb which properties are held fixed?

The volume is constant. The number of atoms of each element is conserved, although the amounts of various chemical species change. As the products have more internal energy but cannot expand the pressure increases significantly.

16.8

Is the dissociation of water pressure sensitive?

Yes, since the total number of moles on the left and right sides of the reaction equation(s) is not the same.

16.9

At 298 K, K = exp(−184) for the water dissociation, what does that imply?

This is an extremely small number, meaning that the reaction tends to go strongly from right to left – in other words, does not tend to go from left to right (dissociation of water) at all.

16.10

If a reaction is insensitive to pressure prove that it is also insensitive to dilution effects at a given T.

Assume the standard reaction we used to develop the expression for the equilibrium constant: $\nu_A\, A + \nu_B\, B \;\;\Leftrightarrow\;\; \nu_C\, C + \nu_D\, D$

let us assume we add an inert component E so the total moles become:

$$n_{tot} = n_A + n_B + n_C + n_D + n_E$$

This will now lower the mole fractions of A, B, C, and D. If the reaction is pressure insensitive then: $\nu_A + \nu_B = \nu_C + \nu_D$ and the equilibrium equation becomes:

$$K = \frac{y_C^{\nu_C}\, y_D^{\nu_D}}{y_A^{\nu_A}\, y_B^{\nu_B}} \left(\frac{P}{P^o}\right)^{\nu_C + \nu_D - \nu_A - \nu_B} = \frac{y_C^{\nu_C}\, y_D^{\nu_D}}{y_A^{\nu_A}\, y_B^{\nu_B}}$$

Since each $y_i = n_i / n_{tot}$ we get:

$$K = \frac{y_C^{\nu_C}\, y_D^{\nu_D}}{y_A^{\nu_A}\, y_B^{\nu_B}} = \frac{(n_C/n_{tot})^{\nu_C}\, (n_D/n_{tot})^{\nu_D}}{(n_A/n_{tot})^{\nu_A}\, (n_B/n_{tot})^{\nu_B}}$$

$$= \frac{n_C^{\nu_C}\, n_D^{\nu_D}}{n_A^{\nu_A}\, n_B^{\nu_B}}\; n_{tot}^{\;\nu_A + \nu_B - \nu_C - \nu_D} = \frac{n_C^{\nu_C}\, n_D^{\nu_D}}{n_A^{\nu_A}\, n_B^{\nu_B}}$$

Now we see that the total number of moles that includes n_E does not enter the equation and thus will not affect any progress of the reaction.

16.11

For a pressure sensitive reaction an inert gas is added (dilution), how does the reaction shift?

Assume the standard reaction we used to develop the expression for the equilibrium constant: $\nu_A\,A + \nu_B\,B \;\Leftrightarrow\; \nu_C\,C + \nu_D\,D$

let us assume we add an inert component E so the total moles become:

$$n_{tot} = n_A + n_B + n_C + n_D + n_E$$

This will now lower the mole fractions of A, B, C, and D. If the reaction is pressure sensitive then: $\nu_A + \nu_B \neq \nu_C + \nu_D$ and the equilibrium equation becomes:

$$K = \frac{y_C^{\nu_C}\, y_D^{\nu_D}}{y_A^{\nu_A}\, y_B^{\nu_B}} \left(\frac{P}{P^o}\right)^{\nu_C + \nu_D - \nu_A - \nu_B}$$

Since each $y_i = n_i / n_{tot}$ we get:

$$K = \frac{(n_C/n_{tot})^{\nu_C}\, (n_D/n_{tot})^{\nu_D}}{(n_A/n_{tot})^{\nu_A}\, (n_B/n_{tot})^{\nu_B}} \left(\frac{P}{P^o}\right)^{\nu_C + \nu_D - \nu_A - \nu_B}$$

$$= \frac{n_C^{\nu_C}\, n_D^{\nu_D}}{n_A^{\nu_A}\, n_B^{\nu_B}} \left(\frac{P}{P^o\, n_{tot}}\right)^{\nu_C + \nu_D - \nu_A - \nu_B}$$

As n_{tot} is raised due to n_E it acts as if the pressure P is lowered thus pushing the reaction towards the side with a larger number of moles.

16.12

In a combustion process is the adiabatic flame temperature affected by reactions?

The adiabatic flame temperature is lower due to dissociation reactions of the products and influenced by other reactions like the water gas reaction.

16.13

In equilibrium Gibbs function of the reactants and the products is the same; how about the energy?

The chemical equilibrium mixture at a given T, P has a certain total internal energy. There is no restriction on its division among the constituents. The conservation of energy from the reactants to the products will determine the temperature so if it takes place in a fixed volume (combustion bomb) then U is constant whereas if it is in a flow (like a steady flow burner) then H is constant. When this is combined with chemical equilibrium it is actually a lengthy procedure to determine both the composition and the temperature in the actual process.

16.14

Does a dissociation process require energy or does it give out energy?

Dissociation reactions require energy and is thus endothermic. Notice from Table A.9 that all the atoms (N, O, H) has a much higher formation enthalpy than the diatomic molecules (which have formation enthalpy equal to zero).

16.15

If I consider the non-frozen (composition can vary) heat capacity, but still assume all components are ideal gases, does that C become a function of temperature? of pressure?

The non-frozen mixture heat capacity will be a function of both T and P, because the mixture composition depends on T and P, while the individual component heat capacities depend only on T.

16.16

What is K for the water gas reaction in Example 16.4 at 1200 K?

Using the result of Example 16.4 and Table A.11

$$\ln K = \frac{1}{2}\,(\ \ln K_I - \ln K_{II}\ )$$
$$= 0.5\ [-35.736 - (-36.363)]\ = +\ 0.3135\ ,\quad K = 1.3682$$

16.17

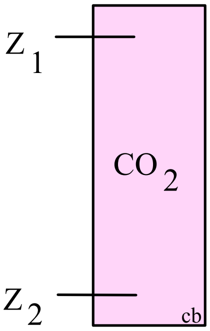
What would happen to the concentrations of the monatomic species like O, N if the pressure is higher in Fig. 16.11

Since those reaction are pressure sensitive (more moles on RHS than on LHS) the higher pressure will push these reactions to the left and reduce the concentrations of the monatomic species.

Equilibrium and Phase Equilibrium

16.18

Carbon dioxide at 15 MPa is injected into the top of a 5-km deep well in connection with an enhanced oil-recovery process. The fluid column standing in the well is at a uniform temperature of 40°C. What is the pressure at the bottom of the well assuming ideal gas behavior?



$(Z_1 - Z_2) = 5000$ m, $P_1 = 15$ MPa

$T = 40\ ^oC$ = constant

Equilibrium at constant T

$$-w_{REV} = 0 = \Delta g + \Delta PE$$

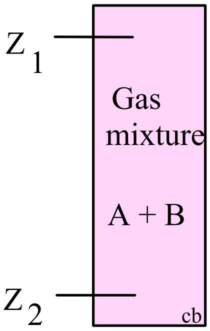
$$= RT \ln (P_2/P_1) + g(Z_2 - Z_1) = 0$$

$$\ln (P_2/P_1) = \frac{9.807\times 5000}{1000\times 0.188\ 92\times 313.2} = 0.8287$$

$$P_2 = 15 \exp(0.8287) = \mathbf{34.36\ MPa}$$

16.19

Consider a 2-km-deep gas well containing a gas mixture of methane and ethane at a uniform temperature of 30°C. The pressure at the top of the well is 14 MPa, and the composition on a mole basis is 90% methane, 10% ethane. Each component is in equilibrium (top to bottom) with dG + g dZ = 0 and assume ideal gas, so for each component Eq.16.10 applies. Determine the pressure and composition at the bottom of the well.



$(Z_1 - Z_2) = 2000$ m, Let $A = CH_4$, $B = C_2H_6$

$P_1 = 14$ MPa, $y_{A1} = 0.90$, $y_{B1} = 0.10$

$T = 30\ ^oC$ = constant

From section 16.1, for A to be at equilibrium between

1 and 2: $W_{REV} = 0 = n_A(\bar{G}_{A1} - \bar{G}_{A2}) + n_A M_A g (Z_1 - Z_2)$

Similarly, for B: $W_{REV} = 0 = n_B(\bar{G}_{B1} - \bar{G}_{B2}) + n_B M_B g (Z_1 - Z_2)$

Using eq. 16.10 for A: $\bar{R}T \ln (P_{A2}/P_{A1}) = M_A g (Z_1 - Z_2)$

with a similar expression for B. Now, ideal gas mixture, $P_{A1} = y_{A1}P$, etc.

Substituting: $\ln\frac{y_{A2}P_2}{y_{A1}P_1} = \frac{M_A g(Z_1\text{-}Z_2)}{\bar{R}T}$ and $\ln\frac{y_{B2}P_2}{y_{B1}P_1} = \frac{M_B g(Z_1\text{-}Z_2)}{\bar{R}T}$

$$\ln (y_{A2} P_2) = \ln(0.9\times14) + \frac{16.04\times9.807(2000)}{1000\times8.3145\times303.2} = 2.6585$$

$\Rightarrow\ y_{A2} P_2 = 14.2748$

$$\ln (y_{B2}P_2) = \ln(0.1\times14) + \frac{30.07\times9.807(2000)}{1000\times8.3145\times303.2} = 0.570\ 43$$

$\Rightarrow\ y_{B2} P_2 = (1 - y_{A2}) P_2 = 1.76903$

Solving: $P_2 =$ **16.044 MPa** & $y_{A2} =$ **0.8897**

16.20

A container has liquid water at 20°C , 100 kPa in equilibrium with a mixture of water vapor and dry air also at 20°C, 100 kPa. How much is the water vapor pressure and what is the saturated water vapor pressure?

From the steam tables we have for saturated liquid:

$$P_g = \mathbf{2.339\ kPa}, \quad v_f = 0.001002\ \text{m}^3/\text{kg}$$

The liquid is at 100 kPa so it is compressed liquid still at 20°C so from Eq.14.15

$$g_{liq} - g_f = \int v\ dP = v_f\,(P - P_g)$$

The vapor in the moist air is at the partial pressure P_v also at 20°C so we assume ideal gas for the vapor

$$g_{vap} - g_g = \int v\ dP = RT \ln \frac{P_v}{P_g}$$

We have the two saturated phases so $g_f = g_g$ ($q = h_{fg} = Ts_{fg}$) and now for equilibrium the two Gibbs function must be the same as

$$g_{vap} = g_{liq} = RT \ln \frac{P_v}{P_g} + g_g = v_f\,(P - P_g) + g_f$$

leaving us with

$$\ln \frac{P_v}{P_g} = v_f\,(P - P_g)/\,RT = \frac{0.001002\ (100 - 2.339)}{0.4615 \times 293.15} = 0.000723$$

$$P_v = P_g\ \exp(0.000723) = \mathbf{2.3407\ kPa}.$$

This is only a minute amount above the saturation pressure. For the moist air applications in Chapter 13 we neglected such differences and assumed the partial water vapor pressure at equilibrium (100% relative humidity) is P_g. The pressure has to be much higher for this to be a significant difference.

16.21

Using the same assumptions as those in developing Eq. d in Example 16.1, develop an expression for pressure at the bottom of a deep column of liquid in terms of the isothermal compressibility, β_T. For liquid water at 20^{o}C, $\beta_T = 0.0005$ [1/MPa]. Use the result of the first question to estimate the pressure in the Pacific ocean at the depth of 3 km.

$$d\, g_T = v° \,(1 - \beta_T P)\, dP_T \qquad\qquad d\, g_T + g\, dz = 0$$

$$v° \,(1 - \beta_T P)\, dP_T + g\, dz = 0 \quad \text{and integrate} \quad \int v°(1-\beta_T P)\, dP_T = - g \int dz$$

$$\int_{P_0}^{P} (1-\beta_T P)\, dP_T = + \frac{g}{v°} \int_{0}^{+H} dz \quad => \qquad P - P_0 - \beta_T \frac{1}{2} [P^2 - P_0^{\ 2}] = \frac{g}{v°} H$$

$$P\,(1 - \frac{1}{2}\beta_T\, P) = P_0 - \frac{1}{2}\beta_T\, P_0^{\ 2} + \frac{g}{v°} H$$

$v° = v_{f\,20°C} = 0.001002$; $H = 3000$ m, $g = 9.80665\ m/s^2$; $\beta_T = 0.0005$ 1/MPa

$$P\,(1 - \frac{1}{2} \times 0.0005P) = 0.101 - \frac{1}{2} \times 0.0005 \times 0.101^2$$

$$+ [9.80665 \times 3000/0.001002] \times 10^{-6}$$

$$= 29.462 \text{ MPa}, \quad \text{which is close to P}$$

Solve by iteration or solve the quadratic equation

P = **29.682 MPa**

Chemical Equilibrium, Equilibrium Constant

16.22

Which of the reactions listed in Table A.11 are pressure sensitive?

Check if: $\nu_A + \nu_B \neq \nu_C + \nu_D$

Reaction	Check	P sensitive?
$H_2 \Leftrightarrow 2H$	$1 < 2$	yes
$O_2 \Leftrightarrow 2O$	$1 < 2$	yes
$N_2 \Leftrightarrow 2N$	$1 < 2$	yes
$2\,H_2O \Leftrightarrow 2\,H_2 + 1\,O_2$	$2 < 3$	yes
$2\,H_2O \Leftrightarrow 2\,H_2 + 2\,OH$	$2 < 4$	yes
$2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$	$2 < 3$	yes
$N_2 + O_2 \Leftrightarrow 2\,NO$	$2 = 2$	no
$N_2 + 2O_2 \Leftrightarrow 2\,NO_2$	$3 > 2$	yes

Most of them have more moles on RHS and thus will move towards the RHS if the pressure is lowered. Only the last one has the opposite and will move towards the LHS if the pressure is lowered.

16.23

Calculate the equilibrium constant for the reaction $O_2 \Leftrightarrow 2O$ at temperatures of 298 K and 6000 K. Verify the result with Table A.11.

Reaction $O_2 \quad \Leftrightarrow \quad 2O$

At 25 °C (298.15 K):

$$\Delta H^0 = 2\bar{h}^0_{f\,O} - 1\bar{h}^0_{f\,O2} = 2(249\ 170) - 1(0) = 498\ 340 \text{ kJ/kmol}$$

$$\Delta S^0 = 2\bar{s}^0_{O} - 1\bar{s}^0_{O2} = 2(161.059) - 1(205.148) = 116.97 \text{ kJ/kmol K}$$

$$\Delta G^0 = \Delta H^0 - T\Delta S^0 = 498\ 340 - 298.15\times116.97 = 463\ 465 \text{ kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = -\frac{463\ 465}{8.3145\times298.15} = \mathbf{-186.961}$$

At 6000 K:

$$\Delta H^0 = 2(249\ 170 + 121\ 264) - (0 + 224\ 210) = 516\ 658 \text{ kJ/kmol}$$

$$\Delta S^0 = 2(224.597) - 1(313.457) = 135.737 \text{ kJ/kmol K}$$

$$\Delta G^0 = 516\ 658 - 6000\times135.737 = -297\ 764 \text{ kJ/kmol}$$

$$\ln K = \frac{+297\ 764}{8.3145\times6000} = \mathbf{+5.969}$$

16.24

Calculate the equilibrium constant for the reaction $H_2 \Leftrightarrow 2H$ at a temperature of 2000 K, using properties from Table A.9. Compare the result with the value listed in Table A.11.

From Table A.9 at 2000 K we find:

$\Delta\bar{h}_{H_2} = 52\ 942$ kJ/kmol; $\bar{s}_{H_2} = 188.419$ kJ/kmol K; $\bar{h}_f^o = 0$

$\Delta\bar{h}_H = 35\ 375$ kJ/kmol; $\bar{s}_H = 154.279$ kJ/kmol K; $\bar{h}_f^o = 217\ 999$ kJ/kmol

$$\Delta G^0 = \Delta H - T\Delta S = H_{RHS} - H_{LHS} - T\,(S^0_{RHS} - S^0_{LHS})$$

$$= 2 \times (35\ 375 + 217\ 999) - 52943 - 2000(2\times 154.279 - 182.419)$$

$$= 213\ 528 \text{ kJ/kmol}$$

$$\ln K = -\Delta G^0/\bar{R}T = -213\ 528 / (8.3145 \times 2000) = -12.8407$$

Table A.11 ln K = −12.841 OK

16.25

For the dissociation of oxygen, $O_2 \Leftrightarrow 2O$, around 2000 K we want a mathematical expression for the equilibrium constant K(T). Assume constant heat capacity, at 2000 K, for O_2 and O from Table A.9 and develop the expression from Eqs. 16.12 and 16.15.

From Eq.16.15 the equilibrium constant is

$$K = \exp\left(- \frac{\Delta G^0}{\bar{R}T} \right) ; \qquad \Delta G^0 = \Delta H^0 - T\, \Delta S^0$$

and the shift is

$$\Delta G^0 = 2\, \bar{h}_O - \bar{h}_{O2} - T(2\bar{s}^o_O - \bar{s}^o_{O_2})$$

Substitute the first order approximation to the functions $\bar{h}$ and $\bar{s}^o$ as

$$\bar{h} = \bar{h}_{2000\text{ K}} + \bar{C}_p\,(T - 2000) \; ; \quad \bar{s}^o = \bar{s}^o_{2000\text{ K}} + \bar{C}_p \ln \frac{T}{2000}$$

The properties are from Table A.9 and $\bar{R}$ = 8.3145 kJ/kmol K

Oxygen O_2: $\bar{h}_{2000\text{ K}}$ = 59 176 kJ/kmol, $\bar{s}^o_{2000\text{ K}}$ = 268.748 kJ/kmol K

$$\bar{C}_p = \frac{\bar{h}_{2200\text{ K}} - \bar{h}_{2200\text{ K}}}{2200 - 1800} = \frac{66\,770 - 51\,674}{400} = 37.74 \text{ kJ/kmol K}$$

Oxygen O: $\bar{h}_{2000\text{ K}}$ = 35 713 + 249 170 = 284 883 kJ/kmol,

$\bar{s}^o_{2000\text{ K}}$ = 201.247 kJ/kmol K

$$\bar{C}_p = \frac{\bar{h}_{2200\text{ K}} - \bar{h}_{2200\text{ K}}}{2200 - 1800} = \frac{39\,878 - 31\,547}{400} = 20.8275 \text{ kJ/kmol K}$$

Substitute and collect terms

$$\frac{\Delta G^0}{\bar{R}T} = \frac{\Delta H^0}{\bar{R}T} - \frac{\Delta S^0}{\bar{R}} = \frac{\Delta H^0_{2000}}{\bar{R}T} + \frac{\Delta \bar{C}_{p\,2000}}{\bar{R}} \left[\frac{T - 2000}{T} - \ln \frac{T}{2000} \right] - \frac{\Delta S^0_{2000}}{\bar{R}}$$

Now we have

$$\Delta H^0_{2000}/\bar{R} = (2 \times 284\,883 - 59\,176)/8.3145 = 61\,409.6 \text{ K}$$

$$\Delta \bar{C}_{p\,2000}/\bar{R} = (2 \times 20.8275 - 37.74)/8.3145 = 0.470864$$

$$\Delta S^0_{2000}/\bar{R} = (2 \times 201.247 - 268.748)/8.3145 = 16.08587$$

so we get

$$\frac{\Delta G^0}{\bar{R}T} = \frac{61\,409.6}{T} + 0.470864 \left[\frac{T - 2000}{T} - \ln \frac{T}{2000} \right] - 16.08587$$

$$= \frac{60\,467.9}{T} - 15.615 - 0.470864 \ln \frac{T}{2000}$$

Now the equilibrium constant K(T) is approximated as

$$K(T) = \exp \left[15.615 - \frac{60\,467.9}{T} + 0.470864 \ln \frac{T}{2000} \right]$$

Remark: We could have chosen to expand the function $\Delta G^0 / \bar{R}T$ as a linear expression instead or even expand the whole $\exp(-\Delta G^0 / \bar{R}T)$ in a linear function.

16.26

Find K for: $CO_2 \Leftrightarrow CO + 1/2 O_2$ at 3000 K using A.11

The elementary reaction in A.11 is : $2CO_2 \Leftrightarrow 2CO + O_2$
so the wanted reaction is (1/2) times that so

$$K = K_{A.11}^{1/2} = \sqrt{\exp(-2.217)} = \sqrt{0.108935} = 0.33$$

or

$$\ln K = 0.5 \ln K_{A.11} = 0.5\ (-2.217) = -1.1085$$
$$K = \exp(-1.1085) = 0.33$$

16.27

Plot to scale the values of ln *K* versus $1/T$ for the reaction $2\ CO_2 \Leftrightarrow 2\ CO + O_2$. Write an equation for ln *K* as a function of temperature.

$$2\ CO_2 \Leftrightarrow 2\ CO + 1\ O_2$$

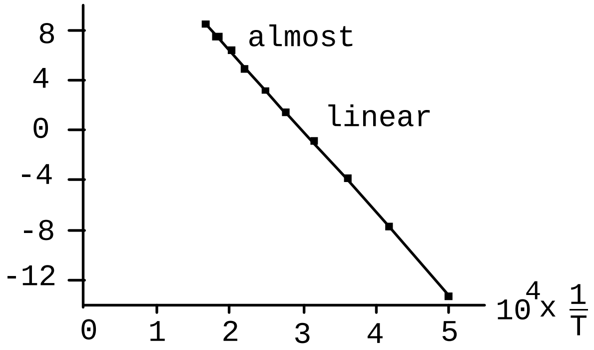
T(K)	$10^4 \times \frac{1}{T}$	ln K	T(K)	$10^4 \times \frac{1}{T}$	ln K
2000	5.000	-13.266	4000	2.500	3.204
2400	4.167	-7.715	4500	2.222	4.985
2800	3.571	-3.781	5000	2.000	6.397
3200	3.125	-0.853	5500	1.818	7.542
3600	2.778	1.408	6000	1.667	8.488

For the range
below ~ 5000 K,

$$\ln K \approx A + B/T$$

Using values at
2000 K & 5000 K

A = 19.5056
B = -65 543 K



16.28

Consider the reaction $2\ CO_2 \Leftrightarrow 2\ CO + O_2$ obtained after heating 1 kmol CO_2 to 3000 K. Find the equilibrium constant from the shift in Gibbs function and verify its value with the entry in Table A.11. What is the mole fraction of CO at 3000 K, 100 kPa?

From Table A.9 we get:

$\Delta\bar{h}_{CO} = 93\ 504$ $\quad \bar{h}^0_{f\ CO} = -110\ 527$ $\quad \bar{s}_{CO} = 273.607$

$\Delta\bar{h}_{CO_2} = 152\ 853$ $\quad \bar{h}^0_{f\ CO_2} = -393\ 522$ $\quad \bar{s}_{CO_2} = 334.17$

$\Delta\bar{h}_{O_2} = 98\ 013$ $\quad \bar{s}_{O_2} = 284.466$

$$\Delta G^0 = \Delta H - T\Delta S = 2\ H_{CO} + H_{O_2} - 2\ H_{CO_2} - T\ (2\bar{s}_{CO} + \bar{s}_{O_2} - 2\bar{s}_{CO_2})$$

$$= 2\ (93\ 504 - 110\ 527) + 98\ 013 + 0 - 2(152\ 853 - 393\ 522)$$

$$-3000(2\times273.607 + 284.466 - 2\times334.17) \quad = 55\ 285$$

$\ln K = -\Delta G^0/\bar{R}T = -55\ 285/\ (8.31451\times3000) = -2.2164$

Table A.11 $\ln K = -2.217$ OK

At 3000 K,

$\ln K = -2.217$

$K = 0.108935$

	$2\ CO_2$	$\Leftrightarrow 2\ CO$	$+ 1\ O_2$
Initial	1	0	0
Change	-2z	+2z	+z
Equil.	1-2z	2z	z

We have $P = P^o = 0.1$ MPa, and $n_{tot} = 1 + z$, so from Eq.15.29

$$K = \frac{y^2_{CO}y_{O_2}}{y^2_{CO_2}}\left(\frac{P}{P^0}\right) = \left(\frac{2z}{1 - 2z}\right)^2 \left(\frac{z}{1 + z}\right)(1) = 0.108935\ ;$$

$$4\ z^3 = 0.108935\ (1 - 2z)^2(1 + z) \quad \Rightarrow \quad z = 0.22$$

$y_{CO} = 2z\ /\ (1 + z) =$ **0.36**

16.29

Assume a diatomic gas like O_2 or N_2 dissociate at a pressure different from P_o. Find an expression for the fraction of the original gas that has dissociated at any T assuming equilibrium.

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\,O$$

Species	O_2	O
Initial	1	0
Change	$-x$	$2x$
Equil.	$1-x$	$2x$

$$n_{tot} = 1 - x + 2x = 1 + x$$

$$y_O = \frac{2x}{1+x} \quad \text{and} \quad y_{O2} = \frac{1-x}{1+x}$$

Substitute this into the equilibrium equation as

$$K_T = \frac{y_O^2}{y_{0_2}} \left(\frac{P}{P_o}\right)^{2-1} = \frac{4x^2}{(1+x)^2}\,\frac{1+x}{1-x}\left(\frac{P}{P_o}\right) = \frac{4x^2}{1-x^2}\left(\frac{P}{P_o}\right)$$

Now solve for x as

$$x^2 = (1-x^2)\ \frac{K_T P_o}{4P}$$

$$x = \sqrt{\frac{K_T}{4(P/P_o) + K_T}}$$

16.30

Consider the dissociation of oxygen, $O_2 \Leftrightarrow 2\ O$, starting with 1 kmol oxygen at 298 K and heating it at constant pressure 100 kPa. At which temperature will we reach a concentration of monatomic oxygen of 10%?

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\ O$$

Species	O_2	O
Initial	1	0
Change	-x	2x
Equil.	1-x	2x

$n_{tot} = 1 - x + 2x = 1 + x$

$$y_O = \frac{2x}{1 + x} = 0.1 \quad \Rightarrow \quad x = 0.1/(2 - 0.1) = 0.0526, \quad y_{O2} = 0.9$$

$$K = \frac{y_O^2}{y_{O_2}} \left(\frac{P}{P_o}\right)^{2-1} = \frac{0.1^2}{0.9}\ 1 = 0.01111 \quad \Rightarrow \quad \ln K = -4.4998$$

Now look in Table A.11: T = **2980 K**

16.31

Redo Problem 16.30, but start with 1 kmol oxygen and 1 kmol helium at 298 K, 100 kPa.

Look at initially 1 mol Oxygen and shift reaction with x

$$O_2 \Leftrightarrow 2\ O$$

Species	O_2	O	He	
Initial	1	0	1	
Change	-x	2x		
Equil.	1-x	2x	1	$n_{tot} = 1 - x + 2x + 1 = 2 + x$

$$y_O = \frac{2x}{2 + x} = 0.1 \quad \Rightarrow \quad x = 0.2/(2 - 0.1) = 0.10526,$$

$$y_{O2} = (1 - x)/(2 + x) = 0.425$$

$$K = \frac{y_O^2}{y_{O_2}} \left(\frac{P}{P_o}\right)^{2-1} = \frac{0.1^2}{0.425}\ 1 = 0.023529 \qquad \Rightarrow \ \ln K = -3.7495$$

Now look in Table A.11: T = **3094 K**

16.32

Calculate the equilibrium constant for the reaction:$2CO_2 \Leftrightarrow 2CO + O_2$ at 3000 K using values from Table A.9 and compare the result to Table A.11.

From Table A.9 we get:

kJ/kmol	kJ/kmol	kJ/kmol K
$\Delta\bar{h}_{CO} = 93\ 504$	$\bar{h}^o_{f\ CO} = -110\ 527$	$\bar{s}_{CO} = 273.607$
$\Delta\bar{h}_{CO_2} = 152\ 853$	$\bar{h}^o_{f\ CO_2} = -393\ 522$	$\bar{s}_{CO_2} = 334.17$
$\Delta\bar{h}_{O_2} = 98\ 013$	$\bar{h}^o_{f\ O_2} = 0$	$\bar{s}_{O_2} = 284.466$

$$\Delta G^0 = \Delta H - T\Delta S = 2\ H_{CO} + H_{O_2} - 2\ H_{CO_2} - T\ (2\bar{s}_{CO} + \bar{s}_{O_2} - 2\bar{s}_{CO_2})$$

$$= 2\ (93\ 504 - 110\ 527) + 98\ 013 + 0 - 2(152\ 853 - 393\ 522)$$

$$-3000(2\times273.607 + 284.466 - 2\times334.17) = 55\ 285\ \text{kJ/kmol}$$

$\ln K = -\Delta G^0/\bar{R}T = -55\ 285/\ (8.31451\times3000) = -2.2164$

Table A.11 $\ln K = -2.217$ OK

16.33

Hydrogen gas is heated from room temperature to 4000 K, 500 kPa, at which state the diatomic species has partially dissociated to the monatomic form. Determine the equilibrium composition at this state.

$H_2 \Leftrightarrow 2\ H$ Equil. $n_{H2} = 1 - x$

$-x \quad +2x$ $n_H = 0 + 2x$

$n = 1 + x$

$$K = \frac{(2x)^2}{(1-x)(1+x)} \left(\frac{P}{P^0}\right)^{2-1}$$ at 4000 K: $\ln K = 0.934 \Rightarrow K = 2.545$

$$\frac{2.545}{4\times(500/100)} = 0.127\ 25 = \frac{x^2}{1-x^2}$$ Solving, $x = 0.3360$

$n_{H2} = 0.664$, $n_H = 0.672$, $n_{tot} = 1.336$

$y_{H2} = 0.497$, $y_H = 0.503$

16.34

Pure oxygen is heated from 25°C to 3200 K in an steady flow process at a constant pressure of 200 kPa. Find the exit composition and the heat transfer.

The only reaction will be the dissociation of the oxygen

$O_2 \Leftrightarrow 2O$; From A.11: $K(3200) = \exp(-3.069) = 0.046467$

Look at initially 1 mol Oxygen and shift reaction with x

$$n_{O2} = 1 - x; \quad n_O = 2x; \quad n_{tot} = 1 + x; \quad y_i = n_i/n_{tot}$$

$$K = \frac{y_O^2}{y_{O_2}} \left(\frac{P}{P_o}\right)^{2-1} = \frac{4x^2}{(1 + x)^2} \frac{1 + x}{1 - x} 2 = \frac{8x^2}{1 - x^2}$$

$$x^2 = \frac{K/8}{1 + K/8} \quad \Rightarrow \quad x = 0.07599;$$

$$y_{O_2} = \frac{1 - x}{1 + x} = 0.859; \quad y_O = 1 - y_{O_2} = 0.141$$

$$\bar{q} = n_{O_2ex}\bar{h}_{O_2ex} + n_{Oex}\bar{h}_{Oex} - \bar{h}_{O_2in} = (1 + x)(y_{O_2}\bar{h}_{O_2} + y_O\bar{h}_O) - 0$$

$$\bar{h}_{O_2} = 106\ 022 \text{ kJ/kmol}; \quad \bar{h}_O = 249\ 170 + 60\ 767 = 309\ 937 \text{ kJ/kmol}$$

$$\Rightarrow \quad \bar{q} = 145\ 015 \text{ kJ/kmol } O_2$$

$$q = \bar{q}/32 = 4532 \text{ kJ/kg} \quad (= 3316.5 \text{ if no reaction})$$

16.35

Nitrogen gas, N_2, is heated to 4000 K, 10 kPa. What fraction of the N_2 is dissociated to N at this state?

$N_2 \iff 2\,N$ @ T = 4000 K, lnK = -12.671

Initial	1	0	$K = 3.14\text{x}10^{-6}$
Change	-x	2x	
Equil.	1-x	2x	$n_{tot} = 1 - x + 2x = 1 + x$

$$y_{N2} = \frac{1 - x}{1 + x}, \qquad y_N = \frac{2x}{1 + x}$$

$$K = \frac{y_N^2}{y_{N2}} \left(\frac{P}{P_o}\right)^{2-1}; \quad \Rightarrow \quad 3.14\text{x}10^{-6} = \frac{4x^2}{1 - x^2} \left(\frac{10}{100}\right) \quad \Rightarrow \quad x = 0.0028$$

$$y_{N2} = \frac{1 - x}{1 + x} = \mathbf{0.9944}, \qquad y_N = \frac{2x}{1 + x} = \mathbf{0.0056}$$

16.36

Find the equilibrium constant for: $CO + 1/2O_2 \Leftrightarrow CO_2$ at 2200 K using Table A.11

The elementary reaction in A.11 is: $2\ CO_2 \Leftrightarrow 2CO + O_2$

The wanted reaction is therefore (−0.5) times that and

$$K = K_{A.11}^{-1/2} = 1 / \sqrt{\exp(-10.232)} = 1 / \sqrt{0.000036} = 166.67$$

or

$$\ln K = -0.5 \ln K_{A.11} = -0.5\ (-10.232) = 5.116$$

$$K = \exp(5.116) = 166.67$$

16.37

Find the equilibrium constant for the reaction $2NO + O_2 \Leftrightarrow 2NO_2$ from the elementary reactions in Table A.11 to answer which of the nitrogen oxides, NO or NO_2, is the more stable at ambient conditions? What about at 2000 K?

$$2\,NO + O_2 \Leftrightarrow 2\,NO_2 \quad (1)$$

$$\text{But } N_2 + O_2 \Leftrightarrow 2\,NO \quad (2)$$

$$N_2 + 2\,O_2 \Leftrightarrow 2\,NO_2 \quad (3)$$

Reaction 1 = Reaction 3 - Reaction 2

$$\Rightarrow \quad \Delta G_1^0 = \Delta G_3^0 - \Delta G_2^0 \quad => \quad \ln K_1 = \ln K_3 - \ln K_2$$

At 25 °C, from Table A.11: $\ln K_1 = -41.355 - (-69.868) = +28.513$

or $K_1 = 2.416 \times 10^{12}$

an extremely large number, which means reaction 1 tends to go very strongly from left to right.

At 2000 K: $\ln K_1 = -19.136 - (-7.825) = -11.311$ or $K_1 = 1.224 \times 10^{-5}$

meaning that reaction 1 tends to go quite strongly from right to left.

16.38

One kilomole Ar and one kilomole O_2 are heated up at a constant pressure of 100 kPa to 3200 K, where it comes to equilibrium. Find the final mole fractions for Ar, O_2, and O.

The only equilibrium reaction listed in the book is dissociation of O_2.

So assuming that we find in Table A.10: $\ln(K) = -3.072$

$$Ar + O_2 \Rightarrow Ar + (1 - x)\, O_2 + 2x\, O$$

The atom balance already shown in above equation can also be done as

Species	Ar	O_2	O
Start	1	1	0
Change	0	-x	2x
Total	1	1-x	2x

The total number of moles is $n_{tot} = 1 + 1\text{-}x + 2x = 2 + x$ so

$$y_{Ar} = 1/(2 + x);\quad y_{O_2} = 1 - x/(2 + x);\quad y_O = 2x/(2 + x)$$

and the definition of the equilibrium constant ($P_{tot} = P_o$) becomes

$$K = e^{-3.072} = 0.04633 = \frac{y_O^2}{y_{0_2}} = \frac{4x^2}{(2 + x)(1 - x)}$$

The equation to solve becomes from the last expression

$$(K + 4)x^2 + Kx - 2K = 0$$

If that is solved we get

$$x = -0.0057 \pm 0.1514 = 0.1457; \qquad \text{x must be positive}$$

$$y_O = 0.1358;\quad y_{0_2} = 0.3981;\quad y_{Ar} = 0.4661$$

16.39

Air (assumed to be 79% nitrogen and 21% oxygen) is heated in a steady state process at a constant pressure of 100 kPa, and some NO is formed. At what temperature will the mole fraction of N.O be 0.001?

$0.79\ N_2 + 0.21\ O_2$ heated at 100 kPa, forms NO

$N_2 + O_2 \Leftrightarrow 2\,NO$

-x -x +2x

$n_{N2} = 0.79 - x$

$n_{O2} = 0.21 - x$

$n_{NO} = 0 + 2x$

$n_{tot} = 1.0$

At exit, $y_{NO} = 0.001 = \frac{2x}{1.0} \quad \Rightarrow \quad x = 0.0005$

$\Rightarrow n_{N2} = 0.7895,\ n_{O2} = 0.2095$

$$K = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{10^{-6}}{0.7895\times0.2095} = 6.046\times10^{-6} \qquad \text{or} \quad \ln K = -12.016$$

From Table A.10, T = **1444 K**

16.40

Assume the equilibrium mole fractions of oxygen and nitrogen are close to those in air, find the equilibrium mole fraction for NO at 3000 K, 500 kPa disregarding dissociations.

Assume the simple reaction to make NO as:

N_2 +	$O_2 \Leftrightarrow$	2 NO	Ar	$n_{N2} = 0.78 - x$
0.78	0.21	0	0.01	$n_{O2} = 0.21 - x$
-x	-x	+2x		$n_{NO} = 2x$, $n_{ar} = 0.01$
0.78–x	0.21–x	2x	0.01	$n_{tot} = 1.0$

From A.11 at 3000 K, $\ln K = -4.205$, $K = 0.014921$

$$K = \frac{4x^2}{(0.78 - x)(0.21 - x)} \left(\frac{P}{P^0}\right)^0 \quad \Rightarrow$$

$$\frac{x^2}{(0.78 - x)(0.21 - x)} = \frac{0.014921}{4} = 0.00373 \quad \text{and} \quad 0 < x < 0.21$$

Solve for x: $x = 0.0230 \quad \Rightarrow \quad y_{NO} = \frac{2x}{1.0} = \mathbf{0.046}$

16.41

The combustion products from burning pentane, C_5H_{12}, with pure oxygen in a stoichiometric ratio exists at 2400 K, 100 kPa. Consider the dissociation of only CO_2 and find the equilibrium mole fraction of CO.

$$C_5H_{12} + 8\ O_2 \rightarrow 5\ CO_2 + 6\ H_2O$$

At 2400 K,

$\ln K = -7.715$

$K = 4.461 \times 10^{-4}$

	$2\ CO_2 \Leftrightarrow$	$2\ CO +$	$1\ O_2$
Initial	5	0	0
Change	-2z	+2z	+z
Equil.	5-2z	2z	z

Assuming $P = P^o = 0.1$ MPa, and $n_{tot} = 5 + z + 6 = 11 + z$

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2} \left(\frac{P}{P^0}\right) = \left(\frac{2z}{5 - 2z}\right)^2 \left(\frac{z}{11 + z}\right) (1) = 4.461 \times 10^{-4} ;$$

Trial & Error (compute LHS for various values of z): $z = 0.291$

$n_{CO_2} = 4.418$; $n_{CO} = 0.582$; $n_{O_2} = 0.291$ => $y_{CO} =$ **0.0515**

16.42

Pure oxygen is heated from 25°C, 100 kPa to 3200 K in a constant volume container. Find the final pressure, composition, and the heat transfer.

As oxygen is heated it dissociates

$$O_2 \Leftrightarrow 2O \qquad \ln K_{eq} = -3.069 \quad \text{from table A.11}$$

C. V. Heater: $U_2 - U_1 = {}_1Q_2 = H_2 - H_1 - P_2v + P_1v$

Per mole O_2: $\quad {}_1\bar{q}_2 = \bar{h}_2 - \bar{h}_1 + \bar{R}[T_1 - (n_2/n_1)T_2]$

Shift x in reaction 1 to have final composition: $(1 - x)O_2 + 2xO$

$n_1 = 1 \qquad n_2 = 1 - x + 2x = 1 + x$

$y_{O_2} = (1 - x)/(1 + x)\ ; \quad y_O = 2x/(1 + x)$

Ideal gas and $V_2 = V_1 \Rightarrow P_2 = P_1n_2T_2/n_1T_1 \quad \Rightarrow \quad P_2/P_o = (1 + x)T_2/T_1$

Substitute the molefractions and the pressure into the equilibrium equation

$$K_{eq} = e^{-3.069} = \frac{y_O^2}{y_{O_2}}\left(\frac{P_2}{P_o}\right) = \left(\frac{2x}{1 + x}\right)^2 \left(\frac{1 + x}{1 - x}\right)\left(\frac{1 + x}{1}\right)\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \frac{4x^2}{1 - x} = \frac{T_1}{T_2}e^{-3.069} = 0.00433 \quad \Rightarrow \quad x = 0.0324$$

The final pressure is then

$$P_2 = P_o(1 + x)\frac{T_2}{T_1} = 100\ (1 + 0.0324) \times \frac{3200}{298.2} = \mathbf{1108\ kPa}$$

$(n_{O_2}) = 0.9676, \quad (n_O) = 0.0648, \quad n_2 = 1.0324$

$${}_1\bar{q}_2 = 0.9676 \times 106022 + 0.0648\ (249170 + 60767) - 0$$
$$+ 8.3145\ (298.15 - 1.0324 \times 3200) = \mathbf{97681\ kJ/kmolO_2}$$

$$y_{O_2} = \frac{0.9676}{1.0324} = \mathbf{0.937}; \qquad y_O = \frac{0.0648}{1.0324} = \mathbf{0.0628}$$

16.43

Combustion of stoichiometric benzene, C_6H_6, and air at 80 kPa with a slight heat loss gives a flame temperature of 2400 K. Consider the dissociation of CO_2 to CO and O_2 as the only equilibrium process possible. Find the fraction of the CO_2 that is dissociated.

Combustion: $C_5H_{12} + 7.5\ O_2 + 7.5 \times 3.76\ N_2 \rightarrow 6\ CO_2 + 3\ H_2O + 28.2\ N_2$

	$2\ CO_2$ ⇔	$2\ CO$ +	$1\ O_2$	also H_2O	N_2
Initial	6	0	0	3	28.2
Change	-2z	+2z	+z	0	0
Equil.	6-2z	2z	z	3	28.2

At 2400 K, ln K = -7.715, K = 4.461×10^{-4} and we have $n_{tot} = 37.2 + z$

Substitute into equilibrium equation,

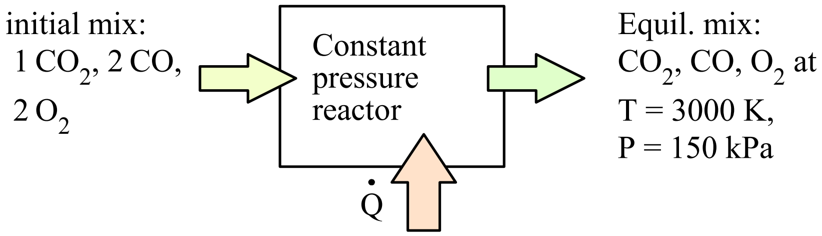
$$K = \frac{4z^2\,z}{(6 - 2z)^2(37.2 + z)}\left(\frac{80}{100}\right)^1 \Rightarrow \frac{z^3}{(3 - z)^2(37.2 + z)} = \ = 0.0005576$$

Solve with limit $0 < z < 3$ gives $z = 0.507$ so then $n_{CO_2} = 6 - 2z = 4.986$

Fraction dissociated = (6 – 4.986)/6 = **0.169**

16.44

A mixture of 1 kmol carbon dioxide, 2 kmol carbon monoxide, and 2 kmol oxygen, at 25°C, 150 kPa, is heated in a constant pressure steady state process to 3000 K. Assuming that only these same substances are present in the exiting chemical equilibrium mixture, determine the composition of that mixture.



Reaction	$2\,CO_2$	$\Leftrightarrow$	$2\,CO$	+	O_2
initial	1		2		2
change	-2x		+2x		+x
equil.	(1 - 2x)		(2 + 2x)		(2 + x)

$n_{tot} = 1 - 2x + 2 + 2x + 2 + x = 5 + x$ so $y = n/\,n_{tot}$

From A.11 at 3000 K: $K = \exp(-2.217) = 0.108935$

For each $n > 0 \quad \Rightarrow \quad -1 < x < +\frac{1}{2}$

$$K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2} \left(\frac{P}{P^0}\right)^1 = 4 \left(\frac{1 + x}{1 - 2x}\right)^2 \left(\frac{2 + x}{5 + x}\right) \left(\frac{150}{100}\right) = 0.108935$$

or $\left(\frac{1 + x}{1 - 2x}\right)^2 \left(\frac{2 + x}{5 + x}\right) = 0.018\,156$, Trial & error: $x = -0.521$

$$\left.\begin{matrix} n_{CO2} = 2.042 & n_{O2} = 1.479 \\ n_{CO} = 0.958 & n_{tot} = 4.479 \end{matrix}\right\} \left\{\begin{matrix} y_{CO2} = 0.4559 \\ y_{CO} = 0.2139 \\ y_{O2} = 0.3302 \end{matrix}\right.$$

16.45

Consider combustion of methane with pure oxygen forming carbon dioxide and water as the products. Find the equilibrium constant for the reaction at 1000 K. Use an average heat capacity of Cp = 52 kJ/kmol K for the fuel and Table A.9 for the other components.

For the reaction equation,

$$CH_4 + 2\,O_2 \Leftrightarrow CO_2 + 2\,H_2O$$

At 1000 K from Table A.9 and A.10 for the fuel at 298 K

$$\begin{aligned}\Delta H^0_{1000\text{ K}} &= 1(-393\,522 + 33\,397) + 2(-241\,826 + 26\,000)\\ &\quad - 1[-74\,873 + 52(1000 - 298.2)] - 2(0 + 22\,703)\\ &= -\,798\,804 \text{ kJ/kmol}\end{aligned}$$

$$\begin{aligned}\Delta S^0_{1000\text{ K}} &= 1\times269.299 + 2\times232.739 - 1(186.251 + \ln\frac{1000}{298.2}) - 2\times243.579\\ &= 487.158 \text{ kJ/kmol K}\end{aligned}$$

$$\begin{aligned}\Delta G^0_{1000\text{ K}} &= \Delta H^0_{1000\text{ K}} - T\ \Delta S^0_{1000\text{ K}}\\ &= -\,798\,804 - 1000 \times 487.158 = -\,1\,285\,962 \text{ kJ/kmol}\end{aligned}$$

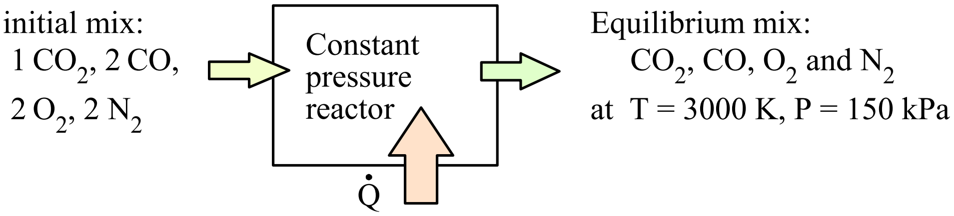
$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = \frac{+\,1\,285\,962}{8.3145\times1000} = +\,154.665\ ,\quad \mathbf{K = 1.4796\ E\ 67}$$

This means the reaction is shifted totally to the right.

16.46

Repeat problem 16.44 for an initial mixture that also includes 2 kmol of nitrogen, which does not dissociate during the process.

This problem has a dilution of the reantant with nitrogen.



Reaction	$2\,CO_2$	$\Leftrightarrow$	$2\,CO$	+	O_2
initial	1		2		2
change	-2x		+2x		+x
equil.	(1-2x)		(2+2x)		(2+x)

From A.11 at 3000 K: $K = \exp(-2.217) = 0.108935$

For each $n > 0 \quad \Rightarrow \quad -1 < x < +\frac{1}{2}$

Equilibrium: $n_{CO2} = (1 - 2x)$, $n_{CO} = (2 + 2x)$, $n_{O2} = (2 + x)$,

$n_{N2} = 2$ so then $n_{tot} = 7 + x$

$$K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = 4\left(\frac{1+x}{1-2x}\right)^2\left(\frac{2+x}{7+x}\right)\left(\frac{150}{100}\right) = 0.108935$$

or $\left(\frac{1+x}{1-2x}\right)^2\left(\frac{2+x}{7+x}\right) = 0.018167$ Trial & error: $x = -0.464$

$$\begin{array}{ll} n_{CO2} = 1.928 & n_{O2} = 1.536 \\ n_{CO} = 1.072 & n_{N2} = 2.0 \\ & n_{TOT} = 6.536 \end{array} \Bigg\} \quad \left\{ \begin{array}{ll} y_{CO2} = 0.295 & y_{O2} = 0.235 \\ y_{CO} = 0.164 & y_{N2} = 0.306 \end{array} \right.$$

16.47

A mixture flows with 2 kmol/s CO_2, 1 kmol/s argon and 1 kmol/s CO at 298 K and it is heated to 3000 K at constant 100 kPa. Assume the dissociation of carbon dioxide is the only equilibrium process to be considered. Find the exit equilibrium composition and the heat transfer rate.

Reaction	$2\,CO_2$	$\Leftrightarrow$	$2\,CO$	+	O_2	Ar
initial	2		1		0	1
change	-2x		+2x		+x	
equil.	2 - 2x		1 + 2x		x	1

From Table A.11:

$$K = \exp(-2.217) = 0.108935 = \frac{y_{CO}^2\, y_{O2}}{y_{CO2}^2}\left(\frac{P}{P_o}\right)^{3-2}$$

$$= \frac{(1+2x)^2}{(4+x)^2}\,\frac{x}{4+x}\,\frac{(4+x)^2}{(2-2x)^2}\left(1\right)^1 = \frac{x}{4}\,\frac{(1+2x)^2}{4+x}\,\frac{1}{(1-x)^2}$$

then

$$0.43574\,(4+x)\,(1-x)^2 = x\,(1+2x)^2$$

trial and error solution gives x = 0.32136

The outlet has: $1.35728\ CO_2 + 1.64272\ CO + 1\ Ar + 0.32136\ O_2$

Energy equation gives from Table A.9 and A.5 (for argon $C_p\Delta T$)

$$\dot{Q} = \sum \dot{n}_{ex}\,\bar{h}_{ex} - \sum \dot{n}_{in}\,\bar{h}_{in}$$

$$= 1.35728\,(152853 - 393522) + 1.64272\,(93504 - 110527)$$

$$+ 1\,(39.948 \times 0.52 \times (3000 - 298)) + 0.32136\,(98013)$$

$$- 2\,(-393522) - 1(-110527) - 1\,(0)$$

= **630 578 kW**

16.48

Catalytic gas generators are frequently used to decompose a liquid, providing a desired gas mixture (spacecraft control systems, fuel cell gas supply, and so forth). Consider feeding pure liquid hydrazine, N_2H_4, to a gas generator, from which exits a gas mixture of N_2, H_2, and NH_3 in chemical equilibrium at 100°C, 350 kPa. Calculate the mole fractions of the species in the equilibrium mixture.

Initially, $\quad 2\,N_2H_4 \rightarrow 1\,N_2 + 1\,H_2 + 2\,NH_3$

Reaction:	N_2 +	$3\,H_2$	$\Leftrightarrow$ $2\,NH_3$	
initial	1	1	2	
change	-x	-3x	+2x	
equil.	(1-x)	(1-3x)	(2+2x)	n_{TOTAL} = (4-2x)

$$K = \frac{y^2_{NH3}}{y_{N2}y^3_{H2}}\left(\frac{P}{P^0}\right)^{-2} = \frac{(2 + 2x)^2(4 - 2x)^2}{(1 - x)(1 - 3x)^3}\left(\frac{350}{100}\right)^{-2}$$

At 100 °C = 373.2 K, for NH_3 use A.5 $\quad \bar{C}_{P0} = 17.03\times2.130 = 36.276$

$$\bar{h}^0_{NH3} = -45\,720 + 36.276(373.2 - 298.2) = -42\,999 \text{ kJ/kmol}$$

$$\bar{s}^0_{NH3} = 192.572 + 36.276 \ln\frac{373.2}{298.2} = 200.71 \text{ kJ/kmol K}$$

Using A.9,

$$\Delta H^0_{100\,C} = 2(-42\,999) - 1(0+2188) - 3(0+2179) = -94\,723 \text{ kJ}$$

$$\Delta S^0_{100\,C} = 2(200.711) - 1(198.155) - 3(137.196) = -208.321 \text{ kJ/K}$$

$$\Delta G^0_{100\,C} = \Delta H^0 - T\Delta S^0 = -94\,723 - 373.2(-208.321) = -16\,978 \text{ kJ}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = \frac{+16\,978}{8.3145\times373.2} = 5.4716 \quad => \quad K = 237.84$$

Therefore,

$$\left[\frac{(1 + x)(2 - x)}{(1 - 3x)}\right]^2 \frac{1}{(1 - x)(1 - 3x)} = \frac{237.84\times3.5^2}{16} = 182.096$$

By trial and error, x = 0.226

$$\left.\begin{array}{ll} n_{N2} = 0.774 & n_{NH3} = 2.452 \\ n_{H2} = 0.322 & n_{TOT} = 3.518 \end{array}\right\} \left\{\begin{array}{l} y_{N2} = 0.2181 \\ y_{H2} = 0.0908 \\ y_{NH3} = 0.6911 \end{array}\right.$$

16.49

Water from the combustion of hydrogen and pure oxygen is at 3800 K and 50 kPa. Assume we only have H_2O, O_2 and H_2 as gases find the equilibrium composition.

With only the given components we have the reaction

$$2\,H_2O \;\Leftrightarrow\; 2H_2 + O_2$$

which at 3800 K has an equilibrium constant from A.11 as $\ln K = -1.906$

Assume we start with 2 kmol water and let it dissociate x to the left then

Species	H_2O	H_2	O_2	
Initial	2	0	0	
Change	-2x	2x	x	
Final	2 – 2x	2x	x	Tot: 2 + x

Then we have

$$K = \exp(-1.906) = \frac{y_{H_2}^2\, y_{O_2}}{y_{H_2O}^2}\left(\frac{P}{P^0}\right)^{2+1-2} = \frac{\left(\frac{2x}{2+x}\right)^2 \frac{x}{2+x}}{\left(\frac{2-2x}{2+x}\right)^2}\,\frac{50}{100}$$

which reduces to

$$0.148674 = \frac{1}{(1-x)^2}\,\frac{4x^3}{2+x}\,\frac{1}{4}\,\frac{1}{2} \quad \text{or} \quad x^3 = 0.297348\ (1-x)^2\,(2+x)$$

Trial and error to solve for x = 0.54 then the concentrations are

$$y_{H2O} = \frac{2-2x}{2+x} = 0.362; \quad y_{O2} = \frac{x}{2+x} = 0.213; \quad y_{H2} = \frac{2x}{2+x} = 0.425$$

16.50

Complete combustion of hydrogen and pure oxygen in a stoichiometric ratio at P_o, T_o to form water would result in a computed adiabatic flame temperature of 4990 K for a steady state setup. How should the adiabatic flame temperature be found if the equilibrium reaction $2H_2 + O_2 \Leftrightarrow 2\ H_2O$ is considered? Disregard all other possible reactions (dissociations) and show the final equation(s) to be solved.

$2H_2 + O_2 \Leftrightarrow 2H_2O$

Species	H_2	O_2	H_2O
Initial	2	1	0
Shift	$-2x$	$-x$	$2x$
Final	$2 - 2x$	$1 - x$	$2x$

$$Keq = \frac{y_{H_2O}^2}{y_{H_2}^2 y_{O_2}} \left(\frac{P}{P^0}\right)^{-1}, \qquad n_{tot} = 2 - 2x + 1 - x + 2x = 3 - x$$

Energy Eq.:

$$H_P = H_R = H_P^o + \Delta H_P = H_R^o = 0$$

$$H_p = (2-2x)\Delta\bar{h}_{H_2} + (1-x)\Delta\bar{h}_{O_2} + 2x(\bar{h}^o_{fH2O} + \Delta\bar{h}_{H2O}) = 0 \qquad (1)$$

Equilibrium constant:

$$Keq = \frac{4x^2}{(3-x)^2}\,\frac{(3-x)^2}{(2-2x)^2}\,\frac{3-x}{1-x} = \frac{x^2(3-x)}{(1-x)^3} = Keq(T) \qquad (2)$$

$\bar{h}^o_{fH2O} = -241\ 826$ kJ/kmol;

$\Delta\bar{h}_{H_2}(T)$, $\Delta\bar{h}_{O_2}(T)$, $\Delta\bar{h}_{H_2O}(T)$ are all from A.9

Keq(T) is from A.11

Trial and Error (solve for x, T) using Eqs. (1) and (2).

$T = ???$, $y_{O_2} = 0.15$; $y_{H_2} = 0.29$; $y_{H_2O} = 0.56$

16.51

The van't Hoff equation

$$d \ln K = \frac{\Delta H^o}{\bar{R}T^2} \, dT_{P^o}$$

relates the chemical equilibrium constant K to the enthalpy of reaction ΔH^o. From the value of K in Table A.11 for the dissociation of hydrogen at 2000 K and the value of ΔH^o calculated from Table A.9 at 2000 K use van't Hoff equation to predict the constant at 2400 K.

$$H_2 \Leftrightarrow 2H$$

$$\Delta H° = 2 \times (35\,375 + 217\,999) - 52\,942 = 453\,806 \text{ kJ/kmol}$$

$$\ln K_{2000} = -12.841;$$

Assume ΔH° is constant and integrate the Van't Hoff equation

$$\ln K_{2400} - \ln K_{2000} = \int_{2400}^{2000} \frac{\Delta H^o}{\bar{R}T^2} \, dT = -\frac{\Delta H°}{\bar{R}} \left(\frac{1}{T_{2400}} - \frac{1}{T_{2000}}\right)$$

$$\ln K_{2400} = \ln K_{2000} + \Delta H° \left(\frac{1}{T_{2400}} - \frac{1}{T_{2000}}\right) / \bar{R}$$

$$= -12.841 + 453\,806 \left(\frac{6-5}{12000}\right) / 8.31451 = -12.841 + 4.548$$

$$= -8.293$$

Table A.11 lists –8.280 (ΔH° not exactly constant)

16.52

Consider the water gas reaction in Example 16.4. Find the equilibrium constant at 500, 1000, 1200 and 1400 K. What can you infer from the result?

As in Example 16.4, III $H_2 + CO_2 \Leftrightarrow H_2O + CO$

I $2\,CO_2 \Leftrightarrow 2\,CO + O_2$

II $2\,H_2O \Leftrightarrow 2\,H_2 + O_2$

Then, $\ln K_{III} = 0.5\,(\ln K_I - \ln K_{II})$

At 500 K, $\ln K_{III} = 0.5\,(-115.234 - (-105.385)) = -4.9245$,
$K = 0.007\,266$

At 1000 K, $\ln K_{III} = 0.5\,(-47.052 - (-46.321)) = -0.3655$,
$K = 0.693\,85$

At 1200 K, $\ln K_{III} = 0.5\,(-35.736 - (-36.363)) = +0.3135$,
$K = 1.3682$

At 1400 K, $\ln K_{III} = 0.5\,(-27.679 - (-29.222)) = +0.7715$,
$K = 2.163$

It is seen that at lower temperature, reaction III tends to go strongly from right to left, but as the temperature increases, the reaction tends to go more strongly from left to right. If the goal of the reaction is to produce more hydrogen, then it is desirable to operate at lower temperature.

16.53

A piston/cylinder contains 0.1 kmol hydrogen and 0.1 kmol Ar gas at 25°C, 200 kPa. It is heated up in a constant pressure process so the mole fraction of atomic hydrogen is 10%. Find the final temperature and the heat transfer needed.

When gas is heated up H_2 splits partly into H as

$H_2 \Leftrightarrow 2H$ and the gas is diluted with Ar

Component	H_2	Ar	H	
Initial	0.1	0.1	0	
Shift	-x	0	2x	
Final	0.1-x	0.1	2x	Total = 0.2 + x

$y_H = 0.1 = 2x /(0.2 + x) \quad \Rightarrow \quad 2x = 0.02 + 0.1x$

$\Rightarrow \quad x = 0.010526 \quad \Rightarrow n_{tot} = 0.21053$

$y_{H2} = 0.425 = [(0.1\text{-}x)/(0.2+x)]; \quad y_{Ar} = 1 – \text{rest} = 0.475$

Do the equilibrium constant:

$$K(T) = \frac{y_H^2}{y_{H2}} \left(\frac{P}{P^0}\right)^{2-1} = \left(\frac{0.01}{0.425}\right) \times \left(\frac{200}{100}\right) = 0.047059$$

ln (K) = –3.056 so from Table A.11 interpolate to get T = 3110 K

To do the energy eq., we look up the enthalpies in Table A.9 at 3110 K

$h_{H2} = 92\ 829.1; \quad h_H = h_f + \Delta h = 217\ 999 + 58\ 447.4 = 276\ 445.4$

$h_{Ar} = 0 + C_P(3110 – 298.15) = 20.7863 \times (3110\text{-}298.13) = 58\ 447.9$

(same as Δh for H)

Now get the total number of moles to get

$n_H = 0.021053; \quad n_{H2} = n_{tot} \times \frac{1\text{-}x}{2+x} = 0.08947; \quad n_{Ar} = 0.1$

Since pressure is constant W = PΔV and Q becomes differences in h

$Q = n\Delta h = 0.08947 \times 92\ 829.1 – 0 + 0.021053 \times 276\ 446.4$

$– 0 + 0.1 \times 58\ 447.9$

= **19 970 kJ**

16.54

A tank contains 0.1 kmol hydrogen and 0.1 kmol of argon gas at 25°C, 200 kPa and the tank keeps constant volume. To what T should it be heated to have a mole fraction of atomic hydrogen, H, of 10%?

For the reaction $H_2 \Leftrightarrow 2H$, $K = \frac{y_H^2}{y_{H_2}} \left(\frac{P}{P^o}\right)^{2-1}$

Assume the dissociation shifts right with an amount x then we get

reaction	H_2	$\Leftrightarrow$	2 H	also,	Ar	
initial	0.1		0		0.1	
change	-x		2x		0	
equil.	0.1 - x		2x		0.1	Tot: 0.2 + x

$$y_H = \frac{2x}{0.2 + x} = 0.10 \quad \Rightarrow \quad x = 0.010526$$

We need to find T so K will take on the proper value, since K depends on P we need to evaluate P first.

$$P_1V = n_1\bar{R}T_1; \quad P_2V = n_2\bar{R}T_2 \quad \Rightarrow \quad P_2 = P_1 \frac{n_2T_2}{n_1T_1}$$

where we have $n_1 = 0.2$ and $n_2 = 0.2 + x = 0.210526$

$$K = \frac{y_H^2}{y_{H_2}} \left(\frac{P}{P^o}\right)^{2-1} = \frac{(2x)^2}{(0.1 - x)\, n_2} \frac{200}{100} \frac{n_2T_2}{0.2 \times 298.15} = 0.0001661\, T_2$$

Now it is trial and error to get T_2 so the above equation is satisfied with K from A.11 at T_2.

3600 K: ln K = -0.611, K = 0.5428, RHS = 0.59796, error = 0.05516

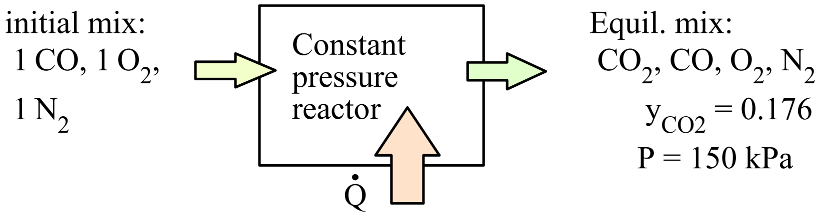
3800 K: ln K = 0.201, K = 1.22262, RHS = 0.63118, error = -0.59144

Linear interpolation between the two to make zero error

$$T = 3600 + 200 \times \frac{0.05516}{0.05516 + 0.59144} = \mathbf{3617\ K}$$

16.55

A gas mixture of 1 kmol carbon monoxide, 1 kmol nitrogen, and 1 kmol oxygen at 25°C, 150 kPa, is heated in a constant pressure process. The exit mixture can be assumed to be in chemical equilibrium with CO_2, CO, O_2, and N_2 present. The mole fraction of CO_2 at this point is 0.176. Calculate the heat transfer for the process.



reaction	$2\ CO_2$	$\Leftrightarrow$	$2\ CO$	+	O_2	also,	N_2
initial	0		1		1		1
change	+2x		-2x		-x		0
equil.	2x		(1-2x)		(1-x)		1

$$y_{CO2} = 0.176 = \frac{2x}{3-x} \quad \Rightarrow \quad x = 0.242\ 65$$

$$\left.\begin{matrix} n_{CO2} = 0.4853 & n_{O2} = 0.7574 \\ n_{CO} = 0.5147 & n_{N2} = 1 \end{matrix}\right\} \left\{\begin{matrix} y_{CO2} = 0.176 \\ y_{CO} = 0.1867 \end{matrix}\right. y_{O2} = 0.2747$$

$$K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = \frac{0.1867^2 \times 0.2747}{0.176^2}\left(\frac{150}{100}\right) = 0.4635$$

From A.11, $T_{PROD} = 3213$ K

From A.10, $H_R = -110\ 527$ kJ

$$H_P = 0.4853(-393\ 522 + 166\ 134) + 0.5147(-110\ 527 + 101\ 447)$$

$$+ 0.7574(0 + 106\ 545) + 1(0 + 100\ 617)$$

$$= +66\ 284 \text{ kJ}$$

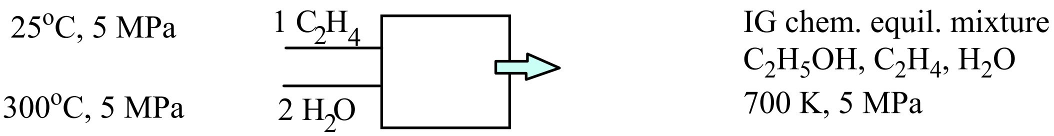
$Q_{CV} = H_P - H_R = 66\ 284 - (-110\ 527) =$ **+176 811 kJ**

16.56

A liquid fuel can be produced from a lighter fuel in a catalytic reactor according to

$$C_2H_4 + H_2O \quad \Leftrightarrow \quad C_2H_5OH$$

Show the equilibrium constant is ln K = –6.691 at 700 K using C_P = 63 kJ/kmol K for ethylene and C_P = 115 kJ/kmol K for ethanol at 500 K.



$$1\ C_2H_4 + 1\ H_2O \Leftrightarrow 1\ C_2H_5OH$$

$$\Delta H^0_{700\text{ K}} = 1(-235\ 000 + 115(700\text{-}298.2)) - 1(+52\ 467 + 63\ (700\text{-}298.2))$$
$$- 1(-241\ 826 + 14\ 190) = -38\ 935\text{ kJ}$$

$$\Delta S^0_{700\text{ K}} = 1(282.444 + 115 \ln\frac{700}{298.2}) - 1(219.330 + 63 \ln\frac{700}{298.2}) - 1(218.739)$$
$$= -111.253\text{ kJ/K}$$

$$\Delta G^0_{700\text{ K}} = \Delta H^0 - T\Delta S^0 = -38\ 935 - 700(-111.253) = +38\ 942\text{ kJ}$$

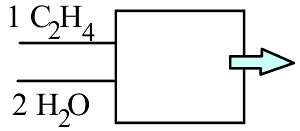
$$\ln K = \frac{-\Delta G^0}{\bar{R}T} = -\frac{38\ 942}{8.31451 \times 700} = \mathbf{-6.691}$$

16.57

A step in the production of a synthetic liquid fuel from organic waste material is the following conversion process at 5 MPa: 1 kmol ethylene gas (converted from the waste) at 25°C and 2 kmol steam at 300°C enter a catalytic reactor. An ideal gas mixture of ethanol, ethylene and water in equilibrium, see previous problem, leaves the reactor at 700 K, 5 MPa. Determine the composition of the mixture.

25°C, 5 MPa

300°C, 5 MPa



IG chem. equil. mixture
C_2H_5OH, C_2H_4, H_2O
700 K, 5 MPa

$$1\ C_2H_4 + 1\ H_2O \Leftrightarrow 1\ C_2H_5OH$$

initial	1	2	0	
change	-x	-x	+x	
equil.	(1-x)	(2-x)	x	total = 3 - x

The reaction rate from the previous problem statement is

$$\ln K = \frac{-\Delta G^0}{\bar{R}T} = -6.691 \quad => \quad K = 0.001\ 242 = \frac{y_{C2H5OH}}{y_{C2H4}\ y_{H2O}} \left(\frac{P}{P^0}\right)^{-1}$$

$$\Rightarrow \left(\frac{x}{1-x}\right)\left(\frac{3-x}{2-x}\right) = 0.001242 \times \frac{5.0}{0.1} = 0.0621$$

By trial and error: x = 0.0404 =>

C_2H_5OH: n = 0.0404, y = 0.01371

C_2H_4: n = 0.9596, y = 0.3242, H_2O: n = 1.9596 , y = 0.6621

16.58

A rigid container initially contains 2 kmol of carbon monoxide and 2 kmol of oxygen at 25°C, 100 kPa. The content is then heated to 3000 K at which point an equilibrium mixture of CO_2, CO, and O_2 exists. Disregard other possible species and determine the final pressure, the equilibrium composition and the heat transfer for the process.

Equilibrium process: $2\ CO + 2\ O_2 \Leftrightarrow 2\ CO_2 + O_2$

Species:	CO	O_2	CO_2	
Initial	2	2	0	
Shift	-2x	-2x+x	2x	
Final	2-2x	2-x	2x :	$n_{tot} = 2 - 2x + 2 - x + 2x = 4 - x$

$$y_{CO} = \frac{2\text{-}2x}{4\text{-}x}; \quad y_{O_2} = \frac{2\text{-}x}{4\text{-}x}; \quad y_{CO_2} = \frac{2x}{4\text{-}x}$$

Energy equation

$$U_2 - U_1 = {}_1Q_2 = H_2 - H_1 - P_2v + P_1v$$

$$= (2 - 2x)\bar{h}_{CO\ 2} + (2 - x)\bar{h}_{O_2 2} + 2x\bar{h}_{CO_2} - 2\bar{h}^o_{fCO_2} - 2\bar{h}^o_{fO_2}$$

$$- \bar{R}\,(4 - x)T_2 + 4\bar{R}T_1$$

Notice P_2 is unknown so write it in terms of T_2 and the number of moles. We flipped the reaction relative to the one in A.11 so then $K_{eq} = 1/\ K_{eq\ A11}$

$$K_{eq} = e^{2.217} = \frac{y^2_{CO_2}}{y_{0_2}y^2_{CO}} \left(\frac{P_2}{P_o}\right)^{-1} = \frac{4x^2}{4(1 - x)^2}\ \frac{4 - x}{2 - x}\ \frac{4T_1}{(4 - x)T_2}$$

$$\Rightarrow \left(\frac{x}{1 - x}\right)^2 \frac{1}{2 - x} = \frac{1}{4}\frac{T_2}{T_1} e^{2.217} = 23.092 \quad \Rightarrow \qquad x = 0.8382;$$

$y_{CO} = \mathbf{0.102}$; $y_{O_2} = \mathbf{0.368}$; $y_{CO_2} = \mathbf{0.53}$

Constant volume and ideal gas approximation $PV = n\bar{R}T$

$$P_2 = P_1(4 - x)\ T_2\ /4T_1 = 100\ \frac{(4 - 0.8382) \times 3000}{4 \times 298.15} = \mathbf{795.4\ kPa}$$

$${}_1Q_2 = 0.3236(-110527 + 93504) + 1.1618(98013) + 1.6764(-393522$$

$$+ 152853) - 2(-110527) - 2(0) + 8.3145[4(298.15) - 3000(3.1618)]$$

$$= \mathbf{-142991\ kJ}$$

16.59

Use the information in Problem 16.81 to estimate the enthalpy of reaction, ΔH^o, at 700 K using Van't Hoff equation (see problem 16.51) with finite differences for the derivatives.

$$d \ln K = [\Delta H°/\bar{R}T^2]\, dT \quad \text{or} \quad \text{solve for } \Delta H^o$$

$$\Delta H° = \bar{R}T^2 \frac{d \ln K}{dT} = \bar{R}T^2 \frac{\Delta \ln K}{\Delta T}$$

$$= 8.31451 \times 700^2 \times \frac{-0.3362 - (-4.607)}{800 - 600} = 86\,998 \text{ kJ/kmol}$$

[Remark: compare this to A.9 values + A.5, A.10,

$$\Delta H^o = H_C + 2H_{H_2} - H_{CH_4} = 0.61 \times 12 \times (700\text{-}298) + 2 \times 11730$$

$$- 2.254 \times 16.04 \times (700\text{-}298) - (-74873) = 86\,739 \text{ kJ/kmol]}$$

16.60

Acetylene gas at 25°C is burned with 140% theoretical air, which enters the burner at 25°C, 100 kPa, 80% relative humidity. The combustion products form a mixture of CO_2, H_2O, N_2, O_2, and NO in chemical equilibrium at 2200 K, 100 kPa. This mixture is then cooled to 1000 K very rapidly, so that the composition does not change. Determine the mole fraction of NO in the products and the heat transfer for the overall process.

$C_2H_2 + 3.5\ O_2 + 13.16\ N_2 + \text{water} \rightarrow$

$2\ CO_2 + 1\ H_2O + 1\ O_2 + 13.16\ N_2 + \text{water}$

water: $P_V = 0.8 \times 3.169 = 2.535$ kPa

$n_V = n_A P_V/P_A = (3.5 + 13.16)\ 2.535/97.465 = 0.433$

So, total H_2O in products is 1.433.

a) reaction: $N_2 + O_2 \ \ \text{<->}\ \ 2\ NO$

change : -x -x +2x

at 2200 K, from A.11: K = 0.001 074

Equil. products: $n_{CO2} = 2$, $n_{H2O} = 1.433$, $n_{O2} = 1 - x$,

$n_{N2} = 13.16 - x$, $n_{NO} = 0 + 2x$, $n_{TOT} = 17.593$

$$K = \frac{(2x)^2}{(1 - x)(13.16 - x)} = 0.001\ 074 \quad => \quad x = 0.0576$$

$$y_{NO} = \frac{2 \times 0.0576}{17.593} = \mathbf{0.006\ 55}$$

b) Final products (same composition) at 1000 K, reactants at 25°C

$H_R = 1(226\ 731 + 0) + 0.433(-241\ 826 + 0) = 122\ 020$ kJ

$H_P = 2(-393\ 522 + 33\ 397) + 1.433(-241\ 826 + 26\ 000)$

$+ 0.9424(0 + 22\ 703) + 13.1024(0 + 21\ 463)$

$+ 0.1152(90\ 291 + 22\ 229)$

$= -713\ 954$ kJ

$Q_{CV} = H_P - H_R = \mathbf{-835\ 974\ kJ}$

16.61

An important step in the manufacture of chemical fertilizer is the production of ammonia, according to the reaction: $N_2 + 3H_2 \Leftrightarrow 2NH_3$ Show that the equilibrium constant is K = 6.202 at 150°C.

$$1\ N_2 + 3\ H_2 \Leftrightarrow 2\ NH_3 \quad \text{at } 150^oC$$

$$\bar{h}^o_{NH3\ 150\ C} = -45\ 720 + 2.13\times17.031(150 - 25) = -41\ 186\ \text{kJ/kmol}$$

$$\bar{s}^o_{NH3\ 150\ C} = 192.572 + 2.13\times17.031\ \ln\frac{423.2}{298.2} = 205.272\ \text{kJ/kmol-K}$$

$$\Delta H^o_{150\ C} = 2(-41\ 186) - 1(0+3649) - 3(0+3636) = -96\ 929\ \text{kJ/kmol}$$

$$\Delta S^0_{150\ C} = 2(205.272) - 1(201.829) - 3(140.860) = -213.865\ \text{kJ/kmol-K}$$

$$\Delta G^0_{150\ C} = -96\ 929 - 423.2(-213.865) = -6421\ \text{kJ/kmol}$$

$$\ln K = \frac{+6421}{8.31451 \times 423.15} = 1.8248,\ \ \mathbf{K = 6.202}$$

16.62

Consider the previous reaction in equilibrium at 150°C, 5 MPa. For an initial composition of 25% nitrogen, 75% hydrogen, on a mole basis, calculate the equilibrium composition

$$\ln K = \frac{+6421}{8.3144\times 423.2} = 1.8248, \qquad \mathbf{K = 6.202}$$

$$n_{NH3} = 2x, \quad n_{N2} = 1 - x, \quad n_{H2} = 3 - 3x, \qquad \text{total } n = 4 - 2x$$

$$K = \frac{y_{NH3}^2}{y_{N2} y_{H2}^3}\left(\frac{P}{P^0}\right)^{-2} = \frac{(2x)^2(4 - 2x)^2}{3^3(1 - x)^4}\left(\frac{P}{P^0}\right)^{-2}$$

$$\text{or} \quad \left(\frac{x}{1-x}\right)^2\left(\frac{2-x}{1-x}\right)^2 = \frac{27}{16}\times 6.202 \times \left(\frac{5}{0.1}\right)^2 = 26165$$

$$\text{or} \quad \left(\frac{x}{1-x}\right)\left(\frac{2-x}{1-x}\right) = 161.755$$

→ Trial & Error:

x = 0.9215

	n	y
NH_3	1.843	0.8544
N_2	0.0785	0.0364
H_2	0.2355	0.1092

16.63

Methane at 25°C, 100 kPa, is burned with 200% theoretical oxygen at 400 K, 100 kPa, in an adiabatic steady state process, and the products of combustion exit at 100 kPa. Assume that the only significant dissociation reaction in the products is that of carbon dioxide going to carbon monoxide and oxygen. Determine the equilibrium composition of the products and also their temperature at the combustor exit.

Combustion: $CH_4 + 4O_2 \rightarrow CO_2 + 2H_2O + 2O_2$

Dissociation:

	$2\,CO_2$ $\Leftrightarrow$	$2\,CO$ +	O_2	$,H_2O$	inert
initial	1	0	2	2	
change	-2x	+2x	+x	0	
equil.	1-2x	2x	2+x	2	$n_{TOT} = 5+x$

$$\text{Equil. Eq'n:}\quad K = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right) = \left(\frac{x}{0.5-x}\right)^2\left(\frac{2+x}{5+x}\right)\left(\frac{P}{P^0}\right)$$

$$\text{or}\qquad \left(\frac{x}{0.5-x}\right)^2\left(\frac{2+x}{5+x}\right) = \frac{K}{(P/P^0)}$$

1st law: $H_P - H_R = 0$

$$(1-2x)(-393\,522 + \Delta\bar{h}_{CO2}) + 2x(-110\,527 + \Delta\bar{h}_{CO})$$

$$+ 2(-241\,826 + \Delta\bar{h}_{H2O}) + (2+x)\Delta\bar{h}_{O2} - 1(-74\,873) - 4(3027) = 0$$

$$\text{or}\quad (1-2x)\Delta\bar{h}_{CO2} + 2x\Delta\bar{h}_{CO} + 2\Delta\bar{h}_{H2O} + (2+x)\Delta\bar{h}_{O2} + 565\,990x - 814\,409 = 0$$

Assume $T_P = 3256$ K. From A.11: $K = 0.6053$

Solving (1) by trial & error, $x = 0.2712$

Substituting x and the $\Delta\bar{h}$ values from A.9 (at 3256 K) into (2)

$$0.4576\times 168\,821 + 0.5424\times 103\,054 + 2\times 140\,914$$

$$+ 2.2712\times 108\,278 + 565\,990\times 0.2712 - 814\,409 \approx 0 \text{ OK}$$

$T_P = 3256$ K & $x = 0.2712$

$n_{CO2} = 0.4576$, $n_{CO} = 0.5424$, $n_{H2O} = 2.0$, $n_{O2} = 2.2712$

$y_{CO2} = 0.0868$, $y_{CO} = 0.1029$, $y_{H2O} = 0.3794$, $y_{O2} = 0.4309$

16.64

Calculate the irreversibility for the adiabatic combustion process described in the previous problem.

From solution of Prob. 16.63, it is found that the product mixture consists of 0.4576 CO_2, 0.5424 CO, 2.0 H_2O & 2.2712 O_2 at 3256 K, 100 kPa. The reactants include

1 CH_4 at 25 °C, 100 kPa and 4 O_2 at 400 K, 100 kPa.

Reactants:

$S_R = 1(186.251) + 4(213.873) = 1041.74$ kJ/K

Products:

	n_i	y_i	$\bar{s}_i^0$	$-\bar{R} \ln \frac{y_i P}{P_0}$	$\bar{S}_i^*$
CO_2	0.4576	0.0868	339.278	+20.322	359.600
CO	0.5424	0.1029	276.660	+18.907	295.567
H_2O	2.0	0.3794	291.099	+8.058	299.157
O_2	2.2712	0.4309	287.749	+7.000	294.749

$S_P = 0.4576(359.600) + 0.5424(295.567) + 2.0(299.157)$

$+ 2.2712(294.749) = 1592.62$ kJ/K

$I = T_0(S_P - S_R) - Q_{CV} = 298.15(1592.62 - 1041.74) - 0 =$ **164 245 kJ**

16.65

One kilomole of carbon dioxide, CO_2, and 1 kmol of hydrogen, H_2 at room temperature, 200 kPa is heated to 1200 K at 200 kPa. Use the water gas reaction to determine the mole fraction of CO. Neglect dissociations of H_2 and O_2.

$$1\ CO_2 + 1\ H_2 \Leftrightarrow 1\ CO + 1\ H_2O$$

Initial	1	1	0	0	
Shift	-x	-x	+x	+x	
Total	1-x	1-x	x	x;	$n_{tot} = 2$

$y_{H2O} = y_{CO} = x/2, \quad y_{H2} = y_{CO2} = (1-x)/2$

From solution to problem 15.36, K = 1.3682

$$\frac{(x/2)(x/2)}{(\frac{1-x}{2})(\frac{1-x}{2})} = K = \frac{x^2}{(1-x)^2} \quad => \quad \frac{x}{1-x} = 1.1697$$

$$x = 1.1697 / 2.1697 = 0.5391$$

$y_{H2O} = y_{CO} = x/2 =$ **0.27**, $\quad y_{H2} = y_{CO2} = (1-x)/2 =$ **0.23**

16.66

Hydrides are rare earth metals, M, that have the ability to react with hydrogen to form a different substance MH_x with a release of energy. The hydrogen can then be released, the reaction reversed, by heat addition to the MH_x. In this reaction only the hydrogen is a gas so the formula developed for the chemical equilibrium is inappropriate. Show that the proper expression to be used instead of Eq. 15.14 is

$$\ln (P_{H2}/P^o) = \Delta G^o/RT$$

when the reaction is scaled to 1 kmol of H_2.

$$M + \frac{1}{2} x\ H_2 \quad \Leftrightarrow \quad MH_x$$

At equilibrium $G_P = G_R$, assume g of the solid is a function of T only.

$$\bar g_{MHx} = \bar h^0_{MHx} - T\bar s^0_{MHx} = \bar g^0_{MHx} , \quad \bar g_M = \bar h^0_M - T\bar s^0_M = \bar g^0_M$$

$$\bar g_{H2} = \bar h^0_{H2} - T\bar s^0_{H2} + \bar R T \ln(P_{H2}/P_o) = \bar g^0_{H2} + \bar R T \ln(P_{H2}/P_o)$$

$$G_P = G_R: \quad \bar g_{MHx} = \bar g_M + \frac{1}{2} x\ \bar g_{H2} = \bar g^0_M + \frac{1}{2} x[\bar g^0_{H2} + \bar R T \ln(P_{H2}/P_o)]$$

$$\Delta\bar G^0 = \bar g^0_{MHx} - \bar g^0_M - x\ \bar g^0_{H2}/2 = \bar g^0_{MHx} - \bar g^0_M$$

Scale to 1 mole of hydrogen

$$\Delta\bar G^0 = (\bar g^0_{MHx} - \bar g^0_M)/(x/2) = \bar R T \ln(P_{H2}/P^o)$$

which is the desired result.

Simultaneous Reactions

16.67

For the process in Problem 16.47 should the dissociation of oxygen also be considered? Verbal answer but supported by numbers.

The dissociation of oxygen

Reaction: $O_2 \leftrightarrow 2\ O$ From A.11: $K_T = \exp(-4.356) = 0.01283$

Equilibrium: $$K_T = 0.01283 = \frac{y_O^2}{y_{O_2}} \left(\frac{P}{P^0}\right) = \frac{y_O^2}{y_{O_2}}$$

So we need to solve two simultaneous reaction equations. Due to the small equilibrium constant the concentration of O is going to be small, but it will have the effect of requiring a larger heat transfer to reach 3000 K.

From problem 16.47 $y_{O2} = 0.074$ so assume this is nearly the same

$$y_O = \sqrt{y_{O2} K_T} = \sqrt{0.074 \times 0.01283} = 0.03$$

so about half of the oxygen is dissociated and we do need to solve the proper equations.

16.68

Which other reactions should be considered in Problem 16.50 and which components will be present in the final mixture?

$2H_2 + O_2 \Leftrightarrow 2H_2O$ Species H_2 O_2 H_2O

Other possible reactions from table A.11

$H_2 \leftrightarrow 2\ H$ $O_2 \leftrightarrow 2\ O$ $2\ H_2O \leftrightarrow H_2 + 2\ OH$

So the final component list most likely has species as:

H_2 O_2 H_2O H O OH

16.69

Ethane is burned with 150% theoretical air in a gas turbine combustor. The products exiting consist of a mixture of CO_2, H_2O, O_2, N_2, and NO in chemical equilibrium at 1800 K, 1 MPa. Determine the mole fraction of NO in the products. Is it reasonable to ignore CO in the products?

Combustion:

$$C_2H_6 + 5.25\ O_2 + 19.74\ N_2 \rightarrow 2\ CO_2 + 3\ H_2O + 1.75\ O_2 + 19.74\ N_2$$

Products at 1800 K, 1 MPa

Equilibrium mixture: CO_2, H_2O, O_2, N_2, NO

	N_2	+ O_2	$\Leftrightarrow$ 2 NO
initial	19.74	1.75	0
change	-x	-x	+2x
equil.	19.74-x	1.75-x	2x

Equil. comp. $n_{CO2} = 2$, $n_{O2} = 1.75\text{-}x$, $n_{NO} = 2x$, $n_{H2O} = 3$, $n_{N2} = 19.74\text{-}x$

$$K = 1.192\times10^{-4} = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{4x^2}{(19.74\text{-}x)(1.75\text{-}x)}$$

Solving, x = 0.031 75

$$y_{NO} = \frac{2\times0.031\ 75}{26.49} = \mathbf{0.0024}$$

b)

	$2\ CO_2$ $\Leftrightarrow$	2 CO +	O_2
initial	2	0	0
change	-2a	+2a	+2x
equil.	2-2a	2a	2x

$$K = 4.194\times10^{-8} = \frac{y_{CO}^2 y_{O2}}{y_{CO2}^2}\left(\frac{P}{P^0}\right)^1 = \left(\frac{2a}{2\text{-}2a}\right)^2\left(\frac{1.75\text{-}x\text{+}a}{26.49\text{+}a}\right)\times\frac{1}{0.1}$$

This equation should be solved simultaneously with the equation solved in part a) (modified to include the unknown a). Since x was found to be small and also a will be very small, the two are practically independent. Therefore, use the value x = 0.031 75 in the equation above, and solve for a.

$$\left(\frac{a}{1\text{-}a}\right)^2\left(\frac{1.75\text{-}0.031\ 75\text{+}a}{26.49\text{+}a}\right) = \left(\frac{0.1}{1.0}\right) \times 4.194 \times10^{-8}$$

Solving, a = 0.000 254 or $y_{CO} = 1.92\times10^{-5}$ negligible for most applications.

16.70

A mixture of 1 kmol water and 1 kmol oxygen at 400 K is heated to 3000 K, 200 kPa, in a steady flow process. Determine the equilibrium composition at the outlet of the heat exchanger, assuming that the mixture consists of H_2O, H_2, O_2, and OH.

Reactions and equilibrium eq'ns the same as in example 16.7 (but different initial composition).

At equil.: $n_{H2O} = 1 - 2a - 2b$, $n_{H2} = 2a + b$, $n_{O2} = 1 + a$

$n_{OH} = 2b$, $n_{TOT} = 2 + a + b$

Since T = 3000 K is the same, the two equilibrium constants are the same:

From Table A.11: $K_1 = 0.002\ 062$, $K_2 = 0.002\ 893$

The two equilibrium equations are

$$K_1 = \left(\frac{2a + b}{1 - 2a - 2b}\right)^2 \frac{1 + a}{2 + a + b}\left(\frac{P}{P^0}\right); \quad K_2 = \frac{2a + b}{2 + a + b}\left(\frac{2b}{1 - 2a - 2b}\right)^2\left(\frac{P}{P^0}\right)$$

which must be solved simultaneously for a & b. If solving manually, it simplifies the solution to divide the first by the second, which leaves a quadratic equation in a & b - can solve for one in terms of the other using the quadratic formula (with the root that gives all positive moles). This reduces the problem to solving one equation in one unknown, by trial & error.

Solving => b = 0.116, a = -0.038 =>

$n_{H2O} = 0.844$, $n_{H2} = 0.0398$, $n_{O2} = 0.962$, $n_{OH} = 0.232$, $n_{TOT} = 2.0778$

$y_{H2O} = 0.4062$, $y_{H2} = 0.0191$, $y_{O2} = 0.4630$, $y_{OH} = 0.1117$

16.71

Assume dry air (79% N_2 and 21% O_2) is heated to 2000 K in a steady flow process at 200 kPa and only the reactions listed in Table A.11 are possible (and their linear combinations). Find the final composition (anything smaller than 1 ppm, parts per million, is neglected) and the heat transfer needed for 1 kmol of air in.

We could have: N_2, N, O_2, O, NO, NO_2

The possible reactions are:

$N_2 \Leftrightarrow 2\,N$ $\quad K(T) = \exp(-41.655) \approx 0$

$O_2 \Leftrightarrow 2\,O$ $\quad K(T) = \exp(-14.619) \approx$ small

$N_2 + O_2 \Leftrightarrow 2\,NO$ $\quad K(T) = \exp(-7.825)$

$N_2 + 2\,O_2 \Leftrightarrow 2\,NO_2$ $\quad K(T) = \exp(-19.136) \approx$ small

So from this we have only NO extra.

	N_2	+ O_2	$\Leftrightarrow$ 2 NO
initial	0.79	0.21	0
change	-x	-x	+2x
equil.	0.79 - x	0.21 - x	2x

$$K = \exp(-7.825) = 0.0004 = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{4x^2}{(0.79 - x)(0.21 - x)}$$

Solving, x = 0.004

$y_{N2} = 0.786, \quad y_{O2} = 0.206, \quad y_{NO} = 0.008$

$$Q = H_{ex} - H_{in} = 0.786\,\Delta\bar{h}_{N2} + 0.206\,\Delta\bar{h}_{O2} + 0.008\,\bar{h}_{NO}$$

$$= 0.786\,(56137) + 0.206\,(59176) + 0.008\,(57859 + 90291)$$

$$= \mathbf{57\,499\ kJ/kmol}$$

If we had solved also for the oxygen O and NO_2 formation we would get approximately (neglect the effect on total moles)

$(y_O^2 / y_{O2})\,(P/P_o) = \exp(-14.619) \quad \Rightarrow \quad y_O \approx 0.0002$

$(y_{NO2}^2 / y_{N2}y_{O2}^2)\,(P/P_o)^{-1} = \exp(-19.136) \quad \Rightarrow \quad y_{NO2} \approx 0.000018$

and the O energy addition would be: 0.0002(35713 + 249170) = 57 kJ/kmol very insignificant.

16.72

One kilomole of water vapor at 100 kPa, 400 K, is heated to 3000 K in a constant pressure steady flow process. Determine the final composition, assuming that H_2O, H_2, H, O_2, and OH are present at equilibrium.

Reactions:

1) $2\,H_2O \Leftrightarrow 2\,H_2 + O_2$ 2) $2\,H_2O \Leftrightarrow H_2 + 2\,OH$

change -2a +2a +a change -2b +b +2b

3) $H_2 \Leftrightarrow 2\,H$

change -c +2c

At equilibrium (3000 K, 100 kPa)

n_{H2O} =	1-2a-2b	n_{O2} =	a	n_H =	2c
n_{H2} =	2a+b-c	n_{OH} =	2b	n_{TOT} =	1+a+b+c

$$\frac{K_1}{(P/P^0)} = \frac{2.062\times10^{-3}}{1} = \left(\frac{2a+b-c}{1-2a-2b}\right)^2 \left(\frac{a}{1+a+b+c}\right)$$

$$\frac{K_2}{(P/P^0)} = \frac{2.893\times10^{-3}}{1} = \left(\frac{2a+b-c}{1+a+b+c}\right)\left(\frac{2b}{1-2a-2b}\right)^2$$

$$\frac{K_3}{(P/P^0)} = \frac{2.496\times10^{-2}}{1} = \frac{(2c)^2}{(2a+b-c)(1+a+b+c)}$$

These three equations must be solved simultaneously for a, b & c:

a = 0.0622, b = 0.0570, c = 0.0327

and $n_{H2O} = 0.7616$ $y_{H2O} = 0.6611$

$n_{H2} = 0.1487$ $y_{H2} = 0.1291$

$n_{O2} = 0.0622$ $y_{O2} = 0.0540$

$n_{OH} = 0.1140$ $y_{OH} = 0.0990$

$n_H = 0.0654$ $y_H = 0.0568$

16.73

Water from the combustion of hydrogen and pure oxygen is at 3800 K and 50 kPa. Assume we only have H_2O, O_2, OH and H_2 as gases with the two simple water dissociation reactions active find the equilibrium composition.

This problem is very similar to Example 16.7 in the text. The only difference is that we have T = 3800 K and P = 50 kPa. From table A.11 we have

$$\ln K_1 = -1.906\;; \quad K_1 = 0.14867; \qquad \ln K_2 = -0.984\;; \quad K_2 = 0.3738$$

$$K_1 = \left(\frac{2a + b}{1 - 2a - 2b}\right)^2 \frac{a}{1 + a + b}\left(\frac{P}{P^0}\right); \quad K_2 = \frac{2a + b}{1 + a + b}\left(\frac{2b}{1 - 2a - 2b}\right)^2\left(\frac{P}{P^0}\right)$$

So we have two equations as

$$\left(\frac{2a + b}{1 - 2a - 2b}\right)^2 \frac{a}{1 + a + b} = K_1 \,/\left(\frac{P}{P^0}\right) = 0.29734 \qquad (1)$$

$$\frac{2a + b}{1 + a + b}\left(\frac{2b}{1 - 2a - 2b}\right)^2 = K_2 \,/\left(\frac{P}{P^0}\right) = 0.7476 \qquad (2)$$

Divide the second equation by the first to give

$$\frac{4b^2}{(2a + b)\, a} = \frac{0.7476}{0.29734} = 2.5143$$

or

$$2a^2 + ba - 1.5909\, b^2 = 0$$

$$a = -(b/4) \pm (1/4)\sqrt{b^2 - 4 \times 2 \times (-1.5909\, b^2)} = 0.676256\, b$$

Now we can do trial and error on equation 1 for only one variable, say b:

$$a = 0.14228, \quad b = 0.2104$$

$n_{H2O} = 1 - 2a - 2b = 0.29464, \quad n_{H2} = 2a + b = 0.49496,$

$n_{O2} = a = 0.14228, \quad n_{OH} = 2b = 0.4208$

16.74

Methane is burned with theoretical oxygen in a steady flow process, and the products exit the combustion chamber at 3200 K, 700 kPa. Calculate the equilibrium composition at this state, assuming that only CO_2, CO, H_2O, H_2, O_2, and OH are present.

Combustion: $CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$

Dissociation reactions:

1) $2\,H_2O \Leftrightarrow 2\,H_2 + O_2$ change $-2a$ $+2a$ $+a$

2) $2\,H_2O \Leftrightarrow H_2 + 2\,OH$ change $-2b$ $+b$ $+2b$

3) $2\,CO_2 \Leftrightarrow 2\,CO + O_2$ change $-2c$ $+2c$ $+c$

At equilibrium:

$N_{H2O} =$ 2-2a-2b $\quad n_{O2} =$ a+c $\quad n_{CO2} =$ 1-2c

$N_{H2} =$ 2a+b $\quad n_{OH} =$ 2b $\quad n_{CO} =$ 2c

$n_{TOT} =$ 3+a+b+c

Products at 3200 K, 700 kPa, from A.11

$$K_1 = 0.007\,328 = \left(\frac{2a+b}{2-2a-2b}\right)^2 \left(\frac{a+c}{3+a+b+c}\right) \left(\frac{700}{100}\right)$$

$$K_2 = 0.012\,265 = \left(\frac{2b}{2-2a-2b}\right)^2 \left(\frac{2a+b}{3+a+b+c}\right) \left(\frac{700}{100}\right)$$

$$K_3 = 0.426\,135 = \left(\frac{2c}{1-2c}\right)^2 \left(\frac{a+c}{3+a+b+c}\right) \left(\frac{700}{100}\right)$$

These 3 equations must be solved simultaneously for a, b, & c. If solving by hand divide the first equation by the second, and solve for c = fct(a,b). This reduces the solution to 2 equations in 2 unknowns. Solving,

$$a = 0.024,\ b = 0.1455,\ c = 0.236$$

Substance:	H_2O	H_2	O_2	OH	CO_2	CO
n	1.661	0.1935	0.260	0.291	0.528	0.472
y	0.4877	0.0568	0.0764	0.0855	0.1550	0.1386

16.75

Butane is burned with 200% theoretical air, and the products of combustion, an equilibrium mixture containing only CO_2, H_2O, O_2, N_2, NO, and NO_2, exit from the combustion chamber at 1400 K, 2 MPa. Determine the equilibrium composition at this state.

Combustion:

$$C_4H_{10} + 13\,O_2 + 48.9\,N_2 \;\rightarrow\; 4\,CO_2 + 5\,H_2O + 6.5\,O_2 + 48.9\,N_2$$

Dissociation:

$$1)\quad N_2 + O_2 \Leftrightarrow 2\,NO \qquad\qquad 2)\quad N_2 + 2O_2 \Leftrightarrow 2\,NO_2$$

$$\text{change}\quad -a \quad -a \quad +2a \qquad \text{change}\quad -b \quad -2b \quad +2b$$

At equilibrium:

$n_{H2O} = 5$ $\quad n_{N2} = 48.9\text{-a-b}$ $\quad n_{NO} = 2a$

$n_{CO2} = 4$ $\quad n_{O2} = 6.5\text{-a-2b}$ $\quad n_{NO2} = 2b$

$n_{TOT} = 64.4\text{-b}$

At 1400 K, from A.11: $\quad K_1 = 3.761\times10^{-6}$, $\;K_2 = 9.026\times10^{-10}$

$$K_1 = \frac{(2a)^2}{(48.9\text{-a-b})(6.5\text{-a-2b})}\;; \qquad K_2 = \frac{(2b)^2(64.4\text{-b})}{(6.5\text{-a-2b})^2(48.9\text{-a-b})}\left(\frac{P}{P^0}\right)^{-1}$$

As K_1 and K_2 are both very small, with $K_2 << K_1$, the unknowns a & b will both be very small, with b << a. From the equilibrium eq.s, for a first trial

$$a \sim \frac{1}{2}\sqrt{K_1\times48.9\times6.5} \sim 0.0173\;; \quad b \sim \frac{1}{2}\times6.5\sqrt{K_2\times\frac{2}{0.1}\times\frac{48.9}{64.4}} \sim 0.000\,38$$

Then by trial & error,

$$\frac{a^2}{(48.9\text{-a-b})(6.5\text{-a-2b})} = \frac{3.761\times10^{-6}}{4} = 0.940\,25\times10^{-6}$$

$$\frac{b^2(64.4\text{-b})}{(6.5\text{-a-2b})^2(48.9\text{-a-b})} = \frac{9.026\times10^{-10}\times(\frac{2}{0.1})}{4} = 45.13\times10^{-10}$$

Solving, $\;a = 0.017\,27$, $\;b = 0.000\,379$

$n_{CO2} = 4$, $\quad n_{H2O} = 5$, $\quad n_{N2} = 48.882$, $\quad n_{O2} = 6.482$,

$y_{CO2} = 0.062\,11$, $\quad y_{H2O} = 0.077\,64$, $\quad y_{N2} = 0.759\,04$, $\quad y_{O2} = 0.100\,65$

$n_{NO} = 0.034\,54$, $\quad n_{NO2} = 0.000\,76$

$y_{NO} = 0.000\,55$, $\quad y_{NO2} = 0.000\,01$

16.76

One kilomole of air (assumed to be 78% nitrogen, 21% oxygen, and 1% argon) at room temperature is heated to 4000 K, 200 kPa. Find the equilibrium composition at this state, assuming that only N_2, O_2, NO, O, and Ar are present.

1 kmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) heated to

4000 K, 200 kPa.

Equil.:

1) $N_2 + O_2 \Leftrightarrow 2\,NO$

change -a -a +2a

2) $O_2 \Leftrightarrow 2\,O$

change -b +2b

$n_{N2} = 0.78\text{-}a$

$n_{O2} = 0.21\text{-}a\text{-}b$

$n_{Ar} = 0.01$

$n_O = 2b$

$n_{NO} = 2a$

$n_{tot} = 1+b$

$$K_1 = 0.0895 = \frac{4a^2}{(0.78\text{-}a)(0.21\text{-}a\text{-}b)}\left(\frac{200}{100}\right)^0$$

$$K_2 = 2.221 = \frac{4b^2}{(1+b)(0.21\text{-}a\text{-}b)}\left(\frac{200}{100}\right)$$

Divide 1st eq'n by 2nd and solve for a as function(b), using

$$X = \frac{K_1}{K_2}\left(\frac{P}{P^0}\right) = 0.0806$$

Get

$$a = \frac{Xb^2}{2(1+b)}\left[-1+\sqrt{1+\frac{4\times0.78(1+b)}{Xb^2}}\right] \qquad (1)$$

Also

$$\frac{b^2}{(1+b)(0.21\text{-}a\text{-}b)} = \frac{K_2}{4(P/P^0)} = 0.277\,63 \qquad (2)$$

Assume b = 0.1280 From (1), get a = 0.0296

Then, check a & b in (2) ⇒ OK

Therefore,

$n_{N2} = 0.7504$ $n_O = 0.2560$ $y_{N2} = 0.6652$ $y_O = 0.2269$

$n_{O2} = 0.0524$ $n_{NO} = 0.0592$ $y_{O2} = 0.0465$ $y_{NO} = 0.0525$

$n_{Ar} = 0.01$ $y_{Ar} = 0.0089$

16.77

Acetylene gas and *x* times theoretical air (*x* > 1) at room temperature and 500 kPa are burned at constant pressure in an adiabatic steady flow process. The flame temperature is 2600 K, and the combustion products are assumed to consist of N_2, O_2, CO_2, H_2O, CO, and NO. Determine the value of *x*.

Combustion:

$$C_2H_2 + 2.5x\, O_2 + 9.4x\, N_2 \rightarrow 2\, CO_2 + H_2O + 2.5(x\text{-}1)O_2 + 9.4x\, N_2$$

Eq. products 2600 K, 500 kPa: N_2, O_2, CO_2, H_2O, CO & NO

2 Reactions: 1) $2\, CO_2 \Leftrightarrow 2\, CO + O_2$ 2) $N_2 + O_2 \Leftrightarrow 2\, NO$

change -2a +2a +a change -b -b +2b

Equil. Comp.: $n_{N2} = 9.4x\text{-}b$, $n_{H2O} = 1$, $n_{CO} = 2a$, $n_{NO} = 2b$

$n_{O2} = 2.5x - 2.5 + a - b$, $n_{CO2} = 2 - 2a$, $n_{TOT} = 11.9x + 0.5 + a$

At 2600 K, from A.11: $K_1 = 3.721 \times 10^{-3}$, $K_2 = 4.913 \times 10^{-3}$

$$\text{EQ1:} \qquad \frac{K_1}{(P/P^o)} = \frac{3.721\times10^{-3}}{5} = \left(\frac{a}{1 - a}\right)^2 \left(\frac{2.5x - 2.5 + a - b}{11.9x + 0.5 + a}\right)$$

$$\text{EQ2:} \qquad K_2 = 4.913\times10^{-3} = \frac{(2b)^2}{(9.4 - b)(2.5x - 2.5 + a - b)}$$

Also, from the energy Eq.: $H_P - H_R = 0$ where

$H_R = 1(+226\ 731) + 0 + 0 = +226\ 731$ kJ

$H_P = (9.4x - b)(0 + 77\ 963) + (2.5x - 2.5 + a - b)(0 + 82\ 225)$

$+ (2 - 2a)(-393\ 522 + 128\ 074) + 1(-241\ 826 + 104\ 520)$

$+ 2a(-110\ 527 + 78\ 679) + 2b(90\ 291 + 80\ 034)$

Substituting H_R and H_P into the energy equation,

EQ3: $988\ 415x + 549\ 425a + 180\ 462b - 1\ 100\ 496 = 0$

which results in 3 equations in the 3 unknowns x, a, b. Assume x = 1.07 then

$$\text{EQ1:} \quad 7.442 \times 10^{-2} = \left(\frac{a}{1\text{-}a}\right)^2 \left(\frac{0.175 + a - b}{13.233 + a}\right)$$

$$\text{EQ2:} \quad 1.2283 \times 10^{-3} = \frac{b^2}{(10.058 - b)(0.175 + a + b)}$$

Solving, a = 0.1595, b = 0.0585 Then checking in EQ3,

$$988\ 415\times1.07 + 549\ 425\times0.1595 + 180\ 462\times0.0585 - 1\ 100\ 496 \approx 0$$

Therefore, x = **1.07**

Gasification

16.78

One approach to using hydrocarbon fuels in a fuel cell is to “reform” the hydrocarbon to obtain hydrogen, which is then fed to the fuel cell. As a part of the analysis of such a procedure, consider the reforming section and determine the equilibrium constant for this reaction at a temperature of 800 K.

The equilibrium constant From Eq.16.15 depends on ΔG^0 and T.

For CH_4, use C_{P0} at average temperature 550 K $\approx \frac{1}{2}(298 + 800)$ K.

Table A.6, $\bar{C}_{P0} = 49.316$ kJ/kmol K

$$\bar{h}^0_{800\text{ K}} = \bar{h}^0_f + \bar{C}_{P0}\Delta T = -74\,873 + 49.316\,(800 - 298.2)$$

$$= -50\,126 \text{ kJ/kmol}$$

$$\bar{s}^0_{800\text{ K}} = 186.251 + 49.316 \ln\frac{800}{298.2} = 234.918 \text{ kJ/kmol K}$$

For $\quad CH_4 + H_2O \Leftrightarrow 3H_2 + CO$

The rest of the properties are from Table A.9 at 800 K.

$$\Delta H^0_{800\text{ K}} = 3(0 + 14\,681) + 1(-110\,527 + 15\,174) - 1(-50\,126)$$

$$- 1(-241\,826 + 18\,002) = +222\,640 \text{ kJ/kmol}$$

$$\Delta S^0_{800\text{ K}} = 3(159.554) + 1(227.277) - 1(234.918) - 1(223.826)$$

$$= +247.195 \text{ kJ/kmol K}$$

$$\Delta G^0 = \Delta H^0 - T\Delta S^0 = 222\,640 - 800(247.195) = +24\,884 \text{ kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = \frac{-24\,884}{8.3145\times 800} = -3.7411 \quad => \quad K = \mathbf{0.0237}$$

16.79

A coal gasifier produces a mixture of 1 CO and $2H_2$ that is fed to a catalytic converter to produce methane. The reaction is $CO + 3H_2 \Leftrightarrow CH_4 + H_2O$. The equilibrium constant at 600 K is $K = 1.83 \times 10^6$. What is the composition of the exit flow assuming a pressure of 600 kPa?

The reaction equation is:

	CO	+	$3\,H_2$	$\Leftrightarrow$	CH_4	+	H_2O	
initial	1		2		0		0	
change	$-x$		$-3x$		$+x$		$+x$	
equil.	$1 - x$		$2 - 3x$		x		x	$n_{TOTAL} = 3 - 2x$

$$K = \frac{y_{CH_4}\, y_{H_2O}}{y_{H_2}^3\, y_{CO}} \left(\frac{P}{P^o}\right)^{1+1-1-3} = \frac{x^2\,(3 - 2x)^2}{(1-x)(2 - 3x)^3} \left(\frac{P}{P^o}\right)^{-2}$$

$$1.83 \times 10^6 \times \left(\frac{600}{100}\right)^2 = 6.588 \times 10^7 = \frac{x^2\,(3 - 2x)^2}{(1-x)(2 - 3x)^3}$$

Trial and error to solve for x.

$x = 0.6654$ LHS $= 6.719 \times 10^7$

$x = 0.66538$ LHS $= 6.41 \times 10^7$

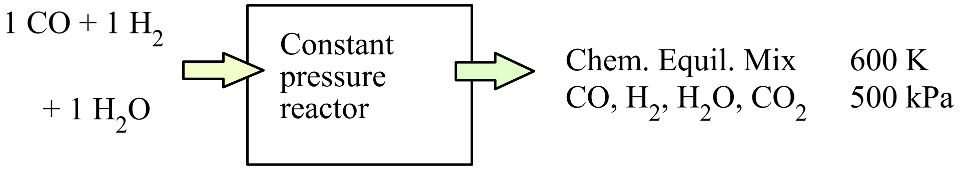
$x = 0.66539$ LHS $= 6.562 \times 10^7$ close enough

$n_{CH4} = 0.66539$, $n_{H2O} = 0.66539$, $n_{CO} = 0.66539$, $n_{H2} = 0.00383$

so we used up nearly all the hydrogen gas.

16.80

Gasification of char (primarily carbon) with steam following coal pyrolysis yields a gas mixture of 1 kmol CO and 1 kmol H_2. We wish to upgrade the hydrogen content of this syngas fuel mixture, so it is fed to an appropriate catalytic reactor along with 1 kmol of H_2O. Exiting the reactor is a chemical equilibrium gas mixture of CO, H_2, H_2O, and CO_2 at 600 K, 500 kPa. Determine the equilibrium composition. Note: see Example 16.4.



$$(1)\ 1\,CO + 1\,H_2O \Leftrightarrow 1\,CO_2 + 1\,H_2$$

$$-x \quad -x \quad +x \quad +x$$

$$(2)\ 2\,H_2O \Leftrightarrow 2\,H_2 + 1\,O_2 \qquad\qquad (3)\ 2\,CO_2 \Leftrightarrow 2\,CO + 1\,O_2$$

$$(1) = \frac{1}{2}(2) - \frac{1}{2}(3)$$

From Table A.11 at 600 K

$$\ln K_1 = \frac{1}{2}\big[-85.79 - (-92.49)\big] = +3.35, \qquad K_1 = 28.503$$

Equilibrium:

$$n_{CO} = 1 - x, \quad n_{H2O} = 1 - x, \quad n_{CO2} = 0 + x, \quad n_{H2} = 1 + x$$

$\sum n = 3$, notice the reaction is pressure insensitive.

$$K = \frac{y_{CO2}\,y_{H2}}{y_{CO}\,y_{H2O}}\left(\frac{P}{P^0}\right)^0 = \frac{y_{CO2}\,y_{H2}}{y_{CO}\,y_{H2O}} = \frac{x(1+x)}{(1-x)^2} = 28.503$$

Solve for x: $x = 0.7794$

	n	y	%
CO	0.2206	0.0735	7.35
H_2O	0.2206	0.0735	7.35
CO_2	0.7794	0.2598	26.0
H_2	1.7794	0.5932	59.3

16.81

The equilibrium reaction as: $CH_4 \Leftrightarrow C + 2H_2$. has ln K = -0.3362 at 800 K and ln$K$ = -4.607 at 600 K. By noting the relation of K to temperature show how you would interpolate ln K in (1/T) to find K at 700 K and compare that to a linear interpolation.

A.11: ln K = – 0.3362 at 800K ln K = –4.607 at 600K

$$\ln K_{700} = \ln K_{800} + \frac{\frac{1}{700} - \frac{1}{800}}{\frac{1}{600} - \frac{1}{800}} \times (-4.607 + 0.3362)$$

$$= -0.3362 + \frac{\frac{800}{700} - 1}{\frac{800}{600} - 1} \times (-4.2708) \quad = \mathbf{-2.1665}$$

Linear interpolation:

$$\ln K_{700} = \ln K_{600} + \frac{700 - 600}{800 - 600} (\ln K_{800} - \ln K_{600})$$

$$= -4.607 + \frac{1}{2} (-0.3362 + 4.607) = \mathbf{-2.4716}$$

16.82

One approach to using hydrocarbon fuels in a fuel cell is to "reform" the hydrocarbon to obtain hydrogen, which is then fed to the fuel cell. As a part of the analysis of such a procedure, consider the reaction $CH_4 + H_2O \Leftrightarrow CO + 3H_2$. One kilomole each of methane and water are fed to a catalytic reformer. A mixture of CH_4, H_2O, H_2, and CO exits in chemical equilibrium at 800 K, 100 kPa; determine the equilibrium composition of this mixture using an equilibrium constant of K = 0.0237.

The reaction equation is:

	CH_4	+	H_2O	$\Leftrightarrow$	$3\,H_2$	+	CO
initial	1		1		0		0
change	-x		-x		+3x		+x
equil.	(1-x)		(1-x)		3x		x

$n_{TOTAL} = 1 - x + 1 - x + 3x + x = 2 + 2x$

$$K = \frac{y_{H2}^3 y_{CO}}{y_{CH4} y_{H2O}} \left(\frac{P}{P^0}\right)^2 = \frac{(3x)^3 x}{(1 - x)(1 - x)(2 + 2x)^2} \left(\frac{100}{100}\right)^2$$

or $$\left(\frac{x}{1 - x}\right)^2 \left(\frac{x}{1 + x}\right)^2 = \frac{4\times0.0237}{27\times1} = 0.003\ 51$$

or $$\frac{x^2}{1 - x^2} = \sqrt{0.003\ 51} = 0.059\ 25 \qquad \text{Solving,} \quad x = 0.2365$$

$$\left.\begin{array}{l} n_{CH4} = 0.7635 \\ n_{H2O} = 0.7635 \\ n_{H2} = 0.7095 \\ n_{CO} = 0.2365 \\ n_{TOT} = 2.473 \end{array}\right\} \left\{\begin{array}{l} y_{CH4} = 0.3087 \\ y_{H2O} = 0.3087 \\ y_{H2} = 0.2870 \\ y_{CO} = 0.0956 \end{array}\right.$$

16.83

Consider a gasifier that receives 4 kmol of carbon-monoxide, 3 kmol hydrogen and 3.76 kmol nitrogen and brings the mixture to equilibrium at 900 K, 1 MPa with the following reaction:

$$2\ CO + 2\ H_2 \ \Leftrightarrow\ CH_4 + CO_2$$

which is the sum of Eq.16.32 and 16.33. If the equilibrium constant is K = 2.679 find the exit composition.

The reaction takes place with the nitrogen as a dilutant.

reaction	2 CO	+	$2\ H_2$	$\Leftrightarrow$	CH_4	+	CO_2	also N_2
initial	4		3		0		0	3.76
change	-2x		-2x		+x		+x	0
equil.	(4-2x)		(3-2x)		x		x	3.76

$n_{TOT} = 10.76 - 2x$

$$K = \frac{y_{CH4}\, y_{CO2}}{y_{CO}^2\, y_{H2}^2} \left(\frac{P}{P^0}\right)^{-2} = \frac{x \times x \times (10.76 - 2x)^2}{(4 - 2x)^2 (3 - 2x)^2} \left(\frac{P}{P^0}\right)^{-2}$$

or take the square-root to get

$$\frac{x(10.76 - 2x)}{(4 - 2x)(3 - 2x)} = \frac{P}{P^0}\sqrt{K} = \frac{1}{0.1}\sqrt{2.679} = 16.368$$

By trial & error or solve the quadratic eq. in x, $x = 1.2781$

$n_{CO} = 1.444$, $n_{H2} = 0.444$, $n_{CH4} = n_{CO2} = 1.278$, $n_{N2} = 3.76$

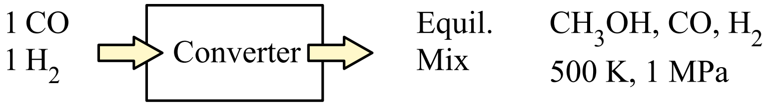
$y_{CO} = 0.176$, $y_{H2} = 0.054$, $y_{CH4} = y_{CO2} = 0.156$, $y_{N2} = 0.458$

16.84

Consider the production of a synthetic fuel (methanol) from coal. A gas mixture of 50% CO and 50% H_2 leaves a coal gasifier at 500 K, 1 MPa, and enters a catalytic converter. A gas mixture of methanol, CO and H_2 in chemical equilibrium with the reaction: $CO + 2H_2 \Leftrightarrow CH_3OH$ leaves the converter at the same temperature and pressure, where it is known that ln K = -5.119.

a. Calculate the equilibrium composition of the mixture leaving the converter.

b. Would it be more desirable to operate the converter at ambient pressure?



Reaction:	CO +	$2\,H_2$ ⇔	CH_3OH
initial	1	1	0
change	-x	-2x	+x
equil.	(1-x)	(1-2x)	x

a) $$K = \frac{y_{CH3OH}}{y_{CO}\,y_{H2}^{2}}\left(\frac{P}{P^0}\right)^{-2} = \left(\frac{x}{1-x}\right)\left(\frac{2-2x}{1-2x}\right)^{2}\left(\frac{P}{P^0}\right)^{-2} \quad => \quad \frac{x(1-x)}{(1-2x)^2} = \frac{K}{4}\left(\frac{P}{P^0}\right)^{2}$$

ln K = -5.119, K = 0.005 98

$$\frac{x(1-x)}{(1-2x)^2} = \frac{0.005\,98}{4}\left(\frac{1}{0.1}\right)^{2} = 0.1495 \quad => \quad x = 0.1045$$

$n_{CH3OH} = x = 0.1045$, $n_{CO} = 1 - x = 0.8955$, $n_{H2} = 1 - 2x = 0.791$

$y_{CH3OH} = 0.0583$, $y_{CO} = 0.5000$, $y_{H2} = \mathbf{0.4417}$

b) For P = 0.1 MPa

$$\frac{x(1-x)}{(1-2x)^2} = \frac{0.005\,98}{4}\left(\frac{0.1}{0.1}\right)^{2} = 0.001\,495$$

x is much smaller (~ 0.0015) **not good**

Ionization

16.85

At 10 000 K the ionization reaction for Ar is: Ar $\Leftrightarrow$ $Ar^+ + e^-$ with equilibrium constant of $K = 4.2 \times 10^{-4}$. What should the pressure be for a mole concentration of argon ions (Ar^+) of 10%?

From the reaction (ionization) we recognize that the concentration of electrons must equal that of argon ions so

$$y_{Ar^+} = y_{e^-} = 0.1 \qquad \text{and} \qquad y_{Ar} = 1 - y_{Ar^+} - y_{e^-} = 0.8$$

Now

$$K = 4.2 \times 10^{-4} = \frac{y_{Ar^+}\, y_{e^-}}{y_{Ar}} \left(\frac{P}{P^o}\right)^{1+1-1} = \frac{0.1 \times 0.1}{0.8} \frac{P}{100}$$

$$P = 0.00042 \times \frac{0.8}{0.1 \times 0.1} \times 100 = \mathbf{3.36\ kPa}$$

16.86

Repeat the previous problem assuming the argon constitutes 1% of a gas mixture where we neglect any reactions of the other gases and find the pressure that will give a mole-concentration of Ar^+ of 0.1%.

Previous problem:

At 10 000 K the ionization reaction for Ar is: $Ar \Leftrightarrow Ar^+ + e^-$ with equilibrium constant of $K = 4.2 \times 10^{-4}$. What should the pressure be for a mole concentration of argon ions (Ar^+) of 10%?

	$Ar \Leftrightarrow$	Ar^+	$+ e^-$	others	
Initial:	0.01	0	0	0.99	
Change	-x	x	x	0	
Final	0.01-x	x	x	0.99	Total = 1 + x

From the reaction (ionization) we recognize that the concentration of electrons must equal that of argon ions so

$$y_{Ar^+} = y_{e^-} = 0.001 = x / (1+x) \quad \Rightarrow \quad x = 0.001001$$

$$y_{Ar} = (0.01 - x)/(1 + x) = 0.00899$$

Now

$$K = 4.2 \times 10^{-4} = \frac{y_{Ar^+}\, y_{e^-}}{y_{Ar}} \left(\frac{P}{P^o}\right)^{1+1-1} = \frac{0.001 \times 0.001}{0.00899} \frac{P}{100}$$

$$P = 0.00042 \times 8990 \times 100 = \mathbf{377\ kPa}$$

16.87

Operation of an MHD converter requires an electrically conducting gas. It is proposed to use helium gas "seeded" with 1.0 mole percent cesium, as shown in Fig. P16.87. The cesium is partly ionized ($Cs \Leftrightarrow Cs^+ + e^-$) by heating the mixture to 1800 K, 1 MPa, in a nuclear reactor to provide free electrons. No helium is ionized in this process, so that the mixture entering the converter consists of He, Cs, Cs^+, and e^-. Determine the mole fraction of electrons in the mixture at 1800 K, where ln *K* = 1.402 for the cesium ionization reaction described.

Reaction:	$Cs \Leftrightarrow$	$Cs^+ +$	e^-,	Also He	ln K = 1.402
initial	0.01	0	0	0.99	=> K = 4.0633
change	-x	+x	+x	0	
Equil	(0.01-x)	x	x	0.99 ;	total: 1 + x

$$K = \frac{y_{e-}\, y_{Cs+}}{y_{Cs}} \left(\frac{P}{P^0}\right) = \left(\frac{x}{0.01 - x}\right) \left(\frac{x}{1 + x}\right) \left(\frac{P}{P^0}\right)$$

$$\text{or} \quad \left(\frac{x}{0.01 - x}\right) \left(\frac{x}{1 + x}\right) = 4.0633 / (1/0.1) = 0.40633$$

Quadratic equation: $x = 0.009767$

$$\Rightarrow \quad y_{e-} = \frac{x}{1 + x} = \mathbf{0.00967}$$

16.88

One kilomole of argon gas at room temperature is heated to 20 000 K, 100 kPa. Assume that the plasma in this condition consists of an equilibrium mixture of Ar, Ar , Ar , and e^{-} according to the simultaneous reactions

(1) Ar $\Leftrightarrow$ $Ar^{+} + e^{-}$ (2) Ar^{+} $\Leftrightarrow$ $Ar^{++} + e^{-}$

The ionization equilibrium constants for these reactions at 20 000 K have been calculated from spectroscopic data as ln K_1 = 3.11 and ln K_2 = -4.92. Determine the equilibrium composition of the plasma.

1) $Ar \Leftrightarrow Ar^{+} + e^{-}$ 2) $Ar^{+} \Leftrightarrow Ar^{++} + e^{-}$

ch. -a +a +a ch. -b +b +b

Equil. Comp.: n_{Ar} = 1-a, n_{Ar+} = a-b, n_{Ar++} = b, n_{e-} = a+b, n_{TOT} = 1+a+b

$$K_1 = \frac{y_{Ar+}y_{e-}}{y_{Ar}}\left(\frac{P}{P^0}\right) = \frac{(a - b)(a + b)}{(1 - a)(1 + a + b)}(1) = 22.421$$

$$K_2 = \frac{y_{Ar++}y_{e-}}{y_{Ar+}}\left(\frac{P}{P^0}\right) = \frac{b(a + b)}{(a - b)(1 + a + b)}(1) = 0.0073$$

By trial & error: a = 0.978 57, b = 0.014 13

n_{Ar} = 0.02143, n_{Ar+} = 0.96444, n_{Ar++} = 0.01413, n_{e-} = 0.9927

y_{Ar} = 0.0107, y_{Ar+} = 0.484, y_{Ar++} = 0.0071, y_{e-} = 0.4982

16.89

At 10 000 K the two ionization reactions for N and Ar as

1: $\text{Ar} \Leftrightarrow \text{Ar}^+ + e^-$ 2: $\text{N} \Leftrightarrow \text{N}^+ + e^-$

have equilibrium constants of $K_1 = 4.2 \times 10^{-4}$ and $K_2 = 6.3 \times 10^{-4}$, respectively. If we start out with 1 kmol Ar and 0.5 kmol N_2, what is the equilibrium composition at a pressure of 10 kPa?

At 10 000 K we assume all the nitrogen is dissociated to N.

Assume we shift the argon ionization with a and the nitrogen ionization with b we get

	Ar	Ar^+	e^-	N	N^+	
Initial	1	0	0	1	0	
Change	-a	a	a + b	-b	b	
Final	1-a	a	a + b	1-b	b	Tot: 2 + a + b

$$K_1 = 4.2 \times 10^{-4} = \frac{y_{Ar+}y_{e-}}{y_{Ar}}\left(\frac{P}{P^0}\right) = \frac{a\,(a + b)}{(1 - a)(2 + a + b)}\left(\frac{10}{100}\right) \quad (1)$$

$$K_2 = 6.3 \times 10^{-4} = \frac{y_{N+}y_{e-}}{y_N}\left(\frac{P}{P^0}\right) = \frac{b\,(a + b)}{(1 - b)(2 + a + b)}\left(\frac{10}{100}\right) \quad (2)$$

Divide the second equation with the first to get

$$\frac{b}{(1 - b)}\frac{(1 - a)}{a} = \frac{K_2}{K_1} = 1.5 \quad \Rightarrow \quad \frac{b - ab}{a - ab} = 1.5$$

$$b - ab = 1.5\,a - 1.5\,ab \quad \Rightarrow \quad b = 1.5\,a - 0.5\,ab = a(1.5 - 0.5\,b)$$

$$a = \frac{b}{1.5 - 0.5\,b} \quad \text{trial and error on equation (1)}$$

$a = 0.059$ and $b = 0.086$

$n_{Ar} = 0.941$, $n_{Ar+} = 0.059$, $n_N = 0.914$, $n_{N+} = 0.086$, $n_{e-} = 0.145$

$y_{Ar} = 0.439$, $y_{Ar+} = 0.027$, $y_N = 0.426$, $y_{N+} = 0.04$, $y_{e-} = 0.068$

16.90

Plot to scale the equilibrium composition of nitrogen at 10 kPa over the temperature range 5000 K to 15 000 K, assuming that N_2, N, N^+, and e^- are present. For the ionization reaction $N \Leftrightarrow N^+ + e^-$, the ionization equilibrium constant *K* has been calculated from spectroscopic data as

T [K]	10000	12 000	14 000	16 000
100*K*	0.0626	1.51	15.1	92

$$1)\ N_2 \Leftrightarrow 2N \qquad\qquad 2)\ N \Leftrightarrow N^+ + e^-$$

$$\text{change} \quad -a \qquad +2a \qquad \text{change} \quad -b \qquad +b \quad +b$$

Equil. Comp.: $n_{N2} = 1 - a$, $n_N = 2a - b$, $n_{N+} = b$, $n_{e-} = b$

EQ1: $$K_1 = \frac{y_N^2}{y_{N2}}\left(\frac{P}{P^0}\right) = \frac{(2a - b)^2}{(1 - a)(1 + a + b)}\left(\frac{P}{P^0}\right)$$

EQ2: $$K_2 = \frac{y_{N+}y_{e-}}{y_N}\left(\frac{P}{P^0}\right) = \frac{b^2}{(2a - b)(1 + a + b)}\left(\frac{P}{P^0}\right)$$

For T < 10 000 K: b ~ 0 so neglect EQ2: $\Rightarrow\ K_1 = \frac{4a^2}{(1-a^2)}\left(\frac{10}{100}\right)$

To extrapolate K_1 above 6000 K: $\ln K_1 \approx 16.845 - \frac{118\ 260}{T}$

(from values at 5000 K & 6000 K)

T(K)	K_1	a	y_N	y_{N2}
5000	0.0011	0.0524	0.0996	0.9004
6000	0.0570	0.3532	0.5220	0.4780
7000	0.9519	0.8391	0.9125	0.0875
8000	7.866	0.9755	0.9876	0.0124
10000	151.26	0.9987	0.9993	0.0007

For T > 10 000 K: $a \approx 1.0$ => $K_2 = \frac{b^2}{(2-b)(2+b)}\left(\frac{10}{100}\right) = \frac{b^2}{(4-b^2)}\,0.1$

T(K)	K_2	b	y_N	y_{N+}
10 000	6.26×10^{-4}	0.1577	0.8538	0.0731
12 000	1.51×10^{-2}	0.7244	0.4862	0.2659
14 000	0.151	1.5512	0.1264	0.4368
16 000	0.92	1.8994	0.0258	0.4871

Note that $b \approx 0$ is not a very good approximation in the vicinity of 10 000 K. In this region, it would be better to solve the original set simultaneously for a & b. The answer would be approximately the same.

Applications

16.91

Are the three reactions in the Zeldovich mechanism pressure sensitive if we look at equilibrium conditions?

No. All three reactions have two moles on the left and right hand sides and the net power to the pressure correction term is zero.

16.92

Assume air is at 3000 K, 1 MPa. Find the time constant for the NO formation. Repeat for 2000 K, 800 kPa.

From the rate in Eq.16.40: $\tau_{NO} = 8 \times 10^{-16}\ T\ (P_o/P)^{1/2} \exp(\frac{58\ 300}{T})$

Case a: $\tau_{NO} = 8 \times 10^{-16} \times 3000\ (100/1000)^{1/2} \exp(\frac{58\ 300}{3000})$

$= \mathbf{2.09 \times 10^{-4}\ s} = 0.209$ ms

Case b: $\tau_{NO} = 8 \times 10^{-16} \times 2000\ (100/800)^{1/2} \exp(\frac{58\ 300}{2000})$

$= \mathbf{2.58\ s}$

Notice the significant difference in time constants.

16.93

Consider air at 2600 K, 1 MPa. Find the equilibrium concentration of NO neglecting dissociations of oxygen and nitrogen.

1 kmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) at 2600 K, 1 MPa.

N_2 +	$O_2 \Leftrightarrow$	2 NO	Ar	$n_{N2} = 0.78 - x$
0.78	0.21	0	0.01	$n_{O2} = 0.21 - x$
-x	-x	+2x		$n_{NO} = 2x$, $n_{ar} = 0.01$
0.78–x	0.21–x	2x	0.01	$n_{tot} = 1.0$

From A.11 at 2600 K, ln K = -5.316, K = 0.0049124

$$K = \frac{4x^2}{(0.78 - x)(0.21 - x)} \left(\frac{P}{P^0}\right)^0 \quad ⇨$$

$$\frac{x^2}{(0.78 - x)(0.21 - x)} = \frac{0.0049124}{4} = 0.001228 \qquad \text{and} 0 < x < 0.21$$

Solve for x: $x = 0.0136$ ⇨ $y_{NO} = \frac{2x}{1.0} = \mathbf{0.0272}$

16.94

Redo the previous Problem but include the dissociation of oxygen and nitrogen.

1 kmol air (0.78 N_2, 0.21 O_2, 0.01 Ar) at 2600 K, 1 MPa.

(1) $O_2 <=> 2\ O$ $\quad$ lnK = -7.52, $\quad$ $K_1 = 0.000542 = (y_O^2/y_{O2})\ P/P_o$

(2) $N_2 <=> 2\ N$ $\quad$ lnK = -28.313, $\quad$ $K_2 = 5.056 \times 10^{-13} = (y_N^2/y_{N2})\ P/P_o$

(3) $N_2 + O_2 <=> 2\ NO$ $\quad$ lnK = -5.316, $\quad$ $K_3 = 0.00491 = (y_{NO}^2/y_{N2}\, y_{O2})$

Call the shifts a,b,c respectively so we get

$n_{O2} = 0.21 - a - c,\quad n_O = 2a,\quad n_{N2} = 0.78 - b - c,\quad n_N = 2b,$

$n_{NO} = 2c,\quad n_{Ar} = 0.01,\quad n_{tot} = 1 + a + b$

From which the mole fractions are formed and substituted into the three equilibrium equations. The result is

$$0.000542\times 0.1 = y_O^2/y_{O2} = 4a^2 / [(1+a+b)(0.21-a-c)]$$

$$5.056 \times 10^{-14} = y_N^2/y_{N2} = 4b^2 / [(1+a+b)(0.79-b-c)]$$

$$0.00491 = y_{NO}^2/y_{N2}\, y_{O2} = 4c^2 / [(0.79-b-c)(0.21-a-c)]$$

which give 3 eqs. for the unknowns (a,b,c). Trial and error assume b = a = 0 solve for c from the last eq. then for a from the first and finally given the (a,c) solve for b from the second equation. The order chosen according to expected magnitude $K_3 > K_1 > K_2$

$a = 0.001626,\ b = 0.99 \times 10^{-7},\ c = 0.01355 =>$

$n_{O2} = 0.1948,\quad n_O = 0.00325,\quad n_{N2} = 0.7665,\quad n_N = 1.98 \times 10^{-7},$

$n_{NO} = 0.0271,\quad n_{Ar} = 0.01$

$y_{NO} = n_{NO} / n_{tot} = 0.0271 / 1.00165 = \mathbf{0.02706}$

Indeed it is a very small effect to include the additional dissociations.

16.95

Calculate the equilibrium constant for the first reaction in the Zeldovich mechanism at 2600 K, 500 kPa. Notice this is not listed in A.11.

The reaction and equilibrium constant are

$$O + N_2 \Leftrightarrow NO + N$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T}\,; \qquad \Delta G^0 = \Delta H^0 - T\Delta S^0$$

$$\Delta H^0 = \bar{h}_{NO} + \bar{h}_N - \bar{h}_O - \bar{h}_{N2} = (80034 + 90291) + (47860 + 472680)$$
$$- \ (48216 + 249170) - 77963 = 315\,516 \text{ kJ/kmol}$$

$$\Delta S^0 = \bar{s}_{NO} + \bar{s}_N - \bar{s}_O - \bar{s}_{N2}$$
$$= 282.822 + 198.322 - 206.714 - 261.615 = 12.815 \text{ kJ/kmol-K}$$

$$\Delta G^0 = 315\,516 - 2600 \times 12.815 = 282\,197 \text{ kJ/kmol}$$

$$\ln K = -\frac{\Delta G^0}{\bar{R}T} = -\frac{282\,197}{8.314472 \times 2600} = -13.054, \qquad K = \mathbf{2.14 \times 10^{-6}}$$

16.96

Find the equilibrium constant for the reaction: $2NO + O_2 \Leftrightarrow 2NO_2$ from the elementary reaction in Table A.11 to answer these two questions. Which of the nitrogen oxides NO or NO_2 is the more stable at 25°C, 100 kPa? At what T do we have an equal amount of each?

The elementary reactions are the last two ones in Table A.11:

a) $N_2 + O_2 \Leftrightarrow 2\,NO$ b) $N_2 + 2O_2 \Leftrightarrow 2\,NO_2$

Actual reaction is : c = b – a $\Rightarrow$ $\ln(K_c) = \ln(K_b) - \ln(K_a)$

At 25°C (approx. 300 K) Table A.11: $\ln(K_a) = -69.868$; $\ln(K_b) = -41.355$

so now:

$$\ln(K_c) = -41.355 + 69.868 = 28.5 \quad \Rightarrow \quad K_c = 2.4 \times 10^{12}$$

meaning reaction is pushed completely to the right and **NO_2 is the stable** compound. Assume we start at room T with 1 kmol NO_2: then

	NO	O_2	NO_2	TOT
start	0	0	1	
change	2x	x	-2x	
Final	2x	x	1-2x	1+x

Equal amount of each

$$y(NO) = \frac{2x}{1+x} = y(NO_2) = \frac{1 - 2x}{1+x} \quad \Rightarrow \quad x = 0.25$$

$$K(T) = \frac{(1 - 2x)^2}{4x^3} = \frac{0.5^2}{4 \times 0.25^3} = 4 \quad \Rightarrow \quad \ln(K) = 1.386$$

We quickly see

$\ln(K)$ at 500 K $= -30.725 + 40.449 = 9.724$

$\ln(K)$ at 1000 K $= -23.039 + 18.709 = -4.33$

Linear interpolation T = 500 + 0.406 × 500 = **703 K**

16.97

If air at 300 K is brought to 2600 K, 1 MPa, instantly, find the formation rate of NO.

The formation rate of NO is given by Eqs.16.39-40 using the equilibrium concentration in Eq.16.41. If we assume NO formation takes place and that the oxygen and nitrogen have not dissociated their concentrations are (we neglect the argon)

$$N_2 + O_2 \Leftrightarrow 2\ NO$$

$$y_{NOe} = 2x, \qquad y_{O2} = 0.21 - x \quad \text{and} \quad y_{N2} = 0.79 - x$$

At 2600 K from A.11

$$\ln K_4 = -5.316 \qquad \Rightarrow \qquad K_4 = 0.0049124$$

so then we have

$$y_{NOe} = 2x = [K_4\ (0.21 - x)\ (0.79 - x)\ ]^{1/2} \quad \text{solve for } x = 0.0136$$

$$y_{NOe} = 2x = 0.0272$$

From the rate in Eq.16.40: $\tau_{NO} = 8 \times 10^{-16}\ T\ (P_o/P)^{1/2} \exp(\frac{58\ 300}{T})$

$$\tau_{NO} = 8 \times 10^{-16} \times 2600\ (100/1000)^{1/2} \exp(\frac{58\ 300}{2600}) = 0.0036\ \text{s}$$

The formation rate is

$$\frac{dy_{NO}}{dt} = \frac{y_{NOe}}{\tau_{NO}} = \frac{0.0272}{0.0036\ \text{s}} = \mathbf{7.56\ s^{-1}}$$

16.98

Estimate the concentration of oxygen atoms in air at 3000 K, 100 kPa and 0.0001 kPa, Compare this to the result in Fig. 16.11.

For the dissociation of oxygen and nitrogen we get from A.11

O_2: ln K = –4.356, N_2: ln K = –22.367

from which it is evident that we need only to consider oxygen.

O_2	⇔	2 O	N_2	Ar	$n_{N2} = 0.78 - x$
0.21		0	0.78	0.01	$n_{O2} = 0.21 - x$
-x		+2x			$n_{NO} = 2x$, $n_{ar} = 0.01$
0.21–x		2x	0.78	0.01	$n_{tot} = 1 + x$

$$K(T) = \frac{y_O^2}{y_{O2}}\left(\frac{P}{P_o}\right)^{2-1} = \left(\frac{2x}{1+x}\right)^2 \frac{1+x}{0.21-x}\frac{P}{P_o} = \frac{4x^2}{1+x}\frac{1}{0.21-x}\frac{P}{P_o}$$

$$\Rightarrow \quad \frac{4x^2}{1+x}\frac{1}{0.21-x} = \frac{P_o}{P}K(T) = \frac{P_o}{P}\exp(-4.356)$$

$P = 100$ kPa: RHS = 0.01283 $\Rightarrow$ $x = 0.0247$, $y_O = \frac{2x}{1+x} = 0.0482$

$P = 0.0001$ kPa: RHS = 12830 $\Rightarrow$ $x = 0.21$, $y_O = \frac{2x}{1+x} = 0.347$

We notice from Fig. 16.11 that at 3000 K all the oxygen is dissociated so

$n_O / n_{\text{atoms in air}} = 0.21$ so $n_O = 0.42$, $y_O = 0.42/1.21 = 0.347$

since air is mainly diatomic so $n_{\text{atoms in air}} \approx 2$ (scale is confusing).

16.99

At what temperature range does air become a plasma?

From Fig. 16.11, we note that air becomes predominantly ions and electrons, a plasma, at about 10-12 000 K.

Review Problems

16.100

In a test of a gas-turbine combustor, saturated-liquid methane at 115 K is to be burned with excess air to hold the adiabatic flame temperature to 1600 K. It is assumed that the products consist of a mixture of CO_2, H_2O, N_2, O_2, and NO in chemical equilibrium. Determine the percent excess air used in the combustion, and the percentage of NO in the products.

$$CH_4 + 2x\,O_2 + 7.52x\,N_2 \quad\rightarrow\quad 1\,CO_2 + 2\,H_2O + (2x\text{-}2)\,O_2 + 7.52x\,N_2$$

Then	N_2	+ O_2	⇔ 2 NO	Also	CO_2	H_2O
initial	7.52x	2x-2	0		1	2
change	-a	-a	+2a		0	0
final	(7.52x-a)	(2x-2-a)	2a		1	2

$$n_{TOT} = 1 + 9.52x$$

1600 K in A.11: $\ln K = -10.55, \quad K = 2.628\times10^{-5}$

$$2.628\times10^{-5}\ K = \frac{y_{NO}^2}{y_{N2}y_{O2}}\left(\frac{P}{P^0}\right)^0 = \frac{y_{NO}^2}{y_{N2}y_{O2}} = \frac{4a^2}{(7.52x - a)(2x - 2 - a)}$$

From A.9 and B.7,

$$H_R = 1\big[-74\,873 + 16.043(-274.7-624.1)\big] + 0 + 0 = -89\,292\text{ kJ}$$

(Air assumed 25 °C)

$$H_P = 1(-393\,522 + 67\,569) + 2(-241\,826 + 52\,907)$$
$$+ (7.52x\text{-}a)(41\,904) + (2x\text{-}2\text{-}a)(44\,267) + 2a(90\,291 + 43\,319)$$
$$= -792\,325 + 403\,652\,x + 181\,049\,a$$

Assume a ~ 0, then from $H_P - H_R = 0 \rightarrow x = 1.7417$ and substitute

$$\frac{a^2}{(13.098 - a)(1.483 - a)} = \frac{2.628\times10^{-5}}{4}, \qquad \text{get} \quad a \approx 0.0113$$

Use this a in the energy equation

$$x = \frac{703\,042 - 181\,049\times0.0113}{403\,652} = 1.7366$$

$$\Rightarrow \frac{a^2}{(13.059\text{-}a)(1.4732\text{-}a)} = \frac{2.628\times10^{-5}}{4},\ a = 0.0112 \qquad \Rightarrow \qquad x = 1.7366$$

% excess air = **73.7 %**

$$\%\ NO = \frac{2\times0.0112\times100}{1+9.52\times1.7366} = \mathbf{0.128\ \%}$$

16.101

Find the equilibrium constant for the reaction in Problem 16.83.

Reaction: $2\,CO + 2\,H_2 \Leftrightarrow CH_4 + CO_2$

For CH_4 at 600 K (formula in Table A.6), $\bar{C}_{P0} = 52.22$

At 900 K

$\bar{h}^0_{CH4} = -74\,873 + 52.22(900 - 298.2) = -43\,446$ kJ/kmol

$\bar{s}^0_{CH4} = 186.251 + 52.22 \ln (900 / 298.2) = 243.936$ kJ/kmol K

(The integrated-equation values are -43 656 and 240.259)

$\Delta H^0_{900\text{ K}} = 1(-43\,446) + 1(-393\,522 + 28\,030)$

$- 2(-110\,527 + 18\,397) - 2(0 + 17\,657) = -259\,993$ kJ

$\Delta S^0_{900\text{ K}} = 1(243.936) + 1(263.646)$

$- 2(231.074) - 2(163.060) = -280.687$ kJ/K

$\Delta G^0_{900\text{ K}} = -259\,993 - 900(-280.687) = -7375$ kJ

$\ln K = \dfrac{+7375}{8.3145 \times 900} = 0.9856, \qquad K = \mathbf{2.679}$

16.102

A space heating unit in Alaska uses propane combustion is the heat supply. Liquid propane comes from an outside tank at -44°C and the air supply is also taken in from the outside at -44°C. The airflow regulator is misadjusted, such that only 90% of the theoretical air enters the combustion chamber resulting in incomplete combustion. The products exit at 1000 K as a chemical equilibrium gas mixture including only CO_2, CO, H_2O, H_2, and N_2. Find the composition of the products. Hint: use the water gas reaction in Example 16.4.

Propane: Liquid, $T_1 = -44^oC = 229.2$ K

Air: $T_2 = -44^oC = 229.2$ K, 90% Theoretical Air

Products: $T_3 = 1000$ K, CO_2, CO, H_2O, H_2, N_2

Theoretical Air: $C_3H_8 + 5O_2 + 18.8N_2 \;=>\; 3CO_2 + 4H_2O + 18.8N_2$

90% Theoretical Air:

$$C_3H_8 + 4.5O_2 + 16.92N_2 \;=>\; aCO_2 + bCO + cH_2O + dH_2 + 16.92N_2$$

Carbon: $a + b = 3$

Oxygen: $2a + b + c = 9$ Where: $2 \le a \le 3$

Hydrogen: $c + d = 4$

Reaction:	CO +	H_2O ↔	CO_2 +	H_2
Initial:	b	c	a	d
Change:	-x	-x	x	x
Equil:	b - x	c - x	a + x	d + x

Chose an Initial guess such as: a = 2, b = 1, c = 4, d = 0

Note: A different initial choice of constants will produce a different value for x, but will result in the same number of moles for each product.

$n_{CO2} = 2 + x$, $n_{CO} = 1 - x$, $n_{H2O} = 4 - x$, $n_{H2} = x$, $n_{N2} = 16.92$

The reaction can be broken down into two known reactions to find K

(1) $2CO_2 \leftrightarrow 2CO + O_2$ @ 1000 K $\ln(K_1) = -47.052$

(2) $2H_2O \leftrightarrow 2H_2 + O_2$ @ 1000 K $\ln(K_2) = -46.321$

For the overall reaction: $\ln K = (\ln(K_2) - \ln(K_1))/2 = 0.3655$; $K = 1.4412$

$$K = \frac{y_{CO2}y_{H2}}{y_{CO}y_{H2O}}\left(\frac{P}{P_o}\right)^{1+1-1-1} = \frac{y_{CO2}y_{H2}}{y_{CO}y_{H2O}} = 1.4412 = \frac{(2+x)x}{(1-4)(4-x)}$$

=> $x = 0.6462$

$\mathbf{n_{CO2} = 2.6462}$ $\mathbf{n_{CO} = 0.3538}$ $\mathbf{n_{N2} = 16.92}$

$\mathbf{n_{H2O} = 3.3538}$ $\mathbf{n_{H2} = 0.6462}$

$\mathbf{y_{CO2} = 0.111}$, $\mathbf{y_{CO} = 0.015}$, $\mathbf{y_{N2} = 0.707}$, $\mathbf{y_{H2O} = 0.140}$, $\mathbf{y_{H2} = 0.027}$

16.103

Derive the van't Hoff equation given in problem 16.51, using Eqs.16.12 and 16.15. Note: the d($\bar{g}$/T) at constant P for each component can be expressed using the relations in Eqs. 16.18 and 16.19.

Eq. 16.12: $\Delta G^0 = v_C \bar{g}^0_C + v_D \bar{g}^0_D - v_A \bar{g}^0_A - v_B \bar{g}^0_B$

Eq. 16.15: $\ln K = \Delta G^0 / \bar{R}T$ Eq. 16.19: $\Delta G^0 = \Delta H° - T\,\Delta S^0$

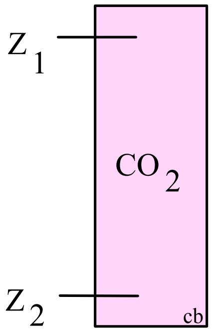
$$\frac{d\ \ln K}{dT} = -\frac{d}{dT}\left(\frac{\Delta G^0}{\bar{R}T}\right) = -\frac{1}{\bar{R}T}\frac{d\Delta G^0}{dT} + \frac{\Delta G^0}{\bar{R}T^2} = \frac{1}{\bar{R}T^2}\left[\,\Delta G^0 - T\frac{d\Delta G^0}{dT}\,\right]$$

$$= \frac{1}{\bar{R}T^2}\left[\Delta G^0 + T\,\Delta S^0\right] \qquad \text{used Eq.16.19} \qquad \frac{d\bar{g}}{dT} = -\,\bar{s}$$

$$= \frac{1}{\bar{R}T^2}\,\Delta H^0$$

16.104

Repeat Problem 16.18 using the generalized charts, instead of ideal gas behavior.



$(Z_1\text{-}Z_2) = 5000$ m, $P_1 = 15$ MPa

$T = 40\ ^oC =$ const

$$T_r = \frac{313.2}{304.1} = 1.03, \quad P_{r1} = \frac{15}{7.38} = 2.033$$

Equilibrium: $-w_{REV} = 0 = \Delta g + \Delta PE$

$$g_2 - g_1 = h_2 - h_1 - T(s_2 - s_1) = g(Z_1 - Z_2) = \frac{9.807\times 5000}{1000} = 49.04 \text{ kJ/kg}$$

From Figures D.2 and D.3,

$h^*_1 - h_1 = RT_c\times 3.54 = 203.4$ kJ/kg ; $s^*_1 - s_1 = R \times 2.61 = 0.4931$ kJ/kg K

$h^*_2 - h^*_1 = 0$; $s^*_2 - s^*_1 = 0 - R\ln(P_2/P_1) = 0.18892\ln(P_2/15)$

Trial and error. Assume $P_2 = 55$ MPa ($P_{r2} = 55/7.38 = 7.45$)

$h^*_2 - h_2 = RT_c\times 3.60 = 206.8$ kJ/kg ; $s^*_2 - s_2 = R \times 2.14 = 0.4043$ kJ/kg K

$\Delta g = -206.8 + 0 + 203.4 - 313.2[-0.4043 - 0.18892\ln(55/15) + 0.4931] = 45.7$

Too low so assume $P_2 = 60$ MPa ($P_{r2} = 60/7.38 = 8.13$)

$h^*_2 - h_2 = RT_c\times 3.57 = 205.1$ kJ/kg ; $s^*_2 - s_2 = R \times 2.11 = 0.3986$ kJ/kg K

$\Delta g = -205.1 + 0 + 203.4 - 313.2[-0.3986 - 0.18892\ln(60/15) + 0.4931] = 50.7$

Make linear interpolation $\Rightarrow$ $P_2 =$ **58 MPa**

16.105

A coal gasifier produces a mixture of 1 CO and $2H_2$ that is then fed to a catalytic converter to produce methane. A chemical-equilibrium gas mixture containing CH_4, CO, H_2, and H_2O exits the reactor at 600 K, 600 kPa. Determine the mole fraction of methane in the mixture.

	CO	$+ 3H_2 \leftrightarrow$	CH_4	$+ H_2O$
Initial	1	2	0	0
Change	-x	-3x	x	x
Equil.	1-x	2-3x	x	x

Total: $n = (1 - x) + (2 - 3x) + x + x = 3 - 2x$

$$K = \frac{y_{CH4}y_{H2O}}{y_{CO}y_{H2}^3}\left(\frac{P}{P_o}\right)^{(1+1-1-3) = -2} = \frac{x^2}{(1-x)(2-3x)^3}\left(\frac{P}{P_o}\right)^{-2}$$

$\ln K = -\Delta G^o/\bar{R}$; $\Delta G^o = \Delta H^o - T\Delta S^o$

$$H_P = n_{CH4}\,[\bar{h}_f^o + \bar{C}_P(T - T_o)] + n_{H2O}\left(\bar{h}_f^o + \Delta\bar{h}\right)$$

$$= [-74873 + 2.254\times16.04(600 - 298.15)] + (-241826 + 10499) = -295290$$

$$H_R = n_{CO}\left(\bar{h}_f^o + \Delta\bar{h}\right) + n_{H2}\left(\bar{h}_f^o + \Delta\bar{h}\right) = 1(-110527 + 8942) + 3(0 + 8799)$$

$$= -75188 \text{ kJ}$$

$$\Delta H^o_{600} = H_P - H_R = -295\,290 - (-75188) = -220\,102 \text{ kJ}$$

$$(\bar{s}_T^o)_{CH4} = \bar{s}_{To}^o + \bar{C}_P\ln(T/T_o) = 186.251 + 2.254\times16.04 \ln(600/298.2) = 211.549$$

$(\bar{s}_T^o)_{H2O} = 213.051$ kJ/kmol-K; $S_P = 424.6$ kJ/K

$(\bar{s}_T^o)_{CO} = 218.321$ kJ/kmol-K, $(\bar{s}_T^o)_{H2} = 151.078$ kJ/kmol-K

$$\Delta S^o_{600} = S_P - S_R = (n\bar{s}_T^o)_{CH4} + (n\bar{s}_T^o)_{H2O} - (n\bar{s}_T^o)_{CO} - (n\bar{s}_T^o)_{H2}$$

$$= (211.549 + 213.051) - (218.321 + 3 \times 151.078) = -246.955 \text{ kJ/K}$$

$\Delta G^o = \Delta H^o - T\Delta S^o = -220\,102 - 600(-246.955) = -71929$ kJ,

$\ln K = -(-71915)/(8.31451\times600) = 14.418 \quad => \quad K = 1.827\times10^6$

Solve for x, $x = 0.6667$, $n_{tot} = 1.6667$, $\mathbf{y_{CH4} = 0.4}$

16.106

One kilomole of liquid oxygen, O_2, at 93 K, and *x* kmol of gaseous hydrogen, H_2, at 25°C, are fed to a combustion chamber. *x* is greater than 2, such that there is excess hydrogen for the combustion process. There is a heat loss from the chamber of 1000 kJ per kmol of reactants. Products exit the chamber at chemical equilibrium at 3800 K, 400 kPa, and are assumed to include only H_2O, H_2, and O.

a. Determine the equilibrium composition of the products and also *x*, the amount of H_2 entering the combustion chamber.

b. Should another substance(s) have been included in part (a) as being present in the products? Justify your answer.

$$x\,H_2 + 1\,O_2 \rightarrow 2\,H_2O + (x-2)\,H_2$$

$$(1)\quad 1\,H_2O \Leftrightarrow 1\,H_2 + 1\,O$$

$$\begin{array}{llllll} \text{shift} & -a & +a & +a & \text{and} & a > 0 \\ \text{Equil} & 2-a & x-2+a & a & a < 2 \text{ and} & n_{tot} = x + a \end{array}$$

$$(2)\quad 2\,H_2O \Leftrightarrow 2\,H_2 + 1\,O_2 \qquad \ln K_2 = -1.906$$

$$(3)\quad 1\,O_2 \Leftrightarrow 2\,O \qquad \ln K_3 = -0.017$$

$$\ln K_1 = 0.5(\ln K_2 + \ln K_3) = -0.9615 \quad \Rightarrow \quad K_1 = 0.3823$$

$$\text{Equil.:}\quad \frac{K_1}{(P/P_o)^1} = \frac{(x-2+a)a}{(2-a)(x+a)} = \frac{0.3823}{4} = 0.95575$$

Energy Eq.: $Q + H_R = H_P$, $Q = (1 + x)(-1000)$ kJ

Table A.9: $\Delta\bar{h}^*_{IG} = -5980$ kJ/kmol [or $= 0.922\times32(93-298.2) = -6054$ kJ/kmol]

Fig. D.2: $T_r = 93/154.6 = 0.601$, $\Delta\bar{h}_f = -5.16\times\bar{R}\times154.6 = -6633$

$$H_R = x(0 + 0) + 1(0 + \Delta\bar{h}^*_{IG} + \Delta\bar{h}_f) = 1(-5980 - 6633) = -12613 \text{ kJ}$$

$$H_P = (2-a)(-241\,826 + 171\,981) + (x-2+a)(0 + 119077)$$
$$+ a(249170 + 73424) = 119077\,x + 511516\,a - 377844$$
$$= Q + H_R = -1000 - 1000\,x - 12613$$

Rearrange eq. to: $x + 4.2599\,a = 3.03331$

Substitute it into the equilibrium eq.: $\dfrac{(1.03331 + 5.2599\,a)\,a}{(2-a)(3.03331-3.2599\,a)} = 0.095575$

Solve $a = 0.198$, LHS $= 0.09547$, $\mathbf{x = 2.1898}$

$$y_{H2O} = \frac{2-a}{x+a} = 0.755, \quad y_{H2} = \frac{x-2+a}{x+a} = 0.162, \quad y_O = \frac{a}{x+a} = 0.083$$

Other substances and reactions: $2\,H_2O \Leftrightarrow H_2 + 2\,OH$, $\ln K = -0.984$,

$H_2 \Leftrightarrow 2\,H$, : $\ln K = 0.201$, $O_2 \Leftrightarrow 2\,O$, : $\ln K = -0.017$

All are significant as K's are of order 1.

16.107

Dry air is heated from 25°C to 4000 K in a 100-kPa constant-pressure process. List the possible reactions that may take place and determine the equilibrium composition. Find the required heat transfer.

Air assumed to be 21% oxygen and 79% nitrogen by volume.

From the elementary reactions we have at 4000 K (A.11)

(1) $O_2 \Leftrightarrow 2\,O$ $\quad K_1 = 2.221 = y_O^2/y_{O2}$

(2) $N_2 \Leftrightarrow 2\,N$ $\quad K_2 = 3.141 \times 10^{-6} = y_N^2/y_{N2}$

(3) $N_2 + O_2 \Leftrightarrow 2\,NO$ $\quad K_3 = 0.08955 = y_{NO}^2/y_{N2}\,y_{O2}$

Call the shifts a,b,c respectively so we get

$n_{O2} = 0.21 - a - c,\quad n_O = 2a,\quad n_{N2} = 0.79 - b - c,\quad n_N = 2b,$

$n_{NO} = 2c,\quad n_{tot} = 1 + a + b$

From which the mole fractions are formed and substituted into the three equilibrium equations. The result is

$$K_1 = 2.221 = y_O^2/y_{O2} = 4a^2 / [(1+a+b)(0.21-a-c)]$$

$$K_2 = 3.141 \times 10^{-6} = y_N^2/y_{N2} = 4b^2 / [(1+a+b)(0.79-b-c)]$$

$$K_3 = 0.08955 = y_{NO}^2/y_{N2}\,y_{O2} = 4c^2 / [(0.79-b-c)(0.21-a-c)]$$

which give 3 eqs. for the unknowns (a,b,c). Trial and error assume b = c = 0 solve for a from K_1 then for c from K_3 and finally given the (a,c) solve for b from K_2. The order chosen according to expected magnitude $K_1 > K_3 > K_2$

$a = 0.15,\quad b = 0.000832,\quad c = 0.0244 \;\Rightarrow$

$n_{O2} = 0.0356,\; n_O = 0.3,\; n_{N2} = 0.765,\; n_N = 0.00167,\; n_{NO} = 0.049$

$$Q = H_{ex} - H_{in} = n_{O2}\Delta\bar{h}_{O2} + n_{N2}\Delta\bar{h}_{N2} + n_O(\bar{h}_{fO} + \Delta\bar{h}_O) + n_N(\bar{h}_{fN} + \Delta\bar{h}_N) + n_{NO}(\bar{h}_{fNO} + \Delta\bar{h}_{NO}) - 0$$

$$= 0.0356 \times 138705 + 0.765 \times 130027 + 0.3(249170 + 77675) + 0.00167(472680 + 77532) + 0.049(90291 + 132671)$$

= **214 306 kJ/kmol air**

[If no reaction $\quad Q = n_{O2}\Delta\bar{h}_{O2} + n_{N2}\Delta\bar{h}_{N2} = 131\,849$ kJ/kmol air]

16.108

Saturated liquid butane (note: use generalized charts) enters an insulated constant pressure combustion chamber at 25°C, and *x* times theoretical oxygen gas enters at the same pressure and temperature. The combustion products exit at 3400 K. Assuming that the products are a chemical equilibrium gas mixture that includes CO, what is *x*?

Butane: $T_1 = 25^oC$, sat. liq., $x_1 = 0$, $T_c = 425.2$ K, $P_c = 3.8$ MPa

$T_{r1} = 0.7$, Figs. D.1and D.2, $P_{r1} = 0.10$, $P_1 = P_{r1}P_c = 380$ kPa

Fig D.2: $\left(\bar{h}_1^* - \bar{h}_1\right)_f = 4.85\ RT_c$

Oxygen: $T_2 = 25^oC$, X * theoretical air Products: $T_3 = 3400$ K

$$C_4H_{10} + 6.5X\ O_2 \Rightarrow 4\ CO_2 + 5\ H_2O + 6.5(X-1)\ O_2$$

$$2CO_2 \Leftrightarrow 2CO + O_2$$

	$2CO_2$	$2CO$	O_2	
Initial	4	0	6.5(X-1)	
Change	-2a	2a	a	
Equil.	4-2a	2a	6.5(X-1) + a	$n_{tot} = 2.5 + a + 6.5X$

$$n_{CO_2} = 4 - 2a,\quad n_{CO} = 2a,\quad n_{O_2} = 6.5(X-1) + a,\quad n_{H2O} = 5,$$

$$y_{CO} = \frac{2a}{2.5 + a + 6.5X},\quad y_{CO2} = \frac{4 - 2a}{2.5 + a + 6.5X},\quad y_{O2} = \frac{6.5(x-1) + a}{2.5 + a + 6.5X}$$

The equilibrium constant is

$$K = \frac{y_{CO}^2 y_{O_2}}{y_{CO_2}^2}\left(\frac{P_1}{P_o}\right)^{2+1-2} = \left(\frac{a}{2-a}\right)^2\left(\frac{6.5X - 6.5 + a}{6.5X - 2.5 + a}\right)\left(\frac{P_1}{P_o}\right)$$

@ $T_3 = 3400$ K Table A.11, $\ln(K) = 0.346$, $K = 1.4134$

$$1.4134 = \left(\frac{a}{2-a}\right)^2\left(\frac{6.5X - 6.5 + a}{6.5X - 2.5 + a}\right)(3.76) \qquad \text{Equation 1.}$$

Need a second equation:

Energy eq.: $Q_{CV} + H_R = H_P + W_{CV}$; $Q_{CV} = 0$, $W_{CV} = 0$

$$H_R = \left(\bar{h}_f^o + \Delta\bar{h}\right)_{C_4H_{10}} = (-126\ 200 - 17\ 146) = -143\ 346\ \text{kJ}$$

Products @ 3400 K:

$$H_P = n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{CO2} + n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{CO} + n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{O2} + n\left(\bar{h}_f^o + \Delta\bar{h}\right)_{H2O}$$

$$= (4 - 2a)(-393\ 522 + 177\ 836) + 2a(-110\ 527 + 108\ 440)$$

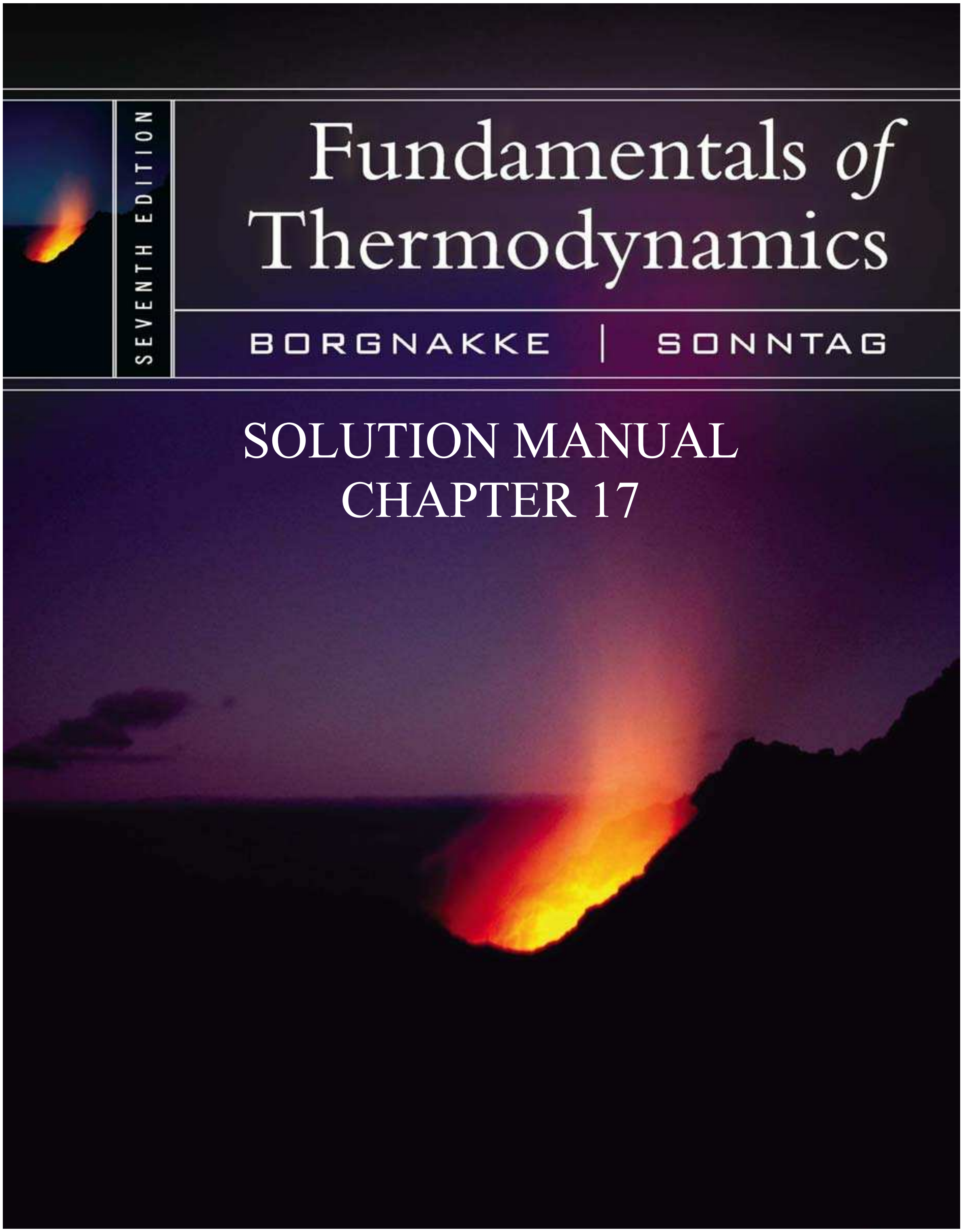
$$+ [6.5(X - 1) + a](0 + 114101) \ \ + 5(-241\ 826 + 149\ 073)$$

$$= -463\ 765 \text{ kJ/kmol}$$

$H_P = H_R$ => $1924820 = 541299a + 741656.5\ X$ Equation 2.

Two equations and two unknowns, solve for X and a.

$a \cong$ **0.87**, $X \cong$ **1.96**



SOLUTION MANUAL
CHAPTER 17

Fundamentals of Thermodynamics 7th Edition
Borgnakke and Sonntag

CONTENT CHAPTER 17

In-Text Concept Questions

17.a

Is stagnation temperature always higher than free stream temperature? Why?

Yes. Since kinetic energy can only be positive we have

$$h_0 = h_1 + \mathbf{V}_1^2/2 \; > \; h_1$$

If it is a gas with constant heat capacity we get

$$T_0 = T_1 + \mathbf{V}_1^2/2C_p$$

17.b

By looking at Eq. 17.25, rank the speed of sound for a solid, a liquid, and a gas.

Speed of sound: $\left(\frac{\partial P}{\partial \rho}\right)_s = c^2$

For a solid and liquid phase the density varies only slightly with temperature and constant s is also nearly constant T. We thus expect the derivative to be very high that is we need very large changes in P to give small changes in density.

A gas is highly compressible so the formula reduces to Eq.17.28 which gives modest values for the speed of sound.

17.c

Does speed of sound in an ideal gas depend on pressure? What about a real gas?

No. For an ideal gas the speed of sound is given by Eq.17.28

$$c = \sqrt{kRT}$$

and is only a function of temperature T.

For a real gas we do not recover the simple expression above and there is a dependency on P particularly in the dense gas region above the critical point.

17.d

Can a convergent adiabatic nozzle produce a supersonic flow?

No. From Eq.17.33 and a nozzle so $dP < 0$ it is required to have $dA > 0$ to reach $M > 1$. A convergent nozzle will have $M = 1$ at the exit, which is the smallest area. For lower back pressures there may be a shock standing in the exit plane.

17.e

To maximize the mass flow rate of air through a given nozzle, which properties should I try to change and in which direction, higher or lower?

The mass flow rate is given by Eq.17.41 and if we have $M = 1$ at the throat then Eq.17.42 gives the maximum mass flow rate possible.

Max flow for:

Higher upstream stagnation pressure
Lower upstream stagnation temperature

17.f

How do the stagnation temperature and pressure change in an isentropic flow?

The stagnation temperature and stagnation pressure are constant.

17.g

Which of the cases in Fig. 17.17 (a-h) have entropy generation and which do not?

a. There is no flow so $s_{gen} = 0$.
b. Subsonic flow, reversible, so $s_{gen} = 0$.
c. Limit for subsonic flow, reversible, so $s_{gen} = 0$.
d. The only supersonic reversible flow solution, so $s_{gen} = 0$.
e. Supersonic reversible in nozzle $s_{gen} = 0$, irreversible outside.
f. Supersonic reversible in nozzle $s_{gen} = 0$, compression outside.
g. Shock stands at exit plane, $s_{gen} > 0$ across shock.
h. Shock is located inside nozzle, $s_{gen} > 0$ across shock.

17.h

How does the stagnation temperature and pressure change in an adiabatic nozzle flow with an efficiency of less than 100%?

The stagnation temperature stays constant (energy eq.)
The stagnation pressure drops (s is generated, less kinetic energy).

17.i

Table A.13 has a column for P_{oy}/P_{ox} why is there not one for T_{oy}/T_{ox}?

The stagnation pressure drops across the shock (irreversible flow) whereas the stagnation temperature is constant (energy equation).

17.j

How high can a gas velocity (Mach number) be and still treat it as incompressible flow within 2% error?

The relative error in the ΔP versus kinetic energy, Eq.17.66, becomes

$$e = \frac{1}{4}\left(\frac{\mathbf{V}}{c_o}\right)^2 = 0.02 \quad \Rightarrow \quad M = \frac{\mathbf{V}}{c_o} = \sqrt{4 \times 0.02} = \mathbf{0.283}$$

Concept-Study Guide Problems

17.1

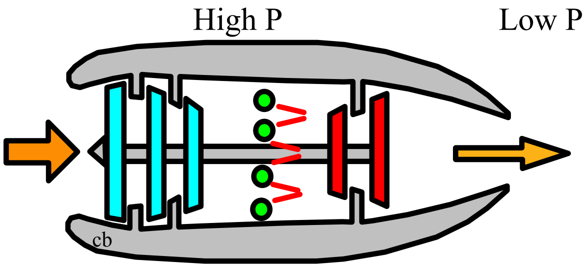
Which temperature does a thermometer or thermocouple measure? Would you ever need to make a correction to that?

Since the probe with the thermocouple in its tip is stationary relative to the moving fluid it will measure something close to the stagnation temperature. If that is high relative to the free stream temperature there will be significant heat transfer (convection and radiation) from the probe and it will measure a little less. For very high accuracy temperature measurements you must make some corrections for these effects.

17.2

The jet engine thrust is found from the overall momentum equation. Where is the actual force acting (it is not a long-range force in the flow)?

The compressor is generating the high pressure flow so the blades pushes hard on the flow and thus a force acts in the forward direction on the shaft holding the rotating blades. The high pressure in the chamber with combustion also has a net force in the forward direction as the flow leaves in the backwards direction so less wall area there. The pressure drop in the turbine means its blades pushes in the other direction but as the turbine exit pressure is higher than the ambient pressure the axial force is less than that of the compressor.



17.3

Most compressors have a small diffuser at the exit to reduce the high gas velocity near the rotating blades and increase the pressure in the exit flow. What does this do to the stagnation pressure?

For a reversible flow (ideal case) the stagnation pressure is constant. However, the reason it is done is to raise the pressure in a near reversible flow (diffuser) rather than let the flow reduce the peak velocities in a less reversible fashion which would lower the stagnation pressure.

17.4

A diffuser is a divergent nozzle used to reduce a flow velocity. Is there a limit for the Mach number for it to work like this?

Yes, the flow must be subsonic. If the flow was supersonic then increasing the flow area would increase the velocity.

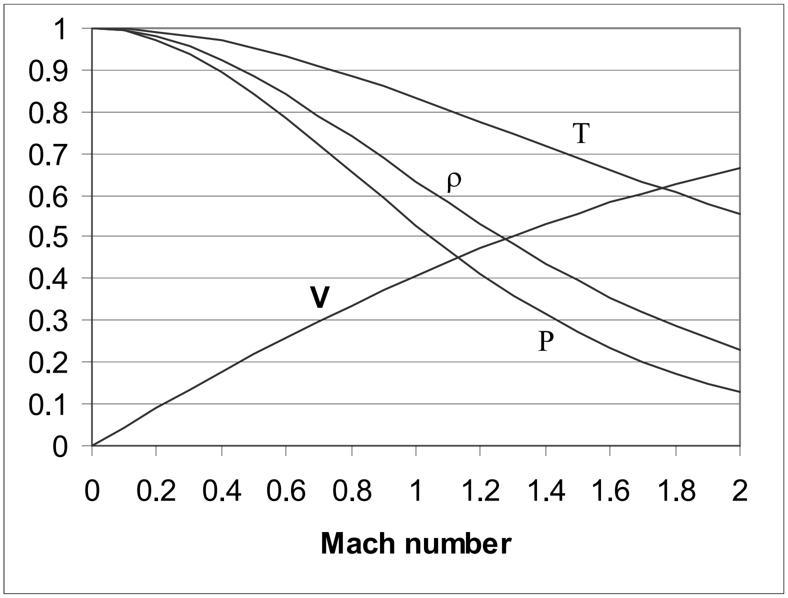
17.5

Sketch the variation in **V**, T, P, ρ and M for a subsonic flow into a convergent nozzle with $M = 1$ at the exit plane?

$$\mathbf{V} = M\,c = M\sqrt{kRT} = \sqrt{2C_p(T_o - T)}$$

Since we do not know the area versus length, we plot it versus mach number M. T, P and ρ relative to the stagnation state is listed in Table A.12 and given in eqs.17.34-36. A small spread sheet (M step 0.1) did the calculations.

Only the first part $0 < M < 1$ is the answer to this question.



The curves are plotted as the variables:

T / T_o

ρ / ρ_o

P / P_o

$V / \sqrt{2C_p\,T_o}$

and for k = 1.4

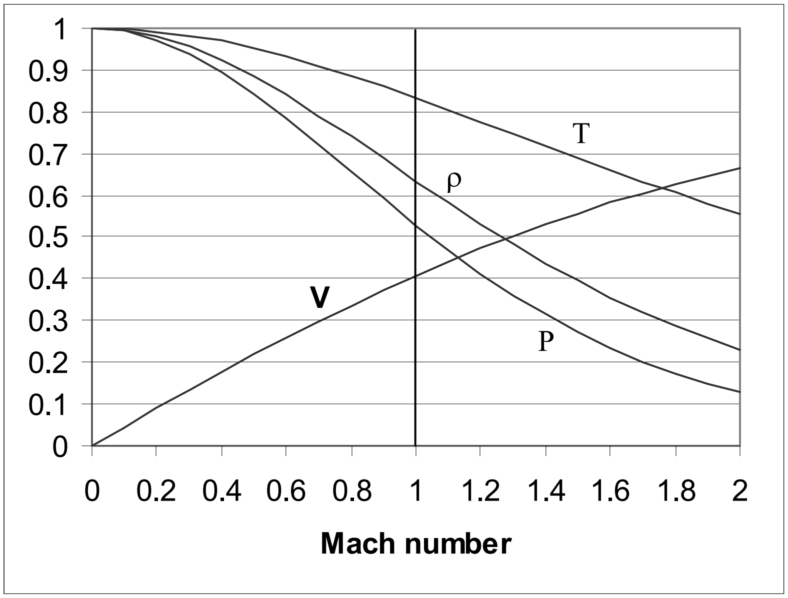
17.6

Sketch the variation in **V**, T, P, ρ and M for a sonic ($M = 1$) flow into a divergent nozzle with $M = 2$ at the exit plane?

$$\mathbf{V} = M\,c = M\sqrt{kRT} = \sqrt{2C_p(T_o - T)}$$

Since we do not know the area versus length, we plot it versus mach number M. T, P and ρ relative to the stagnation state is listed in Table A.12 and given in eqs.17.34-36.

Only the last part $1 < M < 2$ is the answer to this question.



The curves are plotted as the variables:

T / T_o

ρ / ρ_o

P / P_o

$V / \sqrt{2C_p T_o}$

and for $k = 1.4$

17.7

Can any low enough backup pressure generate an isentropic supersonic flow?

No. Only one back pressure corresponds to a supersonic flow, which is the exit pressure at state d in Figure 17.13. However a pressure lower than that can give an isentropic flow in the nozzle, case e, with a drop in pressure outside the nozzle. This is irreversible leading to an increase in s and therefore not isentropic.

17.8

Is there any benefit to operate a nozzle choked?

Yes. Since the mass flow rate is constant (max value) between points c and d in Fig. 17.12 a small variation in the back pressure will not have any influence. The nozzle then provides a constant mass flow rate free of surges up or down which is very useful for flow calibrations or other measurements where a constant mass flow rate is essential.

17.9

Can a shock be located upstream from the throat?

No. The flow adjust so M = 1 at the throat.

17.10

The high velocity exit flow in Example 17.7 is at 183 K. Can that flow be used to cool a room?

Being that cold it sounds like it could. However when the flow enters a room it eventually would have to slow down and then it has the stagnation temperature. If you let the flow run over a surface there will be a boundary layer with zero velocity at the surface and again there the temperature is close to the stagnation temperature.

17.11

A convergent-divergent nozzle is presented for an application that requires a supersonic exit flow. What features of the nozzle do you look at first?

You look at the cross section area change through the nozzle. At the throat M = 1 so in the divergent section the velocity increases and the ratio A/A* determines how the flow changes. The exit area can then tell you what the exit mach number will be and if you can have a reversible flow or not.

17.12

To increase the flow through a choked nozzle, the flow can be heated/cooled or compressed/expanded (four processes) before or after the nozzle. Explain which of these eight possibilities will help and which will not.

The mass flow rate through a choked nozzle is given by Eq.17.42. Since k and R are constant it varies with the upstream stagnation properties P_o and T_o.

After nozzle: Any downstream changes have **no effects**.

Before nozzle: Upstream changes in P_o and T_o has an influence.

a. Heat This lowers mass flow rate (T_o increases)

b. Cool This raises mass flow rate (T_o decreases)

c. Compress. Raises P_o and T_o opposite effects.

Isentropic: $P_{o\,new} = P_o\, r_p$ and $T_{o\,new} = T_o\, (r_p)^{\frac{k-1}{k}}$

$P_{o\,new} / \sqrt{T_{o\,new}} = (r_p)^{\frac{k+1}{2k}} \left[P_o / \sqrt{T_o}\right] > \left[P_o / \sqrt{T_o}\right]$

So the mass flow rate increases

d. Expand. Lowers P_o and T_o opposite effects. Assume isentropic, then mass flow rate decreases.

17.13

Suppose a convergent-divergent nozzle is operated as case h in Fig. 17.17. What kind of nozzle could have the same exit pressure but with a reversible flow?

A convergent nozzle, having subsonic flow everywhere assuming the pressure ratio is higher than the critical.

Stagnation Properties

17.14

A stationary thermometer measures 80°C in an air flow that has a velocity of 200 m/s. What is the actual flow temperature?

We assume that the thermometer measures the stagnation temperature, that is the probe (bulb, thermistor or thermo-couple junction sits stationary).

$$h_0 = h_1 + \mathbf{V}_1^2/2 \qquad \Rightarrow \qquad T_1 = T_0 - \mathbf{V}_1^2/2C_p$$

$$T_1 = 80^{\circ}\mathrm{C} - \frac{200^2}{2000 \times 1.004}\ \mathrm{K} = 60^{\circ}\mathrm{C}$$

17.15

Steam leaves a nozzle with a pressure of 500 kPa, a temperature of 350°C, and a velocity of 250 m/s. What is the isentropic stagnation pressure and temperature?

Stagnation enthalpy from energy equation and values from steam tables B.1.3

$$h_0 = h_1 + \mathbf{V}_1^2/2 = 3167.7 + \frac{250^2}{2000} = 3198.4 \text{ kJ/kg}$$

$$s_0 = s_1 = 7.6329 \text{ kJ/kg K}$$

It can be linearly interpolated from the printed tables

Computer software: $(h_o, s_o) \Rightarrow T_o = \mathbf{365°C}$, $P_o = \mathbf{556\ kPa}$

17.16

Steam at 1600 kPa, 300^{o}C flows so it has a stagnation (total) pressure of 1800 kPa. Find the velocity and the stagnation temperature.

The stagnation state has the same entropy as the inlet state so:

1: 1600 kPa, 300^{o}C has h = 3034.83 kJ/kg, s = 6.8844 kJ/kg-K
Stagnation: 1800 kPa, s_0 = 6.8844 kJ/kg-K has

T_{01} = **316.5^{o}C**, h_{01} = 3066.155 kJ/kg

$$\mathbf{V}_1^2/2 = h_{01} - h_1 = 3066.155 - 3034.83 = 31.325 \text{ kJ/kg}$$

$$\mathbf{V}_1 = \sqrt{2 \times (h_{o1} - h_1)} = \sqrt{2 \times 31.325 \times 1000} = \mathbf{250.3\ m/s}$$

17.17

An object from space enters the earth's upper atmosphere at 5 kPa, 100 K, with a relative velocity of 2000 m/s or more . Estimate the object`s surface temperature.

$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 2000^2/2000 = 2000$ kJ/kg

$h_{o1} = h_1 + 2000 = 100 + 2000 = 2100$ kJ/kg => $T =$ **1875 K**

The value for h_1 from ideal gas table A.7 was estimated since the lowest T in the table is 200 K.

17.18

The products of combustion of a jet engine leave the engine with a velocity relative to the plane of 400 m/s, a temperature of 480°C, and a pressure of 75 kPa. Assuming that $k = 1.32$, $C_p = 1.15$ kJ/kg K for the products, determine the stagnation pressure and temperature of the products relative to the airplane.

Energy Eq.: $h_{o1} - h_1 = \mathbf{V}_1^2/2 = 400^2/2000 = 80$ kJ/kg

$T_{o1} - T_1 = (h_{o1} - h_1)/C_p = 80/1.15 = 69.6$ K

$T_{o1} = 480 + 273.15 + 69.6 =$ **823 K**

Isentropic process relates to the stagnation pressure

$P_{o1} = P_1(T_{o1}/T_1)^{k/(k-1)} = 75(823/753.15)^{4.125} =$ **108 kPa**

17.19

Steam is flowing to a nozzle with a pressure of 400 kPa. The stagnation pressure and temperature are measured to be 600 kPa and 350°C, respectively. What are the flow velocity and temperature?

Stagnation state Table B.1.3: $h_{o1} = 3165.66$ kJ/kg, $s_{o1} = 7.5463$ kJ/kg K

State 1: 400 kPa, $s_1 = s_{o1} = 7.5463$ kJ/kg K

$$T_1 = 250 + (300 - 250)\frac{7.5463 - 7.3788}{7.5661 - 7.3788} = \mathbf{294.7^oC}$$

$$h_1 = 2964.16 + \frac{7.5463 - 7.3788}{7.5661 - 7.3788}(3066.75 - 2964.16) = 3055.9 \text{ kJ/kg}$$

Energy equation gives

$$\mathbf{V}_1^2/2 = h_{o1} - h_1 = 3165.66 - 3055.9 = 109.76 \text{ kJ/kg}$$

$$\mathbf{V}_1 = \sqrt{2 \times (h_{o1} - h_1)} = \sqrt{2 \times 109.76 \times 1000} = \mathbf{468.5\ m/s}$$

17.20

A meteorite melts and burn up at temperatures of 3000 K. If it hits air at 5 kPa, 50 K how high a velocity should it have to experience such a temperature?

Assume we have a stagnation T = 3000 K

$h_1 + \mathbf{V}_1^2/2 = h_{stagn.}$

Use table A.7, $h_{stagn.} = 3525.36$ kJ/kg, $h_1 = 50$ kJ/kg

$\mathbf{V}_1^2/2 = 3525.36 - 50 = 3475.4$ kJ/kg (remember convert to J/kg = m^2/s^2)

$\mathbf{V}_1 = \sqrt{2 \times 3475.4 \times 1000} =$ **2636 m/s**

17.21

Air leaves a compressor in a pipe with a stagnation temperature and pressure of 150°C, 300 kPa, and a velocity of 125 m/s. The pipe has a cross-sectional area of 0.02 m^2. Determine the static temperature and pressure and the mass flow rate.

$$h_{o1} - h_1 = \mathbf{V}_1^2/2 = 125^2/2000 = 7.8125 \text{ kJ/kg}$$

$$T_{o1} - T_1 = (h_{o1} - h_1)/C_p = 7.8125/1.004 = 7.8 \text{ K}$$

$$T_1 = T_{o1} - \Delta T = 150 - 7.8 = \mathbf{142.2\ °C = 415.4\ K}$$

$$P_1 = P_{o1}(T_1/T_{o1})^{k/(k-1)} = 300(415.4/423.15)^{3.5} = \mathbf{281\ kPa}$$

$$\dot{m} = \rho A\mathbf{V} = \frac{A\mathbf{V}}{v} = \frac{P_1 A\mathbf{V}_1}{RT_1} = \frac{281.2(0.02)(125)}{0.287(415.4)} = \mathbf{5.9\ kg/s}$$

17.22

I drive down the highway at 110 km/h on a day with 25°C, 101.3 kPa. I put my hand, cross sectional area 0.01 m^2, flat out the window. What is the force on my hand and what temperature do I feel?

The air stagnates on the hand surface : $h_1 + \mathbf{V}_1^2/2 = h_{stagn.}$

Use constant heat capacity

$$T_{stagn.} = T_1 + \frac{\mathbf{V}_1^2/2}{C_p} = 25 + \frac{0.5 \times 110^2 \times (1000/3600)^2}{1004} = \mathbf{25.465°C}$$

Assume a reversible adiabatic compression

$$P_{stagn.} = P_1\ (T_{stagn.}/T_1)^{k/(k-1)} = 101.3\ (298.615/298.15)^{3.5}$$

$$= \mathbf{101.85\ kPa}$$

17.23

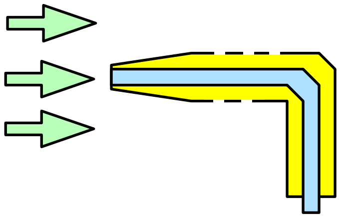
A stagnation pressure of 108 kPa is measured for an airflow where the pressure is 100 kPa and 20°C in the approach flow. What is the incomming velocity?

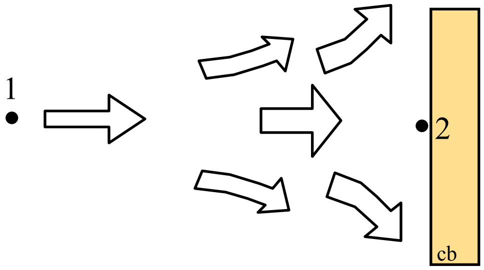
Assume a reversible adiabatic compression

$$T_{o1} = T_1 \times (P_{o1}/P_1)^{(k-1)/k} = 293.15 \times (\frac{108}{100})^{0.2857} = 299.67 \text{ K}$$

$$\mathbf{V}_1^2/2 = h_{o1} - h_1 = C_p\,(T_{o1} - T_1) = 6.543 \text{ kJ/kg}$$

$$\mathbf{V}_1 = \sqrt{2 \times 6.543 \times 1000} = \mathbf{114.4\ m/s}$$





To the left a Pitot tube, blue inner tube measures stagnation pressure and yellow outer tube with holes in it measures static pressure. To the right is a stagnation point on a wall relative to the free stream flow at state 1.

17.24

A 4-cm inner diameter pipe has an inlet flow of 10 kg/s water at 20°C, 200 kPa. After a 90 degree bend as shown in Fig. P17.24, the exit flow is at 20°C, 190 kPa. Neglect gravitational effects and find the anchoring forces F_x and F_y.

$$D = 0.04 \text{ m} \quad \Rightarrow \quad A = \frac{\pi}{4} D^2 = 0.001257 \text{ m}^2$$

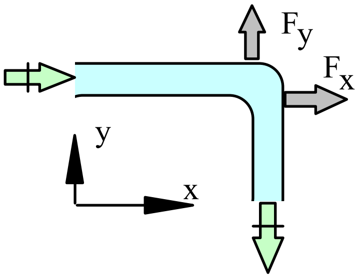
$$\mathbf{V}_{avg} = \frac{\dot{m}}{\rho A} = \frac{10 \times 0.001002}{0.001257} = 7.971 \text{ m/s}$$

Now we can do the x and y direction momentum equations for steady flow and the same magnitude of the velocity, but different directions

X-dir: $\quad 0 = \dot{m}\, \mathbf{V}_{avg\ 1} + F_x - \dot{m} \times 0 + (P_1 - P_o)\, A$

Y-dir: $\quad 0 = \dot{m} \times 0 + F_y - \dot{m} \times (-\mathbf{V}_{avg\ 2}) + (P_2 - P_o)\, A$

$$F_x = -\dot{m}\, \mathbf{V}_{avg\ 1} - (P_1 - P_o)\, A$$
$$= -10 \times 7.97 - 100 \times 0.001257 \times 1000 = \mathbf{-205\ N}$$
$$F_y = -\dot{m}\, \mathbf{V}_{avg\ 2} - (P_2 - P_o)\, A$$
$$= -10 \times 7.97 - 90 \times 0.001257 \times 1000 = \mathbf{-193\ N}$$



17.25

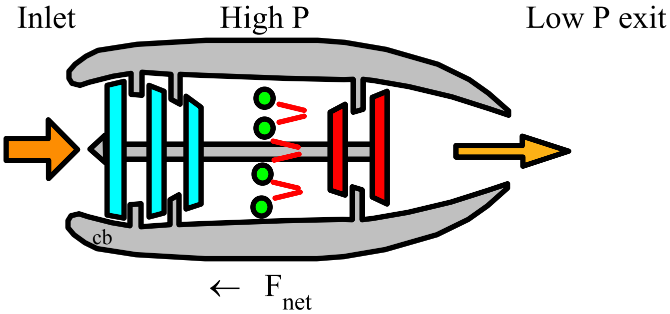
A jet engine receives a flow of 150 m/s air at 75 kPa, 5°C across an area of 0.6 m^2 with an exit flow at 450 m/s, 75 kPa, 600 K. Find the mass flow rate and thrust.

$\dot{m} = \rho A\mathbf{V}$; ideal gas $\rho = P/RT$

$\dot{m} = (P/RT)A\mathbf{V} = \left(\frac{75}{0.287 \times 278.15}\right) \times 0.6 \times 150 = 0.9395 \times 0.6 \times 150$

$= \mathbf{84.555\ kg/s}$

$F_{net} = \dot{m}\,(\mathbf{V}_{ex} - \mathbf{V}_{in}) = 84.555 \times (450 - 150) = \mathbf{25\ 367\ N}$



The shaft must have axial load bearings to transmit thrust to aircraft.

17.26

How large a force must be applied to a squirt gun to have 0.1 kg/s water flow out at 20 m/s? What pressure inside the chamber is needed?

$$F = \frac{d\,m\mathbf{V}}{dt} = \dot{m}\,\mathbf{V} = 0.1 \times 20 \ \text{kg m/s}^2 = \mathbf{2\ N}$$

Eq.17.21: $v\Delta P = 0.5\,\mathbf{V}^2$

$$\Delta P = 0.5\,\mathbf{V}^2/v = 0.5 \times 20^2 / 0.001$$

$$= 200\,000 \text{ Pa} = \mathbf{200\ kPa}$$

17.27

A jet engine at takeoff has air at 20°C, 100 kPa coming at 25 m/s through the 1.0 m diameter inlet. The exit flow is at 1200 K, 100 kPa, through the exit nozzle of 0.4 m diameter. Neglect the fuel flow rate and find the net force (thrust) on the engine.

$$A_1 = \frac{\pi}{4} D^2 = 0.7854 \text{ m}^2; \qquad A_2 = \frac{\pi}{4} D^2 = 0.1257 \text{ m}^2$$

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 293.15}{100} = 0.8409 \text{ m}^3\text{/kg}; \quad v_2 = 3.444 \text{ m}^3\text{/kg}$$

$$\dot{m} = A\mathbf{V}/v = A_1\mathbf{V}_1/v_1 = \frac{0.7854 \times 25}{0.8409} = 48.0 \text{ kg/s}$$

$$\mathbf{V}_2 = \frac{\dot{m}v_2}{A_2} = \frac{48.0 \times 3.444}{0.1257} = 1315 \text{ m/s}$$

Now we can do the x direction momentum equation for steady flow and the same mass flow rate in and out

$$\text{X-dir:} \qquad 0 = \dot{m}\,\mathbf{V}_1 + F_x + (P_1 - P_o)\,A_1 - \dot{m}\,\mathbf{V}_2 - (P_2 - P_o)\,A_2$$

$$\begin{aligned} F_x &= -\,\dot{m}\,\mathbf{V}_1 - (P_1 - P_o)\,A_1 + \dot{m}\,\mathbf{V}_2 + (P_2 - P_o)\,A_2 \\ &= \dot{m}\,(\mathbf{V}_2 - \mathbf{V}_1) - 0 + 0 = 48\,(1315 - 25) = \mathbf{61\,920\ N} \end{aligned}$$

17.28

A water turbine using nozzles is located at the bottom of Hoover Dam 175 m below the surface of Lake Mead. The water enters the nozzles at a stagnation pressure corresponding to the column of water above it minus 20% due to losses. The temperature is 15°C and the water leaves at standard atmospheric pressure. If the flow through the nozzle is reversible and adiabatic, determine the velocity and kinetic energy per kilogram of water leaving the nozzle.

$$\Delta P = \rho g \Delta Z = \frac{g\Delta Z}{v} = \frac{9.807 \times 175}{0.001001 \times 1000} = 1714.5 \text{ kPa}$$

$$\Delta P_{ac} = 0.8\ \Delta P = 1371.6 \text{ kPa}$$

$$v\Delta P = \mathbf{V}_{ex}^2/2 \;\Rightarrow\; \mathbf{V}_{ex} = \sqrt{2v\Delta P}$$

$$\mathbf{V}_{ex} = \sqrt{2 \times 0.001001 \times 1000 \times 1371.6} = \mathbf{62.4\ m/s}$$

$$\mathbf{V}_{ex}^2/2 = v\Delta P = \mathbf{1.373\ kJ/kg}$$

17.29

A water cannon sprays 1 kg/s liquid water at a velocity of 100 m/s horizontally out from a nozzle. It is driven by a pump that receives the water from a tank at 15°C, 100 kPa. Neglect elevation differences and the kinetic energy of the water flow in the pump and hose to the nozzle. Find the nozzle exit area, the required pressure out of the pump and the horizontal force needed to hold the cannon.

$$\dot{m} = \rho A\mathbf{V} = A\mathbf{V}/v \;\Rightarrow\; A = \dot{m}v/\mathbf{V} = 1\,\frac{0.001001}{100} = \mathbf{1.0 \times 10^{-5}\ m^2}$$

$$\dot{W}_p = \dot{m}w_p = \dot{m}v(P_{ex} - P_{in}) = \dot{m}\mathbf{V}_{ex}^2/2$$

$$P_{ex} = P_{in} + \mathbf{V}_{ex}^2/2v = 100 + 100^2/2 \times 1000 \times 0.001 = \mathbf{150\ kPa}$$

$$F = \dot{m}\mathbf{V}_{ex} = 1 \times 100 = \mathbf{100\ N}$$

17.30

An irrigation pump takes water from a lake and discharges it through a nozzle as shown in Fig. P17.30. At the pump exit the pressure is 700 kPa, and the temperature is 20°C. The nozzle is located 10 m above the pump and the atmospheric pressure is 100 kPa. Assuming reversible flow through the system determine the velocity of the water leaving the nozzle.

Assume we can neglect kinetic energy in the pipe in and out of the pump. Incompressible flow so Bernoulli's equation applies ($\mathbf{V}_1 \cong \mathbf{V}_2 \cong \mathbf{V}_3 \cong 0$)

$$v(P_3 - P_2) + (\mathbf{V}_3^2 - \mathbf{V}_2^2)/2 + g(Z_3 - Z_2) = 0$$

$$P_3 = P_2 - \frac{g(Z_3 - Z_2)}{v} = 700 - \frac{9.807(10)}{1000(0.001002)} = 602 \text{ kPa}$$

$$\mathbf{V}_4^2/2 = v(P_3 - P_4)$$

$$\Rightarrow \mathbf{V}_4 = \sqrt{2v(P_3 - P_4)} = \sqrt{2 \times 0.001002 \times 502.1 \times 1000} = \mathbf{31.72\ m/s}$$

17.31

A water tower on a farm holds 1 m^3 liquid water at 20°C, 100 kPa in a tank on top of a 5 m tall tower. A pipe leads to the ground level with a tap that can open a 1.5 cm diameter hole. Neglect friction and pipe losses, and estimate the time it will take to empty the tank for water.

Incompressible flow so we can use Bernoulli Equation.

$$P_e = P_i; \quad \mathbf{V}_i = 0; \quad Z_e = 0; \quad Z_i = H$$

$$\mathbf{V}_e^2/2 = gZ_i \quad \Rightarrow \quad \mathbf{V}_e = \sqrt{2gZ} = \sqrt{2 \times 9.807 \times 5} = 9.9 \text{ m/s}$$

$$\dot{m} = \rho A\mathbf{V}_e = A\mathbf{V}_e/v = \Delta m/\Delta t$$

$$\Delta m = V/v; \quad A = \pi D^2/4 = \pi \times 0.015^2 / 4 = 1.77 \times 10^{-4} \text{ m}^2$$

$$\Rightarrow \quad \Delta t = \Delta m v/A\mathbf{V}_e = V/A\mathbf{V}_e$$

$$\Delta t = \frac{1}{1.77 \times 10^{-4} \times 9.9} = \mathbf{571.6\ sec} = \mathbf{9.53\ min}$$

17.32

Find the speed of sound for air at 100 kPa at the two temperatures 0°C and 30°C. Repeat the answer for carbon dioxide and argon gases.

From eq. 17.28 we have

$$c_0 = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 273.15 \times 1000} = \mathbf{331\ m/s}$$

$$c_{30} = \sqrt{1.4 \times 0.287 \times 303.15 \times 1000} = \mathbf{349\ m/s}$$

For Carbon Dioxide: $R = 0.1889$ kJ/kg K, $k = 1.289$

$$c_0 = \sqrt{1.289 \times 0.1889 \times 273.15 \times 1000} = \mathbf{257.9\ m/s}$$

$$c_{30} = \sqrt{1.289 \times 0.1889 \times 303.15 \times 1000} = \mathbf{271.7\ m/s}$$

For Argon: $R = 0.2081$ kJ/kg K, $k = 1.667$

$$c_0 = \sqrt{1.667 \times 0.2081 \times 273.15 \times 1000} = \mathbf{307.8\ m/s}$$

$$c_{30} = \sqrt{1.667 \times 0.2081 \times 303.15 \times 1000} = \mathbf{324.3\ m/s}$$

17.33

Find the expression for the anchoring force R_x for an incompressible flow like in Figure 17.6. Show that it can be written as

$$R_x = \frac{\mathbf{V}_i - \mathbf{V}_e}{\mathbf{V}_i + \mathbf{V}_e}\left[(P_i - P_o)A_i + (P_e - P_o)A_e\right]$$

Apply the X-dir momentum equation for a steady flow

$$0 = R_x + (P_i - P_o)A_i - (P_e - P_o)A_e + \dot{m}\mathbf{V}_i - \dot{m}\mathbf{V}_e$$

Bernoulli equation for the flow is

$$0.5(\mathbf{V}_e^2 - \mathbf{V}_i^2) + v\,(P_e - P_i) = 0 \qquad \Rightarrow \qquad \mathbf{V}_e - \mathbf{V}_i = \frac{2v\,(P_i - P_e)}{\mathbf{V}_i + \mathbf{V}_e}$$

Continuity equation gives

$$\dot{m} = A_i\mathbf{V}_i\,/v = A_e\mathbf{V}_e\,/v$$

Solve for R_x from the momentum equation

$$R_x = \dot{m}\,(\mathbf{V}_e - \mathbf{V}_i) + (P_e - P_o)A_e - (P_i - P_o)A_i$$

$$= \frac{A_i\mathbf{V}_i}{v}\,\frac{2v\,(P_i - P_e)}{\mathbf{V}_i + \mathbf{V}_e} + (P_e - P_o)A_e - (P_i - P_o)A_i$$

Multiply in and use continuity equation for second term

$$R_x = \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\left[P_iA_i\mathbf{V}_i - P_eA_e\mathbf{V}_e\right] + (P_e - P_o)A_e - (P_i - P_o)A_i$$

$$= \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\Big[P_iA_i\mathbf{V}_i - P_eA_e\mathbf{V}_e + \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_e - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_i$$
$$+ \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_i - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_e\Big]$$

Now put the first four terms together

$$R_x = \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\Big[\tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_i - \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_e$$
$$+ \tfrac{1}{2}(P_e - P_o)A_e\mathbf{V}_i - \tfrac{1}{2}(P_i - P_o)A_i\mathbf{V}_e\Big]$$

$$= \frac{2}{\mathbf{V}_i + \mathbf{V}_e}\left[\tfrac{1}{2}(P_i - P_o)A_i(\mathbf{V}_i - \mathbf{V}_e) + \tfrac{1}{2}(P_e - P_o)A_e(\mathbf{V}_i - \mathbf{V}_e)\right]$$

$$= \frac{\mathbf{V}_i - \mathbf{V}_e}{\mathbf{V}_i + \mathbf{V}_e}\left[(P_i - P_o)A_i + (P_e - P_o)A_e\right]$$

17.34

Estimate the speed of sound for steam directly from Eq. 17.25 and the steam tables for a state of 6 MPa, 400°C. Use table values at 5 and 7 MPa at the same entropy as the wanted state. Eq. 17.25 is then done by finite difference. Find also the answer for the speed of sound assuming steam is an ideal gas.

Eq.17.25: $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = \left(\frac{\Delta P}{\Delta \rho}\right)_s$

State 6 MPa, 400°C ⇒ $s = 6.5407$ kJ/kg K

7 MPa, s ⇒ $v = 0.04205\ \text{m}^3/\text{kg};\quad \rho = 1/v = 23.777\ \text{kg/m}^3$

5 MPa, s ⇒ $v = 0.05467\ \text{m}^3/\text{kg};\quad \rho = 1/v = 18.2909\ \text{kg/m}^3$

$$c^2 = \frac{7000 - 5000}{23.777 - 18.2909} = 364.56 \times 1000 \quad \Rightarrow \quad c = \mathbf{603.8\ m/s}$$

From Table A.8: $C_p = \frac{1338.56 - 1235.3}{50} = 2.0652$ kJ/kg K

$C_v = C_p - R = 2.0652 – 0.4615 = 1.6037$ kJ/kg K

$k = C_p/C_v = 1.288;\quad R = 0.4615$ kJ/kg K (from A.5)

Now do the speed of sound from Eq.17.28

$$c = \sqrt{kRT} = \sqrt{1.288 \times 0.4615 \times 673.15 \times 1000} = \mathbf{632.6\ m/s}$$

17.35

Use the CATT3 software to solve the previous problem.

From Eq. 17.25: $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$

Superheated vapor water at 400°C, 6000 kPa

CATT3: $v = 0.04739$ m^3/kg, $s = 6.541$ kJ/kg K

At P = 6200 kPa & s = 6.541 kJ/kg K: T = 405.1°C, $v = 0.0462$ m^3/kg

At P = 5800 kPa & s = 6.541 kJ/kg K: T = 394.8°C, $v = 0.04866$ m^3/kg

$$c^2 = -(0.04739)^2 \left(\frac{6.2 - 5.8}{0.0462 - 0.04866}\right) \frac{\text{MJ}}{\text{kg}} = 0.36517 \times 10^6 \text{ m}^2/\text{s}^2$$

=> c = **604 m/s**

From Table A.8: $C_p = \frac{1338.56 - 1235.3}{50} = 2.0652$ kJ/kg K

$C_v = C_p - R = 2.0652 – 0.4615 = 1.6037$ kJ/kg K

$k = C_p/C_v = 1.288$; R = 0.4615 kJ/kg K (from A.5)

Now do the speed of sound from Eq.17.28

$c = \sqrt{kRT} = \sqrt{1.288 \times 0.4615 \times 673.15 \times 1000} =$ **632.6 m/s**

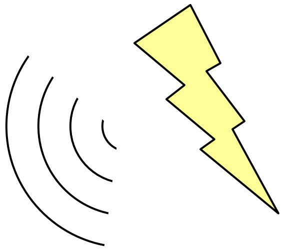
17.36

If the sound of thunder is heard 5 seconds after the lightning is seen and the weather is 20°C. How far away is the lightning taking place?

The sound travels with the speed of sound in air (ideal gas). Use the formula in Eq.17.28

$$L = c \times t = \sqrt{kRT} \times t \;= \sqrt{1.4 \times 0.287 \times 293.15 \times 1000} \times 5 = \mathbf{1716\ m}$$

For every 3 seconds after the lightning the sound travels about 1 km.



17.37

Find the speed of sound for carbon dioxide at 2500 kPa, 60°C using either the tables or the CATT3 software (same procedure as in Problem 17.34) and compare that with Eq.17.28.

From Eq. 17.25: $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$

Superheated carbon dioxide at 60°C, 2500 kPa

CATT3: $v = 0.02291 \text{ m}^3/\text{kg}$, $s = 1.521 \text{ kJ/kg K}$

At $P = 2600$ kPa & $s = 1.521$ kJ/kg K: $T = 63.02^\circ\text{C}$, $v = 0.02221 \text{ m}^3/\text{kg}$

At $P = 2400$ kPa & $s = 1.521$ kJ/kg K: $T = 56.9^\circ\text{C}$, $v = 0.02365 \text{ m}^3/\text{kg}$

$$c^2 = -(0.02291)^2 \left(\frac{2.6 - 2.4}{0.02221 - 0.02365}\right) \frac{\text{MJ}}{\text{kg}} = 7.8983 \times 10^4 \text{ m}^2/\text{s}^2$$

$$\Rightarrow \quad c = \mathbf{270\ m/s}$$

From Table A.5: $k = C_p/C_v = 1.289$; $R = 0.1889$ kJ/kg K

Now do the speed of sound from Eq.17.28

$$c = \sqrt{kRT} = \sqrt{1.289 \times 0.1889 \times 333.15 \times 1000} = \mathbf{284.8\ m/s}$$

17.38

A jet flies at an altitude of 12 km where the air is at -40°C, 45 kPa with a velocity of 900 km/h. Find the Mach number and the stagnation temperature on the nose.

From Table A.5: $k = C_p/C_v = 1.4$; $R = 0.287$ kJ/kg K

Now do the speed of sound from Eq.17.28

$$c = \sqrt{kRT} = \sqrt{1.4 \times 0.287 \times 233.15 \times 1000} = 306 \text{ m/s}$$

$$\mathbf{V} = 900 \text{ km/h} = 900 \text{ (km/h)} \times 1000 \text{ (m/km)} /3600 \text{ (s/h)} = 250 \text{ m/s}$$

$$M = \mathbf{V} / c = 250/306 = \mathbf{0.817}$$

$$h_0 = h_1 + \mathbf{V}_1^2/2 \quad \Rightarrow \quad T_0 = T_1 + \mathbf{V}_1^2 / 2C_p = -45 + \frac{250^2}{2 \times 1004} = \mathbf{-13.9^oC}$$

17.39

The speed of sound in liquid water at 25^{o}C is about 1500 m/s. Find the stagnation pressure and temperature for a M = 0.1 flow at 25^{o}C, 100 kPa. Is it possible to get a significant mach number flow of liquid water?

$$\mathbf{V} = M\,c = 0.1 \times 1500 = 150 \text{ m/s}$$

$$h_0 = h_1 + \mathbf{V}_1^2/2$$

Bernoulli Eq.: $\Delta P = \mathbf{V}_1^2/2v = \dfrac{150^2}{2 \times 0.001} = 11.25 \times 10^6 \text{ Pa} = 11.25 \text{ MPa}$

$$P_0 = P_1 + \Delta P = 100 + 11\,250 = \mathbf{11\,350\ kPa}$$

$$T_0 = T_1 + \mathbf{V}_1^2 / 2C_p = 25 + \frac{150^2}{2 \times 4180} = \mathbf{27.7^{o}C}$$

Remark: Notice the very high pressure. To get a higher velocity you need a higher pressure to accelerate the fluid, that is not feasible for any large flow rate.

17.40

Steam flowing at 15 m/s 1800 kPa, 300°C expands to 1600 kPa in a converging nozzle. Find the exit velocity and area ratio A_e / A_i.

Solve the problem with the steam tables.

Inlet state: $v_i = 0.14021\ \text{m}^3/\text{kg}$, $h_i = 3029.21$ kJ/kg, $s_i = 6.8226$ kJ/kg-K

Exit state: $(P_e, s_e = s_i)$ $v_e = 0.15371\ \text{m}^3/\text{kg}$, $h_e = 3000.995$ kJ/kg

Energy Eq.: $\mathbf{V}_i^2 / 2 + h_i = \mathbf{V}_e^2 / 2 + h_e$; $\mathbf{V}_e^2 = \mathbf{V}_i^2 + 2(h_i - h_e)$

$$\mathbf{V}_e = \sqrt{15 \times 15 + 2000(3029.21 - 3000.995)} = \mathbf{238\ m/s}$$

Same mass flow rate so

$$A_e/A_i = (v_e/v_i)(V_i/V_e) = \frac{0.15371}{0.14021} \times \frac{15}{238} = \mathbf{0.06909}$$

If we solved as ideal gas with constant specific heat we get (k = 1.327)

$$T_e = T_i\ (P_e/P_i)^{(k-1)/k} = 573.15\ (1600/1800)^{0.2464} = 556.76\ \text{K}$$

$$\mathbf{V}_e = \sqrt{\mathbf{V}_i^2 + 2C_p(T_i - T_e)} = \sqrt{15 \times 15 + 2 \times 1872(573.15 - 556.76)}$$
$$= 248\ \text{m/s}$$

$$A_e/A_i = (v_e/v_i)(V_i/V_e) = (P_i/P_e)^{1/k}(V_i/V_e) = \left(\frac{1800}{1600}\right)^{0.7536} \times \frac{15}{248} = 0.0661$$

17.41

A convergent nozzle has a minimum area of 0.1 m^2 and receives air at 175 kPa, 1000 K flowing with 100 m/s. What is the back pressure that will produce the maximum flow rate and find that flow rate?

$$\frac{P^*}{P_o} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528 \qquad \text{Critical Pressure Ratio}$$

Find P_o:

$$h_0 = h_1 + \mathbf{V}_1^2/2 = 1046.22 + 100^2/2000 = 1051.22 \text{ kJ/kg}$$

$$T_0 = T_i + 4.4 = 1004.4 \text{ K} \quad \text{from table A.7}$$

$$P_0 = P_i\,(T_0/T_i)^{k/(k-1)} = 175 \times (1004.4/1000)^{3.5} = 177.71 \text{ kPa}$$

The mass flow rate comes from the throat properties

$$P^* = 0.528\,P_o = 0.528 \times 177.71 = \mathbf{93.83\ kPa}$$

$$T^* = 0.8333\,T_o = 836.97 \text{ K}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{93.83}{0.287 \times 836.97} = 0.3906 \text{ kg/m}^3$$

$$\mathbf{V} = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 836.97} = \mathbf{579.9\ m/s}$$

$$\dot{m} = \rho A \mathbf{V} = 0.3906 \times 0.1 \times 579.9 = \mathbf{22.65\ kg/s}$$

17.42

A convergent-divergent nozzle has a throat area of 100 mm^2 and an exit area of 175 mm^2. The inlet flow is helium at a stagnation pressure of 1 MPa, stagnation temperature of 375 K. What is the back pressure that will give sonic condition at the throat, but subsonic everywhere else?

For this flow we have helium with $k_{He} = 1.667$, so we cannot use the tables for air. We need the solution to the curve labeled c in Fig. 17.13. For critical flow at the throat we have from Table 16.1 last column

$$P^* = 0.4867\ P_o = 486.7 \text{ kPa}$$

Now we need to find the conditions where the area ratio is

$$A_E/A^* = 175/100 = 1.75$$

that is solve for M in Eq. 17.43 given the area ratio. This is nonlinear so we have to iterate on it. Here (k+1)/2(k-1) = 2 so look also at Fig. 17.10 for the general shape.

$$M = 0.4 \;\Rightarrow\; A/A^* = (1/0.4)\,[\,0.75(1 + 0.3333{*}0.4^2)\,]^2 = 1.5602$$

$$M = 0.3 \;\Rightarrow\; A/A^* = (1/0.3)\,[\,0.75(1 + 0.3333{*}0.3^2)\,]^2 = 1.9892$$

$$M = 0.35 \;\Rightarrow\; A/A^* = (1/0.35)\,[\,0.75(1 + 0.3333{*}0.35^2)\,]^2 = 1.7410$$

$$M = 0.34 \;\Rightarrow\; A/A^* = (1/0.34)\,[\,0.75(1 + 0.3333{*}0.34^2)\,]^2 = 1.7844$$

Now do a linear interpolation for the rest to get $M_E = 0.348$;

Eq. 17.35 $\quad P_E/P_o = [1 + 0.3333{*}0.348^2\,]^{-2.5} = 0.9058$

$P_E = 0.9058 \times 1000 =$ **906 kPa**

17.43

To what pressure should the steam in problem 17.40 expand to reach Mach one? Use constant specific heats to solve.

Find stagnation properties

$$T_0 = T_1 + \mathbf{V}_1^2/2C_p = 573.15 + 15^2/(2 \times 1872) = 573.21 \text{ K}$$

$$P_0 = P_1\,(T_0/T_1)^{k/(k-1)} = 1800\,(573.21/573.15)^{4.058} = 1800.765 \text{ kPa}$$

From Eq.17.35 we get (k = 1.327)

$$P = P_0\,[\,1 + \frac{k-1}{2} M^2\,]^{-k/(k-1)} = 1800.765\,[1 + 0.1635]^{-4.058} = \mathbf{974\ kPa}$$

17.44

A jet plane travels through the air with a speed of 1000 km/h at an altitude of 6 km, where the pressure is 40 kPa and the temperature is –12°C. Consider the inlet diffuser of the engine where air leaves with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$\mathbf{V} = 1000$ km/h $= 277.8$ m/s, $\quad v_1 = RT/P = 0.287 \times 261.15/40 = 1.874$ m^3/kg

$h_1 = 261.48$ kJ/kg,

$h_{o1} = 261.48 + 277.8^2/2000 = 300.07$ kJ/kg

$\Rightarrow\ T_{o1} = 299.7$ K,

$P_{o1} = P_1\ (T_{o1}/T_1)^{k/(k-1)} = 40 \times (299.7/261.15)^{3.5} = 64.766$ kPa

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\quad \Rightarrow \quad T_2 =$ **294.7 K**,

$P_2 = P_{o1}\ (T_2/T_{o1})^{k/(k-1)} = 64.766 \times (294.7/299.7)^{3.5} =$ **61 kPa**

$v_2 = RT_2/P_2 = 0.287 \times 294.7/61 = 1.386$ m^3/kg

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.386)(100/277.8) =$ **0.487**

17.45

Air flows into a convergent-divergent nozzle with an exit area of 1.59 times the throat area of 0.005 m^2. The inlet stagnation state is 1 MPa, 600 K. Find the backpressure that will cause subsonic flow throughout the entire nozzle with M = 1 at the throat. What is the mass flow rate?

This corresponds to case c and is a reversible flow.

$A_E/A^* = 1.59$ Look at top in Table A.12 ($M < 1$)

$M_E = 0.4$ and $P_E/P_o = 0.8956$

$P_E = 0.8956\ P_o = 0.8956 \times 1000 = \mathbf{896\ kPa}$

To find the mass flow rate we need the throat conditions, see Table 17.1,

$$T^* = T \frac{2}{k+1} = 600 \times 0.8333 = 500 \text{ K}$$

$$v^* = RT^*/P^* = 0.287 \times 500 / 528.3 = 0.2716 \text{ m}^3\text{/kg}$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 0.287 \times 500 \times 1000} = 448.22 \text{ m/s}$$

$$\dot{m} = A^*c^*/v^* = \frac{0.005 \times 448.22}{0.2716} = \mathbf{8.251\ kg/s}$$

17.46

A nozzle is designed assuming reversible adiabatic flow with an exit Mach number of 2.6 while flowing air with a stagnation pressure and temperature of 2 MPa and 150°C, respectively. The mass flow rate is 5 kg/s, and *k* may be assumed to be 1.40 and constant. Determine the exit pressure, temperature, exit area, and the throat area.

From Table A.12: $M_E = 2.6$

$P_E = 2.0 \times 0.05012 = 0.1002$ MPa

Critical properties from Table 17.1

$T^* = 423.15 \times 0.8333 = \mathbf{352.7\ K}$

$P^* = 2.0 \times 0.5283 = \mathbf{1.057\ MPa}$

$c^* = \sqrt{1.4 \times 1000 \times 0.287 \times 352.7} = 376.5$ m/s

$v^* = RT^*/P^* = 0.287 \times 352.7/1057 = 0.0958\ m^3/kg$

$A^* = \dot{m}\, v^* / c^* = 5 \times 0.0958 / 376.5 = \mathbf{1.272 \times 10^{-3} m^2}$

$A_E = A^* (A_E/A^*) = 1.272 \times 10^{-3} \times 2.896 = \mathbf{3.68 \times 10^{-3} m^2}$

$T_E = T_o\,(T_E / T_o) = 423.15 \times 0.42517 = 179.9$ K

17.47

An air flow at 600 kPa, 600 K, $M = 0.2$ flows into a convergent-divergent nozzle with $M = 1$ at the throat. Assume a reversible flow with an exit area twice the throat area and find the exit pressure and temperature for subsonic exit flow to exist.

To find these properties we need the stagnation properties from the inlet state

From Table A.12: $M_i = 0.2$: $P_i/P_o = 0.9725$, $T_i/T_o = 0.99206$

$$P_o = 600 / 0.9725 = 616.97 \text{ kPa}, \quad T_o = 600 / 0.99206 = 604.8 \text{ K}$$

This flow is case c in Figure 17.13. From Table A.12: $A_E/A^* = 2$

$$P_E/P_o = 0.9360, \quad T_E/T_o = 0.98127$$

$$P_E = 0.9360\ P_o = 0.936 \times 616.97 = \mathbf{577.484\ kPa}$$

$$T_E = 0.98127\ T_o = 0.98127 \times 604.8 = \mathbf{593.5\ K}$$

17.48

Air at 150 kPa, 290 K expands to the atmosphere at 100 kPa through a convergent nozzle with exit area of 0.01 m^2. Assume an ideal nozzle. What is the percent error in mass flow rate if the flow is assumed incompressible?

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{k-1}{k}} = 258.28\ \text{K}$$

$$\mathbf{V}_e^2/2 = h_i - h_e = C_p\,(T_i - T_e) = 1.004\ (290 - 258.28) = 31.83\ \text{kJ/kg}$$

$$\mathbf{V}_e = 252.3\ \text{m/s};\quad v_e = \frac{RT_e}{P_e} = \frac{0.287 \times 258.28}{100} = 0.7412\ \text{m}^3\text{/kg}$$

$$\dot{m} = A\mathbf{V}_e\,/\,v_e = \frac{0.01 \times 252.3}{0.7413} = 3.4\ \text{kg/s}$$

Incompressible Flow: $v_i = RT/P = 0.287 \times 290/150 = 0.55487\ \text{m}^3\text{/kg}$

$$\mathbf{V}_e^2/2 = v\,\Delta P = v_i\,(P_i - P_e) = 0.55487\ (150 - 100) = 27.74\ \text{kJ/kg}$$

$$\Rightarrow\ \mathbf{V}_e = 235\ \text{m/s}\quad \Rightarrow\quad \dot{m} = A\mathbf{V}_e\,/\,v_i = 0.01 \times 235\,/\,0.55487 = 4.23\ \text{kg/s}$$

$$\frac{\dot{m}_{\text{incompressible}}}{\dot{m}_{\text{compressible}}} = \frac{4.23}{3.4} = 1.25$$

about **25% overestimation.**

17.49

Find the exit pressure and temperature for supersonic exit flow to exist in the nozzle flow of Problem 17.47.

We assume a reversible as the possibility which is case d in Figure 17.13.

To find these properties we need the stagnation properties from the inlet state

From Table A.12: $M_i = 0.2$: $P_i/P_o = 0.9725$, $T_i/T_o = 0.99206$

$$P_o = 600 / 0.9725 = 616.97 \text{ kPa}, \quad T_o = 600 / 0.99206 = 604.8 \text{ K}$$

From Table A.12: $A_E/A^* \approx 2$, $P_E/P_o = 0.09352$, $T_E/T_o = 0.50813$

$$P_E = 0.09352\ P_o = 0.09352 \times 616.97 = \mathbf{57.7\ kPa}$$

$$T_E = 0.50813\ T_o = 0.50813 \times 604.8 = \mathbf{307.3\ K}$$

This is significant lower P and T, but then we also have M = 2.2

17.50

Air is expanded in a nozzle from a stagnation state of 2 MPa, 600 K to a backpressure of 1.9 MPa. If the exit cross-sectional area is 0.003 m^2, find the mass flow rate.

This corresponds to case c and is a reversible flow.

$P_E/P_{ox} = 1.9/2.0 = 0.95 \quad \Rightarrow$ Table A.12: $\quad M_E = 0.268$

$T_E = (T/T_o)_E\, T_o = 0.9854 \times 600 = 591.2$ K

$c_E = \sqrt{kRT_E} = \sqrt{1.4 \times 1000 \times 0.287 \times 591.2} = 487.4$ m/s

$\mathbf{V}_E = M_E c_E = 0.268 \times 487.4 = 130.6$ m/s

$v_E = RT/P = 0.287 \times 591.2/1900 = 0.0893\ m^3/kg$

$\dot{m} = A_E \mathbf{V}_E / v_E = 0.002435 \times 130.6/0.0893 =$ **3.561 kg/s**

17.51

A 1-m^3 insulated tank contains air at 1 MPa, 560 K. The tank is now discharged through a small convergent nozzle to the atmosphere at 100 kPa. The nozzle has an exit area of 2×10^{-5} m^2.

a. Find the initial mass flow rate out of the tank.

b. Find the mass flow rate when half the mass has been discharged.

a. The back pressure ratio:

$$P_B/P_{o1} = 100/1000 = 0.1 < (P^*/P_o)_{crit} = 0.5283$$

so the initial flow is choked with the maximum possible flow rate.

$$M_E = 1 \; ; \; P_E = 0.5283 \times 1000 = 528.3 \text{ kPa}$$

$$T_E = T^* = 0.8333 \times 560 = 466.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433 \text{ m/s}$$

$$v_E = RT^*/P_E = 0.287 \times 466.7/528.3 = 0.2535 \text{ m}^3\text{/kg}$$

$$\dot{m}_1 = A\mathbf{V}_E/v_E = 2 \times 10^{-5} \times 433/0.2535 = \mathbf{0.0342 \text{ kg/s}}$$

b. The initial mass is

$$m_1 = P_1V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

with a mass at state 2 as $m_2 = m_1/2 = 3.111$ kg.

Assume an adiabatic reversible expansion of the mass that remains in the tank.

$$P_2 = P_1(v_1/v_2)^k = 100 \times 0.5^{1.4} = 378.9 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 560 \times 0.5^{0.4} = 424 \text{ K}$$

The pressure ratio is still less than critical and the flow thus choked.

$$P_B/P_{o2} = 100/378.9 = 0.264 < (P^*/P_o)_{crit}$$

$$M_E = 1 \; ; \; P_E = 0.5283 \times 378.9 = 200.2 \text{ kPa}$$

$$T_E = T^* = 0.8333 \times 424 = 353.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 353.7} = 377 \text{ m/s}$$

$$\dot{m}_2 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(377)(200.2)}{0.287(353.7)} = \mathbf{0.0149 \text{ kg/s}}$$

17.52

A convergent-divergent nozzle has a throat diameter of 0.05 m and an exit diameter of 0.1 m. The inlet stagnation state is 500 kPa, 500 K. Find the back pressure that will lead to the maximum possible flow rate and the mass flow rate for three different gases as: air; hydrogen or carbon dioxide.

There is a maximum possible flow when M = 1 at the throat,

$$T^* = \frac{2}{k+1} T_o; \quad P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}; \quad \rho^* = \rho_o \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

$$\dot{m} = \rho^* A^* V = \rho^* A^* c = P^* A^* \sqrt{k/RT^*}$$

$$A^* = \pi D^2/4 = 0.001963 \text{ m}^2$$

	k	T^*	P^*	c	ρ^*	$\dot{m}$
a)	1.400	416.7	264.1	448.2	2.209	1.944
b)	1.409	415.1	263.4	1704.5	0.154	0.515
c)	1.289	436.9	273.9	348.9	3.318	2.273

$A_E/A^* = (D_E/D^*)^2 = 4$. There are 2 possible solutions corresponding to points c and d in Fig. 17.13 and Fig. 17.17. For these we have

	Subsonic solution		Supersonic solution	
	M_E	P_E/P_o	M_E	P_E/P_o
a)	0.1466	0.985	2.940	0.0298
b)	0.1464	0.985	2.956	0.0293
c)	0.1483	0.986	2.757	0.0367

$P_B = P_E \cong 0.985 \times 500 = 492.5$ kPa all cases point c

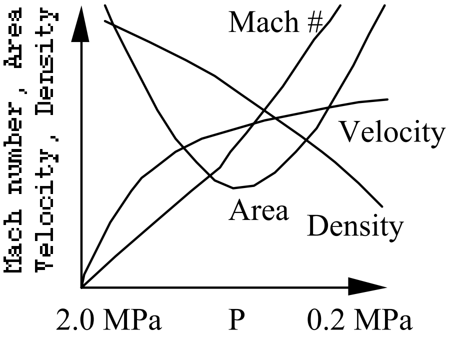
a) $P_B = P_E = 0.0298 \times 500 = 14.9$ kPa, point d

b) $P_B = P_E = 0.0293 \times 500 = 14.65$ kPa, point d

c) $P_B = P_E = 0.0367 \times 500 = 18.35$ kPa, point d

17.53

Air is expanded in a nozzle from a stagnation state of 2 MPa, 600 K, to a static pressure of 200 kPa. The mass flow rate through the nozzle is 5 kg/s. Assume the flow is reversible and adiabatic and determine the throat and exit areas for the nozzle.



$$P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$= 2 \times 0.5283 = 1.056$ MPa

$$T^* = T_o \times \frac{2}{k+1} = 600 \times 0.8333 = 500 \text{ K}$$

$v^* = RT^*/P^* = 0.287 \times 500/1056$

$= 0.1359 \text{ m}^3/\text{kg}$

$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 500} = 448.2$ m/s

$A^* = \dot{m}v^*/c^* = 5 \times 0.1359/448.2 = \mathbf{0.00152\ m^2}$

$P_2/P_o = 200/2000 = 0.1 \qquad \Rightarrow \qquad M_2^* = 1.701 = \mathbf{V}_2/c^*$

Column with mach no. based on throat speed of sound.

$\mathbf{V}_2 = 1.701 \times 448.2 = 762.4$ m/s

$T_2 = T_o\,(T_2/T_o) = 600 \times 0.5176 = 310.56$ K

$v_2 = RT_2/P_2 = 0.287 \times 310.56/200 = 0.4456 \text{ m}^3/\text{kg}$

$A_2 = \dot{m}v_2/\mathbf{V}_2 = 5 \times 0.4456 / 762.4 = \mathbf{0.00292\ m^2}$

17.54

Air flows into a convergent-divergent nozzle with an exit area of 2.0 times the throat area of 0.005 m^2. The inlet stagnation state is 1 MPa, 600 K. Find the backpressure that will cause a reversible supersonic exit flow with M = 1 at the throat. What is the mass flow rate?

This flow is case d in Fig.17.17 the only reversible supersonic flow.

$$A_E/A^* = 2 \quad \text{see Table A.12} \quad (M > 1)$$

$$\Rightarrow \quad M_E = 2.2 \text{ and } P_E/P_o = 0.09399$$

$$P_E = 0.09399 \times 1000 = \mathbf{94\ kPa}$$

To find the mass flow rate we need the throat conditions, see Table 17.1,

$$T^* = T \frac{2}{k+1} = 600 \times 0.8333 = 500 \text{ K}$$

$$P^* = P_o \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 1000\ (0.833333)^{3.5} = 528.3 \text{ kPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{528.3}{0.287 \times 500} = 3.682 \text{ kg/m}^3$$

$$\dot{m} = \rho A\mathbf{V} = \rho^* A^* c^* = \rho^* A^* \sqrt{kRT^*}$$

$$= 3.682 \times 0.005 \sqrt{1.4 \times 0.287 \times 500 \times 1000}$$

$$= \mathbf{8.252\ kg/s}$$

17.55

What is the exit pressure that will allow a reversible subsonic exit flow in the previous problem?

This flow is case c in Fig.17.17 (and c in Fig. 17.13) the only reversible subsonic flow with M = 1 at the throat.

$A_E/A^* = 2$ see Table A.12   ($M < 1$)

$P_E/P_o = 0.9360, \quad T_E/T_o = 0.98127$

$P_E = 0.9360\ P_o = 0.936 \times 1000 =$ **936 kPa**

$(\quad T_E = 0.98127\ T_o = 0.98127 \times 600 = 588.8\text{ K})$

17.56

A flow of helium flows at 500 kPa, 500 K with 100 m/s into a convergent-divergent nozzle. Find the throat pressure and temperature for reversible flow and $M = 1$ at the throat.

We need to find the stagnation properties first (k = 1.667)

$$T_0 = T_1 + \mathbf{V}_1^2/2C_p = 500 + 100^2/(2 \times 5193) = 500.963 \text{ K}$$

$$P_0 = P_1 \, (T_0/T_1)^{k/(k-1)} = 500 \, (500.963/500)^{2.5} = 502.41 \text{ kPa}$$

From the analysis we get Eqs.17.37-38

$$P^* = P_0 \left(\frac{2}{k+1}\right)^{k/(k-1)} = 502.41 \left(\frac{2}{1.667+1}\right)^{2.5} = \mathbf{244.7\ kPa}$$

$$T^* = T_0 \frac{2}{k+1} = 500.963 \times \frac{2}{1.667+1} = \mathbf{375.7\ K}$$

17.57

Assume the same tank and conditions as in Problem 17.51. After some flow out the nozzle flow changes to become subsonic. Find the mass in the tank and the mass flow rate out at that instant.

The initial mass is

$$m_1 = P_1V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

The flow changes to subsonic when the pressure ratio reaches critical.

$$P_B/P_{o3} = 0.5283 \;\Rightarrow\; P_{o3} = 189.3 \text{ kPa}$$

$$v_1/v_3 = (P_{o3}/P_1)^{1/k} = (189.3/1000)^{0.7143} = 0.3046$$

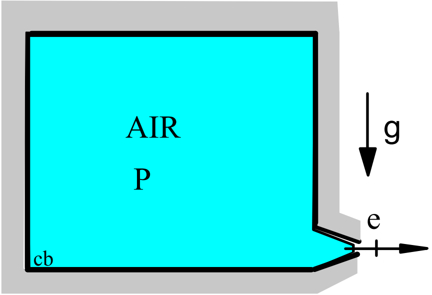
$$m_3 = m_1v_1/v_3 = 6.222 \times 0.3046 = \mathbf{1.895\ kg}$$

$$T_3 = T_1(v_1/v_3)^{k-1} = 560 \times 0.3046^{0.4} = 348 \text{ K}$$

$$P_E = P_B = 100 \text{ kPa} ; \quad M_E = 1$$

$$T_E = 0.8333 \times 348 = 290 \text{ K} \;\; ; \;\; \mathbf{V}_E = \sqrt{kRT_E} = 341.4 \text{ m/s}$$

$$\dot{m}_3 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(341.4)(100)}{0.287(290)} = \mathbf{0.0082\ kg/s}$$



17.58

A given convergent nozzle operates so it is choked with stagnation inlet flow properties of 400 kPa, 400 K. To increase the flow, a reversible adiabatic compressor is added before the nozzle to increase the stagnation flow pressure to 500 kPa. What happens to the flow rate?

Since the nozzle is choked the mass flow rate is given by Eq.17.42. The compressor changes the stagnation pressure and temperature.

Isentropic: $P_{o\ new} = P_o\, r_p$ and $T_{o\ new} = T_o \left(r_p\right)^{\frac{k-1}{k}}$

$$P_{o\ new} / \sqrt{T_{o\ new}} = \left(r_p\right)^{\frac{k+1}{2k}} \left[P_o / \sqrt{T_o}\right]$$

so the mass flow rate is multiplied with the factor

$$\left(r_p\right)^{\frac{k+1}{2k}} = \left(\frac{500}{400}\right)^{\frac{2.4}{2.8}} = 1.21$$

17.59

A 1-m^3 uninsulated tank contains air at 1 MPa, 560 K. The tank is now discharged through a small convergent nozzle to the atmosphere at 100 kPa while heat transfer from some source keeps the air temperature in the tank at 560 K. The nozzle has an exit area of 2×10^{-5} m^2.

a. Find the initial mass flow rate out of the tank.

b. Find the mass flow rate when half the mass has been discharged.

a. Same solution as in 16.52 a)

b. From solution 16.52 b) we have $m_2 = m_1/2 = 3.111$ kg

$P_2 = P_1/2 = 500$ kPa ; $T_2 = T_1$; $\quad P_B/P_2 = 100/500 = 0.2 < (P^*/P_o)_{crit}$

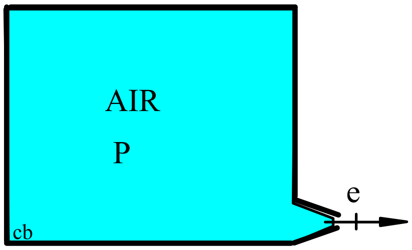
The flow is choked and the velocity is:

$$T_E = T^* = 0.8333 \times 560 = 466.7 \text{ K}$$

$$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433 \text{ m/s}$$

$P_E = 0.5283 \times 500 = 264.2$ kPa ; $M_E = 1$

$$\dot{m}_2 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(433)(264.2)}{0.287(466.7)} = \mathbf{0.01708\ kg/s}$$



17.60

Assume the same tank and conditions as in Problem 17.59. After some flow out the nozzle flow changes to become subsonic. Find the mass in the tank and the mass flow rate out at that instant.

The initial mass is

$$m_1 = P_1V/RT_1 = 1000 \times 1/(0.287 \times 560) = 6.222 \text{ kg}$$

Flow changes to subsonic when the pressure ratio reaches critical.

$P_B/P_o = 0.5283$; $P_3 = P_o = P_B/0.5283 = 100/0.5283 = 189.3$ kPa

$m_3 = m_1P_3/P_1 = \mathbf{1.178\ kg}$;

$T_3 = T_1$

$T_E = T^* = 0.8333 \times 560 = 466.7$ K

$\mathbf{V}_E = c = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 466.7} = 433$ m/s

$$\dot{m}_3 = A\mathbf{V}_E P_E/RT_E = \frac{2\times10^{-5}(433)(189.3)}{0.287(466.7)} = \mathbf{0.01224\ kg/s}$$

Normal Shocks

17.61

The products of combustion, use air, enter a convergent nozzle of a jet engine at a total pressure of 125 kPa, and a total temperature of 650°C. The atmospheric pressure is 45 kPa and the flow is adiabatic, with a rate of 25 kg/s. Determine the exit area of the nozzle.

The critical pressure Table 17.1: $P_{crit} = P_2 = 125 \times 0.5283 = 66 \text{ kPa} > P_{amb}$

The flow is then choked. $T_2 = 923.15 \times 0.8333 = 769.3 \text{ K}$

$$\mathbf{V}_2 = c_2 = \sqrt{1.4 \times 1000 \times 0.287 \times 769.3} = 556 \text{ m/s}$$

$$v_2 = 0.287 \times 769.3/66 = 3.3453 \text{ m}^3\text{/kg}$$

$$A_2 = \dot{m}v_2 / \mathbf{V}_2 = 25 \times 3.3453/556 = \mathbf{0.1504\ m^2}$$

17.62

Redo the previous problem for a mixture with k = 1.3 and molecular mass of 31.

The critical pressure Table 17.1: $P_{crit} = P_2 = 125 \times 0.5457 = 68.2$ kPa > P_{amb}

The flow is then choked. $T_2 = 923.15 \times 0.8696 = 802.8$ K

The gas constant is $R = 8.31451 / 31 = 0.2682$ kJ/kg-K

$$\mathbf{V}_2 = c_2 = \sqrt{1.3 \times 1000 \times 0.2682 \times 802.8} = 529 \text{ m/s}$$

$$v_2 = 0.2682 \times 802.8/68.2 = 3.157 \text{ m}^3\text{/kg}$$

$$A_2 = \dot{m}v_2 / \mathbf{V}_2 = 25 \times 3.157/529 = \mathbf{0.149\ m^2}$$

17.63

At what Mach number will the normal shock occur in the nozzle of Problem 17.52 flowing with air if the back pressure is halfway between the pressures at *c* and *d* in Fig. 17.17?

First find the two pressures that will give exit at c and d. See solution to 16.8 a)

$A_E/A^* = (D_E/D^*)^2 = 4 \quad \Rightarrow$

For case c: $P_E = 492.5$ kPa   For case d: $P_E = 14.9$ kPa

Actual case: $P_E = (492.5 + 14.9)/2 = 253.7$ kPa

Assume $M_x = 2.4 \Rightarrow \quad M_y = 0.5231 ; \quad P_{oy}/P_{ox} = 0.54015$

$A_x/A_x^* = 2.4031 ; \quad A_x/A_y^* = 1.298$

$A_E/A_y^* = (A_E/A_x^*)\ (A_x/A_y^*)\ /\ (A_x/A_x^*) = 4 \times 1.298/2.4031 = 2.1605$

$\Rightarrow \quad M_E = 0.2807 ; \quad P_E/P_{oy} = 0.94675$

$P_E = (P_E/P_{oy})\ (P_{oy}/P_{ox})\ P_{ox} = 0.94675 \times 0.54015 \times 500 = 255.7$ kPa

Repeat if $M_x = 2.5 \quad \Rightarrow \quad P_E = 233.8$ kPa

Interpolate to match the desired pressure => $M_x =$ **2.41**

17.64

Consider the nozzle of Problem 17.53 and determine what back pressure will cause a normal shock to stand in the exit plane of the nozzle. This is case g in Fig. 17.17. What is the mass flow rate under these conditions?

We assume reversible flow up to the shock

Table A.12: $P_E/P_o = 200/2000 = 0.1$; $M_E = 2.1591 = M_x$

Shock functions Table A.13: $M_y = 0.5529$; $P_y/P_x = 5.275$

$P_B = P_y = 5.275 \times P_x = 5.275 \times 200 =$ **1055 kPa**

$\dot{m} =$ **5 kg/s** same as in Problem 17.53 since M = 1 at throat.

17.65

A normal shock in air has upstream total pressure of 500 kPa, stagnation temperature of 500 K and $M_x = 1.2$. Find the downstream stagnation pressure.

From the normal shock relations in Section 17.8 found in Table A.13 we get

$M_x = 1.2$: $\quad P_{o\,y}/P_{o\,x} = 0.9928$

$P_{o\,y} = 0.9928\ P_{o\,x} = 0.9928 \times 500 =$ **496.4 kPa**

Remark: The stagnation temperature would be unchanged (energy equation).

17.66

How much entropy per kg flow is generated in the shock in Example 17.9?

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.32 \ - 0.287 \ln 2.4583$$

$$= 0.27874 - 0.25815 = \mathbf{0.0206\ kJ/kg\ K}$$

Notice that function could have been tabulated also.

17.67

Consider the diffuser of a supersonic aircraft flying at $M = 1.4$ at such an altitude that the temperature is −20°C, and the atmospheric pressure is 50 kPa. Consider two possible ways in which the diffuser might operate, and for each case calculate the throat area required for a flow of 50 kg/s.

a. The diffuser operates as reversible adiabatic with subsonic exit velocity.

b. A normal shock stands at the entrance to the diffuser. Except for the normal shock the flow is reversible and adiabatic, and the exit velocity is subsonic. This is shown in Fig. P17.67.

a. Assume a convergent-divergent diffuser with $M = 1$ at the throat.

Relate the inlet state to the sonic state

$$P_1/P_o = 0.31424 \;;\; P^*/P_{o1} = 0.5283$$

$$P^* = \frac{0.5283}{0.31424}\, 50 = 84 \text{ kPa} \;;\; T^* = \frac{0.8333}{0.71839};\ 253.2 = 293.7 \text{ K}$$

$$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 293.7} = 343.5 \text{ m/s}$$

$$v^* = RT^*/P^* = 0.287 \times 293.7/84 = 1.0035 \ \text{m}^3/\text{kg}$$

$$A^* = \dot{m}v^*/c^* = 50 \times 1.0035/343.5 = \mathbf{0.1461\ m^2}$$

b. Across the shock we have

$$M_y = 0.7397 \;;\; P_y = 50 \times 2.12 = 106 \text{ kPa} \;;$$

$$T_y = 253.2 \times 1.2547 = 317.7 \text{ K}$$

$$P^* = \frac{0.5283}{0.6952} \times 106 = 80.6 \text{ kPa}$$

$$T^* = \frac{0.8333}{0.9011} \times 317.7 = 293.7 \text{ K}, \quad c^* = 343.5 \text{ m/s}$$

$$v^* = 0.287 \times 293.7/80.6 = 1.0458 \text{ m}^3/\text{kg}$$

$$A^* = 50 \times 1.0458/343.5 = \mathbf{0.1522\ m^2}$$

17.68

A flow into a normal shock in air has a total pressure 400 kPa, stagnation temperature of 600 K and $M_x = 1.2$. Find the upstream temperature T_x, the specific entropy generation in the shock and the downstream velocity.

From Table A.12: $M_x = 1.2$ has $T_x/T_o = 0.7764$, $P_x/P_o = 0.41238$

$$T_x = 0.7764\ T_o = 0.7764 \times 600\ \text{K} = \mathbf{465.84\ K}$$

From Table A.13: $T_y/T_x = 1.128$, $P_y/P_x = 1.5133$, $M_y = 0.84217$

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.128 - 0.287 \ln 1.5133$$

$$= 0.12093 - 0.11890 = \mathbf{0.00203\ kJ/kg\ K}$$

From the shock relations we had

$$T_y = 1.128\ T_x = 1.128 \times 465.84\ \text{K} = 525.47\ \text{K}$$

$$\mathbf{V}_y = M_y\, c_y = M_y \sqrt{kRT_y} = 0.84217 \sqrt{1.4 \times 0.287 \times 525.47 \times 1000}$$

$$= 0.84217 \times 459.49 = \mathbf{387\ m/s}$$

17.69

Consider the nozzle in problem 17.42 flowing air. What should the backpressure be for a normal shock to stand at the exit plane? This is case g in Fig.17.17. What is the exit velocity after the shock?

Reversible flow up to the shock with M = 1 at the throat.

$P_{x\,o} = P_o$, $T_{x\,o} = T_o$, $A_E/A^* = 175 / 100 = 1.75$

Table A.12: $M_E = M_x = 2.042$, $P_x/P_{o\,x} = 0.12$, $T_x/ T_{o\,x} = 0.5454$

Now we can do the normal shock from Table A.13

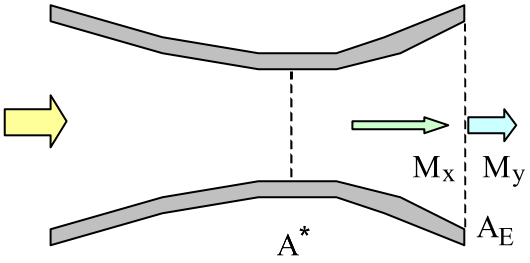
$M_x = 2.042 \;\Rightarrow\; M_y = 0.5704$, $P_y/P_x = 4.6984$, $T_y/T_x = 1.7219$

$T_y = 1.7219\, T_x = 1.7219 \times 0.5454\, T_{x\,o} = 1.7219 \times 0.5454 \times 375 = 352.2 \text{ K}$

$P_y = 4.6984\, P_x = 4.6984 \times 0.12\, P_{x\,o} = 4.6984 \times 0.12 \times 1000 = 563.8 \text{ kPa}$

$\mathbf{V}_y = M_y\, c_y = M_y \sqrt{kRT_y} = 0.5704 \sqrt{1.4 \times 0.287 \times 352.2 \times 1000}$

$= 0.5704 \times 376.2 =$ **214.6 m/s**



17.70

Find the specific entropy generation in the shock of the previous Problem.

Reversible flow up to the shock with M = 1 at the throat.

$P_{x\,o} = P_o$, $T_{x\,o} = T_o$, $A_E/A^* = 175 / 100 = 1.75$

Table A.12: $M_E = M_x = 2.042$, $P_x/P_{o\,x} = 0.12$, $T_x/ T_{o\,x} = 0.5454$

Now we can do the normal shock from Table A.13

$M_x = 2.042 \;\Rightarrow\; M_y = 0.5704$, $P_y/P_x = 4.6984$, $T_y/T_x = 1.7219$

The change in entropy is

$$s_{gen} = s_y - s_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$= 1.004 \ln 1.7219 - 0.287 \ln 4.6984$$

$$= 0.5456 - 0.44405 = \mathbf{0.1015\ kJ/kg\ K}$$

Nozzles, Diffusers, and Orifices

17.71

Steam at 600 kPa, 300°C is fed to a set of convergent nozzles in a steam turbine. The total nozzle exit area is 0.005 m^2 and they have a discharge coefficient of 0.94. The mass flow rate should be estimated from the measurement of the pressure drop across the nozzles, which is measured to be 200 kPa. Determine the mass flow rate.

Inlet B.1.3 $h_i = 3061.6$ kJ/kg, $s_i = 7.3724$ kJ/kg K

Exit: $(P_e, s_{e,s})$ $P_e = P_i - 200 = 400$ kPa, $s_{e,s} = s_i = 7.3724$ kJ/kg K

$\Rightarrow h_{e,s} = 2961$ kJ/kg and $v_{e,s} = 0.5932\ m^3/kg$,

$$\mathbf{V}_{e,s} = \sqrt{2 \times 1000(3061.6 - 2961)} = 448.55 \text{ m/s}$$

$$\dot{m}_s = A\mathbf{V}_{e,s}/v_{e,s} = 0.005 \times 448.55/0.5932 = 3.781 \text{ kg/s}$$

$$\dot{m}_a = C_D\dot{m}_s = 0.94 \times 3.781 = \mathbf{3.554\ kg/s}$$

17.72

Air enters a diffuser with a velocity of 200 m/s, a static pressure of 70 kPa, and a temperature of −6°C. The velocity leaving the diffuser is 60 m/s and the static pressure at the diffuser exit is 80 kPa. Determine the static temperature at the diffuser exit and the diffuser efficiency. Compare the stagnation pressures at the inlet and the exit.

Stagnation T at the inlet

$$T_{o1} = T_1 + \mathbf{V}_1^2/2C_p = 267.15 + 200^2/(2000 \times 1.004) = 287.1 \text{ K}$$

Energy Eq. gives the same stagnation T at exit

$$T_{o2} = T_{o1} \Rightarrow T_2 = T_{o2} - \mathbf{V}_2^2/2C_p = 287.1 - 60^2/(2000 \times 1.004) = 285.3 \text{ K}$$

$$\frac{T_{o1} - T_1}{T_1} = \frac{k-1}{k} \frac{P_{o1} - P_1}{P_1} \qquad \Rightarrow \quad P_{o1} - P_1 = 18.25 \;\Rightarrow\; P_{o1} = 88.3 \text{ kPa}$$

$$\frac{T_{o2} - T_2}{T_2} = \frac{k - 1}{k} \frac{P_{o2} - P_2}{P_2} \qquad \Rightarrow \quad P_{o2} - P_2 = 1.77 \;\Rightarrow\; P_{o2} = 81.8 \text{ kPa}$$

$$T_s^{ex} = T_1\,(P_{o2}/P_1)^{k-1/k} = 267.15 \times 1.0454 = \mathbf{279.3\ K}$$

$$\eta_D = \frac{T_s^{ex} - T_1}{T_{o1} - T_1} = \frac{279.3 - 267.15}{287.1 - 267.15} = \mathbf{0.608}$$

17.73

Repeat Problem 17.44 assuming a diffuser efficiency of 80%.

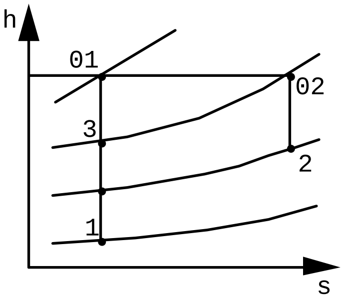
$\mathbf{V}$ = 1000 km/h = 277.8 m/s, $v_1 = RT/P = 0.287 \times 261.15/40 = 1.874\ \mathrm{m^3/kg}$

$h_1 = 261.48$ kJ/kg,

$h_{o1} = 261.48 + 277.8^2/2000 = 300.07$ kJ/kg

$\Rightarrow\ T_{o1} = 299.7$ K,

$P_{o1} = P_1\,(T_{o1}/T_1)^{k/(k-1)} = 40 \times (299.7/261.15)^{3.5} = 64.766$ kPa



Same as problem 17.44, except $\eta_D = 0.80$. We thus have from 17.44

$$\frac{h_3 - h_1}{h_{o1} - h_1} = \frac{h_3 - 261.48}{300.07 - 261.48} = 0.8$$

$\Rightarrow\ h_3 = 292.35$ kJ/kg, $T_3 = 291.9$ K

$P_{o2} = P_3 = P_1\,(T_3/T_1)^{k/(k-1)}$

$= 40\,(291.9/261.15)^{3.5} = 59.06$ kPa

$T_{o2} = T_{o1} = 299.7$ K

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\Rightarrow$ $T_2 =$ **294.7 K**,

$P_2 = P_{o2}\,(T_2/T_{o1})^{k/(k-1)} = 59.06 \times (294.7/299.7)^{3.5} =$ **55.68 kPa**

$v_2 = RT_2/P_2 = 0.287 \times 294.7/55.68 = 1.519\ \mathrm{m^3/kg}$

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.519)\,(100/277.8) =$ **0.444**

17.74

A sharp-edged orifice is used to measure the flow of air in a pipe. The pipe diameter is 100 mm and the diameter of the orifice is 25 mm. Upstream of the orifice, the absolute pressure is 150 kPa and the temperature is 35°C. The pressure drop across the orifice is 15 kPa, and the coefficient of discharge is 0.62. Determine the mass flow rate in the pipeline.

$$\Delta T = T_i\left(\frac{k-1}{k}\right)\frac{\Delta P}{P_i} = 308.15 \times \frac{0.4}{1.4} \times \frac{15}{150} = 8.8 \text{ K}$$

$v_i = RT_i/P_i = 0.5896 \text{ m}^3/\text{kg}$

$P_e = 135 \text{ kPa},\ \ T_e = 299.35 \text{ K},\ \ v_e = 0.6364 \text{ m}^3/\text{kg}$

$\dot{m}_i = \dot{m}_e \ \Rightarrow\ \ \mathbf{V}_i / \mathbf{V}_e = (D_e/D_i)^2\, v_i/v_e = 0.0579$

$h_i - h_e = \mathbf{V}_e^2(1 - 0.0579^2)/2 = C_p(T_i - T_e)$

$\mathbf{V}_{e\,s} = \sqrt{2 \times 1000 \times 1.004 \times 8.8/(1 - 0.0579)^2} = 133.1 \text{ m/s}$

$\dot{m} = C_D\, A\mathbf{V}/v = 0.62\ (\pi/4)\ (0.025)^2\ 133.1 / 0.6364 =$ **0.06365 kg/s**

17.75

A critical nozzle is used for the accurate measurement of the flow rate of air. Exhaust from a car engine is diluted with air so its temperature is 50°C at a total pressure of 100 kPa. It flows through the nozzle with throat area of 700 mm^2 by suction from a blower. Find the needed suction pressure that will lead to critical flow in the nozzle and the mass flow rate.

$P^* = 0.5283\ P_o = \mathbf{52.83\ kPa}$, $T^* = 0.8333\ T_o = 269.3\ \text{K}$

$v^* = RT^*/P^* = 0.287 \times 269.3/52.83 = 1.463\ \text{m}^3/\text{kg}$

$c^* = \sqrt{kRT^*} = \sqrt{1.4 \times 1000 \times 0.287 \times 269.3} = 328.9\ \text{m/s}$

$\dot{m} = Ac^*/v^* = 700 \times 10^{-6} \times 328.9/1.463 = \mathbf{0.157\ kg/s}$

17.76

Air is expanded in a nozzle from 700 kPa, 200°C, to 150 kPa in a nozzle having an efficiency of 90%. The mass flow rate is 4 kg/s. Determine the exit area of the nozzle, the exit velocity, and the increase of entropy per kilogram of air. Compare these results with those of a reversible adiabatic nozzle.

$$T_{2s} = T_1(P_2/P_1)^{(k-1)/k} = 473.2\ (150/700)^{0.286} = 304.6\ \text{K}$$

$$\mathbf{V}_{2s}^{\ 2} = 2 \times 1000 \times 1.004(473.2 - 304.6) = 338400\ \ \text{J/kg}$$

$$\mathbf{V}_2^{\ 2} = 0.9 \times 338400 \ \Rightarrow\ \mathbf{V}_2 = \mathbf{552\ m/s}$$

$$h_2 + \mathbf{V}_2^2/2 = h_1 \ \Rightarrow\ T_2 = T_1 - \mathbf{V}_2^2/2C_p$$

$$T_2 = 473.2 - 552^2/(2 \times 1000 \times 1.004) = 321.4\ \text{K}\ ;$$

$$v_2 = 0.287 \times 321.4/150 = 0.6149\ \ \text{m}^3/\text{kg}$$

$$A_2 = 4 \times 0.6149/552 = 0.00446\ \text{m}^2 = \mathbf{4460\ mm^2}$$

$$s_2 - s_1 = 1.0035\ \ln\left(\frac{321.4}{473.2}\right) - 0.287\ \ln\left(\frac{150}{700}\right) = \mathbf{0.0539\ kJ/kg\ K}$$

17.77

Steam at a pressure of 1 MPa and temperature of 400°C expands in a nozzle to a pressure of 200 kPa. The nozzle efficiency is 90% and the mass flow rate is 10 kg/s. Determine the nozzle exit area and the exit velocity.

First do the ideal reversible adiabatic nozzle

$s_{2s} = s_1 = 7.4651$ kJ/kg K, $h_1 = 3263.9$ kJ/kg

$\Rightarrow$ $T_{2s} = 190.4°C$; $h_{2s} = 2851$ kJ/kg

Now the actual nozzle can be calculated

$h_1 - h_{2ac} = \eta_D(h_1 - h_{2s}) = 0.9(3263.9 - 2851) = 371.6$ kJ/kg

$h_{2ac} = 2892.3$ kJ/kg, $T_2 = 210.9°C$, $v_2 = 1.1062\ m^3/kg$

$\mathbf{V}_2 = \sqrt{2000(3263.9 - 2892.3)} = \mathbf{862\ m/s}$

$A_2 = \dot{m}v_2/\mathbf{V}_2 = 10 \times 1.1062/862 = \mathbf{0.01283\ m^2}$

17.78

Steam at 800 kPa, 350°C flows through a convergent-divergent nozzle that has a throat area of 350 mm^2. The pressure at the exit plane is 150 kPa and the exit velocity is 800 m/s. The flow from the nozzle entrance to the throat is reversible and adiabatic. Determine the exit area of the nozzle, the overall nozzle efficiency, and the entropy generation in the process.

$h_{o1} = 3161.7$ kJ/kg, $s_{o1} = 7.4089$ kJ/kg K

$P^*/P_{o1} = (2/(k+1))^{k/(k-1)} = 0.54099 \Rightarrow P^* = 432.7$ kPa

At *: $(P^*, s^* = s_{o1}) \Rightarrow h^* = 2999.3$ kJ/kg, $v^* = 0.5687$ m^3/kg

$\Delta h = \mathbf{V}^2/2 \Rightarrow \mathbf{V}^* = \sqrt{2000(3161.7\text{-}2999.3)} = 569.9$ m/s

$\dot{m} = A\mathbf{V}^*/v^* = 350 \times 10^{-6} \times 569.9/0.5687 = 0.3507$ kg/s

$h_e = h_{o1} - \mathbf{V}_e^2/2 = 3161.7 - 800^2/2 \times 1000 = 2841.7$ kJ/kg

Exit: P_e, h_e: $v_e = 1.395$ m^3/kg, $s_e = 7.576$ kJ/kg K

$A_e = \dot{m}v_e/\mathbf{V}_e = 0.3507 \times 1.395/800 = \mathbf{6.115 \times 10^{-4}\ m^2}$

$s_{gen} = s_e - s_{o1} = 7.576 - 7.4089 = \mathbf{0.167\ kJ/kg\ K}$

17.79

A convergent nozzle with exit diameter of 2 cm has an air inlet flow of 20°C, 101 kPa (stagnation conditions). The nozzle has an isentropic efficiency of 95% and the pressure drop is measured to 50 cm water column. Find the mass flow rate assuming compressible adiabatic flow. Repeat calculation for incompressible flow.

Convert ΔP to kPa:

$\Delta P = 50 \text{ cm } H_2O = 0.5 \times 9.8064 = 4.903 \text{ kPa}$

$T_0 = 20°C = 293.15 \text{ K} \qquad P_0 = 101 \text{ kPa}$

Assume inlet $V_i = 0 \qquad P_e = P_0 - \Delta P = 101 - 4.903 = 96.097 \text{ kPa}$

$$T_e = T_0 \left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}} = 293.15 \times \left(\frac{96.097}{101}\right)^{0.2857} = 289.01$$

$$\mathbf{V}_e^2/2 = h_i - h_e = C_p (T_i - T_e) = 1.004 \times (293.15 - 289.01)$$

$$= 4.1545 \text{ kJ/kg} = 4254.5 \text{ J/kg} \qquad => \quad \mathbf{V}_e = 91.15 \text{ m/s}$$

$$\mathbf{V}_{e\,ac}^{\,2}/2 = \eta\, \mathbf{V}_{e\,s}^{\,2}/2 = 0.95 \times 4154.5 = 3946.78 \qquad \Rightarrow \quad \mathbf{V}_{e\,ac} = 88.85 \text{ m/s}$$

$$T_{e\,ac} = T_i - \frac{V_{e\,ac}^{\,2}/2}{C_p} = 293.15 - \frac{3.9468}{1.0035} = 289.2 \text{ K}$$

$$\rho_{e\,ac} = \frac{P_e}{RT_p} = \frac{96.097}{0.287 \times 289.2} = 1.158 \text{ kg/m}^3$$

$$\dot{m} = \rho A \mathbf{V} = 1.158 \times \frac{\pi}{4} \times 0.02^2 \times 88.85 = \mathbf{0.0323\ kg/s}$$

17.80

The coefficient of discharge of a sharp-edged orifice is determined at one set of conditions by use of an accurately calibrated gasometer. The orifice has a diameter of 20 mm and the pipe diameter is 50 mm. The absolute upstream pressure is 200 kPa and the pressure drop across the orifice is 82 mm of mercury. The temperature of the air entering the orifice is 25°C and the mass flow rate measured with the gasometer is 2.4 kg/min. What is the coefficient of discharge of the orifice at these conditions?

$$\Delta P = 82 \times 101.325/760 = 10.93 \text{ kPa}$$

$$\Delta T = T_i \left(\frac{k-1}{k}\right) \Delta P/P_i = 298.15 \times \frac{0.4}{1.4} \times 10.93/200 = 4.66$$

$$v_i = RT_i/P_i = 0.4278 \text{ m}^3/\text{kg}, \quad v_e = RT_e/P_e = 0.4455 \text{ m}^3/\text{kg}$$

$$\mathbf{V}_i = \mathbf{V}_e A_e v_i / A_i v_e = 0.1536\, \mathbf{V}_e$$

$$(\mathbf{V}_e^2 - \mathbf{V}_i^2)/2 = \mathbf{V}_e^2(1 - 0.1536^2)/2 = h_i - h_e = C_p \Delta T$$

$$\mathbf{V}_e = \sqrt{2 \times 1000 \times 1.004 \times 4.66/(1 - 0.1536^2)} = 97.9 \text{ m/s}$$

$$\dot{m} = A_e \mathbf{V}_e / v_e = \frac{\pi}{4} \times 0.02^2 \times 97.9/0.4455 = 0.069 \text{ kg/s}$$

$$C_D = 2.4/60 \times 0.069 = \mathbf{0.58}$$

17.81

A convergent nozzle is used to measure the flow of air to an engine. The atmosphere is at 100 kPa, 25°C. The nozzle used has a minimum area of 2000 mm^2 and the coefficient of discharge is 0.95. A pressure difference across the nozzle is measured to 2.5 kPa. Find the mass flow rate assuming incompressible flow. Also find the mass flow rate assuming compressible adiabatic flow.

Assume $\mathbf{V}_i \cong 0$, $v_i = RT_i/P_i = 0.287 \times 298.15/100 = 0.8557 \text{ m}^3\text{/kg}$

Incompressible flow:

$\mathbf{V}_{e,s}^2/2 = h_i - h_{e,s} = v_i(P_i - P_e) = 2.1393$ kJ/kg

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000 \times 2.1393} = 65.41$ m/s

$\dot{m}_s = A\mathbf{V}_{e,s}/v_i = 2000 \times 10^{-6} \times 65.41/0.8557 = 0.153$ kg/s

$\dot{m}_a = C_D\dot{m}_s =$ **0.1454 kg/s**

Compressible flow:

$T_{e,s} = T_i\,(P_e/P_i)^{(k-1)/k} = 298.15(97.5/100)^{0.2857} = 296$ K

$\Delta h = C_p\Delta T = 1.0035 \times 2.15 = 2.1575 = \mathbf{V}_{e,s}^2/2$

$\mathbf{V}_{e,s} = \sqrt{2 \times 1000 \times 2.1575} = 65.69$ m/s

$v_{e,s} = 0.287 \times 296/97.5 = 0.8713 \text{ m}^3\text{/kg}$

$\dot{m}_s = A\mathbf{V}_{e,s}/v_{e,s} = 2000 \times 10^{-6} \times 65.69/0.8713 = 0.1508$ kg/s

$\dot{m}_a = C_D\dot{m}_s =$ **0.1433 kg/s**

Review Problems

17.82

Atmospheric air is at 20°C, 100 kPa with zero velocity. An adiabatic reversible compressor takes atmospheric air in through a pipe with cross-sectional area of 0.1 m^2 at a rate of 1 kg/s. It is compressed up to a measured stagnation pressure of 500 kPa and leaves through a pipe with cross-sectional area of 0.01 m^2. What are the required compressor work and the air velocity, static pressure, and temperature in the exit pipeline?

C.V. compressor out to standing air and exit to stagnation point.

$$\dot{m}\, h_{o1} + \dot{W}_c = \dot{m}(h + \mathbf{V}^2/2)_{ex} = \dot{m}h_{o,ex}$$

$$\dot{m}s_{o1} = \dot{m}s_{o,ex} \;\Rightarrow\; P_{r,o,ex} = P_{r,o1}$$

$$(P_{st,ex}/P_{o1}) = 1.028(500/100) = 5.14$$

$$\Rightarrow\; T_{o,ex} = 463 \text{ K}, \quad h_{o,ex} = 465.38 \text{ kJ/kg}, \quad h_{o1} = 209.45 \text{ kJ/kg}$$

$$\dot{W}_c = \dot{m}(h_{o,ex} - h_{o1}) = 1(465.38 - 209.45) = \mathbf{255.9\ kW}$$

$$P_{ex} = P_{o,ex}(T_{ex}/T_{o,ex})^{k/(k-1)} \qquad T_{ex} = T_{o,ex} - \mathbf{V}_{ex}^2/2C_p$$

$$\dot{m} = 1 \text{ kg/s} = (\rho A\mathbf{V})_{ex} = P_{ex}AV_{ex}/RT_{ex}$$

Now select 1 unknown amongst P_{ex}, T_{ex}, V_{ex} and write the continuity eq. $\dot{m}$ and solve the nonlinear equation. Say, use T_{ex} then

$$\mathbf{V}_{ex} = \sqrt{2C_p(T_{o,ex} - T_{ex})}$$

$$\dot{m} = 1 \text{ kg/s} = P_{o,ex}(T_{ex}/T_{o,ex})^{k/k-1} A\sqrt{2C_p(T_{o,ex} - T_{ex})}/RT_{ex}$$

solve for $T_{ex}/T_{o,ex}$ (close to 1)

$$T_{ex} = \mathbf{462.6\ K} \;\Rightarrow\; \mathbf{V}_{ex} = \mathbf{28.3\ m/s}, \quad P_{ex} = \mathbf{498.6\ kPa}$$

17.83

The nozzle in Problem 17.46 will have a throat area of 0.001272 m^2 and an exit area 2.896 times as large. Suppose the back pressure is raised to 1.4 MPa and that the flow remains isentropic except for a normal shock wave. Verify that the shock mach number (M_x) is close to 2 and find the exit mach number, the temperature and the mass flow rate through the nozzle.

(a) From Table A.12: $M_E = 2.6$

$P_E = 2.0 \times 0.05012 = 0.1002$ MPa

$T^* = 423.15 \times 0.8333 = \mathbf{352.7\ K}$

$P^* = 2.0 \times 0.5283 = \mathbf{1.057\ MPa}$

$c^* = \sqrt{1.4 \times 1000 \times 0.287 \times 352.7} = 376.5$ m/s

$v^* = 0.287 \times 352.7/1057 = 0.0958\ m^3/kg$

$A^* = 5 \times 0.0958/376.5 = \mathbf{1.272 \times 10^{-3} m^2}$

$A_E = 1.272 \times 10^{-3} \times 2.896 = \mathbf{3.68 \times 10^{-3} m^2}$

$T_E = 423.15 \times 0.42517 = 179.9$ K

Assume $M_x = 2$ then

$M_y = 0.57735$, $P_{oy}/P_{ox} = 0.72088$, $A_E/A^*_x = 2.896$

$A_x/A^*_x = 1.6875$, $A_x/A^*_y = 1.2225$,

$A_E/A^*_y = 2.896 \times 1.2225/1.6875 = 2.098$

$\Rightarrow$ $M_E = \mathbf{0.293}$, $P_E/P_{oy} = 0.94171$

$P_E = 0.94171 \times 0.72088 \times 2.0 = 1.357$ MPa, OK close to the 1.4 MPa

$T_E = 0.98298 \times 423.15 = \mathbf{416\ K}$, $\dot{m} = \mathbf{5\ kg/s}$

17.84

At what Mach number will the normal shock occur in the nozzle of Problem 17.53 if the back pressure is 1.4 MPa? (trial and error on M_x)

Relate the inlet and exit conditions to the shock conditions with reversible flow before and after the shock. It becomes trial and error.

Assume $M_x = 1.8 \Rightarrow \quad M_y = 0.6165 ; \quad P_{oy}/P_{ox} = 0.8127$

$A_E/A^*_x = A_2/A^* = 0.002435/0.001516 = 1.6062$

$A_x/A^*_x = 1.439 ; \quad A_x/A^*_y = 1.1694$

$$A_E/A^*_y = (A_E/A^*_x)(A_x/A^*_y)/(A_x/A^*_x) = \frac{1.6062(1.1694)}{1.439} = 1.3053$$

$\Rightarrow \; M_E = 0.5189 ; \; P_E/P_{oy} = 0.8323$

$P_E = (P_E/P_{oy})(P_{oy}/P_{ox})P_{ox} = 0.8323 \times 0.8127 \times 2000 = 1353 \text{ kPa} < 1.4 \text{ MPa}$

So select the mach number a little less

$M_x = 1.7 \Rightarrow M_y = 0.64055 ; \quad P_{oy}/P_{ox} = 0.85573$

$A_x/A^*_x = 1.3376 ; A_x/A^*_y = 1.1446$

$$A_E/A^*_y = (A_E/A^*_x)(A_x/A^*_y)/(A_x/A^*_x) = \frac{1.6062(1.1446)}{1.3376} = 1.3744$$

$\Rightarrow \; M_E = 0.482 ; \; P_E/P_{oy} = 0.853$

$P_E = (P_E/P_{oy})(P_{oy}/P_{ox})P_{ox} = 0.853 \times 0.85573 \times 2000 = 1459.9 \text{ kPa}$

Now interpolate between the two

$M_x = \mathbf{1.756}$ and we check $\Rightarrow M_y = 0.6266 ; \quad P_{oy}/P_{ox} = 0.832$

$A_x/A^*_x = 1.3926 ; \; A_x/A^*_y = 1.1586$

$A_E/A^*_y = 1.6062 \times 1.1586/1.3926 = 1.3363$

$\Rightarrow \; M_E = 0.5 ; \; P_E/P_{oy} = 0.843$

$P_E = 0.843 \times 0.832 \times 2000 = 1402.7$ kPa **OK**

Solution using the Pr or vr functions

17.44

A jet plane travels through the air with a speed of 1000 km/h at an altitude of 6 km, where the pressure is 40 kPa and the temperature is −12°C. Consider the inlet diffuser of the engine where air leaves with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of inlet to exit area of the diffuser, assuming the flow to be reversible and adiabatic.

$$\mathbf{V} = 1000 \text{ km/h} = 277.8 \text{ m/s}, \quad v_1 = \frac{RT}{P} = \frac{0.287 \times 261.15}{40} = 1.874 \text{ m}^3\text{/kg}$$

$$h_1 = 261.48 \text{ kJ/kg}, \;\; P_{r1} = 0.6862$$

$$h_{o1} = 261.48 + 277.8^2/2000 = 300.07 \text{ kJ/kg}$$

$$\Rightarrow \; T_{o1} = 299.7 \text{ K}, \;\; P_{ro1} = 1.1107$$

The ratio of the pressures equals the ratio of the Pr functions when s = constant

$$P_{o1} = P\, P_{ro1}/\, P_{r1} = 40 \times 1.1107/0.6862 = 64.74 \text{ kPa}$$

$$h_2 = 300.07 - 100^2/2000 = 295.07 \qquad \Rightarrow \quad T_2 = \mathbf{294.7\ K}, \;\; P_{r2} = 1.0462$$

$$P_2 = 64.74 \times 1.0462/1.1107 = \mathbf{61\ kPa}$$

$$v_2 = RT_2/P_2 = 0.287 \times 294.7/61 = 1.386 \text{ m}^3\text{/kg}$$

$$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.386)(100/277.8) = \mathbf{0.487}$$

17.73

Repeat Problem 17.44 assuming a diffuser efficiency of 80%.

$$\mathbf{V} = 1000 \text{ km/h} = 277.8 \text{ m/s}, \quad v_1 = \frac{RT}{P} = \frac{0.287 \times 261.15}{40} = 1.874 \text{ m}^3\text{/kg}$$

$h_1 = 261.48$ kJ/kg, $P_{r1} = 0.6862$

$h_{o1} = 261.48 + 277.8^2/2000 = 300.07$ kJ/kg

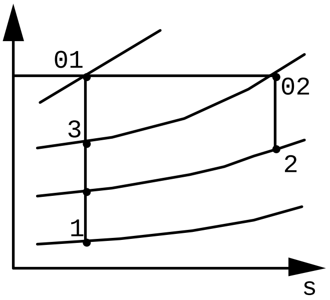
$\Rightarrow\ T_{o1} = 299.7$ K, $P_{ro1} = 1.1107$

Same as problem 17.44, except $\eta_D = 0.80$. We thus have from 17.44

$$\frac{h_3 - h_1}{h_{o1} - h_1} = \frac{h_3 - 261.48}{300.07 - 261.48} = 0.8$$

$\Rightarrow\ h_3 = 292.35$ kJ/kg, $P_{r3} = 1.0129$

$P_{o2} = P_3 = 40 \times 1.0129/0.6862 = 59.04$ kPa



$P_{ro2} = P_{ro1} = 1.1107$

$h_2 = 300.07 - 100^2/2000 = 295.07$ kJ/kg $\Rightarrow$ $T_2 =$ **294.7 K**, $P_{r2} = 1.0462$

$P_2 = P_{o2}\, P_{r2} / P_{ro2} = 59.04 \times 1.0462/1.1107 =$ **55.6 kPa**

$v_2 = RT_2/P_2 = 0.287 \times 294.7/55.6 = 1.521$ m^3/kg

$A_1/A_2 = (v_1/v_2)(V_2/V_1) = (1.874/1.521)\ (100/277.8) =$ **0.444**